Zhejiang University

ICPC Team

Routine Library

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1、几何

1.1 注意

1. 注意舍入方式(0.5 的舍入方向);防止输出-0.

2. 几何题注意多测试不对称数据.

3. 整数几何注意 xmult 和 dmult 是否会出界;
   符点几何注意 eps 的使用.

4. 避免使用斜率;注意除数是否会为 0.

5. 公式一定要化简后再代入.

6. 判断同一个 2*PI 域内两角度差应该是
   abs(a1-a2)<beta||abs(a1-a2)>pi+pi-beta;
   相等应该是
   abs(a1-a2)<eps||abs(a1-a2)>pi+pi-eps;

7. 需要的话尽量使用 atan2,注意:atan2(0,0)=0,
   atan2(1,0)=pi/2,atan2(-1,0)=-pi/2,atan2(0,1)=0,atan2(0,-1)=pi.

8. cross product = |u|*|v|*sin(a)
   dot product = |u|*|v|*cos(a)

9. (P1-P0)x(P2-P0)结果的意义:
   正: <P0,P1>在<P0,P2>顺时针(0,pi)内
   负: <P0,P1>在<P0,P2>逆时针(0,pi)内
   0 : <P0,P1>,<P0,P2>共线,夹角为 0 或 pi

10. 误差限缺省使用 1e-8!

1.2 几何公式

三角形:
1. 半周长 P=(a+b+c)/2
2. 面积 S=aHa/2=absin(C)/2=sqrt(P(P-a)(P-b)(P-c))
3. 中线 Ma=sqrt(2(b^2+c^2)-a^2)/2=sqrt(b^2+c^2+2bccos(A))/2
4. 角平分线 Ta=sqrt(bc((b+c)^2-a^2))/(b+c)=2bccos(A/2)/(b+c)
5. 高线 Ha=bsin(C)=csin(B)=sqrt(b^2-((a^2+b^2-c^2)/(2a))^2)
6. 内切圆半径 r=S/P=asin(B/2)sin(C/2)/sin((B+C)/2)

=4Rsin(A/2)sin(B/2)sin(C/2)=sqrt((P-a)(P-b)(P-c)/P)

=Ptan(A/2)tan(B/2)tan(C/2)

7. 外接圆半径 R=abc/(4S)=a/(2sin(A))=b/(2sin(B))=c/(2sin(C))

四边形:

D1,D2 为对角线,M 对角线中点连线,A 为对角线夹角

1. a^2+b^2+c^2+d^2=D1^2+D2^2+4M^2

2. S=D1D2sin(A)/2

(以下对圆的内接四边形)

3. ac+bd=D1D2

4. S=sqrt((P-a)(P-b)(P-c)(P-d)),P 为半周长

正 n 边形:

R 为外接圆半径,r 为内切圆半径

1. 中心角 A=2PI/n

2. 内角 C=(n-2)PI/n

3. 边长 a=2sqrt(R^2-r^2)=2Rsin(A/2)=2rtan(A/2)

4. 面积 S=nar/2=nr^2tan(A/2)=nR^2sin(A)/2=na^2/(4tan(A/2))

圆:

1. 弧长 l=rA

2. 弦长 a=2sqrt(2hr-h^2)=2rsin(A/2)

3. 弓形高 h=r-sqrt(r^2-a^2/4)=r(1-cos(A/2))=atan(A/4)/2

4. 扇形面积 S1=rl/2=r^2A/2

5. 弓形面积 S2=(rl-a(r-h))/2=r^2(A-sin(A))/2

棱柱:

1. 体积 V=Ah,A 为底面积,h 为高

2. 侧面积 S=lp,l 为棱长,p 为直截面周长

3. 全面积 T=S+2A

棱锥:

1. 体积 V=Ah/3,A 为底面积,h 为高

(以下对正棱锥)

2. 侧面积 S=lp/2,l 为斜高,p 为底面周长

3. 全面积 T=S+A

棱台:

1. 体积 V=(A1+A2+sqrt(A1A2))h/3,A1.A2 为上下底面积,h 为高

(以下为正棱台)

2. 侧面积 S=(p1+p2)l/2,p1.p2 为上下底面周长,l 为斜高

3. 全面积 T=S+A1+A2

圆柱:

1. 侧面积 S=2PIrh
2. 全面积 T=2PIr(h+r)
3. 体积 V=PIr^2h

圆锥:
1. 母线 l=sqrt(h^2+r^2)
2. 侧面积 S=PIrl
3. 全面积 T=PIr(l+r)
4. 体积 V=PIr^2h/3

圆台:
1. 母线 l=sqrt(h^2+(r1-r2)^2)
2. 侧面积 S=PI(r1+r2)l
3. 全面积 T=PIr1(l+r1)+PIr2(l+r2)
4. 体积 V=PI(r1^2+r2^2+r1r2)h/3

球:
1. 全面积 T=4PIr^2
2. 体积 V=4PIr^3/3

球台:
1. 侧面积 S=2PIrh
2. 全面积 T=PI(2rh+r1^2+r2^2)
3. 体积 V=PIh(3(r1^2+r2^2)+h^2)/6

球扇形:
1. 全面积 T=PIr(2h+r0),h 为球冠高,r0 为球冠底面半径
2. 体积 V=2PIr^2h/3

1.3 多边形

```
#include <stdlib.h>
#include <math.h>
#define MAXN 1000
#define offset 10000
#define eps 1e-8
#define zero(x) (((x)>0?(x):-(x))<eps)
#define _sign(x) ((x)>eps?1:((x)<-eps?2:0))
struct point{double x,y;};
struct line{point a,b;};

double xmult(point p1,point p2,point p0){
	return (p1.x-p0.x)*(p2.y-p0.y)-(p2.x-p0.x)*(p1.y-p0.y);
}
```

```
//判定凸多边形,顶点按顺时针或逆时针给出,允许相邻边共线
int is_convex(int n,point* p){
	int i,s[3]={1,1,1};
	for (i=0;i<n&&s[1]|s[2];i++)
		s[_sign(xmult(p[(i+1)%n],p[(i+2)%n],p[i]))]=0;
	return s[1]|s[2];
}

//判定凸多边形,顶点按顺时针或逆时针给出,不允许相邻边共线
int is_convex_v2(int n,point* p){
	int i,s[3]={1,1,1};
	for (i=0;i<n&&s[0]&&s[1]|s[2];i++)
		s[_sign(xmult(p[(i+1)%n],p[(i+2)%n],p[i]))]=0;
	return s[0]&&s[1]|s[2];
}

//判点在凸多边形内或多边形边上,顶点按顺时针或逆时针给出
int inside_convex(point q,int n,point* p){
	int i,s[3]={1,1,1};
	for (i=0;i<n&&s[1]|s[2];i++)
		s[_sign(xmult(p[(i+1)%n],q,p[i]))]=0;
	return s[1]|s[2];
}

//判点在凸多边形内,顶点按顺时针或逆时针给出,在多边形边上返回 0
int inside_convex_v2(point q,int n,point* p){
	int i,s[3]={1,1,1};
	for (i=0;i<n&&s[0]&&s[1]|s[2];i++)
		s[_sign(xmult(p[(i+1)%n],q,p[i]))]=0;
	return s[0]&&s[1]|s[2];
}

//判点在任意多边形内,顶点按顺时针或逆时针给出
//on_edge 表示点在多边形边上时的返回值,offset 为多边形坐标上限
int inside_polygon(point q,int n,point* p,int on_edge=1){
	point q2;
	int i=0,count;
	while (i<n)
		for (count=i=0,q2.x=rand()+offset,q2.y=rand()+offset;i<n;i++)
			if                    (zero(xmult(q,p[i],p[(i+1)%n]))&&(p[i].x-q.x)*(p[(i+1)%n].x-
q.x)<eps&&(p[i].y-q.y)*(p[(i+1)%n].y-q.y)<eps)
				return on_edge;
			else if (zero(xmult(q,q2,p[i])))
```

```
				break;
			else if (xmult(q,p[i],q2)*xmult(q,p[(i+1)%n],q2)<-eps&&xmult(p[i],q,p[(i+1)%n])*xmult(p[i],q2,p[(i+1)%n])<-eps)
				count++;
	return count&1;
}

inline int opposite_side(point p1,point p2,point l1,point l2){
	return xmult(l1,p1,l2)*xmult(l1,p2,l2)<-eps;
}

inline int dot_online_in(point p,point l1,point l2){
	return zero(xmult(p,l1,l2))&&(l1.x-p.x)*(l2.x-p.x)<eps&&(l1.y-p.y)*(l2.y-p.y)<eps;
}

//判线段在任意多边形内,顶点按顺时针或逆时针给出,与边界相交返回 1
int inside_polygon(point l1,point l2,int n,point* p){
	point t[MAXN],tt;
	int i,j,k=0;
	if (!inside_polygon(l1,n,p)||!inside_polygon(l2,n,p))
		return 0;
	for (i=0;i<n;i++)
		if (opposite_side(l1,l2,p[i],p[(i+1)%n])&&opposite_side(p[i],p[(i+1)%n],l1,l2))
			return 0;
		else if (dot_online_in(l1,p[i],p[(i+1)%n]))
			t[k++]=l1;
		else if (dot_online_in(l2,p[i],p[(i+1)%n]))
			t[k++]=l2;
		else if (dot_online_in(p[i],l1,l2))
			t[k++]=p[i];
	for (i=0;i<k;i++)
		for (j=i+1;j<k;j++){
			tt.x=(t[i].x+t[j].x)/2;
			tt.y=(t[i].y+t[j].y)/2;
			if (!inside_polygon(tt,n,p))
				return 0;
		}
	return 1;
}

point intersection(line u,line v){
	point ret=u.a;
	double t=((u.a.x-v.a.x)*(v.a.y-v.b.y)-(u.a.y-v.a.y)*(v.a.x-v.b.x))
			/((u.a.x-u.b.x)*(v.a.y-v.b.y)-(u.a.y-u.b.y)*(v.a.x-v.b.x));
```

```
	ret.x+=(u.b.x-u.a.x)*t;
	ret.y+=(u.b.y-u.a.y)*t;
	return ret;
}

point barycenter(point a,point b,point c){
	line u,v;
	u.a.x=(a.x+b.x)/2;
	u.a.y=(a.y+b.y)/2;
	u.b=c;
	v.a.x=(a.x+c.x)/2;
	v.a.y=(a.y+c.y)/2;
	v.b=b;
	return intersection(u,v);
}

//多边形重心
point barycenter(int n,point* p){
	point ret,t;
	double t1=0,t2;
	int i;
	ret.x=ret.y=0;
	for (i=1;i<n-1;i++)
		if (fabs(t2=xmult(p[0],p[i],p[i+1]))>eps){
			t=barycenter(p[0],p[i],p[i+1]);
			ret.x+=t.x*t2;
			ret.y+=t.y*t2;
			t1+=t2;
		}
	if (fabs(t1)>eps)
		ret.x/=t1,ret.y/=t1;
	return ret;
}
```

1.4 多边形切割

```
//多边形切割
//可用于半平面交
#define MAXN 100
#define eps 1e-8
#define zero(x) (((x)>0?(x):-(x))<eps)
struct point{double x,y;};

double xmult(point p1,point p2,point p0){
```

```
	return (p1.x-p0.x)*(p2.y-p0.y)-(p2.x-p0.x)*(p1.y-p0.y);
}

int same_side(point p1,point p2,point l1,point l2){
	return xmult(l1,p1,l2)*xmult(l1,p2,l2)>eps;
}

point intersection(point u1,point u2,point v1,point v2){
	point ret=u1;
	double t=((u1.x-v1.x)*(v1.y-v2.y)-(u1.y-v1.y)*(v1.x-v2.x))
			/((u1.x-u2.x)*(v1.y-v2.y)-(u1.y-u2.y)*(v1.x-v2.x));
	ret.x+=(u2.x-u1.x)*t;
	ret.y+=(u2.y-u1.y)*t;
	return ret;
}

//将多边形沿 l1,l2 确定的直线切割在 side 侧切割,保证 l1,l2,side 不共线
void polygon_cut(int& n,point* p,point l1,point l2,point side){
	point pp[100];
	int m=0,i;
	for (i=0;i<n;i++){
		if (same_side(p[i],side,l1,l2))
			pp[m++]=p[i];
		if
(!same_side(p[i],p[(i+1)%n],l1,l2)&&!(zero(xmult(p[i],l1,l2))&&zero(xmult(p[(i+1)%n],l1,l2))))
			pp[m++]=intersection(p[i],p[(i+1)%n],l1,l2);
	}
	for (n=i=0;i<m;i++)
		if (!i||!zero(pp[i].x-pp[i-1].x)||!zero(pp[i].y-pp[i-1].y))
			p[n++]=pp[i];
	if (zero(p[n-1].x-p[0].x)&&zero(p[n-1].y-p[0].y))
		n--;
	if (n<3)
		n=0;
}
```

1.5 浮点函数

```
//浮点几何函数库
#include <math.h>
#define eps 1e-8
#define zero(x) (((x)>0?(x):-(x))<eps)
struct point{double x,y;};
struct line{point a,b;};
```

```
//计算 cross product (P1-P0)x(P2-P0)
double xmult(point p1,point p2,point p0){
	return (p1.x-p0.x)*(p2.y-p0.y)-(p2.x-p0.x)*(p1.y-p0.y);
}
double xmult(double x1,double y1,double x2,double y2,double x0,double y0){
	return (x1-x0)*(y2-y0)-(x2-x0)*(y1-y0);
}

//计算 dot product (P1-P0).(P2-P0)
double dmult(point p1,point p2,point p0){
	return (p1.x-p0.x)*(p2.x-p0.x)+(p1.y-p0.y)*(p2.y-p0.y);
}
double dmult(double x1,double y1,double x2,double y2,double x0,double y0){
	return (x1-x0)*(x2-x0)+(y1-y0)*(y2-y0);
}

//两点距离
double distance(point p1,point p2){
	return sqrt((p1.x-p2.x)*(p1.x-p2.x)+(p1.y-p2.y)*(p1.y-p2.y));
}
double distance(double x1,double y1,double x2,double y2){
	return sqrt((x1-x2)*(x1-x2)+(y1-y2)*(y1-y2));
}

//判三点共线
int dots_inline(point p1,point p2,point p3){
	return zero(xmult(p1,p2,p3));
}
int dots_inline(double x1,double y1,double x2,double y2,double x3,double y3){
	return zero(xmult(x1,y1,x2,y2,x3,y3));
}

//判点是否在线段上,包括端点
int dot_online_in(point p,line l){
	return zero(xmult(p,l.a,l.b))&&(l.a.x-p.x)*(l.b.x-p.x)<eps&&(l.a.y-p.y)*(l.b.y-p.y)<eps;
}
int dot_online_in(point p,point l1,point l2){
	return zero(xmult(p,l1,l2))&&(l1.x-p.x)*(l2.x-p.x)<eps&&(l1.y-p.y)*(l2.y-p.y)<eps;
}
int dot_online_in(double x,double y,double x1,double y1,double x2,double y2){
	return zero(xmult(x,y,x1,y1,x2,y2))&&(x1-x)*(x2-x)<eps&&(y1-y)*(y2-y)<eps;
}
```

```
//判点是否在线段上,不包括端点
int dot_online_ex(point p,line l){
	return dot_online_in(p,l)&&(!zero(p.x-l.a.x)||!zero(p.y-l.a.y))&&(!zero(p.x-l.b.x)||!zero(p.y-l.b.y));
}
int dot_online_ex(point p,point l1,point l2){
	return dot_online_in(p,l1,l2)&&(!zero(p.x-l1.x)||!zero(p.y-l1.y))&&(!zero(p.x-l2.x)||!zero(p.y-l2.y));
}
int dot_online_ex(double x,double y,double x1,double y1,double x2,double y2){
	return dot_online_in(x,y,x1,y1,x2,y2)&&(!zero(x-x1)||!zero(y-y1))&&(!zero(x-x2)||!zero(y-y2));
}

//判两点在线段同侧,点在线段上返回 0
int same_side(point p1,point p2,line l){
	return xmult(l.a,p1,l.b)*xmult(l.a,p2,l.b)>eps;
}
int same_side(point p1,point p2,point l1,point l2){
	return xmult(l1,p1,l2)*xmult(l1,p2,l2)>eps;
}

//判两点在线段异侧,点在线段上返回 0
int opposite_side(point p1,point p2,line l){
	return xmult(l.a,p1,l.b)*xmult(l.a,p2,l.b)<-eps;
}
int opposite_side(point p1,point p2,point l1,point l2){
	return xmult(l1,p1,l2)*xmult(l1,p2,l2)<-eps;
}

//判两直线平行
int parallel(line u,line v){
	return zero((u.a.x-u.b.x)*(v.a.y-v.b.y)-(v.a.x-v.b.x)*(u.a.y-u.b.y));
}
int parallel(point u1,point u2,point v1,point v2){
	return zero((u1.x-u2.x)*(v1.y-v2.y)-(v1.x-v2.x)*(u1.y-u2.y));
}

//判两直线垂直
int perpendicular(line u,line v){
	return zero((u.a.x-u.b.x)*(v.a.x-v.b.x)+(u.a.y-u.b.y)*(v.a.y-v.b.y));
}
int perpendicular(point u1,point u2,point v1,point v2){
	return zero((u1.x-u2.x)*(v1.x-v2.x)+(u1.y-u2.y)*(v1.y-v2.y));
```

```
}

//判两线段相交,包括端点和部分重合
int intersect_in(line u,line v){
	if (!dots_inline(u.a,u.b,v.a)||!dots_inline(u.a,u.b,v.b))
		return !same_side(u.a,u.b,v)&&!same_side(v.a,v.b,u);
	return dot_online_in(u.a,v)||dot_online_in(u.b,v)||dot_online_in(v.a,u)||dot_online_in(v.b,u);
}
int intersect_in(point u1,point u2,point v1,point v2){
	if (!dots_inline(u1,u2,v1)||!dots_inline(u1,u2,v2))
		return !same_side(u1,u2,v1,v2)&&!same_side(v1,v2,u1,u2);
	return
dot_online_in(u1,v1,v2)||dot_online_in(u2,v1,v2)||dot_online_in(v1,u1,u2)||dot_online_in(v2,u1,u
2);
}

//判两线段相交,不包括端点和部分重合
int intersect_ex(line u,line v){
	return opposite_side(u.a,u.b,v)&&opposite_side(v.a,v.b,u);
}
int intersect_ex(point u1,point u2,point v1,point v2){
	return opposite_side(u1,u2,v1,v2)&&opposite_side(v1,v2,u1,u2);
}

//计算两直线交点,注意事先判断直线是否平行!
//线段交点请另外判线段相交(同时还是要判断是否平行!)
point intersection(line u,line v){
	point ret=u.a;
	double t=((u.a.x-v.a.x)*(v.a.y-v.b.y)-(u.a.y-v.a.y)*(v.a.x-v.b.x))
			/((u.a.x-u.b.x)*(v.a.y-v.b.y)-(u.a.y-u.b.y)*(v.a.x-v.b.x));
	ret.x+=(u.b.x-u.a.x)*t;
	ret.y+=(u.b.y-u.a.y)*t;
	return ret;
}
point intersection(point u1,point u2,point v1,point v2){
	point ret=u1;
	double t=((u1.x-v1.x)*(v1.y-v2.y)-(u1.y-v1.y)*(v1.x-v2.x))
			/((u1.x-u2.x)*(v1.y-v2.y)-(u1.y-u2.y)*(v1.x-v2.x));
	ret.x+=(u2.x-u1.x)*t;
	ret.y+=(u2.y-u1.y)*t;
	return ret;
}

//点到直线上的最近点
```

```
point ptoline(point p,line l){
	point t=p;
	t.x+=l.a.y-l.b.y,t.y+=l.b.x-l.a.x;
	return intersection(p,t,l.a,l.b);
}
point ptoline(point p,point l1,point l2){
	point t=p;
	t.x+=l1.y-l2.y,t.y+=l2.x-l1.x;
	return intersection(p,t,l1,l2);
}

//点到直线距离
double disptoline(point p,line l){
	return fabs(xmult(p,l.a,l.b))/distance(l.a,l.b);
}
double disptoline(point p,point l1,point l2){
	return fabs(xmult(p,l1,l2))/distance(l1,l2);
}
double disptoline(double x,double y,double x1,double y1,double x2,double y2){
	return fabs(xmult(x,y,x1,y1,x2,y2))/distance(x1,y1,x2,y2);
}

//点到线段上的最近点
point ptoseg(point p,line l){
	point t=p;
	t.x+=l.a.y-l.b.y,t.y+=l.b.x-l.a.x;
	if (xmult(l.a,t,p)*xmult(l.b,t,p)>eps)
		return distance(p,l.a)<distance(p,l.b)?l.a:l.b;
	return intersection(p,t,l.a,l.b);
}
point ptoseg(point p,point l1,point l2){
	point t=p;
	t.x+=l1.y-l2.y,t.y+=l2.x-l1.x;
	if (xmult(l1,t,p)*xmult(l2,t,p)>eps)
		return distance(p,l1)<distance(p,l2)?l1:l2;
	return intersection(p,t,l1,l2);
}

//点到线段距离
double disptoseg(point p,line l){
	point t=p;
	t.x+=l.a.y-l.b.y,t.y+=l.b.x-l.a.x;
	if (xmult(l.a,t,p)*xmult(l.b,t,p)>eps)
		return distance(p,l.a)<distance(p,l.b)?distance(p,l.a):distance(p,l.b);
```

```
	return fabs(xmult(p,l.a,l.b))/distance(l.a,l.b);
}
double disptoseg(point p,point l1,point l2){
	point t=p;
	t.x+=l1.y-l2.y,t.y+=l2.x-l1.x;
	if (xmult(l1,t,p)*xmult(l2,t,p)>eps)
		return distance(p,l1)<distance(p,l2)?distance(p,l1):distance(p,l2);
	return fabs(xmult(p,l1,l2))/distance(l1,l2);
}

//矢量 V 以 P 为顶点逆时针旋转 angle 并放大 scale 倍
point rotate(point v,point p,double angle,double scale){
	point ret=p;
	v.x-=p.x,v.y-=p.y;
	p.x=scale*cos(angle);
	p.y=scale*sin(angle);
	ret.x+=v.x*p.x-v.y*p.y;
	ret.y+=v.x*p.y+v.y*p.x;
	return ret;
}
```

1.6 面积

```
#include <math.h>
struct point{double x,y;};

//计算 cross product (P1-P0)x(P2-P0)
double xmult(point p1,point p2,point p0){
	return (p1.x-p0.x)*(p2.y-p0.y)-(p2.x-p0.x)*(p1.y-p0.y);
}
double xmult(double x1,double y1,double x2,double y2,double x0,double y0){
	return (x1-x0)*(y2-y0)-(x2-x0)*(y1-y0);
}

//计算三角形面积,输入三顶点
double area_triangle(point p1,point p2,point p3){
	return fabs(xmult(p1,p2,p3))/2;
}
double area_triangle(double x1,double y1,double x2,double y2,double x3,double y3){
	return fabs(xmult(x1,y1,x2,y2,x3,y3))/2;
}

//计算三角形面积,输入三边长
double area_triangle(double a,double b,double c){
```

```
	double s=(a+b+c)/2;
	return sqrt(s*(s-a)*(s-b)*(s-c));
}

//计算多边形面积,顶点按顺时针或逆时针给出
double area_polygon(int n,point* p){
	double s1=0,s2=0;
	int i;
	for (i=0;i<n;i++)
		s1+=p[(i+1)%n].y*p[i].x,s2+=p[(i+1)%n].y*p[(i+2)%n].x;
	return fabs(s1-s2)/2;
}
```

1.7 球面

```
#include <math.h>
const double pi=acos(-1);

//计算圆心角 lat 表示纬度,-90<=w<=90,lng 表示经度
//返回两点所在大圆劣弧对应圆心角,0<=angle<=pi
double angle(double lng1,double lat1,double lng2,double lat2){
	double dlng=fabs(lng1-lng2)*pi/180;
	while (dlng>=pi+pi)
		dlng-=pi+pi;
	if (dlng>pi)
		dlng=pi+pi-dlng;
	lat1*=pi/180,lat2*=pi/180;
	return acos(cos(lat1)*cos(lat2)*cos(dlng)+sin(lat1)*sin(lat2));
}

//计算距离,r 为球半径
double line_dist(double r,double lng1,double lat1,double lng2,double lat2){
	double dlng=fabs(lng1-lng2)*pi/180;
	while (dlng>=pi+pi)
		dlng-=pi+pi;
	if (dlng>pi)
		dlng=pi+pi-dlng;
	lat1*=pi/180,lat2*=pi/180;
	return r*sqrt(2-2*(cos(lat1)*cos(lat2)*cos(dlng)+sin(lat1)*sin(lat2)));
}

//计算球面距离,r 为球半径
inline double sphere_dist(double r,double lng1,double lat1,double lng2,double lat2){
	return r*angle(lng1,lat1,lng2,lat2);
```

```
}
```

1.8 三角形

```
#include <math.h>
struct point{double x,y;};
struct line{point a,b;};

double distance(point p1,point p2){
	return sqrt((p1.x-p2.x)*(p1.x-p2.x)+(p1.y-p2.y)*(p1.y-p2.y));
}

point intersection(line u,line v){
	point ret=u.a;
	double t=((u.a.x-v.a.x)*(v.a.y-v.b.y)-(u.a.y-v.a.y)*(v.a.x-v.b.x))
			/((u.a.x-u.b.x)*(v.a.y-v.b.y)-(u.a.y-u.b.y)*(v.a.x-v.b.x));
	ret.x+=(u.b.x-u.a.x)*t;
	ret.y+=(u.b.y-u.a.y)*t;
	return ret;
}

//外心
point circumcenter(point a,point b,point c){
	line u,v;
	u.a.x=(a.x+b.x)/2;
	u.a.y=(a.y+b.y)/2;
	u.b.x=u.a.x-a.y+b.y;
	u.b.y=u.a.y+a.x-b.x;
	v.a.x=(a.x+c.x)/2;
	v.a.y=(a.y+c.y)/2;
	v.b.x=v.a.x-a.y+c.y;
	v.b.y=v.a.y+a.x-c.x;
	return intersection(u,v);
}

//内心
point incenter(point a,point b,point c){
	line u,v;
	double m,n;
	u.a=a;
	m=atan2(b.y-a.y,b.x-a.x);
	n=atan2(c.y-a.y,c.x-a.x);
	u.b.x=u.a.x+cos((m+n)/2);
	u.b.y=u.a.y+sin((m+n)/2);
```

```
	v.a=b;
	m=atan2(a.y-b.y,a.x-b.x);
	n=atan2(c.y-b.y,c.x-b.x);
	v.b.x=v.a.x+cos((m+n)/2);
	v.b.y=v.a.y+sin((m+n)/2);
	return intersection(u,v);
}

//垂心
point perpencenter(point a,point b,point c){
	line u,v;
	u.a=c;
	u.b.x=u.a.x-a.y+b.y;
	u.b.y=u.a.y+a.x-b.x;
	v.a=b;
	v.b.x=v.a.x-a.y+c.y;
	v.b.y=v.a.y+a.x-c.x;
	return intersection(u,v);
}

//重心
//到三角形三顶点距离的平方和最小的点
//三角形内到三边距离之积最大的点
point barycenter(point a,point b,point c){
	line u,v;
	u.a.x=(a.x+b.x)/2;
	u.a.y=(a.y+b.y)/2;
	u.b=c;
	v.a.x=(a.x+c.x)/2;
	v.a.y=(a.y+c.y)/2;
	v.b=b;
	return intersection(u,v);
}

//费马点
//到三角形三顶点距离之和最小的点
point fermentpoint(point a,point b,point c){
	point u,v;
	double step=fabs(a.x)+fabs(a.y)+fabs(b.x)+fabs(b.y)+fabs(c.x)+fabs(c.y);
	int i,j,k;
	u.x=(a.x+b.x+c.x)/3;
	u.y=(a.y+b.y+c.y)/3;
	while (step>1e-10)
		for (k=0;k<10;step/=2,k++)
```

```
			for (i=-1;i<=1;i++)
				for (j=-1;j<=1;j++){
					v.x=u.x+step*i;
					v.y=u.y+step*j;
					if
(distance(u,a)+distance(u,b)+distance(u,c)>distance(v,a)+distance(v,b)+distance(v,c))
						u=v;
				}
	return u;
}
```

1.9 三维几何

```
//三维几何函数库
#include <math.h>
#define eps 1e-8
#define zero(x) (((x)>0?(x):-(x))<eps)
struct point3{double x,y,z;};
struct line3{point3 a,b;};
struct plane3{point3 a,b,c;};

//计算 cross product U x V
point3 xmult(point3 u,point3 v){
	point3 ret;
	ret.x=u.y*v.z-v.y*u.z;
	ret.y=u.z*v.x-u.x*v.z;
	ret.z=u.x*v.y-u.y*v.x;
	return ret;
}

//计算 dot product U . V
double dmult(point3 u,point3 v){
	return u.x*v.x+u.y*v.y+u.z*v.z;
}

//矢量差 U - V
point3 subt(point3 u,point3 v){
	point3 ret;
	ret.x=u.x-v.x;
	ret.y=u.y-v.y;
	ret.z=u.z-v.z;
	return ret;
}
```

```
//取平面法向量
point3 pvec(plane3 s){
	return xmult(subt(s.a,s.b),subt(s.b,s.c));
}
point3 pvec(point3 s1,point3 s2,point3 s3){
	return xmult(subt(s1,s2),subt(s2,s3));
}

//两点距离,单参数取向量大小
double distance(point3 p1,point3 p2){
	return sqrt((p1.x-p2.x)*(p1.x-p2.x)+(p1.y-p2.y)*(p1.y-p2.y)+(p1.z-p2.z)*(p1.z-p2.z));
}

//向量大小
double vlen(point3 p){
	return sqrt(p.x*p.x+p.y*p.y+p.z*p.z);
}

//判三点共线
int dots_inline(point3 p1,point3 p2,point3 p3){
	return vlen(xmult(subt(p1,p2),subt(p2,p3)))<eps;
}

//判四点共面
int dots_onplane(point3 a,point3 b,point3 c,point3 d){
	return zero(dmult(pvec(a,b,c),subt(d,a)));
}

//判点是否在线段上,包括端点和共线
int dot_online_in(point3 p,line3 l){
	return zero(vlen(xmult(subt(p,l.a),subt(p,l.b))))&&(l.a.x-p.x)*(l.b.x-p.x)<eps&&
		(l.a.y-p.y)*(l.b.y-p.y)<eps&&(l.a.z-p.z)*(l.b.z-p.z)<eps;
}
int dot_online_in(point3 p,point3 l1,point3 l2){
	return zero(vlen(xmult(subt(p,l1),subt(p,l2))))&&(l1.x-p.x)*(l2.x-p.x)<eps&&
		(l1.y-p.y)*(l2.y-p.y)<eps&&(l1.z-p.z)*(l2.z-p.z)<eps;
}

//判点是否在线段上,不包括端点
int dot_online_ex(point3 p,line3 l){
	return dot_online_in(p,l)&&(!zero(p.x-l.a.x)||!zero(p.y-l.a.y)||!zero(p.z-l.a.z))&&
		(!zero(p.x-l.b.x)||!zero(p.y-l.b.y)||!zero(p.z-l.b.z));
}
int dot_online_ex(point3 p,point3 l1,point3 l2){
```

```
	return dot_online_in(p,l1,l2)&&(!zero(p.x-l1.x)||!zero(p.y-l1.y)||!zero(p.z-l1.z))&&
		(!zero(p.x-l2.x)||!zero(p.y-l2.y)||!zero(p.z-l2.z));
}

//判点是否在空间三角形上,包括边界,三点共线无意义
int dot_inplane_in(point3 p,plane3 s){
	return zero(vlen(xmult(subt(s.a,s.b),subt(s.a,s.c)))-vlen(xmult(subt(p,s.a),subt(p,s.b)))-
		vlen(xmult(subt(p,s.b),subt(p,s.c)))-vlen(xmult(subt(p,s.c),subt(p,s.a))));
}
int dot_inplane_in(point3 p,point3 s1,point3 s2,point3 s3){
	return zero(vlen(xmult(subt(s1,s2),subt(s1,s3)))-vlen(xmult(subt(p,s1),subt(p,s2)))-
		vlen(xmult(subt(p,s2),subt(p,s3)))-vlen(xmult(subt(p,s3),subt(p,s1))));
}

//判点是否在空间三角形上,不包括边界,三点共线无意义
int dot_inplane_ex(point3 p,plane3 s){
	return dot_inplane_in(p,s)&&vlen(xmult(subt(p,s.a),subt(p,s.b)))>eps&&
		vlen(xmult(subt(p,s.b),subt(p,s.c)))>eps&&vlen(xmult(subt(p,s.c),subt(p,s.a)))>eps;
}
int dot_inplane_ex(point3 p,point3 s1,point3 s2,point3 s3){
	return dot_inplane_in(p,s1,s2,s3)&&vlen(xmult(subt(p,s1),subt(p,s2)))>eps&&
		vlen(xmult(subt(p,s2),subt(p,s3)))>eps&&vlen(xmult(subt(p,s3),subt(p,s1)))>eps;
}

//判两点在线段同侧,点在线段上返回 0,不共面无意义
int same_side(point3 p1,point3 p2,line3 l){
	return dmult(xmult(subt(l.a,l.b),subt(p1,l.b)),xmult(subt(l.a,l.b),subt(p2,l.b)))>eps;
}
int same_side(point3 p1,point3 p2,point3 l1,point3 l2){
	return dmult(xmult(subt(l1,l2),subt(p1,l2)),xmult(subt(l1,l2),subt(p2,l2)))>eps;
}

//判两点在线段异侧,点在线段上返回 0,不共面无意义
int opposite_side(point3 p1,point3 p2,line3 l){
	return dmult(xmult(subt(l.a,l.b),subt(p1,l.b)),xmult(subt(l.a,l.b),subt(p2,l.b)))<-eps;
}
int opposite_side(point3 p1,point3 p2,point3 l1,point3 l2){
	return dmult(xmult(subt(l1,l2),subt(p1,l2)),xmult(subt(l1,l2),subt(p2,l2)))<-eps;
}

//判两点在平面同侧,点在平面上返回 0
int same_side(point3 p1,point3 p2,plane3 s){
	return dmult(pvec(s),subt(p1,s.a))*dmult(pvec(s),subt(p2,s.a))>eps;
}
```

```
int same_side(point3 p1,point3 p2,point3 s1,point3 s2,point3 s3){
	return dmult(pvec(s1,s2,s3),subt(p1,s1))*dmult(pvec(s1,s2,s3),subt(p2,s1))>eps;
}

//判两点在平面异侧,点在平面上返回 0
int opposite_side(point3 p1,point3 p2,plane3 s){
	return dmult(pvec(s),subt(p1,s.a))*dmult(pvec(s),subt(p2,s.a))<-eps;
}
int opposite_side(point3 p1,point3 p2,point3 s1,point3 s2,point3 s3){
	return dmult(pvec(s1,s2,s3),subt(p1,s1))*dmult(pvec(s1,s2,s3),subt(p2,s1))<-eps;
}

//判两直线平行
int parallel(line3 u,line3 v){
	return vlen(xmult(subt(u.a,u.b),subt(v.a,v.b)))<eps;
}
int parallel(point3 u1,point3 u2,point3 v1,point3 v2){
	return vlen(xmult(subt(u1,u2),subt(v1,v2)))<eps;
}

//判两平面平行
int parallel(plane3 u,plane3 v){
	return vlen(xmult(pvec(u),pvec(v)))<eps;
}
int parallel(point3 u1,point3 u2,point3 u3,point3 v1,point3 v2,point3 v3){
	return vlen(xmult(pvec(u1,u2,u3),pvec(v1,v2,v3)))<eps;
}

//判直线与平面平行
int parallel(line3 l,plane3 s){
	return zero(dmult(subt(l.a,l.b),pvec(s)));
}
int parallel(point3 l1,point3 l2,point3 s1,point3 s2,point3 s3){
	return zero(dmult(subt(l1,l2),pvec(s1,s2,s3)));
}

//判两直线垂直
int perpendicular(line3 u,line3 v){
	return zero(dmult(subt(u.a,u.b),subt(v.a,v.b)));
}
int perpendicular(point3 u1,point3 u2,point3 v1,point3 v2){
	return zero(dmult(subt(u1,u2),subt(v1,v2)));
}
```

```
//判两平面垂直
int perpendicular(plane3 u,plane3 v){
	return zero(dmult(pvec(u),pvec(v)));
}
int perpendicular(point3 u1,point3 u2,point3 u3,point3 v1,point3 v2,point3 v3){
	return zero(dmult(pvec(u1,u2,u3),pvec(v1,v2,v3)));
}

//判直线与平面平行
int perpendicular(line3 l,plane3 s){
	return vlen(xmult(subt(l.a,l.b),pvec(s)))<eps;
}
int perpendicular(point3 l1,point3 l2,point3 s1,point3 s2,point3 s3){
	return vlen(xmult(subt(l1,l2),pvec(s1,s2,s3)))<eps;
}

//判两线段相交,包括端点和部分重合
int intersect_in(line3 u,line3 v){
	if (!dots_onplane(u.a,u.b,v.a,v.b))
		return 0;
	if (!dots_inline(u.a,u.b,v.a)||!dots_inline(u.a,u.b,v.b))
		return !same_side(u.a,u.b,v)&&!same_side(v.a,v.b,u);
	return dot_online_in(u.a,v)||dot_online_in(u.b,v)||dot_online_in(v.a,u)||dot_online_in(v.b,u);
}
int intersect_in(point3 u1,point3 u2,point3 v1,point3 v2){
	if (!dots_onplane(u1,u2,v1,v2))
		return 0;
	if (!dots_inline(u1,u2,v1)||!dots_inline(u1,u2,v2))
		return !same_side(u1,u2,v1,v2)&&!same_side(v1,v2,u1,u2);
	return
dot_online_in(u1,v1,v2)||dot_online_in(u2,v1,v2)||dot_online_in(v1,u1,u2)||dot_online_in(v2,u1,u
2);
}

//判两线段相交,不包括端点和部分重合
int intersect_ex(line3 u,line3 v){
	return dots_onplane(u.a,u.b,v.a,v.b)&&opposite_side(u.a,u.b,v)&&opposite_side(v.a,v.b,u);
}
int intersect_ex(point3 u1,point3 u2,point3 v1,point3 v2){
	return
dots_onplane(u1,u2,v1,v2)&&opposite_side(u1,u2,v1,v2)&&opposite_side(v1,v2,u1,u2);
}

//判线段与空间三角形相交,包括交于边界和(部分)包含
```

```
int intersect_in(line3 l,plane3 s){
	return !same_side(l.a,l.b,s)&&!same_side(s.a,s.b,l.a,l.b,s.c)&&
		!same_side(s.b,s.c,l.a,l.b,s.a)&&!same_side(s.c,s.a,l.a,l.b,s.b);
}
int intersect_in(point3 l1,point3 l2,point3 s1,point3 s2,point3 s3){
	return !same_side(l1,l2,s1,s2,s3)&&!same_side(s1,s2,l1,l2,s3)&&
		!same_side(s2,s3,l1,l2,s1)&&!same_side(s3,s1,l1,l2,s2);
}

//判线段与空间三角形相交,不包括交于边界和(部分)包含
int intersect_ex(line3 l,plane3 s){
	return opposite_side(l.a,l.b,s)&&opposite_side(s.a,s.b,l.a,l.b,s.c)&&
		opposite_side(s.b,s.c,l.a,l.b,s.a)&&opposite_side(s.c,s.a,l.a,l.b,s.b);
}
int intersect_ex(point3 l1,point3 l2,point3 s1,point3 s2,point3 s3){
	return opposite_side(l1,l2,s1,s2,s3)&&opposite_side(s1,s2,l1,l2,s3)&&
		opposite_side(s2,s3,l1,l2,s1)&&opposite_side(s3,s1,l1,l2,s2);
}

//计算两直线交点,注意事先判断直线是否共面和平行!
//线段交点请另外判线段相交(同时还是要判断是否平行!)
point3 intersection(line3 u,line3 v){
	point3 ret=u.a;
	double t=((u.a.x-v.a.x)*(v.a.y-v.b.y)-(u.a.y-v.a.y)*(v.a.x-v.b.x))
			/((u.a.x-u.b.x)*(v.a.y-v.b.y)-(u.a.y-u.b.y)*(v.a.x-v.b.x));
	ret.x+=(u.b.x-u.a.x)*t;
	ret.y+=(u.b.y-u.a.y)*t;
	ret.z+=(u.b.z-u.a.z)*t;
	return ret;
}
point3 intersection(point3 u1,point3 u2,point3 v1,point3 v2){
	point3 ret=u1;
	double t=((u1.x-v1.x)*(v1.y-v2.y)-(u1.y-v1.y)*(v1.x-v2.x))
			/((u1.x-u2.x)*(v1.y-v2.y)-(u1.y-u2.y)*(v1.x-v2.x));
	ret.x+=(u2.x-u1.x)*t;
	ret.y+=(u2.y-u1.y)*t;
	ret.z+=(u2.z-u1.z)*t;
	return ret;
}

//计算直线与平面交点,注意事先判断是否平行,并保证三点不共线!
//线段和空间三角形交点请另外判断
point3 intersection(line3 l,plane3 s){
	point3 ret=pvec(s);
```

```
	double t=(ret.x*(s.a.x-l.a.x)+ret.y*(s.a.y-l.a.y)+ret.z*(s.a.z-l.a.z))/
		(ret.x*(l.b.x-l.a.x)+ret.y*(l.b.y-l.a.y)+ret.z*(l.b.z-l.a.z));
	ret.x=l.a.x+(l.b.x-l.a.x)*t;
	ret.y=l.a.y+(l.b.y-l.a.y)*t;
	ret.z=l.a.z+(l.b.z-l.a.z)*t;
	return ret;
}
point3 intersection(point3 l1,point3 l2,point3 s1,point3 s2,point3 s3){
	point3 ret=pvec(s1,s2,s3);
	double t=(ret.x*(s1.x-l1.x)+ret.y*(s1.y-l1.y)+ret.z*(s1.z-l1.z))/
		(ret.x*(l2.x-l1.x)+ret.y*(l2.y-l1.y)+ret.z*(l2.z-l1.z));
	ret.x=l1.x+(l2.x-l1.x)*t;
	ret.y=l1.y+(l2.y-l1.y)*t;
	ret.z=l1.z+(l2.z-l1.z)*t;
	return ret;
}

//计算两平面交线,注意事先判断是否平行,并保证三点不共线!
line3 intersection(plane3 u,plane3 v){
	line3 ret;
	ret.a=parallel(v.a,v.b,u.a,u.b,u.c)?intersection(v.b,v.c,u.a,u.b,u.c):intersection(v.a,v.b,u.a,u.b,u.
c);
	ret.b=parallel(v.c,v.a,u.a,u.b,u.c)?intersection(v.b,v.c,u.a,u.b,u.c):intersection(v.c,v.a,u.a,u.b,u.
c);
	return ret;
}
line3 intersection(point3 u1,point3 u2,point3 u3,point3 v1,point3 v2,point3 v3){
	line3 ret;
	ret.a=parallel(v1,v2,u1,u2,u3)?intersection(v2,v3,u1,u2,u3):intersection(v1,v2,u1,u2,u3);
	ret.b=parallel(v3,v1,u1,u2,u3)?intersection(v2,v3,u1,u2,u3):intersection(v3,v1,u1,u2,u3);
	return ret;
}

//点到直线距离
double ptoline(point3 p,line3 l){
	return vlen(xmult(subt(p,l.a),subt(l.b,l.a)))/distance(l.a,l.b);
}
double ptoline(point3 p,point3 l1,point3 l2){
	return vlen(xmult(subt(p,l1),subt(l2,l1)))/distance(l1,l2);
}

//点到平面距离
double ptoplane(point3 p,plane3 s){
	return fabs(dmult(pvec(s),subt(p,s.a)))/vlen(pvec(s));
```

```
}
double ptoplane(point3 p,point3 s1,point3 s2,point3 s3){
	return fabs(dmult(pvec(s1,s2,s3),subt(p,s1)))/vlen(pvec(s1,s2,s3));
}

//直线到直线距离
double linetoline(line3 u,line3 v){
	point3 n=xmult(subt(u.a,u.b),subt(v.a,v.b));
	return fabs(dmult(subt(u.a,v.a),n))/vlen(n);
}
double linetoline(point3 u1,point3 u2,point3 v1,point3 v2){
	point3 n=xmult(subt(u1,u2),subt(v1,v2));
	return fabs(dmult(subt(u1,v1),n))/vlen(n);
}

//两直线夹角 cos 值
double angle_cos(line3 u,line3 v){
	return dmult(subt(u.a,u.b),subt(v.a,v.b))/vlen(subt(u.a,u.b))/vlen(subt(v.a,v.b));
}
double angle_cos(point3 u1,point3 u2,point3 v1,point3 v2){
	return dmult(subt(u1,u2),subt(v1,v2))/vlen(subt(u1,u2))/vlen(subt(v1,v2));
}

//两平面夹角 cos 值
double angle_cos(plane3 u,plane3 v){
	return dmult(pvec(u),pvec(v))/vlen(pvec(u))/vlen(pvec(v));
}
double angle_cos(point3 u1,point3 u2,point3 u3,point3 v1,point3 v2,point3 v3){
	return dmult(pvec(u1,u2,u3),pvec(v1,v2,v3))/vlen(pvec(u1,u2,u3))/vlen(pvec(v1,v2,v3));
}

//直线平面夹角 sin 值
double angle_sin(line3 l,plane3 s){
	return dmult(subt(l.a,l.b),pvec(s))/vlen(subt(l.a,l.b))/vlen(pvec(s));
}
double angle_sin(point3 l1,point3 l2,point3 s1,point3 s2,point3 s3){
	return dmult(subt(l1,l2),pvec(s1,s2,s3))/vlen(subt(l1,l2))/vlen(pvec(s1,s2,s3));
}
```

1.10 凸包

```
#include <stdlib.h>
#define eps 1e-8
#define zero(x) (((x)>0?(x):-(x))<eps)
```

```
struct point{double x,y;};

//计算 cross product (P1-P0)x(P2-P0)
double xmult(point p1,point p2,point p0){
	return (p1.x-p0.x)*(p2.y-p0.y)-(p2.x-p0.x)*(p1.y-p0.y);
}
//graham 算法顺时针构造包含所有共线点的凸包,O(nlogn)
point p1,p2;
int graham_cp(const void* a,const void* b){
	double ret=xmult(*((point*)a),*((point*)b),p1);
	return zero(ret)?(xmult(*((point*)a),*((point*)b),p2)>0?1:-1):(ret>0?1:-1);
}
void _graham(int n,point* p,int& s,point* ch){
	int i,k=0;
	for (p1=p2=p[0],i=1;i<n;p2.x+=p[i].x,p2.y+=p[i].y,i++)
		if (p1.y-p[i].y>eps||(zero(p1.y-p[i].y)&&p1.x>p[i].x))
			p1=p[k=i];
	p2.x/=n,p2.y/=n;
	p[k]=p[0],p[0]=p1;
	qsort(p+1,n-1,sizeof(point),graham_cp);
	for (ch[0]=p[0],ch[1]=p[1],ch[2]=p[2],s=i=3;i<n;ch[s++]=p[i++])
		for (;s>2&&xmult(ch[s-2],p[i],ch[s-1])<-eps;s--);
}

//构造凸包接口函数,传入原始点集大小 n,点集 p(p 原有顺序被打乱!)
//返回凸包大小,凸包的点在 convex 中
//参数 maxsize 为 1 包含共线点,为 0 不包含共线点,缺省为 1
//参数 clockwise 为 1 顺时针构造,为 0 逆时针构造,缺省为 1
//在输入仅有若干共线点时算法不稳定,可能有此类情况请另行处理!
//不能去掉点集中重合的点
int graham(int n,point* p,point* convex,int maxsize=1,int dir=1){
	point* temp=new point[n];
	int s,i;
	_graham(n,p,s,temp);
	for (convex[0]=temp[0],n=1,i=(dir?1:(s-1));dir?(i<s):i;i+=(dir?1:-1))
		if (maxsize||!zero(xmult(temp[i-1],temp[i],temp[(i+1)%s])))
			convex[n++]=temp[i];
	delete []temp;
	return n;
}
```

1.11 网格

```
#define abs(x) ((x)>0?(x):-(x))
```

```
struct point{int x,y;};

int gcd(int a,int b){
	return b?gcd(b,a%b):a;
}

//多边形上的网格点个数
int grid_onedge(int n,point* p){
	int i,ret=0;
	for (i=0;i<n;i++)
		ret+=gcd(abs(p[i].x-p[(i+1)%n].x),abs(p[i].y-p[(i+1)%n].y));
	return ret;
}

//多边形内的网格点个数
int grid_inside(int n,point* p){
	int i,ret=0;
	for (i=0;i<n;i++)
		ret+=p[(i+1)%n].y*(p[i].x-p[(i+2)%n].x);
	return (abs(ret)-grid_onedge(n,p))/2+1;
}
```

1.12 圆

```
#include <math.h>
#define eps 1e-8
struct point{double x,y;};

double xmult(point p1,point p2,point p0){
	return (p1.x-p0.x)*(p2.y-p0.y)-(p2.x-p0.x)*(p1.y-p0.y);
}

double distance(point p1,point p2){
	return sqrt((p1.x-p2.x)*(p1.x-p2.x)+(p1.y-p2.y)*(p1.y-p2.y));
}

double disptoline(point p,point l1,point l2){
	return fabs(xmult(p,l1,l2))/distance(l1,l2);
}

point intersection(point u1,point u2,point v1,point v2){
	point ret=u1;
	double t=((u1.x-v1.x)*(v1.y-v2.y)-(u1.y-v1.y)*(v1.x-v2.x))
			/((u1.x-u2.x)*(v1.y-v2.y)-(u1.y-u2.y)*(v1.x-v2.x));
```

```
	ret.x+=(u2.x-u1.x)*t;
	ret.y+=(u2.y-u1.y)*t;
	return ret;
}

//判直线和圆相交,包括相切
int intersect_line_circle(point c,double r,point l1,point l2){
	return disptoline(c,l1,l2)<r+eps;
}

//判线段和圆相交,包括端点和相切
int intersect_seg_circle(point c,double r,point l1,point l2){
	double t1=distance(c,l1)-r,t2=distance(c,l2)-r;
	point t=c;
	if (t1<eps||t2<eps)
		return t1>-eps||t2>-eps;
	t.x+=l1.y-l2.y;
	t.y+=l2.x-l1.x;
	return xmult(l1,c,t)*xmult(l2,c,t)<eps&&disptoline(c,l1,l2)-r<eps;
}

//判圆和圆相交,包括相切
int intersect_circle_circle(point c1,double r1,point c2,double r2){
	return distance(c1,c2)<r1+r2+eps&&distance(c1,c2)>fabs(r1-r2)-eps;
}

//计算圆上到点 p 最近点,如 p 与圆心重合,返回 p 本身
point dot_to_circle(point c,double r,point p){
	point u,v;
	if (distance(p,c)<eps)
		return p;
	u.x=c.x+r*fabs(c.x-p.x)/distance(c,p);
	u.y=c.y+r*fabs(c.y-p.y)/distance(c,p)*((c.x-p.x)*(c.y-p.y)<0?-1:1);
	v.x=c.x-r*fabs(c.x-p.x)/distance(c,p);
	v.y=c.y-r*fabs(c.y-p.y)/distance(c,p)*((c.x-p.x)*(c.y-p.y)<0?-1:1);
	return distance(u,p)<distance(v,p)?u:v;
}

//计算直线与圆的交点,保证直线与圆有交点
//计算线段与圆的交点可用这个函数后判点是否在线段上
void intersection_line_circle(point c,double r,point l1,point l2,point& p1,point& p2){
	point p=c;
	double t;
	p.x+=l1.y-l2.y;
```

```
	p.y+=l2.x-l1.x;
	p=intersection(p,c,l1,l2);
	t=sqrt(r*r-distance(p,c)*distance(p,c))/distance(l1,l2);
	p1.x=p.x+(l2.x-l1.x)*t;
	p1.y=p.y+(l2.y-l1.y)*t;
	p2.x=p.x-(l2.x-l1.x)*t;
	p2.y=p.y-(l2.y-l1.y)*t;
}

//计算圆与圆的交点,保证圆与圆有交点,圆心不重合
void intersection_circle_circle(point c1,double r1,point c2,double r2,point& p1,point& p2){
	point u,v;
	double t;
	t=(1+(r1*r1-r2*r2)/distance(c1,c2)/distance(c1,c2))/2;
	u.x=c1.x+(c2.x-c1.x)*t;
	u.y=c1.y+(c2.y-c1.y)*t;
	v.x=u.x+c1.y-c2.y;
	v.y=u.y-c1.x+c2.x;
	intersection_line_circle(c1,r1,u,v,p1,p2);
}
```

1.13 整数函数

```
//整数几何函数库
//注意某些情况下整数运算会出界!
#define sign(a) ((a)>0?1:(((a)<0?-1:0)))
struct point{int x,y;};
struct line{point a,b;};

//计算 cross product (P1-P0)x(P2-P0)
int xmult(point p1,point p2,point p0){
	return (p1.x-p0.x)*(p2.y-p0.y)-(p2.x-p0.x)*(p1.y-p0.y);
}
int xmult(int x1,int y1,int x2,int y2,int x0,int y0){
	return (x1-x0)*(y2-y0)-(x2-x0)*(y1-y0);
}

//计算 dot product (P1-P0).(P2-P0)
int dmult(point p1,point p2,point p0){
	return (p1.x-p0.x)*(p2.x-p0.x)+(p1.y-p0.y)*(p2.y-p0.y);
}
int dmult(int x1,int y1,int x2,int y2,int x0,int y0){
	return (x1-x0)*(x2-x0)+(y1-y0)*(y2-y0);
}
```

```
//判三点共线
int dots_inline(point p1,point p2,point p3){
	return !xmult(p1,p2,p3);
}
int dots_inline(int x1,int y1,int x2,int y2,int x3,int y3){
	return !xmult(x1,y1,x2,y2,x3,y3);
}

//判点是否在线段上,包括端点和部分重合
int dot_online_in(point p,line l){
	return !xmult(p,l.a,l.b)&&(l.a.x-p.x)*(l.b.x-p.x)<=0&&(l.a.y-p.y)*(l.b.y-p.y)<=0;
}
int dot_online_in(point p,point l1,point l2){
	return !xmult(p,l1,l2)&&(l1.x-p.x)*(l2.x-p.x)<=0&&(l1.y-p.y)*(l2.y-p.y)<=0;
}
int dot_online_in(int x,int y,int x1,int y1,int x2,int y2){
	return !xmult(x,y,x1,y1,x2,y2)&&(x1-x)*(x2-x)<=0&&(y1-y)*(y2-y)<=0;
}

//判点是否在线段上,不包括端点
int dot_online_ex(point p,line l){
	return dot_online_in(p,l)&&(p.x!=l.a.x||p.y!=l.a.y)&&(p.x!=l.b.x||p.y!=l.b.y);
}
int dot_online_ex(point p,point l1,point l2){
	return dot_online_in(p,l1,l2)&&(p.x!=l1.x||p.y!=l1.y)&&(p.x!=l2.x||p.y!=l2.y);
}
int dot_online_ex(int x,int y,int x1,int y1,int x2,int y2){
	return dot_online_in(x,y,x1,y1,x2,y2)&&(x!=x1||y!=y1)&&(x!=x2||y!=y2);
}

//判两点在直线同侧,点在直线上返回 0
int same_side(point p1,point p2,line l){
	return sign(xmult(l.a,p1,l.b))*xmult(l.a,p2,l.b)>0;
}
int same_side(point p1,point p2,point l1,point l2){
	return sign(xmult(l1,p1,l2))*xmult(l1,p2,l2)>0;
}

//判两点在直线异侧,点在直线上返回 0
int opposite_side(point p1,point p2,line l){
	return sign(xmult(l.a,p1,l.b))*xmult(l.a,p2,l.b)<0;
}
int opposite_side(point p1,point p2,point l1,point l2){
```

```
	return sign(xmult(l1,p1,l2))*xmult(l1,p2,l2)<0;
}

//判两直线平行
int parallel(line u,line v){
	return (u.a.x-u.b.x)*(v.a.y-v.b.y)==(v.a.x-v.b.x)*(u.a.y-u.b.y);
}
int parallel(point u1,point u2,point v1,point v2){
	return (u1.x-u2.x)*(v1.y-v2.y)==(v1.x-v2.x)*(u1.y-u2.y);
}

//判两直线垂直
int perpendicular(line u,line v){
	return (u.a.x-u.b.x)*(v.a.x-v.b.x)==-(u.a.y-u.b.y)*(v.a.y-v.b.y);
}
int perpendicular(point u1,point u2,point v1,point v2){
	return (u1.x-u2.x)*(v1.x-v2.x)==-(u1.y-u2.y)*(v1.y-v2.y);
}

//判两线段相交,包括端点和部分重合
int intersect_in(line u,line v){
	if (!dots_inline(u.a,u.b,v.a)||!dots_inline(u.a,u.b,v.b))
		return !same_side(u.a,u.b,v)&&!same_side(v.a,v.b,u);
	return dot_online_in(u.a,v)||dot_online_in(u.b,v)||dot_online_in(v.a,u)||dot_online_in(v.b,u);
}
int intersect_in(point u1,point u2,point v1,point v2){
	if (!dots_inline(u1,u2,v1)||!dots_inline(u1,u2,v2))
		return !same_side(u1,u2,v1,v2)&&!same_side(v1,v2,u1,u2);
	return
dot_online_in(u1,v1,v2)||dot_online_in(u2,v1,v2)||dot_online_in(v1,u1,u2)||dot_online_in(v2,u1,u
2);
}

//判两线段相交,不包括端点和部分重合
int intersect_ex(line u,line v){
	return opposite_side(u.a,u.b,v)&&opposite_side(v.a,v.b,u);
}
int intersect_ex(point u1,point u2,point v1,point v2){
	return opposite_side(u1,u2,v1,v2)&&opposite_side(v1,v2,u1,u2);
}
```

2、组合

2.1 组合公式

1. C(m,n)=C(m,m-n)
2. C(m,n)=C(m-1,n)+C(m-1,n-1)

derangement D(n) = n!(1 - 1/1! + 1/2! - 1/3! + ... + (-1)^n/n!)
= (n-1)(D(n-2) - D(n-1))
Q(n) = D(n) + D(n-1)

求和公式,k = 1..n
1. sum(k) = n(n+1)/2
2. sum(2k-1) = n^2
3. sum(k^2) = n(n+1)(2n+1)/6
4. sum((2k-1)^2) = n(4n^2-1)/3
5. sum(k^3) = (n(n+1)/2)^2
6. sum((2k-1)^3) = n^2(2n^2-1)
7. sum(k^4) = n(n+1)(2n+1)(3n^2+3n-1)/30
8. sum(k^5) = n^2(n+1)^2(2n^2+2n-1)/12
9. sum(k(k+1)) = n(n+1)(n+2)/3
10. sum(k(k+1)(k+2)) = n(n+1)(n+2)(n+3)/4
12. sum(k(k+1)(k+2)(k+3)) = n(n+1)(n+2)(n+3)(n+4)/5

2.2 排列组合生成

```
//gen_perm 产生字典序排列 P(n,m)
//gen_comb 产生字典序组合 C(n,m)
//gen_perm_swap 产生相邻位对换全排列 P(n,n)
//产生元素用 1..n 表示
//dummy 为产生后调用的函数,传入 a[]和 n,a[0]..a[n-1]为一次产生的结果
#define MAXN 100
int count;

#include <iostream.h>
void dummy(int* a,int n){
	int i;
	cout<<count++<<": ";
	for (i=0;i<n-1;i++)
		cout<<a[i]<<' ';
	cout<<a[n-1]<<endl;
}
```

```
void _gen_perm(int* a,int n,int m,int l,int* temp,int* tag){
	int i;
	if (l==m)
		dummy(temp,m);
	else
		for (i=0;i<n;i++)
			if (!tag[i]){
				temp[l]=a[i],tag[i]=1;
				_gen_perm(a,n,m,l+1,temp,tag);
				tag[i]=0;
			}
}

void gen_perm(int n,int m){
	int a[MAXN],temp[MAXN],tag[MAXN]={0},i;
	for (i=0;i<n;i++)
		a[i]=i+1;
	_gen_perm(a,n,m,0,temp,tag);
}

void _gen_comb(int* a,int s,int e,int m,int& count,int* temp){
	int i;
	if (!m)
		dummy(temp,count);
	else
		for (i=s;i<=e-m+1;i++){
			temp[count++]=a[i];
			_gen_comb(a,i+1,e,m-1,count,temp);
			count--;
		}
}

void gen_comb(int n,int m){
	int a[MAXN],temp[MAXN],count=0,i;
	for (i=0;i<n;i++)
		a[i]=i+1;
	_gen_comb(a,0,n-1,m,count,temp);
}

void _gen_perm_swap(int* a,int n,int l,int* pos,int* dir){
	int i,p1,p2,t;
	if (l==n)
		dummy(a,n);
```

```
	else{
		_gen_perm_swap(a,n,l+1,pos,dir);
		for (i=0;i<l;i++){
			p2=(p1=pos[l])+dir[l];
			t=a[p1],a[p1]=a[p2],a[p2]=t;
			pos[a[p1]-1]=p1,pos[a[p2]-1]=p2;
			_gen_perm_swap(a,n,l+1,pos,dir);
		}
		dir[l]=-dir[l];
	}
}

void gen_perm_swap(int n){
	int a[MAXN],pos[MAXN],dir[MAXN],i;
	for (i=0;i<n;i++)
		a[i]=i+1,pos[i]=i,dir[i]=-1;
	_gen_perm_swap(a,n,0,pos,dir);
}
```

2.3 生成 gray 码

```
//生成 reflected gray code
//每次调用 gray 取得下一个码
//000...000 是第一个码,100...000 是最后一个码
void gray(int n,int *code){
	int t=0,i;
	for (i=0;i<n;t+=code[i++]);
	if (t&1)
		for (n--;!code[n];n--);
	code[n-1]=1-code[n-1];
}
```

2.4 置换(polya)

```
//求置换的循环节,polya 原理
//perm[0..n-1]为 0..n-1 的一个置换(排列)
//返回置换最小周期,num 返回循环节个数
#define MAXN 1000

int gcd(int a,int b){
	return b?gcd(b,a%b):a;
}
```

```
int polya(int* perm,int n,int& num){
	int i,j,p,v[MAXN]={0},ret=1;
	for (num=i=0;i<n;i++)
		if (!v[i]){
			for (num++,j=0,p=i;!v[p=perm[p]];j++)
				v[p]=1;
			ret*=j/gcd(ret,j);
		}
	return ret;
}
```

2.5 字典序全排列

```
//字典序全排列与序号的转换
int perm2num(int n,int *p){
	int i,j,ret=0,k=1;
	for (i=n-2;i>=0;k*=n-(i--))
		for (j=i+1;j<n;j++)
			if (p[j]<p[i])
				ret+=k;
	return ret;
}

void num2perm(int n,int *p,int t){
	int i,j;
	for (i=n-1;i>=0;i--)
		p[i]=t%(n-i),t/=n-i;
	for (i=n-1;i;i--)
		for (j=i-1;j>=0;j--)
			if (p[j]<=p[i])
				p[i]++;
}
```

2.6 字典序组合

```
//字典序组合与序号的转换
//comb为组合数C(n,m),必要时换成大数,注意处理C(n,m)=0|n<m
int comb(int n,int m){
	int ret=1,i;
	m=m<(n-m)?m:(n-m);
	for (i=n-m+1;i<=n;ret*=(i++));
	for (i=1;i<=m;ret/=(i++));
	return m<0?0:ret;
```

```
}

int comb2num(int n,int m,int *c){
	int ret=comb(n,m),i;
	for (i=0;i<m;i++)
		ret-=comb(n-c[i],m-i);
	return ret;
}

void num2comb(int n,int m,int* c,int t){
	int i,j=1,k;
	for (i=0;i<m;c[i++]=j++)
		for (;t>(k=comb(n-j,m-i-1));t-=k,j++);
}
```

3、结构

3.1 并查集

```
//带路径压缩的并查集,用于动态维护查询等价类
//图论算法中动态判点集连通常用
//维护和查询复杂度略大于 O(1)
//集合元素取值 1..MAXN-1(注意 0 不能用!),默认不等价
#include <string.h>
#define MAXN 100000
#define _ufind_run(x) for(;p[t=x];x=p[x],p[t]=(p[x]?p[x]:x))
#define _run_both _ufind_run(i);_ufind_run(j)

struct ufind{
	int p[MAXN],t;
	void init(){memset(p,0,sizeof(p));}
	void set_friend(int i,int j){_run_both;p[i]=(i==j?0:j);}
	int is_friend(int i,int j){_run_both;return i==j&&i;}
};

//带路径压缩的并查集扩展形式
//用于动态维护查询 friend-enemy 型等价类
//维护和查询复杂度略大于 O(1)
//集合元素取值 1..MAXN-1(注意 0 不能用!),默认无关
#include <string.h>

#define MAXN 100000
#define sig(x) ((x)>0?1:-1)
```

```
#define abs(x) ((x)>0?(x):-(x))
#define _ufind_run(x) for(;p[t=abs(x)];x=sig(x)*p[abs(x)],p[t]=sig(p[t])*(p[abs(x)]?p[abs(x)]:abs(p[t])))
#define _run_both _ufind_run(i);_ufind_run(j)
#define _set_side(x) p[abs(i)]=sig(i)*(abs(i)==abs(j)?0:(x)*j)
#define _judge_side(x) (i==(x)*j&&i)

struct ufind{
	int p[MAXN],t;
	void init(){memset(p,0,sizeof(p));}
	int set_friend(int i,int j){_run_both;_set_side(1);return !_judge_side(-1);}
	int set_enemy(int i,int j){_run_both;_set_side(-1);return !_judge_side(1);}
	int is_friend(int i,int j){_run_both;return _judge_side(1);}
	int is_enemy(int i,int j){_run_both;return _judge_side(-1);}
};
```

3.2 堆

```
//二分堆(binary)
//可插入,获取并删除最小(最大)元素,复杂度均 O(logn)
//可更改元素类型,修改比较符号或换成比较函数
#define MAXN 10000
#define _cp(a,b) ((a)<(b))
typedef int elem_t;

struct heap{
	elem_t h[MAXN];
	int n,p,c;
	void init(){n=0;}
	void ins(elem_t e){
		for (p=++n;p>1&&_cp(e,h[p>>1]);h[p]=h[p>>1],p>>=1);
		h[p]=e;
	}
	int del(elem_t& e){
		if (!n) return 0;
		for (e=h[p=1],c=2;c<n&&_cp(h[c+=(c<n-1&&_cp(h[c+1],h[c]))],h[n]);h[p]=h[c],p=c,c<<=1);
		h[p]=h[n--];return 1;
	}
};

//映射二分堆(mapped)
//可插入,获取并删除任意元素,复杂度均 O(logn)
//插入时提供一个索引值,删除时按该索引删除,获取并删除最小元素时一起获得该索引
```

```
//索引值范围 0..MAXN-1,不能重复,不负责维护索引的唯一性,不在此返回请另外映射
//主要用于图论算法,该索引值可以是节点的下标
//可更改元素类型,修改比较符号或换成比较函数
#define MAXN 10000
#define _cp(a,b) ((a)<(b))
typedef int elem_t;

struct heap{
	elem_t h[MAXN];
	int ind[MAXN],map[MAXN],n,p,c;
	void init(){n=0;}
	void ins(int i,elem_t e){
		for (p=++n;p>1&&_cp(e,h[p>>1]);h[map[ind[p]=ind[p>>1]]=p]=h[p>>1],p>>=1);
		h[map[ind[p]=i]=p]=e;
	}
	int del(int i,elem_t& e){
		i=map[i];if (i<1||i>n) return 0;
		for (e=h[p=i];p>1;h[map[ind[p]=ind[p>>1]]=p]=h[p>>1],p>>=1);
		for (c=2;c<n&&_cp(h[c+=(c<n-1&&_cp(h[c+1],h[c]))],h[n]);h[map[ind[p]=ind[c]]=p]=h[c],p=c,c<<=1);
		h[map[ind[p]=ind[n]]=p]=h[n];n--;return 1;
	}
	int delmin(int& i,elem_t& e){
		if (n<1) return 0;i=ind[1];
		for (e=h[p=1],c=2;c<n&&_cp(h[c+=(c<n-1&&_cp(h[c+1],h[c]))],h[n]);h[map[ind[p]=ind[c]]=p]=h[c],p=c,c<<=1);
		h[map[ind[p]=ind[n]]=p]=h[n];n--;return 1;
	}
};
```

3.3 线段树

线段树应用：
求面积:
1) 坐标离散化
2) 垂直边按 x 坐标排序
3) 从左往右用线段树处理垂直边
   累计每个离散 x 区间长度和线段树长度的乘积

求周长:
1) 坐标离散化
2) 垂直边按 x 坐标排序, 第二关键字为入边优于出边
3) 从左往右用线段树处理垂直边
   在每个离散点上先加入所有入边, 累计线段树长度变化值
   再删除所有出边, 累计线段树长度变化值

4) 水平边按 y 坐标排序，第二关键字为入边优于出边
5) 从上往下用线段树处理水平边
   在每个离散点上先加入所有入边，累计线段树长度变化值
   再删除所有出边，累计线段树长度变化值

```
//线段树
//可以处理加入边和删除边不同的情况
//inc_seg 和 dec_seg 用于加入边
//seg_len 求长度
//t 传根节点(一律为 1)
//l0,r0 传树的节点范围(一律为 1..t)
//l,r 传线段(端点)
#define MAXN 10000
struct segtree{
	int n,cnt[MAXN],len[MAXN];
	segtree(int t):n(t){
		for (int i=1;i<=t;i++)
			cnt[i]=len[i]=0;
	};
	void update(int t,int l,int r);
	void inc_seg(int t,int l0,int r0,int l,int r);
	void dec_seg(int t,int l0,int r0,int l,int r);
	int seg_len(int t,int l0,int r0,int l,int r);
};

int length(int l,int r){
	return r-l;
}

void segtree::update(int t,int l,int r){
	if (cnt[t]||r-l==1)
		len[t]=length(l,r);
	else
		len[t]=len[t+t]+len[t+t+1];
}

void segtree::inc_seg(int t,int l0,int r0,int l,int r){
	if (l0==l&&r0==r)
		cnt[t]++;
	else{
		int m0=(l0+r0)>>1;
		if (l<m0)
			inc_seg(t+t,l0,m0,l,m0<r?m0:r);
		if (r>m0)
```

```
            inc_seg(t+t+1,m0,r0,m0>l?m0:l,r);
        if (cnt[t+t]&&cnt[t+t+1]){
            cnt[t+t]--;
            update(t+t,l0,m0);
            cnt[t+t+1]--;
            update(t+t+1,m0,r0);
            cnt[t]++;
        }
    }
    update(t,l0,r0);
}

void segtree::dec_seg(int t,int l0,int r0,int l,int r){
    if (l0==l&&r0==r)
        cnt[t]--;
    else if (cnt[t]){
        cnt[t]--;
        if (l>l0)
            inc_seg(t,l0,r0,l0,l);
        if (r<r0)
            inc_seg(t,l0,r0,r,r0);
    }
    else{
        int m0=(l0+r0)>>1;
        if (l<m0)
            dec_seg(t+t,l0,m0,l,m0<r?m0:r);
        if (r>m0)
            dec_seg(t+t+1,m0,r0,m0>l?m0:l,r);
    }
    update(t,l0,r0);
}

int segtree::seg_len(int t,int l0,int r0,int l,int r){
    if (cnt[t]||(l0==l&&r0==r))
        return len[t];
    else{
        int m0=(l0+r0)>>1,ret=0;
        if (l<m0)
            ret+=seg_len(t+t,l0,m0,l,m0<r?m0:r);
        if (r>m0)
            ret+=seg_len(t+t+1,m0,r0,m0>l?m0:l,r);
        return ret;
    }
}
```

```
//线段树扩展
//可以计算长度和线段数
//可以处理加入边和删除边不同的情况
//inc_seg 和 dec_seg 用于加入边
//seg_len 求长度,seg_cut 求线段数
//t 传根节点(一律为 1)
//l0,r0 传树的节点范围(一律为 1..t)
//l,r 传线段(端点)
#define MAXN 10000
struct segtree{
	int n,cnt[MAXN],len[MAXN],cut[MAXN],bl[MAXN],br[MAXN];
	segtree(int t):n(t){
		for (int i=1;i<=t;i++)
			cnt[i]=len[i]=cut[i]=bl[i]=br[i]=0;
	};
	void update(int t,int l,int r);
	void inc_seg(int t,int l0,int r0,int l,int r);
	void dec_seg(int t,int l0,int r0,int l,int r);
	int seg_len(int t,int l0,int r0,int l,int r);
	int seg_cut(int t,int l0,int r0,int l,int r);
};

int length(int l,int r){
	return r-l;
}

void segtree::update(int t,int l,int r){
	if (cnt[t]||r-l==1)
		len[t]=length(l,r),cut[t]=bl[t]=br[t]=1;
	else{
		len[t]=len[t+t]+len[t+t+1];
		cut[t]=cut[t+t]+cut[t+t+1];
		if (br[t+t]&&bl[t+t+1])
			cut[t]--;
		bl[t]=bl[t+t],br[t]=br[t+t+1];
	}
}

void segtree::inc_seg(int t,int l0,int r0,int l,int r){
	if (l0==l&&r0==r)
		cnt[t]++;
	else{
		int m0=(l0+r0)>>1;
```

```
		if (l<m0)
			inc_seg(t+t,l0,m0,l,m0<r?m0:r);
		if (r>m0)
			inc_seg(t+t+1,m0,r0,m0>l?m0:l,r);
		if (cnt[t+t]&&cnt[t+t+1]){
			cnt[t+t]--;
			update(t+t,l0,m0);
			cnt[t+t+1]--;
			update(t+t+1,m0,r0);
			cnt[t]++;
		}
	}
	update(t,l0,r0);
}

void segtree::dec_seg(int t,int l0,int r0,int l,int r){
	if (l0==l&&r0==r)
		cnt[t]--;
	else if (cnt[t]){
		cnt[t]--;
		if (l>l0)
			inc_seg(t,l0,r0,l0,l);
		if (r<r0)
			inc_seg(t,l0,r0,r,r0);
	}
	else{
		int m0=(l0+r0)>>1;
		if (l<m0)
			dec_seg(t+t,l0,m0,l,m0<r?m0:r);
		if (r>m0)
			dec_seg(t+t+1,m0,r0,m0>l?m0:l,r);
	}
	update(t,l0,r0);
}

int segtree::seg_len(int t,int l0,int r0,int l,int r){
	if (cnt[t]||(l0==l&&r0==r))
		return len[t];
	else{
		int m0=(l0+r0)>>1,ret=0;
		if (l<m0)
			ret+=seg_len(t+t,l0,m0,l,m0<r?m0:r);
		if (r>m0)
			ret+=seg_len(t+t+1,m0,r0,m0>l?m0:l,r);
```

```
		return ret;
	}
}

int segtree::seg_cut(int t,int l0,int r0,int l,int r){
	if (cnt[t])
		return 1;
	if (l0==l&&r0==r)
		return cut[t];
	else{
		int m0=(l0+r0)>>1,ret=0;
		if (l<m0)
			ret+=seg_cut(t+t,l0,m0,l,m0<r?m0:r);
		if (r>m0)
			ret+=seg_cut(t+t+1,m0,r0,m0>l?m0:l,r);
		if (l<m0&&r>m0&&br[t+t]&&bl[t+t+1])
			ret--;
		return ret;
	}
}
```

3.4 子段和

```
//求 sum{[0..n-1]}
//维护和查询复杂度均为 O(logn)
//用于动态求子段和,数组内容保存在 sum.a[]中
//可以改成其他数据类型
#include <string.h>
#define lowbit(x) ((x)&((x)^((x)-1)))
#define MAXN 10000
typedef int elem_t;

struct sum{
	elem_t a[MAXN],c[MAXN],ret;
	int n;
	void init(int i){memset(a,0,sizeof(a));memset(c,0,sizeof(c));n=i;}
	void update(int i,elem_t v){for (v-=a[i],a[i++]+=v;i<=n;c[i-1]+=v,i+=lowbit(i));}
	elem_t query(int i){for (ret=0;i;ret+=c[i-1],i^=lowbit(i));return ret;}
};
```

3.5 子阵和

```
//求 sum{a[0..m-1][0..n-1]}
```

```
//维护和查询复杂度均为O(logm*logn)
//用于动态求子阵和,数组内容保存在 sum.a[][]中
//可以改成其他数据类型
#include <string.h>
#define lowbit(x) ((x)&((x)^((x)-1)))
#define MAXN 100
typedef int elem_t;

struct sum{
	elem_t a[MAXN][MAXN],c[MAXN][MAXN],ret;
	int m,n,t;
	void init(int i,int j){memset(a,0,sizeof(a));memset(c,0,sizeof(c));m=i,n=j;}
	void update(int i,int j,elem_t v){
		for (v-=a[i][j],a[i++][j++]+=v,t=j;i<=m;i+=lowbit(i))
			for (j=t;j<=n;c[i-1][j-1]+=v,j+=lowbit(j));
	}
	elem_t query(int i,int j){
		for (ret=0,t=j;i;i^=lowbit(i))
			for (j=t;j;ret+=c[i-1][j-1],j^=lowbit(j));
		return ret;
	}
};
```

4、数论

4.1 阶乘最后非 0 位

```
//求阶乘最后非零位,复杂度 O(nlogn)
//返回该位,n 以字符串方式传入
#include <string.h>
#define MAXN 10000

int lastdigit(char* buf){
	const int mod[20]={1,1,2,6,4,2,2,4,2,8,4,4,8,4,6,8,8,6,8,2};
	int len=strlen(buf),a[MAXN],i,c,ret=1;
	if (len==1)
		return mod[buf[0]-'0'];
	for (i=0;i<len;i++)
		a[i]=buf[len-1-i]-'0';
	for (;len;len-=!a[len-1]){
		ret=ret*mod[a[1]%2*10+a[0]]%5;
		for (c=0,i=len-1;i>=0;i--)
			c=c*10+a[i],a[i]=c/5,c%=5;
```

```
	}
	return ret+ret%2*5;
}
```

4.2 模线性方程组

```
#ifdef WIN32
typedef __int64 i64;
#else
typedef long long i64;
#endif
//扩展 Euclid 求解 gcd(a,b)=ax+by
int ext_gcd(int a,int b,int& x,int& y){
	int t,ret;
	if (!b){
		x=1,y=0;
		return a;
	}
	ret=ext_gcd(b,a%b,x,y);
	t=x,x=y,y=t-a/b*y;
	return ret;
}

//计算 m^a, O(loga)，本身没什么用，注意这个按位处理的方法 :-P
int exponent(int m,int a){
	int ret=1;
	for (;a;a>>=1,m*=m)
		if (a&1)
			ret*=m;
	return ret;
}

//计算幂取模 a^b mod n, O(logb)
int modular_exponent(int a,int b,int n){ //a^b mod n
	int ret=1;
	for (;b;b>>=1,a=(int)((i64)a)*a%n)
		if (b&1)
			ret=(int)((i64)ret)*a%n;
	return ret;
}

//求解模线性方程 ax=b (mod n)
//返回解的个数,解保存在 sol[]中
//要求 n>0,解的范围 0..n-1
```

```
int modular_linear(int a,int b,int n,int* sol){
	int d,e,x,y,i;
	d=ext_gcd(a,n,x,y);
	if (b%d)
		return 0;
	e=(x*(b/d)%n+n)%n;
	for (i=0;i<d;i++)
		sol[i]=(e+i*(n/d))%n;
	return d;
}

//求解模线性方程组(中国余数定理)
//   x = b[0] (mod w[0])
//   x = b[1] (mod w[1])
//   ...
//   x = b[k-1] (mod w[k-1])
//要求 w[i]>0,w[i]与 w[j]互质,解的范围 1..n,n=w[0]*w[1]*...*w[k-1]
int modular_linear_system(int b[],int w[],int k){
	int d,x,y,a=0,m,n=1,i;
	for (i=0;i<k;i++)
		n*=w[i];
	for (i=0;i<k;i++){
		m=n/w[i];
		d=ext_gcd(w[i],m,x,y);
		a=(a+y*m*b[i])%n;
	}
	return (a+n)%n;
}
```

4.3 素数

```
//用素数表判定素数,先调用 initprime
int plist[10000],pcount=0;

int prime(int n){
	int i;
	if ((n!=2&&!(n%2))||(n!=3&&!(n%3))||(n!=5&&!(n%5))||(n!=7&&!(n%7)))
		return 0;
	for (i=0;plist[i]*plist[i]<=n;i++)
		if (!(n%plist[i]))
			return 0;
	return n>1;
}
```

```
void initprime(){
	int i;
	for (plist[pcount++]=2,i=3;i<50000;i++)
		if (prime(i))
			plist[pcount++]=i;
}

//miller rabin
//判断自然数 n 是否为素数
//time 越高失败概率越低,一般取 10 到 50
#include <stdlib.h>
#ifdef WIN32
typedef __int64 i64;
#else
typedef long long i64;
#endif

int modular_exponent(int a,int b,int n){ //a^b mod n
	int ret;
	for (;b;b>>=1,a=(int)((i64)a)*a%n)
		if (b&1)
			ret=(int)((i64)ret)*a%n;
	return ret;
}

// Carmicheal number: 561,41041,825265,321197185
int miller_rabin(int n,int time=10){
	if (n==1||(n!=2&&!(n%2))||(n!=3&&!(n%3))||(n!=5&&!(n%5))||(n!=7&&!(n%7)))
		return 0;
	while (time--)
		if (modular_exponent(((rand()&0x7fff<<16)+rand()&0x7fff+rand()&0x7fff)%(n-1)+1,n-1,n)!=1)
			return 0;
	return 1;
}
```

4.4 欧拉函数

```
int gcd(int a,int b){
	return b?gcd(b,a%b):a;
}

inline int lcm(int a,int b){
	return a/gcd(a,b)*b;
```

```
}

//求 1..n-1 中与 n 互质的数的个数
int eular(int n){
    int ret=1,i;
    for (i=2;i*i<=n;i++)
        if (n%i==0){
            n/=i,ret*=i-1;
            while (n%i==0)
                n/=i,ret*=i;
        }
    if (n>1)
        ret*=n-1;
    return ret;
}
```

5、数值计算

5.1 定积分计算(Romberg)

```
/*  Romberg 求定积分
    输入：积分区间[a,b]，被积函数 f(x,y,z)
    输出：积分结果

    f(x,y,z)示例：
    double f0( double x, double l, double t )
    {
        return sqrt(1.0+l*l*t*t*cos(t*x)*cos(t*x));
    }
*/

double Integral(double a, double b, double (*f)(double x, double y, double z), double eps,
                double l, double t)

double Romberg (double a, double b, double (*f)(double x, double y, double z), double eps,
                double l, double t)
{
#define MAX_N   1000

    int i, j, temp2, min;
    double h, R[2][MAX_N], temp4;

    for (i=0; i<MAX_N; i++) {
```

```
        R[0][i] = 0.0;
        R[1][i] = 0.0;
    }
    h = b-a;
    min = (int)(log(h*10.0)/log(2.0)); //h should be at most 0.1
    R[0][0] = ((*f)(a, l, t)+(*f)(b, l, t))*h*0.50;
    i = 1;
    temp2 = 1;
    while (i<MAX_N){
        i++;
        R[1][0] = 0.0;
        for (j=1; j<=temp2; j++)
            R[1][0] += (*f)(a+h*((double)j-0.50), l, t);
        R[1][0] = (R[0][0] + h*R[1][0])*0.50;
        temp4 = 4.0;
        for (j=1; j<i; j++) {
            R[1][j] = R[1][j-1] + (R[1][j-1]-R[0][j-1])/(temp4-1.0);
            temp4 *= 4.0;
        }
        if ((fabs(R[1][i-1]-R[0][i-2])<eps)&&(i>min))
            return R[1][i-1];
        h *= 0.50;
        temp2 *= 2;
        for (j=0; j<i; j++)
            R[0][j] = R[1][j];
    }
    return R[1][MAX_N-1];
}

double Integral(double a, double b, double (*f)(double x, double y, double z), double eps,
                double l, double t)
{
#define pi 3.1415926535897932

    int n;
    double R, p, res;

    n = (int)(floor)(b * t * 0.50 / pi);
    p = 2.0 * pi / t;
    res = b - (double)n * p;
    if (n)
        R = Romberg (a, p, f0, eps/(double)n, l, t);
    R = R * (double)n + Romberg( 0.0, res, f0, eps, l, t );
```

```
	return R/100.0;
}
```

5.2 多项式求根(牛顿法)

```
/* 牛顿法解多项式的根
   输入：多项式系数 c[]，多项式度数 n，求在[a,b]间的根
   输出：根
   要求保证[a,b]间有根
*/

double fabs( double x )
{
	return (x<0)? -x : x;
}

double f(int m, double c[], double x)
{
	int i;
	double p = c[m];

	for (i=m; i>0; i--)
		p = p*x + c[i-1];
	return p;
}

int newton(double x0, double *r,
		double c[], double cp[], int n,
		double a, double b, double eps)
{
	int MAX_ITERATION = 1000;
	int i = 1;
	double x1, x2, fp, eps2 = eps/10.0;

	x1 = x0;
	while (i < MAX_ITERATION) {
		x2 = f(n, c, x1);
		fp = f(n-1, cp, x1);
		if ((fabs(fp)<0.000000001) && (fabs(x2)>1.0))
			return 0;
		x2 = x1 - x2/fp;
		if (fabs(x1-x2)<eps2) {
			if (x2<a || x2>b)
				return 0;
```

```
			*r = x2;
			return 1;
		}
		x1 = x2;
		i++;
	}
	return 0;
}

double Polynomial_Root(double c[], int n, double a, double b, double eps)
{
	double *cp;
	int i;
	double root;

	cp = (double *)calloc(n, sizeof(double));
	for (i=n-1; i>=0; i--) {
		cp[i] = (i+1)*c[i+1];
	}
	if (a>b) {
			root = a; a = b; b = root;
	}
	if ((!newton(a, &root, c, cp, n, a, b, eps)) &&
		(!newton(b, &root, c, cp, n, a, b, eps)))
			newton((a+b)*0.5, &root, c, cp, n, a, b, eps);
	free(cp);
	if (fabs(root)<eps)
		return fabs(root);
	else
		return root;
}
```

5.3 周期性方程(追赶法)

```
/*	追赶法解周期性方程
	周期性方程定义：| a1 b1 c1 ...                  |           =  x1
	                |    a2 b2 c2 ...               |           =  x2
	                |        ...                    | *   X     =  ...
	                | cn-1 ...          an-1 bn-1   |           =  xn-1
	                | bn   cn                  an   |           =  xn
	输入：a[],b[],c[],x[]
	输出：求解结果 X 在 x[]中
*/
```

```
void run()
{
	c[0] /= b[0]; a[0] /= b[0]; x[0] /= b[0];
	for (int i = 1; i < N - 1; i ++) {
		double temp = b[i] - a[i] * c[i - 1];
		c[i] /= temp;
		x[i] = (x[i] - a[i] * x[i - 1]) / temp;
		a[i] = -a[i] * a[i - 1] / temp;
	}
	a[N - 2] = -a[N - 2] - c[N - 2];
	for (int i = N - 3; i >= 0; i --) {
		a[i] = -a[i] - c[i] * a[i + 1];
		x[i] -= c[i] * x[i + 1];
	}
	x[N - 1] -= (c[N - 1] * x[0] + a[N - 1] * x[N - 2]);
	x[N - 1] /= (c[N - 1] * a[0] + a[N - 1] * a[N - 2] + b[N - 1]);
	for (int i = N - 2; i >= 0; i --)
		x[i] += a[i] * x[N - 1];
}
```

6、图论—NP 搜索

6.1 最大团

```
//最大团
//返回最大团大小和一个方案,传入图的大小 n 和邻接阵 mat
//mat[i][j]为布尔量

#define MAXN 60

void clique(int n, int* u, int mat[][MAXN], int size, int& max, int& bb, int* res, int* rr, int* c) {
	int i, j, vn, v[MAXN];
	if (n) {
		if (size + c[u[0]] <= max) return;
		for (i = 0; i < n + size - max && i < n; ++ i) {
			for (j = i + 1, vn = 0; j < n; ++ j)
				if (mat[u[i]][u[j]])
					v[vn ++] = u[j];
			rr[size] = u[i];
			clique(vn, v, mat, size + 1, max, bb, res, rr, c);
			if (bb) return;
		}
	} else if (size > max) {
```

```
            max = size;
            for (i = 0; i < size; ++ i)
                res[i] = rr[i];
            bb = 1;
        }
}

int maxclique(int n, int mat[][MAXN], int *ret) {
        int max = 0, bb, c[MAXN], i, j;
        int vn, v[MAXN], rr[MAXN];
        for (c[i = n - 1] = 0; i >= 0; -- i) {
            for (vn = 0, j = i + 1; j < n; ++ j)
                if (mat[i][j])
                    v[vn ++] = j;
            bb = 0;
            rr[0] = i;
            clique(vn, v, mat, 1, max, bb, ret, rr, c);
            c[i] = max;
        }
        return max;
}
```

6.2 最大团(n<64)(faster)

```
/**
 * WishingBone's ACM/ICPC Routine Library
 *
 * maximum clique solver
 */

#include <vector>

using std::vector;

// clique solver calculates both size and consitution of maximum clique
// uses bit operation to accelerate searching
// graph size limit is 63, the graph should be undirected
// can optimize to calculate on each component, and sort on vertex degrees
// can be used to solve maximum independent set
class clique {
public:
        static const long long ONE = 1;
        static const long long MASK = (1 << 21) - 1;
        char* bits;
```

```
	int n, size, cmax[63];
	long long mask[63], cons;
	// initiate lookup table
	clique() {
		bits = new char[1 << 21];
		bits[0] = 0;
		for (int i = 1; i < 1 << 21; ++i) bits[i] = bits[i >> 1] + (i & 1);
	}
	~clique() {
		delete bits;
	}
	// search routine
	bool search(int step, int size, long long more, long long con);
	// solve maximum clique and return size
	int sizeClique(vector<vector<int> >& mat);
	// solve maximum clique and return constitution
	vector<int> consClique(vector<vector<int> >& mat);
};

// search routine
// step is node id, size is current solution, more is available mask, cons is
constitution mask
bool clique::search(int step, int size, long long more, long long cons) {
	if (step >= n) {
		// a new solution reached
		this->size = size;
		this->cons = cons;
		return true;
	}
	long long now = ONE << step;
	if ((now & more) > 0) {
		long long next = more & mask[step];
		if (size + bits[next & MASK] + bits[(next >> 21) & MASK] + bits[next >>
42] >= this->size
				&& size + cmax[step] > this->size) {
			// the current node is in the clique
			if (search(step + 1, size + 1, next, cons | now)) return true;
		}
	}
	long long next = more & ~now;
	if (size + bits[next & MASK] + bits[(next >> 21) & MASK] + bits[next >> 42]
> this->size) {
		// the current node is not in the clique
		if (search(step + 1, size, next, cons)) return true;
```

```
	}
	return false;
}

// solve maximum clique and return size
int clique::sizeClique(vector<vector<int> >& mat) {
	n = mat.size();
	// generate mask vectors
	for (int i = 0; i < n; ++i) {
		mask[i] = 0;
		for (int j = 0; j < n; ++j) if (mat[i][j] > 0) mask[i] |= ONE << j;
	}
	size = 0;
	for (int i = n - 1; i >= 0; --i) {
		search(i + 1, 1, mask[i], ONE << i);
		cmax[i] = size;
	}
	return size;
}

// solve maximum clique and return constitution
// calls sizeClique and restore cons
vector<int> clique::consClique(vector<vector<int> >& mat) {
	sizeClique(mat);
	vector<int> ret;
	for (int i = 0; i < n; ++i) if ((cons & (ONE << i)) > 0) ret.push_back(i);
	return ret;
}
```

7、图论—连通性

7.1 无向图关键点(dfs 邻接阵)

```
//无向图的关键点,dfs 邻接阵形式,O(n^2)
//返回关键点个数,key[]返回点集
//传入图的大小 n 和邻接阵 mat,不相邻点边权 0
#define MAXN 110

void search(int n,int mat[][MAXN],int* dfn,int* low,int now,int& ret,int* key,int& cnt,int root,int& rd,int* bb){
	int i;
	dfn[now]=low[now]=++cnt;
	for (i=0;i<n;i++)
```

```
			if (mat[now][i]){
				if (!dfn[i]){
					search(n,mat,dfn,low,i,ret,key,cnt,root,rd,bb);
					if (low[i]<low[now])
						low[now]=low[i];
					if (low[i]>=dfn[now]){
						if (now!=root&&!bb[now])
							key[ret++]=now,bb[now]=1;
						else if(now==root)
							rd++;
					}
				}
				else if (dfn[i]<low[now])
					low[now]=dfn[i];
			}
}

int key_vertex(int n,int mat[][MAXN],int* key){
	int ret=0,i,cnt,rd,dfn[MAXN],low[MAXN],bb[MAXN];
	for (i=0;i<n;dfn[i++]=bb[i]=0);
	for (cnt=i=0;i<n;i++)
		if (!dfn[i]){
			rd=0;
			search(n,mat,dfn,low,i,ret,key,cnt,i,rd,bb);
			if (rd>1&&!bb[i])
				key[ret++]=i,bb[i]=1;
		}
	return ret;
}
```

7.2 无向图关键边(dfs 邻接阵)

```
//无向图的关键边,dfs 邻接阵形式,O(n^2)
//返回关键边条数,key[][2]返回边集
//传入图的大小 n 和邻接阵 mat,不相邻点边权 0
#define MAXN 100

void search(int n,int mat[][MAXN],int* dfn,int* low,int now,int& cnt,int key[][2]){
	int i;
	for (low[now]=dfn[now],i=0;i<n;i++)
		if (mat[now][i]){
			if (!dfn[i]){
				dfn[i]=dfn[now]+1;
				search(n,mat,dfn,low,i,cnt,key);
```

```
					if (low[i]>dfn[now])
						key[cnt][0]=i,key[cnt++][1]=now;
					if (low[i]<low[now])
						low[now]=low[i];
				}
				else if (dfn[i]<dfn[now]-1&&dfn[i]<low[now])
					low[now]=lev[i];
			}
}

int key_edge(int n,int mat[][MAXN],int key[][2]){
	int ret=0,i,dfn[MAXN],low[MAXN];
	for (i=0;i<n;dfn[i++]=0);
	for (i=0;i<n;i++)
		if (!dfn[i])
			dfn[i]=1,bridge(n,mat,dfn,low,i,ret,key);
	return ret;
}
```

7.3 无向图的块(bfs 邻接阵)

```
//无向图的块,dfs 邻接阵形式,O(n^2)
//每产生一个块调用 dummy
//传入图的大小 n 和邻接阵 mat,不相邻点边权 0
#define MAXN 100

#include <iostream.h>
void dummy(int n,int* a){
	for (int i=0;i<n;i++)
		cout<<a[i]<<' ';
	cout<<endl;
}

void search(int n,int mat[][MAXN],int* dfn,int* low,int now,int& cnt,int* st,int& sp){
	int i,m,a[MAXN];
	dfn[st[sp++]=now]=low[now]=++cnt;
	for (i=0;i<n;i++)
		if (mat[now][i]){
			if (!dfn[i]){
				search(n,mat,dfn,low,i,cnt,st,sp);
				if (low[i]<low[now])
					low[now]=low[i];
				if (low[i]>=dfn[now]){
					for (st[sp]=-1,a[0]=now,m=1;st[sp]!=i;a[m++]=st[--sp]);
```

```
                    dummy(m,a);
                }
            }
            else if (dfn[i]<low[now])
                low[now]=dfn[i];
        }
}

void block(int n,int mat[][MAXN]){
    int i,cnt,dfn[MAXN],low[MAXN],st[MAXN],sp=0;
    for (i=0;i<n;dfn[i++]=0);
    for (cnt=i=0;i<n;i++)
        if (!dfn[i])
            search(n,mat,dfn,low,i,cnt,st,sp);
}
```

7.4 无向图连通分支(dfs/bfs 邻接阵)

```
//无向图连通分支,dfs 邻接阵形式,O(n^2)
//返回分支数,id 返回 1..分支数的值
//传入图的大小 n 和邻接阵 mat,不相邻点边权 0
#define MAXN 100

void floodfill(int n,int mat[][MAXN],int* id,int now,int tag){
    int i;
    for (id[now]=tag,i=0;i<n;i++)
        if (!id[i]&&mat[now][i])
            floodfill(n,mat,id,i,tag);
}

int find_components(int n,int mat[][MAXN],int* id){
    int ret,i;
    for (i=0;i<n;id[i++]=0);
    for (ret=i=0;i<n;i++)
        if (!id[i])
            floodfill(n,mat,id,i,++ret);
    return ret;
}

//无向图连通分支,bfs 邻接阵形式,O(n^2)
//返回分支数,id 返回 1..分支数的值
//传入图的大小 n 和邻接阵 mat,不相邻点边权 0
#define MAXN 100
```

```
int find_components(int n,int mat[][MAXN],int* id){
    int ret,k,i,j,m;
    for (k=0;k<n;id[k++]=0);
    for (ret=k=0;k<n;k++)
        if (!id[k])
            for (id[k]=-1,ret++,m=1;m;)
                for (m=i=0;i<n;i++)
                    if (id[i]==-1)
                        for (m++,id[i]=ret,j=0;j<n;j++)
                            if (!id[j]&&mat[i][j])
                                id[j]=-1;
    return ret;
}
```

7.5 有向图强连通分支(dfs/bfs 邻接阵)

```
//有向图强连通分支,dfs 邻接阵形式,O(n^2)
//返回分支数,id 返回 1..分支数的值
//传入图的大小 n 和邻接阵 mat,不相邻点边权 0
#define MAXN 100

void search(int n,int mat[][MAXN],int* dfn,int* low,int now,int& cnt,int& tag,int* id,int* st,int&
sp){
    int i,j;
    dfn[st[sp++]=now]=low[now]=++cnt;
    for (i=0;i<n;i++)
        if (mat[now][i]){
            if (!dfn[i]){
                ssearch(n,mat,dfn,low,i,cnt,tag,id,st,sp);
                if (low[i]<low[now])
                    low[now]=low[i];
            }
            else if (dfn[i]<dfn[now]){
                for (j=0;j<sp&&st[j]!=i;j++);
                if (j<cnt&&dfn[i]<low[now])
                    low[now]=dfn[i];
            }
        }
    if (low[now]==dfn[now])
        for (tag++;st[sp]!=now;id[st[--sp]]=tag);
}
int find_components(int n,int mat[][MAXN],int* id){
    int ret=0,i,cnt,sp,st[MAXN],dfn[MAXN],low[MAXN];
    for (i=0;i<n;dfn[i++]=0);
```

```
	for (sp=cnt=i=0;i<n;i++)
		if (!dfn[i])
			search(n,mat,dfn,low,i,cnt,ret,id,st,sp);
	return ret;
}
//有向图强连通分支,bfs 邻接阵形式,O(n^2)
//返回分支数,id 返回 1..分支数的值
//传入图的大小 n 和邻接阵 mat,不相邻点边权 0
#define MAXN 100

int find_components(int n,int mat[][MAXN],int* id){
	int ret=0,a[MAXN],b[MAXN],c[MAXN],d[MAXN],i,j,k,t;
	for (k=0;k<n;id[k++]=0);
	for (k=0;k<n;k++)
		if (!id[k]){
			for (i=0;i<n;i++)
				a[i]=b[i]=c[i]=d[i]=0;
			a[k]=b[k]=1;
			for (t=1;t;)
				for (t=i=0;i<n;i++){
					if (a[i]&&!c[i])
						for (c[i]=t=1,j=0;j<n;j++)
							if (mat[i][j]&&!a[j])
								a[j]=1;
					if (b[i]&&!d[i])
						for (d[i]=t=1,j=0;j<n;j++)
							if (mat[j][i]&&!b[j])
								b[j]=1;
				}
			for (ret++,i=0;i<n;i++)
				if (a[i]&b[i])
					id[i]=ret;
		}
	return ret;
}
```

7.6 有向图最小点基(邻接阵)

```
//有向图最小点基,邻接阵形式,O(n^2)
//返回电集大小和点集
//传入图的大小 n 和邻接阵 mat,不相邻点边权 0
//需要调用强连通分支
#define MAXN 100
```

```
int base_vertex(int n,int mat[][MAXN],int* sets){
	int ret=0,id[MAXN],v[MAXN],i,j;
	j=find_components(n,mat,id);
	for (i=0;i<j;v[i++]=1);
	for (i=0;i<n;i++)
		for (j=0;j<n;j++)
			if (id[i]!=id[j]&&mat[i][j])
				v[id[j]-1]=0;
	for (i=0;i<n;i++)
		if (v[id[i]-1])
			v[id[sets[ret++]=i]-1]=0;
	return ret;
}
```

8、图论—匹配

8.1 二分图最大匹配(hungary 邻接表)

```
//二分图最大匹配,hungary 算法,邻接表形式,复杂度 O(m*e)
//返回最大匹配数,传入二分图大小 m,n 和邻接表 list(只需一边)
//match1,match2 返回一个最大匹配,未匹配顶点 match 值为-1
#include <string.h>
#define MAXN 310
#define _clr(x) memset(x,0xff,sizeof(int)*MAXN)
struct edge_t{
	int from,to;
	edge_t* next;
};

int hungary(int m,int n,edge_t* list[],int* match1,int* match2){
	int s[MAXN],t[MAXN],p,q,ret=0,i,j,k;edge_t* e;
	for (_clr(match1),_clr(match2),i=0;i<m;ret+=(match1[i++]>=0))
		for (_clr(t),s[p=q=0]=i;p<=q&&match1[i]<0;p++)
			for (e=list[k=s[p]];e&&match1[i]<0;e=e->next)
				if (t[j=e->to]<0){
					s[++q]=match2[j],t[j]=k;
					if (s[q]<0)
						for (p=j;p>=0;j=p)
							match2[j]=k=t[j],p=match1[k],match1[k]=j;
				}
	return ret;
}
```

8.2 二分图最大匹配(hungary 邻接阵)

```
//二分图最大匹配,hungary 算法,邻接阵形式,复杂度 O(m*m*n)
//返回最大匹配数,传入二分图大小 m,n 和邻接阵 mat,非零元素表示有边
//match1,match2 返回一个最大匹配,未匹配顶点 match 值为-1
#include <string.h>
#define MAXN 310
#define _clr(x) memset(x,0xff,sizeof(int)*MAXN)

int hungary(int m,int n,int mat[][MAXN],int* match1,int* match2){
	int s[MAXN],t[MAXN],p,q,ret=0,i,j,k;
	for (_clr(match1),_clr(match2),i=0;i<m;ret+=(match1[i++]>=0))
		for (_clr(t),s[p=q=0]=i;p<=q&&match1[i]<0;p++)
			for (k=s[p],j=0;j<n&&match1[i]<0;j++)
				if (mat[k][j]&&t[j]<0){
					s[++q]=match2[j],t[j]=k;
					if (s[q]<0)
						for (p=j;p>=0;j=p)
							match2[j]=k=t[j],p=match1[k],match1[k]=j;
				}
	return ret;
}
```

8.3 二分图最大匹配(hungary 正向表)

```
//二分图最大匹配,hungary 算法,正向表形式,复杂度 O(m*e)
//返回最大匹配数,传入二分图大小 m,n 和正向表 list,buf(只需一边)
//match1,match2 返回一个最大匹配,未匹配顶点 match 值为-1
#include <string.h>
#define MAXN 310
#define _clr(x) memset(x,0xff,sizeof(int)*MAXN)

int hungary(int m,int n,int* list,int* buf,int* match1,int* match2){
	int s[MAXN],t[MAXN],p,q,ret=0,i,j,k,l;
	for (_clr(match1),_clr(match2),i=0;i<m;ret+=(match1[i++]>=0))
		for (_clr(t),s[p=q=0]=i;p<=q&&match1[i]<0;p++)
			for (l=list[k=s[p]];l<list[k+1]&&match1[i]<0;l++)
				if (t[j=buf[l]]<0){
					s[++q]=match2[j],t[j]=k;
					if (s[q]<0)
						for (p=j;p>=0;j=p)
							match2[j]=k=t[j],p=match1[k],match1[k]=j;
				}
```

```
	return ret;
}
```

8.4 二分图最佳匹配(kuhn_munkras 邻接阵)

```
//二分图最佳匹配,kuhn munkras 算法,邻接阵形式,复杂度 O(m*m*n)
//返回最佳匹配值,传入二分图大小 m,n 和邻接阵 mat,表示权值
//match1,match2 返回一个最佳匹配,未匹配顶点 match 值为-1
//一定注意 m<=n,否则循环无法终止
//最小权匹配可将权值取相反数
#include <string.h>
#define MAXN 310
#define inf 1000000000
#define _clr(x) memset(x,0xff,sizeof(int)*n)

int kuhn_munkras(int m,int n,int mat[][MAXN],int* match1,int* match2){
	int s[MAXN],t[MAXN],l1[MAXN],l2[MAXN],p,q,ret=0,i,j,k;
	for (i=0;i<m;i++)
		for (l1[i]=-inf,j=0;j<n;j++)
			l1[i]=mat[i][j]>l1[i]?mat[i][j]:l1[i];
	for (i=0;i<n;l2[i++]=0);
	for (_clr(match1),_clr(match2),i=0;i<m;i++){
		for (_clr(t),s[p=q=0]=i;p<=q&&match1[i]<0;p++)
			for (k=s[p],j=0;j<n&&match1[i]<0;j++)
				if (l1[k]+l2[j]==mat[k][j]&&t[j]<0){
					s[++q]=match2[j],t[j]=k;
					if (s[q]<0)
						for (p=j;p>=0;j=p)
							match2[j]=k=t[j],p=match1[k],match1[k]=j;
				}
		if (match1[i]<0){
			for (i--,p=inf,k=0;k<=q;k++)
				for (j=0;j<n;j++)
					if (t[j]<0&&l1[s[k]]+l2[j]-mat[s[k]][j]<p)
						p=l1[s[k]]+l2[j]-mat[s[k]][j];
			for (j=0;j<n;l2[j]+=t[j]<0?0:p,j++);
			for (k=0;k<=q;l1[s[k++]]-=p);
		}
	}
	for (i=0;i<m;i++)
		ret+=mat[i][match1[i]];
	return ret;
}
```

8.5 一般图匹配(邻接表)

```
//一般图最大匹配,邻接表形式,复杂度 O(n*e)
//返回匹配顶点对数,match 返回匹配,未匹配顶点 match 值为-1
//传入图的顶点数 n 和邻接表 list
#define MAXN 100
struct edge_t{
	int from,to;
	edge_t* next;
};

int aug(int n,edge_t* list[],int* match,int* v,int now){
	int t,ret=0;edge_t* e;
	v[now]=1;
	for (e=list[now];e;e=e->next)
		if (!v[t=e->to]){
			if (match[t]<0)
				match[now]=t,match[t]=now,ret=1;
			else{
				v[t]=1;
				if (aug(n,list,match,v,match[t]))
					match[now]=t,match[t]=now,ret=1;
				v[t]=0;
			}
			if (ret)
				break;
		}
	v[now]=0;
	return ret;
}

int graph_match(int n,edge_t* list[],int* match){
	int v[MAXN],i,j;
	for (i=0;i<n;i++)
	v[i]=0,match[i]=-1;
	for (i=0,j=n;i<n&&j>=2;)
		if (match[i]<0&&aug(n,list,match,v,i))
			i=0,j-=2;
		else
			i++;
	for (i=j=0;i<n;i++)
		j+=(match[i]>=0);
	return j/2;
}
```

8.6 一般图匹配(邻接阵)

```
//一般图最大匹配,邻接阵形式,复杂度O(n^3)
//返回匹配顶点对数,match返回匹配,未匹配顶点match值为-1
//传入图的顶点数n和邻接阵mat
#define MAXN 100

int aug(int n,int mat[][MAXN],int* match,int* v,int now){
	int i,ret=0;
	v[now]=1;
	for (i=0;i<n;i++)
		if (!v[i]&&mat[now][i]){
			if (match[i]<0)
				match[now]=i,match[i]=now,ret=1;
			else{
				v[i]=1;
				if (aug(n,mat,match,v,match[i]))
					match[now]=i,match[i]=now,ret=1;
				v[i]=0;
			}
			if (ret)
				break;
		}
	v[now]=0;
	return ret;
}

int graph_match(int n,int mat[][MAXN],int* match){
	int v[MAXN],i,j;
	for (i=0;i<n;i++)
	v[i]=0,match[i]=-1;
	for (i=0,j=n;i<n&&j>=2;)
		if (match[i]<0&&aug(n,mat,match,v,i))
			i=0,j-=2;
		else
			i++;
	for (i=j=0;i<n;i++)
		j+=(match[i]>=0);
	return j/2;
}
```

8.7 一般图匹配(正向表)

```
//一般图最大匹配,正向表形式,复杂度 O(n*e)
//返回匹配顶点对数,match 返回匹配,未匹配顶点 match 值为-1
//传入图的顶点数 n 和正向表 list,buf
#define MAXN 100

int aug(int n,int* list,int* buf,int* match,int* v,int now){
	int i,t,ret=0;
	v[now]=1;
	for (i=list[now];i<list[now+1];i++)
		if (!v[t=buf[i]]){
			if (match[t]<0)
				match[now]=t,match[t]=now,ret=1;
			else{
				v[t]=1;
				if (aug(n,list,buf,match,v,match[t]))
					match[now]=t,match[t]=now,ret=1;
				v[t]=0;
			}
			if (ret)
				break;
		}
	v[now]=0;
	return ret;
}

int graph_match(int n,int* list,int* buf,int* match){
	int v[MAXN],i,j;
	for (i=0;i<n;i++)
	v[i]=0,match[i]=-1;
	for (i=0,j=n;i<n&&j>=2;)
		if (match[i]<0&&aug(n,list,buf,match,v,i))
			i=0,j-=2;
		else
			i++;
	for (i=j=0;i<n;i++)
		j+=(match[i]>=0);
	return j/2;
}
```

9、图论—网络流

9.1 最大流(邻接阵)

```
//求网络最大流,邻接阵形式
//返回最大流量,flow 返回每条边的流量
//传入网络节点数 n,容量 mat,源点 source,汇点 sink

#define MAXN 100
#define inf 1000000000

int max_flow(int n,int mat[][MAXN],int source,int sink,int flow[][MAXN]){
	int pre[MAXN],que[MAXN],d[MAXN],p,q,t,i,j;
	if (source==sink) return inf;
	for (i=0;i<n;i++)
		for (j=0;j<n;flow[i][j++]=0);
	for (;;){
		for (i=0;i<n;pre[i++]=0);
		pre[t=source]=source+1,d[t]=inf;
		for (p=q=0;p<=q&&!pre[sink];t=que[p++])
			for (i=0;i<n;i++)
				if (!pre[i]&&j=mat[t][i]-flow[t][i])
					pre[que[q++]=i]=t+1,d[i]=d[t]<j?d[t]:j;
				else if (!pre[i]&&j=flow[i][t])
					pre[que[q++]=i]=-t-1,d[i]=d[t]<j?d[t]:j;
		if (!pre[sink]) break;
		for (i=sink;i!=source;)
			if (pre[i]>0)
				flow[pre[i]-1][i]+=d[sink],i=pre[i]-1;
			else
				flow[i][-pre[i]-1]-=d[sink],i=-pre[i]-1;
	}
	for (j=i=0;i<n; j+=flow[source][i++]);
	return j;
}
```

9.2 上下界最大流(邻接阵)

```
//求上下界网络最大流,邻接阵形式
//返回最大流量,-1 表示无可行流,flow 返回每条边的流量
//传入网络节点数 n,容量 mat,流量下界 bf,源点 source,汇点 sink
//MAXN 应比最大结点数多 2,无可行流返回-1 时 mat 未复原!

#define MAXN 100
#define inf 1000000000

int limit_max_flow(int n,int mat[][MAXN],int bf[][MAXN],int source,int sink,int flow[][MAXN]){
```

```
	int i,j,sk,ks;
	if (source==sink) return inf;
	for (mat[n][n+1]=mat[n+1][n]=mat[n][n]=mat[n+1][n+1]=i=0;i<n;i++)
		for (mat[n][i]=mat[i][n]=mat[n+1][i]=mat[i][n+1]=j=0;j<n;j++)
			mat[i][j]-=bf[i][j],mat[n][i]+=bf[j][i],mat[i][n+1]+=bf[i][j];
	sk=mat[source][sink],ks=mat[sink][source],mat[source][sink]=mat[sink][source]=inf;
	for (i=0;i<n+2;i++)
		for (j=0;j<n+2;flow[i][j++]=0);
	_max_flow(n+2,mat,n,n+1,flow);
	for (i=0;i<n;i++)
		if (flow[n][i]<mat[n][i]) return -1;
	flow[source][sink]=flow[sink][source]=0,mat[source][sink]=sk,mat[sink][source]=ks;
	_max_flow(n,mat,source,sink,flow);
	for (i=0;i<n;i++)
		for (j=0;j<n;j++)
			mat[i][j]+=bf[i][j],flow[i][j]+=bf[i][j];
	for (j=i=0;i<n;j+=flow[source][i++]);
	return j;
}
```

9.3 上下界最小流(邻接阵)

```
//求上下界网络最小流,邻接阵形式
//返回最大流量,-1 表示无可行流,flow 返回每条边的流量
//传入网络节点数 n,容量 mat,流量下界 bf,源点 source,汇点 sink
//MAXN 应比最大结点数多 2,无可行流返回-1 时 mat 未复原!

#define MAXN 100
#define inf 1000000000
int limit_min_flow(int n,int mat[][MAXN],int bf[][MAXN],int source,int sink,int flow[][MAXN]){
	int i,j,sk,ks;
	if (source==sink) return inf;
	for (mat[n][n+1]=mat[n+1][n]=mat[n][n]=mat[n+1][n+1]=i=0;i<n;i++)
		for (mat[n][i]=mat[i][n]=mat[n+1][i]=mat[i][n+1]=j=0;j<n;j++)
			mat[i][j]-=bf[i][j],mat[n][i]+=bf[j][i],mat[i][n+1]+=bf[i][j];
	sk=mat[source][sink],ks=mat[sink][source],mat[source][sink]=mat[sink][source]=inf;
	for (i=0;i<n+2;i++)
		for (j=0;j<n+2;flow[i][j++]=0);
	_max_flow(n+2,mat,n,n+1,flow);
	for (i=0;i<n;i++)
		if (flow[n][i]<mat[n][i]) return -1;
	flow[source][sink]=flow[sink][source]=0,mat[source][sink]=sk,mat[sink][source]=ks;
	_max_flow(n,mat,sink,source,flow);
```

```
	for (i=0;i<n;i++)
		for (j=0;j<n;j++)
			mat[i][j]+=bf[i][j],flow[i][j]+=bf[i][j];
	for (j=i=0;i<n; j+=flow[source][i++]);
	return j;
}
```

9.4 最大流无流量(邻接阵)

```
//求网络最大流,邻接阵形式
//返回最大流量
//传入网络节点数 n,容量 mat,源点 source,汇点 sink
//注意 mat 矩阵被修改

#define MAXN 100
#define inf 1000000000

int max_flow(int n,int mat[][MAXN],int source,int sink){
	int v[MAXN],c[MAXN],p[MAXN],ret=0,i,j;
	for (;;){
		for (i=0;i<n;i++)
			v[i]=c[i]=0;
		for (c[source]=inf;;){
			for (j=-1,i=0;i<n;i++)
				if (!v[i]&&c[i]&&(j==-1||c[i]>c[j]))
					j=i;
			if (j<0) return ret;
			if (j==sink) break;
			for (v[j]=1,i=0;i<n;i++)
				if (mat[j][i]>c[i]&&c[j]>c[i])
					c[i]=mat[j][i]<c[j]?mat[j][i]:c[j],p[i]=j;
		}
		for (ret+=j=c[i=sink];i!=source;i=p[i])
			mat[p[i]][i]-=j,mat[i][p[i]]+=j;
	}
}
```

9.5 最小费用最大流(邻接阵)

```
//求网络最小费用最大流,邻接阵形式
//返回最大流量,flow 返回每条边的流量,netcost 返回总费用
//传入网络节点数 n,容量 mat,单位费用 cost,源点 source,汇点 sink
```

```
#define MAXN 100
#define inf 1000000000

int min_cost_max_flow(int n,int mat[][MAXN],int cost[][MAXN],int source,int sink,int flow[][MAXN],int& netcost){
	int pre[MAXN],min[MAXN],d[MAXN],i,j,t,tag;
	if (source==sink) return inf;
	for (i=0;i<n;i++)
		for (j=0;j<n;flow[i][j++]=0);
	for (netcost=0;;){
		for (i=0;i<n;i++)
			pre[i]=0,min[i]=inf;
		for (pre[source]=source+1,min[source]=0,d[source]=inf,tag=1;tag;)
			for (tag=t=0;t<n;t++)
				if (d[t])
					for (i=0;i<n;i++)
						if (j=mat[t][i]-flow[t][i]&&min[t]+cost[t][i]<min[i])
							tag=1,min[i]=min[t]+cost[t][i],pre[i]=t+1,d[i]=d[t]<j?d[t]:j;
						else if (j=flow[i][t]&&min[t]<inf&&min[t]-cost[i][t]<min[i])
							tag=1,min[i]=min[t]-cost[i][t],pre[i]=-t-1,d[i]=d[t]<j?d[t]:j;
		if (!pre[sink])  break;
		for (netcost+=min[sink]*d[i=sink];i!=source;)
			if (pre[i]>0)
				flow[pre[i]-1][i]+=d[sink],i=pre[i]-1;
			else
				flow[i][-pre[i]-1]-=d[sink],i=-pre[i]-1;
	}
	for (j=i=0;i<n; j+=flow[source][i++]);
	return j;
}
```

10、 图论—应用

10.1 欧拉回路(邻接阵)

```
//求欧拉回路或欧拉路,邻接阵形式,复杂度O(n^2)
//返回路径长度,path返回路径(有向图时得到的是反向路径)
//传入图的大小n和邻接阵mat,不相邻点边权0
//可以有自环与重边,分为无向图和有向图

#define MAXN 100

void find_path_u(int n,int mat[][MAXN],int now,int& step,int* path){
```

```
    int i;
    for (i=n-1;i>=0;i--)
        while (mat[now][i]){
            mat[now][i]--,mat[i][now]--;
            find_path_u(n,mat,i,step,path);
        }
    path[step++]=now;
}

void find_path_d(int n,int mat[][MAXN],int now,int& step,int* path){
    int i;
    for (i=n-1;i>=0;i--)
        while (mat[now][i]){
            mat[now][i]--;
            find_path_d(n,mat,i,step,path);
        }
    path[step++]=now;
}

int euclid_path(int n,int mat[][MAXN],int start,int* path){
    int ret=0;
    find_path_u(n,mat,start,ret,path);
//  find_path_d(n,mat,start,ret,path);
    return ret;
}
```

10.2 树的前序表转化

```
//将用边表示的树转化为前序表示的树
//传入节点数 n 和邻接表 list[],邻接表必须是双向的,会在函数中释放
//pre[]返回前序表,map[]返回前序表中的节点到原来节点的映射
#define MAXN 10000
struct node{
    int to;
    node* next;
};

void prenode(int n,node* list[],int* pre,int* map,int* v,int now,int last,int& id){
    node* t;
    int p=id++;
    for (v[map[p]=now]=1,pre[p]=last;list[now];){
        t=list[now],list[now]=t->next;
        if (!v[t->to])
            prenode(n,list,pre,map,v,t->to,p,id);
```

```
	}
}

void makepre(int n,node* list[],int* pre,int* map){
	int v[MAXN],id=0,i;
	for (i=0;i<n;v[i++]=0);
	prenode(n,list,pre,map,v,0,-1,id);
}
```

10.3 树的优化算法

```
//最大顶点独立集
int max_node_independent(int n,int* pre,int* set){
	int c[MAXN],i,ret=0;
	for (i=0;i<n;i++)
		c[i]=set[i]=0;
	for (i=n-1;i>=0;i--)
		if (!c[i]){
			set[i]=1;
			if (pre[i]!=-1)
				c[pre[i]]=1;
			ret++;
		}
	return ret;
}

//最大边独立集
int max_edge_independent(int n,int* pre,int* set){
	int c[MAXN],i,ret=0;
	for (i=0;i<n;i++)
		c[i]=set[i]=0;
	for (i=n-1;i>=0;i--)
		if (!c[i]&&pre[i]!=-1&&!c[pre[i]]){
			set[i]=1;
			c[pre[i]]=1;
			ret++;
		}
	return ret;
}

//最小顶点覆盖集
int min_node_cover(int n,int* pre,int* set){
	int c[MAXN],i,ret=0;
	for (i=0;i<n;i++)
```

```
		c[i]=set[i]=0;
	for (i=n-1;i>=0;i--)
		if (!c[i]&&pre[i]!=-1&&!c[pre[i]]){
			set[i]=1;
			c[pre[i]]=1;
			ret++;
		}
	return ret;
}

//最小顶点支配集
int min_node_dominant(int n,int* pre,int* set){
	int c[MAXN],i,ret=0;
	for (i=0;i<n;i++)
		c[i]=set[i]=0;
	for (i=n-1;i>=0;i--)
		if (!c[i]&&(pre[i]==-1||!set[pre[i]])){
			if (pre[i]!=-1){
				set[pre[i]]=1;
				c[pre[i]]=1;
				if (pre[pre[i]]!=-1)
					c[pre[pre[i]]]=1;
			}
			else
				set[i]=1;
			ret++;
		}
	return ret;
}
```

10.4 拓扑排序(邻接阵)

```
//拓扑排序,邻接阵形式,复杂度O(n^2)
//如果无法完成排序,返回0,否则返回1,ret返回有序点列
//传入图的大小n和邻接阵mat,不相邻点边权0
#define MAXN 100

int toposort(int n,int mat[][MAXN],int* ret){
	int d[MAXN],i,j,k;
	for (i=0;i<n;i++)
		for (d[i]=j=0;j<n;d[i]+=mat[j++][i]);
	for (k=0;k<n;ret[k++]=i){
		for (i=0;d[i]&&i<n;i++);
		if (i==n)
```

```
			return 0;
		for (d[i]=-1,j=0;j<n;j++)
			d[j]-=mat[i][j];
	}
	return 1;
}
```

10.5 最佳边割集

```
//最佳边割集
#define MAXN 100
#define inf 1000000000

int max_flow(int n,int mat[][MAXN],int source,int sink){
	int v[MAXN],c[MAXN],p[MAXN],ret=0,i,j;
	for (;;){
		for (i=0;i<n;i++)
			v[i]=c[i]=0;
		for (c[source]=inf;;){
			for (j=-1,i=0;i<n;i++)
				if (!v[i]&&c[i]&&(j==-1||c[i]>c[j]))
					j=i;
			if (j<0) return ret;
			if (j==sink) break;
			for (v[j]=1,i=0;i<n;i++)
				if (mat[j][i]>c[i]&&c[j]>c[i])
					c[i]=mat[j][i]<c[j]?mat[j][i]:c[j],p[i]=j;
		}
		for (ret+=j=c[i=sink];i!=source;i=p[i])
			mat[p[i]][i]-=j,mat[i][p[i]]+=j;
	}
}

int best_edge_cut(int n,int mat[][MAXN],int source,int sink,int set[][2],int& mincost){
	int m0[MAXN][MAXN],m[MAXN][MAXN],i,j,k,l,ret=0,last;
	if (source==sink)
		return -1;
	for (i=0;i<n;i++)
		for (j=0;j<n;j++)
			m0[i][j]=mat[i][j];
	for (i=0;i<n;i++)
		for (j=0;j<n;j++)
			m[i][j]=m0[i][j];
	mincost=last=max_flow(n,m,source,sink);
```

```
	for (k=0;k<n&&last;k++)
		for (l=0;l<n&&last;l++)
			if (m0[k][l]){
				for (i=0;i<n+n;i++)
					for (j=0;j<n+n;j++)
						m[i][j]=m0[i][j];
				m[k][l]=0;
				if (max_flow(n,m,source,sink)==last-mat[k][l]){
					set[ret][0]=k;
					set[ret++][1]=l;
					m0[k][l]=0;
					last-=mat[k][l];
				}
			}
	return ret;
}
```

10.6 最佳点割集

```
//最佳顶点割集
#define MAXN 100
#define inf 1000000000

int max_flow(int n,int mat[][MAXN],int source,int sink){
	int v[MAXN],c[MAXN],p[MAXN],ret=0,i,j;
	for (;;){
		for (i=0;i<n;i++)
			v[i]=c[i]=0;
		for (c[source]=inf;;){
			for (j=-1,i=0;i<n;i++)
				if (!v[i]&&c[i]&&(j==-1||c[i]>c[j]))
					j=i;
			if (j<0) return ret;
			if (j==sink) break;
			for (v[j]=1,i=0;i<n;i++)
				if (mat[j][i]>c[i]&&c[j]>c[i])
					c[i]=mat[j][i]<c[j]?mat[j][i]:c[j],p[i]=j;
		}
		for (ret+=j=c[i=sink];i!=source;i=p[i])
			mat[p[i]][i]-=j,mat[i][p[i]]+=j;
	}
}

int best_vertex_cut(int n,int mat[][MAXN],int* cost,int source,int sink,int* set,int& mincost){
```

```
	int m0[MAXN][MAXN],m[MAXN][MAXN],i,j,k,ret=0,last;
	if (source==sink||mat[source][sink])
		return -1;
	for (i=0;i<n+n;i++)
		for (j=0;j<n+n;j++)
			m0[i][j]=0;
	for (i=0;i<n;i++)
		for (j=0;j<n;j++)
			if (mat[i][j])
				m0[i][n+j]=inf;
	for (i=0;i<n;i++)
		m0[n+i][i]=cost[i];
	for (i=0;i<n+n;i++)
		for (j=0;j<n+n;j++)
			m[i][j]=m0[i][j];
	mincost=last=max_flow(n+n,m,source,n+sink);
	for (k=0;k<n&&last;k++)
		if (k!=source&&k!=sink){
			for (i=0;i<n+n;i++)
				for (j=0;j<n+n;j++)
					m[i][j]=m0[i][j];
			m[n+k][k]=0;
			if (max_flow(n+n,m,source,n+sink)==last-cost[k]){
				set[ret++]=k;
				m0[n+k][k]=0;
				last-=cost[k];
			}
		}
	return ret;
}
```

10.7 最小边割集

```
//最小边割集
#define MAXN 100
#define inf 1000000000

int max_flow(int n,int mat[][MAXN],int source,int sink){
	int v[MAXN],c[MAXN],p[MAXN],ret=0,i,j;
	for (;;){
		for (i=0;i<n;i++)
			v[i]=c[i]=0;
		for (c[source]=inf;;){
			for (j=-1,i=0;i<n;i++)
```

```
				if (!v[i]&&c[i]&&(j==-1||c[i]>c[j]))
					j=i;
			if (j<0) return ret;
			if (j==sink) break;
			for (v[j]=1,i=0;i<n;i++)
				if (mat[j][i]>c[i]&&c[j]>c[i])
					c[i]=mat[j][i]<c[j]?mat[j][i]:c[j],p[i]=j;
		}
		for (ret+=j=c[i=sink];i!=source;i=p[i])
			mat[p[i]][i]-=j,mat[i][p[i]]+=j;
	}
}

int min_edge_cut(int n,int mat[][MAXN],int source,int sink,int set[][2]){
	int m0[MAXN][MAXN],m[MAXN][MAXN],i,j,k,l,ret=0,last;
	if (source==sink)
		return -1;
	for (i=0;i<n;i++)
		for (j=0;j<n;j++)
			m0[i][j]=(mat[i][j]!=0);
	for (i=0;i<n;i++)
		for (j=0;j<n;j++)
			m[i][j]=m0[i][j];
	last=max_flow(n,m,source,sink);
	for (k=0;k<n&&last;k++)
		for (l=0;l<n&&last;l++)
			if (m0[k][l]){
				for (i=0;i<n+n;i++)
					for (j=0;j<n+n;j++)
						m[i][j]=m0[i][j];
				m[k][l]=0;
				if (max_flow(n,m,source,sink)<last){
					set[ret][0]=k;
					set[ret++][1]=l;
					m0[k][l]=0;
					last--;
				}
			}
	return ret;
}
```

10.8 最小点割集

//最小顶点割集

```
#define MAXN 100
#define inf 1000000000

int max_flow(int n,int mat[][MAXN],int source,int sink){
	int v[MAXN],c[MAXN],p[MAXN],ret=0,i,j;
	for (;;){
		for (i=0;i<n;i++)
			v[i]=c[i]=0;
		for (c[source]=inf;;){
			for (j=-1,i=0;i<n;i++)
				if (!v[i]&&c[i]&&(j==-1||c[i]>c[j]))
					j=i;
			if (j<0) return ret;
			if (j==sink) break;
			for (v[j]=1,i=0;i<n;i++)
				if (mat[j][i]>c[i]&&c[j]>c[i])
					c[i]=mat[j][i]<c[j]?mat[j][i]:c[j],p[i]=j;
		}
		for (ret+=j=c[i=sink];i!=source;i=p[i])
			mat[p[i]][i]-=j,mat[i][p[i]]+=j;
	}
}

int min_vertex_cut(int n,int mat[][MAXN],int source,int sink,int* set){
	int m0[MAXN][MAXN],m[MAXN][MAXN],i,j,k,ret=0,last;
	if (source==sink||mat[source][sink])
		return -1;
	for (i=0;i<n+n;i++)
		for (j=0;j<n+n;j++)
			m0[i][j]=0;
	for (i=0;i<n;i++)
		for (j=0;j<n;j++)
			if (mat[i][j])
				m0[i][n+j]=inf;
	for (i=0;i<n;i++)
		m0[n+i][i]=1;
	for (i=0;i<n+n;i++)
		for (j=0;j<n+n;j++)
			m[i][j]=m0[i][j];
	last=max_flow(n+n,m,source,n+sink);
	for (k=0;k<n&&last;k++)
		if (k!=source&&k!=sink){
			for (i=0;i<n+n;i++)
				for (j=0;j<n+n;j++)
```

```
                m[i][j]=m0[i][j];
            m[n+k][k]=0;
            if (max_flow(n+n,m,source,n+sink)<last){
                set[ret++]=k;
                m0[n+k][k]=0;
                last--;
            }
        }
    return ret;
}
```

10.9 最小路径覆盖

```
//最小路径覆盖,O(n^3)
//求解最小的路径覆盖图中所有点,有向图无向图均适用
//注意此问题等价二分图最大匹配,可以用邻接表或正向表减小复杂度
//返回最小路径条数,pre 返回前指针(起点-1),next 返回后指针(终点-1)
#include <string.h>
#define MAXN 310
#define _clr(x) memset(x,0xff,sizeof(int)*n)

int hungary(int n,int mat[][MAXN],int* match1,int* match2){
    int s[MAXN],t[MAXN],p,q,ret=0,i,j,k;
    for (_clr(match1),_clr(match2),i=0;i<n;ret+=(match1[i++]>=0))
        for (_clr(t),s[p=q=0]=i;p<=q&&match1[i]<0;p++)
            for (k=s[p],j=0;j<n&&match1[i]<0;j++)
                if (mat[k][j]&&t[j]<0){
                    s[++q]=match2[j],t[j]=k;
                    if (s[q]<0)
                        for (p=j;p>=0;j=p)
                            match2[j]=k=t[j],p=match1[k],match1[k]=j;
                }
    return ret;
}

inline int path_cover(int n,int mat[][MAXN],int* pre,int* next){
    return n-hungary(n,mat,next,pre);
}
```

11、 图论—支撑树

11.1 最小生成树(kruskal 邻接表)

```
//无向图最小生成树,kruskal 算法,邻接表形式,复杂度 O(mlogm)
//返回最小生成树的长度,传入图的大小 n 和邻接表 list
//可更改边权的类型,edge[][2]返回树的构造,用边集表示
//如果图不连通,则对各连通分支构造最小生成树,返回总长度
#include <string.h>
#define MAXN 200
#define inf 1000000000
typedef double elem_t;
struct edge_t{
	int from,to;
	elem_t len;
	edge_t* next;
};

#define _ufind_run(x) for(;p[t=x];x=p[x],p[t]=(p[x]?p[x]:x))
#define _run_both _ufind_run(i);_ufind_run(j)
struct ufind{
	int p[MAXN],t;
	void init(){memset(p,0,sizeof(p));}
	void set_friend(int i,int j){_run_both;p[i]=(i==j?0:j);}
	int is_friend(int i,int j){_run_both;return i==j&&i;}
};

#define _cp(a,b) ((a).len<(b).len)
struct heap_t{int a,b;elem_t len;};
struct minheap{
	heap_t h[MAXN*MAXN];
	int n,p,c;
	void init(){n=0;}
	void ins(heap_t e){
		for (p=++n;p>1&&_cp(e,h[p>>1]);h[p]=h[p>>1],p>>=1);
		h[p]=e;
	}
	int del(heap_t& e){
		if (!n) return 0;
		for                                        (e=h[p=1],c=2;c<n&&_cp(h[c+=(c<n-
1&&_cp(h[c+1],h[c]))],h[n]);h[p]=h[c],p=c,c<<=1);
		h[p]=h[n--];return 1;
	}
};

elem_t kruskal(int n,edge_t* list[],int edge[][2]){
	ufind u;minheap h;
	edge_t* t;heap_t e;
```

```
	elem_t ret=0;int i,m=0;
	u.init(),h.init();
	for (i=0;i<n;i++)
		for (t=list[i];t;t=t->next)
			if (i<t->to)
				e.a=i,e.b=t->to,e.len=t->len,h.ins(e);
	while (m<n-1&&h.del(e))
		if (!u.is_friend(e.a+1,e.b+1))
			edge[m][0]=e.a,edge[m][1]=e.b,ret+=e.len,u.set_friend(e.a+1,e.b+1);
	return ret;
}
```

11.2 最小生成树(kruskal 正向表)

```
//无向图最小生成树,kruskal 算法,正向表形式,复杂度 O(mlogm)
//返回最小生成树的长度,传入图的大小 n 和正向表 list,buf
//可更改边权的类型,edge[][2]返回树的构造,用边集表示
//如果图不连通,则对各连通分支构造最小生成树,返回总长度
#include <string.h>
#define MAXN 200
#define inf 1000000000
typedef double elem_t;
struct edge_t{
	int to;
	elem_t len;
};

#define _ufind_run(x) for(;p[t=x];x=p[x],p[t]=(p[x]?p[x]:x))
#define _run_both _ufind_run(i);_ufind_run(j)
struct ufind{
	int p[MAXN],t;
	void init(){memset(p,0,sizeof(p));}
	void set_friend(int i,int j){_run_both;p[i]=(i==j?0:j);}
	int is_friend(int i,int j){_run_both;return i==j&&i;}
};

#define _cp(a,b) ((a).len<(b).len)
struct heap_t{int a,b;elem_t len;};
struct minheap{
	heap_t h[MAXN*MAXN];
	int n,p,c;
	void init(){n=0;}
	void ins(heap_t e){
		for (p=++n;p>1&&_cp(e,h[p>>1]);h[p]=h[p>>1],p>>=1);
```

```
		h[p]=e;
	}
	int del(heap_t& e){
		if (!n) return 0;
		for (e=h[p=1],c=2;c<n&&_cp(h[c+=(c<n-1&&_cp(h[c+1],h[c]))],h[n]);h[p]=h[c],p=c,c<<=1);
		h[p]=h[n--];return 1;
	}
};

elem_t kruskal(int n,int* list,edge_t* buf,int edge[][2]){
	ufind u;minheap h;
	heap_t e;elem_t ret=0;
	int i,j,m=0;
	u.init(),h.init();
	for (i=0;i<n;i++)
		for (j=list[i];j<list[i+1];j++)
			if (i<buf[j].to)
				e.a=i,e.b=buf[j].to,e.len=buf[j].len,h.ins(e);
	while (m<n-1&&h.del(e))
		if (!u.is_friend(e.a+1,e.b+1))
			edge[m][0]=e.a,edge[m][1]=e.b,ret+=e.len,u.set_friend(e.a+1,e.b+1);
	return ret;
}
```

11.3 最小生成树(prim+binary_heap 邻接表)

```
//无向图最小生成树,prim 算法+二分堆,邻接表形式,复杂度 O(mlogm)
//返回最小生成树的长度,传入图的大小 n 和邻接表 list
//可更改边权的类型,pre[]返回树的构造,用父结点表示,根节点(第一个)pre 值为-1
//必须保证图的连通的!
#define MAXN 200
#define inf 1000000000
typedef double elem_t;
struct edge_t{
	int from,to;
	elem_t len;
	edge_t* next;
};

#define _cp(a,b) ((a).d<(b).d)
struct heap_t{elem_t d;int v;};
struct heap{
	heap_t h[MAXN*MAXN];
```

```
	int n,p,c;
	void init(){n=0;}
	void ins(heap_t e){
		for (p=++n;p>1&&_cp(e,h[p>>1]);h[p]=h[p>>1],p>>=1);
		h[p]=e;
	}
	int del(heap_t& e){
		if (!n) return 0;
		for (e=h[p=1],c=2;c<n&&_cp(h[c+=(c<n-1&&_cp(h[c+1],h[c]))],h[n]);h[p]=h[c],p=c,c<<=1);
		h[p]=h[n--];return 1;
	}
};

elem_t prim(int n,edge_t* list[],int* pre){
	heap h;
	elem_t min[MAXN],ret=0;
	edge_t* t;heap_t e;
	int v[MAXN],i;
	for (i=0;i<n;i++)
		min[i]=inf,v[i]=0,pre[i]=-1;
	h.init();e.v=0,e.d=0,h.ins(e);
	while (h.del(e))
		if (!v[e.v])
			for (v[e.v]=1,ret+=e.d,t=list[e.v];t;t=t->next)
				if (!v[t->to]&&t->len<min[t->to])
					pre[t->to]=t->from,min[e.v=t->to]=e.d=t->len,h.ins(e);
	return ret;
}
```

11.4 最小生成树(prim+binary_heap 正向表)

```
//无向图最小生成树,prim 算法+二分堆,正向表形式,复杂度 O(mlogm)
//返回最小生成树的长度,传入图的大小 n 和正向表 list,buf
//可更改边权的类型,pre[]返回树的构造,用父结点表示,根节点(第一个)pre 值为-1
//必须保证图的连通的!
#define MAXN 200
#define inf 1000000000
typedef double elem_t;
struct edge_t{
	int to;
	elem_t len;
};
```

```
#define _cp(a,b) ((a).d<(b).d)
struct heap_t{elem_t d;int v;};
struct heap{
	heap_t h[MAXN*MAXN];
	int n,p,c;
	void init(){n=0;}
	void ins(heap_t e){
		for (p=++n;p>1&&_cp(e,h[p>>1]);h[p]=h[p>>1],p>>=1);
		h[p]=e;
	}
	int del(heap_t& e){
		if (!n) return 0;
		for (e=h[p=1],c=2;c<n&&_cp(h[c+=(c<n-1&&_cp(h[c+1],h[c]))],h[n]);h[p]=h[c],p=c,c<<=1);
		h[p]=h[n--];return 1;
	}
};

elem_t prim(int n,int* list,edge_t* buf,int* pre){
	heap h;heap_t e;
	elem_t min[MAXN],ret=0;
	int v[MAXN],i,j;
	for (i=0;i<n;i++)
		min[i]=inf,v[i]=0,pre[i]=-1;
	h.init();e.v=0,e.d=0,h.ins(e);
	while (h.del(e))
		if (!v[i=e.v])
			for (v[i]=1,ret+=e.d,j=list[i];j<list[i+1];j++)
				if (!v[buf[j].to]&&buf[j].len<min[buf[j].to])
					pre[buf[j].to]=i,min[e.v=buf[j].to]=e.d=buf[j].len,h.ins(e);
	return ret;
}
```

11.5 最小生成树(prim+mapped_heap 邻接表)

```
//无向图最小生成树,prim 算法+映射二分堆,邻接表形式,复杂度 O(mlogn)
//返回最小生成树的长度,传入图的大小 n 和邻接表 list
//可更改边权的类型,pre[]返回树的构造,用父结点表示,根节点(第一个)pre 值为-1
//必须保证图的连通的!
#define MAXN 200
#define inf 1000000000
typedef double elem_t;
struct edge_t{
	int from,to;
```

```
	elem_t len;
	edge_t* next;
};

#define _cp(a,b) ((a)<(b))
struct heap{
	elem_t h[MAXN+1];
	int ind[MAXN+1],map[MAXN+1],n,p,c;
	void init(){n=0;}
	void ins(int i,elem_t e){
		for (p=++n;p>1&&_cp(e,h[p>>1]);h[map[ind[p]=ind[p>>1]]=p]=h[p>>1],p>>=1);
		h[map[ind[p]=i]=p]=e;
	}
	int del(int i,elem_t& e){
		i=map[i];if (i<1||i>n) return 0;
		for (e=h[p=i];p>1;h[map[ind[p]=ind[p>>1]]=p]=h[p>>1],p>>=1);
		for (c=2;c<n&&_cp(h[c+=(c<n-1&&_cp(h[c+1],h[c]))],h[n]);h[map[ind[p]=ind[c]]=p]=h[c],p=c,c<<=1);
		h[map[ind[p]=ind[n]]=p]=h[n];n--;return 1;
	}
	int delmin(int& i,elem_t& e){
		if (n<1) return 0;i=ind[1];
		for (e=h[p=1],c=2;c<n&&_cp(h[c+=(c<n-1&&_cp(h[c+1],h[c]))],h[n]);h[map[ind[p]=ind[c]]=p]=h[c],p=c,c<<=1);
		h[map[ind[p]=ind[n]]=p]=h[n];n--;return 1;
	}
};

elem_t prim(int n,edge_t* list[],int* pre){
	heap h;
	elem_t min[MAXN],ret=0,e;
	edge_t* t;
	int v[MAXN],i;
	for (h.init(),i=0;i<n;i++)
		min[i]=(i?inf:0),v[i]=0,pre[i]=-1,h.ins(i,min[i]);
	while (h.delmin(i,e))
		for (v[i]=1,ret+=e,t=list[i];t;t=t->next)
			if (!v[t->to]&&t->len<min[t->to])
				pre[t->to]=t->from,h.del(t->to,e),h.ins(t->to,min[t->to]=t->len);
	return ret;
}
```

11.6 最小生成树(prim+mapped_heap 正向表)

```
//无向图最小生成树,prim 算法+映射二分堆,正向表形式,复杂度 O(mlogn)
//返回最小生成树的长度,传入图的大小 n 和正向表 list,buf
//可更改边权的类型,pre[]返回树的构造,用父结点表示,根节点(第一个)pre 值为-1
//必须保证图的连通的!
#define MAXN 200
#define inf 1000000000
typedef double elem_t;
struct edge_t{
	int to;
	elem_t len;
};

#define _cp(a,b) ((a)<(b))
struct heap{
	elem_t h[MAXN+1];
	int ind[MAXN+1],map[MAXN+1],n,p,c;
	void init(){n=0;}
	void ins(int i,elem_t e){
		for (p=++n;p>1&&_cp(e,h[p>>1]);h[map[ind[p]=ind[p>>1]]=p]=h[p>>1],p>>=1);
		h[map[ind[p]=i]=p]=e;
	}
	int del(int i,elem_t& e){
		i=map[i];if (i<1||i>n) return 0;
		for (e=h[p=i];p>1;h[map[ind[p]=ind[p>>1]]=p]=h[p>>1],p>>=1);
		for (c=2;c<n&&_cp(h[c+=(c<n-1&&_cp(h[c+1],h[c]))],h[n]);h[map[ind[p]=ind[c]]=p]=h[c],p=c,c<<=1);
		h[map[ind[p]=ind[n]]=p]=h[n];n--;return 1;
	}
	int delmin(int& i,elem_t& e){
		if (n<1) return 0;i=ind[1];
		for (e=h[p=1],c=2;c<n&&_cp(h[c+=(c<n-1&&_cp(h[c+1],h[c]))],h[n]);h[map[ind[p]=ind[c]]=p]=h[c],p=c,c<<=1);
		h[map[ind[p]=ind[n]]=p]=h[n];n--;return 1;
	}
};

elem_t prim(int n,int* list,edge_t* buf,int* pre){
	heap h;
	elem_t min[MAXN],ret=0,e;
	int v[MAXN],i,j;
	for (h.init(),i=0;i<n;i++)
		min[i]=(i?inf:0),v[i]=0,pre[i]=-1,h.ins(i,min[i]);
	while (h.delmin(i,e))
		for (v[i]=1,ret+=e,j=list[i];j<list[i+1];j++)
```

```
			if (!v[buf[j].to]&&buf[j].len<min[buf[j].to])
				pre[buf[j].to]=i,h.del(buf[j].to,e),h.ins(buf[j].to,min[buf[j].to]=buf[j].len);
	return ret;
}
```

11.7 最小生成树(prim 邻接阵)

```
//无向图最小生成树,prim 算法,邻接阵形式,复杂度 O(n^2)
//返回最小生成树的长度,传入图的大小 n 和邻接阵 mat,不相邻点边权 inf
//可更改边权的类型,pre[]返回树的构造,用父结点表示,根节点(第一个)pre 值为-1
//必须保证图的连通的!
#define MAXN 200
#define inf 1000000000
typedef double elem_t;

elem_t prim(int n,elem_t mat[][MAXN],int* pre){
	elem_t min[MAXN],ret=0;
	int v[MAXN],i,j,k;
	for (i=0;i<n;i++)
		min[i]=inf,v[i]=0,pre[i]=-1;
	for (min[j=0]=0;j<n;j++){
		for (k=-1,i=0;i<n;i++)
			if (!v[i]&&(k==-1||min[i]<min[k]))
				k=i;
		for (v[k]=1,ret+=min[k],i=0;i<n;i++)
			if (!v[i]&&mat[k][i]<min[i])
				min[i]=mat[pre[i]=k][i];
	}
	return ret;
}
```

11.8 最小树形图(邻接阵)

```
//多源最小树形图,edmonds 算法,邻接阵形式,复杂度 O(n^3)
//返回最小生成树的长度,构造失败返回负值
//传入图的大小 n 和邻接阵 mat,不相邻点边权 inf
//可更改边权的类型,pre[]返回树的构造,用父结点表示
//传入时 pre[]数组清零,用-1 标出源点
#include <string.h>
#define MAXN 120
#define inf 1000000000
typedef int elem_t;
```

```
elem_t edmonds(int n,elem_t mat[][MAXN*2],int* pre){
	elem_t ret=0;
	int c[MAXN*2][MAXN*2],l[MAXN*2],p[MAXN*2],m=n,t,i,j,k;
	for (i=0;i<n;l[i]=i,i++);
	do{
		memset(c,0,sizeof(c)),memset(p,0xff,sizeof(p));
		for (t=m,i=0;i<m;c[i][i]=1,i++);
		for (i=0;i<t;i++)
			if (l[i]==i&&pre[i]!=-1){
				for (j=0;j<m;j++)
					if (l[j]==j&&i!=j&&mat[j][i]<inf&&(p[i]==-1||mat[j][i]<mat[p[i]][i]))
						p[i]=j;
				if ((pre[i]=p[i])==-1)
					return -1;
				if (c[i][p[i]]){
					for (j=0;j<=m;mat[j][m]=mat[m][j]=inf,j++);
					for (k=i;l[k]!=m;l[k]=m,k=p[k])
						for (j=0;j<m;j++)
							if (l[j]==j){
								if (mat[j][k]-mat[p[k]][k]<mat[j][m])
									mat[j][m]=mat[j][k]-mat[p[k]][k];
								if (mat[k][j]<mat[m][j])
									mat[m][j]=mat[k][j];
							}
					c[m][m]=1,l[m]=m,m++;
				}
				for (j=0;j<m;j++)
					if (c[i][j])
						for (k=p[i];k!=-1&&l[k]==k;c[k][j]=1,k=p[k]);
			}
	}
	while (t<m);
	for (;m-->n;pre[k]=pre[m])
		for (i=0;i<m;i++)
			if (l[i]==m){
				for (j=0;j<m;j++)
					if (pre[j]==m&&mat[i][j]==mat[m][j])
						pre[j]=i;
				if (mat[pre[m]][m]==mat[pre[m]][i]-mat[pre[i]][i])
					k=i;
			}
	for (i=0;i<n;i++)
		if (pre[i]!=-1)
			ret+=mat[pre[i]][i];
```

```
    return ret;
}
```

12、 图论—最短路径

12.1 最短路径(单源 bellman_ford 邻接阵)

```
//单源最短路径,bellman_ford 算法,邻接阵形式,复杂度 O(n^3)
//求出源 s 到所有点的最短路径,传入图的大小 n 和邻接阵 mat
//返回到各点最短距离 min[]和路径 pre[],pre[i]记录 s 到 i 路径上 i 的父结点,pre[s]=-1
//可更改路权类型,路权可为负,若图包含负环则求解失败,返回 0
//优化:先删去负边使用 dijkstra 求出上界,加速迭代过程
#define MAXN 200
#define inf 1000000000
typedef int elem_t;

int bellman_ford(int n,elem_t mat[][MAXN],int s,elem_t* min,int* pre){
    int v[MAXN],i,j,k,tag;
    for (i=0;i<n;i++)
        min[i]=inf,v[i]=0,pre[i]=-1;
    for (min[s]=0,j=0;j<n;j++){
        for (k=-1,i=0;i<n;i++)
            if (!v[i]&&(k==-1||min[i]<min[k]))
                k=i;
        for (v[k]=1,i=0;i<n;i++)
            if (!v[i]&&mat[k][i]>=0&&min[k]+mat[k][i]<min[i])
                min[i]=min[k]+mat[pre[i]=k][i];
    }
    for (tag=1,j=0;tag&&j<=n;j++)
        for (tag=i=0;i<n;i++)
            for (k=0;k<n;k++)
                if (min[k]+mat[k][i]<min[i])
                    min[i]=min[k]+mat[pre[i]=k][i],tag=1;
    return j<=n;
}
```

12.2 最短路径(单源 dijkstra+bfs 邻接表)

```
//单源最短路径,用于路权相等的情况,dijkstra 优化为 bfs,邻接表形式,复杂度 O(m)
//求出源 s 到所有点的最短路径,传入图的大小 n 和邻接表 list,边权值 len
//返回到各点最短距离 min[]和路径 pre[],pre[i]记录 s 到 i 路径上 i 的父结点,pre[s]=-1
//可更改路权类型,但必须非负且相等!
```

```
#define MAXN 200
#define inf 1000000000
typedef int elem_t;
struct edge_t{
	int from,to;
	edge_t* next;
};

void dijkstra(int n,edge_t* list[],elem_t len,int s,elem_t* min,int* pre){
	edge_t* t;
	int i,que[MAXN],f=0,r=0,p=1,l=1;
	for (i=0;i<n;i++)
		min[i]=inf;
	min[que[0]=s]=0,pre[s]=-1;
	for (;r<=f;l++,r=f+1,f=p-1)
		for (i=r;i<=f;i++)
			for (t=list[que[i]];t;t=t->next)
				if (min[t->to]==inf)
					min[que[p++]=t->to]=len*l,pre[t->to]=que[i];
}
```

12.3 最短路径(单源 dijkstra+bfs 正向表)

```
//单源最短路径,用于路权相等的情况,dijkstra 优化为 bfs,正向表形式,复杂度 O(m)
//求出源 s 到所有点的最短路经,传入图的大小 n 和正向表 list,buf,边权值 len
//返回到各点最短距离 min[]和路径 pre[],pre[i]记录 s 到 i 路径上 i 的父结点,pre[s]=-1
//可更改路权类型,但必须非负且相等!
#define MAXN 200
#define inf 1000000000
typedef int elem_t;

void dijkstra(int n,int* list,int* buf,elem_t len,int s,elem_t* min,int* pre){
	int i,que[MAXN],f=0,r=0,p=1,l=1,t;
	for (i=0;i<n;i++)
		min[i]=inf;
	min[que[0]=s]=0,pre[s]=-1;
	for (;r<=f;l++,r=f+1,f=p-1)
		for (i=r;i<=f;i++)
			for (t=list[que[i]];t<list[que[i]+1];t++)
				if (min[buf[t]]==inf)
					min[que[p++]=buf[t]]=len*l,pre[buf[t]]=que[i];
}
```

12.4 最短路径(单源 dijkstra+binary_heap 邻接表)

```
//单源最短路径,dijkstra 算法+二分堆,邻接表形式,复杂度 O(mlogm)
//求出源 s 到所有点的最短路径,传入图的大小 n 和邻接表 list
//返回到各点最短距离 min[]和路径 pre[],pre[i]记录 s 到 i 路径上 i 的父结点,pre[s]=-1
//可更改路权类型,但必须非负!
#define MAXN 200
#define inf 1000000000
typedef int elem_t;
struct edge_t{
	int from,to;
	elem_t len;
	edge_t* next;
};

#define _cp(a,b) ((a).d<(b).d)
struct heap_t{elem_t d;int v;};
struct heap{
	heap_t h[MAXN*MAXN];
	int n,p,c;
	void init(){n=0;}
	void ins(heap_t e){
		for (p=++n;p>1&&_cp(e,h[p>>1]);h[p]=h[p>>1],p>>=1);
		h[p]=e;
	}
	int del(heap_t& e){
		if (!n) return 0;
		for                                        (e=h[p=1],c=2;c<n&&_cp(h[c+=(c<n-
1&&_cp(h[c+1],h[c]))],h[n]);h[p]=h[c],p=c,c<<=1);
		h[p]=h[n--];return 1;
	}
};

void dijkstra(int n,edge_t* list[],int s,elem_t* min,int* pre){
	heap h;
	edge_t* t;heap_t e;
	int v[MAXN],i;
	for (i=0;i<n;i++)
		min[i]=inf,v[i]=0,pre[i]=-1;
	h.init();min[e.v=s]=e.d=0,h.ins(e);
	while (h.del(e))
		if (!v[e.v])
			for (v[e.v]=1,t=list[e.v];t;t=t->next)
				if (!v[t->to]&&min[t->from]+t->len<min[t->to])
```

```
                    pre[t->to]=t->from,min[e.v=t->to]=e.d=min[t->from]+t->len,h.ins(e);
}
```

12.5 最短路径(单源 dijkstra+binary_heap 正向表)

```
//单源最短路径,dijkstra 算法+二分堆,正向表形式,复杂度 O(mlogm)
//求出源 s 到所有点的最短路经,传入图的大小 n 和正向表 list,buf
//返回到各点最短距离 min[]和路径 pre[],pre[i]记录 s 到 i 路径上 i 的父结点,pre[s]=-1
//可更改路权类型,但必须非负!
#define MAXN 200
#define inf 1000000000
typedef int elem_t;
struct edge_t{
	int to;
	elem_t len;
};

#define _cp(a,b) ((a).d<(b).d)
struct heap_t{elem_t d;int v;};
struct heap{
	heap_t h[MAXN*MAXN];
	int n,p,c;
	void init(){n=0;}
	void ins(heap_t e){
		for (p=++n;p>1&&_cp(e,h[p>>1]);h[p]=h[p>>1],p>>=1);
		h[p]=e;
	}
	int del(heap_t& e){
		if (!n) return 0;
		for                                    (e=h[p=1],c=2;c<n&&_cp(h[c+=(c<n-
1&&_cp(h[c+1],h[c]))],h[n]);h[p]=h[c],p=c,c<<=1);
		h[p]=h[n--];return 1;
	}
};

void dijkstra(int n,int* list,edge_t* buf,int s,elem_t* min,int* pre){
	heap h;heap_t e;
	int v[MAXN],i,t,f;
	for (i=0;i<n;i++)
		min[i]=inf,v[i]=0,pre[i]=-1;
	h.init();min[e.v=s]=e.d=0,h.ins(e);
	while (h.del(e))
		if (!v[e.v])
			for (v[f=e.v]=1,t=list[f];t<list[f+1];t++)
```

```
			if (!v[buf[t].to]&&min[f]+buf[t].len<min[buf[t].to])
				pre[buf[t].to]=f,min[e.v=buf[t].to]=e.d=min[f]+buf[t].len,h.ins(e);
}
```

12.6 最短路径(单源 dijkstra+mapped_heap 邻接表)

```
//单源最短路径,dijkstra 算法+映射二分堆,邻接表形式,复杂度 O(mlogn)
//求出源 s 到所有点的最短路经,传入图的大小 n 和邻接表 list
//返回到各点最短距离 min[]和路径 pre[],pre[i]记录 s 到 i 路径上 i 的父结点,pre[s]=-1
//可更改路权类型,但必须非负!
#define MAXN 200
#define inf 1000000000
typedef int elem_t;
struct edge_t{
	int from,to;
	elem_t len;
	edge_t* next;
};

#define _cp(a,b) ((a)<(b))
struct heap{
	elem_t h[MAXN+1];
	int ind[MAXN+1],map[MAXN+1],n,p,c;
	void init(){n=0;}
	void ins(int i,elem_t e){
		for (p=++n;p>1&&_cp(e,h[p>>1]);h[map[ind[p]=ind[p>>1]]=p]=h[p>>1],p>>=1);
		h[map[ind[p]=i]=p]=e;
	}
	int del(int i,elem_t& e){
		i=map[i];if (i<1||i>n) return 0;
		for (e=h[p=i];p>1;h[map[ind[p]=ind[p>>1]]=p]=h[p>>1],p>>=1);
		for                                            (c=2;c<n&&_cp(h[c+=(c<n-
1&&_cp(h[c+1],h[c]))],h[n]);h[map[ind[p]=ind[c]]=p]=h[c],p=c,c<<=1);
		h[map[ind[p]=ind[n]]=p]=h[n];n--;return 1;
	}
	int delmin(int& i,elem_t& e){
		if (n<1) return 0;i=ind[1];
		for                                   (e=h[p=1],c=2;c<n&&_cp(h[c+=(c<n-
1&&_cp(h[c+1],h[c]))],h[n]);h[map[ind[p]=ind[c]]=p]=h[c],p=c,c<<=1);
		h[map[ind[p]=ind[n]]=p]=h[n];n--;return 1;
	}
};

void dijkstra(int n,edge_t* list[],int s,elem_t* min,int* pre){
```

```
	heap h;
	edge_t* t;elem_t e;
	int v[MAXN],i;
	for (h.init(),i=0;i<n;i++)
		min[i]=((i==s)?0:inf),v[i]=0,pre[i]=-1,h.ins(i,min[i]);
	while (h.delmin(i,e))
		for (v[i]=1,t=list[i];t;t=t->next)
			if (!v[t->to]&&min[i]+t->len<min[t->to])
				pre[t->to]=i,h.del(t->to,e),min[t->to]=e=min[i]+t->len,h.ins(t->to,e);
}
```

12.7 最短路径(单源 dijkstra+mapped_heap 正向表)

```
//单源最短路径,dijkstra 算法+映射二分堆,正向表形式,复杂度 O(mlogn)
//求出源 s 到所有点的最短路经,传入图的大小 n 和正向表 list,buf
//返回到各点最短距离 min[]和路径 pre[],pre[i]记录 s 到 i 路径上 i 的父结点,pre[s]=-1
//可更改路权类型,但必须非负!
#define MAXN 200
#define inf 1000000000
typedef int elem_t;
struct edge_t{
	int to;
	elem_t len;
};

#define _cp(a,b) ((a)<(b))
struct heap{
	elem_t h[MAXN+1];
	int ind[MAXN+1],map[MAXN+1],n,p,c;
	void init(){n=0;}
	void ins(int i,elem_t e){
		for (p=++n;p>1&&_cp(e,h[p>>1]);h[map[ind[p]=ind[p>>1]]=p]=h[p>>1],p>>=1);
		h[map[ind[p]=i]=p]=e;
	}
	int del(int i,elem_t& e){
		i=map[i];if (i<1||i>n) return 0;
		for (e=h[p=i];p>1;h[map[ind[p]=ind[p>>1]]=p]=h[p>>1],p>>=1);
		for                                              (c=2;c<n&&_cp(h[c+=(c<n-
1&&_cp(h[c+1],h[c]))],h[n]);h[map[ind[p]=ind[c]]=p]=h[c],p=c,c<<=1);
		h[map[ind[p]=ind[n]]=p]=h[n];n--;return 1;
	}
	int delmin(int& i,elem_t& e){
		if (n<1) return 0;i=ind[1];
		for                                    (e=h[p=1],c=2;c<n&&_cp(h[c+=(c<n-
```

```
1&&_cp(h[c+1],h[c]))],h[n]);h[map[ind[p]=ind[c]]=p]=h[c],p=c,c<<=1);
        h[map[ind[p]=ind[n]]=p]=h[n];n--;return 1;
    }
};

void dijkstra(int n,int* list,edge_t* buf,int s,elem_t* min,int* pre){
    heap h;elem_t e;
    int v[MAXN],i,t;
    for (h.init(),i=0;i<n;i++)
        min[i]=((i==s)?0:inf),v[i]=0,pre[i]=-1,h.ins(i,min[i]);
    while (h.delmin(i,e))
        for (v[i]=1,t=list[i];t<list[i+1];t++)
            if (!v[buf[t].to]&&min[i]+buf[t].len<min[buf[t].to])

    pre[buf[t].to]=i,h.del(buf[t].to,e),min[buf[t].to]=e=min[i]+buf[t].len,h.ins(buf[t].to,e);
}
```

12.8 最短路径(单源 dijkstra 邻接阵)

```
//单源最短路径,dijkstra 算法,邻接阵形式,复杂度 O(n^2)
//求出源 s 到所有点的最短路经,传入图的顶点数 n,(有向)邻接矩阵 mat
//返回到各点最短距离 min[]和路径 pre[],pre[i]记录 s 到 i 路径上 i 的父结点,pre[s]=-1
//可更改路权类型,但必须非负!
#define MAXN 200
#define inf 1000000000
typedef int elem_t;

void dijkstra(int n,elem_t mat[][MAXN],int s,elem_t* min,int* pre){
    int v[MAXN],i,j,k;
    for (i=0;i<n;i++)
        min[i]=inf,v[i]=0,pre[i]=-1;
    for (min[s]=0,j=0;j<n;j++){
        for (k=-1,i=0;i<n;i++)
            if (!v[i]&&(k==-1||min[i]<min[k]))
                k=i;
        for (v[k]=1,i=0;i<n;i++)
            if (!v[i]&&min[k]+mat[k][i]<min[i])
                min[i]=min[k]+mat[pre[i]=k][i];
    }
}
```

12.9 最短路径(多源 floyd_warshall 邻接阵)

```
//多源最短路径,floyd_warshall 算法,复杂度 O(n^3)
//求出所有点对之间的最短路径,传入图的大小和邻接阵
//返回各点间最短距离 min[]和路径 pre[],pre[i][j]记录 i 到 j 最短路径上 j 的父结点
//可更改路权类型,路权必须非负!
#define MAXN 200
#define inf 1000000000
typedef int elem_t;

void floyd_warshall(int n,elem_t mat[][MAXN],elem_t min[][MAXN],int pre[][MAXN]){
	int i,j,k;
	for (i=0;i<n;i++)
		for (j=0;j<n;j++)
			min[i][j]=mat[i][j],pre[i][j]=(i==j)?-1:i;
	for (k=0;k<n;k++)
		for (i=0;i<n;i++)
			for (j=0;j<n;j++)
				if (min[i][k]+min[k][j]<min[i][j])
					min[i][j]=min[i][k]+min[k][j],pre[i][j]=pre[k][j];
}
```

13、 应用

13.1 Joseph 问题

```
// Joseph's Problem
// input: n,m          -- the number of persons, the inteval between persons
// output:        -- return the reference of last person

int josephus0(int n, int m)
{
	if (n == 2) return (m%2) ? 2 : 1;
	int v = (m+josephus0(n-1,m)) % n;
	if (v == 0) v = n;
	return v;
}
int josephus(int n, int m)
{
	if (m == 1) return n;
	if (n == 1) return 1;
	if (m >=n) return josephus0(n,m);
	int l = (n/m)*m;
	int j = josephus(n - (n/m), m);
	if (j <= n-l) return l+j;
```

```
	j -= n-l;
	int t = (j/(m-1))*m;
	if ((j % (m-1)) == 0) return t-1;
	return t + (j % (m-1));
}
```

13.2 N 皇后构造解

```
//N 皇后构造解,n>=4

void even1(int n,int *p){
	int i;
	for (i=1;i<=n/2;i++)
		p[i-1]=2*i;
	for (i=n/2+1;i<=n;i++)
		p[i-1]=2*i-n-1;
}

void even2(int n,int *p){
	int i;
	for (i=1;i<=n/2;i++)
		p[i-1]=(2*i+n/2-3)%n+1;
	for (i=n/2+1;i<=n;i++)
		p[i-1]=n-(2*(n-i+1)+n/2-3)%n;
}

void generate(int,int*);
void odd(int n,int *p){
	generate(n-1,p),p[n-1]=n;
}

void generate(int n,int *p){
	if (n&1)
		odd(n,p);
	else if (n%6!=2)
		even1(n,p);
	else
		even2(n,p);
}
```

13.3 布尔母函数

```
//布尔母函数
```

```
//判 m[]个价值为 w[]的货币能否构成 value
//适合 m[]较大 w[]较小的情况
//返回布尔量
//传入货币种数 n,个数 m[],价值 w[]和目标值 value
#define MAXV 100000

int genfunc(int n,int* m,int* w,int value){
	int i,j,k,c;
	char r[MAXV];
	for (r[0]=i=1;i<=value;r[i++]=0);
	for (i=0;i<n;i++){
		for (j=0;j<w[i];j++){
			c=m[i]*r[k=j];
			while ((k+=w[i])<=value)
				if (r[k])
					c=m[i];
				else if (c)
					r[k]=1,c--;
			if (r[value])
				return 1;
		}
	}
	return 0;
}
```

13.4 第 k 元素

```
//取第 k 个元素,k=0..n-1
//平均复杂度 O(n)
//注意 a[]中的顺序被改变

#define _cp(a,b) ((a)<(b))
typedef int elem_t;
elem_t kth_element(int n,elem_t* a,int k){
	elem_t t,key;
	int l=0,r=n-1,i,j;
	while (l<r){
		for (key=a[((i=l-1)+(j=r+1))>>1];i<j;){
			for (j--;_cp(key,a[j]);j--);
			for (i++;_cp(a[i],key);i++);
			if (i<j) t=a[i],a[i]=a[j],a[j]=t;
		}
		if (k>j) l=j+1;
		else r=j;
```

```
	}
	return a[k];
}
```

13.5 幻方构造

```
//幻方构造(l!=2)
#define MAXN 100

void dllb(int l,int si,int sj,int sn,int d[][MAXN]){
	int n,i=0,j=l/2;
	for (n=1;n<=l*l;n++){
		d[i+si][j+sj]=n+sn;
		if (n%l){
			i=(i)?(i-1):(l-1);
			j=(j==l-1)?0:(j+1);
		}
		else
			i=(i==l-1)?0:(i+1);
	}
}

void magic_odd(int l,int d[][MAXN]){
	dllb(l,0,0,0,d);
}

void magic_4k(int l,int d[][MAXN]){
	int i,j;
	for (i=0;i<l;i++)
		for (j=0;j<l;j++)

	d[i][j]=((i%4==0||i%4==3)&&(j%4==0||j%4==3)||(i%4==1||i%4==2)&&(j%4==1||j%4==2))?
(l*l-(i*l+j)):(i*l+j+1);
}

void magic_other(int l,int d[][MAXN]){
	int i,j,t;
	dllb(l/2,0,0,0,d);
	dllb(l/2,l/2,l/2,l*l/4,d);
	dllb(l/2,0,l/2,l*l/2,d);
	dllb(l/2,l/2,0,l*l/4*3,d);
	for (i=0;i<l/2;i++)
		for (j=0;j<l/4;j++)
			if (i!=l/4||j)
```

```
				t=d[i][j],d[i][j]=d[i+l/2][j],d[i+l/2][j]=t;
	t=d[l/4][l/4],d[l/4][l/4]=d[l/4+l/2][l/4],d[l/4+l/2][l/4]=t;
	for (i=0;i<l/2;i++)
		for (j=l-l/4+1;j<l; j++)
			t=d[i][j],d[i][j]=d[i+l/2][j],d[i+l/2][j]=t;
}

void generate(int l,int d[][MAXN]){
	if (l%2)
		magic_odd(l,d);
	else if (l%4==0)
		magic_4k(l,d);
	else
		magic_other(l,d);
}
```

13.6 模式匹配(kmp)

```
//模式匹配,kmp 算法,复杂度 O(m+n)
//返回匹配位置,-1 表示匹配失败,传入匹配串和模式串和长度
//可更改元素类型,更换匹配函数
#define MAXN 10000
#define _match(a,b) ((a)==(b))
typedef char elem_t;

int pat_match(int ls,elem_t* str,int lp,elem_t* pat){
	int fail[MAXN]={-1},i=0,j;
	for (j=1;j<lp;j++){
		for (i=fail[j-1];i>=0&&!_match(pat[i+1],pat[j]);i=fail[i]);
		fail[j]=(_match(pat[i+1],pat[j])?i+1:-1);
	}
	for (i=j=0;i<ls&&j<lp;i++)
		if (_match(str[i],pat[j]))
			j++;
		else if (j)
			j=fail[j-1]+1,i--;
	return j==lp?(i-lp):-1;
}
```

13.7 逆序对数

```
//序列逆序对数,复杂度 O(nlogn)
//传入序列长度和内容,返回逆序对数
```

```
//可更改元素类型和比较函数
#include <string.h>
#define MAXN 1000000
#define _cp(a,b) ((a)<=(b))
typedef int elem_t;
elem_t _tmp[MAXN];

int inv(int n,elem_t* a){
	int l=n>>1,r=n-l,i,j;
	int ret=(r>1?(inv(l,a)+inv(r,a+l)):0);
	for (i=j=0;i<=l;_tmp[i+j]=a[i],i++)
		for (ret+=j;j<r&&(i==l||!_cp(a[i],a[l+j]));_tmp[i+j]=a[l+j],j++);
	memcpy(a,_tmp,sizeof(elem_t)*n);
	return ret;
}
```

13.8 字符串最小表示

```
/*
	求字符串的最小表示
	输入：字符串
	返回：字符串最小表示的首字母位置(0...size-1)
*/
template <class T>
int MinString(vector <T> &str)
{
	int i, j, k;
	vector <T> ss(str.size() << 1);
	for (i = 0; i < str.size(); i ++) ss[i] = ss[i + str.size()] = str[i];
	for (i = k = 0, j = 1; k < str.size() && i < str.size() && j < str.size(); ) {
		for (k = 0; k < str.size() && ss[i + k] == ss[j + k]; k ++);
		if (k < str.size()) {
			if (ss[i + k] > ss[j + k])
				i += k + 1;
			else j += k + 1;
			if (i == j) j ++;
		}
	}
	return i < j ? i : j;
}
```

13.9 最长公共单调子序列

```
// 最长公共递增子序列， 时间复杂度 O(n^2 * logn)，空间 O(n^2)

/**
 * n 为 a 的大小, m 为 b 的大小
 * 结果在 ans 中
 * "define _cp(a,b) ((a)<(b))"求解最长严格递增序列
 */
#define MAXN 1000
#define _cp(a,b) ((a)<(b))
typedef int elem_t;

elem_t DP[MAXN][MAXN];
int num[MAXN], p[1<<20];

int LIS(int n, elem_t *a, int m, elem_t *b, elem_t *ans){
	int i, j, l, r, k;

	DP[0][0] = 0;
	num[0] = (b[0] == a[0]);
	for(i = 1; i < m; i++) {
		num[i] = (b[i] == a[0]) || num[i-1];
		DP[i][0] = 0;
	}
	for(i = 1; i < n; i++){
		if(b[0] == a[i] && !num[0]) {
			num[0] = 1;
			DP[0][0] = i<<10;
		}

		for(j = 1; j < m; j++){
			for(k=((l=0)+(r=num[j-1]-1))>>1; l<=r; k=(l+r)>>1)
				if(_cp(a[DP[j-1][k]>>10], a[i]))
					l=k+1;
				else
					r=k-1;

			if(l < num[j-1] && i == (DP[j-1][l]>>10) ){
				if(l >= num[j]) DP[j][num[j]++] = DP[j-1][l];
				else DP[j][l] = _cp(a[DP[j][l]>>10],a[i]) ? DP[j][l] : DP[j-1][l];
			}
			if(b[j] == a[i]){
				for(k=((l=0)+(r=num[j]-1))>>1; l<=r; k=(l+r)>>1)
					if(_cp(a[DP[j][k]>>10], a[i]))
						l=k+1;
```

```
				else
					r=k-1;

			DP[j][l] = (i<<10) + j;
			num[j] += (l>=num[j]);
			p[DP[j][l]] = l ? DP[j][l-1] : -1;
		}
	}
}

for (k=DP[m-1][i=num[m-1]-1];i>=0;ans[i--]=a[k>>10],k=p[k]);
return num[m-1];
}
```

13.10 最长子序列

```
//最长单调子序列,复杂度 O(nlogn)
//注意最小序列覆盖和最长序列的对应关系,例如
//"define _cp(a,b) ((a)>(b))"求解最长严格递减序列,则
//"define _cp(a,b) (!((a)>(b)))"求解最小严格递减序列覆盖
//可更改元素类型和比较函数
#define MAXN 10000
#define _cp(a,b) ((a)>(b))
typedef int elem_t;

int subseq(int n,elem_t* a){
	int b[MAXN],i,l,r,m,ret=0;
	for (i=0;i<n;b[l]=i++,ret+=(l>ret))
		for (m=((l=1)+(r=ret))>>1;l<=r;m=(l+r)>>1)
			if (_cp(a[b[m]],a[i]))
				l=m+1;
			else
				r=m-1;
	return ret;
}

int subseq(int n,elem_t* a,elem_t* ans){
	int b[MAXN],p[MAXN],i,l,r,m,ret=0;
	for (i=0;i<n;p[b[l]=i++]=b[l-1],ret+=(l>ret))
		for (m=((l=1)+(r=ret))>>1;l<=r;m=(l+r)>>1)
			if (_cp(a[b[m]],a[i]))
				l=m+1;
			else
				r=m-1;
```

```
	for (m=b[i=ret];i;ans[--i]=a[m],m=p[m]);
	return ret;
}
```

13.11 最大子串匹配

```
//最大子串匹配,复杂度 O(mn)
//返回最大匹配值,传入两个串和串的长度,重载返回一个最大匹配
//注意做字符串匹配是串末的'\0'没有置!
//可更改元素类型,更换匹配函数和匹配价值函数
#include <string.h>
#define MAXN 100
#define max(a,b) ((a)>(b)?(a):(b))
#define _match(a,b) ((a)==(b))
#define _value(a,b) 1
typedef char elem_t;

int str_match(int m,elem_t* a,int n,elem_t* b){
	int match[MAXN+1][MAXN+1],i,j;
	memset(match,0,sizeof(match));
	for (i=0;i<m;i++)
		for (j=0;j<n;j++)
			match[i+1][j+1]=max(max(match[i][j+1],match[i+1][j]),
								(match[i][j]+_value(a[i],b[i]))*_match(a[i],b[j]));
	return match[m][n];
}

int str_match(int m,elem_t* a,int n,elem_t* b,elem_t* ret){
	int match[MAXN+1][MAXN+1],last[MAXN+1][MAXN+1],i,j,t;
	memset(match,0,sizeof(match));
	for (i=0;i<m;i++)
		for (j=0;j<n;j++){
			match[i+1][j+1]=(match[i][j+1]>match[i+1][j]?match[i][j+1]:match[i+1][j]);
			last[i+1][j+1]=(match[i][j+1]>match[i+1][j]?3:1);
			if ((t=(match[i][j]+_value(a[i],b[i]))*_match(a[i],b[j]))>match[i+1][j+1])
				match[i+1][j+1]=t,last[i+1][j+1]=2;
		}
	for (;match[i][j];i-=(last[t=i][j]>1),j-=(last[t][j]<3))
		ret[match[i][j]-1]=(last[i][j]<3?a[i-1]:b[j-1]);
	return match[m][n];
}
```

13.12 最大子段和

```
//求最大子段和,复杂度 O(n)
//传入串长 n 和内容 list[]
//返回最大子段和,重载返回子段位置(maxsum=list[start]+...+list[end])
//可更改元素类型
typedef int elem_t;

elem_t maxsum(int n,elem_t* list){
	elem_t ret,sum=0;
	int i;
	for (ret=list[i=0];i<n;i++)
		sum=(sum>0?sum:0)+list[i],ret=(sum>ret?sum:ret);
	return ret;
}

elem_t maxsum(int n,elem_t* list,int& start,int& end){
	elem_t ret,sum=0;
	int s,i;
	for (ret=list[start=end=s=i=0];i<n;i++,s=(sum>0?s:i))
		if ((sum=(sum>0?sum:0)+list[i])>ret)
			ret=sum,start=s,end=i;
	return ret;
}
```

13.13 最大子阵和

```
//求最大子阵和,复杂度 O(n^3)
//传入阵的大小 m,n 和内容 mat[][]
//返回最大子阵和,重载返回子阵位置(maxsum=list[s1][s2]+...+list[e1][e2])
//可更改元素类型
#define MAXN 100
typedef int elem_t;

elem_t maxsum(int m,int n,elem_t mat[][MAXN]){
	elem_t matsum[MAXN][MAXN+1],ret,sum;
	int i,j,k;
	for (i=0;i<m;i++)
		for (matsum[i][j=0]=0;j<n;j++)
			matsum[i][j+1]=matsum[i][j]+mat[i][j];
	for (ret=mat[0][j=0];j<n;j++)
		for (k=j;k<n;k++)
			for (sum=0,i=0;i<m;i++)
				sum=(sum>0?sum:0)+matsum[i][k+1]-matsum[i][j],ret=(sum>ret?sum:ret);
	return ret;
}
```

```
elem_t maxsum(int m,int n,elem_t mat[][MAXN],int& s1,int& s2,int& e1,int& e2){
	elem_t matsum[MAXN][MAXN+1],ret,sum;
	int i,j,k,s;
	for (i=0;i<m;i++)
		for (matsum[i][j=0]=0;j<n;j++)
			matsum[i][j+1]=matsum[i][j]+mat[i][j];
	for (ret=mat[s1=e1=0][s2=e2=j=0];j<n;j++)
		for (k=j;k<n;k++)
			for (sum=0,s=i=0;i<m;i++,s=(sum>0?s:i))
				if ((sum=(sum>0?sum:0)+matsum[i][k+1]-matsum[i][j])>ret)
					ret=sum,s1=s,s2=i,e1=j,e2=k;
	return ret;
}
```

14、 其它

14.1 大数(只能处理正数)

```
#include <iostream.h>
#include <string.h>

#define DIGIT4
#define DEPTH	10000
#define MAX	100
typedef int bignum_t[MAX+1];

int read(bignum_t a,istream& is=cin){
	char buf[MAX*DIGIT+1],ch;
	int i,j;
	memset((void*)a,0,sizeof(bignum_t));
	if (!(is>>buf)) return 0;
	for (a[0]=strlen(buf),i=a[0]/2-1;i>=0;i--)
		ch=buf[i],buf[i]=buf[a[0]-1-i],buf[a[0]-1-i]=ch;
	for (a[0]=(a[0]+DIGIT-1)/DIGIT,j=strlen(buf);j<a[0]*DIGIT;buf[j++]='0');
	for (i=1;i<=a[0];i++)
		for (a[i]=0,j=0;j<DIGIT;j++)
			a[i]=a[i]*10+buf[i*DIGIT-1-j]-'0';
	for (;!a[a[0]]&&a[0]>1;a[0]--);
	return 1;
}

void write(const bignum_t a,ostream& os=cout){
```

```
	int i,j;
	for (os<<a[i=a[0]],i--;i;i--)
		for (j=DEPTH/10;j;j/=10)
			os<<a[i]/j%10;
}

int comp(const bignum_t a,const bignum_t b){
	int i;
	if (a[0]!=b[0])
		return a[0]-b[0];
	for (i=a[0];i;i--)
		if (a[i]!=b[i])
			return a[i]-b[i];
	return 0;
}

int comp(const bignum_t a,const int b){
	int c[12]={1};
	for (c[1]=b;c[c[0]]>=DEPTH;c[c[0]+1]=c[c[0]]/DEPTH,c[c[0]]%=DEPTH,c[0]++);
	return comp(a,c);
}

int comp(const bignum_t a,const int c,const int d,const bignum_t b){
	int i,t=0,O=-DEPTH*2;
	if (b[0]-a[0]<d&&c)
		return 1;
	for (i=b[0];i>d;i--){
		t=t*DEPTH+a[i-d]*c-b[i];
		if (t>0) return 1;
		if (t<O) return 0;
	}
	for (i=d;i;i--){
		t=t*DEPTH-b[i];
		if (t>0) return 1;
		if (t<O) return 0;
	}
	return t>0;
}

void add(bignum_t a,const bignum_t b){
	int i;
	for (i=1;i<=b[0];i++)
		if ((a[i]+=b[i])>=DEPTH)
			a[i]-=DEPTH,a[i+1]++;
```

```
	if (b[0]>=a[0])
		a[0]=b[0];
	else
		for (;a[i]>=DEPTH&&i<a[0];a[i]-=DEPTH,i++,a[i]++);
	a[0]+=(a[a[0]+1]>0);
}

void add(bignum_t a,const int b){
	int i=1;
	for (a[1]+=b;a[i]>=DEPTH&&i<a[0];a[i+1]+=a[i]/DEPTH,a[i]%=DEPTH,i++);
	for (;a[a[0]]>=DEPTH;a[a[0]+1]=a[a[0]]/DEPTH,a[a[0]]%=DEPTH,a[0]++);
}

void sub(bignum_t a,const bignum_t b){
	int i;
	for (i=1;i<=b[0];i++)
		if ((a[i]-=b[i])<0)
			a[i+1]--,a[i]+=DEPTH;
	for (;a[i]<0;a[i]+=DEPTH,i++,a[i]--);
	for (;!a[a[0]]&&a[0]>1;a[0]--);
}

void sub(bignum_t a,const int b){
	int i=1;
	for                                         (a[1]-=b;a[i]<0;a[i+1]+=(a[i]-DEPTH+1)/DEPTH,a[i]-=(a[i]-
DEPTH+1)/DEPTH*DEPTH,i++);
	for (;!a[a[0]]&&a[0]>1;a[0]--);
}

void sub(bignum_t a,const bignum_t b,const int c,const int d){
	int i,O=b[0]+d;
	for (i=1+d;i<=O;i++)
		if ((a[i]-=b[i-d]*c)<0)
			a[i+1]+=(a[i]-DEPTH+1)/DEPTH,a[i]-=(a[i]-DEPTH+1)/DEPTH*DEPTH;
	for (;a[i]<0;a[i+1]+=(a[i]-DEPTH+1)/DEPTH,a[i]-=(a[i]-DEPTH+1)/DEPTH*DEPTH,i++);
	for (;!a[a[0]]&&a[0]>1;a[0]--);
}

void mul(bignum_t c,const bignum_t a,const bignum_t b){
	int i,j;
	memset((void*)c,0,sizeof(bignum_t));
	for (c[0]=a[0]+b[0]-1,i=1;i<=a[0];i++)
		for (j=1;j<=b[0];j++)
			if ((c[i+j-1]+=a[i]*b[j])>=DEPTH)
```

```
                    c[i+j]+=c[i+j-1]/DEPTH,c[i+j-1]%=DEPTH;
        for (c[0]+=(c[c[0]+1]>0);!c[c[0]]&&c[0]>1;c[0]--);
}

void mul(bignum_t a,const int b){
        int i;
        for (a[1]*=b,i=2;i<=a[0];i++){
                a[i]*=b;
                if (a[i-1]>=DEPTH)
                        a[i]+=a[i-1]/DEPTH,a[i-1]%=DEPTH;
        }
        for (;a[a[0]]>=DEPTH;a[a[0]+1]=a[a[0]]/DEPTH,a[a[0]]%=DEPTH,a[0]++);
        for (;!a[a[0]]&&a[0]>1;a[0]--);
}

void mul(bignum_t b,const bignum_t a,const int c,const int d){
        int i;
        memset((void*)b,0,sizeof(bignum_t));
        for (b[0]=a[0]+d,i=d+1;i<=b[0];i++)
                if ((b[i]+=a[i-d]*c)>=DEPTH)
                        b[i+1]+=b[i]/DEPTH,b[i]%=DEPTH;
        for (;b[b[0]+1];b[0]++,b[b[0]+1]=b[b[0]]/DEPTH,b[b[0]]%=DEPTH);
        for (;!b[b[0]]&&b[0]>1;b[0]--);
}

void div(bignum_t c,bignum_t a,const bignum_t b){
        int h,l,m,i;
        memset((void*)c,0,sizeof(bignum_t));
        c[0]=(b[0]<a[0]+1)?(a[0]-b[0]+2):1;
        for (i=c[0];i;sub(a,b,c[i]=m,i-1),i--)
                for (h=DEPTH-1,l=0,m=(h+l+1)>>1;h>l;m=(h+l+1)>>1)
                        if (comp(b,m,i-1,a)) h=m-1;
                        else l=m;
        for (;!c[c[0]]&&c[0]>1;c[0]--);
        c[0]=c[0]>1?c[0]:1;
}

void div(bignum_t a,const int b,int& c){
        int i;
        for (c=0,i=a[0];i;c=c*DEPTH+a[i],a[i]=c/b,c%=b,i--);
        for (;!a[a[0]]&&a[0]>1;a[0]--);
}

void sqrt(bignum_t b,bignum_t a){
```

```
	int h,l,m,i;
	memset((void*)b,0,sizeof(bignum_t));
	for (i=b[0]=(a[0]+1)>>1;i;sub(a,b,m,i-1),b[i]+=m,i--)
		for (h=DEPTH-1,l=0,b[i]=m=(h+l+1)>>1;h>l;b[i]=m=(h+l+1)>>1)
			if (comp(b,m,i-1,a)) h=m-1;
			else l=m;
	for (;!b[b[0]]&&b[0]>1;b[0]--);
	for (i=1;i<=b[0];b[i++]>>=1);
}

int length(const bignum_t a){
	int t,ret;
	for (ret=(a[0]-1)*DIGIT,t=a[a[0]];t;t/=10,ret++);
	return ret>0?ret:1;
}

int digit(const bignum_t a,const int b){
	int i,ret;
	for (ret=a[(b-1)/DIGIT+1],i=(b-1)%DIGIT;i;ret/=10,i--);
	return ret%10;
}

int zeronum(const bignum_t a){
	int ret,t;
	for (ret=0;!a[ret+1];ret++);
	for (t=a[ret+1],ret*=DIGIT;!(t%10);t/=10,ret++);
	return ret;
}

void comp(int* a,const int l,const int h,const int d){
	int i,j,t;
	for (i=l;i<=h;i++)
		for (t=i,j=2;t>1;j++)
			while (!(t%j))
				a[j]+=d,t/=j;
}

void convert(int* a,const int h,bignum_t b){
	int i,j,t=1;
	memset(b,0,sizeof(bignum_t));
	for (b[0]=b[1]=1,i=2;i<=h;i++)
		if (a[i])
			for (j=a[i];j;t*=i,j--)
				if (t*i>DEPTH)
```

```
					mul(b,t),t=1;
	mul(b,t);
}

void combination(bignum_t a,int m,int n){
	int* t=new int[m+1];
	memset((void*)t,0,sizeof(int)*(m+1));
	comp(t,n+1,m,1);
	comp(t,2,m-n,-1);
	convert(t,m,a);
	delete []t;
}

void permutation(bignum_t a,int m,int n){
	int i,t=1;
	memset(a,0,sizeof(bignum_t));
	a[0]=a[1]=1;
	for (i=m-n+1;i<=m;t*=i++)
		if (t*i>DEPTH)
			mul(a,t),t=1;
	mul(a,t);
}

#define SGN(x) ((x)>0?1:((x)<0?-1:0))
#define ABS(x) ((x)>0?(x):-(x))

int read(bignum_t a,int &sgn,istream& is=cin){
	char str[MAX*DIGIT+2],ch,*buf;
	int i,j;
	memset((void*)a,0,sizeof(bignum_t));
	if (!(is>>str)) return 0;
	buf=str,sgn=1;
	if (*buf=='-') sgn=-1,buf++;
	for (a[0]=strlen(buf),i=a[0]/2-1;i>=0;i--)
		ch=buf[i],buf[i]=buf[a[0]-1-i],buf[a[0]-1-i]=ch;
	for (a[0]=(a[0]+DIGIT-1)/DIGIT,j=strlen(buf);j<a[0]*DIGIT;buf[j++]='0');
	for (i=1;i<=a[0];i++)
		for (a[i]=0,j=0;j<DIGIT;j++)
			a[i]=a[i]*10+buf[i*DIGIT-1-j]-'0';
	for (;!a[a[0]]&&a[0]>1;a[0]--);
	if (a[0]==1&&!a[1]) sgn=0;
	return 1;
}
```

14.2 分数

```
struct frac{
	int num,den;
};

double fabs(double x){
	return x>0?x:-x;
}

int gcd(int a,int b){
	int t;
	if (a<0)
		a=-a;
	if (b<0)
		b=-b;
	if (!b)
		return a;
	while (t=a%b)
		a=b,b=t;
	return b;
}

void simplify(frac& f){
	int t;
	if (t=gcd(f.num,f.den))
		f.num/=t,f.den/=t;
	else
		f.den=1;
}

frac f(int n,int d,int s=1){
	frac ret;
	if (d<0)
		ret.num=-n,ret.den=-d;
	else
		ret.num=n,ret.den=d;
	if (s)
		simplify(ret);
	return ret;
}

frac convert(double x){
	frac ret;
```

```
	for (ret.den=1;fabs(x-int(x))>1e-10;ret.den*=10,x*=10);
	ret.num=(int)x;
	simplify(ret);
	return ret;
}

int fraqcmp(frac a,frac b){
	int g1=gcd(a.den,b.den),g2=gcd(a.num,b.num);
	if (!g1||!g2)
		return 0;
	return b.den/g1*(a.num/g2)-a.den/g1*(b.num/g2);
}

frac add(frac a,frac b){
	int g1=gcd(a.den,b.den),g2,t;
	if (!g1)
		return f(1,0,0);
	t=b.den/g1*a.num+a.den/g1*b.num;
	g2=gcd(g1,t);
	return f(t/g2,a.den/g1*(b.den/g2),0);
}

frac sub(frac a,frac b){
	return add(a,f(-b.num,b.den,0));
}

frac mul(frac a,frac b){
	int t1=gcd(a.den,b.num),t2=gcd(a.num,b.den);
	if (!t1||!t2)
		return f(1,1,0);
	return f(a.num/t2*(b.num/t1),a.den/t1*(b.den/t2),0);
}

frac div(frac a,frac b){
	return mul(a,f(b.den,b.num,0));
}
```

14.3 矩阵

```
define MAXN 100

#define fabs(x) ((x)>0?(x):-(x))
#define zero(x) (fabs(x)<1e-10)
```

```
struct mat{
	int n,m;
	double data[MAXN][MAXN];
};

int mul(mat& c,const mat& a,const mat& b){
	int i,j,k;
	if (a.m!=b.n)
		return 0;
	c.n=a.n,c.m=b.m;
	for (i=0;i<c.n;i++)
		for (j=0;j<c.m;j++)
			for (c.data[i][j]=k=0;k<a.m;k++)
				c.data[i][j]+=a.data[i][k]*b.data[k][j];
	return 1;
}

int inv(mat& a){
	int i,j,k,is[MAXN],js[MAXN];
	double t;
	if (a.n!=a.m)
		return 0;
	for (k=0;k<a.n;k++){
		for (t=0,i=k;i<a.n;i++)
			for (j=k;j<a.n;j++)
				if (fabs(a.data[i][j])>t)
					t=fabs(a.data[is[k]=i][js[k]=j]);
		if (zero(t))
			return 0;
		if (is[k]!=k)
			for (j=0;j<a.n;j++)
				t=a.data[k][j],a.data[k][j]=a.data[is[k]][j],a.data[is[k]][j]=t;
		if (js[k]!=k)
			for (i=0;i<a.n;i++)
				t=a.data[i][k],a.data[i][k]=a.data[i][js[k]],a.data[i][js[k]]=t;
		a.data[k][k]=1/a.data[k][k];
		for (j=0;j<a.n;j++)
			if (j!=k)
				a.data[k][j]*=a.data[k][k];
		for (i=0;i<a.n;i++)
			if (i!=k)
				for (j=0;j<a.n;j++)
					if (j!=k)
						a.data[i][j]-=a.data[i][k]*a.data[k][j];
```

```
		for (i=0;i<a.n;i++)
			if (i!=k)
				a.data[i][k]*=-a.data[k][k];
	}
	for (k=a.n-1;k>=0;k--){
		for (j=0;j<a.n;j++)
			if (js[k]!=k)
				t=a.data[k][j],a.data[k][j]=a.data[js[k]][j],a.data[js[k]][j]=t;
		for (i=0;i<a.n;i++)
			if (is[k]!=k)
				t=a.data[i][k],a.data[i][k]=a.data[i][is[k]],a.data[i][is[k]]=t;
	}
	return 1;
}

double det(const mat& a){
	int i,j,k,sign=0;
	double b[MAXN][MAXN],ret=1,t;
	if (a.n!=a.m)
		return 0;
	for (i=0;i<a.n;i++)
		for (j=0;j<a.m;j++)
			b[i][j]=a.data[i][j];
	for (i=0;i<a.n;i++){
		if (zero(b[i][i])){
			for (j=i+1;j<a.n;j++)
				if (!zero(b[j][i]))
					break;
			if (j==a.n)
				return 0;
			for (k=i;k<a.n;k++)
				t=b[i][k],b[i][k]=b[j][k],b[j][k]=t;
			sign++;
		}
		ret*=b[i][i];
		for (k=i+1;k<a.n;k++)
			b[i][k]/=b[i][i];
		for (j=i+1;j<a.n;j++)
			for (k=i+1;k<a.n;k++)
				b[j][k]-=b[j][i]*b[i][k];
	}
	if (sign&1)
		ret=-ret;
	return ret;
```

```
}
```

14.4 线性方程组

```
#define MAXN 100
#define fabs(x) ((x)>0?(x):-(x))
#define eps 1e-10

//列主元 gauss 消去求解 a[][]x[]=b[]
//返回是否有唯一解,若有解在 b[]中
int gauss_cpivot(int n,double a[][MAXN],double b[]){
	int i,j,k,row;
	double maxp,t;
	for (k=0;k<n;k++){
		for (maxp=0,i=k;i<n;i++)
			if (fabs(a[i][k])>fabs(maxp))
				maxp=a[row=i][k];
		if (fabs(maxp)<eps)
			return 0;
		if (row!=k){
			for (j=k;j<n;j++)
				t=a[k][j],a[k][j]=a[row][j],a[row][j]=t;
			t=b[k],b[k]=b[row],b[row]=t;
		}
		for (j=k+1;j<n;j++){
			a[k][j]/=maxp;
			for (i=k+1;i<n;i++)
				a[i][j]-=a[i][k]*a[k][j];
		}
		b[k]/=maxp;
		for (i=k+1;i<n;i++)
			b[i]-=b[k]*a[i][k];
	}
	for (i=n-1;i>=0;i--)
		for (j=i+1;j<n;j++)
			b[i]-=a[i][j]*b[j];
	return 1;
}

//全主元 gauss 消去解 a[][]x[]=b[]
//返回是否有唯一解,若有解在 b[]中
int gauss_tpivot(int n,double a[][MAXN],double b[]){
	int i,j,k,row,col,index[MAXN];
	double maxp,t;
```

```
    for (i=0;i<n;i++)
        index[i]=i;
    for (k=0;k<n;k++){
        for (maxp=0,i=k;i<n;i++)
            for (j=k;j<n;j++)
                if (fabs(a[i][j])>fabs(maxp))
                    maxp=a[row=i][col=j];
        if (fabs(maxp)<eps)
            return 0;
        if (col!=k){
            for (i=0;i<n;i++)
                t=a[i][col],a[i][col]=a[i][k],a[i][k]=t;
            j=index[col],index[col]=index[k],index[k]=j;
        }
        if (row!=k){
            for (j=k;j<n;j++)
                t=a[k][j],a[k][j]=a[row][j],a[row][j]=t;
            t=b[k],b[k]=b[row],b[row]=t;
        }
        for (j=k+1;j<n;j++){
            a[k][j]/=maxp;
            for (i=k+1;i<n;i++)
                a[i][j]-=a[i][k]*a[k][j];
        }
        b[k]/=maxp;
        for (i=k+1;i<n;i++)
            b[i]-=b[k]*a[i][k];
    }
    for (i=n-1;i>=0;i--)
        for (j=i+1;j<n;j++)
            b[i]-=a[i][j]*b[j];
    for (k=0;k<n;k++)
        a[0][index[k]]=b[k];
    for (k=0;k<n;k++)
        b[k]=a[0][k];
    return 1;
}
```

14.5 线性相关

```
//判线性相关(正交化)
//传入 m 个 n 维向量
#include <math.h>
#define MAXN 100
```

```
#define eps 1e-10

int linear_dependent(int m,int n,double vec[][MAXN]){
	double ort[MAXN][MAXN],e;
	int i,j,k;
	if (m>n)
		return 1;
	for (i=0;i<m;i++){
		for (j=0;j<n;j++)
			ort[i][j]=vec[i][j];
		for (k=0;k<i;k++){
			for (e=j=0;j<n;j++)
				e+=ort[i][j]*ort[k][j];
			for (j=0;j<n;j++)
				ort[i][j]-=e*ort[k][j];
			for (e=j=0;j<n;j++)
				e+=ort[i][j]*ort[i][j];
			if (fabs(e=sqrt(e))<eps)
				return 1;
			for (j=0;j<n;j++)
				ort[i][j]/=e;
		}
	}
	return 0;
}
```

14.6 日期

```
//日期函数
int days[12]={31,28,31,30,31,30,31,31,30,31,30,31};
struct Date{
  int year, month, day;
};

//判闰年
inline int leap(int year){
  return (year%4==0&&year%100!=0)||year%400==0;
}

//判合法性
inline int legal(Date a){
  if(a.month<0||a.month>12)
    return 0;
  if(a.month==2)
    return a.day>0 && a.day<=28+leap(a.year);
  return a.day>0 && a.day<=days[a.month-1];
}
```

```
//比较日期大小
inline int datecmp(Date a, Date b){
   if(a.year != b.year)
      return a.year - b.year;
   if(a.month != b.month)
      return a.month - b.month;
   return a.day - b.day;
}

//返回指定日期是星期几
int weekday(Date a){
  int tm = a.month>=3 ? (a.month-2) : (a.month+10);
  int ty = a.month>=3 ? a.year : (a.year-1);
  return (ty+ty/4-ty/100+ty/400+(int)(2.6*tm-0.2)+a.day)%7;
}

//日期转天数偏移
int date2int(Date a){
  int ret=a.year*365+(a.year-1)/4-(a.year-1)/100+(a.year-1)/400;
  days[1]+=leap(a.year);
  for(int i=0; i<a.month-1; ret+=days[i++]);
  days[1]=28;
  return ret+a.day;
}

//天数偏移转日期
Date int2date(int a){
  Date ret;
  ret.year = a/146097*400;
  for(a%=146097; a>=365+leap(ret.year); a-=365+leap(ret.year),ret.year++);
  days[1] += leap(ret.year);
  for(ret.month=1; a>=days[ret.month-1]; a-=days[ret.month-1],ret.month++);
  days[1]=28;
  ret.day=a+1;
  return ret;
}
```