BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: BE
BRANCH: CSE

SEMESTER : III
SESSION : MO/16

SUBJECT: CS4101 DISCRETE MATHEMATICAL STRUCTURE

TIME: 03:00 Hours

FULL MARKS: 60

INSTRUCTIONS:
1. The question paper contains 7 questions each of 12 marks and total 84 marks.
2. Candidates may attempt any 5 questions maximum of 60 marks.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

Q.1(a) Show that $(\sim p \wedge (p \vee q)) \Rightarrow q$ is a tautology. [2]

(b) Using Tautologies prove that $(\sim (p \wedge \sim q) \wedge (\sim q \vee r) \wedge (\sim r)) \Rightarrow \sim p$ [4]

(c) Show that $\exists x (p(x) \Rightarrow q(x)) \equiv \forall x\, p(x) \Rightarrow \exists x\, q(x)$ [6]

Q.2(a) State if the argument given below is valid or not [2]

I will become famous or I will not become a writer
I will become a writer

I will become famous

(b) Say $p(x,y) \equiv x.y = 0$ *where x, y are real numbers* [4]
Check if the following statements are true or false
$a.\ \forall x \forall y\, p(x,y)$ $b.\ \forall x\ \exists y\ p(x,y)$ $c. \exists x\ \exists y\ p(x,y)$

(c) If a sequence a_n satisfies $a_{n+1} = \dfrac{a_n}{a_n + 1}$. Show using Mathematical Induction $a_n = \dfrac{a_0}{na_0 + 1}$ [6]

Q.3(a) Let A = {1,2,3,4}. Determine whether the relation R on set A is reflexive, irreflexive, symmetric, asymmetric, antisymmetric, or transitive. [2]
R = {(1,1), (1,3), (3,1), (1,2), (3,3),(4,4)}. Justify your answer.

(b) Let A be a given finite set and P(A) its power set. Let $\subseteq$ be the inclusion relation (subset relation) on P(A) i.e. (P(A), $\subseteq$) is a poset. Draw the Hasse diagram for [4]
a. A= {1} b. A = {1,2} c. A = {1,2,3} .

(c) Let S = (1,2,3,4) and A =S×S. Define the following relation R on A : (a,b) R (c,d) iff a+b = c+d. [6]
Show that
a. R is and Equivalence relation.
b. Compute $\dfrac{A}{R}$.

Q.4(a) Define transitive closure of a relation R on set A. [2]

(b) Show that the function g from N×N to N given as g(x,y) = xy. Check whether the function is one-one and/or onto with proper justification. [4]

(c) Let A = {1,2,3,4,5} and relation R on set A be given as $M_R = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$. Find $\dfrac{A}{R^{\infty}}$. [6]

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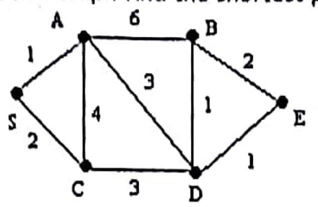
Q.5(a) Define undirected Tree. [2]

(b) Define Hamiltonian Path and circuit in Graph using suitable example. [4]

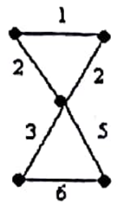
(c) In the Graph find the shortest path, using suitable algorithm, between S and E. [6]



Q.6(a) Define rooted tree. [2]

(b) Explain pre-order tree traversing. Design a binary on which if pre-order traversing is run the output string is CATSANDDOGS. [4]

(c) Find the minimum spanning tree using suitable algorithm. [6]



Q.7(a) Does the following table define a semi-group or a monoid? [2]

*	a	b	c
a	a	c	b
b	c	b	a
c	b	a	c

(b) Prove that in a Group $< G,\circ >$ [4]

a. Inverse of every element is unique.

b. $(a \circ b)^{-1} = b^{-1} o\, a^{-1} \quad \forall a,b \in G$

(c) Let $H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ be parity check matrix. Determine the (3,6) group code function $e_H : B^3 \rightarrow B^6$. Find the minimum distance of e_H and how many errors e_H can detect. [6]

*****02-12-2016 E******

BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(MID SEMESTER EXAMINATION)

CLASS: BE
BRANCH: CS

SEMESTER: III
SESSION: MO/2017

SUBJECT: CS4101 DISCRETE MATHEMATICAL STRUCTURE

TIME: 1.5 HOURS

FULL MARKS: 25

INSTRUCTIONS:
1. The total marks of the questions are 30.
2. Candidates may attempt for all 30 marks.
3. In those cases where the marks obtained exceed 25 marks, the excess will be ignored.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. The missing data, if any, may be assumed suitably.

Q1 (a) Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logcally equivalent by developing a series of logical equivalences. [2]

(b) Show that $\forall x\,(P(x) \wedge Q(x))$ and $\forall x\, P(x) \wedge \forall x Q(x))$ are logically equivalent. [3]

Q2 (a) When the mixed quantifiers (i) $\forall x \exists y\, P(x,y)$ (ii) $\forall x \forall y\, P(x,y)$ are false? [2]

(b) Show that the hypothesis "It is not sunny this afternoon and it is colder than yesterday", "We will go swimming only if it is sunny", "If we do not go swimming, then we will take a canoe trip", and "If we take a canoe trip, then we will be home by sunset" lead to the conclusion "We will be home by sunset". [3]

Q3 (a) Find the matrix representing the relation R^2, where the matrix representing R is [2]

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

(b) If the relation R on a set A is transitive then $R^n \subseteq R$, where n is a natural number. [3]

Q4 Let $R = \{(a,d),(b,a),(b,c),(c,a),(c,d),(d,c)\}$ be the relation defined on a set $A = \{a,b,c,d\}$. Find the transitive closure of R using Warshall's Algorithm. [5]

Q5 (a) Define equivalence classes. What are the equivalence classes of 0 and 1 for congruence modulo 4? [2]

(b) Let R be an equivalence relation on a set A. Prove that the statements (i) a R b (ii) $[a]=[b]$ (iii) $[a] \cap [b] \neq \phi$ are equivalent. [3]

Q6 (a) How many reflexive and symmetric relations are there on a set with n elements? [2]

(b) Prove by contradiction that the number of primes are infinite. [3]

::::19-09-2017 E ::::

BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: BE
BRANCH: CSE

SEMESTER : III
SESSION : MO/17

SUBJECT: CS4101 DISCRETE MATHEMATICAL STRUCTURES

TIME: 3HRS

FULL MARKS: 60

INSTRUCTIONS:
1. The question paper contains 7 questions each of 12 marks and total 84 marks.
2. Candidates may attempt any 5 questions maximum of 60 marks.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

1(a) State Modus ponens law and Hypothetical syllogism law. [2]

(b) Let $L(x,y)$ be the statement "x loves y" where the domain for both x and y consists of all people in the world. Use quantifiers to express the statement: "There is somebody whom no one loves". [4]

(c) Show that the following argument is valid: [6]
If x is human then x is omnivorous. Rohit is not omnivorous. Therefore, Rohit is not human.

2(a) Suppose that the relation R on a set A is represented by the matrix [2]

$$M_R = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

Is R possesses the reflexive and symmetric property?

(b) Prove that the transitive closure of a relation R equals the connectivity relation R^*. [4]

(c) Find the zero-one matrix of the transitive closure of the relation R where [6]

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

3(a) What do you mean by the term Walk and Path in a graph. [2]

(b) The number of vertices of odd degree in a graph is always even. [4]

(c) Show that a simple graph with n vertices and k components can have at most $\frac{(n-k)(n-k+1)}{2}$. [6]

4(a) Define Euler circuit and Hamiltonian circuit. [2]

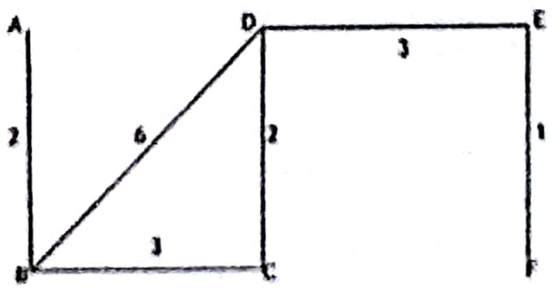
(b) Explain Konigsberg bridge problem using graph theory. [4]

(c) Prove that a given connected graph G is an Euler graph if and only if all vertices of G are of even degree. [6]

5(a) If in a graph G there is one and only one path between every pair of vertices, then prove that G is a tree. [2]

(b) Prove that a tree with n vertices have (n-1) edges. [4]

(c) Find a minimum spanning tree using Kruskal's algorithm of the labeled connected graph shown in the figure given below: [6]



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6(a) Define group. Give an example of a group. [2]

(b) If G is a group such that $(ab)^2 = a^2b^2,\ \forall a,b \in G$. Show that G must be abelian. [4]

(c) Define subgroup of a group G. Prove that a non-void subset H of a group G is a subgroup of G if and only if $a,b \in H \Rightarrow ab^{-1} \in H$. [6]

7(a) Consider the poset {1,2,3,4,5,6,7,8,9,10,11,12} with integer division as the partial order. Draw Hasse diagram for this poset. [2]

b) Proof by contraposition that if $x^2(y+3)$ is even then either x is even or y is odd, for all $x, y \in Z$. [4]

(c) If R and S are relations from A to B, prove that [6]

(i) $R^{-1} \subseteq S^{-1}$ when $R \subseteq S$

(ii) $(R \cup S)^{-1} = R^{-1} \cup S^{-1}$

:::::27/11/2017:::::E

BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(MID SEMESTER EXAMINATION)

CLASS: BE
BRANCH: CSE

SEMESTER: III
SESSION : MO/2018

SUBJECT : CS4101-DISCRETE MATHEMATICS STRUCTURE

TIME: 1.5 HOURS

FULL MARKS: 25

INSTRUCTIONS:
1. The total marks of the questions are 30.
2. Candidates may attempt for all 30 marks.
3. In those cases where the marks obtained exceed 25 marks, the excess will be ignored.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. The missing data, if any, may be assumed suitably.

Q1 (a) What is a statement? [2]
(b) Write the negations of the following statements: [3]
i) Raju plays cricket on Sunday and Sachin do walking on Saturday.
ii) 5 < x < 20.
iii) If n is divisible by 6 then n is divisible by 2 and n is divisible by 3.

Q2 (a) Use the Principle of Mathematical Induction to verify that, for any positive integer n, $6^n - 1$ is divisible by 5. [5]

Q3 (a) Express the statement "there is a number x such that when it is added to any number, the result is that number, and if it is multiplied by any number, the result is x" as a logical expression. [2]
(b) Without using truth tables, prove that $\sim(p \vee (\sim p \wedge q))$ and $(\sim p \wedge \sim q)$ are logically equivalent. [3]

Q4 (a) In each of the following situation, indicate whether *f=O(g)* or *f=Ω(g)* or *f=Θ(g)* and justify your answer. [5]
i) $f(n) = n \log n$, $g(n) = 10n \log 10n$
ii) *f(n) =n-100, g(n) =n-200*

Q5 (a) What is the difference between a relation and a function? Give an example of a relation which is not a function. [2]
(b) Define Transitive closure. Find the transitive closure of the following relation R={(1,1), (1,2), (2,2), (2,4), (3,2), (4,1), (4,3)}. [3]

Q6 (a) Give the definition of Equivalence Relation and an example. [2]
(b) Prove that *f(x)=Θ(g(x))* if and only if *f(x)=O(g(x))* and *g(x)=O(f(x))*. [3]

:::: 10/09/2018 E ::::::

BIRLA INSTITUTE OF TECHNOLOGY, MESRA, RANCHI
(END SEMESTER EXAMINATION)

CLASS: BE
BRANCH: CSE

SEMESTER : III
SESSION : MO/18

SUBJECT: CS4101-DISCRETE MATHEMATICS STRUCTURE

TIME: 03:00

FULL MARKS: 60

INSTRUCTIONS:
1. The question paper contains 7 questions each of 12 marks and total 84 marks.
2. Candidates may attempt any 5 questions maximum of 60 marks.
3. The missing data, if any, may be assumed suitably.
4. Before attempting the question paper, be sure that you have got the correct question paper.
5. Tables/Data hand book/Graph paper etc. to be supplied to the candidates in the examination hall.

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Q.1(a) Suppose the predicate $F(x, y, t)$ is used to represent the statement that person x can fool person y at time t. [2]

(b) Write the following statements in symbolic form: [4]
(i) "If my computations are correct and I pay the electric bill, then I will run out of money. If I don't pay the electric bill, the power will be turned off. Therefore, if I don't run out of money and the power is still on, then my computations are incorrect." Convert this argument into logical notations using the variables c, b, r, p for propositions of computations, electric bills, out of money and the power respectively.
(ii) The sum of two positive integers is always positive.

(c) Prove or Disprove: $[(p \wedge q) \rightarrow r] \rightarrow [\sim r \rightarrow (\sim p \vee \sim q)]$ is a tautology. [6]

Q.2(a) Prove that, for any integers x, y, the product xy is even if and only if either x is even or y is even. [4]

(b) Prove that the following expression is true for every natural numbers n, [8]

$$1.n + 2.(n-1) + 3.(n-2) + \cdots + (n-1).2 + n.1 = \frac{1}{6}n(n+1)(n+2)$$

Q.3(a) Prove that, if f is $o(g)$ (little-o) then f is $O(g)$ (Big-O). [4]

(b) Prove that, if a and r are real numbers and $r = 0$, then [4]

$$\sum_{j=0}^{n} ar^j = \begin{cases} \dfrac{ar^{n+1} - a}{r-1} & if\ r \neq 1 \\ (n+1)a & if\ r = 1 \end{cases}$$

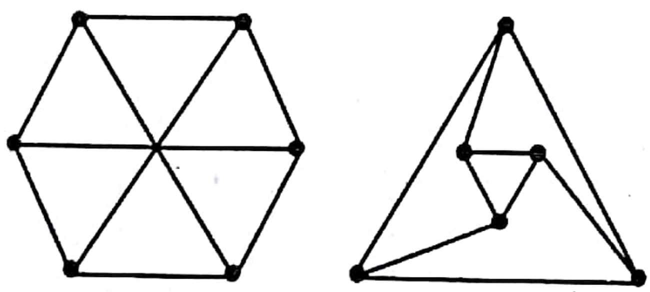
(c) Show that $\log_a x \in o(x)$ where a is a positive number different from 1. [4]

Q.4(a) Show that the "greater than or equal" relation $(\geq)$ is a partial ordering on the set of integers. [6]

(b) Find the reflexive, symmetric and transitive closures of the following relation [6]
$R = \{(1,1), (1,2), (2,2), (2,4), (3,2), (4,1)\}$.

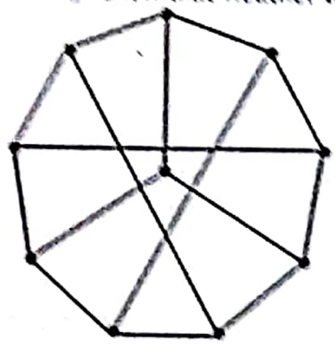
Q.5(a) Prove that, if a (p, q) graph G is k-connected then $q \geq \frac{pk}{2}$ [2]

(b) Draw the complements of the following two graphs. Are these complements isomorphic to each other? [4]



PTO

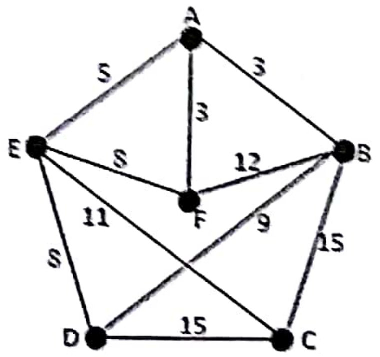
(c) i) Is the complete bipartite graph $K_{[illegible]}$ Hamiltonian? Justify your answer. [6]
ii) Show that weather the given graph is Hamiltonian or not. Justify your answer.



Q.6(a) Find the no. of vertices of degree 1 in a binary tree. [2]

(b) Let the tree T has 50 edges; the removal of certain edges from T yields two disjoint trees T_1 and T_2 such that the number of vertices in T_1 equals the number of edges of T_2. Determine the number of edges and vertices of T_1 and T_2. [4]

(c) Using Kruskal's Algorithm find a minimal spanning tree of the weighted graph given below [6]



Q.7(a) Prove that order of the subgroup divides the order of the group. [4]

(b) If $(G_1, *_1)$ and $(G_2, *_2)$ are groups, then show that $G = G_1 \times G_2$ i.e., $(G, *)$ is a group with binary operation $*$ defined by $(a_1, b_1) * (a_2, b_2) = (a_1 *_1 b_1, a_2 *_2 b_2)$. [8]

******26.11.18******E