Earthquake Nowcasting in Taiwan: Analysis of Interevent Count Distribution

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Introduction

What is an Earthquake?

An earthquake is a sudden shaking of the ground which occurs when two tectonics plates suddenly slide against each other. The space between the plates or the surface where they come into contact or slide is called the fault or the fault plane. The location below the surface where the earthquake begins is called the hypocenter and its corresponding point on the earth surface is called the epicenter. Sometimes an earthquake has fore-shocks, which are smaller earthquakes that occur in the same place as the larger earthquake that follows. The larger earthquake is called the 'mainshock'. The mainshocks are usually followed by aftershocks. The size of the earthquake is called the magnitude which is measured on the Richter scale.

Differences between Forecasting, Prediction and Nowcasting

Earthquake Prediction is a branch of seismology for determining when and where an earthquake will occur along with how intense it will be. It is empirical in nature and usually relies on observations for a short-term prediction. An earthquake must define three elements, the date, the location and the intensity or magnitude of the earthquake. It determines the parameters for the next major earthquake. However, there is not enough scientific evidence to prove an earthquake.[1]

On the other hand, a **forecast** is a probabilistic statement which estimates the likelihood of an earthquake based on data factors such as the frequency of occurrence and the magnitude of earthquakes in a given area over many years. It usually employs timeseries analysis and we apply trend methods to forecast.

Earthquake nowcasting is a method to evaluate the current state of seismic hazard from large earthquakes.[2] It uses the count of the number of small earthquakes since the last large earthquake in a defined region (in our case Taiwan) to estimate the current hazard level in the region. While forecasting is the calculation of probabilities for the future, nowcasting is the calculation of present state of the system.

We will be applying the method of Earthquake Nowcasting to the country of Taiwan in order to estimate the current state of seismic hazard from large earthquakes by calculating the **Earthquake Potential Score** (EPS).

Is Taiwan prone to Earthquakes?

Taiwan lies near the junction of two tectonic plates in the South China Sea on the circum-Pacific seismic zone. Earthquake occurrence here is quite common and strong earthquakes also occur frequently. According to the data provided by the Central Weather Bureau (CWB) seismic network, in the years till 1990 the average number of earthquakes was 2200 annually out of which 214 were felt. Geologists have identified 42 active faults on the island but most of the earthquakes detected in Taiwan are due to the convergence of the Philippine sea plate and the Eurasian Plate to the east of the island.[3]

Preprocessing the Available Dataset

The dataset that we utilized for this report basically constituted of the various earthquakes that have occurred in Taiwan with respect to the following conditions: **latitude 21ºN-26ºN, longitude 120ºE-122ºE and time period 1963-2020.** The software Python (Jupyter Notebook) has been utilized to filter out the unnecessary data and extract the useful data. First, the dataset had been imported to the software where all the large earthquakes (M≥6) were extracted from the dataset, as shown in Table 1.

The interevent counts (Natural Time) for each large earthquake were found out and then added to this dataset. The **interevent counts** are described as the cumulative counts of "small" events (4.0≤M<6.0) between two successive "large" earthquakes (say, M≥6), as shown in Table 2.

	Date (mm/dd/yr)	Time	Lat	Lon	Depth	Mag
0	1964-01-18	12:04:40	23.1000	120.5000	33.0	6.8
1	1965-05-17	17:19:25.900000	22.5000	121.3000	21.0	6.2
2	1970-11-14	07:58:19.800000	22.7110	121.3440	28.0	6.1
3	1972-04-24	09:57:21.700000	23.6380	121.5510	33.0	6.9
4	1972-09-22	19:57:27.400000	22.3310	121.1970	33.0	6.2
...	...	...	...	...	...	...
38	2013-06-02	05:43:03.800000	23.8318	121.0590	17.0	6.1
39	2013-10-31	12:02:09.350000	23.5436	121.4580	14.8	6.1
40	2016-02-05	19:57:27	22.9064	120.6061	18.9	6.2
41	2018-02-06	15:50:43.040000	24.1140	121.7269	12.9	6.1
42	2019-04-18	05:01:06.490000	24.0374	121.6501	20.0	6.3

43 rows × 6 columns

Table 1: Large Earthquake Dataset

	Date (mm/dd/yr)	Time	Lat	Lon	Depth	Mag	Intervent Counts
0	1964-01-18	12:04:40	23.1000	120.5000	33.0	6.8	10
1	1965-05-17	17:19:25.900000	22.5000	121.3000	21.0	6.2	24
2	1970-11-14	07:58:19.800000	22.7110	121.3440	28.0	6.1	81
3	1972-04-24	09:57:21.700000	23.6380	121.5510	33.0	6.9	22
4	1972-09-22	19:57:27.400000	22.3310	121.1970	33.0	6.2	19
...	...	...	...	...	...	...	...
39	2013-10-31	12:02:09.350000	23.5436	121.4580	14.8	6.1	17
40	2016-02-05	19:57:27	22.9064	120.6061	18.9	6.2	128
41	2018-02-06	15:50:43.040000	24.1140	121.7269	12.9	6.1	126
42	2019-04-18	05:01:06.490000	24.0374	121.6501	20.0	6.3	109
43	NaT	NaN	NaN	NaN	NaN	NaN	83

44 rows × 7 columns

Table 2: Dataset with Interevent Counts

All the large earthquakes that had a zero interevent count were discarded from the dataset. A final preprocessed dataset is given in Table 3 with all the relevant information required. The last value in the dataset (83 interevent counts) has null values because this is the intervent count for the upcoming earthquake which has not occurred yet.

	Date (mm/dd/yr)	Time	Lat	Lon	Depth	Mag	Intervent Counts
0	1964-01-18	12:04:40	23.1000	120.5000	33.0	6.8	10
1	1965-05-17	17:19:25.900000	22.5000	121.3000	21.0	6.2	24
2	1970-11-14	07:58:19.800000	22.7110	121.3440	28.0	6.1	81
3	1972-04-24	09:57:21.700000	23.6380	121.5510	33.0	6.9	22
4	1972-09-22	19:57:27.400000	22.3310	121.1970	33.0	6.2	19
...	...	...	...	...	...	...	...
33	2013-10-31	12:02:09.350000	23.5436	121.4580	14.8	6.1	17
34	2016-02-05	19:57:27	22.9064	120.6061	18.9	6.2	128
35	2018-02-06	15:50:43.040000	24.1140	121.7269	12.9	6.1	126
36	2019-04-18	05:01:06.490000	24.0374	121.6501	20.0	6.3	109
37	NaT	NaN	NaN	NaN	NaN	NaN	83

38 rows × 7 columns

Table 3: Dataset-Non-Zero Interevent Counts

After discarding the large earthquakes with zero interevent counts, it was observed that the region of Taiwan had undergone **37 cycles of large earthquakes** in the time period from 1963-2020.

The dataset in Table 3 is now modelled with 8 candidate distributions that include Weibull, Normal (Gaussian), Exponentiated Weibull, Lognormal, Exponential, Gamma, Inverse Gaussian and Inverse Weibull distributions. This is done to find out that distribution which best fits the given data using the various Goodness of Fit Tests such as KS (Kolmogorov-Smirnov) Test and AIC scores.

Estimation of Parameters and Goodness of Fit Tests

The two methods that we used to estimate the parameters were: Method of Moments (MoM) and Maximum Likelihood Estimation (MLE). The parameters of the various candidate distributions were estimated using the **actuar** and **fitdistrplus** libraries on **R**. Initially, we had tried to perform the estimation on Python, however certain statistical distributions were not available, and the estimations of the parameters were only limited to Maximum Likelihood Estimation method. The KS statistic, p-value and AIC scores were also obtained using the same software packages. The summary of the results is given in Tables 4 and 5.

MLE: Maximum likelihood estimation is used to make inferences about the population that is most likely to have generated the sample. The likelihood function is known as the joint density function of the observed data. Our goal is to maximize this likelihood function and obtain the optimum values of the parameters.

MOM: Method of Moments estimation starts by indicating the population moments as functions of the parameters to be estimated. Those expressions are then equated to the sample moments to obtain the parameters.

The parameters estimated using MLE are chosen over those estimated by MOM as MLE is known to provide better estimates. Also, due to certain limitations in the software packages in R only 7 of the 8 distributions' parameters can be estimated by MOM (Only MLE estimates can be obtained for the Exponential Weibull Distribution).

KS Test: The KS test is a non-parametric goodness-of-fit test used to validate a continuous fully defined distribution for a sample of data given, or whether an underlying probability distribution differs from a hypothesized distribution.

AIC: The Akaike Information Criterion (AIC) is a mathematical method for evaluating how well a specific model fits the original data from which it was generated, i.e., given a number of models for a dataset, AIC helps us compare these models and estimate quality of each model, relative to each of the other. Thus, AIC can be used for model selection.

The KS test is used as the measure of determining the best fit distribution for the Interevent Counts Data, rather than the AIC test. This is because the AIC scores do not vary much from one distribution to another, making it difficult for us to make a clear distinction between the fitting of the various candidate distributions, one of the reasons for this is the parametric nature of AIC tests where the result depend on the number of parameters while the KS test being non-parametric will have an upper hand. Not only that the KS Test is free from a priori distributional assumption and it performs well under a wide range of population distributions.[4] The AIC score also provides better results in the case of a large sample size, however our sample size consists of merely **37** datapoints.

Distribution	**Estimated Parameters (MLE)**	**KS Statistic Value**	**KS P-value**	**AIC**
Exp. Weibull	Kappa= 0.8186113 Lambda=46.3922652 Alpha= 1.6679260	0.075206	0.9849	396.6046
Log-Normal	Meanlog= 3.793244 sdlog= 1.124995	0.08184828	0.9653	398.4172
Weibull	Shape= 1.06983 Scale= 74.80112	0.08388776	0.957	394.9955
Gamma	Shape= 1.14987772 Rate = 0.01579841	0.08672685	0.9436	394.8383
Exponential	Rate= 0.01373932	0.08928736	0.9296	393.2745
Inv. Weibull	Shape= 0.7205068 Scale= 24.2254196	0.1649457	0.2665	415.1995
Normal	Mean=72.78378, Sd=70.74550	0.1654269	0.2634	424.174
Inv. Gauss	Mean= 72.71221 Shape 23.01678	0.2013781	0.09947	414.0056

Table 4: Results of MLE estimated parameters for the candidate distributions and Goodness of Fit Test Results using R

Distribution	Estimated Parameters (MoM)	KS Statistic Value	KS P-value	AIC
Gamma	Shape= 1.05845300 Rate = 0.01454243	0.07893	0.9753	394.9938
Weibull	Shape= 1.02893 Scale= 73.63527	0.081619	0.9662	395.0902
Exponential	Rate = 0.01373932	0.089287	0.9296	393.2745
Log-Normal	Meanlog= 3.954920 sdlog= 0.815565	0.15392	0.3447	409.471
Normal	Mean=72.78378 Sd=70.74550	0.16543	0.2634	424.174
Inv. Gauss	Mean= 72.71221 Shape= 23.01678	0.20138	0.09947	414.0056
Inv. Weibull	Shape= 2.556288 Scale= 49.535455	0.35941	0.0001412	415.1995

Table 5: Results of MOM estimated parameters for the candidate distributions and Goodness of Fit Test using R

The results from the KS Test indicate that the Exponentiated Weibull Distribution is the best candidate fit for the given dataset, owing to the largest KS p-value and smallest KS distance. The five best candidate distributions that fit this data are Exp. Weibull, Log-Normal, Weibull, Gamma and Exponential (all having a KS p-value > 0.9). The normal distribution on the other hand which describes a lot of natural phenomena shows a **terrible fit** for the Interevent Counts Data (having a p-value of merely 0.2634).

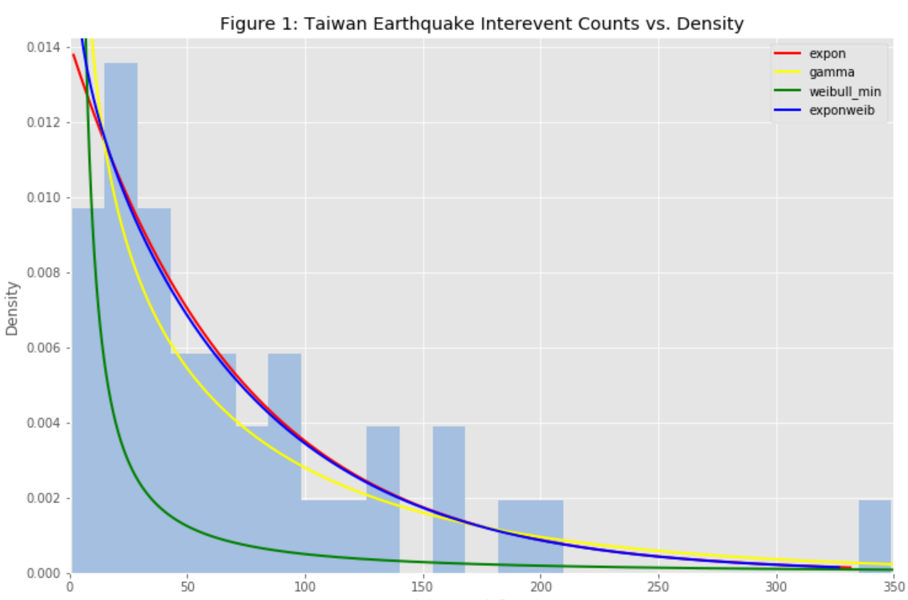


Figure 1 is shown below plotting the data on a histogram plot and along with the top distributions' density functions on Python.

The lognormal curve has been excluded owing to a few limitations of the Python libraries. Figure 1 gives us assurance that the KS Test results are accurate, and the best fit distributions obtained by the KS Test show a good fit on the histogram plotted data.

Brief Analysis of the Earthquakes in Taiwan

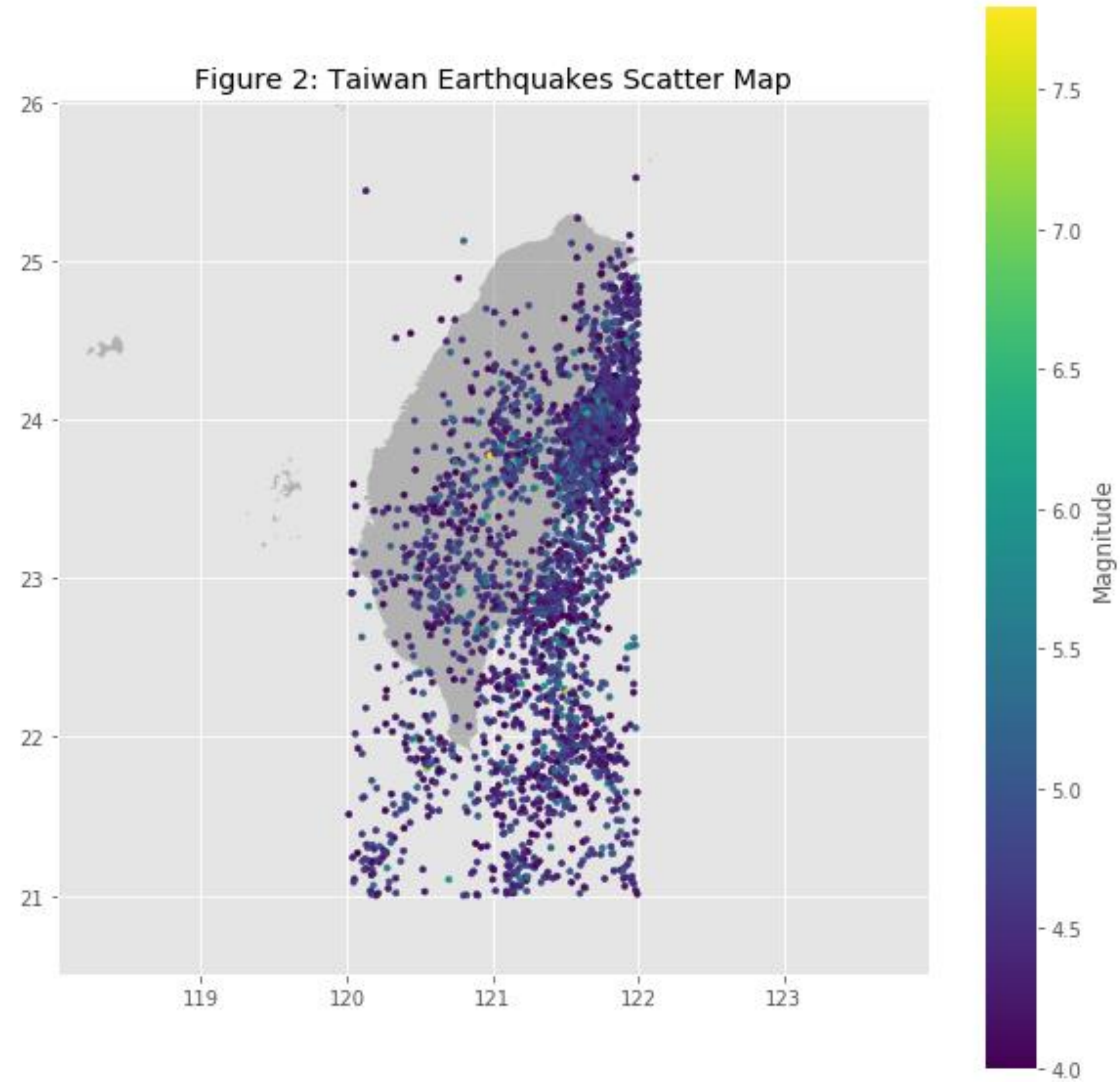
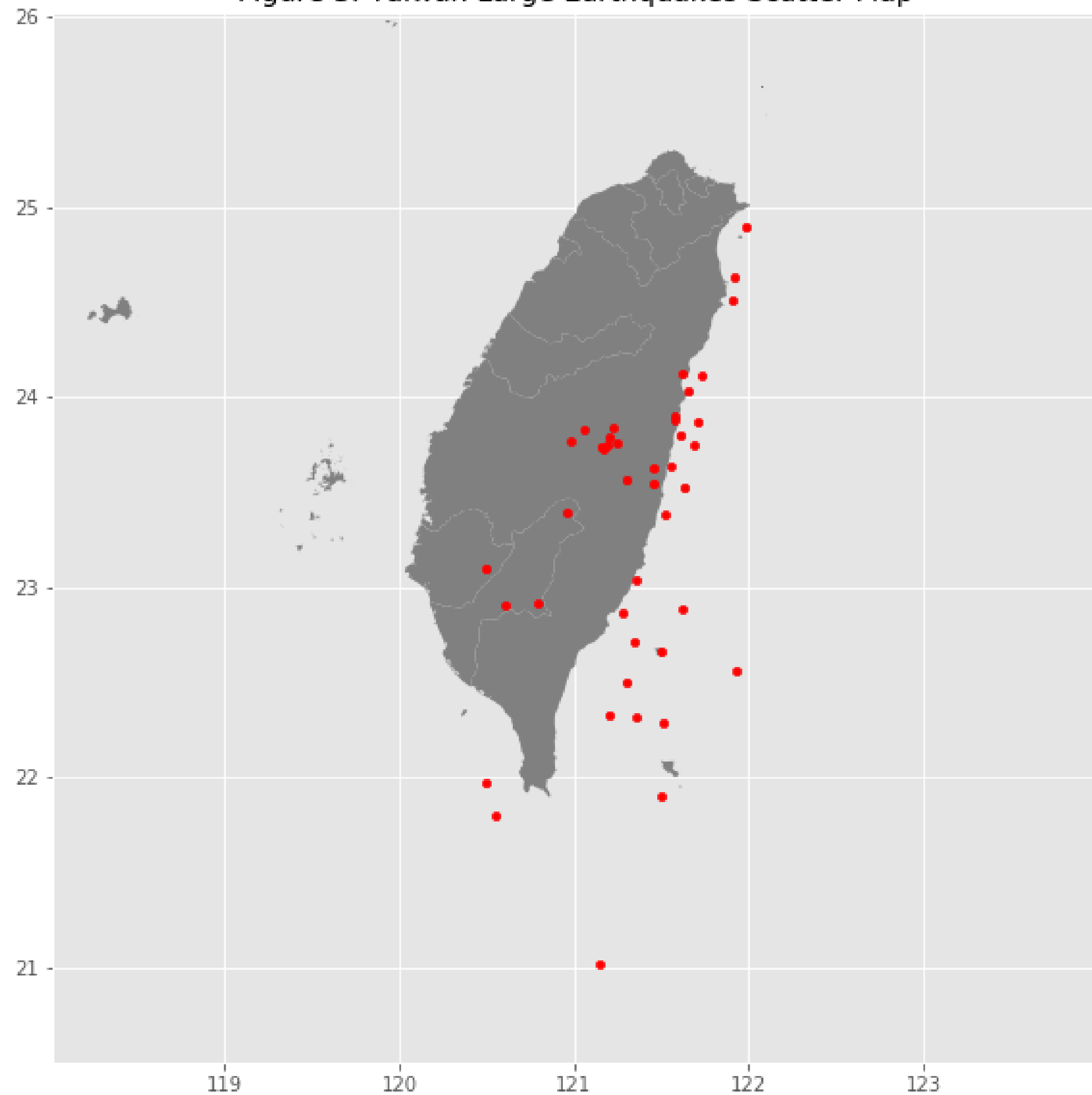


Figure 2 shows a basic scatter diagram of the earthquakes that have occurred in Taiwan since 1963. A majority of the earthquakes are small earthquakes which have magnitudes lying between 4 and 6. This scatter plot has been made using the Python library **geopandas** through a shape file of Taiwan's map with respect to its latitude and longitude that has been obtained from **MIT GeoWeb**[5]. Figure 3 is a scatter diagram indicating the large magnitude earthquakes. These earthquakes are represented by red dots on the map of Taiwan. Majority of the country's large earthquakes seemed to have occurred in the south-eastern and eastern parts of the country.

Figure 3: Taiwan Large Earthquakes Scatter Map



<u>Earthquake Potential Score (EPS)</u>

The EPS is a quantification of the current state of earthquake hazard for the chosen region and it indicates the current level of seismic progress through the regional earthquake cycle of recurring events in the region.

We know that when the EPS score is close to 1 (or close to 100 on a scale of 100), it indicates that there is an extremely high probability that a large earthquake is bound to occur, possibly even on the same day.

If n(t) is the number of small earthquakes since the last large earthquake, the EPS score is defined to be: $\mathbf{EPS = P\{X \leq n(t)\}}$, where P is the CDF of small earthquakes occurring between large earthquakes. EPS is a monotonic function whose value lies between [0,1] and resets to 0 at the onset of every large earthquake.[4]

EPS is therefore the current level of hazard and assigns a number between 0% and 100% to every region so defined, thus providing a unique measure. Physically, the EPS corresponds to an estimate of the level of progress through the earthquake cycle in the defined region at the current time. The nowcasting approach however yields no information regarding the future. This means that a large earthquake may take place at EPS scores of even as less as 30-40 for a particular country/ city.

To illustrate the nowcasting approach in the study region, we computed EPS values for the two seismically exposed megacities in Taiwan, Taipei and Tainan.

We have considered a radius of R = 100, 150 and 200 km around these cities and found all the earthquakes that are lying within these regions by using the latitude and longitudes given in the dataset for each earthquake. **The small earthquakes are considered to have magnitudes between [4,6) whereas for the large earthquakes M ≥ 6, as considered previously.**

The city center coordinates for the two cities are:

Taipei City Center - 25.0330° N, 121.5654° E

Tainan City Center - 22.9997° N, 120.2270° E

The EPS scores have been obtained through R by using the best fit distribution i.e., Exponentiated Weibull distribution with the parameters as stated in Table 4.

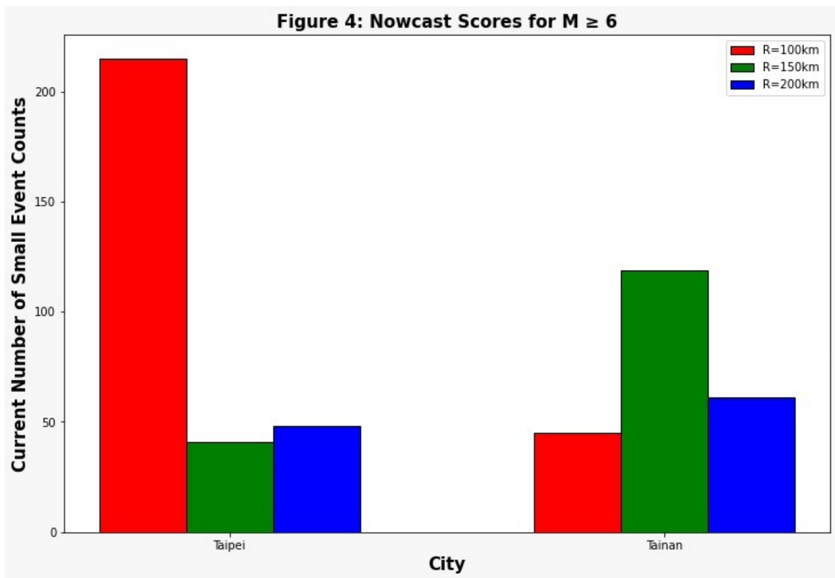
The data is shown for the various radii taken keeping the city center coordinates as the center of the circular region.

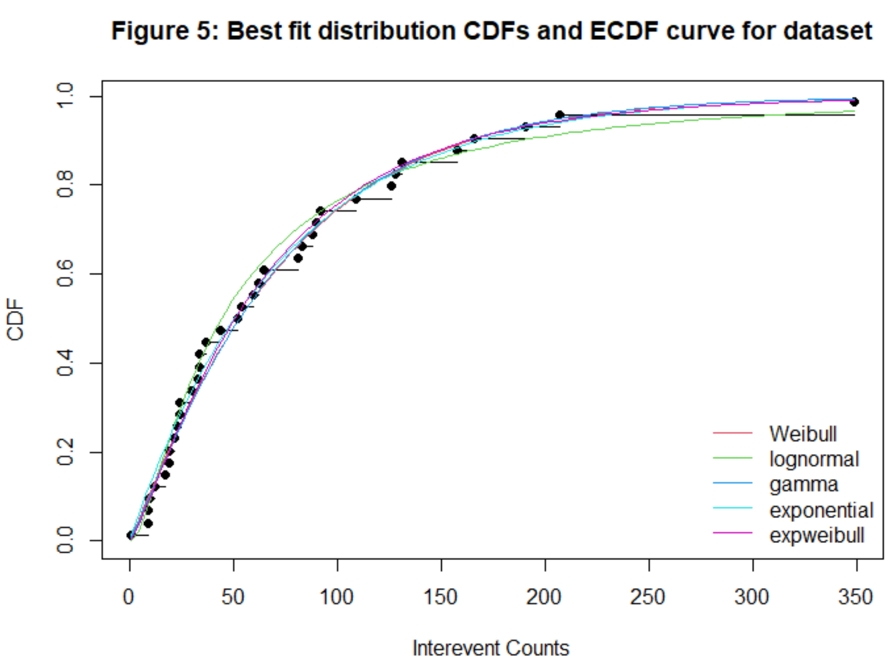
City	Radius (R)	Date of Last Large Event (mm/dd/yy)	Magnitude of Last Large Event	Current Interevent Count	Number of Cycles	Earthquake Potential Score (EPS)
	100 km	5/15/2002	6.2	215	3	**0.9505872**
Taipei	150 km	4/18/2019	6.3	41	16	**0.4206074**
	200 km	4/18/2019	6.3	48	22	**0.4779803**

	100 km	2/5/2016	6.2	45	4	**0.4541065**
Tainan	150 km	2/5/2016	6.2	119	20	**0.8155418**
	200 km	4/18/2019	6.3	61	32	**0.5699152**

Table 6: Nowcast Values for Taipei and Tainan with R = 100 km, 150 km and 200 km

The EPS scores for Taipei (R = 100 km) and Tainan (R = 150 km) indicate that the progress of the current earthquake cycle is almost complete suggesting a higher risk of imminent large scale earthquakes. The other EPS scores indicate roughly 50% progress of the earthquake cycle in the various regions surrounding the 2 cities. A bar graph in Figure 4 gives us a pictorial comparison of the intervent counts for different values of R.

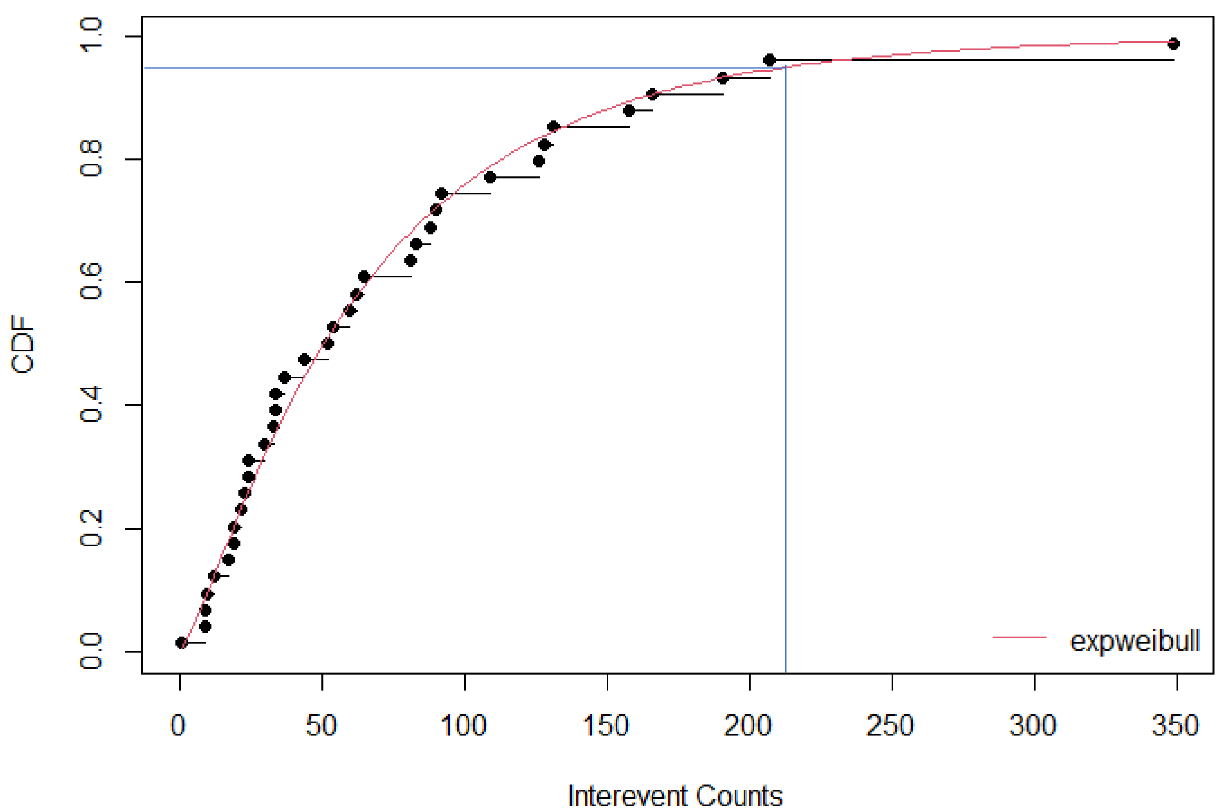




CDF plots are utilised to obtain the EPS information.[2] Figure 5 shows the CDF plots that have been obtained from R for all the best fit curves with respect to the ECDF curve of the dataset of Interevent Counts.

The curves shown in the CDF Plot (Figure 5) indicate the KS Test Validity and give us assurance of an accurate EPS score. We will now use the Exponentiated Weibull distribution to obtain the EPS score for Taipei for R = 100 km in Figure 6.

Figure 6: Exponentiated Weibull Distribution used to obtain EPS Score



For 215 Intervent Counts it can be observed that the EPS score from the Exponentiated Weibull Distribution is approximately 95 on a scale of 100.

Data Driven Models based Analysis

Motivation and Problem Statement being considered:

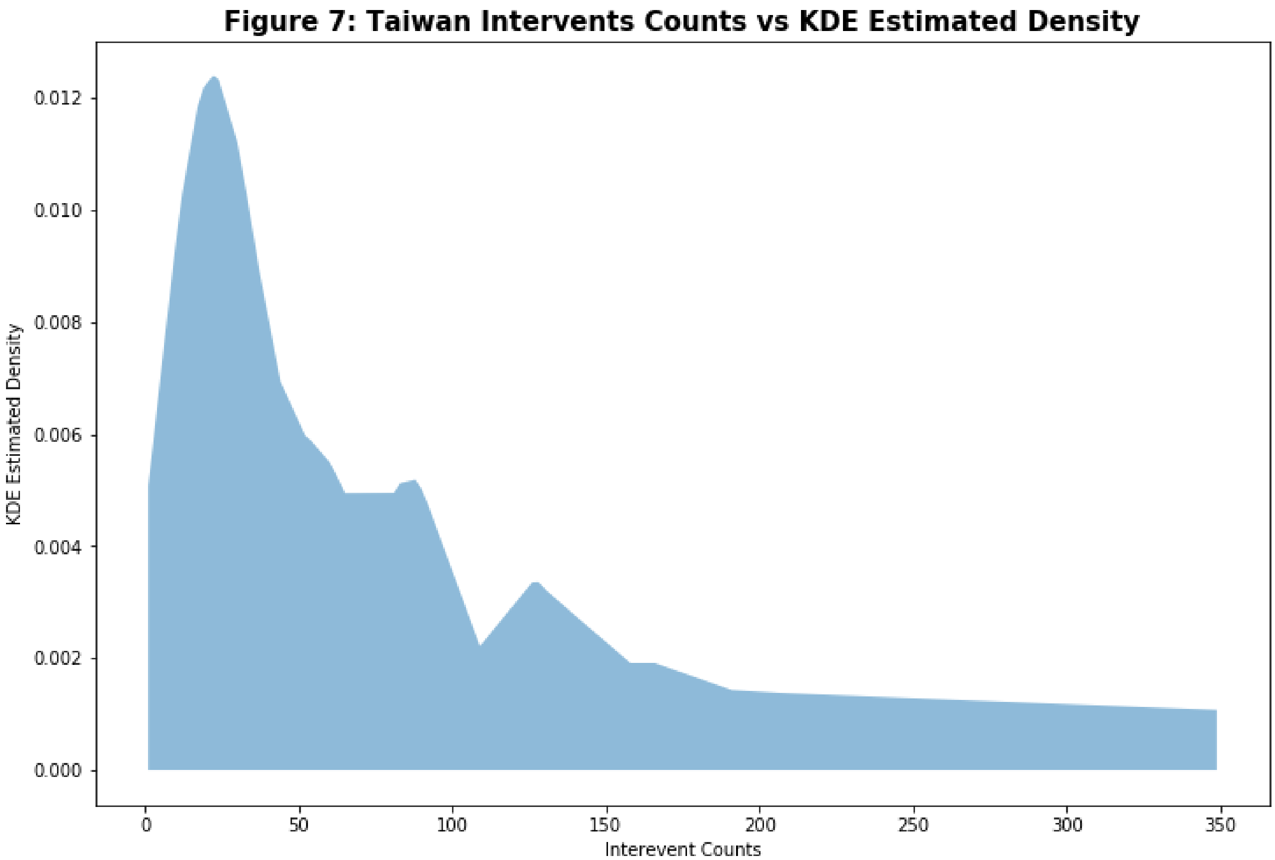
We wanted to verify our MLE + KS based estimation even further, to solve this problem we used a data-driven non-parametric based density estimation known as Kernel Density Estimation.

Methodology:

Kernel Density Estimation in crude terms can be explained as a modification of histogram-based density estimation where instead of stacking the blocks aligned with the *bins* of histogram, we stack the blocks aligned with the point they represent, thus placing a distribution per point.

The free parameters of kernel density estimation are the kernel, which specifies the shape of the distribution placed at each point, and the kernel bandwidth, which controls the size of the kernel at each point.

For our problem we used the Kernel as Gaussian distribution and to select the bandwidth parameter we did a cross-validation hyperparameter search which resulted in best bandwidth corresponding to our problem as 10.



As is evident from Figure 7, Kernel density estimation also provided an estimation which closely resembles our MLE+KS based estimation thus even further verifying our MLE+KS based estimation.

Conclusion

We have used Maximum Likelihood (MLE) and Method of Moments (MoM) for the estimation of parameters. Further we used two tests for finding the best-fit distribution: Kolmogorov-Smirnov test (K-S test) and Akaike Information Criterion (AIC test).

As mentioned earlier preferring parameters obtained via MLE and considering K-S test superior, we get Exp. Weibull, Log-normal, Weibull, Gamma and exponential as significant fits compared to others, where **Exp. Weibull** is the best fit with parameters: **Kappa= 0.8186113, Lambda=46.3922652 & Alpha= 1.667926**. Normal distribution cannot describe the Interevent Counts Data owing to a low KS p-value and is a terrible fit for the dataset.

Further to calculate the EPS scores in Taiwan, we used the obtained Exp. Weibull distribution with the corresponding parameters. Our cities of concern are Tainan and Taipei, and by using the CDF from above distribution we get the EPS values for both the cities within 100 km, 150 km and 200 km of the center point of each city. The results clearly indicate that the area under 100 KM radius for Taipei is at highest risk for an imminent large earthquake to occur as the seismic cycle has reached **95% towards completion**. For Tainan, the area under 150 km is at a higher risk of an imminent large earthquake due to **81.5% seismic cycle completion**. KDE is then utilized as a post-processing method to validate and strengthen our results, and the distribution obtained by KDE closely resembles our MLE+KS test result. (Compare **Figure 1** and **Figure 7**)

<u>Density Functions of the Candidate Distributions</u>

Distribution	Density Function	Interpretation	Conditions
Normal	$\frac{1}{\beta\sqrt{2\pi}} exp\ (-\frac{1}{2}\left(\frac{t-\alpha}{\beta}\right)^2)$	β– *scale*	$t > 0$, $-\infty < \alpha < \infty$, $\beta > 0$
Exponential	$e^{-t/\alpha}/\alpha$	α - *scale*	$t > 0$, $\alpha > 0$
Weibull	$\beta t^{\beta-1} e^{\ -t/\alpha\ ^{\beta}} /\alpha^{\beta}$	α – *scale*, β– *shape*	$t > 0$, $\alpha > 0$, $\beta > 0$
Gamma	$t^{\beta-1}e^{-t/\alpha}/\ \Gamma\ \ \beta\ \ \alpha^{\beta}$	α – *scale*, β– *shape*	$t > 0$, $\alpha > 0$, $\beta > 0$
Log Normal	$\frac{exp\left[-\frac{ln\,t-\alpha\ ^2}{2\beta^2}\right]}{t\beta\sqrt{2\pi}}$	α – *log - scale*, β– *shape*	$t > 0$, $-\infty < \alpha < \infty$, $\beta > 0$
Inverse Gaussian	$\frac{\sqrt{\beta}\,exp\left[-\frac{\beta\ t-\alpha\ ^2}{2\alpha^2 t}\right]}{\sqrt{2\pi t^3}}$	β/α – *shape*	$t > 0$, $\alpha > 0$, $\beta > 0$
Inverse Weibull	$\beta\alpha^{\beta}t^{-\beta-1}e^{\ -t/\alpha\ ^{-\beta}}$	$1/\alpha$ – *scale*, β– *shape*	$t > 0$, $\alpha > 0$, $\beta > 0$
Exponentiated Weibull	$\frac{\beta\gamma}{\alpha}\left(\frac{t}{\alpha}\right)^{\beta-1} e^{-\ t/\alpha\ ^{\beta}}\left(1-e^{-\ t/\alpha\ ^{\beta}}\right)^{\gamma-1}$	α– *scale*, β–*shape*, γ- *shape*	$t > 0$, $\alpha > 0$, $\beta > 0$, $\gamma > 0$

References

1. Wood, H. O., & Gutenberg, B. (1935). EARTHQUAKE PREDICTION. Science, 82(2123), 219–220. doi:10.1126/science.82.2123.219

2. John B Rundle, DL Turcotte, A Donnellan, L Grant Ludwig, Molly Luginbuhl, and G Gong. Nowcasting earthquakes. Earth and Space Science, 3(11):480–486, 2016.

3. Yu, S.-B., Chen, H.-Y., & Kuo, L.-C. (1997). Velocity field of GPS stations in the Taiwan area. Tectonophysics, 274(1-3), 41–59. doi:10.1016/s0040-1951(96)00297-1

4. Pasari, S., & Sharma, Y. (2020). Contemporary Earthquake Hazards in the West-Northwest Himalaya: A Statistical Perspective through Natural Times. Seismological Research Letters. doi:10.1785/0220200104

5. Hijmans, R. and University of California, Berkeley, Museum of Vertebrate Zoology. (2015). First-level Administrative Divisions, Taiwan, 2015. UC Berkeley, Museum of Vertebrate Zoology. Available at: http://purl.stanford.edu/gm351ph7903

Codes and Datasets Used for the Report: GitHub Link