

# ASSIGNMENT 1

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## 1. Reduction of the N-queens problem to SAT

Code for the reduction of the N-queens to SAT problem is available in *src/reduce\_nq\_sat.py*.

## 2. Reduction of the dominating set problem to SAT

### Description

Input: Graph  $G = \{V, E\}$  and integer  $k$ .

Problem: Does in graph  $G$  contains a dominating set  $\{DS\}$  of size  $\geq k$ ?

Variables:  $x_i^r$  is true if node  $i$  is the  $r$ -th node of the dominating set, where  $1 \leq i \leq n$ ,  $1 \leq r \leq k$  and  $n = |V|$ .

Clauses:

1. Some node is the  $r$ -th node of the  $\{DS\}$ :

For each  $r$ :  $x_1^r \vee x_2^r \vee \dots \vee x_n^r$

2. No node is both the  $r$ -th and  $s$ -th node of the  $\{DS\}$ :

For each  $i$ ,  $r < s$ :  $\neg x_i^r \vee \neg x_i^s$

3. Only one node can be the  $r$ -th node of the  $\{DS\}$ :

For each  $r$ ,  $i < j$ :  $\neg x_i^r \vee \neg x_j^r$

4. At least one of the  $i$ -th node or one of its neighbour nodes must be in the  $\{DS\}$ :

For each  $i$  and its neighbours<sup>1</sup>:  $(x_i^1 \vee x_{adj(i,1)}^1 \vee x_{adj(i,2)}^1 \vee \dots \vee x_{adj(i,m)}^1) \vee \dots \vee (x_i^r \vee x_{adj(i,1)}^r \vee x_{adj(i,2)}^r \vee \dots \vee x_{adj(i,m)}^r)$

### Code

Code for the reduction of the dominating set to SAT problem is available in *src/reduce\_ds\_sat.py*.

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<sup>1</sup>Node  $adj(i, m)$  is the  $m$ -th neighbour of the  $i$ -th node.

## Solutions

We have manually performed search with bisection and found the following sizes of the minimal dominating sets:

1. *g1.col*: 40
2. *g2.col*: 3
3. *g3.col*: 15
4. *g4.col*: Not found any solution
5. *g5.col*: 5

## 3. Other

We have also implemented reduction of the vertex cover (*src/reduce\_vc\_sat.py*) and clique problem (*src/reduce\_clique\_sat.py*) the to SAT.