Syllabus

MA 205 **Discrete Mathematics** 3-1-0-4

Module I
Mathematical logic and Mathematical Reasoning, Compound Statements, Propositional Equivalences, Predicates and Quantifiers, Methods of Proof, Mathematical Induction, Well-ordering principal, Recursive Definition and Algorithms. [9L]

Module II
Recurrence Relations, Classification of Recurrence Relations and their solutions by Characteristic Root method, Generating function and their various aspects, Utility of Generating function in solving Recurrence Relations. [9L]

Module III
Set, Operations on Set, Computer representation of Set, Relations, Properties/Classification of Relations, Closure operations on Relations, Matrix representation of Relations, Digraphs. Functions and their Representation, Classification of Functions, Warshall's algorithm, Discrete Numeric Functions, Growth of Functions, Big O, Big Q, Hash Function, Growth Functions. [9L]

Module IV
Binary Operations, Groups, Product and Quotients of Groups, Semi group, Products and Quotients of Semi groups, Permutation Group, Composition of Permutation, Inverse Permutation, Cyclic Permutation, Transposition, Even and Odd Permutation, Coding of Binary Information and Error Correction, Decoding and Error Correction. [9L]

Module V
Introduction to Graph, Graph Terminologies and their Representation, Connected & Disconnected graphs, Isomorphic Graph, Euler & Hamilton graphs. Introduction to Trees, Versatility of Trees, Tree traversal. Spanning Trees, Minimum Spanning Tree. [9L]

Text Books:

1. **Mott, Joe L., Abraham Kandel, and Theodore P. Baker** Discrete Mathematics for Computer Scientists & Mathematicians, PHI, 2nd edition 2002.
2. **Swapan Kumar Chakraborty and Bikash Kanti Sarkar:** Discrete Mathematics, Oxford Univ. Publication, 2010.
3. **Kolman, Bernard, Robert C. Busby, and Sharon Ross.** Discrete mathematical structures, Prentice-Hall, Inc., 2003.

Reference Books:

1. **Bikash Kanti Sarkar and Swapan Kumar Chakraborty,** *Combinatorics and Graph Theory*, PHI, 2016.
2. **Seymour Lipschuz and Mark Lipson,** *Discrete Mathematics*, Shaum's outlines, 2003.
3. **Liu, Chung Laung,** *Elements of Discrete mathematis*, Mcgraw Hill, 2nd edition, 2001.
4. Bondy and Murty, Grapg Theory with Applications, American Elsevier,1979.
5. Robin J. Wilson, Introduction to Graph Theory, Pearson, 2010.

BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(MID SEMESTER EXAMINATION)

CLASS: BE
BRANCH: IT

SEMESTER : III
SESSION : MO/15

SUBJECT : IT3021 DISCRETE MATHEMATICS & GRAPH THEORY

TIME: 1.5 HOURS

FULL MARKS: 25

INSTRUCTIONS:
1. The total value of the questions are 30 marks.
2. Candidates may attempt for all 30 marks.
3. In those cases where the marks obtain exceed 25 marks. The excess will be ignored.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. The missing data, If any, may be assumed suitably.

Q1. (a) Let p : Arnab is rich and q : Arnab is happy. Express, using simple verbal sentence, the following statements. [3]
(i) $p \vee \sim q$ (ii) $q \Leftrightarrow \sim p$ (iii) $\sim \sim p$

(b) Let p : Rina is tall and q : Rina is beautiful. Write the following statements in symbolic form. [2]
(i) Rina is tall and beautiful (ii) It is false that Rina is short or beautiful

Q2. Investigate the following compound propositions for tautologies: [5]
i) $(p \wedge \sim q) \vee (q \wedge \sim p)$ ii) $(p \rightarrow q) \vee (q \rightarrow p)$

Q3. Using principle of mathematical induction, show that [5]

Q4. Find the solution of the linear nonhomogeneous recurrence relation $a_r + 5a_{r-1} + 6a_{r-2} = 3r^2$. [5]

Q5. Let $(i) f(r) = 3^r, (ii)\ f(r) = r \cdot 3^r,\ (iii)\ f(r) = (r^2+1) \cdot 3^r$. Find the form of the particular solutions of the linear nonhomogeneous recurrence relation $a_{r+2} - 6a_{r+1} + 9a_r = f(r)$, for each case. Solve the same for $f(r) = r \cdot 3^r$. [5]

Q6. Using generating function method, find the solution of the recurrence relation $y_{n+2} - 4y_{n+1} + 3y_n = 0$ subjected to the initial conditions $y_0 = 2$ and $y_1 = 4$. [5]

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BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: BE
BRANCH: IT

SEMESTER : III
SESSION : MO/15

SUBJECT: IT3021: DISCRETE MATHEMATICS AND GRAPH THEORY

TIME: 03:00

FULL MARKS: 60

INSTRUCTIONS:
1. The question paper contains 7 questions each of 12 marks and total 84 marks.
2. Candidates may attempt any 5 questions maximum of 60 marks.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

Q.1(a) Consider the following: [5]
p : He is rich and q : He is generous
Write the new propositions using conjunction ($\wedge$), disjunction ($\vee$), and negation (~).

Q.1(b) Investigate the following compound proposition for a truth table: [7]
$(p \wedge \sim q) \vee (\sim p \wedge q)$

Q.2(a) Using principle of mathematical induction, prove that $\forall n \in N$, $\frac{n^5}{5} + \frac{n^3}{3} + \frac{7n}{15}$ is a natural number. [6]

Q.2(b) Show, by principle of mathematical induction, that any positive integer n, greater than or equal to 2 is either prime or product of primes. [6]

Q.3(a) Solve the recurrence relation $a_{r+2} - 6a_{r+1} + 9a_r = 3 \cdot 2^r + 7 \cdot 3^r$, $n \geq 0$, subjected to the initial conditions $a_0 = 1, a_1 = 4$. [6]

Q.3(b) Assume that there exists a valid codeword of an $n-$digit number in decimal notation having an even number of 0's. Let a_r denote the member of valid codeword of length n . If the sequence $\{a_r\}$ satisfies the recurrence relation $a_r = 8a_{r-1} + 10^{r-1}$, with the initial condition $a_1 = 9$, then find an explicit formula for a_r using the generating function. [6]

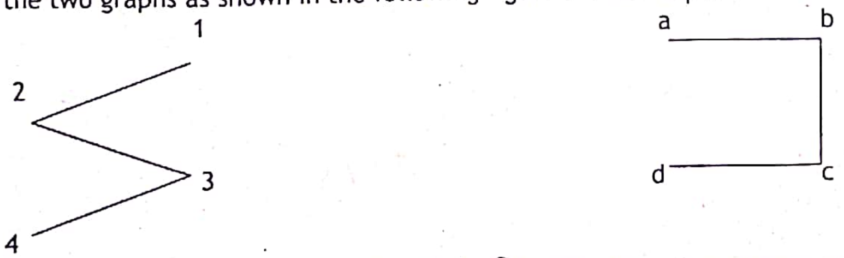
Q.4(a) Define digraph. [4]
Let $A = \{1,2,3,4\}$ and $R = \{(1,1),(1,2),(2,1),(2,2),(2,3),(2,4),(3,4),(4,1),(4,4)\}$. Construct the digraph.

Q.4(b). Let G be a simple graph with 12 edges. If G has 6 vertices of degree 3 and the rest of the vertices have degree less than 3. Determine the (i) minimum number of vertices and (ii) maximum number of vertices. [8]

Q.5(a) For each of the following degree sequences, investigate and explain vividly if there exists a graph whose degree sequences is given. If possible, draw the graph for both simple and nonsimple. [8]
(i) $(5,4,3,2,1,1)$ (ii) $(3,3,3,1)$

Q.5(b) Show that the maximum number of edges in a simple undirected graph with n vertices is $n(n-1)/2$. [4]

Q.6(a) Show that the two graphs as shown in the following figure are isomorphic. [8]



Q.6(b) Show that the degree of a vertex of a simple graph G having n vertices cannot exceed $n-1$. [4]

Q.7 Write short notes of the following: [12]
(i) Labelled Trees (ii) Tree Searching (iii) Spanning Trees

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BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(MID SEMESTER EXAMINATION)

CLASS: BE
BRANCH: IT

SEMESTER : III
SESSION : MO/16

SUBJECT: IT3021: DISCRETE MATHEMATICS AND GRAPH THEORY

TIME: 1.5 HOURS

FULL MARKS: 25

INSTRUCTIONS:
1. The total value of the questions are 30 marks.
2. Candidates may attempt for all 30 marks.
3. In those cases where the marks obtain exceed 25 marks. The excess will be ignored.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. The missing data, If any, may be assumed suitably.

Q1 (a) Express the following statements in symbolic form: [2]
(i) If Kuntal is not in a good mood or he is not busy, then he will go to Bengaluru.
(ii) If Bidhan knows object-oriented programming and oracle, then he will get a job.

(b) Consider p : Debu is rich and q : Debu is generous. Write the new propositions using conjunction $(\wedge)$, disjunction $(\vee)$ and negation $(\sim)$. [3]

Q2 Investigate the following compound propositions for tautologies: [5]
i) $(p \wedge \sim q) \vee (q \wedge \sim p)$ ii) $(p \rightarrow q) \vee (q \rightarrow p)$

Q3 Find an explicit formula of the Fibonacci sequence $0, 1, 1, 3, 5, 8, 13, \ldots\ldots$, using the recurrence relation of the sequence. [5]

Q4 The solution of a recurrence relation $Aa_r + Ba_{n-1} + Ca_{r-2} = f(r)$ is $a_r = c_1 \cdot 3^r + c_2 \cdot 4^r + 2$, where $f(r) = 6$ for all r. Determine $A,\ B,\ C$. [5]

Q5 (a) Find the sequence of the generating function $\dfrac{1}{(1+2x)^{1/2}}$. $(1+2x)^{-1/2}$ [2]

(b) Provide a closed formula for the generating function $\dfrac{x^3}{(1+3x)}$. [3]

Q6 Solve the recurrence relation $a_{n+2} - 2a_{n+1} + a_n = 2^n$, $r \geq 2$, using the generating function method, under the initial conditions $a_0 = 2$ and $a_1 = 1$. [5]

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2)

$(1+x)^{-n} =$

BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: BE
BRANCH: IT

SEMESTER : III
SESSION : MO/16

SUBJECT: IT3021 DISCRETE MATHEMATICS AND GRAPH THEORY

TIME: 03:00

FULL MARKS: 60

INSTRUCTIONS:
1. The question paper contains 7 questions each of 12 marks and total 84 marks.
2. Candidates may attempt any 5 questions maximum of 60 marks.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.
6. Nothing should be written on front or back of the question paper except tick marking.

Q.1(a) How many positive integers not exceeding 100 are divisible either by 4 or by 6? [2]

(b) Find the value of $\lfloor -3.6 \rfloor$. If x and y are real numbers, then find max (x, y) + min (x, y). [4]

(c) Prove that if a set A has n elements then its power set has 2^n elements. Find the number of elements in the power set of {{{}}, 1, {2,3}} . [6]

Q.2(a) How many bits are needed to represent the integer n? [2]

(b) Suppose A and B are two sets. Find the number of relations from A to B. Find a recurrence relation that satisfies the sequence: 2, 4, 6, 8, 10 ,.... [4]

(c) Let A= {1,2,3} and R= {(1,2), (2,3), (3,1)} Find the reflexive symmetric and transitive closure of R. [6]

Q.3(a) Define tautology. [2]

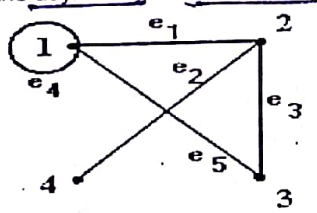
(b) Convert the proposition: $(p \leftrightarrow r) \rightarrow q$ to a CNF. [4]

(c) On a multiple choice test with 100 questions and 5 answers per question, how many different ways can the test be completed? State the concept of Pigeon-Hole principle and explain it with example. [6]

Q.4(a) If the height of a tree is the length of the longest root-to-leaf path in it, find the maximum and minimum number of nodes in a binary tree of height 5. [2]

(b) Does there exist a graph with degree sequence 1; 1; 2; 3? (degree sequence is the ordered list of vertex degrees in a graph). Explain your answer. [4]

(c) Find the adjacency and incidence matrix of the following graph. [6]



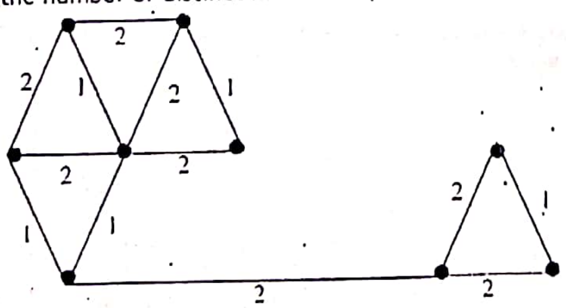
Q.5(a) A complete graph K_n has 28 edges. What is the value of n? [2]

(b) What is bipartite graph? Draw $K_{3;3}$. Find the maximum number of edges in a bipartite graph on 12 vertices. [4]

(c) For a simple graph with n vertices, what is the minimum number of edges required to ensure that the graph is connected ? [6]

Q.6(a) What is spanning tree? [2]

(b) Find the number of distinct minimum spanning trees for the weighted graph given below. [4]



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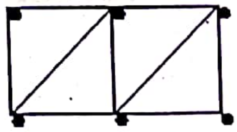
(c) Define Hamiltonian graph. Find a Hamiltonian cycle (if exists) from the graph given below. [6]



Q.7(a) Define planar graph and explain its importance. [2]

(b) A connected planar graph has 10 vertices, each of degree 3. Into how many regions, does a representation of this planar graph split the plan? [4]

(c) Prove that: an n-vertex graph is tree if its chromatic polynomial is: $p_G(k) = k(k-1)^{n-1}$ [6]

*****21/11/16******E

BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(NONCOLLEGIATE EXAMINATION)

CLASS : BE
BRANCH : IT

SEMESTER : NC2016
SESSION : SP/2016

SUBJECT: IT4101 DISCRETE MATHEMATICS & GRAPH THEORY

TIME : 3 HOURS

FULL MARKS: 100

INSTRUCTIONS :

1. The question paper contains 7 questions each of 20 marks and total 140 marks.
2. Candidates may attempt all questions.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Successful candidates are entitled to 'C' grade only.

Q1. (a) Is $[(p \Rightarrow r) \wedge (q \Rightarrow r)] \Rightarrow [(p \vee q) \Rightarrow r]$ a tautology. [4]

(b) Prove that $\sqrt{2}$ is not a rational number. [6]

(c) Is the following argument valid? [10]
"Every computer has a CD drive. Some Computer has a Floppy drive. Therefore some computers have both CD and a Floppy drive."
Use propositional logic.

$a_n + a_{n-1} = 2n-3$

Q2. (a) Prove that for any integer n, the number $n^5 - n$ is divisible by 5. [5]

(b) Express the generating function for the sequence 1,-2, 3,-4, ... [5]

(c) Solve the recursive relation $a = -a_{n-1} + 2n-3$, $n \geq 1$ given $a_0 = 1$. [10]

Q3. (a) How many distinguishable permutations of the letter in the word BANANA are there?. [5]

(b) How many different seven person committees can be formed each containing three women from an available set of 20 women and four men from an available set of 30 men? [5]

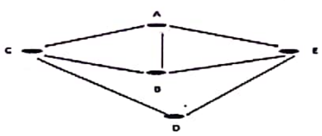
(c) State and prove Pigeonhole principle. [5]

(d) Describe an algorithm which, given n real numbers $a_1, a_2, \ldots, a_n$, output the number of a_i which lie in the range 85-90, inclusive. [5]

Q4. (a) Draw a graph with five vertices v_1, v_2, v_3, v_4, v_5 such that deg $v_1 = 3$, v_2 is an odd vertex, deg $v_3 = 2$, and v_4 and v_5 are adjacent. [5]

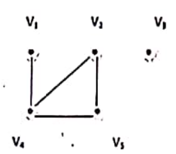
(b) Describe isomorphism with respect to graph with examples. [5]

(c) Is the following graph Hamiltonian? Justify. [10]



Give an counter example.

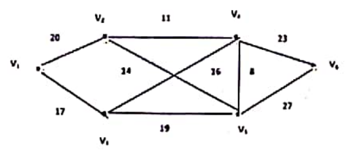
Q5. (a) Consider the graph [5]



Find A^2 where A is the adjacency matrix.

(b) Show that the number of vertices of odd degree in a graph is always even. [5]

(c) Find the shortest path between v_1 to v_6 in the following diagram using a suitable algorithm [10]



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Q6. Explain the following with examples. [6+6+8]

(i) Spanning tree
(ii) Bellman's algorithm
(iii) Depth-First search

Q7. (a) Show that the complete graph K_4 is planar. [5]

(b) Explain the following with examples. [5+5+5]

(i) Coloring graphs
(ii) Flows and Cuts
(iii) Maximal Flows

*****01/09/16*****

BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(MID SEMESTER EXAMINATION)

CLASS: BE
BRANCH: IT

SEMESTER: III
SESSION : MO/2017

SUBJECT : IT3021: DISCRETE MATHEMATICS AND GRAPH THEORY

TIME: 1.5 HOURS

FULL MARKS: 25

INSTRUCTIONS:
1. The total marks of the questions are 30.
2. Candidates may attempt for all 30 marks.
3. In those cases where the marks obtained exceed 25 marks, the excess will be ignored.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. The missing data, if any, may be assumed suitably.

Q1 (a) Express the following statements in symbolic form: [2]
(i) If Kuntal is not in a good mood or he is not busy, then he will go to Bengaluru.
(ii) If Bidhan knows object-oriented programming and oracle, then he will get a job.

(b) Consider p : Debu is rich and q : Debu is generous. Write the new propositions using conjunction $(\wedge)$, disjunction $(\vee)$ and negation $(\sim)$. [3]

Q2 Investigate the following compound propositions for tautologies: [5]
i) $(p \wedge \sim q) \vee (q \wedge \sim p)$ ii) $(p \rightarrow q) \vee (q \rightarrow p)$

Q3 Find an explicit formula of the Fibonacci sequence $0, 1, 1, 3, 5, 8, 13, \ldots\ldots$, using the recurrence relation of the sequence. [5]

Q4 The solution of a recurrence relation $pa_n + qa_{n-1} + ra_{n-2} = f(n)$ is $a_n = c_1 \cdot 3^n + c_2 \cdot 4^n + 2$, where $f(n) = 6$ for all n. Determine $p,\ q,\ r$. [5]

Q5 (a) Find the sequence of the generating function $\frac{1}{(1+2x)^{1/2}}$. [2]

(b) Provide a closed formula for the generating function $\frac{x^3}{(1+3x)}$. [3]

Q6 Solve the recurrence relation $a_{n+2} - 2a_{n+1} + a_n = 2^n$, $r \geq 2$, using the generating function method, under the initial conditions $a_0 = 2$ and $a_1 = 1$. [5]

::::19-09-2017 E ::::

BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: BE
BRANCH: IT

SEMESTER : III
SESSION : MO/17

SUBJECT: IT3021: DISCRETE MATHEMATICS AND GRAPH THEORY

TIME: 3. Hours

FULL MARKS: 60

INSTRUCTIONS:
1. The question paper contains 7 questions each of 12 marks and total 84 marks.
2. Candidates may attempt any 5 questions maximum of 60 marks.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

Q.1(a) Express the following statements in symbolic form: [4]
(i) If Kalyani is not in a good mood or he is not busy, then he will go to Bengaluru.
(ii) (ii) If Asim knows object-oriented programming and oracle, then he will get a job.

Q.1(b) Among the two restaurants next to each other, one has a sign that says 'good food is not cheap' and the other has a sign that 'Cheap food is not good'. Justify the signs in respect to their equivalence. [6]

Q.1(c) Find the generating function of a_r, the number of ways to select r objects from n objects with unlimited repetitions, and also find a_r. [2]

Q.2(a) Using principle of mathematical induction, show that [6]

$$a+ar+ar^2+....+ar^{n-1}=\frac{a(1-r^n)}{1-r}, \quad r\neq 0 .$$

Q.2(b) Let $N=n^2+n+41$. Show that there are some values of n for which N is a prime number, and others for which it is not. It follows that there is no inductive step which would show that n^2+n+41 is a prime number for all possible n. [6]

Q.3(a) Find the generating function of the Fibonacci sequence $\{a_n\}$ defined by [6]

$a_n=a_{n-1}+a_{n-2}$; $a_0=0$, $a_1=1$.

Q.3(b) Solve the recurrence relation $a_r-3a_{n-1}+2a_{r-2}=0$, $r\geq 2$ using the generating function method, under the initial conditions $a_0=2$ and $a_1=3$. [6]

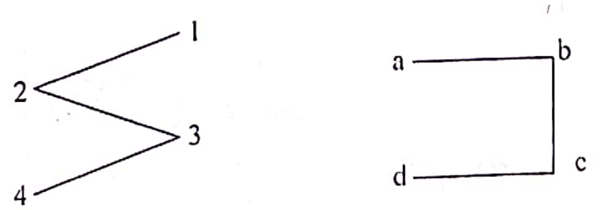
Q.4(a) Define digraph. [4]
Let $A=\{1,2,3,4\}$ and $R=\{(1,1),(1,2),(2,1),(2,2),(2,3),(2,4),(3,4),(4,1),(4,4)\}$.
Construct the digraph.

Q.4(b) Let G be a simple graph with 12 edges. If G has 6 vertices of degree 3 and the rest of the vertices have degree less than 3. Determine the (i) minimum number of vertices and (ii) maximum number of vertices. [8]

Q.5(a) Let G be a simple graph having n vertices and p components. Then G may have $\frac{1}{2}(n-p)(n-p+1)$ edges. Justify the validation of the given simple graph G. [8]

Q.5(b) In a graph, an Euler path can also be a Hamiltonian path in some cases. Investigate it. If true, give an example of such case. [4]

Q.6(a) Show that the two graphs as shown in the following figure are isomorphic. [8]



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Q.6(b) For each of the following degree sequences, determine if there exists a graph. Draw the graph. (i) (7, 6, 5, 4, 4, 3, 2, 1) (ii) (5, 5, 4, 4, 3, 2, 2, 1, 1) [4]

Q.7 Write short notes of the following: [12]

(i) Labelled Trees (ii) Tree Searching (iii) Spanning Trees

:::::27-11-2017 E:::::

BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(SHORT SEMESTER EXAMINATION)

CLASS: BE
BRANCH: IT

SEMESTER : SS/17
SESSION : 16-17

SUBJECT: IT3021-DISCRETE MATHEMATICS AND GRAPH THEORY

TIME: 3 HOURS

FULL MARKS: 100

INSTRUCTIONS:
1. The question paper contains 7 questions each of 20 marks and total 140 marks.
2. Candidates may attempt any 5 questions maximum of 100 marks.
3. 4. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

Q.1(a) State De-Morgan's Laws. Derive truth table for implication $x \Rightarrow y \equiv (\neg x) \vee y$. [12]

(b) If F_1, F_2 and F_3 are propositional formulae such that $F_1 \wedge F_2 \rightarrow F_3$ and $F_1 \wedge F_1 \rightarrow \sim F2$ are both tautologies, then prove that the conjunction $F_1 \wedge F_2$ is not satisfiable. [8]

Q.2(a) State the first principle of mathematical induction. Prove that $10^{2n-1} + 1$ is divisible by 11 for all natural numbers n. [10]

(b) Find an explicit formula for the following recurrence sequence. [10]

$$f(0) = 1, \quad f(1) = 2$$
$$f(n) = 4\,f(n-1) - 4\,f(n-2) \text{ for } n \geq 2$$

Q.3(a) State Pigeon-Hole principle and its application. Applying this principle decide the number of persons need to be present in a room to ensure that there are at least 2 persons born in same month. [10]

(b) Define algorithm. What resources are necessary to assess an algorithm? Write linear-search algorithm and measure its complexity. [10]

Q.4(a) Define graph. Explain its applications. [8]

(b) For any two vertices *u* and *v* of a graph G, if G contains a walk from *u* to *v*, then G contains a path from *u* to v. [6]

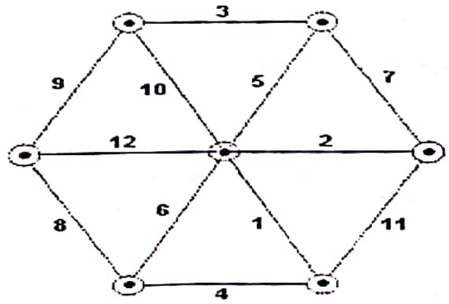
(c) For a simple graph with n vertices, what is the minimum number of edges required to ensure that the graph is connected? [6]

Q.5(a) Prove or disprove: *A simple graph with n vertices (n >2) has at least two vertices with same degree.* [10]

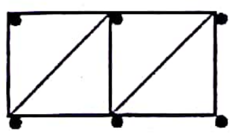
(b) Show that the maximum number of edges in a simple undirected graph with n vertices is $\frac{n(n-1)}{2}$. [10]

Q.6(a) What is spanning tree? How do you find it from a graph? [8]

(b) Draw the minimal spanning tree and calculate its total weight. [12]



Q.7(a) What is Hamiltonian graph? Find a Hamiltonian path/circuit from this graph if possible [12]



(b) Write short note on Euler graph. [8]

*****14/06/17*****

BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(MID SEMESTER EXAMINATION)

CLASS: BE | **SEMESTER: III**
BRANCH: IT | **SESSION : MO/2018**

SUBJECT : IT3021 DISCRETE MATHEMATICS AND GRAPH THEORY

TIME: 1.5 HOURS | **FULL MARKS: 25**

INSTRUCTIONS:
1. The total marks of the questions are 30.
2. Candidates may attempt for all 30 marks.
3. In those cases where the marks obtained exceed 25 marks, the excess will be ignored.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. The missing data, if any, may be assumed suitably.

Q1 (a) Express the following statements in symbolic form: [2]
(i) If Kuntal is not in a good mood or he is not busy, then he will go to Bengaluru.
(ii) If Bidhan knows object-oriented programming and oracle, then he will get a job.

(b) Consider p : Debu is rich and q : Debu is generous. Write the new propositions using conjunction $(\wedge)$, disjunction $(\vee)$ and negation $(\sim)$. [3]

Q2 (a) Investigate the tautology of the propositional statement (symbolic), i.e., $(p \wedge \sim q) \vee (\sim p \wedge q)$ [2]

(b) Show that the statement $2+4+\ldots+2n=(n+2)(n-1)$, for all $n \geq 1$, satisfies the inductive step but possesses no basis. [3]

Q3 Find an explicit formula of the Fibonacci sequence 0, 1, 1, 2, 3, 5, 8, 13, ..., using the recurrence relation of the sequence. [5]

Q4 The solution of a recurrence relation $\alpha y_n + \beta y_{n-1} + \gamma y_{n-2} = f(n)$ is $y_n = c_1 \cdot 3^n + c_2 \cdot 4^n + 2$, where $f(n)=6$ for all n. Determine $\alpha,\ \beta,\ \gamma$. [5]

Q5 (a) Find the coefficient of x^6 in the following product of polynomials like $(1+x+x^2)(x+x^2+x^3)(x^3+x^5)$ [2]

(b) Provide a closed formula for the generating function $\dfrac{x^3}{(1+3x)}$. [3]

Q6 Solve the recurrence relation $a_{n+2} - 5a_{n+1} + 6a_n = 2,\ n \geq 0$, using the generating function method, subjected to the initial conditions $a_0 = 1$ and $a_1 = 3$. [5]

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BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

SEMESTER : IV
SESSION : MO/14

CLASS: BE
BRANCH: IT

SUBJECT: IT2021 DISCRETE MATHEMATICS AND GRAPH THEORY

FULL MARKS: 60

TIME: 3.00 HOURS

INSTRUCTIONS:
1. The question paper contains 7 questions each of 12 marks and total 84 marks.
2. Candidates may attempt any 5 questions maximum of 60 marks.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

Q1. (a) Let p : It is a hot day and q : The temperature is $45^\circ C$. Write in simple sentences the following: [2]
(i) $p \wedge q$ (ii) $\sim p \wedge \sim q$ (iii) $\sim(\sim p \wedge \sim q)$

(b) Consider the following: [4]
p : He is rich and q : He is generous
Write the new propositions using conjunction ($\wedge$), disjunction ($\vee$) and negation ($\sim$).

(c) Show that the following propositions are equivalent to $p \rightarrow q$: [6]
(i) $\sim(p \wedge \sim q)$ (ii) $(\sim p \vee q)$ (iii) $\sim q \rightarrow \sim p$

Q2. (a) Prove by induction that the expression for the number of diagonals in a polygon of n sides is $\frac{n(n-3)}{2}$. [2]

(b) Show, by principle of mathematical induction, that any positive integer n greater than or equal to 2 is either prime or product of primes. [4]

(c) Let $N = n^2 - n + 41$. Show that there exists some value of n for which N is a prime number, others for which it is not. It follows that there is no inductive step which would show that N is a prime number for all possible n. [6]

Q3. (a) Solve the recurrence relation $a_{n+2} - 6a_{n+1} + 9a_n = 3\cdot 2^n + 7\cdot 3^n$, $n \ge 0$, subjected to the initial conditions $a_0 = 1, a_1 = 4$. [6]

(b) Assume that there exists a valid codeword of an n-digit number in decimal notation having an even number of 0's. Let a_n denote the number of valid codeword of length n. If the sequence $\{a_n\}$ satisfies the recurrence relation $a_n = 8a_{n-1} + 10^{n-1}$, with the initial condition $a_1 = 9$, then find an explicit formula for a_n using the generating function. [6]

Q4. (a) Draw a 3 – regular graph of five vertices. [2

(b) Define digraph [4
Let $A = \{1,2,3,4\}$ and $R = \{(1,1),$ [illegible]$,(2,1),(2,2),(2,3),(2,4),(3,4),(4,1),(4,4)\}$.
Construct the digraph.

(c) Let G be a simple graph with 12 edges. If G has 6 vertices of degree 3 and the rest of the vertices have degree less than 3. Determine the (i) minimum number of vertices and (ii) maximum number of vertices. [6

Q5. (a) Prove that in any graph, there are an even number of vertices of odd degree.

(b) Show that the maximum number of edges in a simple undirected graph with n vertices is $n(n-1)/2$.

(c) For each of the following degree sequences, investigate and explain briefly if there exists a graph whose degree sequences is given. If possible, draw the graph for both simple and nonsimple:
(i) $(5,4,3,2,1,1)$ (ii) $(3,3,3,1)$

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SUBJECT: BT3021 BIOLOGICAL SCIENCES

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[2]

Q6. (a) Show that the two graphs as shown in the following figure are isomorphic.

(b) Show that the degree of a vertex of a simple graph G having n vertices cannot exceed $n-$

Q7. Write short notes of the following:

(i) Labelled Trees (ii) Tree Searching (iii) Spanning Trees

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BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: BE
BRANCH: IT

SEMESTER : III
SESSION : MO/18

SUBJECT: IT3021-DISCRETE MATHEMATICS AND GRAPH THEORY

TIME: 03:00

FULL MARKS: 60

INSTRUCTIONS:
1. The question paper contains 7 questions each of 12 marks and total 84 marks.
2. Candidates may attempt any 5 questions maximum of 60 marks.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

Q.1(a) Let p: Babu is rich, q: Babu is happy. Give a simple verbal sentence which describes each of the following proposition: [4]

(i) $p \vee \sim q$ (ii) $\sim p \to q$ (iii) $\sim \sim p$ (iv) $(\sim p \wedge q) \to p$

(b) Investigate the following compound proposition for a truth table as a tautology: [8]

$$(p \wedge q) \vee (p \wedge r)$$

Q.2(a) Using principle of mathematical induction, show that [6]

$$\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\dots+\frac{1}{\sqrt{n}} > \sqrt{n}, \quad \text{for } n \geq 2 .$$

(b) Let $N = n^2 + n + 41$. Show that there are some values of n for which N is a prime number, and others for which it is not. It follows that there is no inductive step which would show that $n^2 + n + 41$ is a prime number for all possible n. [6]

Q.3(a) Find the generating function of the Fibonacci sequence $\{a_n\}$ defined by [6]

$$a_n = a_{n-1} + a_{n-2} ; \quad a_0 = 0, \ a_1 = 1 .$$

(b) Solve the recurrence relation $a_r = 4a_{r-1} - 4a_{r-2} + 4^r$, $r \geq 2$ using the generating function method, under the initial conditions $a_0 = 2$ and $a_1 = 8$. [6]

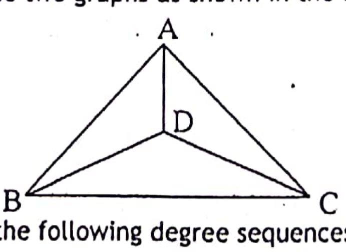
Q.4(a) Prove that the maximum number of edges in a simple disconnected graph G with n vertices and k components is $\frac{(n-k)(n-k+1)}{2}$. [8]

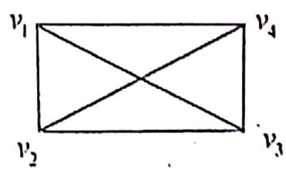
(b) Let G be a simple graph with 12 edges. If G has 6 vertices of degree 3 and the rest of the vertices have degree less than 3. Determine the (i) minimum number of vertices and (ii) maximum number of vertices. [4]

Q.5(a) Define Euler and Hamiltonian graphs with vivid description citing its construction. [8]

(b) In a graph, prove that an Euler path can also be a Hamiltonian path in some cases. Investigate it. If true, give an example of such case. [4]

Q.6(a) Show that the two graphs as shown in the following figure are isomorphic. [8]





(b) For each of the following degree sequences, determine if there exists a graph. Draw the graph. [4]

(i) (5, 5, 4, 3, 2, 1) (ii) (5, 4, 3, 2, 1, 1)

Q.7 Write short notes of the following: [12]

(i) Labelled Trees (ii) Tree Searching (iii) Spanning Trees

******26.11.18******E

BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(MID SEMESTER EXAMINATION)

CLASS: BE
BRANCH: IT

SEMESTER: III
SESSION : MO/2018

SUBJECT : IT3021 DISCRETE MATHEMATICS AND GRAPH THEORY

TIME: 1.5 HOURS

FULL MARKS: 25

INSTRUCTIONS:
1. The total marks of the questions are 30.
2. Candidates may attempt for all 30 marks.
3. In those cases where the marks obtained exceed 25 marks, the excess will be ignored.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. The missing data, if any, may be assumed suitably.

Q1 (a) Express the following statements in symbolic form: [2]

(i) If Kuntal is not in a good mood or he is not busy, then he will go to Bengaluru.

(ii) If Bidhan knows object-oriented programming and oracle, then he will get a job.

(b) Consider p : Debu is rich and q : Debu is generous. Write the new propositions using conjunction $(\wedge)$, disjunction $(\vee)$ and negation $(\sim)$. [3]

Q2 (a) Investigate the tautology of the propositional statement (symbolic), i.e., $(p \wedge \sim q) \vee (\sim p \wedge q)$ ~~[3]~~ 2

(b) Show that the statement $2+4+\ldots+2n=(n+2)(n-1)$, for all $n \geq 1$, satisfies the inductive step but possesses no basis. [3]

Q3 Find an explicit formula of the Fibonacci sequence 0, 1, 1, 2, 3, 5, 8, 13, ..., using the recurrence relation of the sequence. [5]

Q4 The solution of a recurrence relation $\alpha y_n + \beta y_{n-1} + \gamma y_{n-2} = f(n)$ is $y_n = c_1 \cdot 3^n + c_2 \cdot 4^n + 2$, where $f(n)=6$ for all n. Determine α, β, γ. [5]

Q5 (a) Find the coefficient of x^6 in the following product of polynomials like $(1+x+x^2)(x+x^2+x^3)(x^3+x^5)$ [2]

(b) Provide a closed formula for the generating function $\dfrac{x^3}{(1+3x)}$. [3]

Q6 Solve the recurrence relation $a_{n+2} - 5a_{n+1} + 6a_n = 2,\ n \geq 0$, using the generating function method, subjected to the initial conditions $a_0 = 1$ and $a_1 = 3$. [5]

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BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(MID SEMESTER EXAMINATION)

CLASS: BE
BRANCH: CSE

SEMESTER: III
SESSION : MO/2018

SUBJECT : CS4101-DISCRETE MATHEMATICS STRUCTURE

TIME: 1.5 HOURS

FULL MARKS: 25

INSTRUCTIONS:
1. The total marks of the questions are 30.
2. Candidates may attempt for all 30 marks.
3. In those cases where the marks obtained exceed 25 marks, the excess will be ignored.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. The missing data, if any, may be assumed suitably.

Q1 (a) What is a statement? [2]
(b) Write the negations of the following statements: [3]
i) Raju plays cricket on Sunday and Sachin do walking on Saturday.
ii) $5 < x < 20$.
iii) If n is divisible by 6 then n is divisible by 2 and n is divisible by 3.

Q2 (a) Use the Principle of Mathematical Induction to verify that, for any positive integer n, $6^n - 1$ is divisible by 5. [5]

Q3 (a) Express the statement "there is a number x such that when it is added to any number, the result is that number, and if it is multiplied by any number, the result is x" as a logical expression. [2]
(b) Without using truth tables, prove that $\sim(p \vee (\sim p \wedge q))$ and $(\sim p \wedge \sim q)$ are logically equivalent. [3]

Q4 (a) In each of the following situation, indicate whether *f=O(g)* or *f=Ω(g)* or *f=Θ(g)* and justify your answer. [5]
i) $f(n) = n \log n$, $g(n) = 10n \log 10n$
ii) $f(n) = n-100$, $g(n) = n-200$

Q5 (a) What is the difference between a relation and a function? Give an example of a relation which is not a function. [2]
(b) Define Transitive closure. Find the transitive closure of the following relation [3]
R={(1,1), (1,2), (2,2), (2,4), (3,2), (4,1), (4,3)}.

Q6 (a) Give the definition of Equivalence Relation and an example. [2]
(b) Prove that $f(x)=\Theta(g(x))$ if and only if $f(x)=O(g(x))$ and $g(x)=O(f(x))$. [3]

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