

## [报告]SGU Matrix Multiplication

[Source]

<http://acm.sgu.ru/problem.php?contest=0&problem=196>

[Description]

Let us consider an undirected graph  $G = \langle V, E \rangle$  which has  $N$  vertices and  $M$  edges. Incidence matrix of this graph is an  $N \times M$  matrix  $A = \{a_{ij}\}$ , such that  $a_{ij}$  is 1 if  $i$ -th vertex is one of the ends of  $j$ -th edge and 0 in the other case. Your task is to find the sum of all elements of the matrix  $A^T A$  where  $A^T$  is  $A$  transposed, i.e. an  $M \times N$  matrix obtained from  $A$  by turning its columns to rows and vice versa.

[Solution]

大家都 A 了，我就先说一下当时的想法。第一个，暴力  $N^3$ ，直接否定；然后想优化，没想起来…。最后找规律，题目要求求矩阵乘，而且该矩阵是点与边的关联矩阵，于是想到应该是与顶点的度有关系。按照一次的关系没找到，二次平方刚好符合，然后自己造的 sample 也过了，就水了一下。

以下是参考某牛博客的证明， $ans = \sum d[i]^2$ ;

$A$  是图  $G$  的关联矩阵，求  $B = A^T A$  的元素和。

对于图的关联矩阵，有性质：

$$ans = \sum_{i,j} B_{ij} = \sum_{i,j} \sum_k A_{ki} A_{kj} = \sum_k \sum_{i,j} A_{ki} A_{kj} = \sum_{k=0}^n d(v_k)^2$$

$(0 \leq k < n; 0 \leq i, j < m)$

其中  $d(v_k)$  为定点  $k$  的度数。复杂度为  $O(n+m)$ 。

[Code]

```
#include<iostream>
#include<cstdio>
#include<string.h>
#include<string>
#include<math.h>
#include<algorithm>
```

```
#include<queue>
using namespace std;

int a[100009];
int m,n;

int main() {
    int u,v;
    long long ans = 0;
    scanf("%d%d",&n,&m);
    for(int i=1;i<=m;i++) {
        scanf("%d%d",&u,&v);
        a[u]++;
        a[v]++;
    }
    for(int i=1;i<=n;i++) {
        ans+=a[i]*a[i];
    }
    printf("%lld\n",ans);
    return 0;
}
```