Logical Representation

The Standard in Knowledge Representation

Predicate Logic
& First Order Predicate Calculus

- A language for expressing facts and rules,
  - and
- A mathematics for manipulating truth values and well-formed expressions

Uses of Logic in AI

- Representation
  - of problem states
  - of rules
- Reasoning
  - Sound Problem Solving
  - Theorem proving systems
  - Refutation
- Deductive Data Retrieval
- Expert Systems
- Inference

Three Brands of Inference

Abduction
Induction
Deduction

Abduction

- Approximate Reasoning
- Plausible Inference
- Reverse Causal Reasoning
  - if P -> Q, and Q, Abduce P
- Not logically sound, but economical

Induction

(Mathematician's Favorite)

- To Reason from specific cases to general rules
- Very Difficult to Mechanize
- Major problem for Machine Learning

Robins fly
sparrows fly → birds fly
hawks fly

Deduction
(as per Sherlock)

- Given a set of
  - Assumptions (Facts)
  - Logical Rules
  - Universal Laws of Logic
- We can find all new facts which logically follow from our assumptions

BACKGROUND

- Logical Expressions are built out of primitives:
  - constants
  - variables
  - predicates
  - connectives
  - quantifiers
- According to well-formedness rules

Propositional vs Predicate Logic

- Propositional Logic works with constants
- P is True
- Q is True
- P^Q is true

- Predicates are a little more complicated...

Constants

- Symbols which denote objects and individuals in the world:
  - JOHN
  - MARY
  - TABLE
  - BLOCK

Variables

- Indefinite References to objects and individuals in the world:
  - x
  - z
  - y

Predicates (or functions)

- Functions denoting aspects of objects or relationships between individuals:
  - LOVES(John, Mary)
  - RED(Ball1)
  - INSIDE(x,Box23)

Connectives

- Logical operators which compute (or constrain) truth values:
  - & AND
  - | OR
  - ~ NOT
  - => IMPLIES

Quantifiers

- Logical Meta-operators which assert across the entire population of objects:

  ∀ For All

  ∃ There Exists

Literals

- Well-formed expressions which state facts about individuals in the world:
    - Loves(Mary John) &Married(John)
    - Male(Terry) & ~Married(Terry)
- No Variables
- No Quantifiers

Formulae

- Well-formed expressions which assert rules (generalized) about relations between objects:
- Sometimes the quantifiers are implicit
    - ∃x Person(x)
    - ∀ y boy(y) => male (y)
    - ∀x ∃y Man(x) & Woman(y) & Loves(x,y)

How to Use Logic?

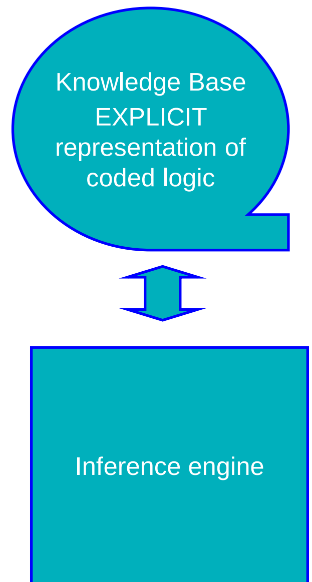
- Decide on Objects, Predicates
- Translate Facts & Rules about the domain to Expressions
- The Rest is Mechanical...
  - Can represent logical formula in list
  - Can manipulate lists mechanically in LISP to follow logical rules...

Old AI Extremist Position

- all human thought can be reduced to logic
- All logic can be mechanized
- THEREFORE: all human thought can be mechanized through logic

Practical use of logic

Logic is IMPLICIT in your code via If/then/else, case control logic predicates (evenp) etc.



Problems remain

- Choices of predicates
  - depends on human idiosyncracies
- Translation:
  - Ambiguity, Quantifier Scoping
- Mechanics:
  - Combinatorial Explosion

Mechanics of Logic:

Manipulating Expressions without Lying

- Double Negation
  - ~~Man(x) = Man(x)
- DeMorgan's Laws
  - ~(P & Q) = (~P | ~Q)
  - ~(P | Q) = (~P & ~Q)

Mechanics of Logic:

- Commutativity
  - P & Q = Q & P
  - P | Q = Q | P
- Distributivity
  - X & (Y | Z) = (X & Y) | (X & Z)
  - X | (Y & Z) = (X | Y) & (X | Z)
- Associativity
  - (A & B) & C = A & (B & C)

Mechanics of Logic:

Formulae can also be mucked with

- Demorgans Laws for Quantification
- $\sim\exists$ x P(x) = $\forall$ x ~ P(x)
- ~ $\forall$x Q(x) = $\exists$ x ~Q(x)

Laws of Inference

Domain-Independent Universally Sound Rules

- Modus Ponens (Forward)
  - IF P => Q and P is true,
  - then infer Q
- Modus Tollens
  - If P => Q and Q is False,
  - then Infer ~P
- Syllogism
  - If P => Q and Q => R
  - then Infer P => R

Logical Problem Solving

Initial State	Operators	Goal
Statements of fact, Object Database	Formulae for domain Rules of Inference	Proof of some truth, refutation

Combinatorial Explosion!

- There Are Too Many True Facts
- There Are Too Many Equiv. Representations
- There are Too Many Operators
- Its easy to make infinite loops creating more and more "knowledge spam".

$\forall x \exists y$ person(x)=> person(y) & parent(x,y)

Resolution

one law of inference to mechanize logic

The Resolution Principle

- One way of representing things
  - Conjunctive Normal Form
- One Inference Rule
  - Disjunctive Syllogism

Disjunctive Syllogism

Subsumes other Laws of inference

- Two Disjuncts yield something new
  (A | B | C) and (D | ~B | E) yields (A | C | D | E)

  if a term is in one disjunct and its negative in another combine them together without that term

  in this case B and ~B cancel out

Disjunctive Syllogism

Subsumes other Laws of inference

- Modus Ponens
  - (P) and (~P | Q) yields (Q)
- Modus Tollens
  - (~Q) and (~P | Q) Yields (~P)
- Chaining
  - (~ P | Q) and (~Q | R) yields (~P | R)