Notation

MIP formulation

- Variables
- Constraints
- Objective

Extended mip formulation

- Additional variables

Notation

- a set $T = \{0, \ldots, m-1\}$ of m machine types
- a set $P = \{0, \ldots, n-1\}$ of n time periods

Production variables

- $x_p^t \in \{0, 1\}$ for $t \in T,\ p \in P$ with $x_p^t = 1 \Leftrightarrow$ an item of machine type t is produced in time period p

State variables

- $y_p^t \in \{0, 1\}$ for $t \in T,\ p \in P$ with $y_p^t = 1 \Leftrightarrow$ the machine is ready to produce an item of machine type t in time period p.

Stock variables

- $s_p^t \in \mathbb{R}_{\geq 0}$ for $t \in T,\ p \in P \cup \{-1\}$ with $s_p^t = \ell$ representing that ℓ items of machine type t are kept in stock in time period p

Transition variables

- $u_p^{ij} \in \{0, 1\}$ for $i \in T,\ j \in T,\ p \in P$ with $u_p^{ij} = 1 \Leftrightarrow$ the production changes from machine type i to machine type j in time period p

Initial stock constraints

- ensure an empty stock for each machine type before production begins

$$s^t_{-1} = 0 \text{ for } t \in T \tag{1}$$

Demand constraints

- ensure that the demand for each machine type is met by production and stock
- let $d_p^t \in \{0, 1\}$ be the given demand of machine type t in time period p

$$s_{p-1}^t + x_p^t = d_p^t + s_p^t \text{ for } t \in T,\ p \in P \tag{2}$$

State constraints

- ensures that the machine is ready to produce machine type t in time period p when producing machine type t in p

$$x_p^t \leq y_p^t \text{ for } t \in T,\ p \in P \tag{3}$$

Configuration constraints

- ensure that the machine is in exactly one state at any time

$$\sum_{t \in T} y_p^t = 1 \text{ for } p \in P \qquad (4)$$

Transition constraints

- ensure that the values of the transition variables are set correctly

$$u_p^{ij} \geq y_{p-1}^i + y_p^j - 1 \text{ for } i \in T,\ j \in T,\ p \in P \setminus \{0\} \qquad (5)$$

Entire formulation

- let $h \in \mathbb{R}$ be the stocking cost representing the cost for stocking an item (of any machine type) for one time period
- let $c_{ij} \in \mathbb{R}$ for $i \in T,\ j \in T$ be the transition cost representing the cost for changing the machine configuration from machine type i to machine type j

$$\min \sum_{t \in T} \sum_{p \in P} h s_p^t + \sum_{i \in T} \sum_{j \in T} \sum_{p \in P \setminus \{0\}} c_{ij} u_p^{ij}$$

$$\textbf{s.t.} \quad (1) - (5).$$

Predecessor state variables

- $v_p^t \in \{0, 1\}$ for $t \in T,\ p \in P \setminus \{0\}$ with $v_p^t = 1 \Leftrightarrow$ the machine is ready to produce an item of machine type t in time period p but not in time period $p - 1$

Successor state variables

- $w_p^t \in \{0, 1\}$ for $t \in T,\ p \in P \setminus \{0\}$ with $w_p^t = 1 \Leftrightarrow$ the machine is ready to produce an item of machine type t in time period p but not in time period $p + 1$