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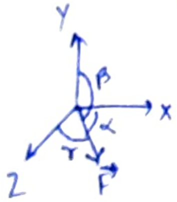
If two forces are represented as their magnitude and direction in the form of adjacent sides of a parallelogram (either head to tail or tail to head), then the corresponding diagonal of the parallelogram represents the resultant force.

$\vec{c} = \vec{a} + \vec{b}$

Direction cosines are the cosines of angles between the given vector and the coordinate axes.

For eg:- $\vec{F} = 2\hat{i} + 3\hat{j} + 5\hat{k}$

then $|\vec{F}| = \sqrt{2^2+3^2+5^2} = \sqrt{4+9+25} = \sqrt{38}$



$\cos\alpha = \frac{2\hat{i}}{|\vec{F}|} = \frac{2}{\sqrt{38}}$

$\cos\beta = \frac{3}{\sqrt{38}}$

$\cos\gamma = \frac{5}{\sqrt{38}}$

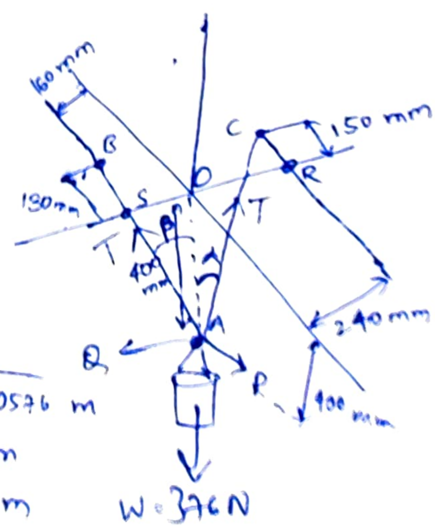
Moment of a couple is dependent on the perpendicular distance of the two forces which constitute the couple. Hence we can shift the couple anywhere on the body until and unless the perpendicular distance between the two constituting forces of the couple remain same. Hence a couple is a free vector.

(4) →

(5) Ans →

The centre of mass is always situated somewhere on the centroidal axis. Hence, calculating the first moment about the centroidal axis will always be zero.

(6) Ans →



Given, $W = 376\,N$

$\vec{AC} = \vec{AO} + \vec{OC}$

$d_{CR} = .15\ m$

$d_{OR} = .24\ m$

$\therefore d_{OC} = \sqrt{(.15)^2 + (.24)^2}\ m = \sqrt{.0225 + .0576}\ m$

$= \sqrt{.0801}\ m$

$= 0.283\ m$

$d_{AC}\cos\alpha = d_{AO}$

$= .4\ m$

Now, $d_{AC} = \sqrt{d_{OA}^2 + d_{OC}^2} = \sqrt{(.4)^2 + (.283)^2} = \sqrt{.16 + .0801} = \sqrt{.2401}$

$= 0.49\ m$

Hence, $\cos\alpha = \dfrac{d_{AO}}{d_{AC}} = \dfrac{0.4}{0.49} = 0.8163$

(c)(continued)

$d_{AB}\cos\beta = d_{AO} \Rightarrow \cos\beta = \frac{d_{AO}}{d_{AB}} = \frac{0.4}{d_{AB}}$

Now, for d_{AB},

$d_{AB} = \sqrt{d_{OB}^2 + d_{AO}^2}$

Now, for d_{OB},

$d_{OB} = \sqrt{(.16)^2 + (.13)^2} = \sqrt{.0169 + 0.0256} = \sqrt{0.0425}$

$= 0.206$ m

$\Rightarrow d_{AB} = \sqrt{(0.206)^2 + (0.4)^2} = \sqrt{.0425 + 0.16} = \sqrt{.2025} = 0.45$

$\therefore \cos\beta = \frac{0.4}{d_{AB}} = \frac{0.4}{0.45} = 0.8888$ m

Now,

$T_{AC} = T_{AB}$ (given) $= T$

$T\cos\alpha + T\cos\beta = 2W$

$\Rightarrow T\cos\alpha + T\cos\beta = 376$ ——— (i)

$\Rightarrow T = \frac{376}{\cos\alpha + \cos\beta} = \frac{376}{0.8163 + 0.8888} = \frac{376}{1.7051} = 220.51$ N

Now,

$T_{OC} = T_{AC}\sin\alpha$

$= T\sin\alpha = T\sqrt{1-\cos^2\alpha}$

$= T\sqrt{1 - 0.666} = 220.51 \times \sqrt{0.334}$

$= 220.51 \times 0.578 = 127.439$ N

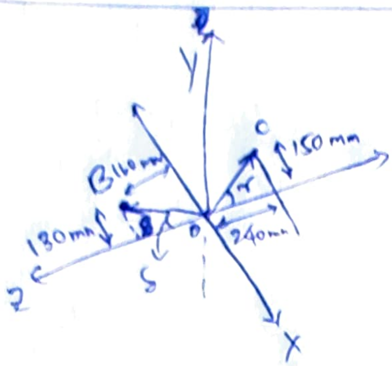
Similarly, $T_{OB} = T_{AB}\sin\beta = T\sqrt{1-\cos^2\beta}$

$= 220.51 \times \sqrt{1 - 0.7899} = 220.51 \times \sqrt{.2101}$

$= 220.51 \times 0.4584$

$= 101.074$ N

(G) Ans → (contd.)



$\cos\delta = \frac{160\ mm}{d_{OB}}$

$= \frac{160}{206} = 0.777$

And, $\cos\gamma = \frac{240\ mm}{d_{OC}} = \frac{240}{283}$

$= 0.848$

Now, equating the forces,

$T_{OC}\cos\gamma = T_{OB}\cos\delta + Q$ — (ii)

$T_{OC}\sin\gamma + T_{OB}\sin\delta = P$ — (iii)

From Equ (ii),

$Q = T_{OC}\cos\gamma - T_{OB}\cos\delta$

$= (127.439)(0.848) - (101.074)(0.777)$

$= 108.068 - 78.534$

$= 29.534\ N$

$\therefore Q = 29.534\ \hat{k}$ — Ans.

Similarly, $P = T_{OC}\sqrt{1-\cos^2\gamma} + T_{OB}\sqrt{1-\cos^2\delta}$

$= (127.439)\sqrt{1-.719} + 101.074 \times \sqrt{1-0.6037}$

$= (127.439)(\sqrt{.281}) + 101.074 \times \sqrt{.3963}$

$= 127.439 \times .5300 + 101.074 \times 0.6295$

$= 67.555 + 63.626\ N$

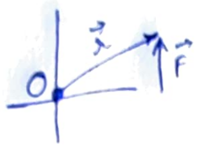
$= 131.181\ N$

$\therefore P = 131.181\ \hat{i}$ — Ans

(7) →

Given, $F = 4\hat{i} - 3\hat{j} + 5\hat{k}$

(a) Position vector of $A = \vec{OA} = \vec{r} = 2\hat{i} + 3\hat{j} - 4\hat{k}$



Moment about origin

$= \vec{r} \times \vec{F}$

$= (2\hat{i} + 3\hat{j} - 4\hat{k}) \times (4\hat{i} - 3\hat{j} + 5\hat{k})$

$= \hat{i}(15-12) - \hat{j}(10+16) + \hat{k}(-6-12)$

$= 3\hat{i} - 26\hat{j} - 18\hat{k}$. Ans

Rough

$$\begin{vmatrix} i & j & k \\ 2 & 3 & -4 \\ 4 & -3 & 5 \end{vmatrix}$$

$\hat{i}(15-12)$
$-\hat{j}(10+16)$
$k(-6-12)$

(b) Given, $\vec{r} = -8\hat{i} + 6\hat{j} - 10\hat{k}$

$\vec{F} = 4\hat{i} - 3\hat{j} + 5\hat{k}$

$\therefore$ Moment about origin $= \vec{r} \times \vec{F}$

$= (-8\hat{i} + 6\hat{j} - 10\hat{k}) \times (4\hat{i} - 3\hat{j} + 5\hat{k})$

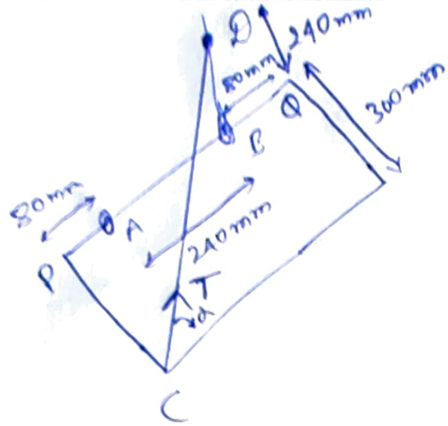
$= \hat{i}(30-30) - \hat{j}(-40-(-40))$
$+ \hat{k}(+24-24)$

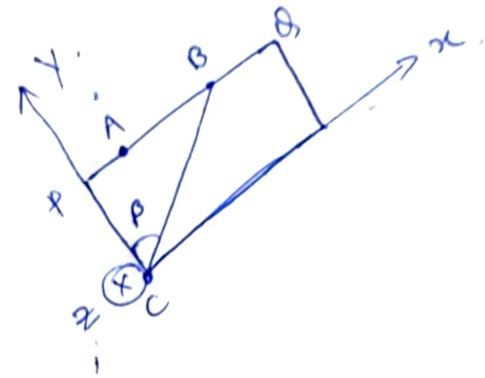
$= 0\hat{i} + 0\hat{j} + 0\hat{k}$

Rough

$$\begin{vmatrix} i & j & k \\ -8 & 6 & -10 \\ 4 & -3 & 5 \end{vmatrix}$$

(9) Ans →





Given, $T = 200$ N

Let, α be the angle between the wire DC and the rectangular plate's surface.

Now, $200 \sin\alpha$ is the vertically perpendicular force on the rectangular plate.

$$d_{BC} = \sqrt{d_{PC}^2 + d_{PB}^2} = \sqrt{(.3)^2 + (.32)^2} \text{ metres}$$

$$= \sqrt{.09 + .1024}\text{ m} = \sqrt{.1924}\text{ m}$$

$$= 0.4386 \text{ m}$$

$$d_{CD} = \sqrt{d_{BC}^2 + d_{BD}^2} = \sqrt{(0.4386)^2 + (.24)^2}$$

$$= \sqrt{.1924 + .0576} = \sqrt{0.25} = 0.5 \text{ m}$$

$$\therefore \sin\alpha = \frac{d_{BD}}{d_{CD}} = \frac{.24}{0.5} = 0.48$$

Now, $T_{CD}\cos\alpha$ is the component of T_{CD} along the plane of rectangular plate.

$$T_{CB} = 200\cos\alpha = 200 \times \sqrt{1-(0.48)^2} = 200 \times \sqrt{.7696}$$

$$= 200 \times 0.877 \text{ N} = 175.4 \text{ N} \quad \text{(a)}$$

(9) Ans

Now, $\cos\beta = \frac{300\ mm}{0.4386\ m}$ $\left[i.e.\ \frac{d_{PC}}{d_{BC}} \right]$

$= \frac{300\ mm}{438.6\ mm} = 0.6839$

$\therefore$ ~~Sin~~ $\sin\beta = \sqrt{1-\cos^2\beta} = \sqrt{1-0.4677} = \sqrt{0.5323}$

$= 0.7296$

Now, the ~~for~~ tension in the cable CD can be represented as,

~~$T_{CB}\cos\beta\,(\hat{j})$~~

$T_{CB}\cos\beta\ \hat{j} + T_{CB}\sin\beta\ \hat{i} + T_{CD}\sin\alpha\ \hat{k}$

$= \underset{\times\cos\alpha}{(200)}(0.6839)\ \hat{j} + \underset{\times\cos\alpha}{(200)}(0.7296)\ \hat{i} + 200\,(0.48)\ \hat{k}$

Using Eqn ⓐ,

$=$ ~~200~~ $(175.4)(0.6839)\ \hat{j} + (175.4)(0.7296)\ \hat{i} + (200)(0.48)\ \hat{k}$

$= 120.9\ \hat{j} + 127.97\ \hat{i} + 96\ \hat{k}$

$T = 127.97\ \hat{i} + 120.9\ \hat{j} + 96\ \hat{k}$ —— (11)

Now, For moment about A,

Moment because of T_x i.e. $127.97\ N\ \hat{i}$

$= d_{PC} \times T_x = (0.3\ m \times 127.97\ N)\ \hat{k}$

$= 38.391\ Nm\ \hat{k}$ —— ~~Ans.~~ (111)

Moment because of T_y i.e. $120.9\ N\ \hat{j}$

$= d_{AP} \times 120.9\ N\!m\,(-\hat{k})$

(9) Ans →

$d_{AP} \times 120.9 \quad (-\hat{k})$

$= (.08)\,m \times 120.9\,N \quad (-\hat{k})$

$= 9.672 \quad Nm \quad (-\hat{k})$ —— (IV)

Now Moment becz of T_3 ie $96\,N\,\hat{k}$,

$= \cancel{d_{AC} \times} \; (d_{AP} \times 96)\,\hat{j} + (d_{PC}) \times 96 \; (-\hat{i})$

$= (0.08\,m \times 96)\,\hat{j} + (.3)\,m \times 96\,(-\hat{i})$

$= 7.68\,\hat{j} + 28.8\,(-\hat{i})$ —— (V)

Now, using (III), (IV) & (V),

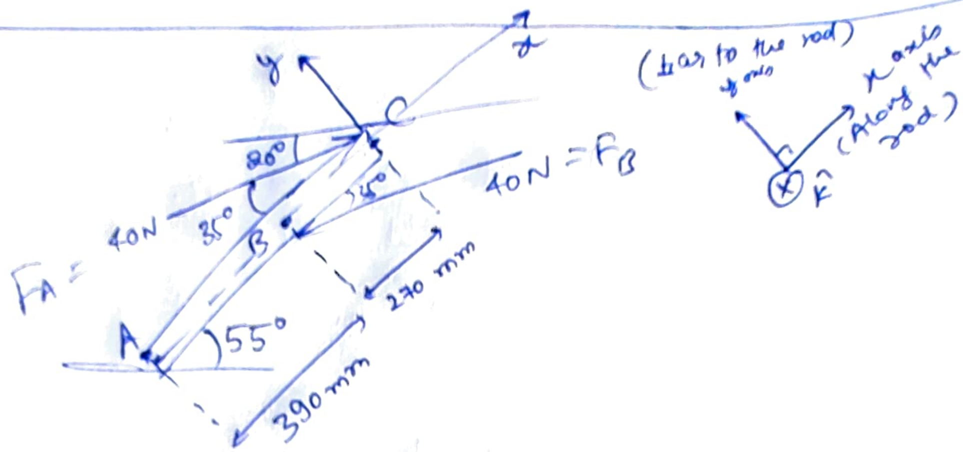
Total moment $= 38.391\,Nm\,\hat{k} + 9.672\,Nm\,(-\hat{k}) + 7.68\,Nm\,\hat{j} + 28.8\,Nm\,(-\hat{i})$

$= -(28.8\,Nm)\,\hat{i} + (7.68\,Nm)\,\hat{j} + (38.391 - 9.672)\,Nm\,\hat{k}$

$= -(28.8\,Nm)\,\hat{i} + (7.68\,Nm)\,\hat{j} + (28.719)\,Nm\,\hat{k}$

Ans

(10) Ans →



ⓐ $F_A = 40\cos 35^\circ \hat{i} + (40\sin 35^\circ)(-\hat{j})$

$F_B = 40\cos 35^\circ(-\hat{i}) + (40\sin 35)\hat{j}$

$F_{net} = 0$

Moment of all the components about A,

For F_A, (Assuming $\vec{r} = (390+270)mm\ \hat{i}$

Moment about A = $(40\cos 35^\circ \hat{i})\cdot(0) +$

$(390+270)mm\,\hat{i} \times 40\sin 35^\circ(-\hat{j})$

$= 0 + (0.66)\cdot(40\sin 35)\ -\hat{k}$

$= 26.4\sin 35^\circ\ -\hat{k}$ —— (I)

For F_B, Here $\vec{r} = .39\ m\ \hat{i}$

Moment about A = ~~0 + 40 sin 35°~~

$= 0 + (.39 + \cancel{.27})m\,\hat{i} \times 40\sin 35^\circ\,\hat{j}$

$= 15.6\sin 35^\circ\ \hat{k}$ —— (II)

Now, using (I) & (II), we get

∴ Net moment = $26.4\sin 35^\circ\ -\hat{k} + 15.6\sin 35^\circ\ \hat{k}$

$= 10.8\sin 35^\circ(-\hat{k})$ Ans

(10)

(b) → Moment of $\vec{F_A}$ about point A,

$= \vec{r}_{AC} \times \vec{F_A}$

$= (.39 + 0.27)(40) \sin 35^\circ$ Nm $-\hat{k}$

$= 26.4 \sin 35^\circ$ Nm $(-\hat{k})$ [as angle between $\vec{r}_{AC}$ and $\vec{F_A} = 35^\circ$]

Moment of $\vec{F_B}$ about point A,

$= \vec{r}_{AC} \times \vec{F_B}$

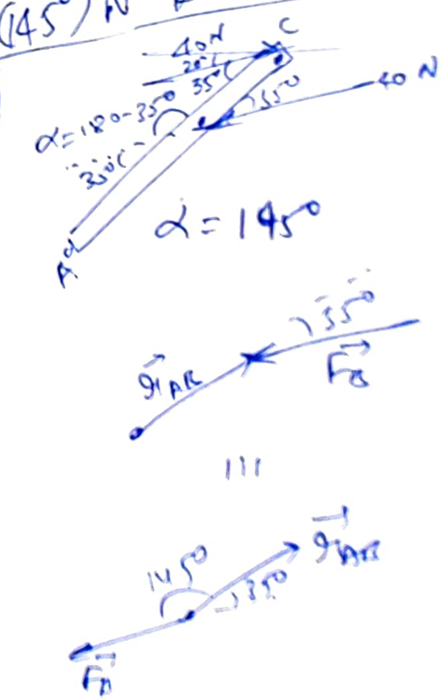
$= (.39)$m $(40) \sin(145^\circ)$ N $\hat{k}$

$= (0.39)(40) \sin 35$ Nm $\hat{k}$

[$\because \sin(180^\circ - \alpha) = \sin \alpha$]

& $\sin(145^\circ) = \sin(180^\circ - 35^\circ)$

$= 15.6 \sin 35^\circ$ Nm $\hat{k}$



$\therefore$ Net moment about A due to couple

$= (-26.4 \sin 35^\circ) + (15.6 \sin 35^\circ)$ Nm $\hat{k}$

$= (10.8 \sin 35^\circ)$ Nm $(-\hat{k})$ Ans

1 Ans→ Radius of gyration → If the total mass of a body is assumed to be at a point P, then the distance k for which $M_{total} \cdot k^2$ = Moment of inertia of the body about the axis passing through its centre of mass. Then this value of k is the radius of gyration of the body.

Moment of inertia ~~is the net~~ about any axis is the ~~st~~ integral summation of all the masses dm and the square of their distances from the ~~coc~~ given axis.

Mathematically,

$$\text{Moment of inertia} = \int_{\text{given object}} dm \cdot r^2 .$$

Parallel axis theorem states that if we know the moment of inertia about any axis then we can also calculate the moment of inertia for any other axis parallel to this axis.

if $I_A = Mk^2$ then $I_B = Mk^2 + Md^2$
(i.e. $I_A + Md^2$)