Copyright Notice

Classification

Motivations

Classification

Question	Answer "y"	
Is this email spam?	no	yes
Is the transaction fraudulent?	no	yes
Is the tumor malignant?	no	yes

y can only be one of two values

"binary classification"

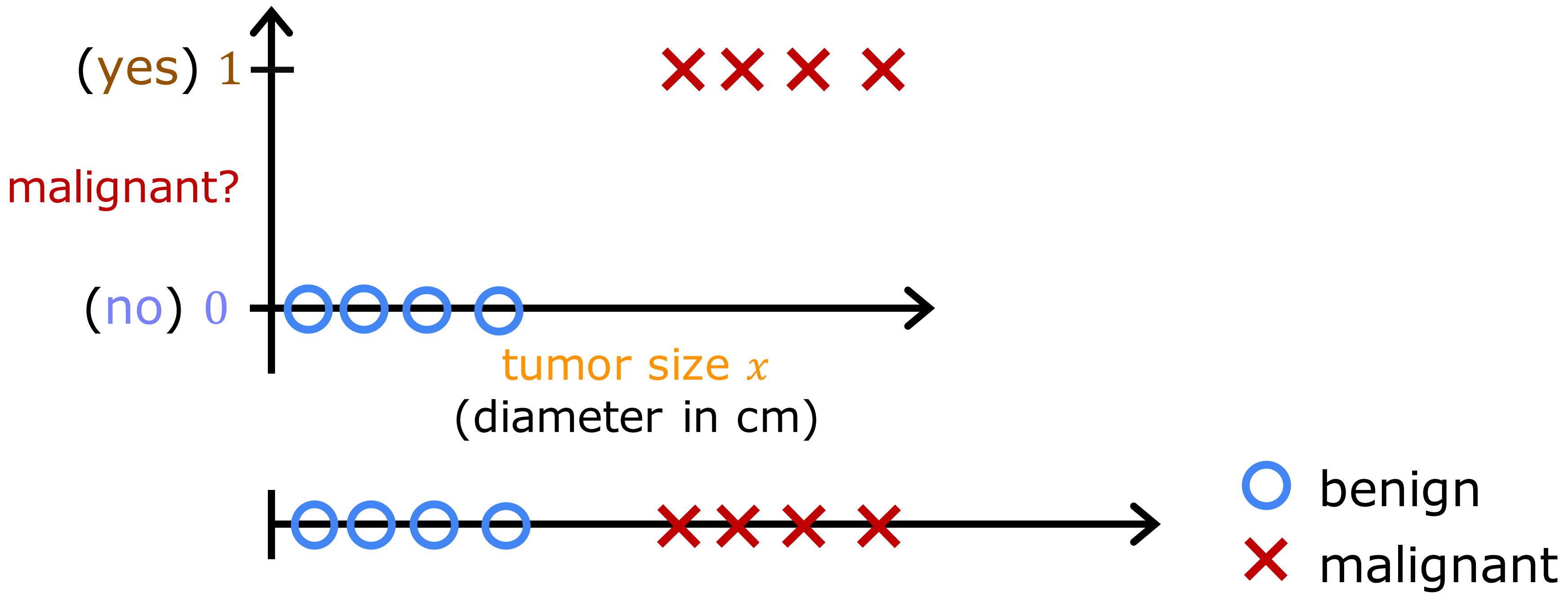
class = category

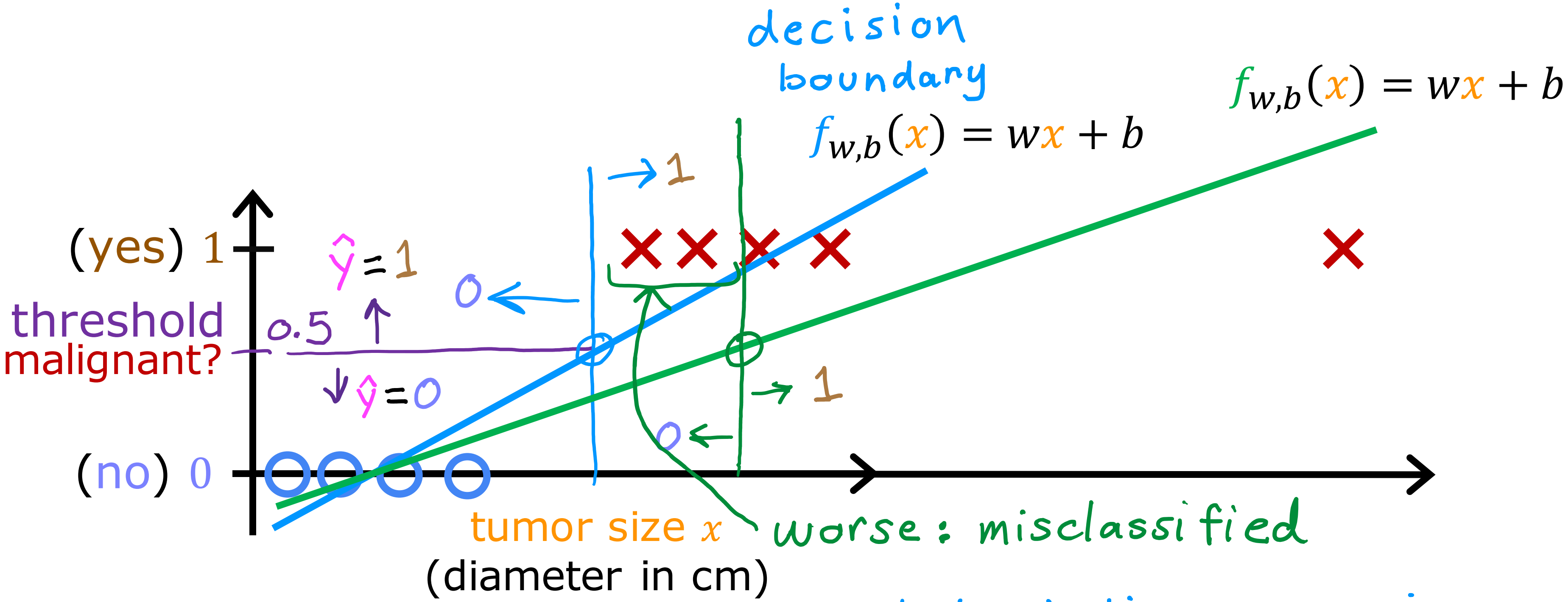
false	true
0	1

useful for classification

"negative class"
≠ "bad"
absence

"positive class"
≠ "good"
presence


(yes) 1
malignant?
(no) 0
tumor size x
(diameter in cm)
benign
malignant



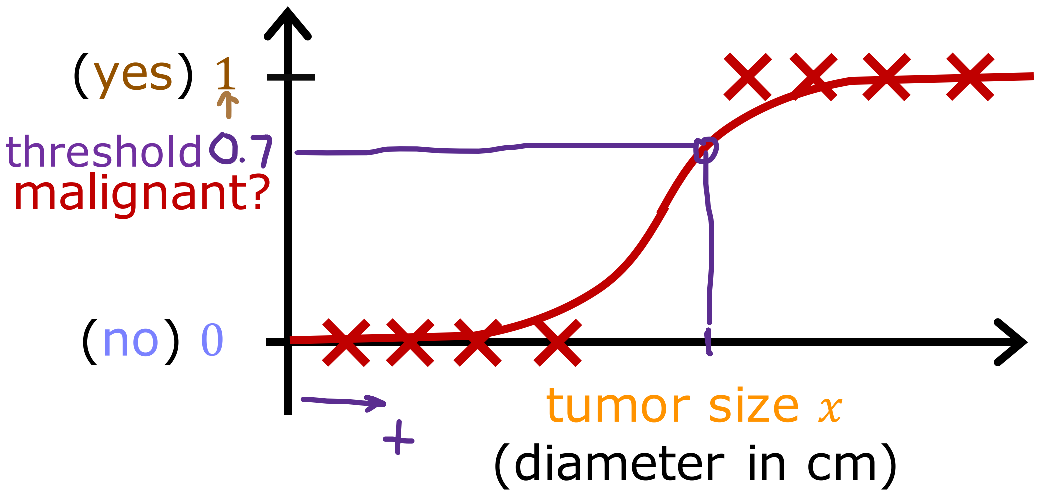
if $f_{w,b}(x) < 0.5 \rightarrow \hat{y} = 0$

if $f_{w,b}(x) \geq 0.5 \rightarrow \hat{y} = 1$

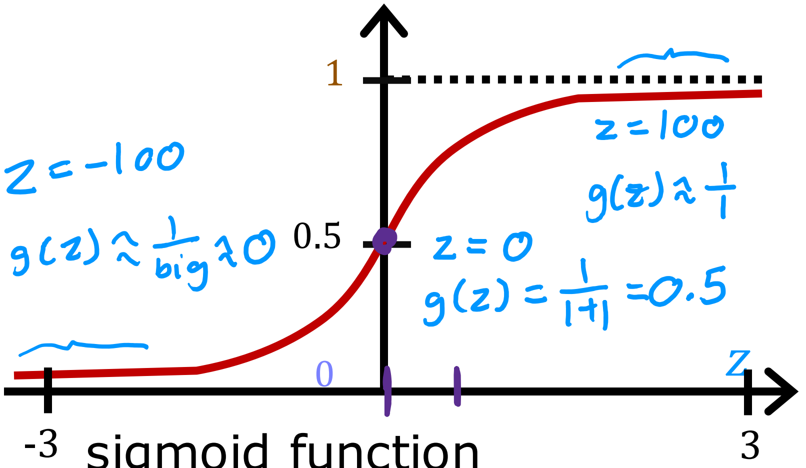
next: $\underbrace{\text{logistic regression}}_{\text{classification}}$

Classification

Logistic Regression



Want outputs between 0 and 1



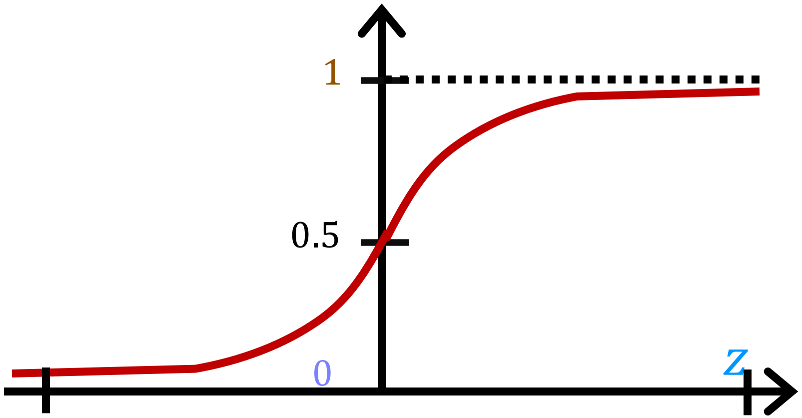
sigmoid function

logistic function

outputs between 0 and 1

$$g(z) = \frac{1}{1+e^{-z}} \qquad 0 < g(z) < 1$$

Want outputs between 0 and 1



sigmoid function

logistic function

outputs between 0 and 1

$g(z) = \frac{1}{1+e^{-z}}$ $\quad 0 < g(z) < 1$

$f_{\vec{w},b}(\vec{x})$

$\boxed{\vec{w} \cdot \vec{x} + b}$

$\downarrow$

z

$\downarrow$

$\boxed{g(z) = \frac{1}{1+e^{-z}}}$

$$f_{\vec{w},b}(\vec{x}) = g(\underbrace{\vec{w} \cdot \vec{x} + b}_{z}) = \frac{1}{1 + e^{-(\vec{w}\cdot\vec{x}+b)}}$$

"logistic regression"

$e \approx 2.7$

Interpretation of logistic regression output

$$f_{\vec{w},b}(\vec{x}) = \frac{1}{1+e^{-(\vec{w}\cdot\vec{x}+b)}}$$

"probability" that class is 1

Example:

x is "tumor size"

y is 0 (not malignant)

or 1 (malignant)

$$f_{\vec{w},b}(\vec{x}) = 0.7$$

70% chance that y is 1

P(1)

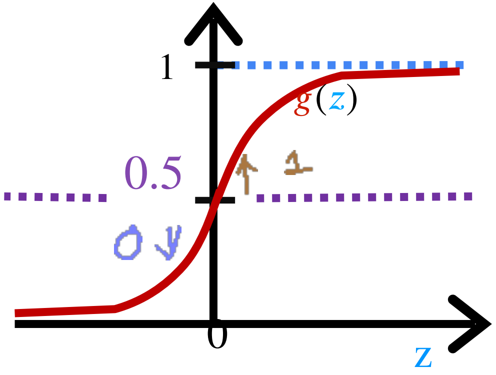
$$f_{\vec{w},b}(\vec{x}) = P(y=1|\vec{x};\vec{w},b)$$

Probability that y is 1,
given input $\vec{x}$, parameters $\vec{w},b$

$$P(y=0) + P(y=1) = 1$$

Classification

Decision Boundary



$f_{\vec{w},b}(\vec{x})$

$$\boxed{z = \vec{w} \cdot \vec{x} + b}$$

$$\boxed{g(z) = \frac{1}{1 + e^{-z}}}$$

$$f_{\vec{w},b}(\vec{x}) = g(\underbrace{\vec{w} \cdot \vec{x} + b}_{z}) = \frac{1}{1 + e^{-(\vec{w} \cdot \vec{x} + b)}}$$

$$= P(y = 1 \mid x; \vec{w}, b)$$ 0.7 0.3

Is $f_{\vec{w},b}(\vec{x}) \overset{\text{threshold}}{\geq} 0.5$?

Yes: $\hat{y} = 1$ No: $\hat{y} = 0$

When is $f_{\vec{w},b}(\vec{x}) \geq 0.5$? $g(z) \geq 0.5$

$z \geq 0$	$z < 0$
$\vec{w} \cdot \vec{x} + b \geq 0$	$\vec{w} \cdot \vec{x} + b < 0$
$\hat{y} = 1$	$\hat{y} = 0$

Decision boundary

$$f_{\vec{w},b}(\vec{x}) = g(z) = g(\underset{1}{w_1} x_1 + \underset{1}{w_2} x_2 + \underset{-3}{b})$$

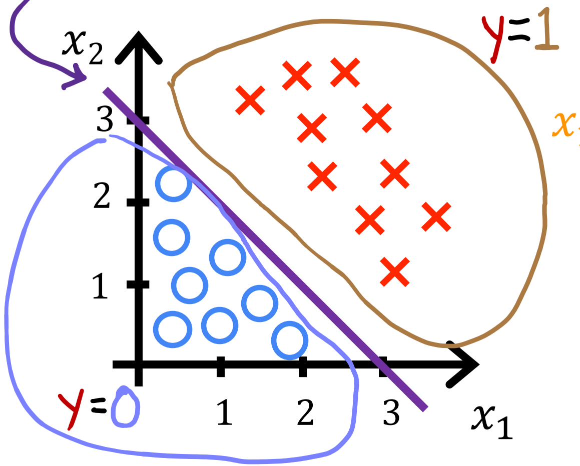
Decision boundary $\quad z = \vec{w} \cdot \vec{x} + b = 0$

$$z = x_1 + x_2 - 3 = 0$$

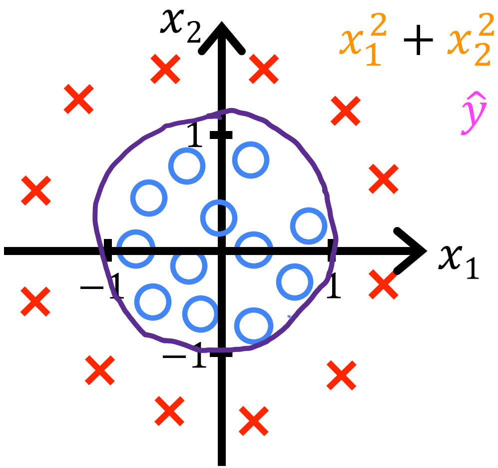
$$x_1 + x_2 = 3$$

$$x_1 + x_2 \geq 3$$

$$x_1 + x_2 < 3$$



Non-linear decision boundaries



$x_1^2 + x_2^2 \geq 1$

$\hat{y} = 1$

$x_1^2 + x_2^2 < 1$

$\hat{y} = 0$

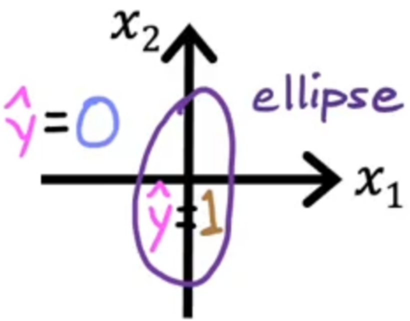
$$\overbrace{\underset{1}{w_1} x_1^2 + \underset{1}{w_2} x_2^2 + \underset{-1}{b}}^{z}$$

decision boundary

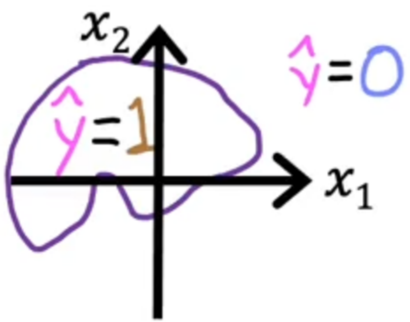
$z = x_1^2 + x_2^2 - 1 = 0$

$x_1^2 + x_2^2 = 1$

Non-linear decision boundaries



$$f_{\vec{w},b}(\vec{x}) = g(z) = g(w_1x_1 + w_2x_2 + w_3x_1^2 + w_4x_1x_2 + w_5x_2^2 + w_6x_1^3 + \cdots + b)$$



Cost Function

Cost Function for Logistic Regression

Training set

tumor size (cm) x_1	...	patient's age x_n	malignant? y
10		52	1
2		73	0
5		55	0
12		49	1
...		...	...

$i=1$
⋮
$i=m$

$i = 1, \ldots, m$ ← training examples

$j = 1, \ldots, n$ ← features

target y is 0 or 1

$$f_{\vec{w},b}(\vec{x}) = \frac{1}{1 + e^{-(\vec{w}\cdot\vec{x}+b)}}$$

How to choose $\vec{w} = [w_1 \quad w_2 \quad \cdots \quad w_n]$ and b?

Squared error cost

cost

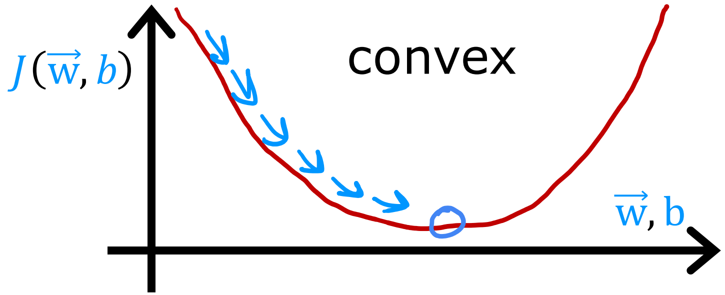
$$J(\vec{w},b) = \frac{1}{m}\sum_{i=1}^{m} \frac{1}{2}\left(f_{\vec{w},b}\left(\vec{x}^{(i)}\right) - y^{(i)}\right)^2$$

average of training set

loss $\boxed{L\left(f_{\vec{w},b}\left(\vec{x}^{(i)}\right), y^{(i)}\right)}$

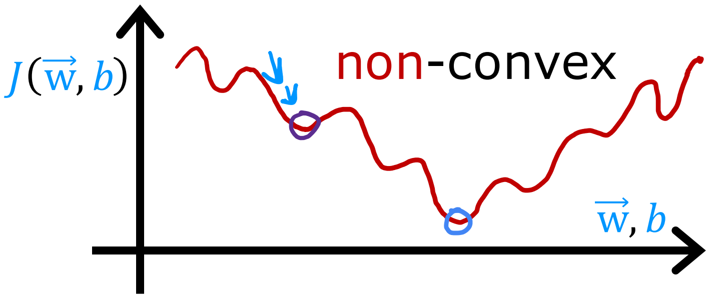
linear regression

$$f_{\vec{w},b}(\vec{x}) = \vec{w} \cdot \vec{x} + b$$



logistic regression

$$f_{\vec{w},b}(\vec{x}) = \frac{1}{1 + e^{-(\vec{w}\cdot\vec{x}+b)}}$$

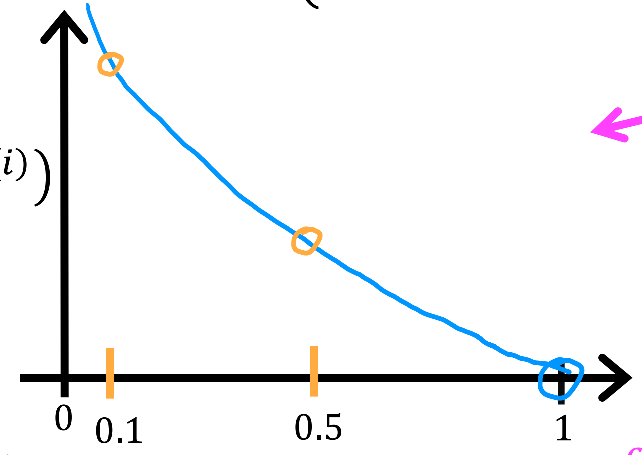


Logistic loss function

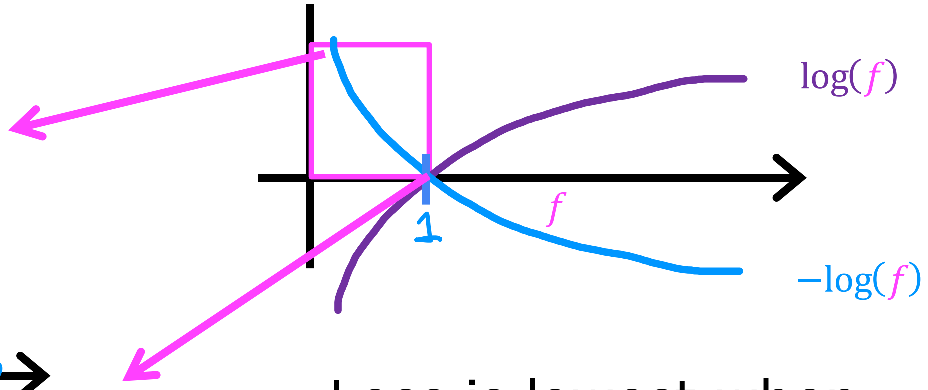
$$L\left(f_{\vec{\mathrm{w}},b}\left(\vec{\mathrm{x}}^{(i)}\right), y^{(i)}\right) = \begin{cases} -\log\left(f_{\vec{\mathrm{w}},b}\left(\vec{\mathrm{x}}^{(i)}\right)\right) & \text{if } y^{(i)} = 1 \\ -\log\left(1 - f_{\vec{\mathrm{w}},b}\left(\vec{\mathrm{x}}^{(i)}\right)\right) & \text{if } y^{(i)} = 0 \end{cases}$$

$L\left(f_{\vec{\mathrm{w}},b}\left(\vec{\mathrm{x}}^{(i)}\right), y^{(i)}\right)$

if $y^{(i)} = 1$



$f_{\vec{\mathrm{w}},b}\left(\vec{\mathrm{x}}^{(i)}\right)$



As $f_{\vec{\mathrm{w}},b}\left(\vec{\mathrm{x}}^{(i)}\right) \rightarrow 1$ then $\text{loss} \rightarrow 0$

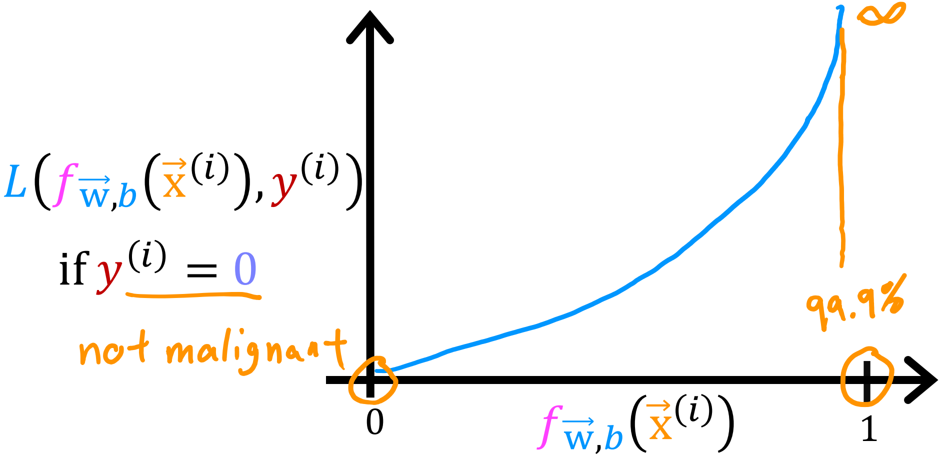
As $f_{\vec{\mathrm{w}},b}\left(\vec{\mathrm{x}}^{(i)}\right) \rightarrow 0$ then $\text{loss} \rightarrow \infty$

Loss is lowest when $f_{\vec{\mathrm{w}},b}\left(\vec{\mathrm{x}}^{(i)}\right)$ predicts close to true label $y^{(i)}$.

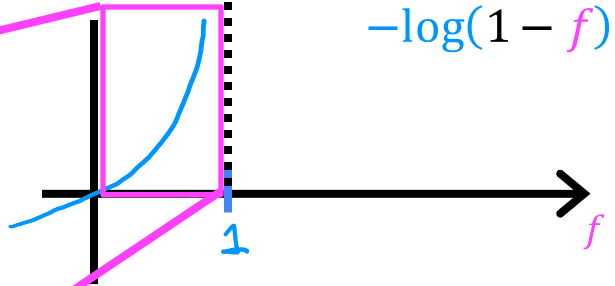
Logistic loss function

$$L\left(f_{\vec{w},b}\left(\vec{x}^{(i)}\right), y^{(i)}\right) = \begin{cases} -\log\left(f_{\vec{w},b}\left(\vec{x}^{(i)}\right)\right) & \text{if } y^{(i)} = 1 \\ -\log\left(1 - f_{\vec{w},b}\left(\vec{x}^{(i)}\right)\right) & \text{if } y^{(i)} = 0 \end{cases}$$

As $f_{\vec{w},b}(\vec{x}^{(i)}) \to 0$ then loss $\to 0$



As $f_{\vec{w},b}(\vec{x}^{(i)}) \to 1$ then loss $\to \infty$



The further prediction $f_{\vec{w},b}(\vec{x}^{(i)})$ is from target $y^{(i)}$, the higher the loss.

Cost

$$J(\vec{w},b) = \frac{1}{m}\sum_{i=1}^{m} L(\underbrace{f_{\vec{w},b}(\vec{x}^{(i)}),y^{(i)}}_{\text{loss}})$$

$$= \begin{cases} -\log\left(f_{\vec{w},b}(\vec{x}^{(i)})\right) & \text{if } y^{(i)} = 1 \\ -\log\left(1 - f_{\vec{w},b}(\vec{x}^{(i)})\right) & \text{if } y^{(i)} = 0 \end{cases}$$

convex

↳ can reach a global minimum

find w, b that minimize cost J

Cost Function

Simplified Cost Function for Logistic Regression

Simplified loss function

$$L\left(f_{\vec{w},b}\left(\vec{x}^{(i)}\right), y^{(i)}\right) = \begin{cases} \boxed{-\log\left(f_{\vec{w},b}\left(\vec{x}^{(i)}\right)\right)} & \text{if } y^{(i)} = 1 \\ -\log\left(1 - f_{\vec{w},b}\left(\vec{x}^{(i)}\right)\right) & \text{if } y^{(i)} = 0 \end{cases}$$

$$\boxed{L\left(f_{\vec{w},b}\left(\vec{x}^{(i)}\right), y^{(i)}\right) = -\underset{1}{y^{(i)}}\log\left(f_{\vec{w},b}\left(\vec{x}^{(i)}\right)\right) - \left(1 - y^{(i)}\right)\log\left(1 - f_{\vec{w},b}\left(\vec{x}^{(i)}\right)\right)}$$

$$\underbrace{(1-1)}_{0}$$

if $y^{(i)} = 1$:

$$L\left(f_{\vec{w},b}\left(\vec{x}^{(i)}\right), y^{(i)}\right) = -1 \log\left(f(\vec{x})\right)$$

Simplified loss function

$$L\left(f_{\vec{\mathrm{w}},b}\left(\vec{\mathrm{x}}^{(i)}\right), y^{(i)}\right) = \begin{cases} -\log\left(f_{\vec{\mathrm{w}},b}\left(\vec{\mathrm{x}}^{(i)}\right)\right) & \text{if } y^{(i)} = 1 \\ \boxed{-\log\left(1 - f_{\vec{\mathrm{w}},b}\left(\vec{\mathrm{x}}^{(i)}\right)\right)} & \text{if } y^{(i)} = 0 \end{cases}$$

$$\boxed{L\left(f_{\vec{\mathrm{w}},b}\left(\vec{\mathrm{x}}^{(i)}\right), y^{(i)}\right) = -\,y^{(i)}\log\left(f_{\vec{\mathrm{w}},b}\left(\vec{\mathrm{x}}^{(i)}\right)\right) - \left(1 - y^{(i)}\right)\log\left(1 - f_{\vec{\mathrm{w}},b}\left(\vec{\mathrm{x}}^{(i)}\right)\right)}$$

0 $(1-0)$

if $y^{(i)} = 1$:

$$L\left(f_{\vec{\mathrm{w}},b}\left(\vec{\mathrm{x}}^{(i)}\right), y^{(i)}\right) = -1 \log\left(f(\vec{x})\right)$$

if $y^{(i)} = 0$:

$$L\left(f_{\vec{\mathrm{w}},b}\left(\vec{\mathrm{x}}^{(i)}\right), y^{(i)}\right) = \underbrace{-\,(1-0)\log\left(1-f(\vec{x})\right)}$$

Simplified cost function

loss

$$L\left(f_{\vec{w},b}\left(\vec{x}^{(i)}\right), y^{(i)}\right) = -\underbrace{y^{(i)}\log\left(f_{\vec{w},b}\left(\vec{x}^{(i)}\right)\right) - \left(1 - y^{(i)}\right)\log\left(1 - f_{\vec{w},b}\left(\vec{x}^{(i)}\right)\right)}$$

cost

$$J(\vec{w}, b) = \frac{1}{m}\sum_{i=1}^{m}\left[\underbrace{L\left(f_{\vec{w},b}\left(\vec{x}^{(i)}\right), y^{(i)}\right)}\right]$$

convex

(single global minimum)

$$= -\frac{1}{m}\sum_{i=1}^{m}\left[y^{(i)}\log\left(f_{\vec{w},b}\left(\vec{x}^{(i)}\right)\right) + \left(1 - y^{(i)}\right)\log\left(1 - f_{\vec{w},b}\left(\vec{x}^{(i)}\right)\right)\right]$$

Gradient Descent

Gradient Descent Implementation

Training logistic regression

Find $\vec{\mathrm{w}}, b$

Given new $\vec{\mathrm{x}}$, output $f_{\vec{\mathrm{w}},b}(\vec{\mathrm{x}}) = \frac{1}{1+e^{-(\vec{\mathrm{w}}\cdot\vec{\mathrm{x}}+b)}}$

$$P(y = 1|\vec{\mathrm{x}}; \vec{\mathrm{w}}, b)$$

Gradient descent

cost

$$J(\vec{\mathrm{w}},b) = -\frac{1}{m}\sum_{i=1}^{m}\left[y^{(i)}\log\left(f_{\vec{\mathrm{w}},b}\left(\vec{\mathrm{x}}^{(i)}\right)\right) + \left(1-y^{(i)}\right)\log\left(1-f_{\vec{\mathrm{w}},b}\left(\vec{\mathrm{x}}^{(i)}\right)\right)\right]$$

repeat {

j=1...n

$$w_j = w_j - \alpha\underbrace{\frac{\partial}{\partial w_j}J(\vec{\mathrm{w}},b)}$$

$$\frac{\partial}{\partial w_j}J(\vec{\mathrm{w}},b) = \frac{1}{m}\sum_{i=1}^{m}\left(f_{\vec{\mathrm{w}},b}\left(\vec{\mathrm{x}}^{(i)}\right) - y^{(i)}\right)x_j^{(i)}$$

$$b = b - \alpha\underbrace{\frac{\partial}{\partial b}J(\vec{\mathrm{w}},b)}$$

$$\frac{\partial}{\partial b}J(\vec{\mathrm{w}},b) = \frac{1}{m}\sum_{i=1}^{m}\left(f_{\vec{\mathrm{w}},b}\left(\vec{\mathrm{x}}^{(i)}\right) - y^{(i)}\right)$$

} simultaneous updates

Gradient descent for logistic regression

repeat {

looks like linear regression!

$$w_j = w_j - \alpha \left[\frac{1}{m} \sum_{i=1}^{m} \left(f_{\vec{w},b}(\vec{x}^{(i)}) - y^{(i)} \right) x_j^{(i)} \right]$$

$$b = b - \alpha \left[\frac{1}{m} \sum_{i=1}^{m} \left(f_{\vec{w},b}(\vec{x}^{(i)}) - y^{(i)} \right) \right]$$

} simultaneous updates

Same concepts:

- Monitor gradient descent (learning curve)
- Vectorized implementation
- Feature scaling

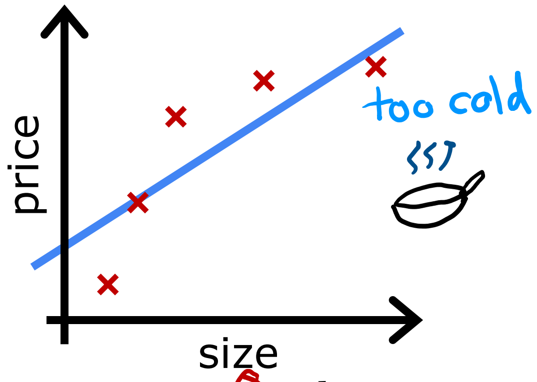
Linear regression $f_{\vec{w},b}(\vec{x}) = \vec{w} \cdot \vec{x} + b$

Logistic regression $f_{\vec{w},b}(\vec{x}) = \dfrac{1}{1 + e^{(-\vec{w} \cdot \vec{x} + b)}}$

Regularization to Reduce Overfitting

The Problem of Overfitting

Regression example

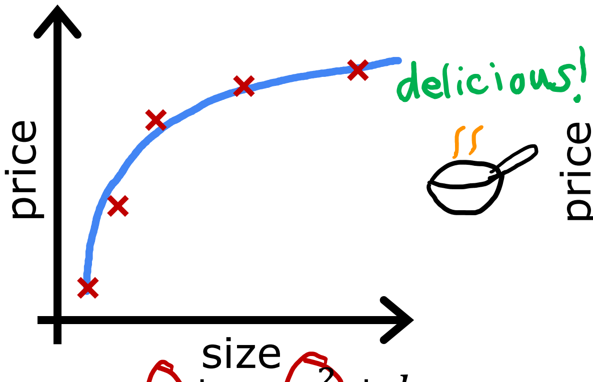


$w_1 x + b$

underfit

- Does not fit the training set well

high bias

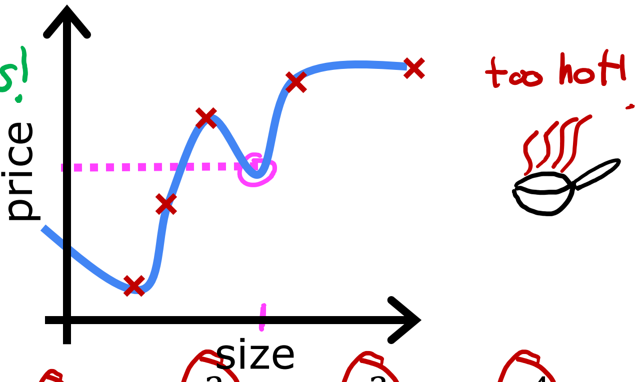


$w_1 x + w_2 x^2 + b$

just right

- Fits training set pretty well

generalization



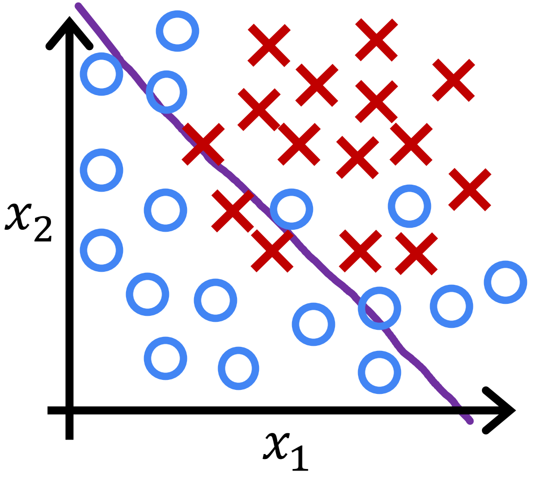
$w_1 x + w_2 x^2 + w_3 x^3 + w_4 x^4 + b$

overfit

- Fits the training set extremely well

high variance

Classification

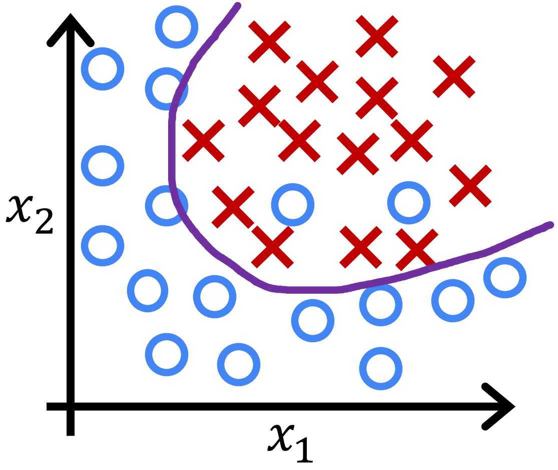


$$z = w_1 x_1 + w_2 x_2 + b$$

$$f_{\vec{w},b}(\vec{x}) = g(z)$$

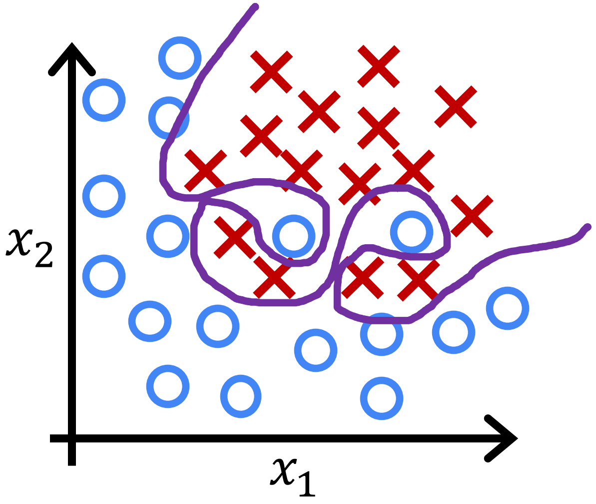
g is the sigmoid function

underfit high bias



$$z = w_1 x_1 + w_2 x_2 + w_3 x_1^2 + w_4 x_2^2 + w_5 x_1 x_2 + b$$

just right



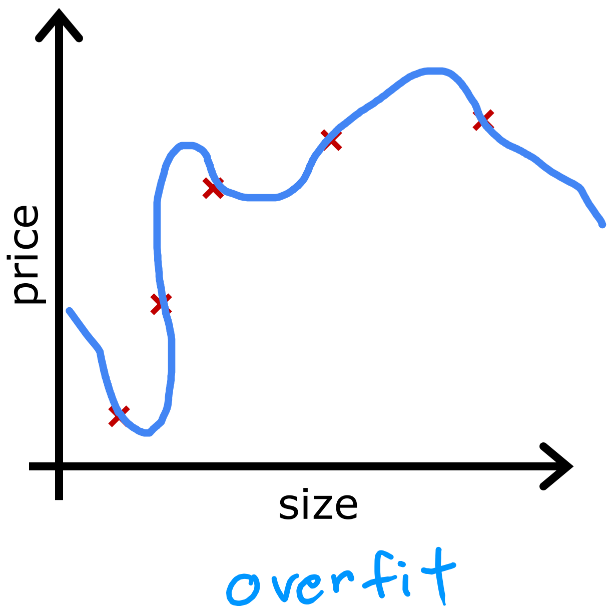
$$z = w_1 x_1 + w_2 x_2 + w_3 x_1^2 x_2 + w_4 x_1^2 x_2^2 + w_5 x_1^2 x_2^3 + w_6 x_1^3 x_2 + \cdots + b$$

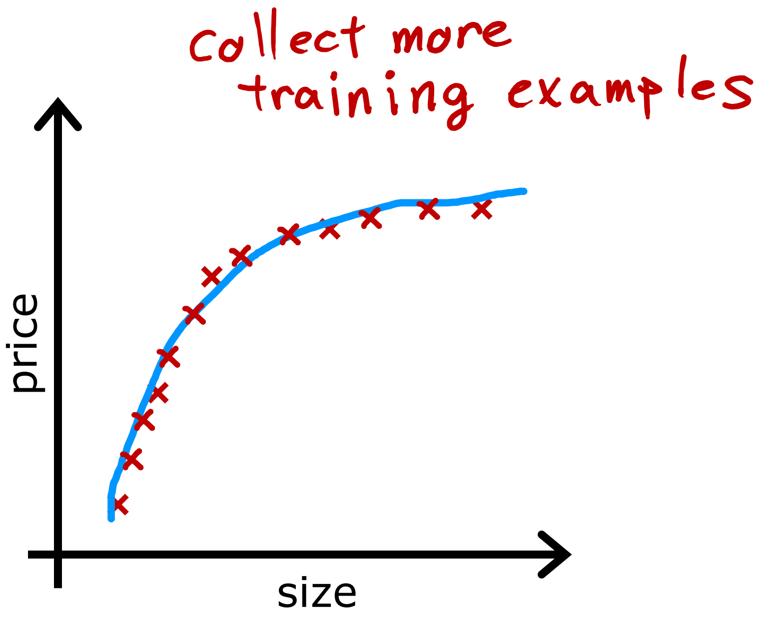
overfit

Regularization to Reduce Overfitting

Addressing Overfitting

Collect more training examples





Select features to include/exclude

size	bedrooms	floors	age	avg income	...	distance to coffee shop	price
x_1	x_2	x_3	x_4	x_5		x_{100}	y

all features

+

insufficient data

↓

overfit

selected features

size
bedrooms
age

just right

feature selection

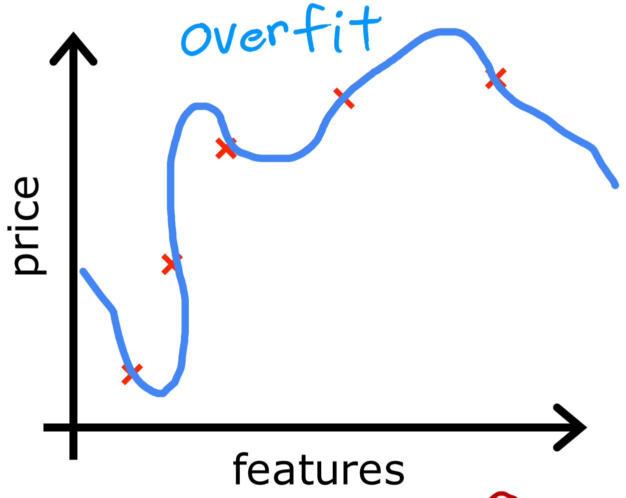
disadvantage

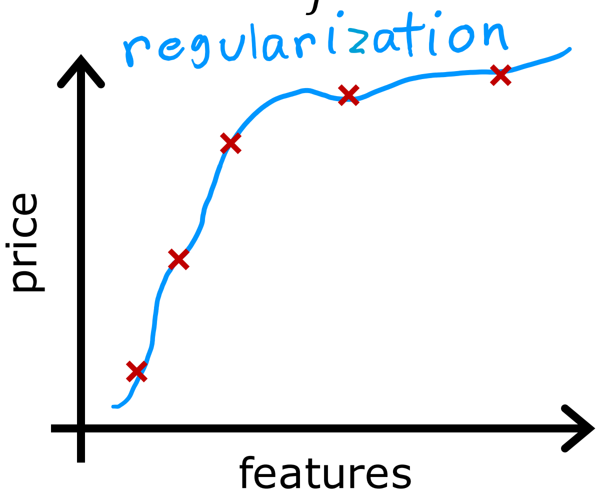
↓

useful features could be lost

Regularization

Reduce the size of parameters w_j





$$f(x) = 28x - 385x^2 + 39x^3 - \underbrace{\cancel{174}x^4}_{0} + 10$$

eliminate feature

large values for w_j

$$f(x) = 13x - 0.23x^2 + \underbrace{0.000014}x^3 - \underbrace{0.0001}\,x^4 + 10$$

small values for w_j

Addressing overfitting

Options

1. Collect more data

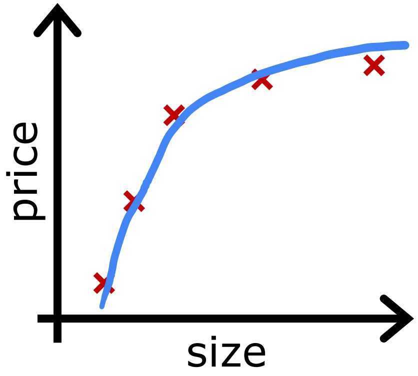
2. Select features
   – Feature selection

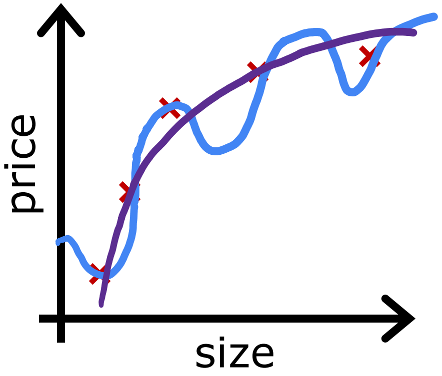
3. Reduce size of parameters
   – "Regularization"

Regularization to Reduce Overfitting

Cost Function with Regularization

Intuition





$w_1 x + w_2 x^2 + b$

$w_1 x + w_2 x^2 + \underbrace{\cancel{w_3 x^3}}_{\approx 0} + \underbrace{\cancel{w_4 x^4}}_{\approx 0} + b$

make w_3, w_4 really small (≈ 0)

$$\min_{\vec{w},b} \frac{1}{2m} \sum_{i=1}^{m} \left(f_{\vec{w},b}\left(\vec{x}^{(i)}\right) - y^{(i)}\right)^2 + 1000 \underbrace{w_3^2}_{0.001} + 1000 \underbrace{w_4^2}_{0.002}$$

Regularization

small values $w_1, w_2, \cdots, w_n, b$

simpler model $\quad w_3 \approx 0$

less likely to overfit $\quad w_4 \approx 0$

size	bedrooms	floors	age	avg income	...	distance to coffee shop	price
x_1	x_2	x_3	x_4	x_5		x_{100}	y

n features $\quad n = 100$

$w_1, w_1, w_2, \cdots, w_{100}, b$

$$J(\vec{w}, b) = \frac{1}{2m}\left[\sum_{i=1}^{m}\left(f_{\vec{w},b}\left(\vec{x}^{(i)}\right) - y^{(i)}\right)^2 + \overbrace{\frac{\lambda}{2m}\sum_{j=1}^{n} w_j^2}^{\text{regularization term}}\right]$$

"lambda"

regularization parameter

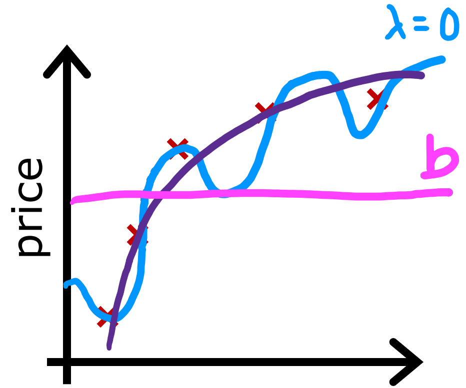
Regularization

$$\min_{\vec{w},b} J(\vec{w},b) = \min_{\vec{w},b}\left[\overbrace{\frac{1}{2m}\sum_{i=1}^{m}\left(f_{\vec{w},b}(\vec{x}^{(i)}) - y^{(i)}\right)^2}^{\text{mean squared error}} + \overbrace{\frac{\lambda}{2m}\sum_{j=1}^{n} w_j^2}^{\text{regularization term}}\right]$$

fit data

keep w_j small

λ balances both goals



choose $\lambda = 10^{10}$

$$f_{\vec{w},b}(\vec{x}) = \underset{\approx 0}{\cancel{w_1 x}} + \underset{\approx 0}{\cancel{w_2 x^2}} + \underset{\approx 0}{\cancel{w_3 x^3}} + \underset{\approx 0}{\cancel{w_4 x^4}} + b$$

$f(x) = b$

Choose λ

Regularization to Reduce Overfitting

Regularized Linear Regression

Regularized linear regression

$$\min_{\vec{w},b} J(\vec{w},b) = \min_{\vec{w},b}\left[\frac{1}{2m}\sum_{i=1}^{m}\left(f_{\vec{w},b}(\vec{x}^{(i)}) - y^{(i)}\right)^2 + \frac{\lambda}{2m}\sum_{j=1}^{n} w_j^2\right]$$

Gradient descent

repeat {

$$w_j = w_j - \alpha\frac{\partial}{\partial w_j}J(\vec{w},b) \qquad = \frac{1}{m}\sum_{i=1}^{m}\left(f_{\vec{w},b}(\vec{x}^{(i)}) - y^{(i)}\right)x_j^{(i)} + \frac{\lambda}{m}w_j$$

$j=1,\ldots,n$

$$b = b - \alpha\frac{\partial}{\partial b}J(\vec{w},b) \qquad = \frac{1}{m}\sum_{i=1}^{m}\left(f_{\vec{w},b}(\vec{x}^{(i)}) - y^{(i)}\right)$$

} simultaneous update

Implementing gradient descent

repeat {

$$w_j = w_j - \alpha \left[\frac{1}{m} \sum_{i=1}^{m} \left[\left(f_{\vec{w},b}(\vec{x}^{(i)}) - y^{(i)} \right) x_j^{(i)} \right] + \frac{\lambda}{m} w_j \right]$$

$$b = b - \alpha \frac{1}{m} \sum_{i=1}^{m} \left(f_{\vec{w},b}(\vec{x}^{(i)}) - y^{(i)} \right)$$

} simultaneous update $j = 1 \ldots n$

$$w_j = \underbrace{1 w_j - \alpha \frac{\lambda}{m} w_j}_{w_j \underbrace{\left(1 - \alpha \frac{\lambda}{m}\right)}_{\text{shrink } w_j}} \underbrace{- \alpha \frac{1}{m} \sum_{i=1}^{m} \left(f_{w,b}(\vec{x}^{(i)}) - y^{(i)} \right) x_j^{(i)}}_{\text{usual update}}$$

$$\alpha \frac{\lambda}{m}$$

$$0.01 \frac{1}{50} = 0.0002$$

$$w_j \underbrace{(1 - 0.0002)}_{0.9998}$$

How we get the derivative term (optional)

$$\frac{\partial}{\partial w_j} J(\vec{w}, b) = \frac{\partial}{\partial w_j}\left[\frac{1}{2m}\sum_{i=1}^{m}\left(\underbrace{f(\vec{x}^{(i)})}_{\vec{w}\cdot\vec{x}^{(i)}+b} - y^{(i)}\right)^2 + \frac{\lambda}{2m}\sum_{j=1}^{n} w_j^2\right]$$

$$= \frac{1}{\cancel{2}m}\sum_{i=1}^{m}\left[\left(\vec{w}\cdot\vec{x}^{(i)} + b - y^{(i)}\right)\cancel{2}x_j^{(i)}\right] + \frac{\lambda}{\cancel{2}m}\cancel{2}w_j \quad \text{No } \sum_{j=1}^{n}$$

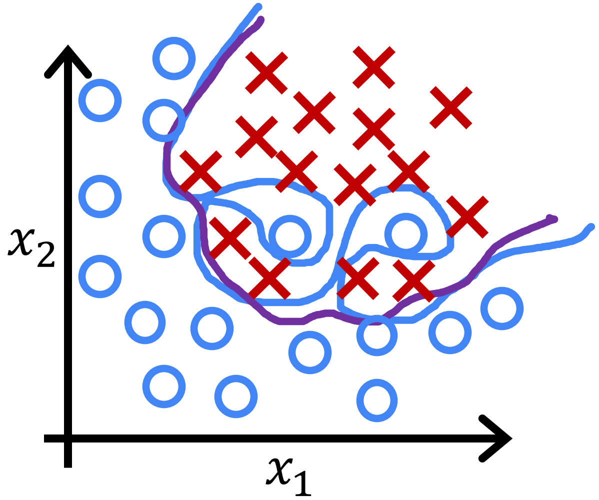
$$= \frac{1}{m}\sum_{i=1}^{m}\left[\left(\underbrace{\vec{w}\cdot\vec{x}^{(i)} + b}_{f(\vec{x})} - y^{(i)}\right)x_j^{(i)}\right] + \frac{\lambda}{m}w_j$$

$$= \frac{1}{m}\sum_{i=1}^{m}\left[\left(f_{\vec{w},b}(\vec{x}^{(i)}) - y^{(i)}\right)x_j^{(i)}\right] + \frac{\lambda}{m}w_j$$

Regularization to Reduce Overfitting

Regularized Logistic Regression

Regularized logistic regression



$$z = w_1x_1 + w_2x_2 + w_3x_1^2x_2 + w_4x_1^2x_2^2 + w_5x_1^2x_2^3 + \cdots + b$$

$$f_{\vec{\mathrm{w}},b}(\vec{\mathrm{x}}) = \frac{1}{1+e^{-z}}$$

Cost function

$$\underbrace{J(\vec{\mathrm{w}},b) = -\frac{1}{m}\sum_{i=1}^{m}\left[y^{(i)}\log\left(f_{\vec{\mathrm{w}},b}(\vec{\mathrm{x}}^{(i)})\right) + (1-y^{(i)})\log\left(1-f_{\vec{\mathrm{w}},b}(\vec{\mathrm{x}}^{(i)})\right)\right] + \frac{\lambda}{2m}\sum_{j=1}^{n} w_j^2}$$

$$\min_{\vec{w},b} J(\vec{\mathrm{w}},b) \rightarrow w_j \downarrow$$

Regularized logistic regression

$$J(\vec{w},b) = -\frac{1}{m}\sum_{i=1}^{m}\left[y^{(i)}\log\left(f_{\vec{w},b}(\vec{x}^{(i)})\right) + (1-y^{(i)})\log\left(1-f_{\vec{w},b}(\vec{x}^{(i)})\right)\right] + \frac{\lambda}{2m}\sum_{j=1}^{n} w_j^2$$

$\min_{\vec{w},b}$

Gradient descent

repeat {

$$w_j = w_j - \alpha\frac{\partial}{\partial w_j}J(\vec{w},b)$$

$j=1\ldots n$

$$b = b - \alpha\frac{\partial}{\partial b}J(\vec{w},b)$$

}

Looks same as for linear regression!

$$= \frac{1}{m}\sum_{i=1}^{m}\left[\left(f_{\vec{w},b}(\vec{x}^{(i)}) - y^{(i)}\right)x_j^{(i)}\right] + \frac{\lambda}{m}w_j$$

logistic regression

$$= \frac{1}{m}\sum_{i=1}^{m}\left(f_{\vec{w},b}(\vec{x}^{(i)}) - y^{(i)}\right)$$