Long Term Statistical Analysis of US Asset Classes

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Preface

The statistical analysis of the economic and financial situation together with long term investment studies has a history of more than one decade. One of the first studies on long term investments in U.S. equities and bonds appeared 1994 by Jeremy Siegel. Robert Shiller explored in 2000 the U.S. equity market and in 2005 the U.S. housing markets in view of upcoming market bubbles, which resulted in the dot com crisis 2000 and in the worldwide recession 2008/2009, respectively. Dimson, Marsh and Staunton published in 2002 a book investigating the long term investment opportunities in 19 countries and three global regions around the world. The Ibbotson SBBI Classic Yearbook is another source in book form with market results for bills, bonds, stocks and inflation. In addition several reports from banks are available online. These reports describe and investigate long term market behavior with the special view for investments in the different asset classes considered.

In his book *Stocks for the Long Run* Jeremy Siegel [1994, 2006] tried to support the hypothesis that investing in stocks always beats investing in bonds or other asset classes. The stock and bond markets investigated in his book date back to 1802. Time diversification and horizon based asset allocation are the key concept behind his book. The reason for this is that stocks are less riskier over longer periods than shorter ones.

In his book *Irrational Exuberance* Robert Shiller [2000] investigated the U.S. stock market and demonstrated that the market was overvalued at that time. Shiller got right when the NASDAQ peaked and the stock markets collapsed right after. The 2nd edition of his book [2005] analyzed the U.S. housing bubble which lead to the worldwide recession of 2008/2009.

Dimson, Marsh and Staunton [2002] provide in their book entitled *Triumph of the Optimists* an overview of long run rates of financial returns and related statistical quantities. The book includes the analysis of the equity risk premium, the long-run risks of equity investment, international diversification and many other strategic issues in investment.

Ibbotson's yearbook [2011] is one of the definite sources on long term market results for stocks, bonds, bills and inflation. The yearbook investigates the long run perspective of the U.S. financial market, describes the basic and derived series of the asset class time series, graphically displays index values, computes returns and performs a basic statistical analysis of the data. Furthermore, the yearbook considers some more specific aspects like firm size and returns, growth and value investing as well as liquidity investing. Further topics include the use of historical data in forecasting and optimization.

Several banks have contributed to the analysis and discussions in long term investments. This includes reports mostly in form of yearbooks on global investment returns. Examples are the *Global Investment Returns Yearbook* from Credit Suisse and the *Global Investment Returns Yearbook* from ABN AMRO. Clariden Leu has published a study on a *Historical Investment Analysis* ranging back to 1950 and Barclays has published the *Equity Gilt Studies* and *The Commodity Refiner* reports. The *Deutsche Bank* has contributed a report on *100 Years of Corporate Bond Returns*.

In this report, we investigate the long term dynamic evolution of bill, bond, stock and commodity prices. Amongst other analyses we introduce new stress and stability metrics to overcome traditional risk measures such as volatility, value at risk, and drawdown statistics. With these new measures we identify and quantify break points by Bayesian methods, detect extremes and outlying values by robust principal component approaches and explore non-stationary and multi-resolution aspects by wavelet analysis. The data set consists of a long-run study starting at the end of 1925 and covering the main asset classes from the point of view of an U.S. Investor.

The remaining of this report is organized as follows. In Part I we present and discuss the financial time series that we used in this investigation. In Part II we compute performance and risk measures as used in most investment analysis today. In Part III we investigate the dependencies between different asset classes. In Part IV we introduce new measures for an alternative computation of stress and stability measures, and in Part V we analyze portfolios consisting of different asset classes.

This is the first issue of the Rmetrics U.S. Handbook. It is produced by the Econophysics Group at ETH and the Rmetrics Associtіon Zurich.

Diethelm Würtz
Zurich, October 2011

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Major U.S. Asset Classes

Chapter 1

Markets and Data

Data sets for this investigation were taken from public and commercial databases. Major sources for commercially available data sets are the Ibbotson SBBI Yearbook [2011], the Center for Research in Security Prices, CRSP [2010] and the Global Financial Services, GFD [2011]. Major sources for publicly available data sets are the Federal Reserve database, FRED2 in St. Louis, the National Bureau of Statistical Research, NBER and Robert Shiller's web page with data from his book "Irrational Exuberance". The Shiller online data set allows to download long term U.S. stock market and housing data on a monthly, and older data on an annual, frequency base. The COW database, where COW abbreviates *Correlates of Wars*, is a project which collects and provides all kind of information about political conflicts and wars.

1.1 Description of Databases

In the following we describe various data sources that are well suited to obtain long term time series.

SBBI Database: Ibbotson's SBBI database is to our knowledge the most widely used monthly data set in performance analysis by financial advisors and fund managers for long term investment studies. It covers stocks, bonds, bills and inflation information, therefore the name SBBI. The time series are going back to the end of 1926. The SBBI is available through Morningstar [2011], including financial time series for U.S. large cap stocks, U.S. small cap stocks, U.S. long term government bonds, U.S. long term corporate bonds, domestic high yield corporate bonds, U.S. intermediate term bonds, U.S. 30-day treasury bills, and U.S. inflation.

CRSP Database: The CRSP, Center for Research in Security Prices, database provides data sets widely used in scientific investigations and publications. The data starts in the same year (1926) as the SBBI data sets and has monthly data records as well. Especially CRSP's Decile Indices on stocks are the datasets used when capitalization effects plays an important role in the stock market analysis.

GFD Database: The GFD, *Global Financial Database*, specializes in providing financial and economical data with some selected data sets that extend from the 1200's to present. The data they offer has been compiled from original sources but also from academic journals, news periodicals, books and numerous other archival sources, GFD [2011]. Recent data sets are recorded on a daily basis, the data sets going longer back in history have usually monthly, quarterly or annual frequencies. GFD provides time series of several indices for asset allocation tasks. These include a U.S. treasury bills index which is calculated from the yields on bills ranging from 1919 to date and a U.S. long term government index which is build from the Federal Reserve Board's 10-15 year treasury bond index and after 1941 from the index of bonds with a maturity of 10 years. GFD provides also data for the SP500 and the DJIA total return index. The SP500 data records after 1970 are (official) numbers from Standard and Poors, those before 1970 are calculated by GFD based on information from the Cowles Commission and from Standard and Poors itself. The data records for the DJIA index are from Down Jones. In the case of commodities, GFD comes with time series for the major commodity indices and several time series for commodity prices.

FRED2 Database: The FRED2, *Federal Reserve St. Louis Database*, allows to download thousands of economic and financial data sets covering U.S. financial data, interest rates, exchange rates and consumer and producer price indicies amongst others. Selected data sets from the NBER, *National Bureau of Economic Research*, and the BLS, *Bureau of Labor Statistics*, can also be downloaded from FRED2. From BLS this is includes the all urban consumer price index, CPI, which is exactly the same data as provided by Ibbotson. The NBER business cycles for recessions are also available in time series format on the FRED2 server. In addition Moodys seasoned AAA and BAA corporate monthly yield long term data sets can be downloaded from the FRED2 database. They are identical to those delivered by GFD.

NBER Database: The NBER, National Bureau of Statistical Research, is responsible for the U.S. business cycles data describing expansions and contractions in the U.S. economy. We describe the economic situation by considering the monthly time series data of the NBER for recessions, of the consumer price index measuring inflation and of the gross domestic product. The datsets are most easily available from the FRED2 data base of the Federal Reserve in St. Louis.

BLS Database: The BLS, Bureau of Labor Statistics, provides long term data sets on labour statistics and related time series. The all urban consumer price index, short CPI, provides monthly information on changes in the prices paid by urban consumers for a representative basket of goods and services. The numbers are calculated by the Bureau of Labor Statistics, BLS. The data provided by BLS is based upon a 1982 base of 100, the monthly data set we have downloaded from the FRED2 database ranges back to 1913. The CPI is also available from Shiller's online data set, from the "US Inflation Calculator" website and from GFD. Historical inflation rates on a monthly frequency ranging from 1914 to present are listed in a table on the "US Inflation Calculator" website. The rates of inflation are calculated using the CPI from BLS.

COW Database: The COW, *Correlates of Wars*, can be used to explore and to explain relations between market data and political conflicts. Therefore we listed some calendar dates of wars and political crisis.

1.2 Description of Data Sets

In this section we briefly describe the time series we used for the analyses in the following chapters.

BILL: As a proxy for money market investments we use the three month U.S. treasury bills from the FRED2 database. Index levels are calculated from the FRED2 Treasury Bill Rates TB3MS ranging back to January 1934. For earlier dates we refer to the SBBI/GFD databases. We abrreviate this time series in the following by *BILL*.

ITGOV: As a proxy for U.S. intermediate term government bonds we use *Barclays Capital U.S. 3 to 7 Years Treasury Bond Index*. Index levels are calculated from iShares' investable ETF Fund IEI ranging back until May 2007. For earlier dates we refer to the SBBI database. We abrreviate this time series in the following by *ITGOV*.

LTGOV: As a proxy for the U.S. long term government bonds we use *Barclays Capital U.S. 10 to 20 Years Treasury Bond Index*. Index levels are calculated from iShares' investable ETF Fund TLH ranging back until May 2007. For earlier dates we refer to the SBBI database. We abrreviate this time series in the following by *LTGOV*.

LTCORP: As a proxy for the U.S. long term corporate bonds we use the *iBoxx USD Liquid Investment Grade Index*. Index levels are calculated from iShares' investable ETF Fund LQD ranging back until August 2002. For earlier dates we refer to the SBBI database. We abrreviate this time series in the following by *LTCORP*.

HYCORP: As a proxy for the U.S. high yield corporate bonds we use the *iBoxx USD Liquid High Yield Index*. Index levels are calculated from iShares' investable ETF Fund LQD ranging back until May 2007. For earlier dates we refer to the Morningstar database. We abrreviate this time series in the following by *HYCORP*.

LCAP: As a proxy for the U.S. large cap stocks we use the *SP500* total return index. Index levels are calculated from iShares' investable ETF Fund IVV ranging back until March 2001. For earlier dates we refer to the Standard and Poors, SBBI and GFD databases. We abrreviate this time series in the following by *LCAP*.

SCAP: As a proxy for the U.S. small cap stocks we use the *Russell2000* total return index. Index levels are calculated from iShares' investable ETF Fund IWM ranging back until March 2001. For earlier dates we refer to SBBI and GFD databases. We abrreviate this time series in the following by *SCAP*.

CRB: As a proxy for commodities we use the *Thompson Reuters Jeffries CRB* total return index. Index levels are taken from the datafile provided on Jeffries website ranging back until January 1994. For earlier dates we refer to BLS, CRB and GFD databases. We abrreviate this time series in the following by *CRB*.

CPI: As a proxy for inflation we take the U.S. Consumer Price Index as available from the FRED2 database. The data is also part of Ibbotson's SBBI Classical Yearbook and Global Financial's GFD databases. We abrreviate this time series in the following by *CPI*.

1.3 SUMMARY

The following tables summarize information about the data providers (1.1) and the data sets (1.2) discussed in the previous sections, as well as information about business cycle expansion and contraction dates (1.3), about political conflicts and wars (1.4) and about economic and financial disasters (1.5).

Table of Data Providers

```
Commercially Provided:
    SBBI      Ibbotson SBBI Yearbook                     www.morningstar.com
    CRSP      Center for Research in Security Prices     www.crsp.com
    GFD       Global Financial Data                      www.globalfinancialdata.com

Public Available:
    FRED2     Federal Reserve St. Louis                  research.stlouisfed.org
    NBER      National Bureau of Statistical Research    www.nber.org
    COW       Correlates of Wars                         www.correlatesofwar.org
```

FIGURE 1.1: Listing of database symbols, the names of the databases, and the web address of the major commercial and public data sources providing long term time series.

Table of Data Sets

```
Symbol:     Provider:   Description:

BILL        SBBI        U.S. 30-day Treasury Bill
ITGOV       SBBI        U.S. Intermediate Term Bonds
LTGOV       SBBI        U.S. Long Term Government Bonds
LTCORP      SBBI        U.S. Long Term Corporate Bonds
HYCORP      SBBI        Domestic High Yield Corporate Bonds

LCAP        SBBI        U.S. Large Cap Stocks
SCAP        SBBI        U.S. Small Cap Stocks

CRB         GFD         Commodity Research Bureau Index

CPI         FRED2       U.S. Consumer Price Index
```

FIGURE 1.2: Listing of symbols, data providers, and short description of the time series used in this investigation.

Table of Business Cycle Expansion and Contractions

Peak	Trough	Contraction	Expansion		Cycle
		PT	TP	TT	PP
October 1926(III)	November 1927 (IV)	13	27	40	41
August 1929(III)	March 1933 (I)	43	21	64	34
May 1937(II)	June 1938 (II)	13	50	63	93
February 1945(I)	October 1945 (IV)	8	80	88	93
November 1948(IV)	October 1949 (IV)	11	37	48	45
July 1953(II)	May 1954 (II)	10	45	55	56
August 1957(III)	April 1958 (II)	8	39	47	49
April 1960(II)	February 1961 (I)	10	24	34	32
December 1969(IV)	November 1970 (IV)	11	106	117	116
November 1973(IV)	March 1975 (I)	16	36	52	47
January 1980(I)	July 1980 (III)	6	58	64	74
July 1981(III)	November 1982 (IV)	16	12	28	18
July 1990(III)	March 1991(I)	8	92	100	108
March 2001(I)	November 2001 (IV)	8	120	128	128
December 2007 (IV)	June 2009 (II)	18	73	91	81
Average, all cycles:					
1919-1945 (6 cycles)		18	35	53	53
1945-2009 (11 cycles)		11	59	73	66

FIGURE 1.3: Listing of the U.S. recession periods back to 1923 together with the average of all cycles for the periods 1919-1945, and 1919-1945. The numbers count the length of the recession periods where `PT` abbreviates "Peak to Trough", `TP` "Previous Trough to Peak", `TT`" Trough from Previous Trough", and `PP` "Peak from Previous Peak". *Source: www.nber.org.*

Table of Political Conflicts and Wars

```
1936 Treaty of Versailles
1939 Poland Attack
1944 D Day
1950 Korean War
1956 Suez Crisis
1961 Berlin Wall
1963 Vietnam War Major Expansions
1973 Jom Kippur War
1990 Gulf War
2001 September 11 Attack
```

FIGURE 1.4: Listing of the years of the major political crisis and wars since 1925. *Source: www.correlatesofwar.org.*

Table of Economic and Financial Disasters

```
1929 Great Depression
1987 Black Monday
1997 Asia FX Crisis (Thailand)
1998 LCTM Disaster
2000 Dot Com Bubble
2008 Sub Prime Crisis
```

FIGURE 1.5: Listing of the years of the major economic and financial disasters since 1925.

Part II

Asset Statistics

Chapter 2

Inroduction

In this Part we compute statistical performance and risk measures of the U.S. asset classes which are usually found in fund reports. We analyze the growth of the wealth indices, the total returns, the risk premiums, the effect of holding periods, the drawdowns and recovery times, and the market timing effects.

In the following we will treat wealth indices in chapter 3 and total returns for the different asset classes in chapter 4. We will look at the risk premiums in chapter 5, at the holding periods in chapter 6, and at the drawdowns and recovery time in chapter 7. The market timing will be discussed in chapter 8.

CHAPTER 3

WEALTH INDICES

The wealth index is the cumulated growth of 1 USD over time and the returns describe its growth rate. Wealth indices are calculated either from (i) the index (or price) series by normalizing the records such that the first value at December 1925 is one, or directly from (ii) the returns series. The approach that is better suited dependens on the form in which the data is available from the database. For comparison reasons we calculate besides the nominal wealth also the inflation adjusted or so called real wealth series.

Monthly wealth indices W_t^m are calculated from the monthly indices I_t^m or from the monthly returns series R_t^m

$$W_t^m = \frac{I_t^m}{I_{t=0}^m}$$

$$R_t^m = \frac{I_t^m - I_{t-1}^m}{I_{t-1}^m}$$

$$W_t^m = W_{t-1}^m (1 + R_t^m) = W_{t=0}^m \prod_{\tau=1}^{t} (1 + R_\tau^m)$$

Here t numbers the end-of-month dates, where $t = 0$ belongs to the starting month *1925-12-31*, the millenium date *1999-12-31* is $t = 889$, and m stands for monthly.

Monthly real total returns or inflation adjusted total returns are calculated from the geometric difference between nominal returns and the inflation rate, CPI.

$$R_t^{real} = \frac{1 + R_t}{1 + R_t^{CPI}} - 1$$

From this we calculate the inflation adjusted wealth indices which are also called real wealth indices.

We have calculated the nominal wealth indices for bills, bonds, stocks, and commodities for four different time horizons starting 1925, 1950, 1975, and 2000. The initial wealth for each time horizon is 1 USD. In addition, tables give the values of the wealth indices on a five year basis, and a one year basis for the most recent index, starting 2000. We have also calculated the real wealth indices of the above mentioned asset classes for the same time horizons in order to show the effect of inflation. The initial wealth is again 1 USD. We conclude this section with graphs that visually compare the nominal and real wealth indices for every asset class.

3.1 NOMINAL WEALTH INDEX CHARTS

Nominal Wealth Index 1925

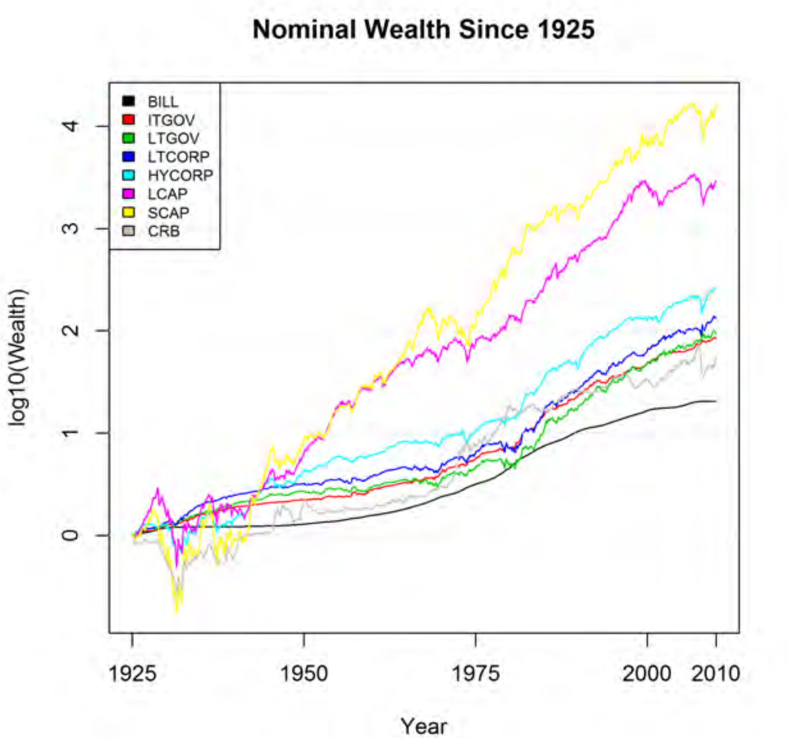

FIGURE 3.1: Nominal monthly wealth series for 8 asset classes describing the growth of 1 USD over time. The series frequency is given by end of month dates, starting in December 1925.

Nominal Wealth Index 1950

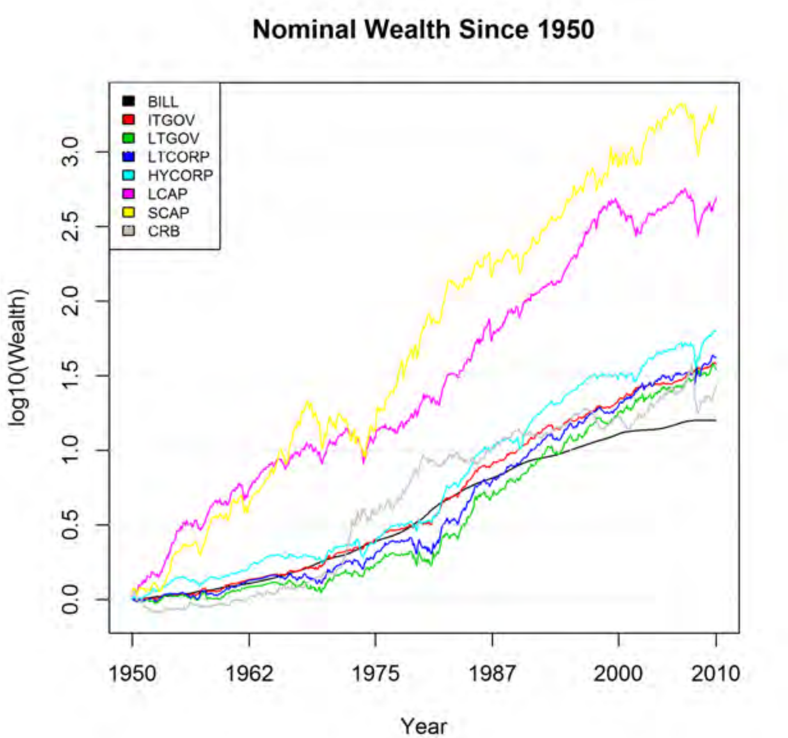

FIGURE 3.2: Nominal monthly wealth series for 8 asset classes describing the growth of 1 USD over time. The series frequency is given by end of month dates, starting in December 1950.

Nominal Wealth Index 1975

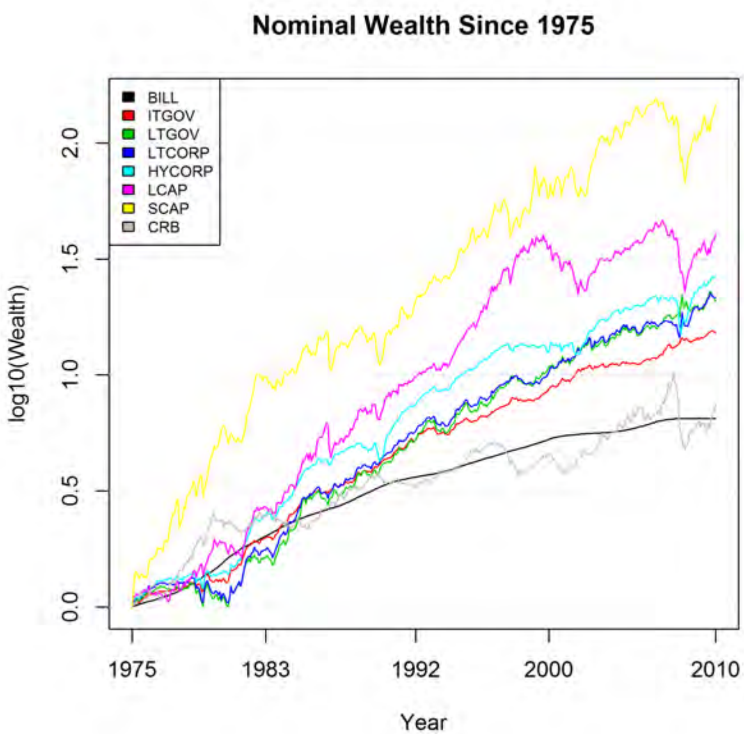

FIGURE 3.3: Nominal monthly wealth series for 8 asset classes describing the growth of 1 USD over time. The series frequency is given by end of month dates, starting in December 1975.

Nominal Wealth Index 2000

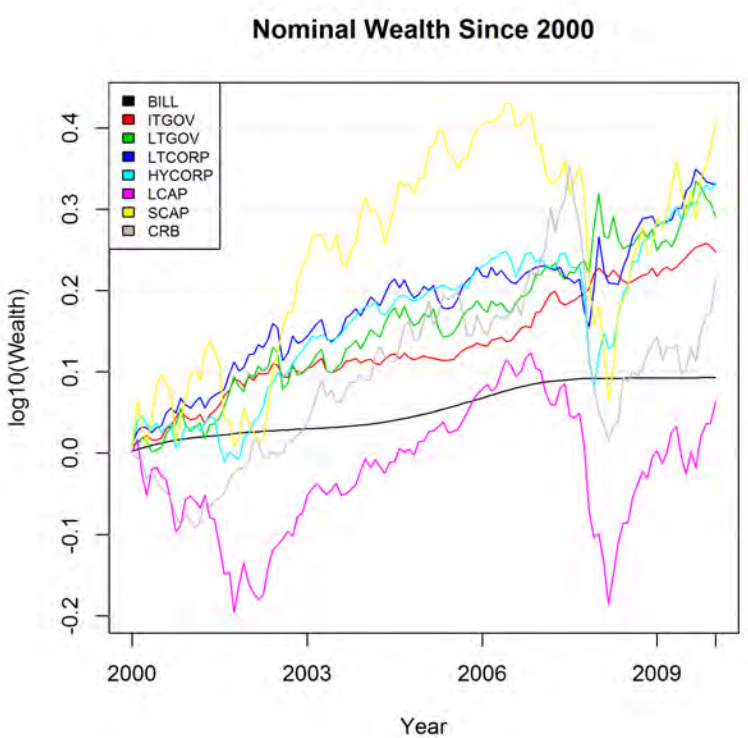

FIGURE 3.4: Nominal monthly wealth series for 8 asset classes describing the growth of 1 USD over time. The series frequency is given by end of month dates, starting in December 2000.

3.2 NOMINAL WEALTH INDEX TABLES

Nominal Wealth Index Table 1925/1950

GMT	BILL	ITGOV	LTGOV	LTCORP	HYCORP	LCAP	SCAP	CRB
1925-12-31	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
1930-12-31	1.2	1.3	1.3	1.3	1.1	1.5	0.5	0.5
1935-12-31	1.2	1.6	1.6	2.0	1.4	1.8	1.0	0.7
1940-12-31	1.2	1.9	2.1	2.5	1.5	1.8	0.9	0.7
1945-12-31	1.2	2.1	2.5	2.9	3.5	4.0	6.0	1.1
1950-12-31	1.3	2.2	2.7	3.2	4.2	6.4	8.7	2.1
1955-12-31	1.4	2.4	2.9	3.5	5.9	18.6	17.4	1.7
1960-12-31	1.6	2.8	3.0	3.8	6.0	28.5	28.8	1.8
1965-12-31	1.8	3.2	3.5	4.6	8.3	53.0	72.6	2.3
1970-12-31	2.4	4.2	3.5	4.8	8.3	62.5	104.2	3.0
1975-12-31	3.2	5.7	4.7	6.5	10.0	73.1	107.2	7.3
1980-12-31	4.6	7.3	5.1	7.3	13.6	140.5	524.0	17.5
1985-12-31	7.5	15.1	11.0	16.5	34.7	279.1	1241.2	17.6
1990-12-31	10.4	23.6	18.4	27.2	43.8	517.5	1277.4	26.3
1995-12-31	12.9	36.1	34.0	48.3	102.5	1113.9	3822.4	31.2
2000-12-31	16.6	48.6	48.8	64.0	126.4	2586.5	6402.2	33.8
2005-12-31	18.4	62.7	70.7	99.9	193.2	2663.0	13706.1	49.3
2010-12-31	20.4	84.1	92.9	133.4	266.6	2982.2	16054.7	55.2

FIGURE 3.5: The nominal monthly increase of 1 USD over time. The numbers are given every 5 years at the end of the year.

GMT	BILL	ITGOV	LTGOV	LTCORP	HYCORP	LCAP	SCAP	CRB
1950-12-31	1.0	1.0	1.0	1.0	1.0	1.1	1.1	1.1
1955-12-31	1.1	1.1	1.1	1.1	1.4	3.1	2.2	0.9
1960-12-31	1.2	1.3	1.1	1.2	1.5	4.7	3.6	0.9
1965-12-31	1.4	1.5	1.3	1.4	2.0	8.8	9.2	1.2
1970-12-31	1.9	1.9	1.3	1.5	2.0	10.3	13.2	1.5
1975-12-31	2.5	2.5	1.7	2.0	2.4	12.1	13.5	3.7
1980-12-31	3.6	3.3	1.9	2.3	3.3	23.2	66.1	8.8
1985-12-31	5.8	6.8	4.1	5.2	8.3	46.1	156.7	8.9
1990-12-31	8.1	10.6	6.8	8.5	10.5	85.5	161.2	13.3
1995-12-31	10.0	16.2	12.6	15.2	24.6	184.1	482.5	15.7
2000-12-31	12.9	21.8	18.1	20.1	30.3	427.5	808.1	17.0
2005-12-31	14.3	28.2	26.3	31.3	46.3	440.2	1730.1	24.8
2010-12-31	15.9	37.8	34.6	41.8	63.9	492.9	2026.5	27.8

FIGURE 3.6: The nominal monthly increase of 1 USD over time. The numbers are given every 5 years at the end of the year.

Nominal Wealth Index Table 1975/2000

GMT	BILL	ITGOV	LTGOV	LTCORP	HYCORP	LCAP	SCAP	CRB
1975-12-31	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
1980-12-31	1.5	1.3	1.1	1.2	1.4	1.9	4.8	2.4
1985-12-31	2.4	2.7	2.5	2.7	3.5	3.8	11.4	2.4
1990-12-31	3.3	4.2	4.1	4.4	4.4	7.0	11.7	3.6
1995-12-31	4.1	6.5	7.6	7.8	10.3	15.1	35.0	4.2
2000-12-31	5.3	8.7	10.9	10.3	12.7	35.0	58.6	4.6
2005-12-31	5.9	11.3	15.8	16.1	19.4	36.1	125.4	6.7
2010-12-31	6.5	15.1	20.7	21.5	26.8	40.4	146.8	7.5

FIGURE 3.7: The nominal monthly increase of 1 USD over time. The numbers are given every 5 years at the end of the year.

GMT	BILL	ITGOV	LTGOV	LTCORP	HYCORP	LCAP	SCAP	CRB
2000-12-31	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
2001-12-31	1.0	1.1	1.1	1.1	1.1	0.9	1.3	0.8
2002-12-31	1.1	1.2	1.3	1.3	1.1	0.7	1.1	1.0
2003-12-31	1.1	1.3	1.3	1.4	1.4	0.9	1.7	1.1
2004-12-31	1.1	1.3	1.4	1.5	1.5	1.0	2.1	1.2
2005-12-31	1.1	1.3	1.5	1.6	1.6	1.0	2.2	1.5
2006-12-31	1.2	1.4	1.5	1.7	1.7	1.2	2.5	1.4
2007-12-31	1.2	1.5	1.7	1.7	1.7	1.3	2.4	1.7
2008-12-31	1.2	1.7	2.1	1.8	1.3	0.8	1.5	1.1
2009-12-31	1.2	1.6	1.8	1.9	1.9	1.0	1.9	1.4
2010-12-31	1.2	1.8	2.0	2.1	2.2	1.2	2.6	1.6

FIGURE 3.8: The nominal monthly increase of 1 USD over time. The numbers are given every 5 years at the end of the year.

3.3 REAL WEALTH INDEX CHARTS

Real Wealth Index 1925

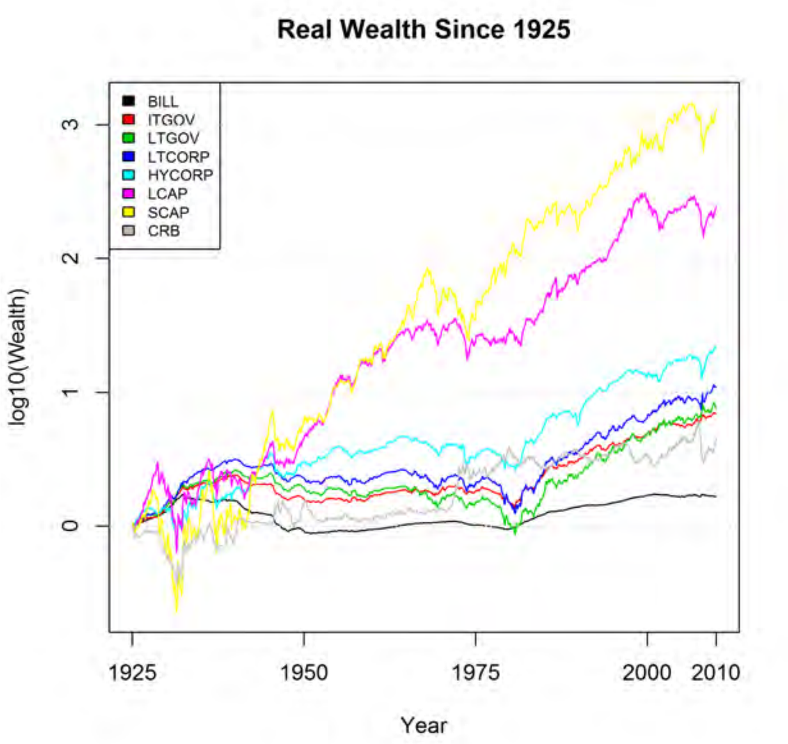

FIGURE 3.9: Real monthly wealth series for 8 asset classes describing the growth of 1 USD over time. The series frequency is given by end of month dates, starting in December 1925.

Real Wealth Index 1950

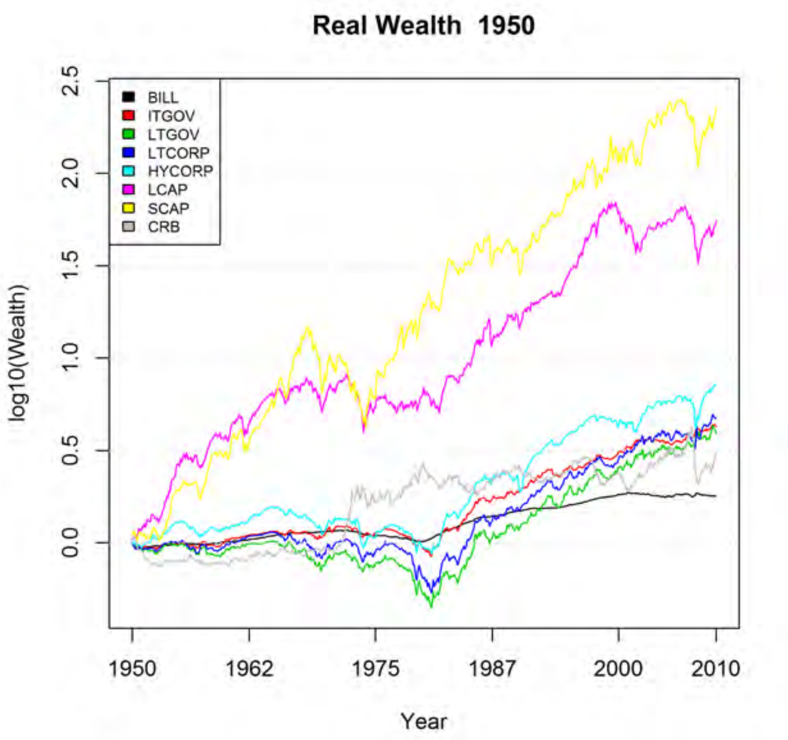

FIGURE 3.10: Real monthly wealth series for 8 asset classes describing the growth of 1 USD over time. The series frequency is given by end of month dates, starting in December 1950.

Real Wealth Index 1975

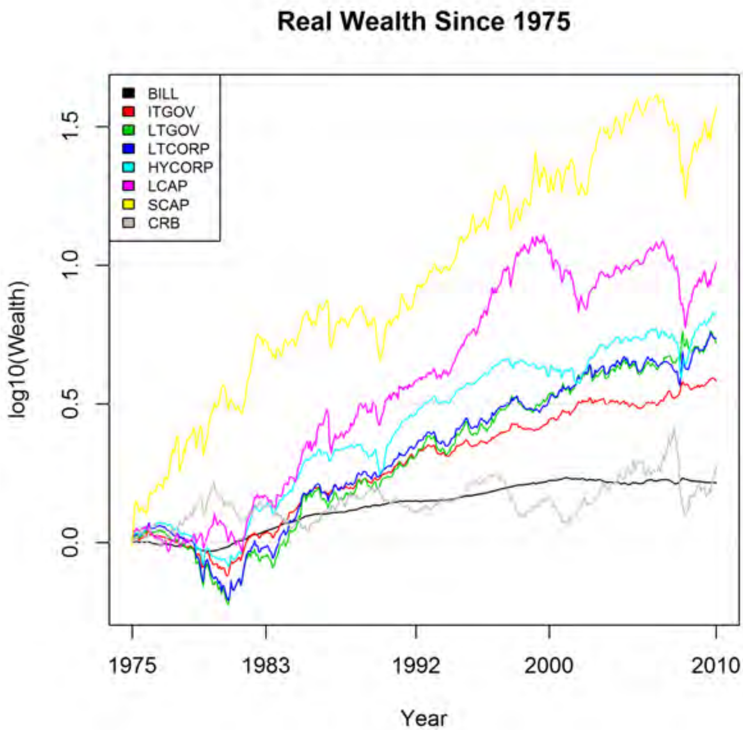

FIGURE 3.11: Real monthly wealth series for 8 asset classes describing the growth of 1 USD over time. The series frequency is given by end of month dates, starting in December 1950.

Real Wealth Index 2000

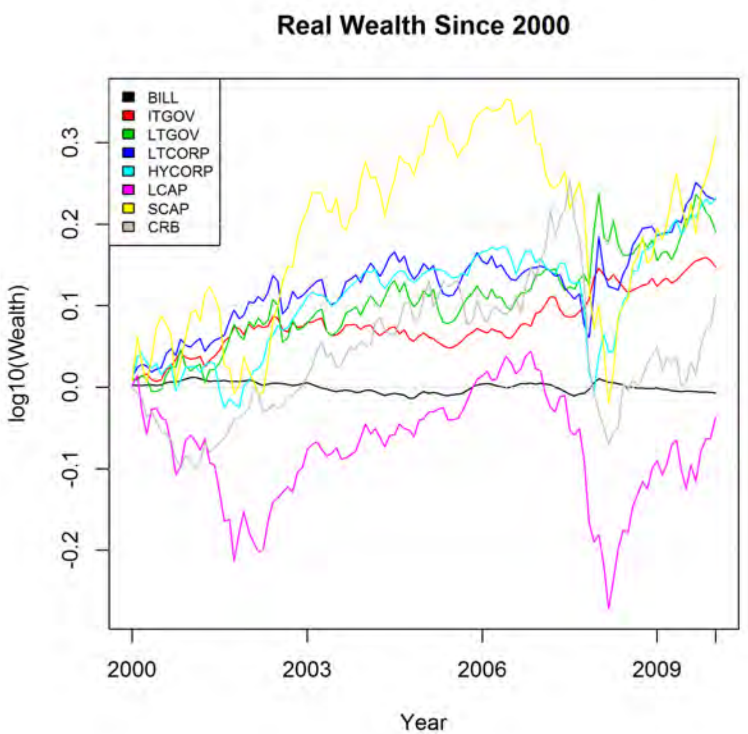

FIGURE 3.12: Real monthly wealth series for 8 asset classes describing the growth of 1 USD over time. The series frequency is given by end of month dates, starting in December 2000.

3.4 REAL WEALTH INDEX TABLES

Real Monthly Wealth Index Table 1925/1950

GMT	BILL	ITGOV	LTGOV	LTCORP	HYCORP	LCAP	SCAP	CRB
1925-12-31	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
1930-12-31	1.3	1.4	1.4	1.5	1.3	1.7	0.6	0.6
1935-12-31	1.6	2.1	2.1	2.6	1.8	2.3	1.3	0.9
1940-12-31	1.5	2.4	2.6	3.1	1.9	2.3	1.1	0.8
1945-12-31	1.2	2.0	2.5	2.9	3.4	3.9	5.9	1.0
1950-12-31	0.9	1.6	1.9	2.3	3.0	4.6	6.2	1.5
1955-12-31	0.9	1.6	1.9	2.4	4.0	12.4	11.6	1.2
1960-12-31	0.9	1.7	1.8	2.3	3.6	17.1	17.3	1.1
1965-12-31	1.0	1.8	1.9	2.6	4.7	29.8	40.8	1.3
1970-12-31	1.1	1.9	1.6	2.2	3.7	28.1	46.9	1.3
1975-12-31	1.0	1.8	1.5	2.1	3.2	23.6	34.6	2.4
1980-12-31	1.0	1.5	1.1	1.5	2.8	29.1	108.7	3.6
1985-12-31	1.2	2.5	1.8	2.7	5.7	45.7	203.3	2.9
1990-12-31	1.4	3.2	2.5	3.6	5.9	69.2	170.9	3.5
1995-12-31	1.5	4.2	4.0	5.6	12.0	129.9	445.7	3.6
2000-12-31	1.7	5.0	5.0	6.6	13.0	266.1	658.6	3.5
2005-12-31	1.7	5.7	6.4	9.1	17.6	242.2	1246.6	4.5
2010-12-31	1.7	6.9	7.6	10.9	21.8	243.6	1311.2	4.5

FIGURE 3.13: The real or inflation adjusted increase of 1 USD over time. The numbers are given every 5 years at the end of the year.

GMT	BILL	ITGOV	LTGOV	LTCORP	HYCORP	LCAP	SCAP	CRB
1950-12-31	1.0	1.0	1.0	1.0	1.0	1.0	1.1	1.1
1955-12-31	1.0	1.0	1.0	1.0	1.3	2.8	2.0	0.8
1960-12-31	1.0	1.1	0.9	1.0	1.2	3.9	3.0	0.8
1965-12-31	1.1	1.1	1.0	1.1	1.6	6.8	7.1	0.9
1970-12-31	1.2	1.2	0.8	0.9	1.2	6.4	8.2	0.9
1975-12-31	1.1	1.1	0.8	0.9	1.1	5.4	6.0	1.6
1980-12-31	1.0	0.9	0.5	0.7	0.9	6.6	18.9	2.5
1985-12-31	1.3	1.5	0.9	1.2	1.9	10.4	35.4	2.0
1990-12-31	1.5	2.0	1.3	1.6	1.9	15.8	29.8	2.4
1995-12-31	1.6	2.6	2.0	2.4	4.0	29.6	77.6	2.5
2000-12-31	1.8	3.1	2.6	2.8	4.3	60.7	114.7	2.4
2005-12-31	1.8	3.5	3.3	3.9	5.8	55.2	217.1	3.1
2010-12-31	1.8	4.3	3.9	4.7	7.2	55.6	228.4	3.1

FIGURE 3.14: The real or inflation adjusted increase of 1 USD over time. The numbers are given every 5 years at the end of the year.

Real Monthly Wealth Index Table 1975/2000

GMT	BILL	ITGOV	LTGOV	LTCORP	HYCORP	LCAP	SCAP	CRB
1975-12-31	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
1980-12-31	0.9	0.8	0.7	0.8	0.9	1.2	3.1	1.5
1985-12-31	1.2	1.4	1.2	1.3	1.8	1.9	5.7	1.2
1990-12-31	1.4	1.8	1.7	1.8	1.8	2.9	4.8	1.5
1995-12-31	1.5	2.3	2.7	2.8	3.7	5.4	12.6	1.5
2000-12-31	1.7	2.8	3.5	3.3	4.0	11.1	18.6	1.5
2005-12-31	1.6	3.2	4.4	4.5	5.5	10.1	35.2	1.9
2010-12-31	1.6	3.8	5.2	5.4	6.8	10.2	37.0	1.9

FIGURE 3.15: The real or inflation adjusted increase of 1 USD over time. The numbers are given every 5 years at the end of the year.

GMT	BILL	ITGOV	LTGOV	LTCORP	HYCORP	LCAP	SCAP	CRB
2000-12-31	1	1.0	1.0	1.0	1.0	1.0	1.0	1.0
2001-12-31	1	1.1	1.0	1.1	1.1	0.9	1.2	0.8
2002-12-31	1	1.2	1.2	1.3	1.0	0.7	1.0	0.9
2003-12-31	1	1.2	1.2	1.3	1.3	0.8	1.6	1.0
2004-12-31	1	1.2	1.3	1.4	1.4	0.9	1.9	1.1
2005-12-31	1	1.2	1.3	1.4	1.4	0.9	1.9	1.3
2006-12-31	1	1.2	1.3	1.4	1.5	1.0	2.2	1.2
2007-12-31	1	1.2	1.4	1.4	1.4	1.0	2.0	1.4
2008-12-31	1	1.4	1.7	1.5	1.1	0.7	1.3	0.9
2009-12-31	1	1.3	1.4	1.5	1.5	0.8	1.6	1.1
2010-12-31	1	1.4	1.6	1.7	1.7	0.9	2.0	1.3

FIGURE 3.16: The real or inflation adjusted increase of 1 USD over time. The numbers are given every 5 years at the end of the year.

3.5 NOMINAL VS. REAL INDICES

Nominal vs. Real Monthly Wealth Index Chart I

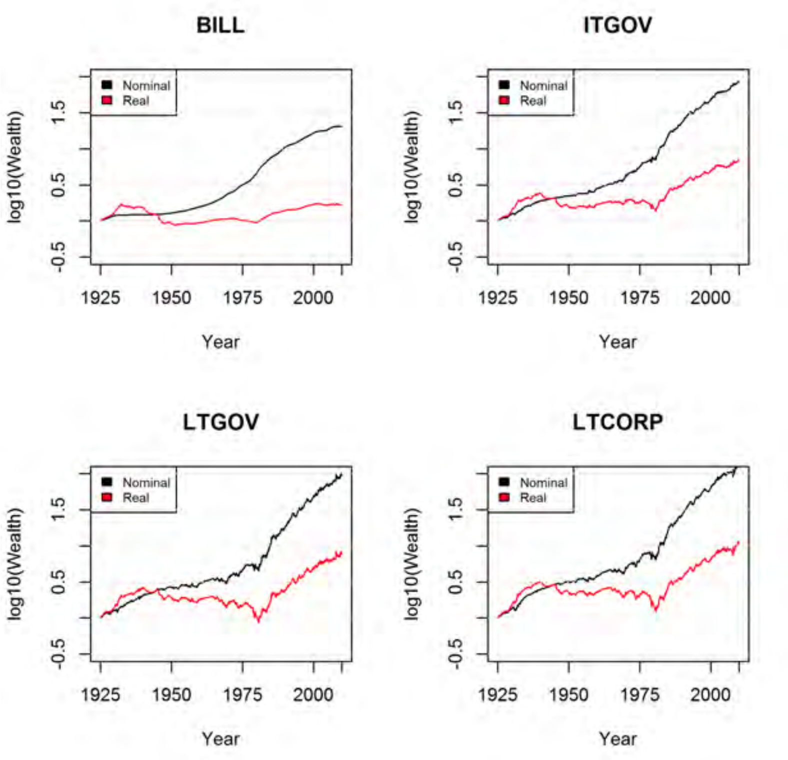

FIGURE 3.17: Log of real or inflation adjusted wealth series for bills (BILL), intermediate term (ITGOV) and long term gvernment bonds (LTGOV), and long term corporate bonds (LTCORP).

Nominal vs. Real Monthly Wealth Index Chart II

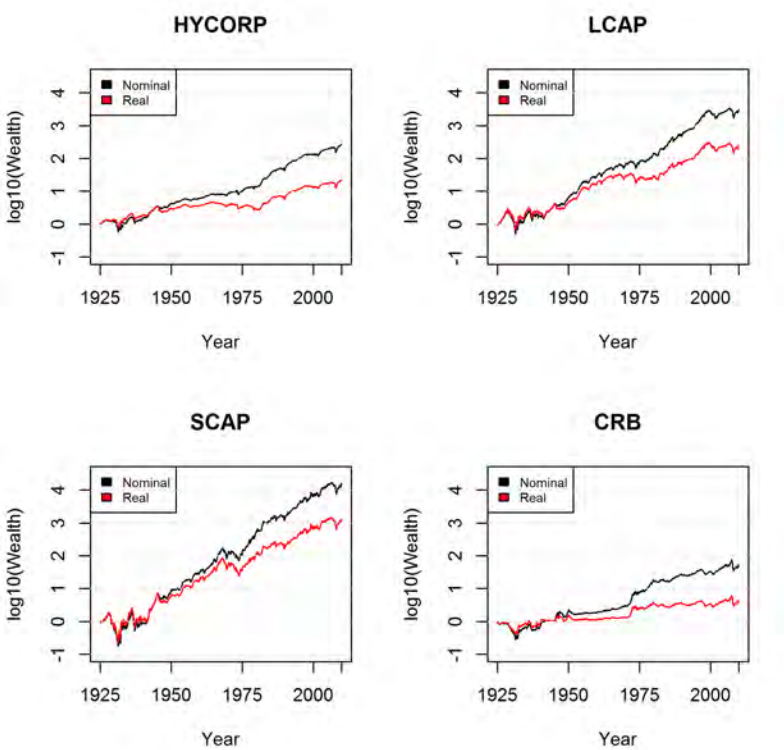

FIGURE 3.18: Log of real or inflation adjusted wealth series for high yield corporate bonds (HYCORP), large cap stocks (LCAP), small cap stocks (SCAP), and commodities (CRB).

Chapter 4

Total Returns

Monthly total returns are calculated as geometric returns from prices or indices. Within the following chapters we use monthly total returns, if otherwise not explicitly remarked, then in some cases we use monthly price returns. The monthly total returns are also called *monthly nominal total returns* to distinguish them from *monthly real total returns* which are adjusted for inflation.

Monthly nominal total returns R_t^m are the geometric returns directly calculated from dividend adjusted price or index series.

$$R_t^m = \frac{I_t^m - I_{t-1}^m}{I_{t-1}^m}$$

Monthly real total returns or inflation adjusted total returns are calculated as the geometric difference between nominal returns and the inflation rate, CPI.

$$R_t^{real} = \frac{1 + R_t}{1 + R_t^{CPI}} - 1$$

We show plots of the monthly nominal and real total returns for bills, bonds, stocks, and commodities. Further graphs show the density of the monthly nominal total returns and box plots were created to display extreme and outlying monthly nominal total return values. Information about statistical quantities are given in form of tables next to every plot.

4.1 NOMINAL MONTHLY TOTAL RETURNS SERIES

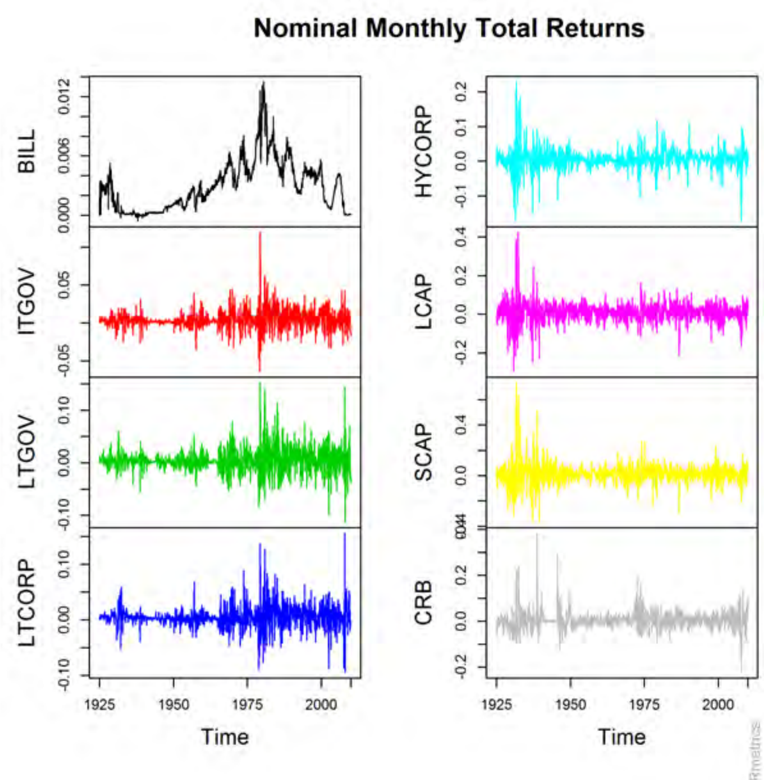

FIGURE 4.1: Nominal monthly total returns statistics since 1925 for all 8 asset classes.

4.2 NOMINAL MONTHLY TOTAL RETURNS STATISTICS

	BILL	ITGOV	LTGOV	LTCORP	HYCORP	LCAP	SCAP	CRB
Minimum	-0.06	-6.41	-11.24	-9.49	-17.28	-29.72	-36.74	-22.27
Maximum	1.35	11.98	15.23	15.60	22.94	42.56	73.48	38.36
1. Quartile	0.08	-0.08	-0.65	-0.36	-0.66	-1.72	-2.61	-1.36
3. Quartile	0.43	0.97	1.54	1.28	1.82	3.94	4.98	2.05
Mean	0.30	0.44	0.47	0.50	0.60	0.94	1.29	0.47
Median	0.27	0.28	0.32	0.40	0.68	1.30	1.45	0.25
Variance	0.06	1.62	5.70	4.56	9.48	30.65	70.50	16.04
Stdev	0.25	1.27	2.39	2.14	3.08	5.54	8.40	4.01
Skewness	1.03	0.92	0.62	0.74	0.08	0.35	1.21	1.73
Kurtosis	1.27	8.95	5.35	7.60	9.53	9.33	12.30	14.31

FIGURE 4.2: Nominal monthly total returns statistics: Minimum and maximum returns for each asset, 25% and 75% quantiles, mean and median, variance and standard deviation, and skewness and kurtosis. Values are given in percent.

4.3 REAL MONTHLY TOTAL RETURNS

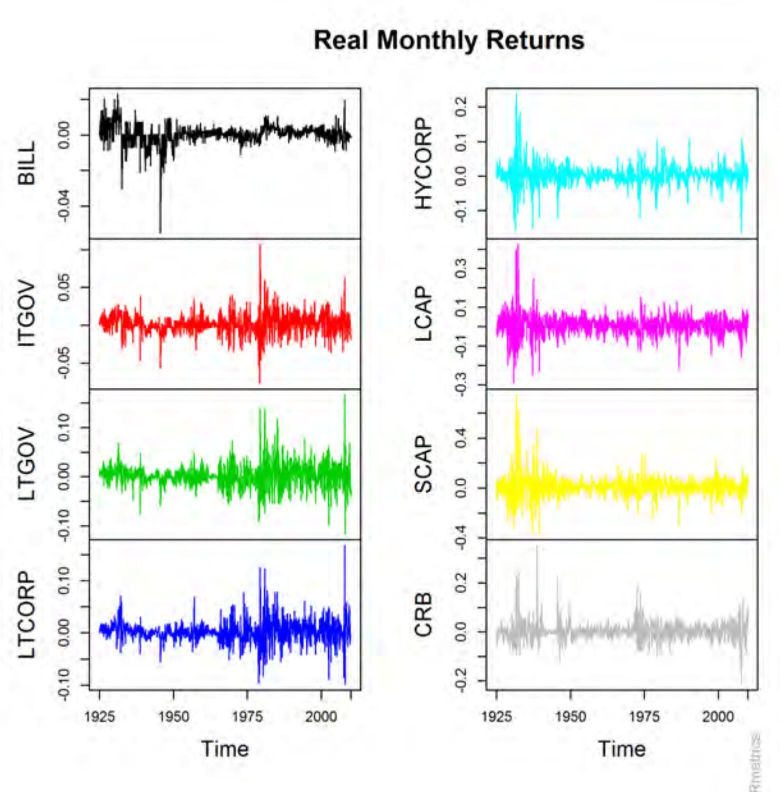

FIGURE 4.3: Real monthly total returns since 1925 for all 8 asset classes.

4.4 REAL MONTHLY TOTAL RETURNS STATISTICS

	BILL	ITGOV	LTGOV	LTCORP	HYCORP	LCAP	SCAP	CRB
Minimum	-5.52	-7.71	-11.63	-9.88	-16.44	-29.25	-36.74	-21.48
Maximum	2.33	10.74	16.66	16.81	23.85	42.56	74.77	35.41
1. Quartile	-0.15	-0.50	-1.04	-0.80	-0.99	-2.16	-2.94	-1.66
3. Quartile	0.29	0.98	1.51	1.27	1.67	3.66	4.85	1.84
Mean	0.05	0.20	0.23	0.26	0.35	0.69	1.04	0.22
Median	0.06	0.14	0.18	0.22	0.36	0.94	1.30	0.07
Variance	0.29	1.97	6.20	5.03	9.55	30.87	70.50	14.75
Stdev	0.54	1.40	2.49	2.24	3.09	5.56	8.40	3.84
Skewness	-1.52	0.30	0.54	0.68	0.29	0.45	1.26	1.54
Kurtosis	14.38	5.37	5.23	7.41	9.30	9.30	12.53	12.58

FIGURE 4.4: Real monthly total returns statistics: Minimum and maxium returns for each asset, 25% and 75% quantiles, mean and median, variance and standard deviation, and skewness and kurtosis.

4.5 DENSITY OF MONTHLY TOTAL RETURNS

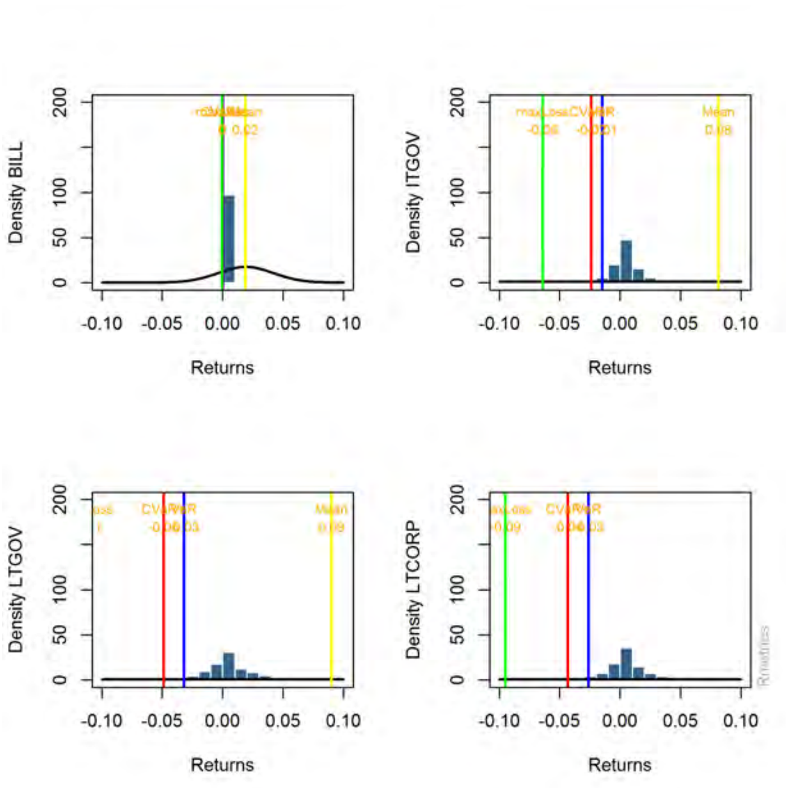

FIGURE 4.5: Density of monthly total returns for treasury bills (BILL), intermediate government bonds (ITGOV), and long term government (LTGOV) and corporate bonds (LTGOV).

	VaR	VaRplus	maxLoss	mean	sd
BILL	0.01	0.00	-0.06	1.90	2.30
ITGOV	-1.47	-2.37	-6.41	8.14	37.97
LTGOV	-3.17	-4.87	-11.24	9.01	96.17
LTCORP	-2.59	-4.31	-9.49	12.97	113.47

FIGURE 4.6: Statistics of the density of monthly total returns: VaR Value at Risk, VaRplus Value at Risk plus, maxLoss maximum loss, mean Mean, and sd Standard Deviation. Note the returns are given in percent. Values are given in percent.

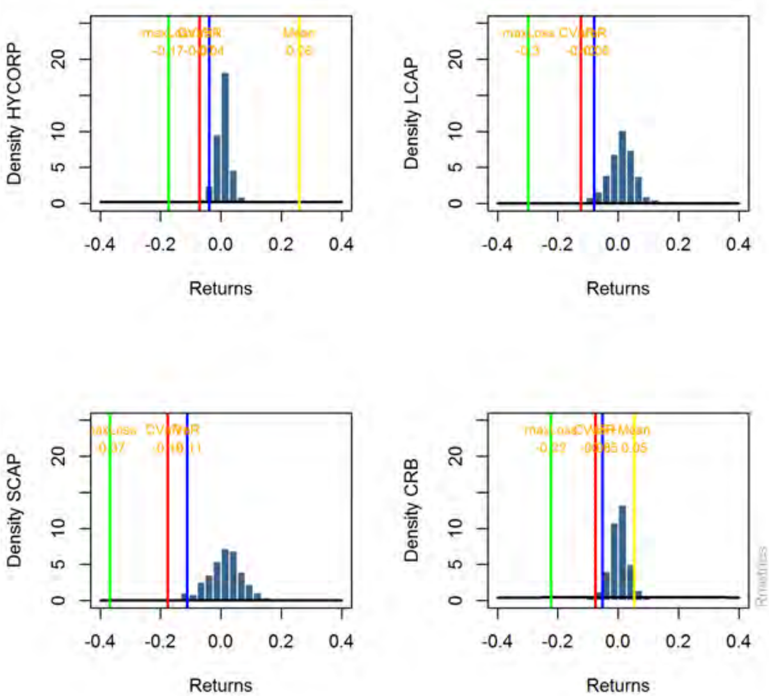

FIGURE 4.7: Density of monthly total returns for domestic high yield corporate bonds (HYCORP), large cap (LCAP) and small cap SCAP) stock indices, and CRB commodity index.

	VaR	VaRplus	maxLoss	mean	sd
HYCORP	-3.88	-7.08	-17.28	26.01	218.26
LCAP	-7.88	-12.29	-29.72	291.99	4647.77
SCAP	-11.16	-17.60	-36.74	1572.35	26085.62
CRB	-5.23	-7.53	-22.27	5.31	92.79

FIGURE 4.8: Statistics of the density of monthly total returns: VaR Value at Risk, VaRplus Value at Risk plus, maxLoss maximum loss, mean Mean, and sd Standard Deviation. Note the returns are given in percent.

4.6 EXTREME AND OUTLYING RETURNS

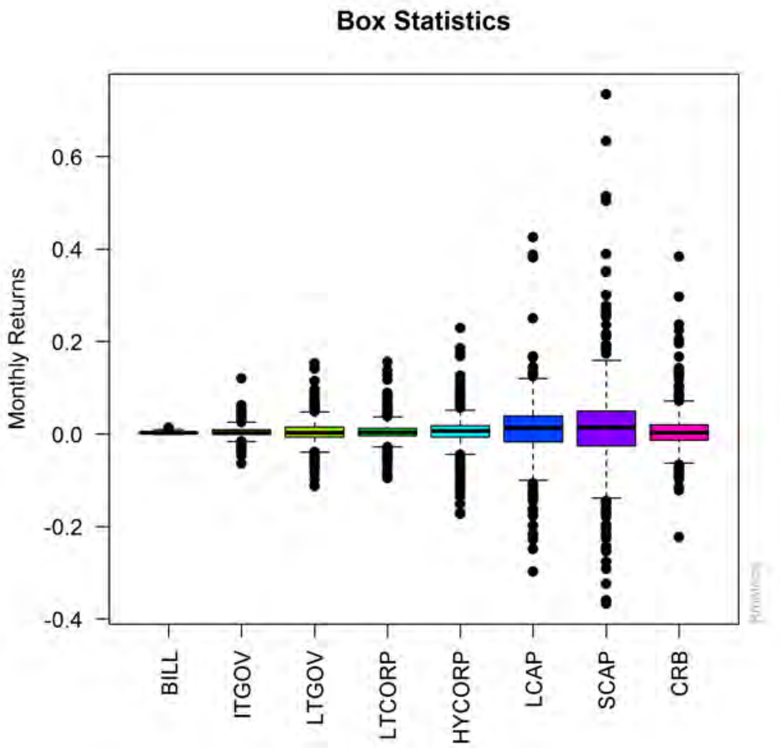

FIGURE 4.9: Extreme and outlying returns: The box is drawn from the 25th to the 75th percentile and splitted with a line at the median. The distance of the whiskers (thin lines) to the 25th or the 75th percentile is 1.5 times the length of the box.

	BILL	ITGOV	LTGOV	LTCORP	HYCORP	LCAP	SCAP	CRB
lower whisker	-0.06	-1.64	-3.88	-2.77	-4.37	-9.97	-13.79	-6.28
lower hinge	0.08	-0.08	-0.65	-0.36	-0.66	-1.72	-2.61	-1.36
median	0.27	0.28	0.32	0.40	0.68	1.30	1.45	0.25
upper hinge	0.43	0.97	1.54	1.28	1.82	3.94	4.98	2.05
upper whisker	0.95	2.54	4.81	3.72	5.16	12.06	15.92	7.15

FIGURE 4.10: Box plot statistics. Values are given in percent.

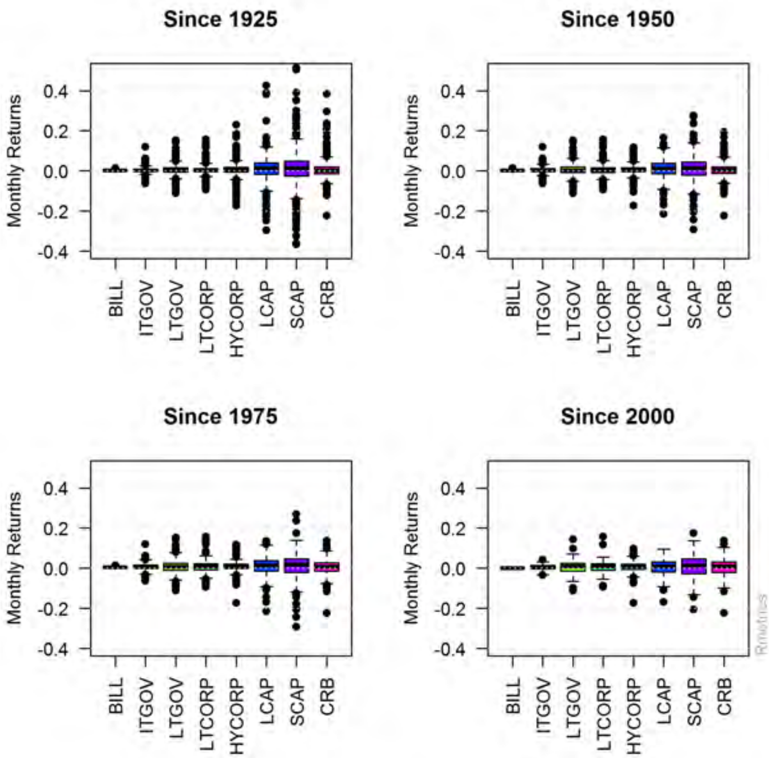

FIGURE 4.11: Extreme and outlying returns for four different horizons starting 1925, 1950, 1975, and 2000. The boxes are drawn from the 25th to the 75th percentile and splitted with a line at the median. The whiskers (thin lines) give the range of returns for the 75th percentile up to the maximum value and the 25th percentile down to the minimum value.

CHAPTER 5

RISK PREMIUMS

The risk premium is the geometric difference between the total returns of two assets. From the basic asset classes we can define several kinds of risk premiums. Here we investigate the following risk premium measures: (i) equity risk premium, (ii) small stock premium, (iii) bond default premium, and (iv) bond horizon premium.

The historical *risk premium* between two assets, say A and B, is calculated as the geometric difference between the total returns (real or nominal) of asset A and asset B.

$$R_t^{premium} = \frac{1 + R_t^A}{1 + R_t^B} - 1$$

We define the *equity risk premium* as the geometric difference in nominal total returns between the large cap stocks, *LCAP*, and the U.S. treasury bills, *BILL*.

$$R_t^{ERP} = \frac{1 + R_t^{LCAP}}{1 + R_t^{BILL}} - 1$$

The *small stock premium* is taken between the nominal total returns of the small cap stocks, *SCAP*, and the large cap stock, *LCAP*.

$$R_t^{SSP} = \frac{1 + R_t^{SCAP}}{1 + R_t^{LCAP}} - 1$$

The *bond default premium* describes the difference in the nominal total returns of the long term corporate bonds, *LTCORP*, and the long term government bonds, *LTGOV*.

$$R_t^{BDP} = \frac{1 + R_t^{LTCORP}}{1 + R_t^{LTGOV}} - 1$$

And finally, the *bond horizon premium* is defined to measure the difference in nominal total returns of the long term government bonds, *LTGOV*, and the U.S. treasury Bills, *BILL*.

$$R_t^{BHP} = \frac{1 + R_t^{LTGOV}}{1 + R_t^{BILL}} - 1$$

For every risk premium measure we will show a graph of the returns and wealth series. Further graphs are given that compare the wealth series to those of the underlying asset classes, together with a table showing the wealth values on a five year basis.

5.1 EQUITY RISK PREMIUM

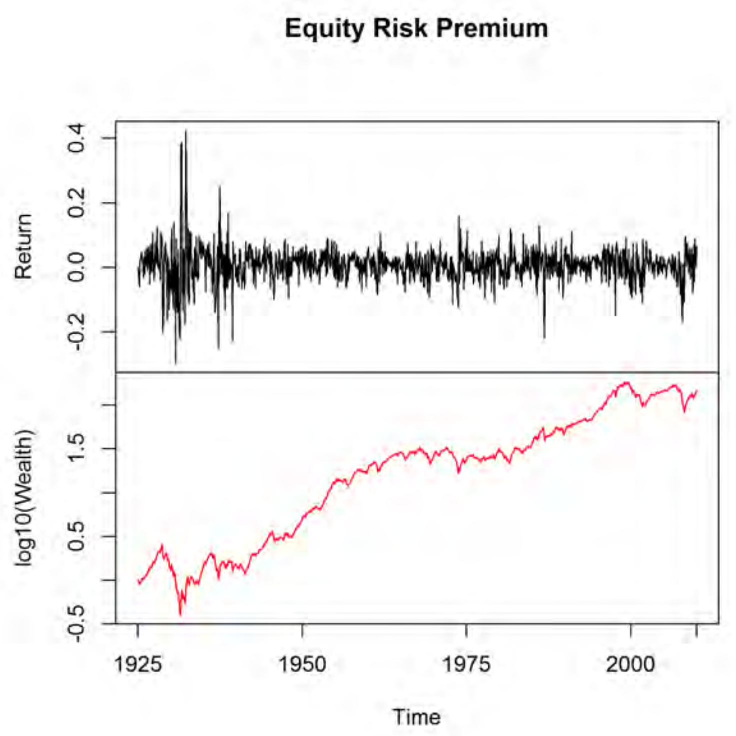

FIGURE 5.1: Equity risk premium between the large cap stocks *LCAP* and the U.S. treasury bills, *BILL*.

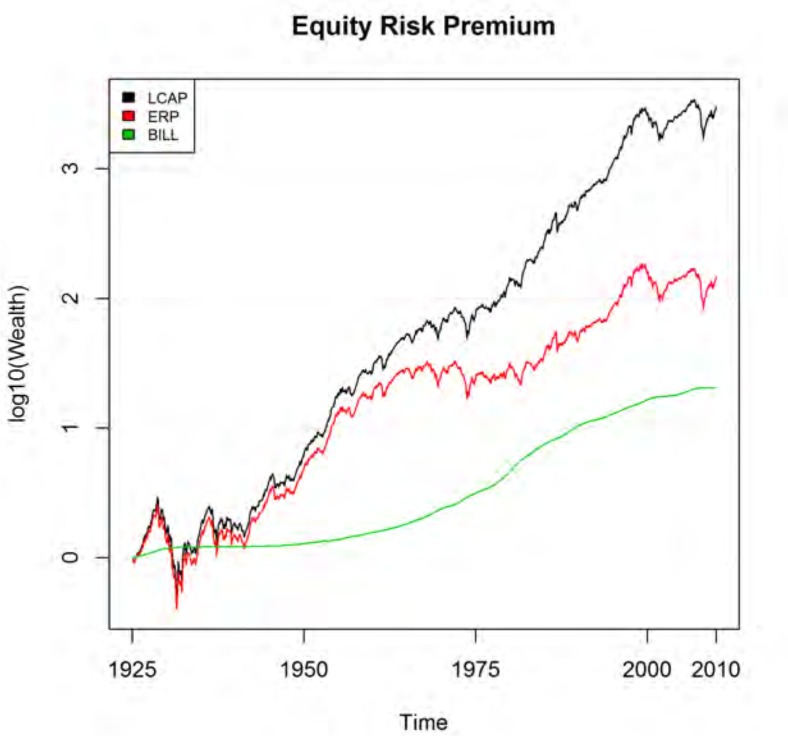

FIGURE 5.2: Equity risk premium between the large cap stocks *LCAP* and the U.S. treasury bills, *BILL*.

```
           LCAP   RP BILL
1925-12-31  1.0  1.0  1.0
1930-12-31  1.5  1.3  1.2
1935-12-31  1.8  1.5  1.2
1940-12-31  1.8  1.5  1.2
1945-12-31  4.0  3.2  1.2
1950-12-31  6.4  4.9  1.3
1955-12-31 18.6 13.4  1.4
1960-12-31 28.5 18.1  1.6
1965-12-31 53.0 29.0  1.8
```

```
             LCAP    RP BILL
1970-12-31   62.5  26.2  2.4
1975-12-31   73.1  23.2  3.2
1980-12-31  140.5  30.6  4.6
1985-12-31  279.1  37.2  7.5
1990-12-31  517.5  49.6 10.4
1995-12-31 1113.9  86.5 12.9
2000-12-31 2586.5 156.2 16.6
2005-12-31 2663.0 144.7 18.4
2010-12-31 2982.2 146.1 20.4
```

5.2 SMALL STOCK PREMIUM

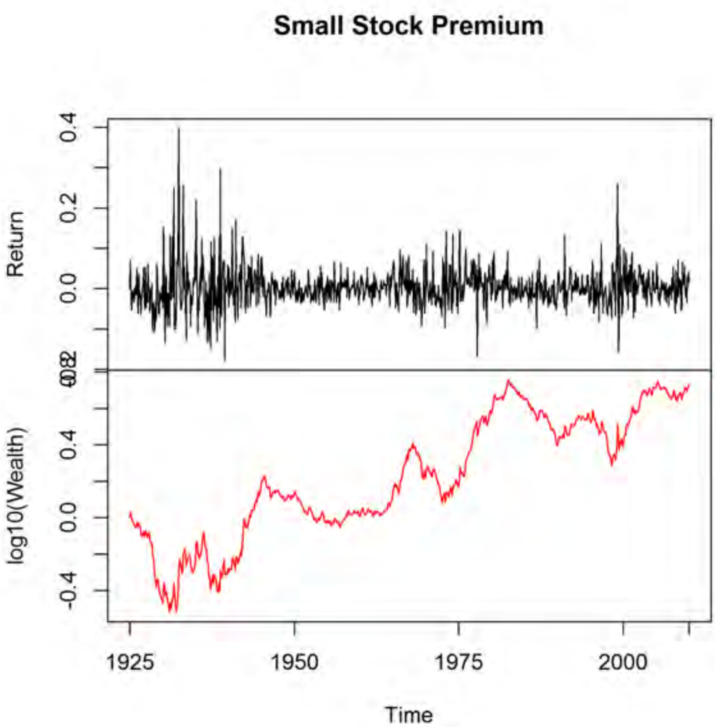

FIGURE 5.3: Small stock premium between the nominal total returns of the small cap stocks *SCAP* and the large cap stocks *LCAP*.

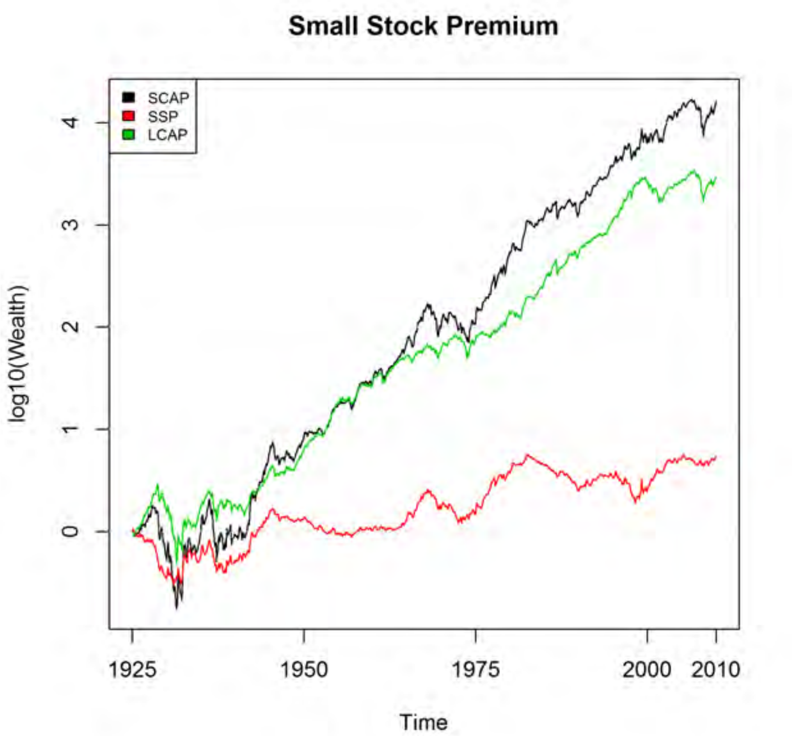

FIGURE 5.4: Small stock premium between the nominal total returns of the small cap stocks *SCAP* and the large cap stocks *LCAP*.

	SCAP	RP	LCAP
1925-12-31	1.0	1.0	1.0
1930-12-31	0.5	0.3	1.5
1935-12-31	1.0	0.6	1.8
1940-12-31	0.9	0.5	1.8
1945-12-31	6.0	1.5	4.0
1950-12-31	8.7	1.4	6.4
1955-12-31	17.4	0.9	18.6
1960-12-31	28.8	1.0	28.5
1965-12-31	72.6	1.4	53.0

	SCAP	RP	LCAP
1970-12-31	104.2	1.7	62.5
1975-12-31	107.2	1.5	73.1
1980-12-31	524.0	3.7	140.5
1985-12-31	1241.2	4.4	279.1
1990-12-31	1277.4	2.5	517.5
1995-12-31	3822.4	3.4	1113.9
2000-12-31	6402.2	2.5	2586.5
2005-12-31	13706.1	5.1	2663.0
2010-12-31	16054.7	5.4	2982.2

5.3 BOND DEFAULT PREMIUM

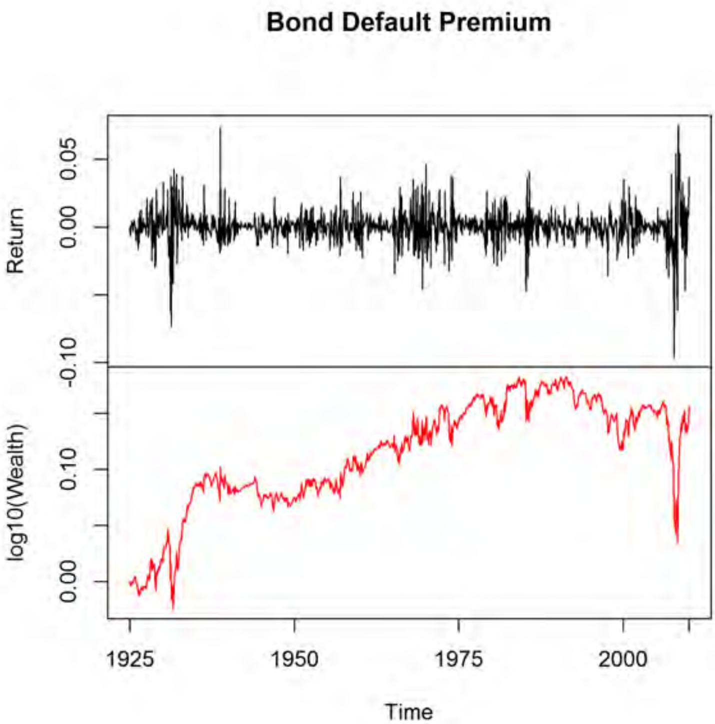

FIGURE 5.5: Bond default premium between the long term corporate bonds *LTCORP*, and the long term government bonds *LTGOV*.

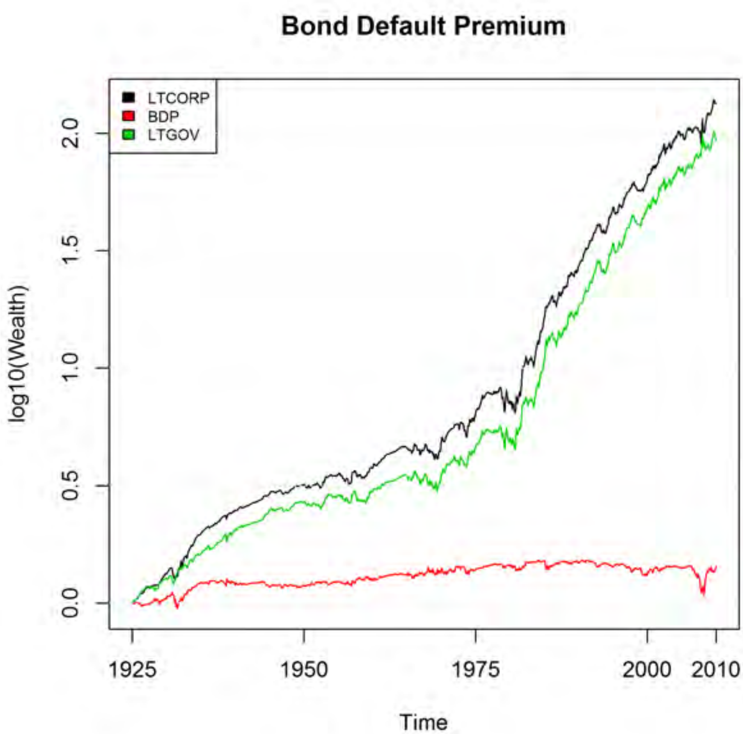

FIGURE 5.6: Bond default premium between the long term corporate bonds *LTCORP*, and the long term government bonds *LTGOV*.

```
           LTCORP  RP LTGOV
1925-12-31    1.0 1.0   1.0
1930-12-31    1.3 1.0   1.3
1935-12-31    2.0 1.2   1.6
1940-12-31    2.5 1.2   2.1
1945-12-31    2.9 1.2   2.5
1950-12-31    3.2 1.2   2.7
1955-12-31    3.5 1.2   2.9
1960-12-31    3.8 1.2   3.0
1965-12-31    4.6 1.3   3.5
```

```
           LTCORP  RP LTGOV
1970-12-31    4.8 1.4   3.5
1975-12-31    6.5 1.4   4.7
1980-12-31    7.3 1.4   5.1
1985-12-31   16.5 1.5  11.0
1990-12-31   27.2 1.5  18.4
1995-12-31   48.3 1.4  34.0
2000-12-31   64.0 1.3  48.8
2005-12-31   99.9 1.4  70.7
2010-12-31  133.4 1.4  92.9
```

5.4 BOND HORIZON PREMIUM

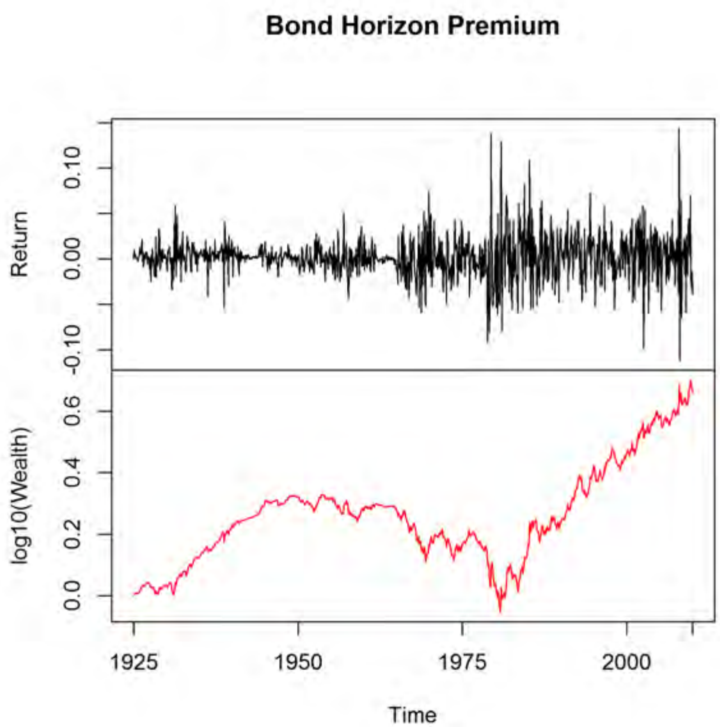

FIGURE 5.7: Bond horizon premium between the long term government bonds *LTGOV*, and the U.S. treasury Bills *BILL*.

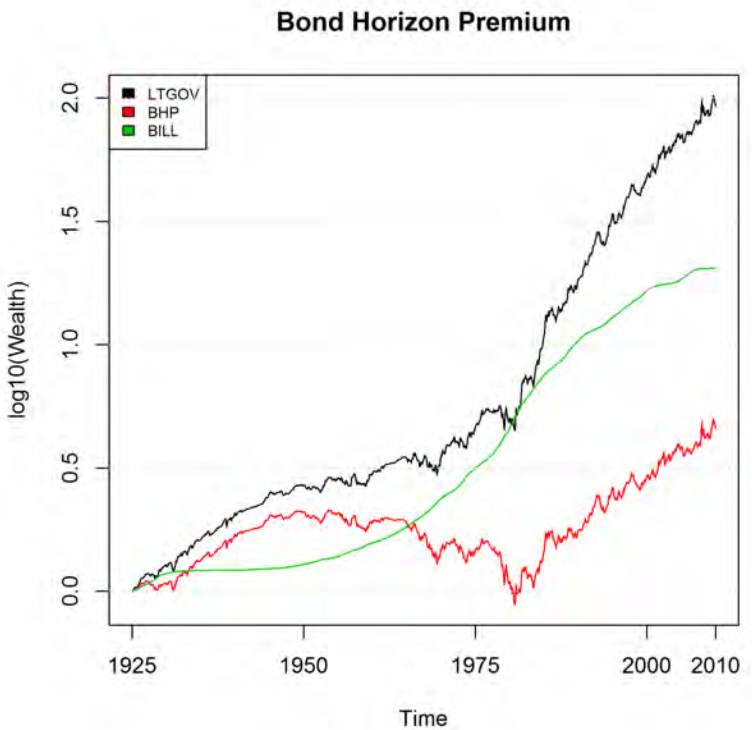

FIGURE 5.8: Bond horizon premium between the long term government bonds *LTGOV*, and the U.S. treasury Bills *BILL*.

```
           LTGOV  RP BILL
1925-12-31   1.0 1.0  1.0
1930-12-31   1.3 1.1  1.2
1935-12-31   1.6 1.3  1.2
1940-12-31   2.1 1.7  1.2
1945-12-31   2.5 2.0  1.2
1950-12-31   2.7 2.1  1.3
1955-12-31   2.9 2.1  1.4
1960-12-31   3.0 1.9  1.6
1965-12-31   3.5 1.9  1.8
```

```
           LTGOV  RP BILL
1970-12-31   3.5 1.4  2.4
1975-12-31   4.7 1.5  3.2
1980-12-31   5.1 1.1  4.6
1985-12-31  11.0 1.5  7.5
1990-12-31  18.4 1.8 10.4
1995-12-31  34.0 2.6 12.9
2000-12-31  48.8 2.9 16.6
2005-12-31  70.7 3.8 18.4
2010-12-31  92.9 4.6 20.4
```

Chapter 6

Holding Periods

In this chapter we investigate the returns for different holding periods, which are annualized returns over a given window. We will consider holding periods over a wide range up to 25 years.

Annual total returns R_t^a and *annual total returns over n-year periods* $R_t^{a,n}$ are calculated from monthly total returns by the following formulas:

$$R_t^a = \prod_{\tau=1}^{12} (1 + R_{t+1-\tau}^m) - 1$$

$$R_t^{a,n} = \Big(\prod_{\tau=1}^{12*n} (1 + R_{t+1-\tau}^m)\Big)^{1/n} - 1$$

Returns for different holding periods are nothing else than calculating the returns over a given window with a fixed length, and then annualizing the obtained return values. If the window has a length of n years, then the first result can be calculated n years after the first measurement and then for every point in time until the last measurement.

We use monthly total returns to compute annualized nominal and inflation adjusted (real) returns for holding periods of 1 year, 5 years, 10 years, a decade, and 20 years. Additionally we calculate annualized real returns for the risk premiums between all eight asset classes and the U.S. treasury bills for holding periods of 1 year and 10 years. The reults are visualized and tables provide numbers on a five year basis.

We have also calculated the annualized nominal mean returns and standard deviations, as well as the worst annualized nominal returns and the negative periods in percent on rolling investment horizons ranging from 1 year to 20 years. Those tables give information on the minimum investment horizon to achieve always a positive worst return.

There is also a figure for every asset that summarizes the most important findings and visualizes the turning points of the original index. A table next to every plot shows more detailed numbers about the distribution of the annualized nominal returns for holding periods of 1 year to 25 years.

ANNUAL TOTAL RETURNS

1 Year Annual Total Returns

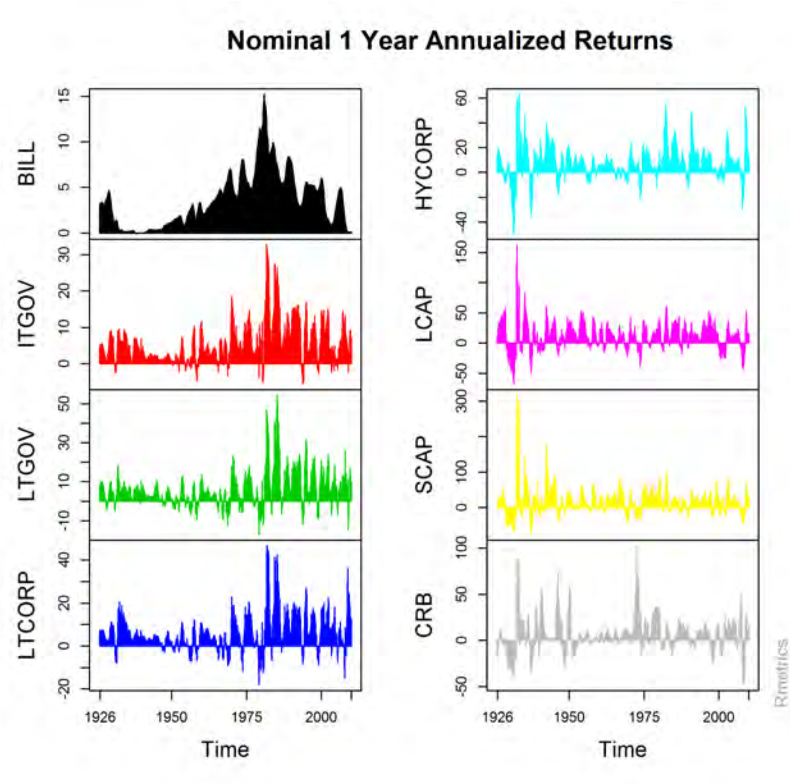

FIGURE 6.1: 1 year annual total return series for the 8 asset classes.

1 Year Annual Total Returns Statistics

	BILL	ITGOV	LTGOV	LTCORP	HYCORP	LCAP	SCAP	CRB
1926-12-31	3.27	5.38	7.77	7.37	14.43	11.62	0.28	-18.05
1931-12-31	1.08	-2.31	-5.32	-1.85	-30.92	-43.33	-49.75	-29.75
1936-12-31	0.18	3.05	7.49	6.74	22.46	33.92	64.81	15.90
1941-12-31	0.04	0.49	0.93	2.73	3.63	-11.59	-9.00	47.20
1946-12-31	0.36	1.01	-0.09	1.72	-9.85	-8.07	-11.63	59.42
1951-12-31	1.48	0.37	-3.94	-2.69	-0.90	24.02	7.80	-9.19
1956-12-31	2.47	-0.42	-5.59	-6.82	-6.89	6.56	4.28	7.93
1961-12-31	2.12	1.88	0.97	4.82	4.48	26.89	32.09	8.40
1966-12-31	4.75	4.69	3.65	0.20	-4.31	-10.06	-7.01	3.11
1971-12-31	4.38	8.74	13.24	11.01	13.14	14.31	16.50	5.28
1976-12-31	5.08	12.88	16.76	18.65	21.61	23.84	57.38	12.60
1981-12-31	14.72	9.44	1.86	-1.24	10.43	-4.91	13.88	-5.31
1986-12-31	6.16	15.13	24.52	19.85	17.45	18.47	6.85	-1.16
1991-12-31	5.60	15.46	19.26	19.89	46.19	30.55	44.63	-4.03
1996-12-31	5.20	2.09	-0.92	1.40	11.35	23.07	17.62	12.03
2001-12-31	3.82	7.61	3.70	10.65	5.28	-11.88	22.77	-17.18
2006-12-31	4.71	3.03	1.42	3.24	8.91	15.79	16.17	-2.85
2010-12-31	0.11	7.12	10.14	12.44	12.58	15.06	31.26	17.59

FIGURE 6.2: 1 year annual total return series for the 8 asset classes. We have printed the values every five years. Values are given in percent.

5 Years Annual Total Returns

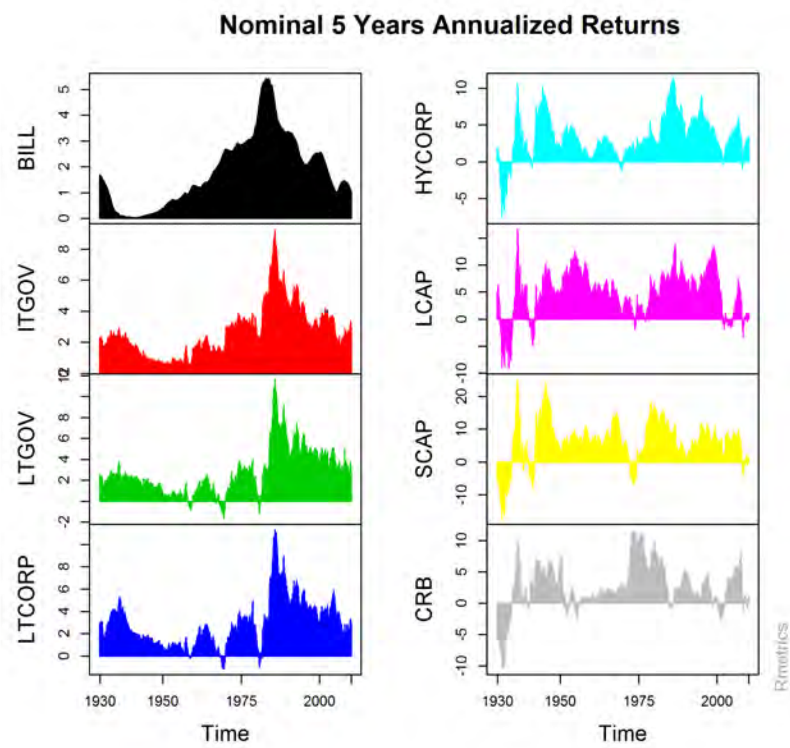

FIGURE 6.3: 5 years annual total return series for the 8 asset classes.

5 Years Annual Total Returns Statistics

	BILL	ITGOV	LTGOV	LTCORP	HYCORP	LCAP	SCAP	CRB
1930-12-31	1.69	2.32	2.43	2.84	1.32	4.25	-6.43	-5.96
1935-12-31	0.26	2.36	2.48	4.13	1.86	1.55	7.23	2.33
1940-12-31	0.04	1.82	2.48	2.27	0.80	0.25	-1.33	-0.25
1945-12-31	0.13	0.92	1.93	1.68	8.90	8.15	20.79	4.83
1950-12-31	0.40	0.68	0.69	0.88	1.94	4.84	3.79	7.19
1955-12-31	0.74	0.72	0.64	0.99	3.45	11.31	7.22	-2.06
1960-12-31	1.27	1.67	0.58	0.68	0.20	4.36	5.16	0.46
1965-12-31	1.53	1.40	1.31	1.89	3.24	6.42	9.67	2.44
1970-12-31	2.69	2.51	-0.01	0.61	-0.07	1.66	3.69	2.50
1975-12-31	2.85	3.15	3.03	2.96	1.88	1.59	0.28	9.54
1980-12-31	3.82	2.51	0.84	1.17	3.17	6.75	17.20	9.09
1985-12-31	5.02	7.61	8.09	8.56	9.81	7.10	9.01	0.04
1990-12-31	3.36	4.56	5.23	5.09	2.38	6.37	0.29	4.11
1995-12-31	2.13	4.32	6.35	5.93	8.86	7.97	11.58	1.70
2000-12-31	2.55	3.04	3.68	2.85	2.12	8.79	5.29	0.81
2005-12-31	1.06	2.58	3.79	4.55	4.34	0.29	7.91	3.85
2010-12-31	1.04	2.98	2.77	2.93	3.27	1.14	1.59	1.15

FIGURE 6.4: 5 years annual total return series for the 8 asset classes. We have printed the values ever five years.

10 Years Annual Total Returns

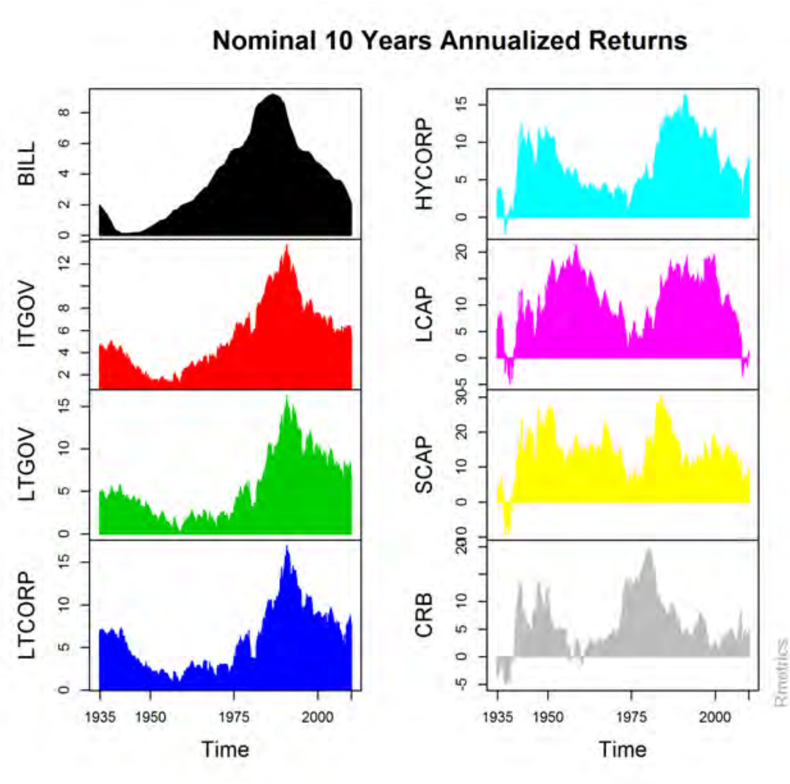

FIGURE 6.5: 10 years annual total return series for the 8 asset classes.

10 Years Annual Total Returns Statistics

	BILL	ITGOV	LTGOV	LTCORP	HYCORP	LCAP	SCAP	CRB
1935-12-31	1.96	4.73	4.97	7.08	3.20	5.86	0.34	-3.77
1940-12-31	0.31	4.21	5.02	6.49	2.68	1.80	5.81	2.07
1945-12-31	0.18	2.75	4.46	3.99	9.77	8.42	19.18	4.57
1950-12-31	0.53	1.60	2.63	2.57	11.02	13.38	25.37	12.37
1955-12-31	1.14	1.40	1.33	1.87	5.46	16.69	11.29	4.98
1960-12-31	2.02	2.40	1.22	1.67	3.66	16.16	12.75	-1.61
1965-12-31	2.82	3.09	1.89	2.58	3.45	11.06	15.33	2.92
1970-12-31	4.26	3.95	1.30	2.51	3.17	8.18	13.72	5.00
1975-12-31	5.61	5.74	3.02	3.59	1.81	3.27	3.98	12.27
1980-12-31	6.77	5.73	3.90	4.16	5.10	8.44	17.53	19.50
1985-12-31	9.03	10.31	8.99	9.84	13.29	14.33	27.75	9.14
1990-12-31	8.55	12.52	13.75	14.09	12.42	13.93	9.32	4.15
1995-12-31	5.56	9.07	11.91	11.32	11.45	14.84	11.90	5.88
2000-12-31	4.73	7.48	10.26	8.95	11.17	17.46	17.49	2.52
2005-12-31	3.64	5.70	7.60	7.53	6.55	9.11	13.62	4.69
2010-12-31	2.11	5.63	6.66	7.61	7.75	1.43	9.63	5.04

FIGURE 6.6: 10 years annual total return series for the 8 asset classes. We have printed the values ever five years.

20 Years Annual Total Returns

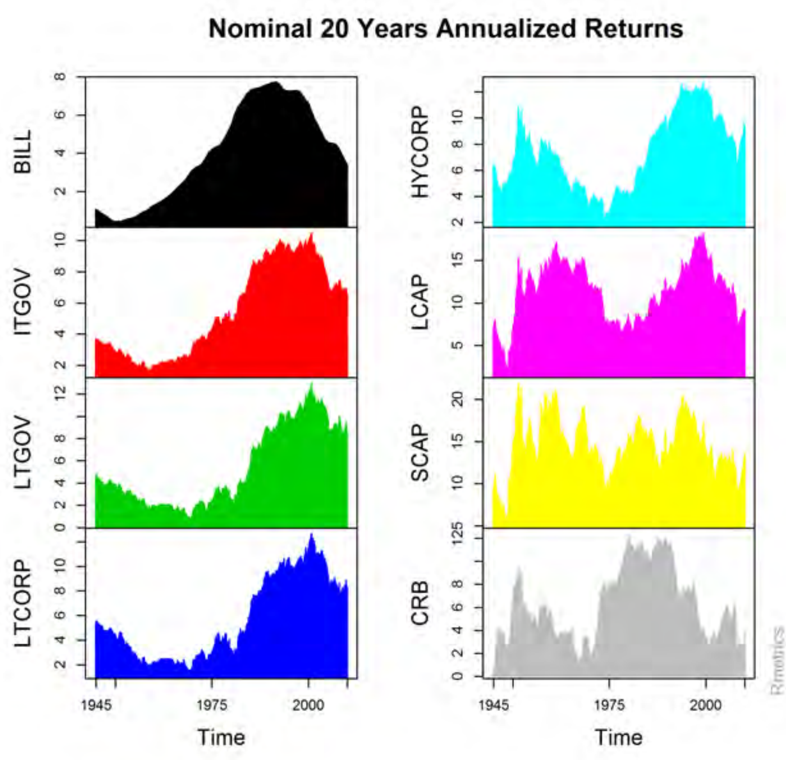

FIGURE 6.7: 20 years annual total return series for the 8 asset classes.

20 Years Annual Total Returns Statistics

	BILL	ITGOV	LTGOV	LTCORP	HYCORP	LCAP	SCAP	CRB
1945-12-31	1.07	3.74	4.71	5.52	6.44	7.13	9.36	0.31
1950-12-31	0.42	2.90	3.82	4.51	6.77	7.43	15.17	7.10
1955-12-31	0.66	2.07	2.88	2.92	7.60	12.48	15.17	4.78
1960-12-31	1.27	2.00	1.92	2.12	7.28	14.76	18.89	5.15
1965-12-31	1.98	2.24	1.61	2.23	4.45	13.84	13.29	3.95
1970-12-31	3.13	3.17	1.26	2.09	3.42	12.10	13.23	1.65
1975-12-31	4.21	4.41	2.46	3.08	2.63	7.10	9.51	7.49
1980-12-31	5.51	4.84	2.59	3.33	4.13	8.31	15.61	12.02
1985-12-31	7.31	8.00	5.97	6.66	7.39	8.66	15.25	10.69
1990-12-31	7.66	9.07	8.71	9.01	8.70	11.15	13.35	11.56
1995-12-31	7.28	9.69	10.44	10.58	12.37	14.59	19.57	7.50
2000-12-31	6.63	9.97	11.99	11.49	11.79	15.68	13.33	3.33
2005-12-31	4.59	7.37	9.74	9.41	8.97	11.94	12.76	5.28
2010-12-31	3.41	6.55	8.45	8.28	9.45	9.15	13.49	3.77

FIGURE 6.8: 20 years annual total return series for the 8 asset classes. We have printed the values ever five years.

INFLATION ADJUSTED ANNUAL RETURNS

Inflation Adjusted: 1 Year Annual Returns

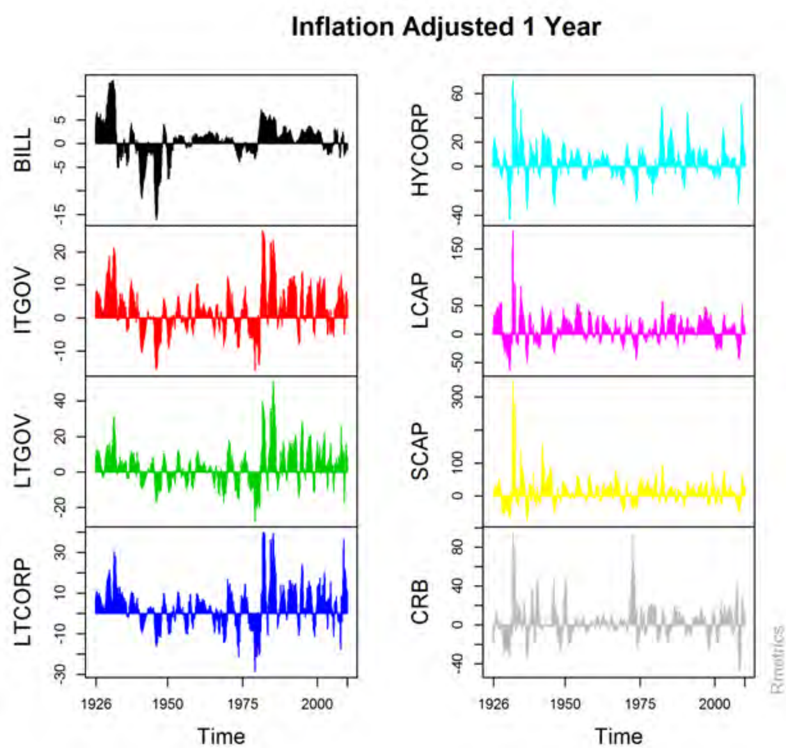

FIGURE 6.9: Inflation adjusted 1-year annual total return series for the 8 asset classes.

Inflation Adjusted: 1 Year Annual Total Returns Statistics

	BILL	ITGOV	LTGOV	LTCORP	HYCORP	LCAP	SCAP	CRB
1926-12-31	4.41	6.56	9.00	8.59	15.74	12.88	1.42	-17.12
1931-12-31	11.45	7.72	4.40	8.23	-23.83	-37.51	-44.58	-22.53
1936-12-31	-1.23	1.58	5.95	5.22	20.70	32.02	62.45	14.24
1941-12-31	-9.01	-8.59	-8.19	-6.57	-5.72	-19.58	-17.22	33.88
1946-12-31	-15.04	-14.48	-15.41	-13.87	-23.68	-22.19	-25.18	34.96
1951-12-31	-4.28	-5.31	-9.38	-8.21	-6.52	16.98	1.70	-14.32
1956-12-31	-0.50	-3.31	-8.34	-9.52	-9.58	3.49	1.28	4.80
1961-12-31	1.43	1.20	0.30	4.12	3.78	26.05	31.20	7.67
1966-12-31	1.24	1.17	0.18	-3.16	-7.51	-13.07	-10.13	-0.34
1971-12-31	1.08	5.31	9.65	7.51	9.56	10.72	12.82	1.96
1976-12-31	0.20	7.64	11.35	13.14	15.99	18.11	50.09	7.36
1981-12-31	5.32	0.46	-6.48	-9.34	1.37	-12.70	4.55	-13.07
1986-12-31	5.02	13.87	23.18	18.54	16.18	17.16	5.68	-2.26
1991-12-31	2.45	12.02	15.73	16.32	41.87	26.67	40.35	-6.89
1996-12-31	1.81	-1.18	-4.11	-1.86	7.80	19.11	13.84	8.45
2001-12-31	2.24	5.97	2.11	8.95	3.69	-13.24	20.90	-18.45
2006-12-31	2.11	0.48	-1.07	0.68	6.21	12.92	13.30	-5.28
2010-12-31	-1.36	5.54	8.52	10.79	10.93	13.35	29.32	15.88

FIGURE 6.10: 1 year annual total return series for the 8 asset classes. We have printed the values ever five years.

Inflation Adjusted: 5 Years Annual Returns

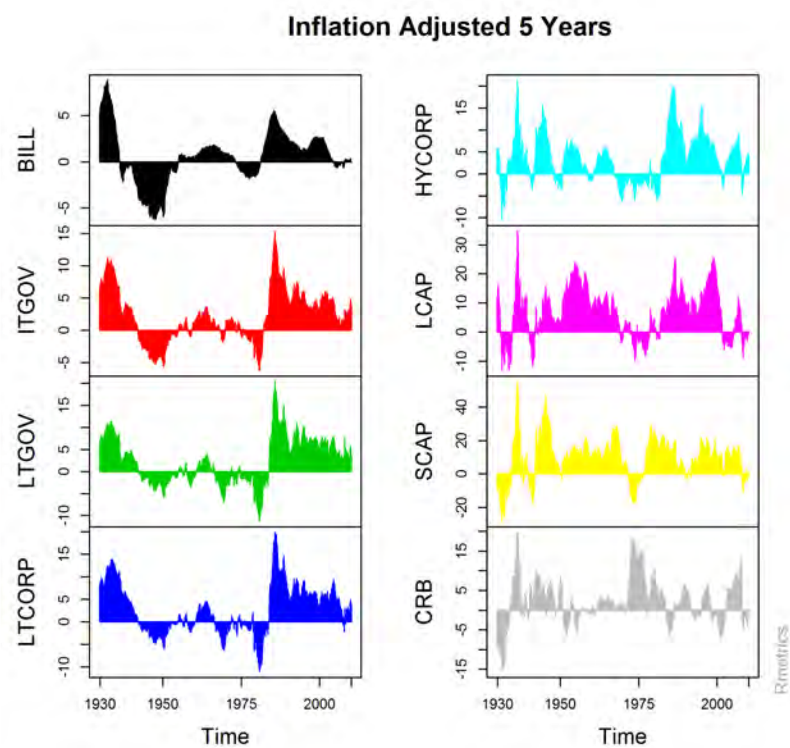

FIGURE 6.11: Inflation adjusted 5-years annual total return series for the 8 asset classes.

Inflation Adjusted: 5-Years Annual Total Returns Statistics

	BILL	ITGOV	LTGOV	LTCORP	HYCORP	LCAP	SCAP	CRB
1930-12-31	5.63	6.93	7.17	8.03	4.85	11.00	-10.57	-9.67
1935-12-31	3.68	8.05	8.30	11.82	7.01	6.34	18.60	7.98
1940-12-31	-0.35	3.22	4.57	4.13	1.17	0.07	-3.07	-0.93
1945-12-31	-4.74	-3.23	-1.27	-1.76	12.69	11.14	38.63	4.42
1950-12-31	-5.39	-4.87	-4.86	-4.49	-2.47	3.15	1.09	7.83
1955-12-31	0.07	0.04	-0.12	0.57	5.54	22.17	13.37	-5.40
1960-12-31	0.41	1.20	-0.96	-0.77	-1.70	6.64	8.27	-1.19
1965-12-31	1.75	1.49	1.31	2.48	5.22	11.79	18.72	3.59
1970-12-31	0.83	0.48	-4.41	-3.22	-4.52	-1.20	2.78	0.45
1975-12-31	-1.03	-0.45	-0.68	-0.82	-2.89	-3.42	-5.91	12.26
1980-12-31	-1.33	-3.80	-6.91	-6.30	-2.56	4.33	25.74	8.95
1985-12-31	5.20	10.46	11.44	12.42	15.02	9.42	13.34	-4.54
1990-12-31	2.60	4.99	6.35	6.05	0.65	8.65	-3.41	4.09
1995-12-31	1.47	5.87	10.03	9.17	15.30	13.42	21.13	0.63
2000-12-31	2.57	3.54	4.83	3.17	1.69	15.42	8.12	-0.89
2005-12-31	-0.37	2.66	5.09	6.64	6.22	-1.86	13.61	5.23
2010-12-31	-0.08	3.78	3.38	3.69	4.38	0.11	1.01	0.12

FIGURE 6.12: Inflation adjusted 5-years annual total return series for the 8 asset classes. We have printed the values every five years.

Inflation Adjusted: 10 Years Annual Returns

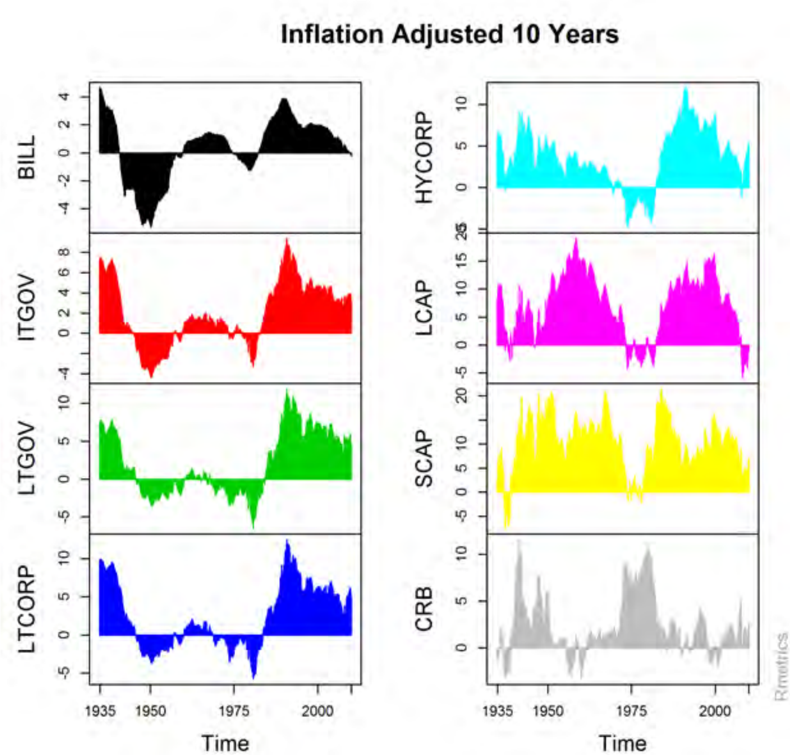

FIGURE 6.13: Inflation adjusted 10-years annual total return series for the 8 asset classes.

Inflation Adjusted: 10-Years Annual Total Returns Statistics

	BILL	ITGOV	LTGOV	LTCORP	HYCORP	LCAP	SCAP	CRB
1935-12-31	4.65	7.49	7.74	9.91	5.92	8.65	2.99	-1.24
1940-12-31	1.65	5.61	6.42	7.91	4.05	3.16	7.22	3.43
1945-12-31	-2.57	-0.06	1.61	1.14	6.77	5.46	15.92	1.71
1950-12-31	-5.07	-4.05	-3.08	-3.14	4.84	7.07	18.38	6.11
1955-12-31	-2.70	-2.44	-2.52	-1.99	1.46	12.26	7.06	1.00
1960-12-31	0.24	0.62	-0.54	-0.10	1.86	14.14	10.79	-3.32
1965-12-31	1.08	1.34	0.17	0.84	1.70	9.19	13.37	1.17
1970-12-31	1.29	0.98	-1.59	-0.41	0.23	5.09	10.47	2.01
1975-12-31	-0.11	0.01	-2.56	-2.03	-3.71	-2.32	-1.66	6.19
1980-12-31	-1.18	-2.14	-3.85	-3.60	-2.73	0.38	8.77	10.59
1985-12-31	1.88	3.08	1.85	2.64	5.87	6.84	19.38	1.98
1990-12-31	3.89	7.69	8.87	9.19	7.60	9.04	4.63	-0.32
1995-12-31	2.04	5.43	8.18	7.60	7.73	11.01	8.17	2.34
2000-12-31	2.02	4.70	7.40	6.13	8.28	14.41	14.44	-0.13
2005-12-31	1.09	3.10	4.96	4.89	3.93	6.43	10.83	2.12
2010-12-31	-0.22	3.22	4.23	5.15	5.29	-0.88	7.13	2.64

FIGURE 6.14: Inflation adjusted 10-years annual total return series for the 8 asset classes. We have printed the values ever five years.

Inflation Adjusted: 20 Years Annual Returns

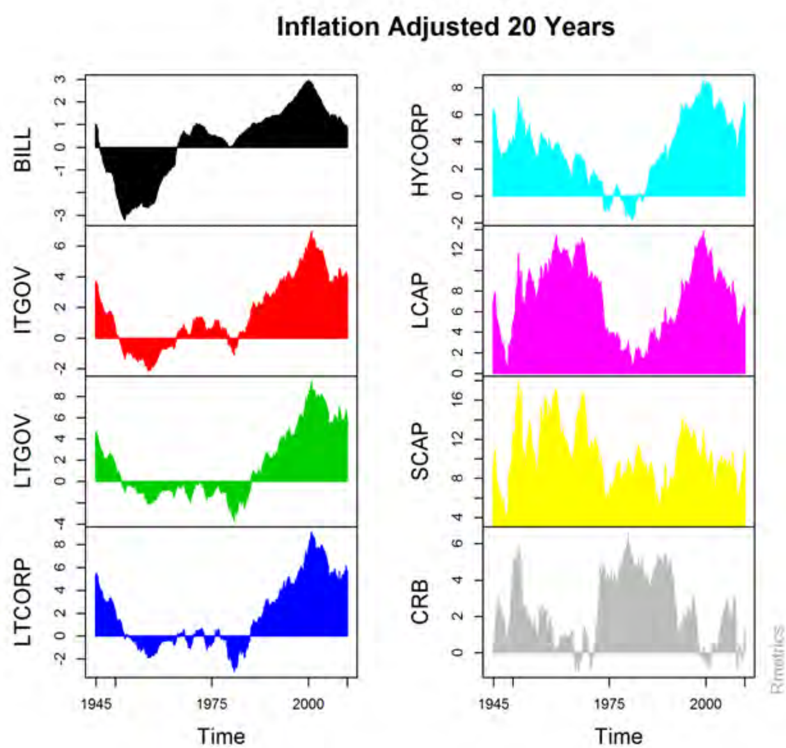

FIGURE 6.15: Inflation adjusted 20-years annual total return series for the 8 asset classes.

Inflation Adjusted: 20-Years Annual Total Returns Statistics

	BILL	ITGOV	LTGOV	LTCORP	HYCORP	LCAP	SCAP	CRB
1945-12-31	0.98	3.65	4.63	5.43	6.35	7.04	9.26	0.22
1950-12-31	-1.77	0.66	1.56	2.24	4.44	5.09	12.66	4.76
1955-12-31	-2.63	-1.26	-0.48	-0.44	4.08	8.80	11.40	1.35
1960-12-31	-2.45	-1.74	-1.82	-1.63	3.34	10.55	14.52	1.29
1965-12-31	-0.83	-0.57	-1.19	-0.59	1.58	10.71	10.17	1.08
1970-12-31	0.76	0.80	-1.07	-0.26	1.04	9.52	10.63	-0.69
1975-12-31	0.48	0.67	-1.21	-0.60	-1.04	3.27	5.59	3.65
1980-12-31	0.04	-0.59	-2.73	-2.02	-1.26	2.71	9.62	6.22
1985-12-31	0.88	1.54	-0.38	0.28	0.96	2.16	8.35	4.07
1990-12-31	1.32	2.66	2.31	2.60	2.31	4.62	6.68	4.99
1995-12-31	1.96	4.25	4.97	5.09	6.79	8.91	13.63	2.16
2000-12-31	2.95	6.18	8.13	7.65	7.94	11.69	9.42	-0.23
2005-12-31	1.56	4.26	6.56	6.24	5.81	8.69	9.49	2.23
2010-12-31	0.89	3.96	5.80	5.64	6.78	6.49	10.72	1.24

FIGURE 6.16: Inflation adjusted 20-years annual total return series for the 8 asset classes. We have printed the values ever five years.

PREMIUM

Premium: 1 Year Annual Returns

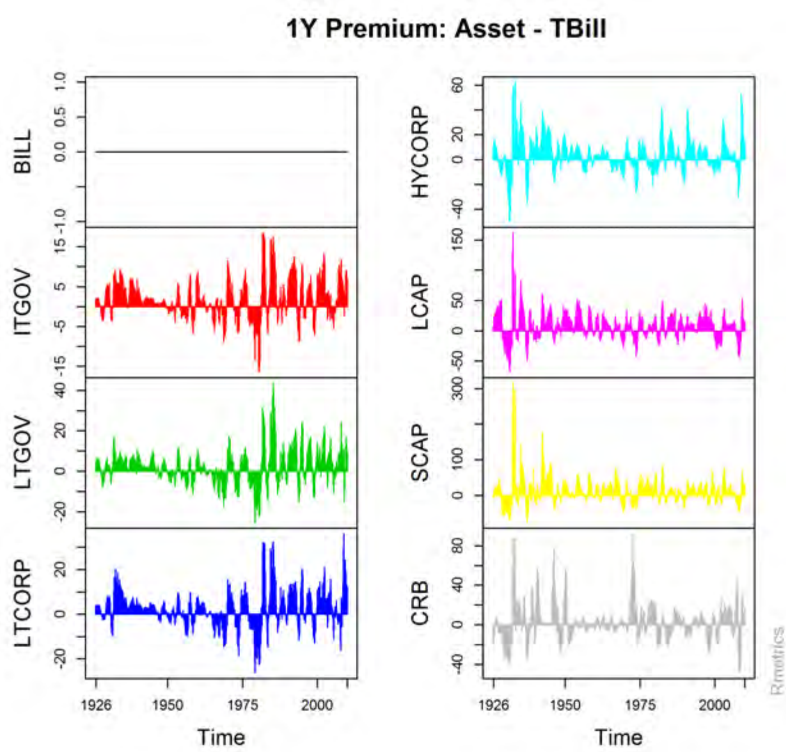

FIGURE 6.17: Premium: 1 year annual total return series for the 8 asset classes.

Premium: 1 Year Annual Total Returns Statistics

	BILL	ITGOV	LTGOV	LTCORP	HYCORP	LCAP	SCAP	CRB
1926-12-31	0	2.06	4.35	3.98	10.82	8.09	-2.89	-20.64
1931-12-31	0	-3.36	-6.34	-2.90	-31.64	-43.94	-50.28	-30.49
1936-12-31	0	2.85	7.29	6.53	22.23	33.69	64.52	15.69
1941-12-31	0	0.44	0.88	2.69	3.61	-11.63	-9.04	47.13
1946-12-31	0	0.66	-0.43	1.39	-10.16	-8.39	-11.94	58.85
1951-12-31	0	-1.08	-5.34	-4.09	-2.33	22.21	6.24	-10.53
1956-12-31	0	-2.81	-7.89	-9.07	-9.13	3.98	1.77	5.34
1961-12-31	0	-0.24	-1.11	2.65	2.30	24.24	29.33	6.16
1966-12-31	0	-0.07	-1.06	-4.35	-8.65	-14.16	-11.24	-1.55
1971-12-31	0	4.15	8.49	6.35	8.39	9.50	11.62	0.87
1976-12-31	0	7.44	11.13	12.90	15.73	17.85	49.77	7.16
1981-12-31	0	-4.61	-11.20	-13.91	-3.72	-17.10	-0.72	-17.46
1986-12-31	0	8.44	17.29	12.89	10.64	11.60	0.67	-6.91
1991-12-31	0	9.34	12.93	13.53	38.46	23.63	36.96	-9.11
1996-12-31	0	-2.95	-5.82	-3.61	5.84	17.00	11.79	6.47
2001-12-31	0	3.65	-0.14	6.58	1.41	-15.13	18.26	-20.24
2006-12-31	0	-1.61	-3.14	-1.40	4.03	10.58	10.94	-7.21
2010-12-31	0	6.98	10.02	12.30	12.47	14.91	31.12	17.46

FIGURE 6.18: 1 year annual total return series for the 8 asset classes. We have printed the values ever five years.

Premium: 10 Year Annual Returns

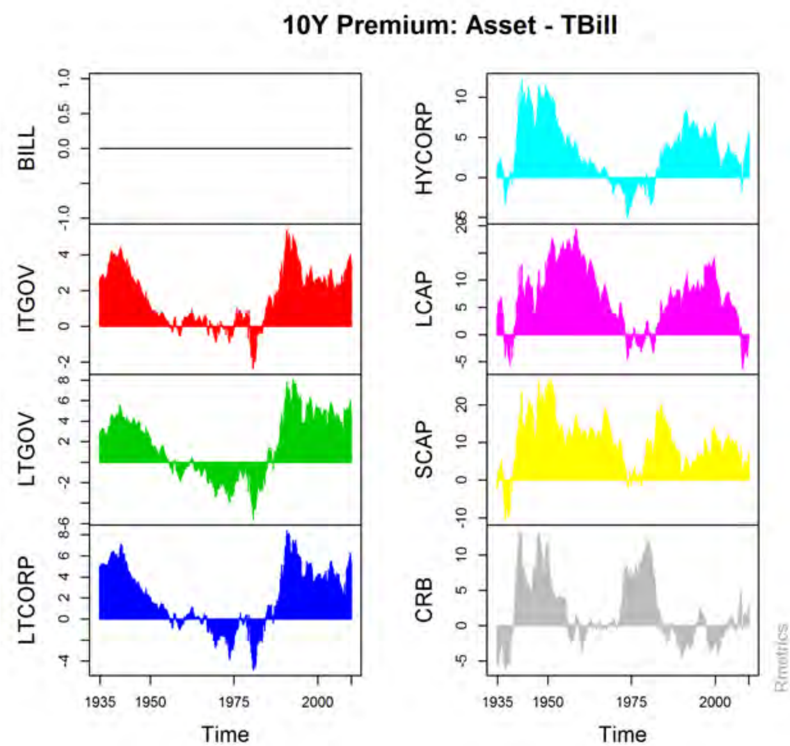

FIGURE 6.19: Premium: 10 years annual total return series for the 8 asset classes.

Premium: 10 Years Annual Total Returns Statistics

	BILL	ITGOV	LTGOV	LTCORP	HYCORP	LCAP	SCAP	CRB
1935-12-31	0	2.71	2.94	5.02	1.22	3.83	-1.59	-5.63
1940-12-31	0	3.89	4.69	6.16	2.37	1.49	5.48	1.76
1945-12-31	0	2.58	4.28	3.81	9.59	8.22	18.98	4.38
1950-12-31	0	1.07	2.09	2.04	10.43	12.78	24.70	11.77
1955-12-31	0	0.27	0.19	0.73	4.28	15.38	10.04	3.80
1960-12-31	0	0.38	-0.79	-0.34	1.61	13.87	10.53	-3.55
1965-12-31	0	0.26	-0.90	-0.23	0.61	8.02	12.17	0.10
1970-12-31	0	-0.30	-2.84	-1.68	-1.05	3.76	9.07	0.73
1975-12-31	0	0.11	-2.46	-1.92	-3.60	-2.22	-1.55	6.31
1980-12-31	0	-0.98	-2.70	-2.45	-1.56	1.57	10.07	11.92
1985-12-31	0	1.17	-0.03	0.74	3.91	4.87	17.18	0.10
1990-12-31	0	3.65	4.79	5.10	3.57	4.96	0.72	-4.05
1995-12-31	0	3.33	6.01	5.45	5.60	8.80	6.02	0.31
2000-12-31	0	2.63	5.26	4.03	6.15	12.15	12.18	-2.11
2005-12-31	0	1.99	3.82	3.76	2.81	5.28	9.63	1.01
2010-12-31	0	3.45	4.46	5.39	5.54	-0.66	7.36	2.87

FIGURE 6.20: Premium: 10 years annual total return series for the 8 asset classes. We have printed the values ever five years.

HOLDING PERIODS

Mean / StDev: Nominal Holding Periods

	BILL	ITGOV	LTGOV	LTCORP	HYCORP	LCAP	SCAP	CRB
1 Year	3.68	5.51	5.85	6.25	7.56	12.10	17.13	6.35
2 Years	3.70	5.45	5.65	6.03	7.02	10.82	14.29	5.62
3 Years	3.72	5.44	5.59	5.94	6.84	10.38	13.48	5.51
4 Years	3.73	5.44	5.57	5.93	6.83	10.16	13.24	5.45
5 Years	3.74	5.42	5.55	5.93	6.82	10.08	13.24	5.50
6 Years	3.74	5.42	5.55	5.93	6.86	10.12	13.31	5.61
7 Years	3.76	5.44	5.55	5.95	6.99	10.31	13.55	5.74
8 Years	3.78	5.46	5.55	5.95	7.07	10.44	13.67	5.83
9 Years	3.80	5.47	5.54	5.93	7.10	10.54	13.73	5.86
10 Years	3.83	5.47	5.53	5.90	7.13	10.67	13.83	5.88
11 Years	3.85	5.46	5.51	5.87	7.14	10.78	13.88	5.91
12 Years	3.87	5.46	5.50	5.84	7.16	10.86	13.88	5.91
13 Years	3.90	5.46	5.49	5.83	7.22	10.97	14.00	5.98
14 Years	3.92	5.46	5.48	5.81	7.28	11.04	14.11	6.05
15 Years	3.94	5.46	5.46	5.79	7.32	11.10	14.20	6.12
16 Years	3.97	5.46	5.44	5.77	7.34	11.16	14.28	6.15
17 Years	3.99	5.46	5.42	5.76	7.37	11.22	14.37	6.17
18 Years	4.02	5.47	5.41	5.75	7.37	11.26	14.41	6.18
19 Years	4.04	5.48	5.39	5.73	7.37	11.29	14.42	6.21
20 Years	4.07	5.48	5.37	5.71	7.33	11.32	14.40	6.24

FIGURE 6.21: Minimum holding periods: annualized nominal mean of total returns over all periods. Values are given in percent.

	BILL	ITGOV	LTGOV	LTCORP	HYCORP	LCAP	SCAP	CRB
1 Year	3.08	5.53	8.83	8.69	13.33	21.95	36.48	18.37
2 Years	3.01	4.29	6.25	6.29	9.78	15.21	22.67	12.96
3 Years	2.95	3.78	5.19	5.26	8.07	12.16	18.07	10.23
4 Years	2.90	3.55	4.75	4.77	6.96	10.26	15.97	8.63
5 Years	2.87	3.40	4.50	4.45	6.00	8.83	13.61	7.49
6 Years	2.85	3.28	4.27	4.16	5.22	7.61	11.32	6.52
7 Years	2.83	3.21	4.11	3.98	4.67	6.73	9.50	5.77
8 Years	2.81	3.17	4.03	3.87	4.26	6.23	8.18	5.40
9 Years	2.80	3.13	3.94	3.76	4.01	5.93	7.33	5.21
10 Years	2.78	3.10	3.87	3.69	3.91	5.68	6.73	4.98
11 Years	2.76	3.08	3.83	3.63	3.90	5.49	6.49	4.79
12 Years	2.75	3.06	3.80	3.59	3.84	5.26	6.24	4.62
13 Years	2.73	3.04	3.78	3.55	3.71	5.00	5.84	4.34
14 Years	2.71	3.02	3.75	3.51	3.53	4.74	5.30	4.07
15 Years	2.69	3.00	3.72	3.48	3.36	4.47	4.69	3.86
16 Years	2.67	2.98	3.69	3.45	3.23	4.21	4.14	3.68
17 Years	2.64	2.96	3.66	3.42	3.09	3.98	3.63	3.52
18 Years	2.62	2.95	3.63	3.38	2.97	3.79	3.39	3.41
19 Years	2.59	2.93	3.60	3.35	2.88	3.61	3.27	3.29
20 Years	2.57	2.90	3.55	3.31	2.80	3.41	3.17	3.13

FIGURE 6.22: Minimum holding period: annualized nominal standard deviations of total returns over all periods. Values are given in percent.

Worst / Negative Periods: Nominal Holding Periods

	BILL	ITGOV	LTGOV	LTCORP	HYCORP	LCAP	SCAP	CRB
1 Year	-0.06	-5.55	-17.10	-18.17	-49.33	-67.56	-75.90	-48.17
2 Years	-0.04	-0.85	-10.01	-10.99	-33.47	-54.34	-57.55	-34.47
3 Years	-0.02	-0.41	-6.03	-6.90	-22.67	-42.35	-51.94	-30.41
4 Years	-0.01	0.42	-4.90	-4.94	-18.82	-27.52	-41.62	-24.51
5 Years	0.04	0.66	-3.27	-2.31	-14.65	-17.36	-31.63	-19.91
6 Years	0.07	0.83	-2.09	-1.34	-9.92	-12.04	-24.40	-16.71
7 Years	0.09	0.88	-1.51	-0.73	-6.33	-6.12	-19.91	-15.15
8 Years	0.09	0.94	-1.04	-0.03	-2.68	-7.72	-9.41	-8.59
9 Years	0.11	1.08	-0.44	0.68	-1.64	-6.65	-12.60	-5.71
10 Years	0.13	1.17	-0.08	0.60	-2.20	-4.95	-9.26	-5.13
11 Years	0.15	1.18	0.50	0.67	-1.28	-4.48	-6.69	-4.46
12 Years	0.17	1.20	-0.12	0.94	-0.53	-4.06	-5.76	-4.37
13 Years	0.18	1.27	0.47	1.02	-0.17	-4.26	-5.47	-4.46
14 Years	0.19	1.31	0.19	1.06	1.03	-1.40	-3.84	-2.51
15 Years	0.20	1.38	0.22	1.02	1.51	-0.41	-1.96	-2.69
16 Years	0.23	1.35	0.29	1.11	2.15	1.11	-1.21	-0.31
17 Years	0.27	1.47	0.95	1.41	2.33	1.36	0.79	0.09
18 Years	0.32	1.48	0.59	1.44	1.86	1.43	3.91	0.25
19 Years	0.37	1.51	0.66	1.30	1.65	1.87	5.63	0.24
20 Years	0.42	1.58	0.46	1.33	2.08	1.89	5.42	0.31

FIGURE 6.23: Minimum holding period: worst annualized nominal return over all periods. Values are given in percent.

	BILL	ITGOV	LTGOV	LTCORP	HYCORP	LCAP	SCAP	CRB
1 Year	2.67	8.42	21.88	19.11	22.38	26.63	28.32	32.87
2 Years	2.20	1.40	14.53	12.53	21.44	19.34	21.04	25.65
3 Years	1.42	0.41	11.05	9.53	16.84	17.75	18.05	23.43
4 Years	0.21	0.00	8.42	6.26	12.01	14.48	17.35	20.64
5 Years	0.00	0.00	5.61	3.95	8.21	13.10	13.31	16.11
6 Years	0.00	0.00	2.84	1.47	5.47	9.58	10.42	13.79
7 Years	0.00	0.00	1.92	0.85	4.26	5.86	8.10	10.45
8 Years	0.00	0.00	0.76	0.22	2.59	3.56	6.16	10.04
9 Years	0.00	0.00	0.33	0.00	1.86	4.27	3.94	9.30
10 Years	0.00	0.00	0.11	0.00	1.77	5.76	3.22	8.54
11 Years	0.00	0.00	0.00	0.00	1.91	3.71	4.61	7.98
12 Years	0.00	0.00	0.11	0.00	0.46	3.30	5.92	6.49
13 Years	0.00	0.00	0.00	0.00	0.12	2.66	4.97	4.62
14 Years	0.00	0.00	0.00	0.00	0.00	2.11	4.22	2.81
15 Years	0.00	0.00	0.00	0.00	0.00	0.24	2.38	0.83
16 Years	0.00	0.00	0.00	0.00	0.00	0.00	0.84	0.24
17 Years	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
18 Years	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
19 Years	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
20 Years	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

FIGURE 6.24: Minimum holding period: negative periods in percent.

Holding Periods: BILL

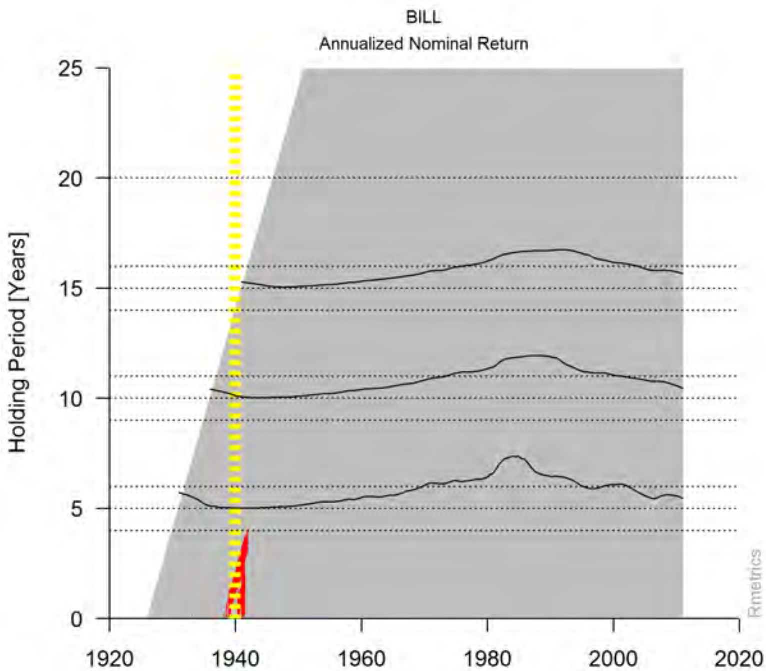

FIGURE 6.25: Holding Periods. First let's have a look at the dashed line that starts at a holding period of 5 years; indicated on the y-axis. The first annualized return can be calculated where the dashed line hits the gray background, which is at the end of 1930; indicated on the x-axis. The solid line shows a scaled version of the annualized returns for a period of 5 years for every point in time and uses the dashed line as zero line. The upper and under dashed lines have a distance of one year to the middle dashed line. We calculated the "annulized" returns from 1 month up to 300 months which leads to 300 potential vertical lines similar to the one for the 5 years holding period just explained. Showing scaled version of the annualized returns for every of the 300 holding periods does of course not make much sense because of overlapping. Therefore we made a gray point for every positive annualized return which leads to the gray background and a red point for every negative value which leads to the red structures. The yellow lines show the turning points as explained in **??**.

Holding Periods: BILL

	mean	sd	5% quant	worst	best	% neg
1 Year	3.68	3.08	0.09	-0.06	15.22	2.67
2 Years	3.70	3.01	0.13	-0.04	13.49	2.20
3 Years	3.72	2.95	0.13	-0.02	12.62	1.42
4 Years	3.73	2.90	0.13	-0.01	11.82	0.21
5 Years	3.74	2.87	0.12	0.04	11.14	0.00
6 Years	3.74	2.85	0.11	0.07	10.93	0.00
7 Years	3.76	2.83	0.13	0.09	10.49	0.00
8 Years	3.78	2.81	0.15	0.09	10.06	0.00
9 Years	3.80	2.80	0.16	0.11	9.61	0.00
10 Years	3.83	2.78	0.17	0.13	9.20	0.00
11 Years	3.85	2.76	0.19	0.15	9.04	0.00
12 Years	3.87	2.75	0.21	0.17	8.96	0.00
13 Years	3.90	2.73	0.24	0.18	8.80	0.00
14 Years	3.92	2.71	0.26	0.19	8.58	0.00
15 Years	3.94	2.69	0.29	0.20	8.33	0.00
16 Years	3.97	2.67	0.32	0.23	8.15	0.00
17 Years	3.99	2.64	0.35	0.27	8.14	0.00
18 Years	4.02	2.62	0.40	0.32	8.07	0.00
19 Years	4.04	2.59	0.44	0.37	7.93	0.00
20 Years	4.07	2.57	0.49	0.42	7.73	0.00
21 Years	4.10	2.54	0.54	0.47	7.61	0.00
22 Years	4.12	2.51	0.56	0.52	7.56	0.00
23 Years	4.14	2.47	0.61	0.56	7.48	0.00
24 Years	4.17	2.44	0.67	0.59	7.38	0.00
25 Years	4.19	2.40	0.73	0.62	7.25	0.00

FIGURE 6.26: Holding periods of bills. Values are given in percent.

Holding Periods: ITGOV

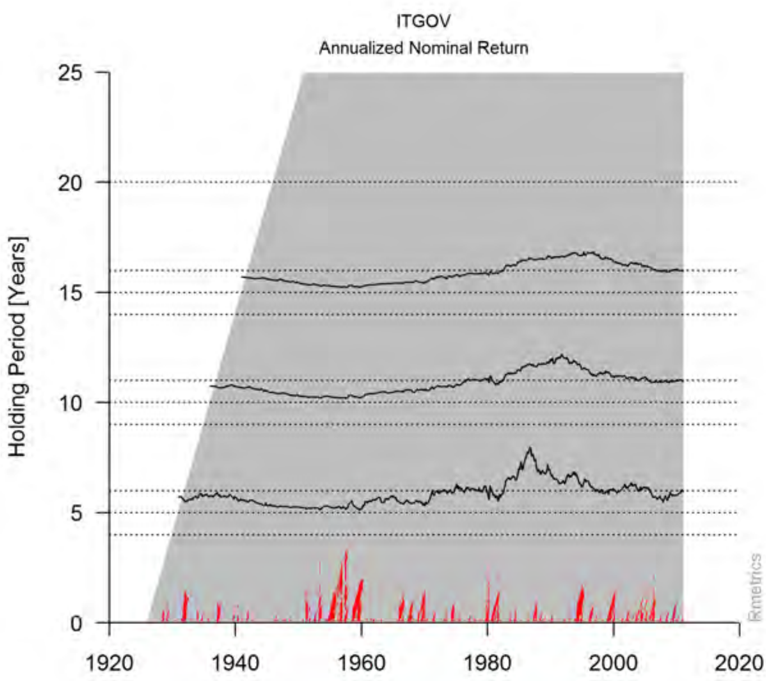

FIGURE 6.27: Holding periods of intermediate term government bonds.

Holding Periods: ITGOV

	mean	sd	5% quant	worst	best	% neg
1 Year	5.51	5.53	-0.79	-5.55	32.70	8.42
2 Years	5.45	4.29	0.78	-0.85	23.10	1.40
3 Years	5.44	3.78	1.25	-0.41	18.41	0.41
4 Years	5.44	3.55	1.40	0.42	18.92	0.00
5 Years	5.42	3.40	1.36	0.66	19.46	0.00
6 Years	5.42	3.28	1.37	0.83	16.41	0.00
7 Years	5.44	3.21	1.42	0.88	15.50	0.00
8 Years	5.46	3.17	1.46	0.94	14.53	0.00
9 Years	5.47	3.13	1.48	1.08	13.63	0.00
10 Years	5.47	3.10	1.48	1.17	13.73	0.00
11 Years	5.46	3.08	1.52	1.18	13.78	0.00
12 Years	5.46	3.06	1.54	1.20	13.60	0.00
13 Years	5.46	3.04	1.56	1.27	12.76	0.00
14 Years	5.46	3.02	1.59	1.31	12.15	0.00
15 Years	5.46	3.00	1.63	1.38	11.36	0.00
16 Years	5.46	2.98	1.71	1.35	11.26	0.00
17 Years	5.46	2.96	1.79	1.47	11.29	0.00
18 Years	5.47	2.95	1.86	1.48	10.73	0.00
19 Years	5.48	2.93	1.97	1.51	10.55	0.00
20 Years	5.48	2.90	1.99	1.58	10.50	0.00
21 Years	5.48	2.88	2.04	1.72	10.54	0.00
22 Years	5.49	2.85	2.10	1.92	10.22	0.00
23 Years	5.49	2.82	2.15	1.89	10.15	0.00
24 Years	5.49	2.79	2.21	1.96	9.86	0.00
25 Years	5.50	2.75	2.25	2.08	9.43	0.00

FIGURE 6.28: Holding periods of intermediate term government bonds. Values are given in percent.

Holding Periods: LTGOV

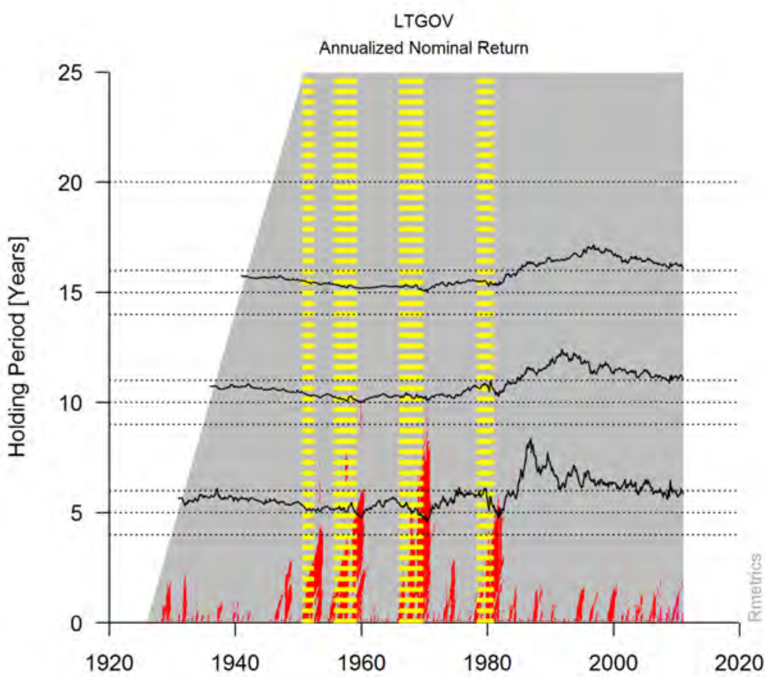

FIGURE 6.29: Holding periods of long term government bonds.

Holding Periods: LTGOV

	mean	sd	5% quant	worst	best	% neg
1 Year	5.85	8.83	-5.91	-17.10	54.41	21.88
2 Years	5.65	6.25	-2.43	-10.01	38.53	14.53
3 Years	5.59	5.19	-1.64	-6.03	25.42	11.05
4 Years	5.57	4.75	-1.08	-4.90	25.04	8.42
5 Years	5.55	4.50	-0.03	-3.27	24.60	5.61
6 Years	5.55	4.27	0.40	-2.09	19.90	2.84
7 Years	5.55	4.11	0.55	-1.51	18.01	1.92
8 Years	5.55	4.03	0.73	-1.04	17.69	0.76
9 Years	5.54	3.94	1.00	-0.44	16.26	0.33
10 Years	5.53	3.87	1.08	-0.08	16.34	0.11
11 Years	5.51	3.83	1.01	0.50	16.24	0.00
12 Years	5.50	3.80	1.12	-0.12	16.60	0.11
13 Years	5.49	3.78	1.14	0.47	14.66	0.00
14 Years	5.48	3.75	1.21	0.19	14.88	0.00
15 Years	5.46	3.72	1.32	0.22	14.00	0.00
16 Years	5.44	3.69	1.41	0.29	13.98	0.00
17 Years	5.42	3.66	1.40	0.95	14.48	0.00
18 Years	5.41	3.63	1.42	0.59	13.08	0.00
19 Years	5.39	3.60	1.50	0.66	12.93	0.00
20 Years	5.37	3.55	1.49	0.46	12.98	0.00
21 Years	5.34	3.51	1.46	0.64	13.06	0.00
22 Years	5.31	3.46	1.52	0.74	12.60	0.00
23 Years	5.29	3.42	1.62	0.70	12.28	0.00
24 Years	5.26	3.38	1.57	0.71	12.19	0.00
25 Years	5.24	3.33	1.60	0.89	11.69	0.00

FIGURE 6.30: Holding periods of long term government bonds. Values are given in percent.

Holding Periods: LTCORP

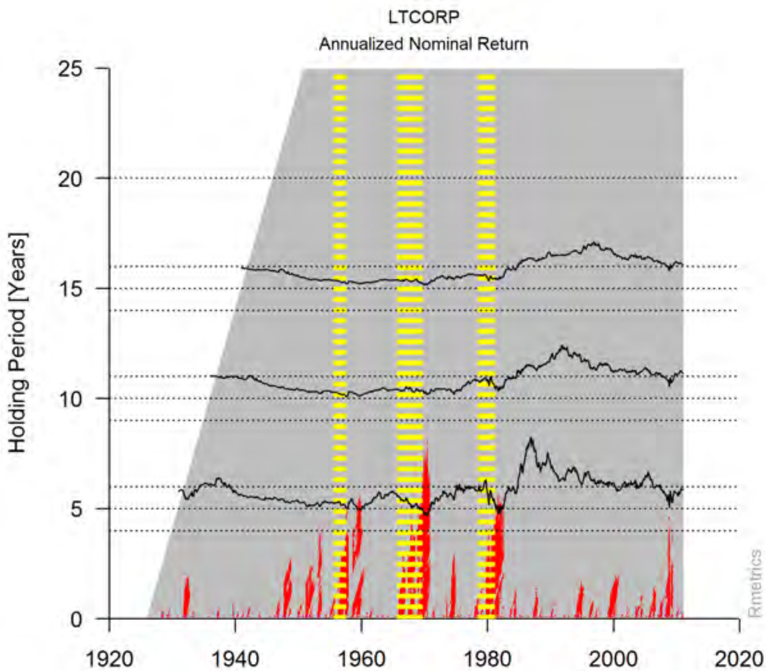

FIGURE 6.31: Holding periods of long term corporate bonds.

Holding Periods: LTCORP

	mean	sd	5% quant	worst	best	% neg
1 Year	6.25	8.69	-5.94	-18.17	46.74	19.11
2 Years	6.03	6.29	-2.34	-10.99	33.60	12.53
3 Years	5.94	5.26	-0.98	-6.90	23.85	9.53
4 Years	5.93	4.77	-0.40	-4.94	25.03	6.26
5 Years	5.93	4.45	0.35	-2.31	23.92	3.95
6 Years	5.93	4.16	0.92	-1.34	19.64	1.47
7 Years	5.95	3.98	1.06	-0.73	18.88	0.85
8 Years	5.95	3.87	1.30	-0.03	18.24	0.22
9 Years	5.93	3.76	1.47	0.68	16.61	0.00
10 Years	5.90	3.69	1.53	0.60	16.94	0.00
11 Years	5.87	3.63	1.63	0.67	16.69	0.00
12 Years	5.84	3.59	1.61	0.94	16.60	0.00
13 Years	5.83	3.55	1.73	1.02	14.59	0.00
14 Years	5.81	3.51	1.68	1.06	14.96	0.00
15 Years	5.79	3.48	1.70	1.02	14.17	0.00
16 Years	5.77	3.45	1.83	1.11	14.08	0.00
17 Years	5.76	3.42	1.87	1.41	14.13	0.00
18 Years	5.75	3.38	1.90	1.44	12.94	0.00
19 Years	5.73	3.35	1.93	1.30	12.52	0.00
20 Years	5.71	3.31	1.99	1.33	12.66	0.00
21 Years	5.69	3.27	2.01	1.36	12.80	0.00
22 Years	5.67	3.23	2.06	1.48	12.56	0.00
23 Years	5.65	3.19	2.13	1.36	12.28	0.00
24 Years	5.63	3.14	2.09	1.35	12.11	0.00
25 Years	5.62	3.09	2.09	1.46	11.68	0.00

FIGURE 6.32: Holding periods of long term corporate bonds. Values are given in percent.

Holding Periods: HYCORP

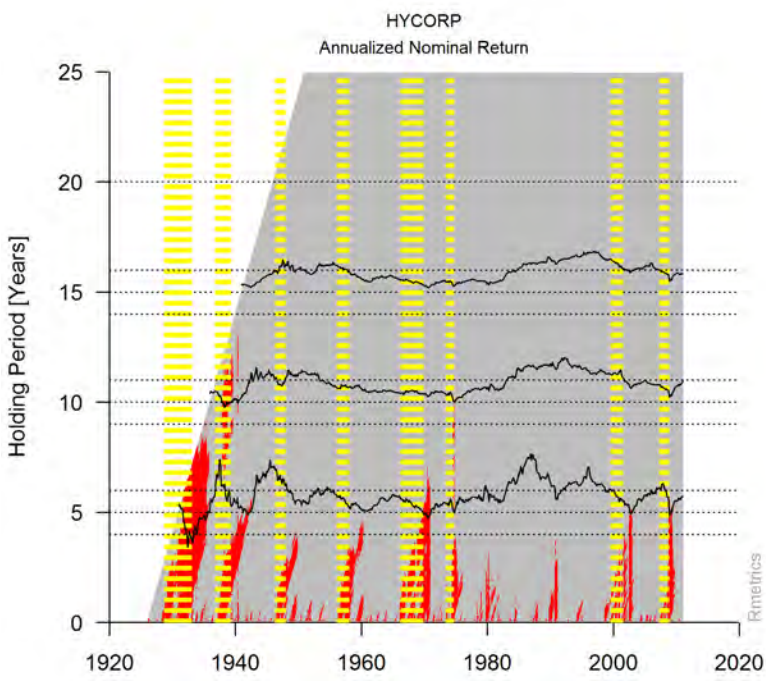

FIGURE 6.33: Holding periods of high yield corporate bonds.

Holding Periods: HYCORP

	mean	sd	5% quant	worst	best	% neg
1 Year	7.56	13.33	-11.60	-49.33	63.11	22.38
2 Years	7.02	9.78	-8.18	-33.47	38.18	21.44
3 Years	6.84	8.07	-5.15	-22.67	31.95	16.84
4 Years	6.83	6.96	-3.03	-18.82	27.04	12.01
5 Years	6.82	6.00	-1.66	-14.65	23.89	8.21
6 Years	6.86	5.22	-0.65	-9.92	21.26	5.47
7 Years	6.99	4.67	0.51	-6.33	20.36	4.26
8 Years	7.07	4.26	0.92	-2.68	18.11	2.59
9 Years	7.10	4.01	1.40	-1.64	16.92	1.86
10 Years	7.13	3.91	1.42	-2.20	16.41	1.77
11 Years	7.14	3.90	1.77	-1.28	16.48	1.91
12 Years	7.16	3.84	1.99	-0.53	16.26	0.46
13 Years	7.22	3.71	2.20	-0.17	15.63	0.12
14 Years	7.28	3.53	2.11	1.03	15.23	0.00
15 Years	7.32	3.36	2.59	1.51	14.90	0.00
16 Years	7.34	3.23	2.92	2.15	14.88	0.00
17 Years	7.37	3.09	3.10	2.33	14.29	0.00
18 Years	7.37	2.97	3.28	1.86	14.34	0.00
19 Years	7.37	2.88	3.41	1.65	13.56	0.00
20 Years	7.33	2.80	3.47	2.08	12.74	0.00
21 Years	7.29	2.73	3.56	2.44	12.76	0.00
22 Years	7.28	2.67	3.63	2.62	12.68	0.00
23 Years	7.26	2.60	3.65	2.73	12.76	0.00
24 Years	7.25	2.52	3.65	2.83	12.39	0.00
25 Years	7.24	2.45	3.61	3.04	11.88	0.00

FIGURE 6.34: Holding periods of high yield corporate bonds. Values are given in percent.

Holding Periods: LCAP

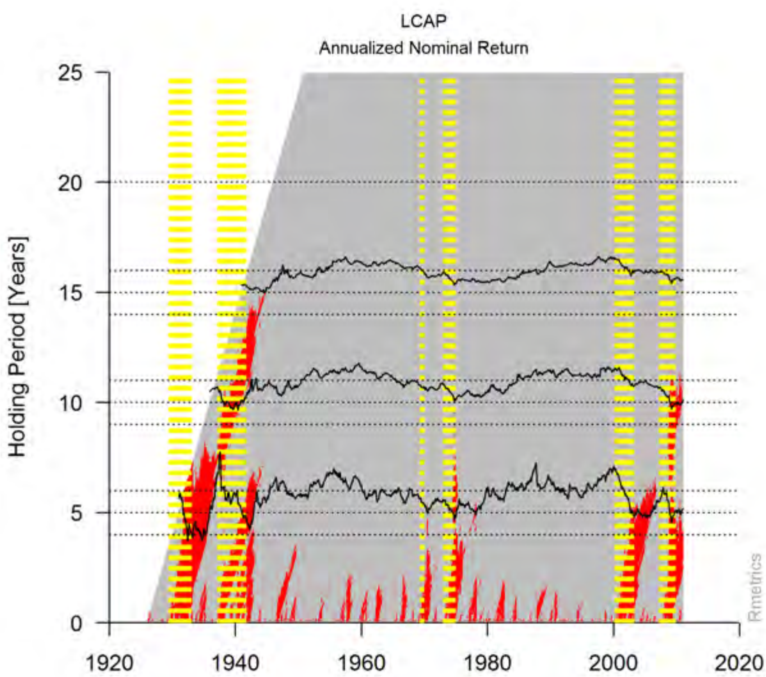

FIGURE 6.35: Holding periods of large cap equities.

Holding Periods: LCAP

	mean	sd	5% quant	worst	best	% neg
1 Year	12.10	21.95	-23.87	-67.56	162.91	26.63
2 Years	10.82	15.21	-15.73	-54.34	57.14	19.34
3 Years	10.38	12.16	-9.92	-42.35	43.35	17.75
4 Years	10.16	10.26	-7.70	-27.52	42.29	14.48
5 Years	10.08	8.83	-6.60	-17.36	36.12	13.10
6 Years	10.12	7.61	-3.37	-12.04	26.97	9.58
7 Years	10.31	6.73	-0.48	-6.12	25.82	5.86
8 Years	10.44	6.23	0.67	-7.72	22.99	3.56
9 Years	10.54	5.93	0.64	-6.65	21.27	4.27
10 Years	10.67	5.68	-0.29	-4.95	21.43	5.76
11 Years	10.78	5.49	0.37	-4.48	19.62	3.71
12 Years	10.86	5.26	2.07	-4.06	19.45	3.30
13 Years	10.97	5.00	2.87	-4.26	19.94	2.66
14 Years	11.04	4.74	3.02	-1.40	20.68	2.11
15 Years	11.10	4.47	2.87	-0.41	19.68	0.24
16 Years	11.16	4.21	3.61	1.11	19.66	0.00
17 Years	11.22	3.98	4.40	1.36	19.76	0.00
18 Years	11.26	3.79	5.31	1.43	19.17	0.00
19 Years	11.29	3.61	5.56	1.87	18.27	0.00
20 Years	11.32	3.41	5.59	1.89	18.25	0.00
21 Years	11.32	3.23	6.00	3.10	17.90	0.00
22 Years	11.35	3.04	6.04	4.40	17.75	0.00
23 Years	11.36	2.85	6.41	4.81	16.92	0.00
24 Years	11.39	2.59	7.14	4.54	17.09	0.00
25 Years	11.40	2.32	7.79	5.62	17.25	0.00

FIGURE 6.36: Holding periods of large cap stocks. Values are given in percent.

Holding Periods: SCAP

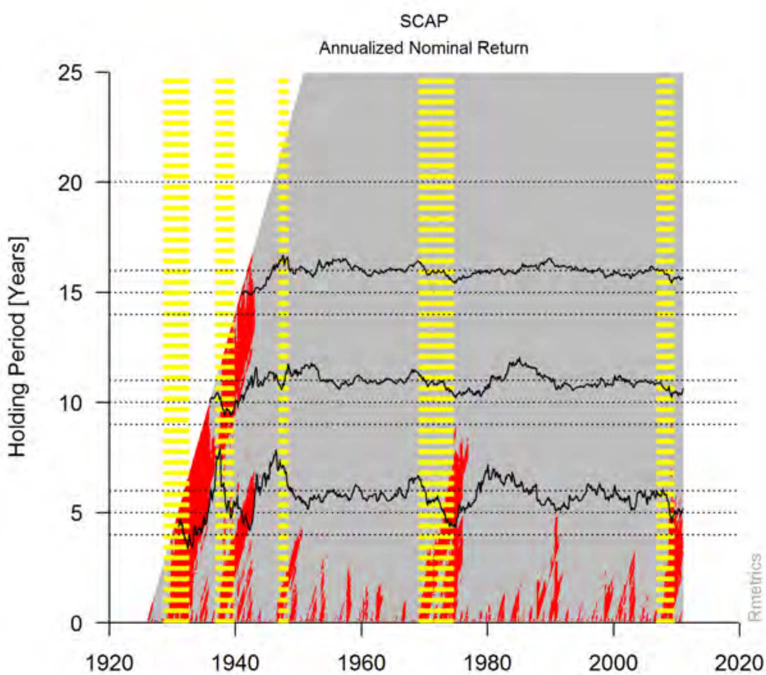

FIGURE 6.37: Holding periods of small cap equities.

Holding Periods: SCAP

	mean	sd	5% quant	worst	best	% neg
1 Year	17.13	36.48	-33.82	-75.90	316.50	28.32
2 Years	14.29	22.67	-25.65	-57.55	108.06	21.04
3 Years	13.48	18.07	-15.60	-51.94	88.93	18.05
4 Years	13.24	15.97	-10.81	-41.62	76.37	17.35
5 Years	13.24	13.61	-11.85	-31.63	56.79	13.31
6 Years	13.31	11.32	-8.63	-24.40	47.21	10.42
7 Years	13.55	9.50	-3.88	-19.91	40.43	8.10
8 Years	13.67	8.18	-0.71	-9.41	38.22	6.16
9 Years	13.73	7.33	1.01	-12.60	35.32	3.94
10 Years	13.83	6.73	2.94	-9.26	30.58	3.22
11 Years	13.88	6.49	1.54	-6.69	29.85	4.61
12 Years	13.88	6.24	-0.83	-5.76	27.76	5.92
13 Years	14.00	5.84	0.52	-5.47	30.10	4.97
14 Years	14.11	5.30	2.15	-3.84	30.86	4.22
15 Years	14.20	4.69	3.98	-1.96	24.61	2.38
16 Years	14.28	4.14	5.93	-1.21	24.88	0.84
17 Years	14.37	3.63	8.20	0.79	22.75	0.00
18 Years	14.41	3.39	8.49	3.91	23.50	0.00
19 Years	14.42	3.27	8.70	5.63	23.08	0.00
20 Years	14.40	3.17	8.84	5.42	21.90	0.00
21 Years	14.36	3.12	8.27	5.77	21.31	0.00
22 Years	14.37	3.03	8.16	6.74	21.54	0.00
23 Years	14.38	3.00	8.41	7.22	21.83	0.00
24 Years	14.42	2.91	8.94	7.16	21.30	0.00
25 Years	14.45	2.82	9.03	6.76	20.81	0.00

FIGURE 6.38: Holding periods of small cap stocks. Values are given in percent.

Holding Periods: CRB

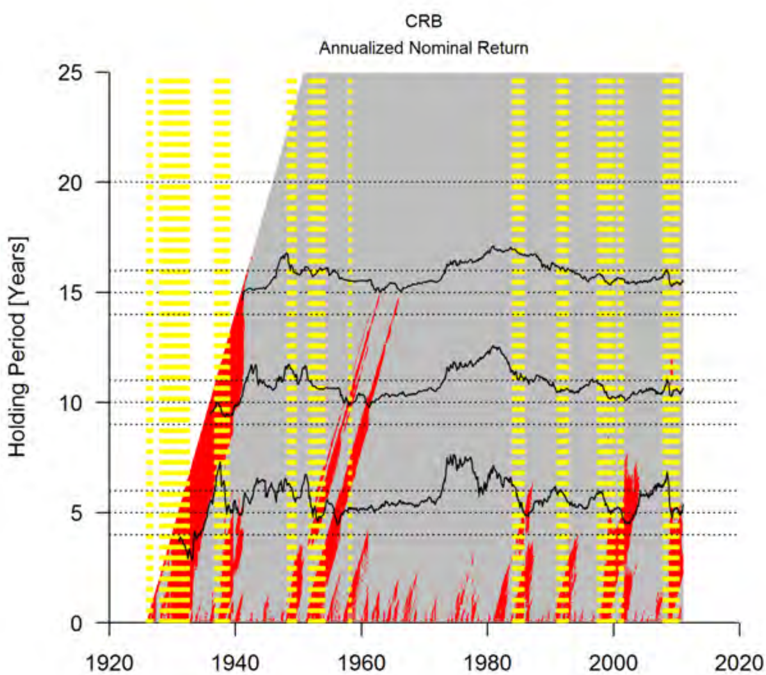

FIGURE 6.39: Holding periods of commodities.

Holding Periods: CRB

	mean	sd	5% quant	worst	best	% neg
1 Year	6.35	18.37	-22.58	-48.17	102.57	32.87
2 Years	5.62	12.96	-14.65	-34.47	55.71	25.65
3 Years	5.51	10.23	-7.20	-30.41	41.05	23.43
4 Years	5.45	8.63	-6.75	-24.51	29.71	20.64
5 Years	5.50	7.49	-5.96	-19.91	24.88	16.11
6 Years	5.61	6.52	-3.26	-16.71	22.65	13.79
7 Years	5.74	5.77	-3.01	-15.15	20.43	10.45
8 Years	5.83	5.40	-2.71	-8.59	21.74	10.04
9 Years	5.86	5.21	-1.99	-5.71	22.62	9.30
10 Years	5.88	4.98	-1.16	-5.13	20.17	8.54
11 Years	5.91	4.79	-1.24	-4.46	18.95	7.98
12 Years	5.91	4.62	-0.58	-4.37	18.17	6.49
13 Years	5.98	4.34	0.31	-4.46	17.24	4.62
14 Years	6.05	4.07	0.54	-2.51	15.88	2.81
15 Years	6.12	3.86	1.02	-2.69	15.20	0.83
16 Years	6.15	3.68	1.13	-0.31	14.14	0.24
17 Years	6.17	3.52	1.18	0.09	13.47	0.00
18 Years	6.18	3.41	1.35	0.25	13.27	0.00
19 Years	6.21	3.29	1.55	0.24	12.71	0.00
20 Years	6.24	3.13	2.07	0.31	12.40	0.00
21 Years	6.27	2.96	2.24	1.13	11.71	0.00
22 Years	6.27	2.82	2.52	1.43	11.78	0.00
23 Years	6.27	2.71	2.44	1.65	11.33	0.00
24 Years	6.29	2.61	2.76	1.47	10.80	0.00
25 Years	6.30	2.52	2.80	2.13	10.68	0.00

FIGURE 6.40: Holding periods of commodity research bureau index. Values are given in percent.

CHAPTER 7

DRAWDOWNS AND RECOVERY

Drawdowns determine the periods where an index moves down. During the down periods we can measure several quantities like the actual depth of the drawdown, the maximal depth of the drawdown as well as the time to reach the maximal depth and the time to recover from there. Alternative measures to quantify drawdowns are e.g. the drawdown at risk, *DaR*, or the conditional drawdown at risk, *CDaR*. Maximum drawdowns and recovery times describe the largest cumulative decline in an asset value from a historical peak and the time it requires to to recover from that decline. Drawdown depths and recovery times depend on the starting date of the investment and on the selected holding period by the investor. Therefore we address different starting times and consider holding periods over a wide range up to 20 years.

Drawdown risk can be observed in several flavors like the actual drawdown (in our case the monthly drawdown), the maximum drawdown, the drawdown value at a given significance level, we call it *DaR*, drawdown at risk, or *CDaR*, the conditional drawdown at risk. The *DaR* and *CDaR* expressions are defined in the same spirit as the value at risk, *VaR*, and the conditional value at risk, *CVaR*, that are used to quantify returns.

To compute *monthly drawdowns* one compares the actual value of an index (e.g. the wealth index) with its historical maximum. Then one can display the result in a time series chart at each point in time (T).

$$DD(T) = \frac{W_T}{\max_{t \in (0,T)} W_t} - 1$$

Drawdown periods can be examined by following the downward moves and doing a bookkeeping of the length to its recovery and the maximum depth during that time.

Drawdown periods for bonds are expected to be smaller in depth and shorter in recovery time compared to those of equities. Corporate bonds and commodities have usually a similar drawdown statistics compared to assets characterizing the stock market.

For every asset class the monthly drawdowns are calculated and plotted. Histograms show the mean monthly drawdowns and information about quantiles. We also determined all the drawdown periods and present them in a table with information about the duration of those periods and the maximum depth. We think that such a list gives much more detailed information than a time series plot can return. By varying the time horizon, rolling monthly maximum drawdowns give information about the maximum drawdowns on different scales. If the horizon is n years, then the first result can be calculated n years after the first measurement and then for every month until the last measurement. We were looking on horizons of 1 year, 5 years, 10 years and 20 years. Histograms show the mean maximum drawdowns and information about quantiles over the considered investment horizons.

7.1 MONTHLY DRAWDOWN SERIES

Monthly Drawdowns - Charts I

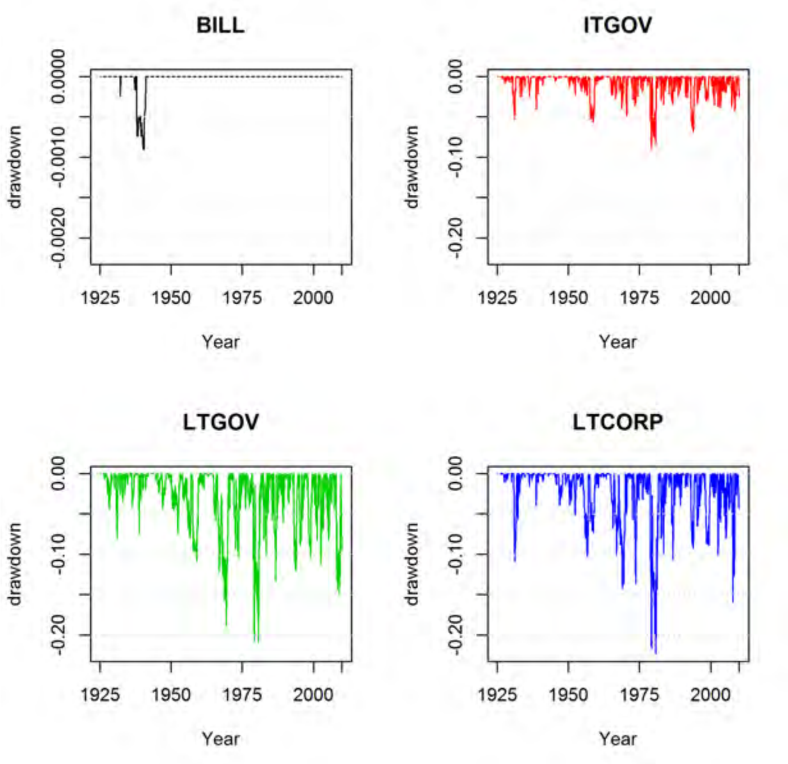

FIGURE 7.1: For comparability the drawdown axis has the same scale for the asset classes ITGOV, LTGOV and LTCORP. For readability the drawdown axis of the BILL's has a minimum value that is 100 times smaller than for the others. These plots show montly drawdowns.

Monthly Drawdowns - Charts II

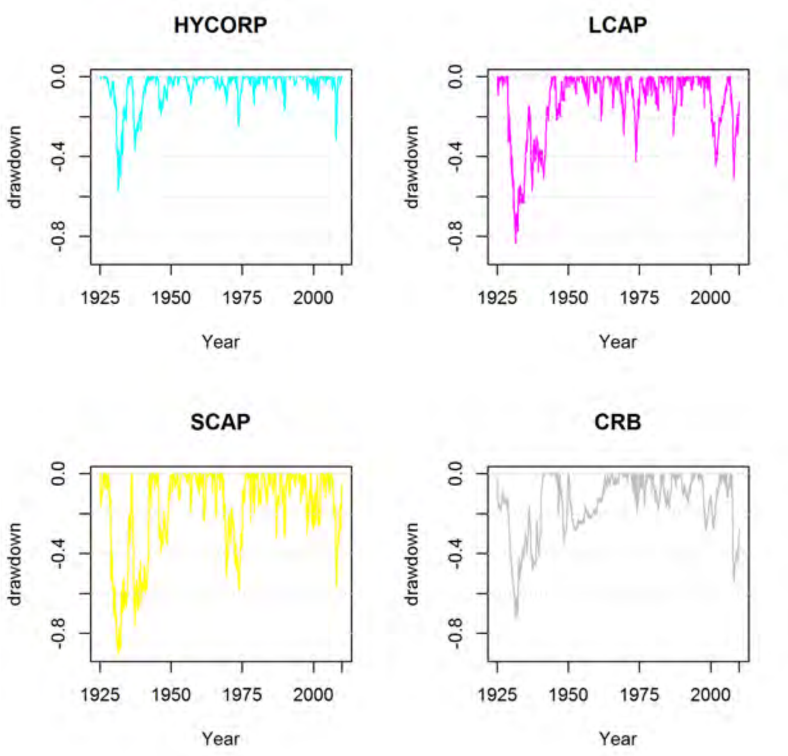

FIGURE 7.2: For comparability the drawdown axis has the same scale for the asset classes HYCORP, LCAP, SCAP and CRB.

7.2 MONTHLY DRAWDOWN STATATISTICS

Monthly Drawdowns - Statistics I

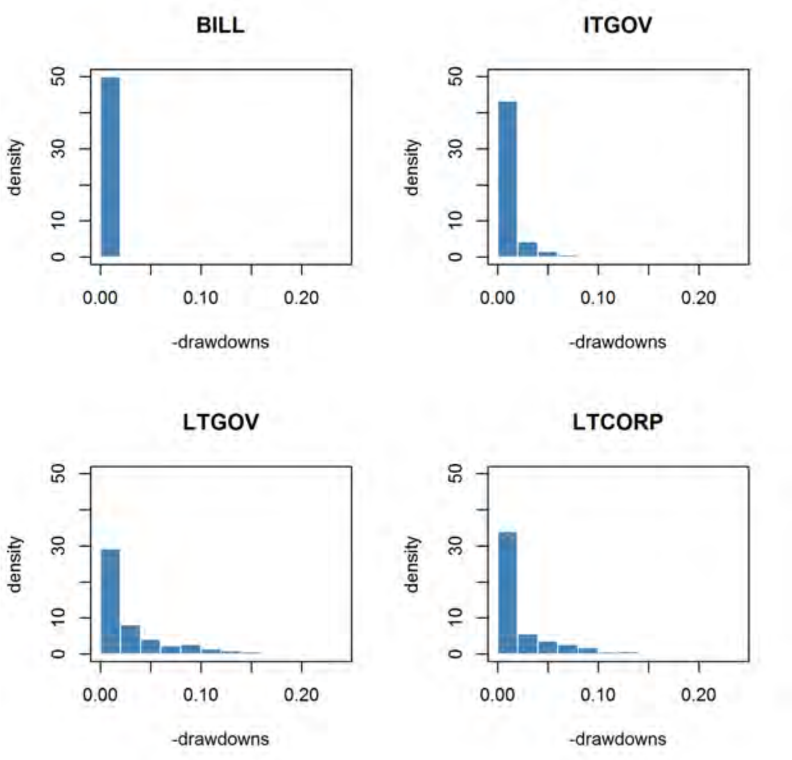

FIGURE 7.3: Histogram plot of monthly drawdowns. To conveniently compare the different asset classes the histograms have the same scales on the x and y axes. We also made sure that the width of the bars is the same for all assets.

	Mean	90%	95%	99%
BILL	0.0000	0.0000	0.0000	0.0007
ITGOV	0.0076	0.0252	0.0394	0.0641
LTGOV	0.0276	0.0844	0.1114	0.1523
LTCORP	0.0219	0.0741	0.0931	0.1497

FIGURE 7.4: Table of monthly drawdown statistics. Listed are the mean, and the 90%, 95%, and 99% quantiles.

Monthly Drawdowns - Statistics II

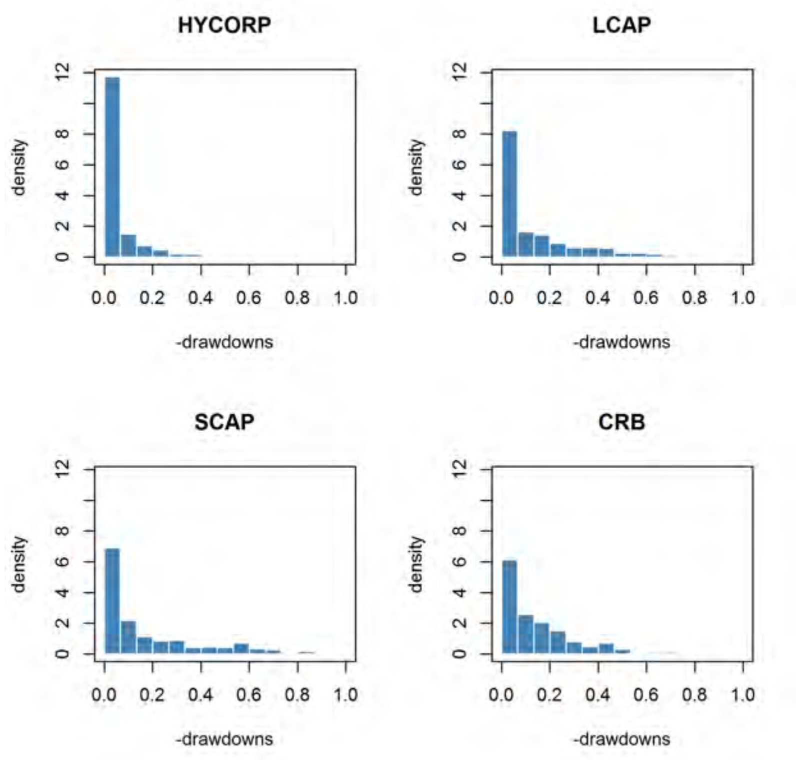

FIGURE 7.5: Histogram plot of monthly drawdowns. To conveniently compare the different asset classes the histograms have the same scales on the x and y axes. We also made sure that the width of the bars is the same for all assets.

```
         Mean    90%    95%    99%
HYCORP 0.0499 0.1559 0.2423 0.4245
LCAP   0.1305 0.4041 0.5131 0.7262
SCAP   0.1747 0.5487 0.6359 0.8538
CRB    0.1433 0.3839 0.4565 0.6511
```

FIGURE 7.6: Table of monthly drawdown statistics. Listed are the mean, and the 90%, 95%, and 99% quantiles.

7.3 DRAWDOWN DEPTHS AND LENGTHS

Drawdown Periods: Treasury Bill Index

	From	Trough	To	Depth	Length	ToTrough	Recovery
1	1938-11-30	1941-04-30	1942-02-28	-0.00090112	40	30	10
2	1933-02-28	1933-02-28	1933-03-31	-0.00024849	2	1	1
3	1938-03-31	1938-03-31	1938-04-30	-0.00016387	2	1	1
4	1938-07-31	1938-07-31	1938-09-30	-0.00016387	3	1	2

FIGURE 7.7: Table of BILL - Drawdowns sorted by depth. `From` denotes the first date where the drawdown gets negative. `To` denotes the first date where the drawdown gets positive. `NA` means that `To` is the end of the series, the `Length` is the number of months from `From` to `To` plus 1.

	From	Trough	To	Depth	Length	ToTrough	Recovery
1	1938-11-30	1941-04-30	1942-02-28	-0.00090112	40	30	10
2	1938-07-31	1938-07-31	1938-09-30	-0.00016387	3	1	2
3	1938-03-31	1938-03-31	1938-04-30	-0.00016387	2	1	1
4	1933-02-28	1933-02-28	1933-03-31	-0.00024849	2	1	1

FIGURE 7.8: Table of BILL - Drawdowns sorted by length

Drawdown Periods: Intermediate Term Government Bond Index

	From	Trough	To	Depth	Length	ToTrough	Recovery
1	1979-07-31	1980-02-29	1980-04-30	-0.088899	10	8	2
2	1980-06-30	1981-08-31	1981-11-30	-0.084909	18	15	3
3	1994-02-28	1994-11-30	1995-05-31	-0.069126	16	10	6
4	1958-06-30	1959-08-31	1960-03-31	-0.056901	22	15	7
5	1931-06-30	1932-01-31	1932-08-31	-0.053112	15	8	7
6	1971-04-30	1971-06-30	1971-10-31	-0.049736	7	3	4
7	2009-01-31	2009-06-30	2009-11-30	-0.042900	11	6	5
8	1939-08-31	1939-09-30	1939-12-31	-0.040509	5	2	3
9	1969-05-31	1969-09-30	1970-02-28	-0.039952	10	5	5
10	2004-04-30	2004-05-31	2004-10-31	-0.038137	7	2	5
11	2003-06-30	2003-08-31	2004-02-29	-0.037894	9	3	6
12	2008-04-30	2008-05-31	2008-10-31	-0.037519	7	2	5
13	1974-03-31	1974-04-30	1974-10-31	-0.036066	8	2	6
14	2001-11-30	2002-03-31	2002-06-30	-0.035015	8	5	3
15	1984-02-29	1984-05-31	1984-07-31	-0.034920	6	4	2
16	1987-03-31	1987-09-30	1987-11-30	-0.034394	9	7	2
17	1996-02-29	1996-05-31	1996-10-31	-0.033413	9	4	5
18	1998-11-30	1999-05-31	2000-06-30	-0.030881	20	7	13

FIGURE 7.9: Table of ITGOV - Drawdowns sorted by depth.

	From	Trough	To	Depth	Length	ToTrough	Recovery
1	1958-06-30	1959-08-31	1960-03-31	-0.056901	22	15	7
2	1998-11-30	1999-05-31	2000-06-30	-0.030881	20	7	13
3	1980-06-30	1981-08-31	1981-11-30	-0.084909	18	15	3
4	1954-09-30	1955-07-31	1956-01-31	-0.018583	17	11	6
5	1994-02-28	1994-11-30	1995-05-31	-0.069126	16	10	6
6	1931-06-30	1932-01-31	1932-08-31	-0.053112	15	8	7
7	1967-04-30	1967-06-30	1968-05-31	-0.027117	14	3	11
8	2005-09-30	2006-05-31	2006-08-31	-0.019920	12	9	3
9	1936-12-31	1937-03-31	1937-11-30	-0.024367	12	4	8
10	1965-12-31	1966-02-28	1966-10-31	-0.022796	11	3	8
11	2009-01-31	2009-06-30	2009-11-30	-0.042900	11	6	5
12	1946-03-31	1946-04-30	1946-12-31	-0.005807	10	2	8
13	1969-05-31	1969-09-30	1970-02-28	-0.039952	10	5	5
14	1979-07-31	1980-02-29	1980-04-30	-0.088899	10	8	2
15	1996-02-29	1996-05-31	1996-10-31	-0.033413	9	4	5
16	1987-03-31	1987-09-30	1987-11-30	-0.034394	9	7	2
17	2003-06-30	2003-08-31	2004-02-29	-0.037894	9	3	6
18	1957-04-30	1957-07-31	1957-11-30	-0.023709	8	4	4

FIGURE 7.10: Table of ITGOV - Drawdowns sorted by length.

Drawdown Periods: Long Term Government Bond Index

	From	Trough	To	Depth	Length	ToTrough	Recovery
1	1979-07-31	1981-09-30	1982-07-31	-0.209725	37	27	10
2	1967-02-28	1970-05-31	1971-03-31	-0.187929	50	40	10
3	2009-01-31	2009-12-31	2010-08-31	-0.149030	20	12	8
4	1987-03-31	1987-09-30	1988-01-31	-0.134120	11	7	4
5	1994-02-28	1994-10-31	1995-05-31	-0.120926	16	9	7
6	2003-06-30	2003-07-31	2004-09-30	-0.112087	16	2	14
7	1958-06-30	1959-12-31	1960-12-31	-0.109625	31	19	12
8	1972-12-31	1974-08-31	1974-12-31	-0.104165	25	21	4
9	1998-10-31	1999-12-31	2000-07-31	-0.103980	22	15	7
10	1983-05-31	1984-05-31	1984-09-30	-0.102564	17	13	4
11	2010-09-30	2010-12-31	2010-12-31	-0.096167	5	4	NA
12	1996-01-31	1996-05-31	1996-11-30	-0.089610	11	5	6
13	2005-09-30	2006-04-30	2006-11-30	-0.083052	15	8	7
14	2001-11-30	2002-03-31	2002-07-31	-0.082548	9	5	4
15	1931-07-31	1931-12-31	1932-04-30	-0.081126	10	6	4
16	1954-08-31	1957-07-31	1957-12-31	-0.080132	41	36	5
17	1950-09-30	1953-05-31	1953-12-31	-0.074462	40	33	7
18	1939-08-31	1939-09-30	1940-03-31	-0.073509	8	2	6

FIGURE 7.11: Table of LTGOV - Drawdowns sorted by depth.

	From	Trough	To	Depth	Length	ToTrough	Recovery
1	1967-02-28	1970-05-31	1971-03-31	-0.187929	50	40	10
2	1954-08-31	1957-07-31	1957-12-31	-0.080132	41	36	5
3	1950-09-30	1953-05-31	1953-12-31	-0.074462	40	33	7
4	1979-07-31	1981-09-30	1982-07-31	-0.209725	37	27	10
5	1958-06-30	1959-12-31	1960-12-31	-0.109625	31	19	12
6	1972-12-31	1974-08-31	1974-12-31	-0.104165	25	21	4
7	1998-10-31	1999-12-31	2000-07-31	-0.103980	22	15	7
8	1977-09-30	1978-06-30	1979-05-31	-0.041333	21	10	11
9	2009-01-31	2009-12-31	2010-08-31	-0.149030	20	12	8
10	1928-04-30	1929-03-31	1929-10-31	-0.044490	19	12	7
11	1947-09-30	1947-12-31	1949-02-28	-0.044034	18	4	14
12	1965-08-31	1966-08-31	1966-12-31	-0.059205	17	13	4
13	1983-05-31	1984-05-31	1984-09-30	-0.102564	17	13	4
14	1946-04-30	1946-09-30	1947-07-31	-0.023602	16	6	10
15	2003-06-30	2003-07-31	2004-09-30	-0.112087	16	2	14
16	1994-02-28	1994-10-31	1995-05-31	-0.120926	16	9	7
17	2005-09-30	2006-04-30	2006-11-30	-0.083052	15	8	7
18	1937-03-31	1937-03-31	1938-01-31	-0.041119	11	1	10

FIGURE 7.12: Table of LTGOV - Drawdowns sorted for length.

Drawdown Periods: Long Term Corporate Bond Index

	From	Trough	To	Depth	Length	ToTrough	Recovery
1	1979-07-31	1981-09-30	1982-08-31	-0.223844	38	27	11
2	2008-02-29	2008-10-31	2008-12-31	-0.159246	11	9	2
3	1967-02-28	1969-12-31	1970-12-31	-0.142500	47	35	12
4	1973-12-31	1974-08-31	1975-01-31	-0.136006	14	9	5
5	2009-01-31	2009-04-30	2009-07-31	-0.126882	7	4	3
6	1931-09-30	1932-02-29	1932-11-30	-0.109337	15	6	9
7	1987-03-31	1987-09-30	1988-01-31	-0.106573	11	7	4
8	1956-03-31	1957-08-31	1957-12-31	-0.102927	22	18	4
9	2003-06-30	2003-07-31	2004-03-31	-0.101141	10	2	8
10	1983-05-31	1984-05-31	1984-08-31	-0.098414	16	13	3
11	1994-02-28	1994-10-31	1995-04-30	-0.092182	15	9	6
12	1999-02-28	2000-05-31	2000-11-30	-0.089423	22	16	6
13	2005-07-31	2006-05-31	2006-10-31	-0.080406	16	11	5
14	1958-06-30	1959-09-30	1960-07-31	-0.073158	26	16	10
15	1965-11-30	1966-08-31	1967-01-31	-0.067789	15	10	5
16	1996-02-29	1996-04-30	1996-10-31	-0.065018	9	3	6
17	2004-04-30	2004-05-31	2004-08-31	-0.060120	5	2	3
18	1933-02-28	1933-04-30	1933-06-30	-0.056862	5	3	2

FIGURE 7.13: Table of LTCORP - Drawdowns sorted for depth.

	From	Trough	To	Depth	Length	ToTrough	Recovery
1	1967-02-28	1969-12-31	1970-12-31	-0.142500	47	35	12
2	1979-07-31	1981-09-30	1982-08-31	-0.223844	38	27	11
3	1958-06-30	1959-09-30	1960-07-31	-0.073158	26	16	10
4	1999-02-28	2000-05-31	2000-11-30	-0.089423	22	16	6
5	1956-03-31	1957-08-31	1957-12-31	-0.102927	22	18	4
6	1951-02-28	1951-06-30	1952-08-31	-0.039358	19	5	14
7	1947-08-31	1947-11-30	1948-12-31	-0.039303	17	4	13
8	2005-07-31	2006-05-31	2006-10-31	-0.080406	16	11	5
9	1983-05-31	1984-05-31	1984-08-31	-0.098414	16	13	3
10	1965-11-30	1966-08-31	1967-01-31	-0.067789	15	10	5
11	1994-02-28	1994-10-31	1995-04-30	-0.092182	15	9	6
12	1931-09-30	1932-02-29	1932-11-30	-0.109337	15	6	9
13	1973-12-31	1974-08-31	1975-01-31	-0.136006	14	9	5
14	2006-12-31	2007-07-31	2007-12-31	-0.044686	13	8	5
15	1946-04-30	1946-11-30	1947-03-31	-0.013556	12	8	4
16	1952-12-31	1953-05-31	1953-10-31	-0.051247	11	6	5
17	1987-03-31	1987-09-30	1988-01-31	-0.106573	11	7	4
18	2008-02-29	2008-10-31	2008-12-31	-0.159246	11	9	2

FIGURE 7.14: Table of LTCORP - Drawdowns sorted by length.

Drawdown Periods: Domestic High Yield Corporate Bond Index

	From	Trough	To	Depth	Length	ToTrough	Recovery
1	1928-05-31	1932-05-31	1935-11-30	-0.570250	91	49	42
2	1937-03-31	1938-03-31	1942-08-31	-0.373109	66	13	53
3	2007-06-30	2008-11-30	2009-09-30	-0.319898	28	18	10
4	1974-03-31	1974-09-30	1976-01-31	-0.252077	23	7	16
5	1946-02-28	1947-05-31	1949-12-31	-0.192455	47	16	31
6	1990-08-31	1990-10-31	1991-03-31	-0.171644	8	3	5
7	1979-09-30	1980-03-31	1980-06-30	-0.144143	10	7	3
8	1956-03-31	1957-11-30	1960-03-31	-0.139352	49	21	28
9	1968-10-31	1970-06-30	1971-01-31	-0.138604	28	21	7
10	2001-03-31	2002-07-31	2003-03-31	-0.120067	25	17	8
11	1999-05-31	2000-11-30	2001-02-28	-0.089131	22	19	3
12	1989-09-30	1990-02-28	1990-07-31	-0.080460	11	6	5
13	1984-02-29	1984-05-31	1984-09-30	-0.075210	8	4	4
14	1966-02-28	1966-08-31	1967-01-31	-0.072274	12	7	5
15	1998-08-31	1998-10-31	1999-04-30	-0.070404	9	3	6
16	1987-04-30	1987-10-31	1988-01-31	-0.069379	10	7	3
17	1967-02-28	1967-12-31	1968-09-30	-0.063540	20	11	9
18	1973-11-30	1973-11-30	1973-12-31	-0.060103	2	1	1

FIGURE 7.15: Table of HYCORP - Drawdowns sorted by depth.

	From	Trough	To	Depth	Length	ToTrough	Recovery
1	1928-05-31	1932-05-31	1935-11-30	-0.570250	91	49	42
2	1937-03-31	1938-03-31	1942-08-31	-0.373109	66	13	53
3	1956-03-31	1957-11-30	1960-03-31	-0.139352	49	21	28
4	1946-02-28	1947-05-31	1949-12-31	-0.192455	47	16	31
5	1968-10-31	1970-06-30	1971-01-31	-0.138604	28	21	7
6	2007-06-30	2008-11-30	2009-09-30	-0.319898	28	18	10
7	2001-03-31	2002-07-31	2003-03-31	-0.120067	25	17	8
8	1974-03-31	1974-09-30	1976-01-31	-0.252077	23	7	16
9	1999-05-31	2000-11-30	2001-02-28	-0.089131	22	19	3
10	1967-02-28	1967-12-31	1968-09-30	-0.063540	20	11	9
11	1951-02-28	1951-06-30	1952-03-31	-0.052699	14	5	9
12	1994-02-28	1994-04-30	1995-02-28	-0.046828	13	3	10
13	1980-07-31	1980-09-30	1981-06-30	-0.057026	12	3	9
14	1966-02-28	1966-08-31	1967-01-31	-0.072274	12	7	5
15	1953-03-31	1953-05-31	1954-01-31	-0.040868	11	3	8
16	1989-09-30	1990-02-28	1990-07-31	-0.080460	11	6	5
17	1987-04-30	1987-10-31	1988-01-31	-0.069379	10	7	3
18	1979-09-30	1980-03-31	1980-06-30	-0.144143	10	7	3

FIGURE 7.16: Table of HYCORP - Drawdowns sorted by length.

Drawdown Periods: Large Cap Equity Index

	From	Trough	To	Depth	Length	ToTrough	Recovery
1	1929-09-30	1932-06-30	1945-01-31	-0.834089	185	34	151
2	2007-11-30	2009-02-28	2010-12-31	-0.509487	39	16	NA
3	2000-09-30	2002-09-30	2006-10-31	-0.447329	74	25	49
4	1973-01-31	1974-09-30	1976-06-30	-0.426287	42	21	21
5	1987-09-30	1987-11-30	1989-05-31	-0.295327	21	3	18
6	1968-12-31	1970-06-30	1971-03-31	-0.292531	28	19	9
7	1962-01-31	1962-06-30	1963-04-30	-0.222819	16	6	10
8	1946-06-30	1946-11-30	1949-10-31	-0.217593	41	6	35
9	1980-12-31	1982-07-31	1982-10-31	-0.169149	23	20	3
10	1966-02-28	1966-09-30	1967-03-31	-0.156408	14	8	6
11	1998-07-31	1998-08-31	1998-11-30	-0.153690	5	2	3
12	1957-08-31	1957-12-31	1958-07-31	-0.149547	12	5	7
13	1990-06-30	1990-10-31	1991-02-28	-0.147028	9	5	4
14	1977-01-31	1978-02-28	1978-07-31	-0.141290	19	14	5
15	1956-08-31	1957-02-28	1957-07-31	-0.102426	12	7	5
16	1980-03-31	1980-03-31	1980-06-30	-0.098660	4	1	3
17	1926-02-28	1926-03-31	1926-07-31	-0.093700	6	2	4
18	1978-09-30	1978-10-31	1979-03-31	-0.093449	7	2	5

FIGURE 7.17: Table of LCAP - Drawdowns sorted by depth.

	From	Trough	To	Depth	Length	ToTrough	Recovery
1	1929-09-30	1932-06-30	1945-01-31	-0.834089	185	34	151
2	2000-09-30	2002-09-30	2006-10-31	-0.447329	74	25	49
3	1973-01-31	1974-09-30	1976-06-30	-0.426287	42	21	21
4	1946-06-30	1946-11-30	1949-10-31	-0.217593	41	6	35
5	2007-11-30	2009-02-28	2010-12-31	-0.509487	39	16	NA
6	1968-12-31	1970-06-30	1971-03-31	-0.292531	28	19	9
7	1980-12-31	1982-07-31	1982-10-31	-0.169149	23	20	3
8	1987-09-30	1987-11-30	1989-05-31	-0.295327	21	3	18
9	1977-01-31	1978-02-28	1978-07-31	-0.141290	19	14	5
10	1962-01-31	1962-06-30	1963-04-30	-0.222819	16	6	10
11	1966-02-28	1966-09-30	1967-03-31	-0.156408	14	8	6
12	1953-01-31	1953-08-31	1954-01-31	-0.087283	13	8	5
13	1960-01-31	1960-10-31	1960-12-31	-0.083783	12	10	2
14	1956-08-31	1957-02-28	1957-07-31	-0.102426	12	7	5
15	1957-08-31	1957-12-31	1958-07-31	-0.149547	12	5	7
16	1983-12-31	1984-05-31	1984-08-31	-0.074145	9	6	3
17	1990-06-30	1990-10-31	1991-02-28	-0.147028	9	5	4
18	1971-05-31	1971-10-31	1971-12-31	-0.079147	8	6	2

FIGURE 7.18: Table of LCAP - Drawdowns sorted by length.

Drawdown Periods: Small Cap Equity Index

	From	Trough	To	Depth	Length	ToTrough	Recovery
1	1928-12-31	1932-05-31	1937-01-31	-0.90321	98	42	56
2	1937-04-30	1938-03-31	1943-04-30	-0.75900	73	12	61
3	1969-01-31	1974-12-31	1976-12-31	-0.58355	96	72	24
4	2007-06-30	2009-02-28	2010-12-31	-0.56877	44	21	NA
5	1946-06-30	1947-05-31	1950-09-30	-0.40270	52	12	40
6	1987-09-30	1987-11-30	1989-05-31	-0.32552	21	3	18
7	1989-10-31	1990-10-31	1991-10-31	-0.32090	25	13	12
8	1998-05-31	1998-08-31	1999-12-31	-0.30625	20	4	16
9	2000-03-31	2000-11-30	2003-07-31	-0.27735	41	9	32
10	1978-09-30	1978-10-31	1979-03-31	-0.24512	7	2	5
11	1962-04-30	1962-10-31	1963-05-31	-0.23902	14	7	7
12	1966-05-31	1966-10-31	1967-02-28	-0.21732	10	6	4
13	1980-02-29	1980-03-31	1980-07-31	-0.20108	6	2	4
14	1957-07-31	1957-12-31	1958-06-30	-0.19470	12	6	6
15	1943-06-30	1943-11-30	1944-02-29	-0.17060	9	6	3
16	1926-02-28	1926-03-31	1927-02-28	-0.16441	13	2	11
17	1981-07-31	1981-09-30	1982-10-31	-0.16397	16	3	13
18	1983-07-31	1984-07-31	1985-02-28	-0.15117	20	13	7

FIGURE 7.19: Table of SCAP - Drawdowns sorted by depth.

	From	Trough	To	Depth	Length	ToTrough	Recovery
1	1928-12-31	1932-05-31	1937-01-31	-0.903211	98	42	56
2	1969-01-31	1974-12-31	1976-12-31	-0.583548	96	72	24
3	1937-04-30	1938-03-31	1943-04-30	-0.759002	73	12	61
4	1946-06-30	1947-05-31	1950-09-30	-0.402704	52	12	40
5	2007-06-30	2009-02-28	2010-12-31	-0.568768	44	21	NA
6	2000-03-31	2000-11-30	2003-07-31	-0.277345	41	9	32
7	1989-10-31	1990-10-31	1991-10-31	-0.320899	25	13	12
8	1987-09-30	1987-11-30	1989-05-31	-0.325519	21	3	18
9	1983-07-31	1984-07-31	1985-02-28	-0.151173	20	13	7
10	1998-05-31	1998-08-31	1999-12-31	-0.306250	20	4	16
11	1981-07-31	1981-09-30	1982-10-31	-0.163967	16	3	13
12	1951-10-31	1952-04-30	1952-12-31	-0.071229	15	7	8
13	1953-03-31	1953-09-30	1954-05-31	-0.137618	15	7	8
14	1962-04-30	1962-10-31	1963-05-31	-0.239019	14	7	7
15	1996-06-30	1996-07-31	1997-06-30	-0.147012	13	2	11
16	1926-02-28	1926-03-31	1927-02-28	-0.164408	13	2	11
17	1956-05-31	1956-09-30	1957-04-30	-0.045951	12	5	7
18	1957-07-31	1957-12-31	1958-06-30	-0.194701	12	6	6

FIGURE 7.20: Table of SCAP - Drawdowns sorted by length.

Drawdown Periods: CRB Commodity Index

	From	Trough	To	Depth	Length	ToTrough	Recovery
1	1926-01-31	1932-06-30	1942-02-28	-0.718700	194	78	116
2	2008-07-31	2009-02-28	2010-12-31	-0.540221	31	8	NA
3	1947-12-31	1949-06-30	1950-12-31	-0.349990	37	19	18
4	1951-03-31	1953-06-30	1965-12-31	-0.288365	178	28	150
5	1997-10-31	1999-02-28	2004-01-31	-0.283697	76	17	59
6	1947-04-30	1947-06-30	1947-11-30	-0.202836	8	3	5
7	1974-10-31	1975-02-28	1976-05-31	-0.202593	20	5	15
8	1980-12-31	1982-06-30	1983-08-31	-0.180604	33	19	14
9	1984-06-30	1986-06-30	1987-05-31	-0.173669	36	25	11
10	1977-05-31	1977-08-31	1978-03-31	-0.162725	11	4	7
11	1990-10-31	1993-01-31	1994-05-31	-0.144552	44	28	16
12	1974-03-31	1974-05-31	1974-07-31	-0.137022	5	3	2
13	2006-08-31	2006-09-30	2007-09-30	-0.119249	14	2	12
14	1973-08-31	1973-10-31	1974-01-31	-0.099083	6	3	3
15	1980-03-31	1980-03-31	1980-06-30	-0.094791	4	1	3
16	1976-07-31	1976-09-30	1977-01-31	-0.080010	7	3	4
17	1988-07-31	1988-09-30	1988-11-30	-0.071838	5	3	2
18	2006-02-28	2006-02-28	2006-04-30	-0.066468	3	1	2

FIGURE 7.21: Table of CRB - Drawdowns sorted for depth.

	From	Trough	To	Depth	Length	ToTrough	Recovery
1	1926-01-31	1932-06-30	1942-02-28	-0.718700	194	78	116
2	1951-03-31	1953-06-30	1965-12-31	-0.288365	178	28	150
3	1997-10-31	1999-02-28	2004-01-31	-0.283697	76	17	59
4	1990-10-31	1993-01-31	1994-05-31	-0.144552	44	28	16
5	1947-12-31	1949-06-30	1950-12-31	-0.349990	37	19	18
6	1984-06-30	1986-06-30	1987-05-31	-0.173669	36	25	11
7	1980-12-31	1982-06-30	1983-08-31	-0.180604	33	19	14
8	2008-07-31	2009-02-28	2010-12-31	-0.540221	31	8	NA
9	1966-08-31	1967-03-31	1968-10-31	-0.058163	27	8	19
10	1974-10-31	1975-02-28	1976-05-31	-0.202593	20	5	15
11	1970-11-30	1971-09-30	1971-12-31	-0.041653	14	11	3
12	2006-08-31	2006-09-30	2007-09-30	-0.119249	14	2	12
13	1977-05-31	1977-08-31	1978-03-31	-0.162725	11	4	7
14	1944-04-30	1944-10-31	1944-12-31	-0.011887	9	7	2
15	1994-08-31	1994-11-30	1995-03-31	-0.037703	8	4	4
16	2004-04-30	2004-06-30	2004-11-30	-0.059335	8	3	5
17	1947-04-30	1947-06-30	1947-11-30	-0.202836	8	3	5
18	1983-09-30	1983-10-31	1984-03-31	-0.061071	7	2	5

FIGURE 7.22: Table of CRB - Drawdowns sorted by length.

7.4 ROLLING MAXIMUM DRAWDOWNS

Rolling Maximum Drawdowns: BILL

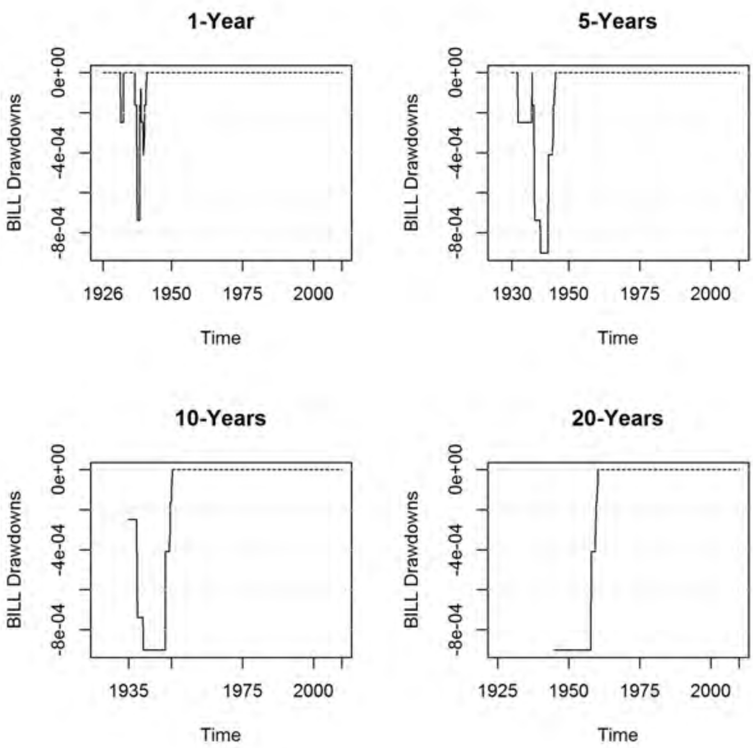

FIGURE 7.23: Rolling Maximum Drawdowns: BILL. These plots show maximum drawdowns.

Histogram of Rolling Maximum Drawdowns: BILL

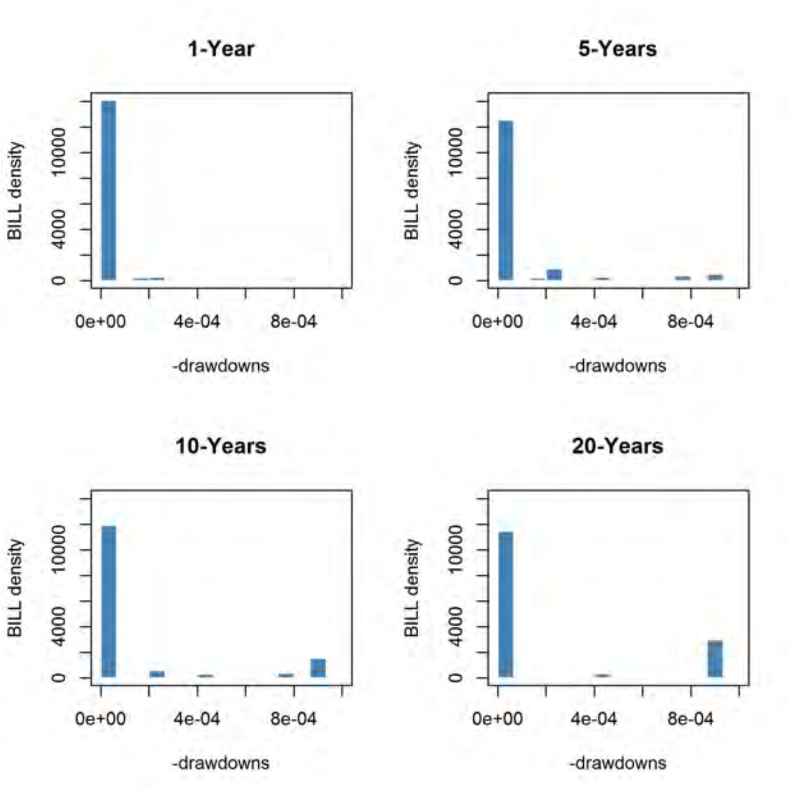

FIGURE 7.24: Histogram of Rolling Maximum Drawdowns: BILL. Values within the following tables are not given in percent. The following plots and statistics are based on maximum drawdowns.

```
            Mean   90%   95%   99%
1-Year    0e+00 0e+00 2e-04 6e-04
5-Years   1e-04 2e-04 7e-04 9e-04
10-Years 1e-04 9e-04 9e-04 9e-04
20-Years 2e-04 9e-04 9e-04 9e-04
```

Rolling Maximum Drawdowns: ITGOV

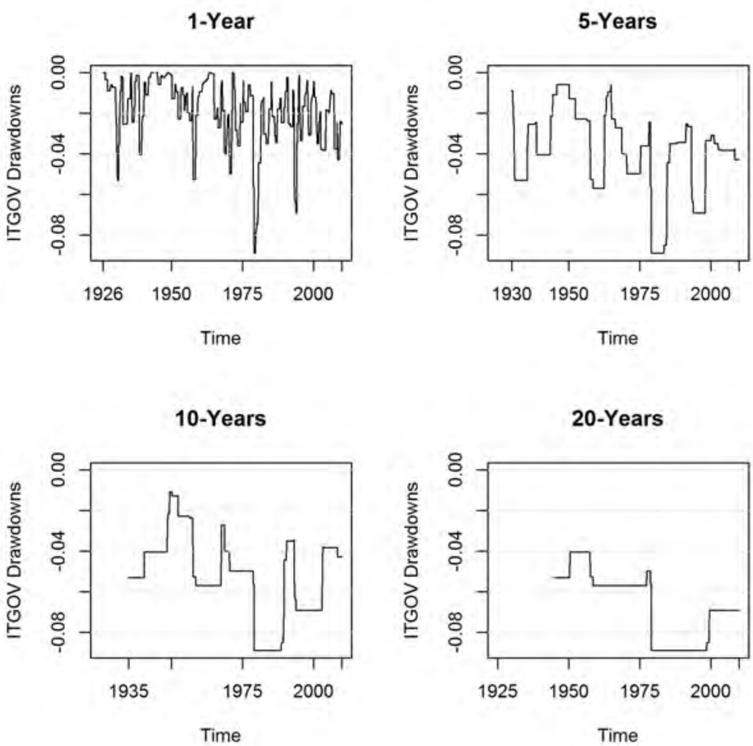

FIGURE 7.25: Rolling Maximum Drawdowns: ITGOV.

Histogram of Rolling Maximum Drawdowns: ITGOV

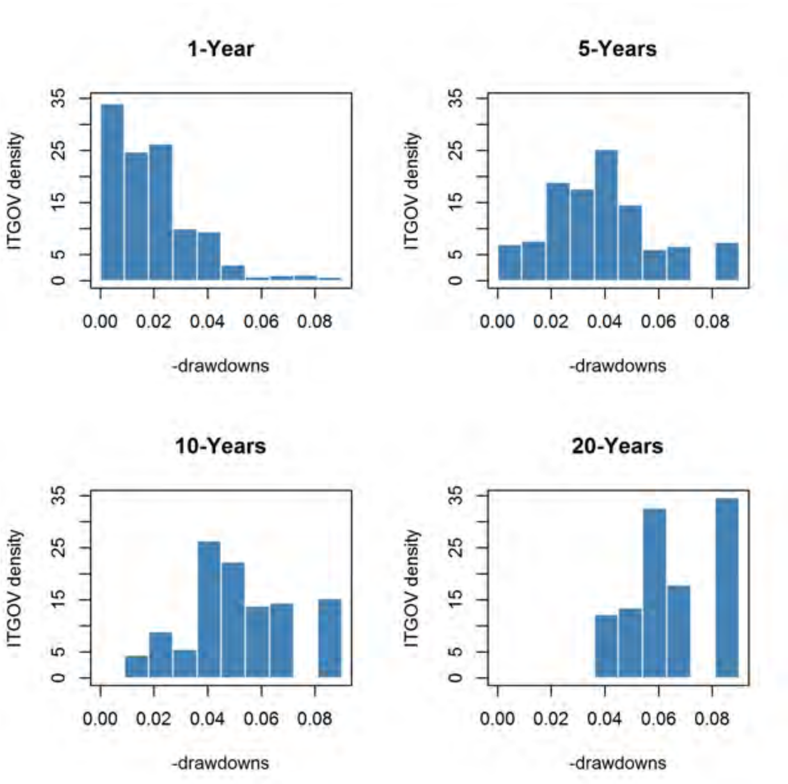

FIGURE 7.26: Histogram of Rolling Maximum Drawdowns: ITGOV

```
           Mean    90%    95%    99%
1-Year   0.0190 0.0381 0.0497 0.0766
5-Years  0.0393 0.0691 0.0889 0.0889
10-Years 0.0520 0.0889 0.0889 0.0889
20-Years 0.0665 0.0889 0.0889 0.0889
```

Rolling Maximum Drawdowns: LTGOV

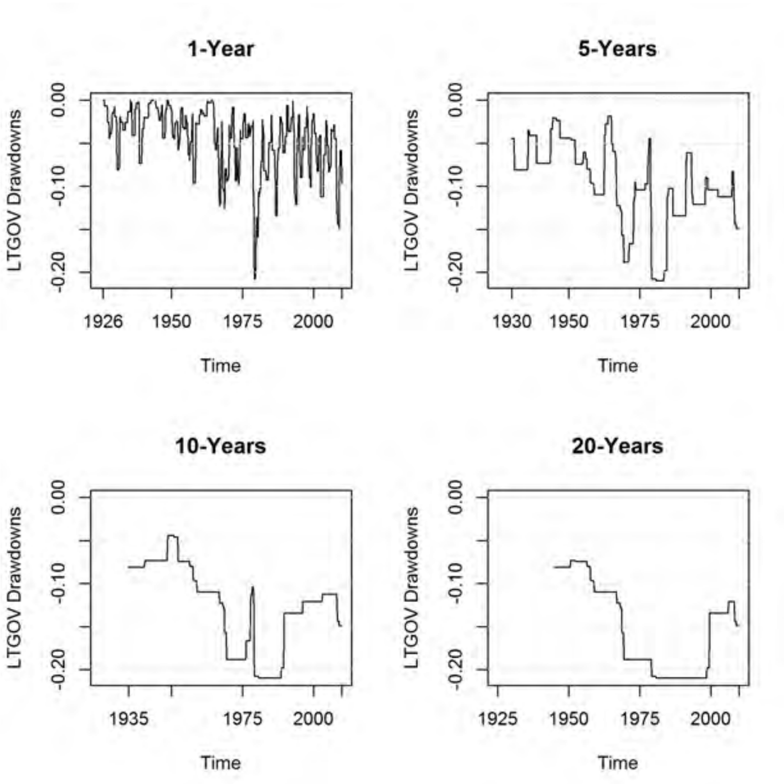

FIGURE 7.27: Rolling Maximum Drawdowns: LTGOV

Histogram of Rolling Maximum Drawdowns: LTGOV

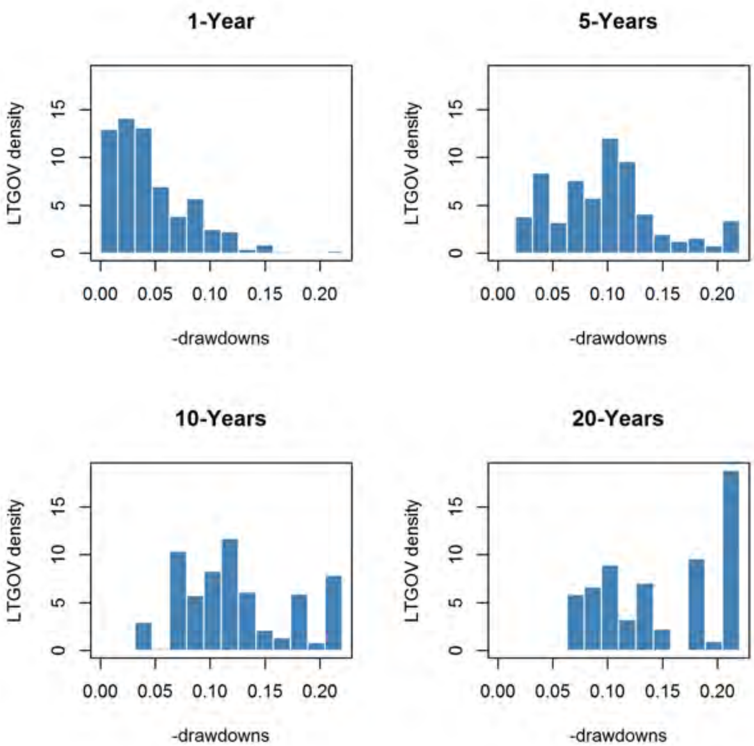

FIGURE 7.28: Histogram of Rolling Maximum Drawdowns: LTGOV

```
             Mean    90%    95%    99%
1-Year   0.0458 0.0957 0.1129 0.1581
5-Years  0.0983 0.1665 0.2076 0.2097
10-Years 0.1249 0.2097 0.2097 0.2097
20-Years 0.1507 0.2097 0.2097 0.2097
```

Rolling Maximum Drawdowns: LTCORP

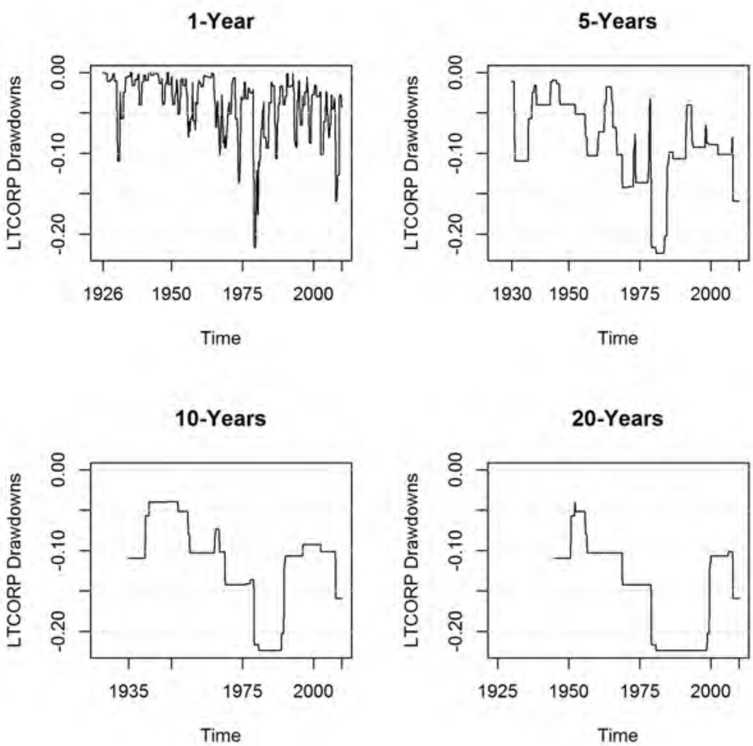

FIGURE 7.29: Rolling Maximum Drawdowns: LTCORP

Histogram of Rolling Maximum Drawdowns: LTCORP

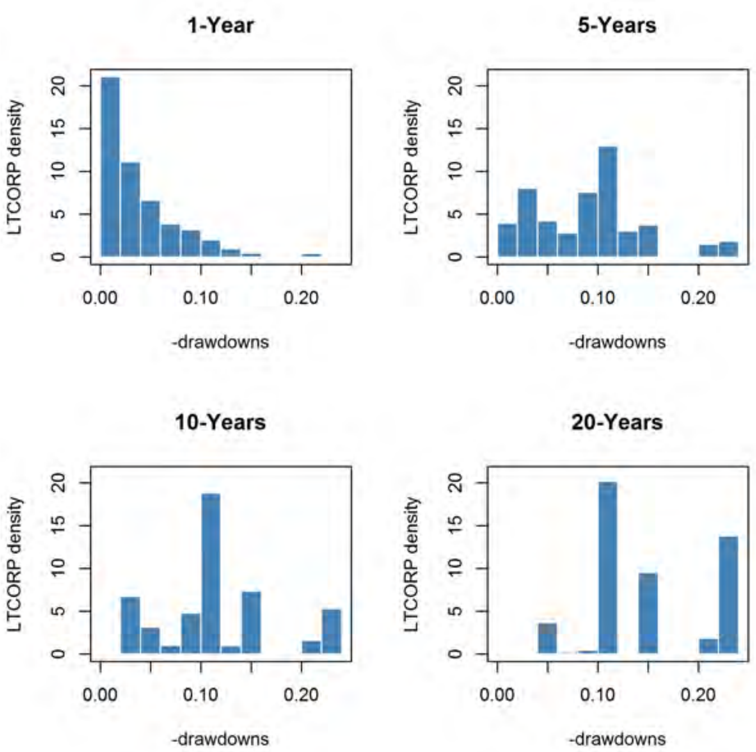

Figure 7.30: Histogram of Rolling Maximum Drawdowns: LTCORP

```
           Mean    90%    95%    99%
1-Year   0.0388 0.0922 0.1093 0.1740
5-Years  0.0909 0.1425 0.2168 0.2238
10-Years 0.1135 0.2238 0.2238 0.2238
20-Years 0.1455 0.2238 0.2238 0.2238
```

Rolling Maximum Drawdowns: HYCORP

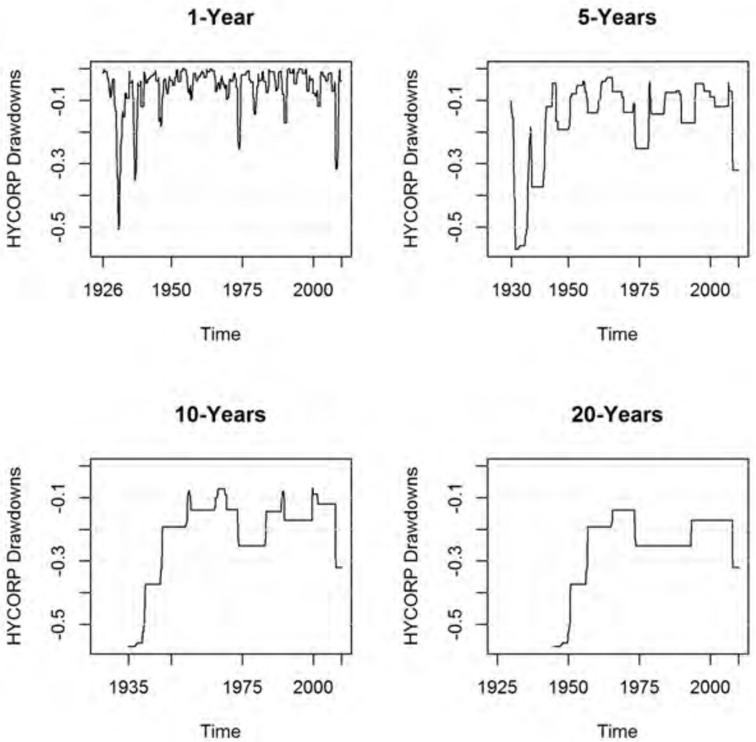

FIGURE 7.31: Rolling maximum drawdowns: HYCORP

Histogram of Rolling Maximum Drawdowns: HYCORP

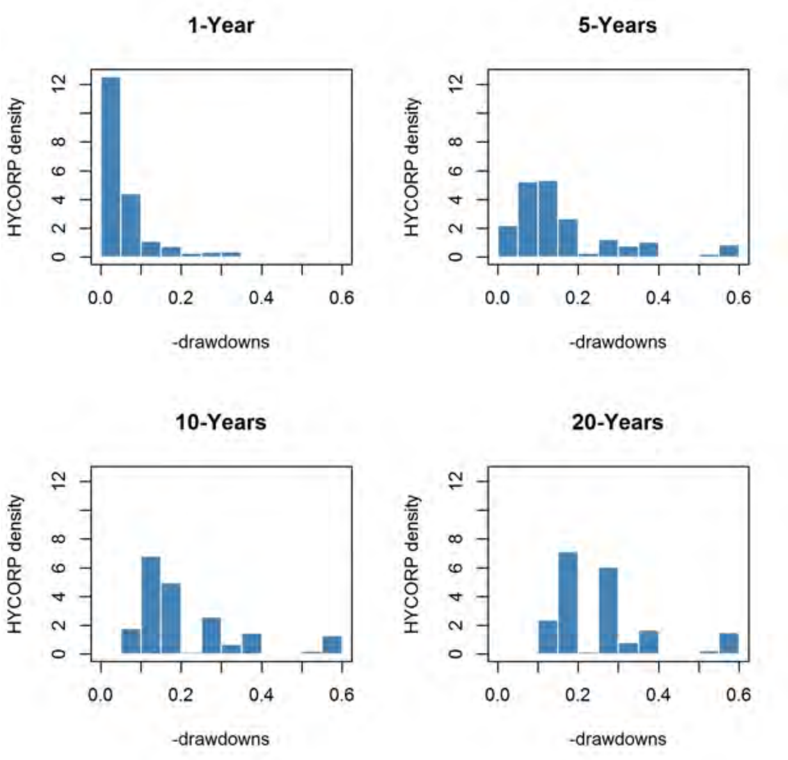

FIGURE 7.32: Histogram of rolling maximum drawdowns: HYCORP

```
             Mean    90%    95%    99%
1-Year    0.0610 0.1441 0.2182 0.3433
5-Years   0.1655 0.3731 0.5071 0.5703
10-Years  0.2138 0.3731 0.5588 0.5703
20-Years  0.2529 0.3731 0.5588 0.5703
```

Rolling Maximum Drawdowns: LCAP

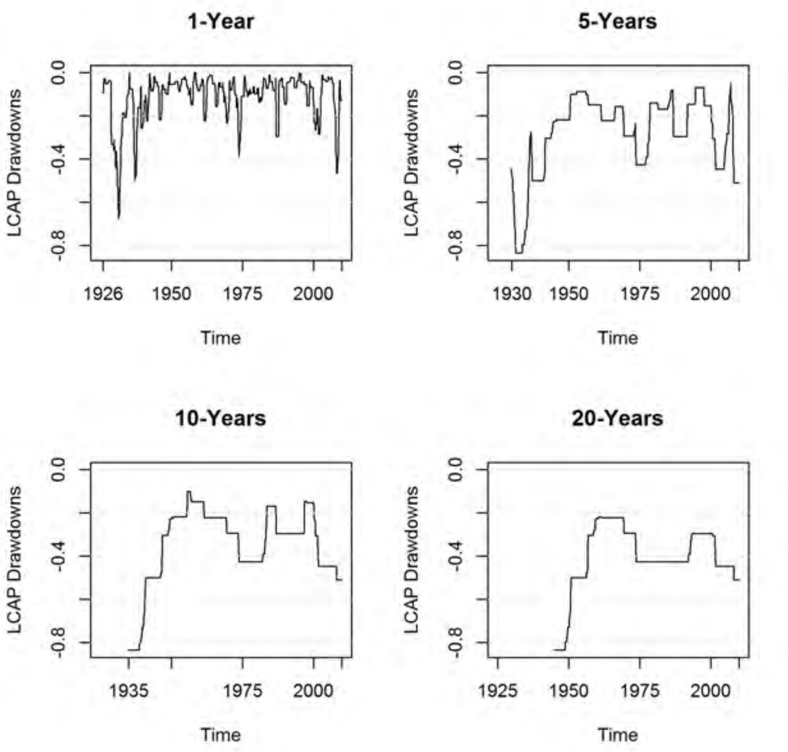

FIGURE 7.33: Rolling Maximum Drawdowns: LCAP

Histogram of Rolling Maximum Drawdowns: LCAP

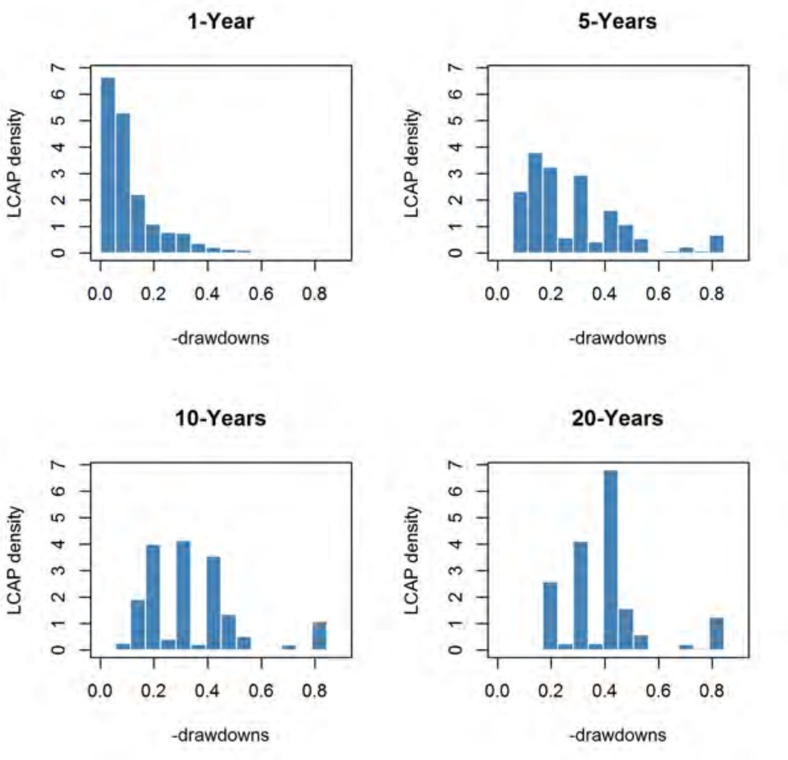

FIGURE 7.34: Histogram of Rolling Maximum Drawdowns: LCAP

```
           Mean    90%    95%    99%
1-Year   0.1159 0.2614 0.3606 0.5175
5-Years  0.2830 0.5004 0.7151 0.8341
10-Years 0.3470 0.5095 0.8337 0.8341
20-Years 0.4085 0.5095 0.8341 0.8341
```

Rolling Maximum Drawdowns: SCAP

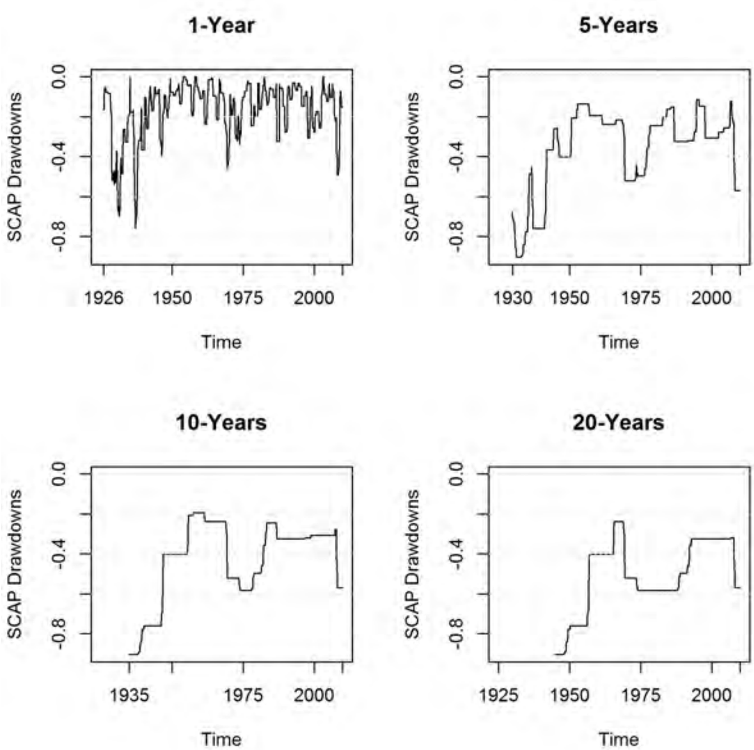

FIGURE 7.35: Rolling Maximum Drawdowns: SCAP

Histogram of Rolling Maximum Drawdowns: SCAP

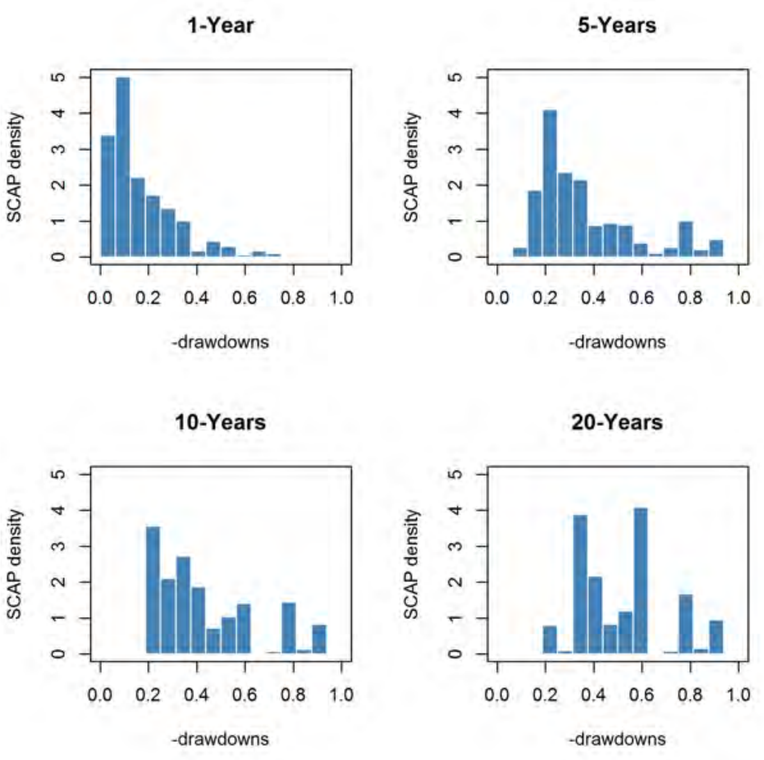

FIGURE 7.36: Histogram of Rolling Maximum Drawdowns: SCAP

```
           Mean   90%    95%    99%
1-Year   0.1684 0.360 0.4849 0.6696
5-Years  0.3624 0.759 0.7770 0.9032
10-Years 0.4286 0.759 0.8882 0.9032
20-Years 0.5084 0.759 0.8991 0.9032
```

Rolling Maximum Drawdowns: CRB

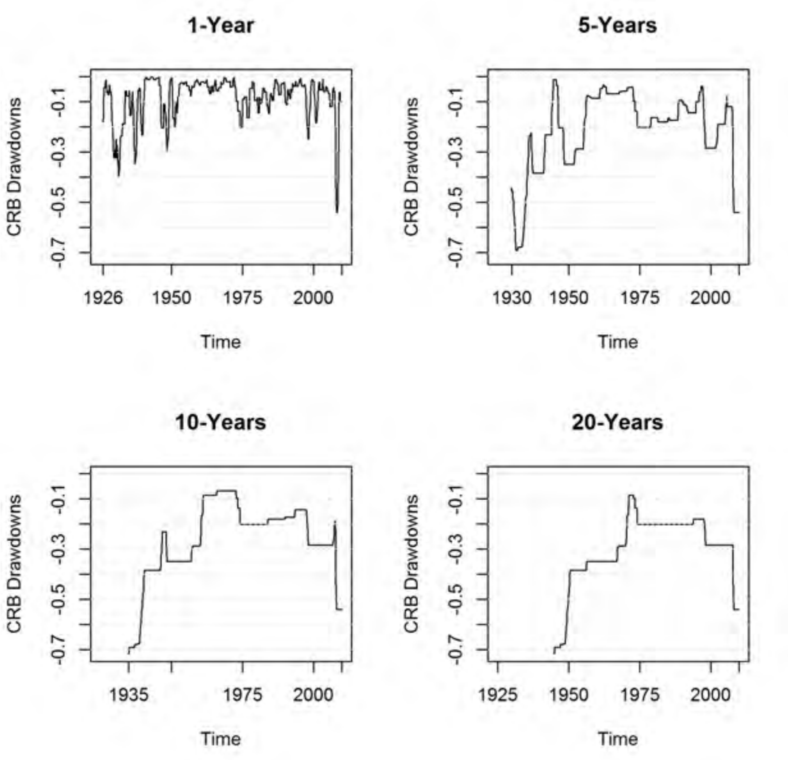

FIGURE 7.37: Rolling Maximum Drawdowns: CRB

Histogram of Rolling Maximum Drawdowns: CRB

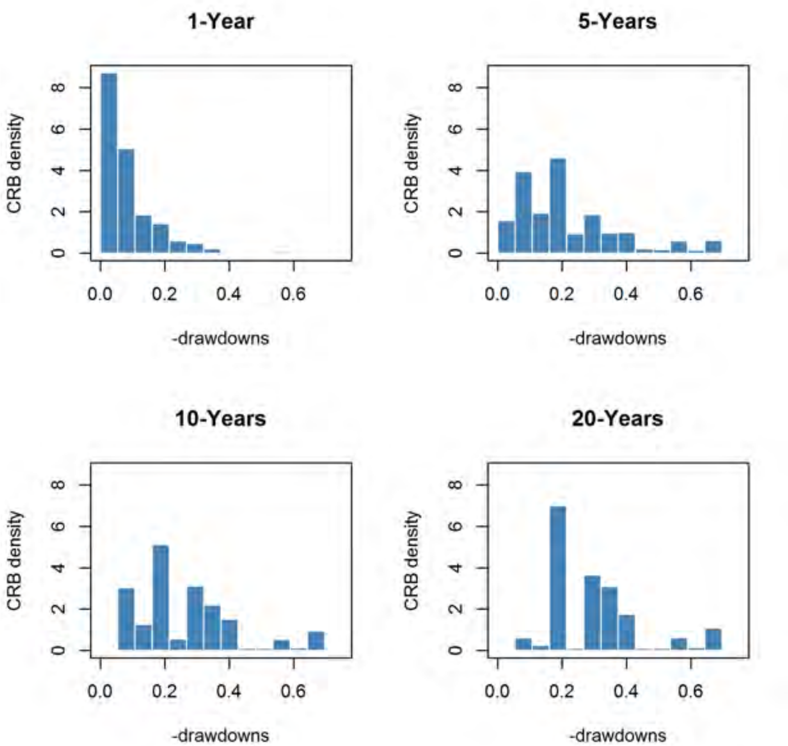

FIGURE 7.38: Histogram of Rolling Maximum Drawdowns: CRB

```
            Mean    90%    95%    99%
1-Year    0.0874 0.2026 0.2763 0.4153
5-Years   0.2144 0.3838 0.5402 0.6801
10-Years 0.2639 0.4636 0.6778 0.6920
20-Years 0.3028 0.5402 0.6799 0.6920
```

Chapter 8

Market Timing

In general, market timing deals with strategies of making buy or sell decisions of financial assets. In this analysis we concentrate on the aspect of missing trades in order to compute the resulting loss in performance of such decissions and examine the returns in specific months.

Martin Enz [2008] discussed the observation, that many investors use market timing to their detriment and that investors tend to act in procyclical manner. As a consequence investors often lose patience and sell at low prices. This results in the fact, that they miss the best months at the beginning of a market recovery. Typical examples for *missed trades* are the market recovery after the oil crisis 1975, the recovery after the Asian crisis during October/November 1998, or the recovery after the IT bubble burst in April 2003, amongst others. Another effect which may influence investors in their decisions is the *month of year effect*. Their is the believe that returns in specific months are different from others, one example is the end of year rally in December. We calculated the impact of missed trades and the month of year effect as follows:

Missed Trades: The effect missing the best trades after market recoveries are calculated by setting the returns of the best n-th month to zero.

Month of Year Effect: This effect can be investigated by calculating for each calendar month the mean of the associated returns and their volatilities.

We model the reduction of the wealth due to missed trades for four time periods starting 1925, 1950, 1975, and 2000. We select the best 1% trades and set their return to zero. Then we compare the wealth for all trades and missing 1% of the best trades.

To quantify the month of year effect we have calculated for every month the mean and median monthly returns, and the standard deviations of the monthly returns. This was done for the four periods starting in January of the years 1926, 1951, 1976, and 2001.

8.1 MISSED TRADES

BILL: Market timing plot

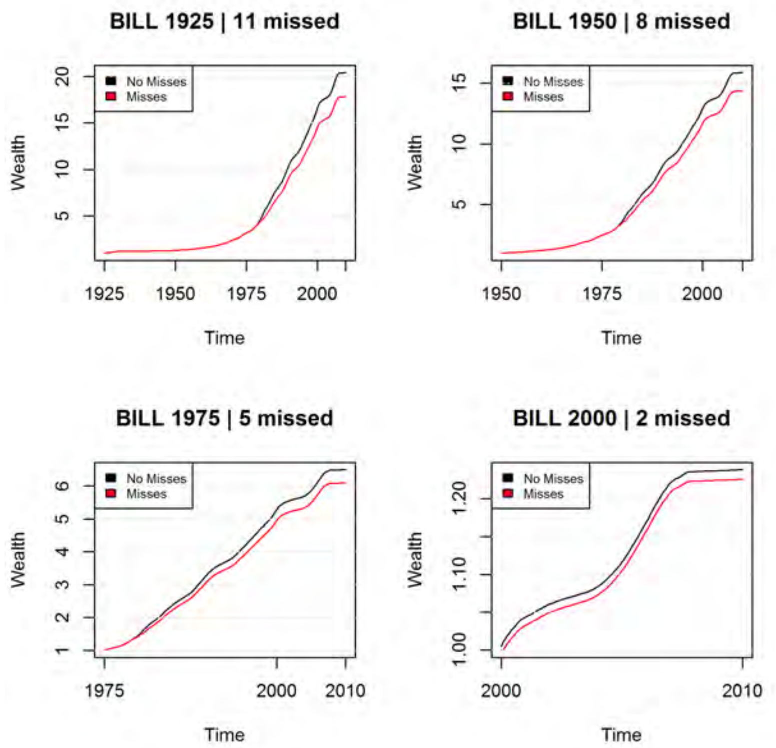

FIGURE 8.1: BILL market timing since 1925, 1950, 1975 and 2000.

BILL: Market timing statistics

```
              BILL
1981-06-30 1.35
1980-12-31 1.31
1981-08-31 1.28
1980-04-30 1.26
1981-07-31 1.24
1981-09-30 1.24
1981-10-31 1.21
1980-03-31 1.21
1981-03-31 1.21
1981-05-31 1.15
1982-04-30 1.13
              BILL
1981-06-30 1.35
1980-12-31 1.31
1981-08-31 1.28
1980-04-30 1.26
1981-07-31 1.24
1981-09-30 1.24
1981-10-31 1.21
1980-03-31 1.21
              BILL
1981-06-30 1.35
1980-12-31 1.31
1981-08-31 1.28
1980-04-30 1.26
1981-07-31 1.24
              BILL
2001-01-31 0.54
2000-12-31 0.50
```

FIGURE 8.2: Best monthly returns (in percent) for the periods starting in 1925, 1950, 1975, and 2000.

ITGOV: Market timing plot

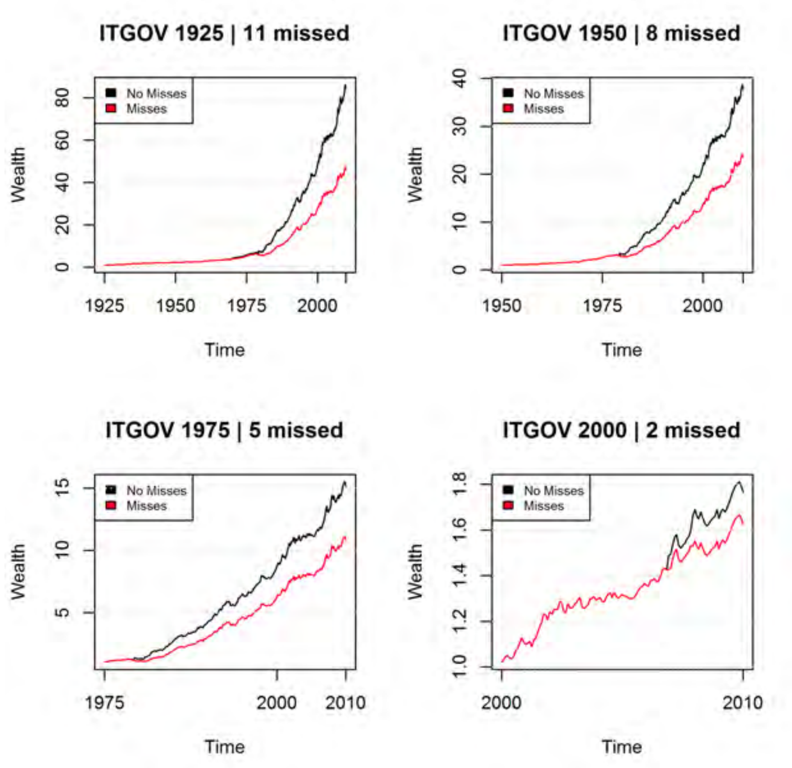

FIGURE 8.3: ITGOV market timing since 1925, 1950, 1975 and 2000.

ITGOV: Market timing statistics

```
            ITGOV
1980-04-30 11.98
1981-11-30  6.24
1981-10-31  6.11
1982-10-31  5.31
1980-05-31  4.90
1985-05-31  4.85
1982-08-31  4.69
1982-07-31  4.64
1970-11-30  4.51
1970-02-28  4.39
2008-11-30  4.30
            ITGOV
1980-04-30 11.98
1981-11-30  6.24
1981-10-31  6.11
1982-10-31  5.31
1980-05-31  4.90
1985-05-31  4.85
1982-08-31  4.69
1982-07-31  4.64
            ITGOV
1980-04-30 11.98
1981-11-30  6.24
1981-10-31  6.11
1982-10-31  5.31
1980-05-31  4.90
            ITGOV
2008-11-30  4.30
2007-11-30  4.25
```

FIGURE 8.4: ITGOV market timing statistics.

LTGOV: Market timing plot

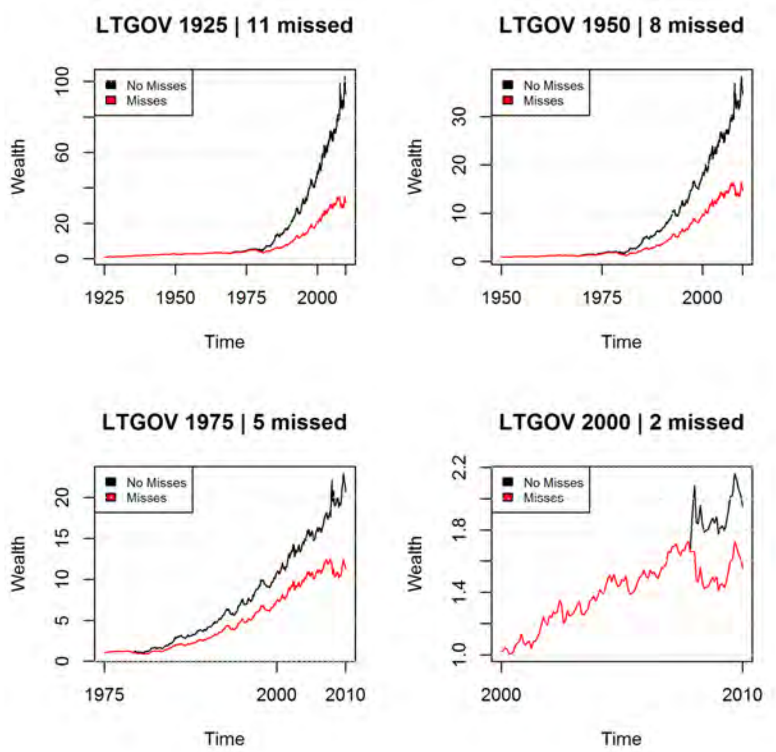

FIGURE 8.5: LTGOV market timing since 1925, 1950, 1975 and 2000.

LTGOV: Market timing statistics

```
             LTGOV
1980-04-30 15.23
2008-11-30 14.43
1981-11-30 14.10
1986-02-28 11.45
2008-12-31  9.67
1985-05-31  8.96
1981-10-31  8.29
1970-11-30  7.91
1995-05-31  7.90
1982-08-31  7.81
1986-03-31  7.70
```

```
             LTGOV
1980-04-30 15.23
2008-11-30 14.43
1981-11-30 14.10
1986-02-28 11.45
2008-12-31  9.67
1985-05-31  8.96
1981-10-31  8.29
1970-11-30  7.91
```

```
             LTGOV
1980-04-30 15.23
2008-11-30 14.43
1981-11-30 14.10
1986-02-28 11.45
2008-12-31  9.67
```

```
             LTGOV
2008-11-30 14.43
2008-12-31  9.67
```

FIGURE 8.6: LTGOV market timing statistics.

LTCORP: Market timing plot

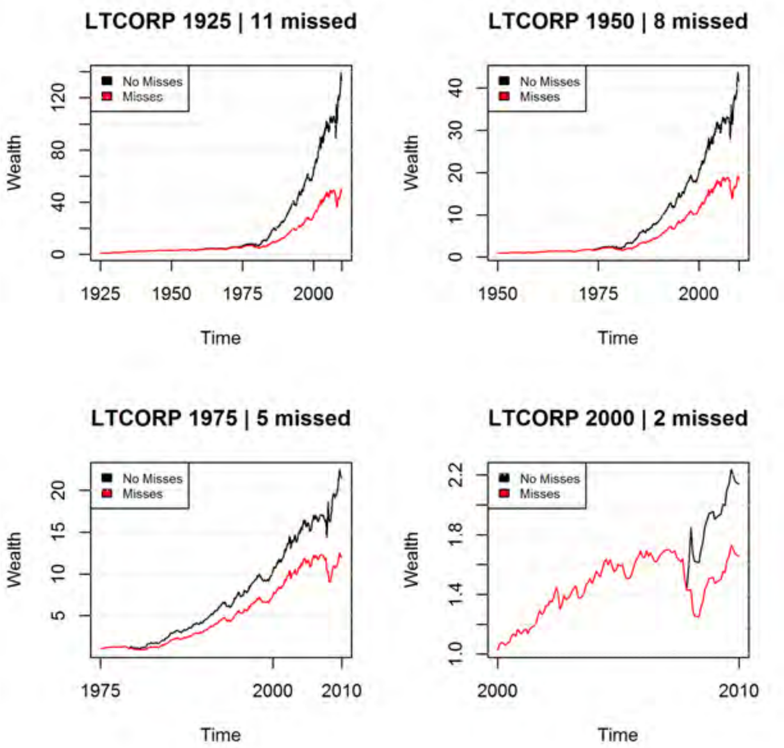

FIGURE 8.7: LTCORP market timing since 1925, 1950, 1975 and 2000.

LTCORP: Market timing statistics

```
            LTCORP
2008-12-31   15.60
1980-04-30   13.76
1981-11-30   12.67
2008-11-30   11.74
1974-10-31    8.85
1982-08-31    8.37
1985-05-31    8.20
1982-10-31    7.59
1986-02-28    7.52
1957-12-31    6.85
1995-05-31    6.31
```

```
            LTCORP
2008-12-31   15.60
1980-04-30   13.76
1981-11-30   12.67
2008-11-30   11.74
1974-10-31    8.85
1982-08-31    8.37
1985-05-31    8.20
1982-10-31    7.59
```

```
            LTCORP
2008-12-31   15.60
1980-04-30   13.76
1981-11-30   12.67
2008-11-30   11.74
1982-08-31    8.37
```

```
            LTCORP
2008-12-31   15.60
2008-11-30   11.74
```

FIGURE 8.8: LTCORP market timing statistics.

HYCORP: Market timing plot

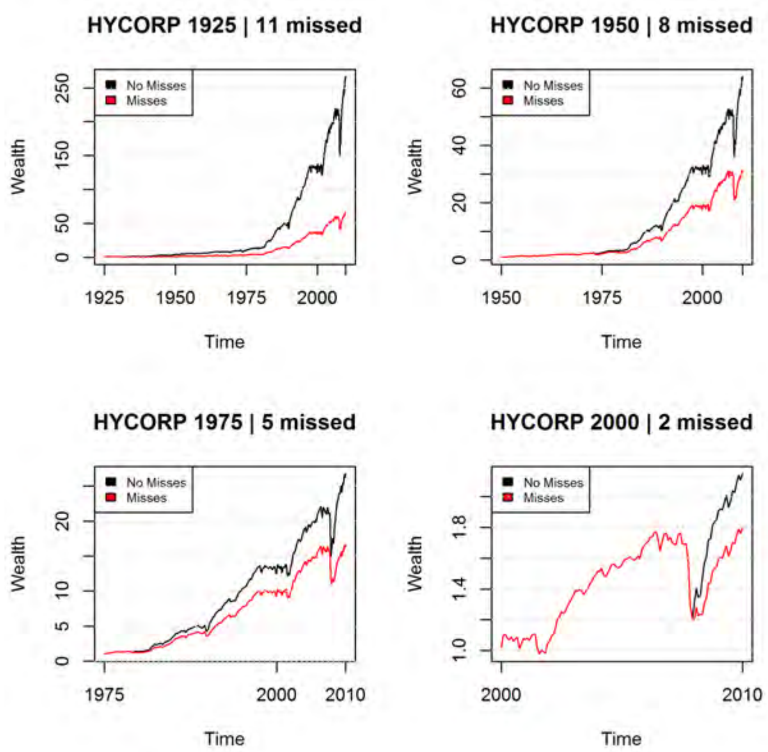

FIGURE 8.9: HYCORP market timing since 1925, 1950, 1975 and 2000.

HYCORP: Market timing statistics

```
              HYCORP
1932-08-31     22.94
1934-01-31     18.55
1933-05-31     17.39
1932-07-31     16.84
1933-04-30     12.63
1980-04-30     11.67
1991-02-28     10.94
1936-01-31      9.90
2008-12-31      9.68
1939-09-30      9.30
1938-04-30      9.14
              HYCORP
1980-04-30     11.67
1991-02-28     10.94
2008-12-31      9.68
2009-04-30      9.09
1982-08-31      8.69
1981-11-30      8.41
1973-12-31      8.35
1974-10-31      7.60

              HYCORP
1980-04-30     11.67
1991-02-28     10.94
2008-12-31      9.68
2009-04-30      9.09
1982-08-31      8.69

              HYCORP
2008-12-31      9.68
2009-04-30      9.09
```

FIGURE 8.10: HYCORP market timing statistics.

LCAP: Market timing plot

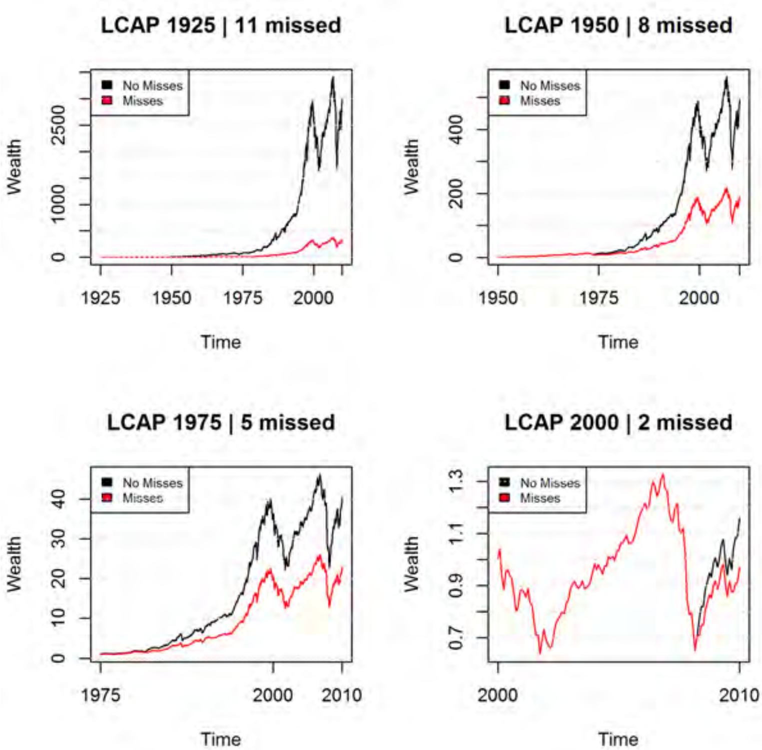

FIGURE 8.11: LCAP market timing since 1925, 1950, 1975 and 2000.

LCAP: Market timing statistics

```
               LCAP
1933-04-30 42.56
1932-08-31 38.70
1932-07-31 38.15
1938-06-30 25.02
1933-05-31 16.83
1939-09-30 16.73
1974-10-31 16.57
1938-04-30 14.46
1931-06-30 14.20
1987-01-31 13.43
1933-06-30 13.38
               LCAP
1974-10-31 16.57
1987-01-31 13.43
1982-08-31 12.67
1975-01-31 12.51
1976-01-31 11.99
1991-12-31 11.43
1982-10-31 11.26
1984-08-31 11.25

               LCAP
1987-01-31 13.43
1982-08-31 12.67
1976-01-31 11.99
1991-12-31 11.43
1982-10-31 11.26

              LCAP
2009-04-30 9.57
2010-09-30 8.92
```

FIGURE 8.12: LCAP market timing statistics.

SCAP: Market timing plot

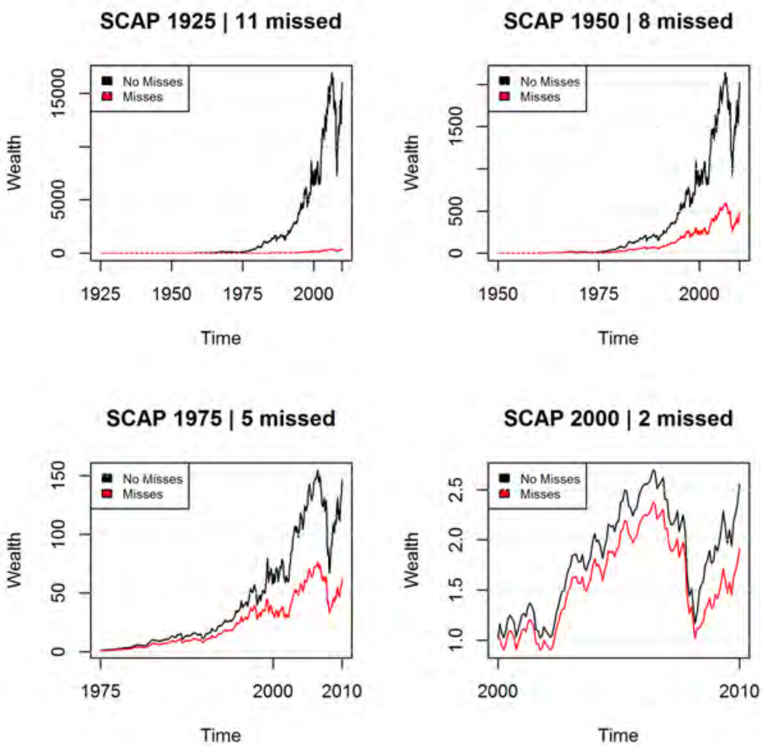

FIGURE 8.13: SCAP market timing since 1925, 1950, 1975 and 2000.

SCAP: Market timing statistics

```
                SCAP
1932-08-31 73.48
1933-05-31 63.37
1939-09-30 51.44
1933-04-30 50.36
1934-01-31 38.92
1932-07-31 35.24
1938-06-30 34.97
1936-01-31 30.10
1938-04-30 27.76
1975-01-31 27.67
1976-01-31 26.84
                SCAP
1975-01-31 27.67
1976-01-31 26.84
2000-02-29 23.58
1967-01-31 18.38
2009-04-30 17.39
1971-01-31 15.92
1968-04-30 14.61
1976-02-29 13.90
                SCAP
1976-01-31 26.84
2000-02-29 23.58
2009-04-30 17.39
1976-02-29 13.90
2001-01-31 13.80
                SCAP
2009-04-30 17.39
2001-01-31 13.80
```

FIGURE 8.14: SCAP market timing statistics.

CRB: Market timing plot

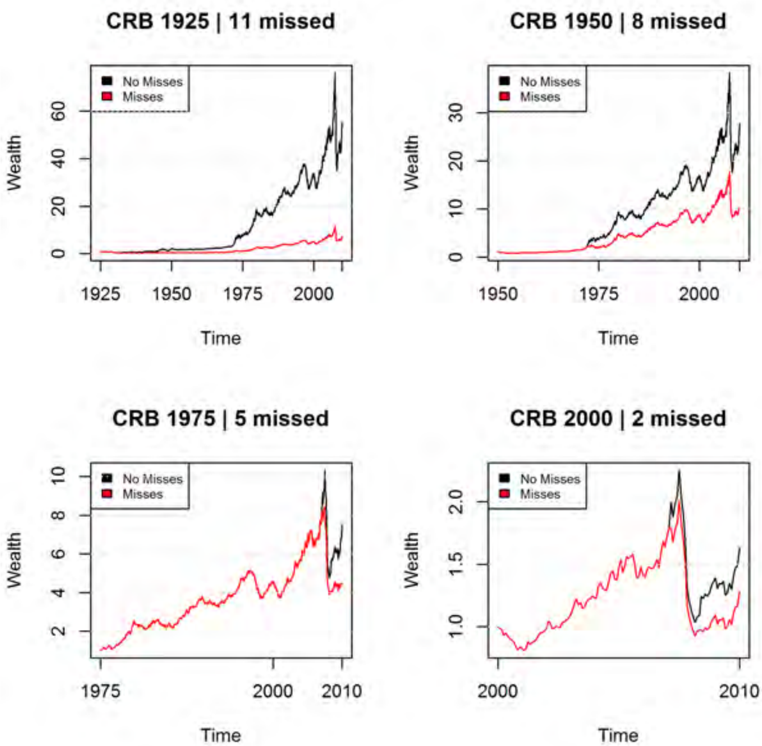

FIGURE 8.15: CRB - Market timing since 1925, 1950, 1975 and 2000.

CRB: Market timing statistics

```
                CRB
1939-09-30 38.36
1946-07-31 29.64
1933-05-31 23.70
1932-08-31 22.29
1933-04-30 20.45
1973-07-31 19.52
1974-07-31 16.68
1933-07-31 14.24
2009-05-31 13.80
1973-05-31 13.07
1950-07-31 12.79
                CRB
1973-07-31 19.52
1974-07-31 16.68
2009-05-31 13.80
1973-05-31 13.07
2008-02-29 11.92
2010-12-31 10.41
1975-07-31 10.41
1973-02-28 10.25
                CRB
2009-05-31 13.80
2008-02-29 11.92
2010-12-31 10.41
2008-06-30  9.79
2010-09-30  8.60
                CRB
2009-05-31 13.80
2008-02-29 11.92
```

FIGURE 8.16: CRB market timing statistics.

8.2 MONTH OF YEAR EFFECT

BILL: Month of Year Effect Chart

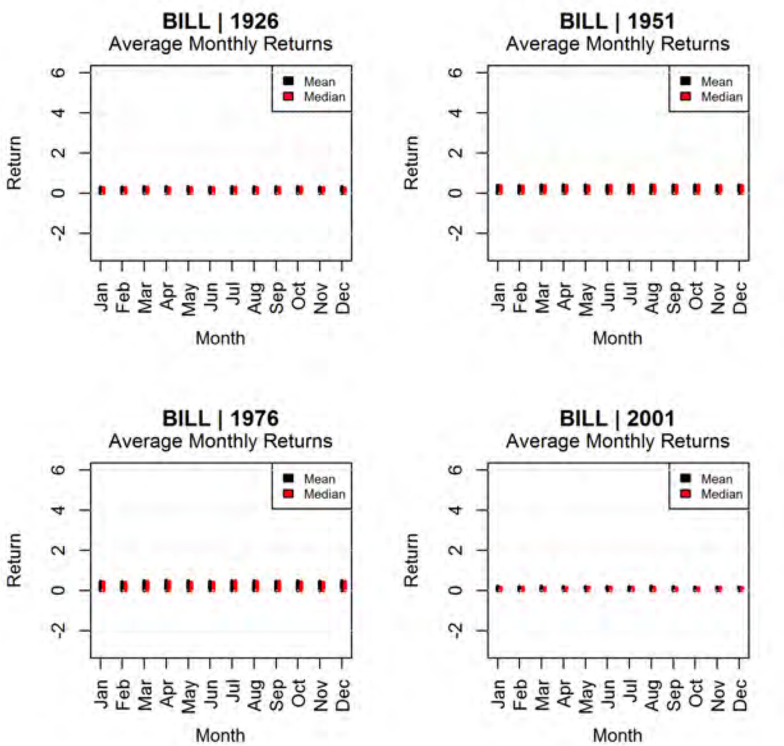

FIGURE 8.17: BILL - Month of year effect. The four periods since 1926, 1951, 1976 and 2001, each sequence starts in January.

BILL: Month of Year Effect Statistics

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
mean	0.29	0.28	0.30	0.30	0.30	0.29	0.30	0.30	0.30	0.31	0.29	0.30
median	0.27	0.25	0.29	0.25	0.26	0.27	0.28	0.26	0.27	0.29	0.27	0.24
sd	0.23	0.23	0.26	0.27	0.26	0.25	0.26	0.25	0.25	0.26	0.25	0.26

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
mean	0.38	0.36	0.39	0.39	0.39	0.38	0.39	0.39	0.39	0.40	0.37	0.40
median	0.37	0.35	0.38	0.37	0.35	0.37	0.39	0.40	0.38	0.39	0.36	0.38
sd	0.22	0.22	0.25	0.27	0.25	0.24	0.25	0.24	0.24	0.25	0.24	0.25

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
mean	0.43	0.42	0.46	0.47	0.45	0.44	0.45	0.45	0.44	0.46	0.43	0.45
median	0.42	0.39	0.43	0.43	0.40	0.40	0.45	0.43	0.44	0.42	0.39	0.44
sd	0.24	0.25	0.29	0.30	0.28	0.27	0.27	0.26	0.26	0.29	0.27	0.28

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
mean	0.19	0.18	0.19	0.18	0.18	0.17	0.18	0.19	0.17	0.16	0.16	0.15
median	0.15	0.14	0.12	0.12	0.14	0.14	0.14	0.14	0.13	0.12	0.14	0.13
sd	0.18	0.15	0.16	0.16	0.15	0.14	0.14	0.15	0.14	0.14	0.15	0.14

FIGURE 8.18: Monthly returns statistics (in percent) per month for periods starting in January of the years 1926, 1951, 1976 and 2001.

ITGOV: Month of Year Effect Chart

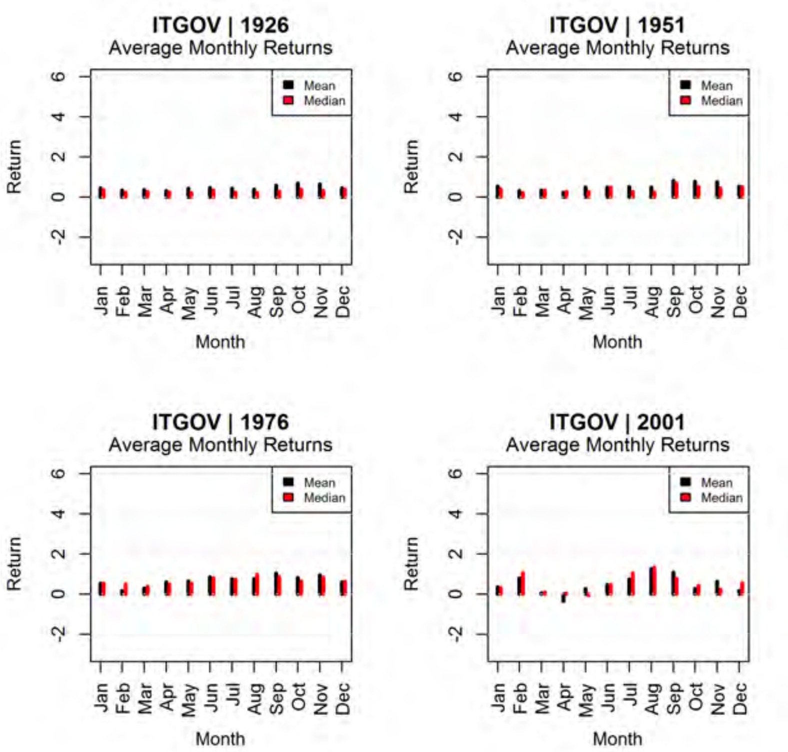

FIGURE 8.19: ITGOV - Month of year effect. The four periods since 1926, 1951, 1976 and 2001, each sequence starts in January.

ITGOV: Month of Year Effect Statistics

```
        Jan  Feb  Mar  Apr  May  Jun  Jul  Aug  Sep  Oct  Nov  Dec
mean   0.43 0.32 0.34 0.30 0.41 0.45 0.41 0.38 0.56 0.67 0.63 0.43
median 0.34 0.22 0.27 0.24 0.20 0.33 0.21 0.19 0.29 0.36 0.31 0.36
sd     0.95 1.34 1.04 1.77 1.30 1.00 1.25 1.30 1.21 1.45 1.40 1.10
        Jan  Feb  Mar  Apr  May  Jun  Jul  Aug  Sep  Oct  Nov  Dec
mean   0.52 0.31 0.32 0.22 0.49 0.49 0.50 0.49 0.82 0.76 0.75 0.52
median 0.40 0.20 0.32 0.25 0.25 0.47 0.27 0.23 0.67 0.50 0.43 0.51
sd     1.08 1.57 1.16 2.07 1.46 1.09 1.46 1.49 1.28 1.63 1.63 1.19

        Jan  Feb  Mar  Apr  May  Jun  Jul  Aug  Sep  Oct  Nov  Dec
mean   0.53 0.14 0.28 0.60 0.63 0.84 0.75 0.74 1.04 0.82 0.94 0.59
median 0.52 0.48 0.37 0.47 0.52 0.79 0.72 0.97 0.85 0.64 0.84 0.61
sd     1.28 1.84 1.24 2.52 1.80 1.01 1.68 1.60 1.30 1.97 1.81 1.21

        Jan  Feb  Mar   Apr   May  Jun  Jul  Aug  Sep  Oct  Nov  Dec
mean   0.35 0.78 0.03 -0.35  0.27 0.45 0.72 1.24 1.08 0.26 0.62 0.16
median 0.31 1.06 0.09  0.02 -0.06 0.46 1.04 1.32 0.77 0.41 0.22 0.55
sd     1.27 1.14 1.19  1.89  1.30 0.72 1.80 0.64 1.35 0.96 2.24 1.57
```

FIGURE 8.20: ITGOV: Month of year statistics.

LTGOV: Month of Year Effect Chart

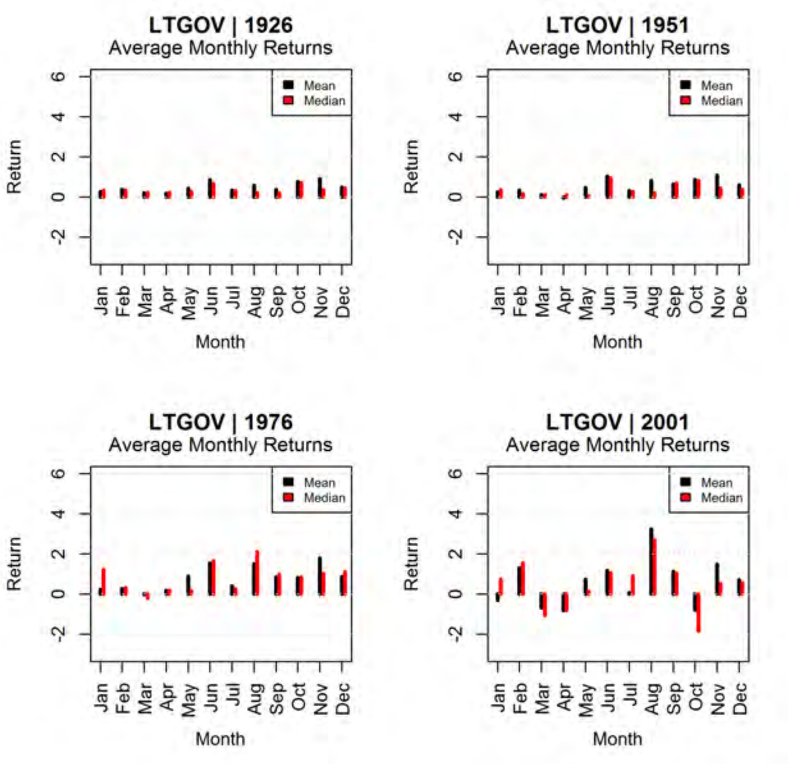

FIGURE 8.21: LTGOV - Month of year effect. The four periods since 1926, 1951, 1976 and 2001, each sequence starts in January.

LTGOV: Month of Year Effect Statistics

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
mean	0.25	0.37	0.20	0.18	0.41	0.85	0.32	0.57	0.37	0.77	0.89	0.48
median	0.33	0.32	0.21	0.21	0.27	0.65	0.31	0.20	0.19	0.71	0.35	0.44
sd	2.37	2.37	2.15	2.75	2.44	1.71	2.39	2.30	2.16	2.56	2.85	2.42

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
mean	0.23	0.32	0.11	-0.04	0.45	1.02	0.31	0.81	0.64	0.85	1.08	0.58
median	0.34	0.16	0.10	0.08	0.05	0.95	0.27	0.20	0.67	0.80	0.44	0.36
sd	2.76	2.73	2.44	3.12	2.81	1.96	2.75	2.66	2.38	2.92	3.32	2.79

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
mean	0.21	0.27	-0.02	0.17	0.87	1.53	0.39	1.50	0.83	0.82	1.77	0.86
median	1.21	0.30	-0.21	0.18	0.15	1.64	0.24	2.11	0.96	0.84	1.00	1.10
sd	3.29	3.31	2.80	3.78	3.33	1.98	3.30	2.97	2.71	3.49	3.74	3.26

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
mean	-0.32	1.30	-0.69	-0.81	0.72	1.16	0.07	3.24	1.12	-0.79	1.47	0.71
median	0.72	1.53	-1.05	-0.81	0.12	1.06	0.90	2.71	1.04	-1.83	0.51	0.55
sd	4.14	1.60	3.26	3.89	2.81	1.65	3.98	1.63	2.52	2.76	5.26	4.57

FIGURE 8.22: LTGOV: Month of year statistics.

LTCORP: Month of Year Effect Chart

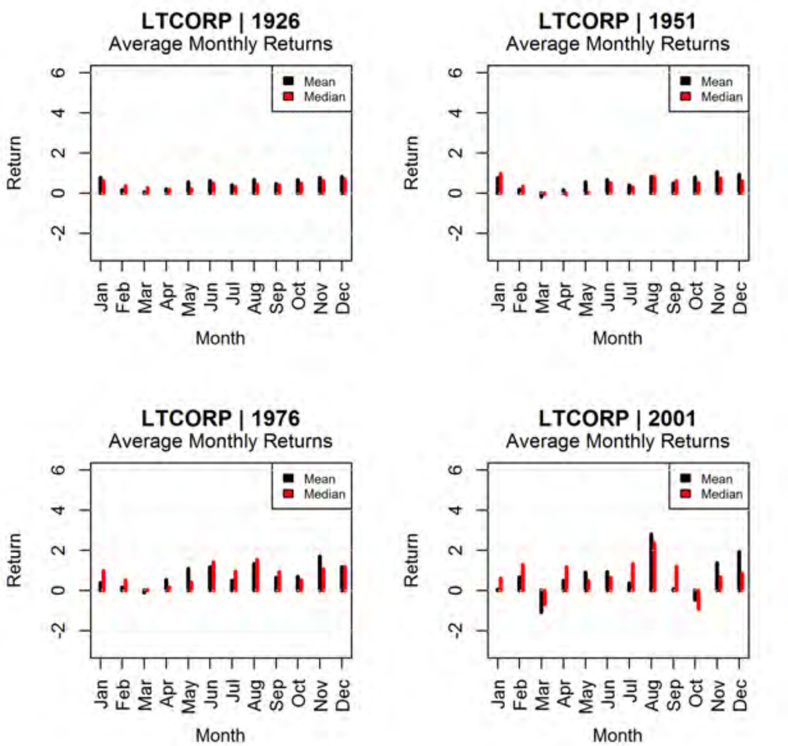

FIGURE 8.23: LTCORP - Month of year effect. The four periods since 1926, 1951, 1976 and 2001, each sequence starts in January.

LTCORP: Month of Year Effect Statistics

```
        Jan  Feb  Mar  Apr  May  Jun  Jul  Aug  Sep  Oct  Nov  Dec
mean   0.77 0.15 0.03 0.19 0.54 0.61 0.39 0.66 0.46 0.65 0.77 0.82
median 0.59 0.34 0.25 0.18 0.20 0.48 0.30 0.44 0.40 0.49 0.61 0.67
sd     2.14 2.03 1.48 2.41 2.14 1.54 2.22 2.09 2.02 2.43 2.37 2.46
        Jan  Feb   Mar   Apr  May  Jun  Jul  Aug  Sep  Oct  Nov  Dec
mean   0.75 0.18 -0.17  0.15 0.54 0.66 0.39 0.82 0.49 0.79 1.05 0.93
median 0.97 0.32 -0.04 -0.07 0.02 0.51 0.29 0.81 0.59 0.50 0.74 0.60
sd     2.44 2.27  1.62  2.84 2.43 1.79 2.63 2.34 2.35 2.81 2.72 2.85

        Jan  Feb   Mar  Apr  May  Jun  Jul  Aug  Sep  Oct  Nov  Dec
mean   0.38 0.15 -0.12 0.52 1.08 1.17 0.49 1.32 0.63 0.68 1.70 1.17
median 0.96 0.51  0.02 0.16 0.39 1.41 0.94 1.52 0.93 0.51 1.06 1.17
sd     2.76 2.71  1.76 3.49 2.97 1.92 3.06 2.51 2.89 3.14 3.01 3.31

        Jan  Feb   Mar  Apr  May  Jun  Jul  Aug  Sep   Oct  Nov  Dec
mean   0.04 0.66 -1.08 0.48 0.90 0.92 0.34 2.80 0.06 -0.47 1.35 1.94
median 0.58 1.27 -0.69 1.16 0.46 0.64 1.32 2.34 1.18 -0.93 0.65 0.84
sd     3.64 1.84  1.60 2.81 2.61 2.09 3.96 1.30 3.92  2.59 3.90 5.23
```

FIGURE 8.24: LTCORP: Month of year statistics.

HYCORP: Month of Year Effect Chart

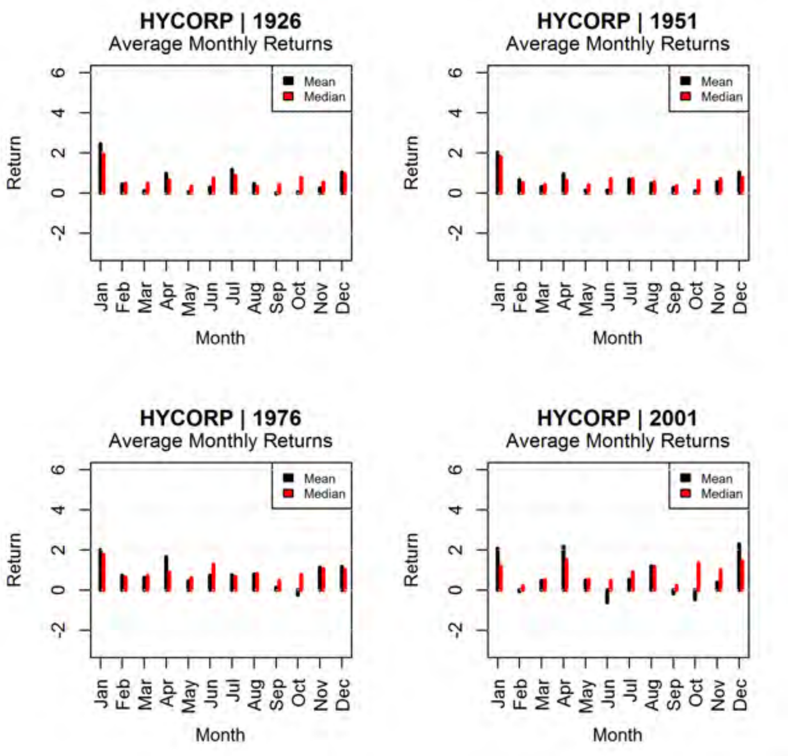

FIGURE 8.25: HYCORP - Month of year effect. The four periods since 1926, 1951, 1976 and 2001, each sequence starts in January.

HYCORP: Month of Year Effect Statistics

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
mean	2.44	0.44	0.10	0.96	0.03	0.29	1.17	0.46	-0.05	0.05	0.23	1.04
median	1.93	0.47	0.48	0.64	0.33	0.73	0.85	0.31	0.39	0.77	0.52	0.95
sd	3.00	2.73	2.92	3.38	3.79	2.46	2.79	3.28	3.33	3.17	2.55	2.52

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
mean	2.03	0.65	0.31	0.94	0.12	0.13	0.67	0.48	0.26	0.09	0.54	1.04
median	1.79	0.51	0.42	0.62	0.40	0.70	0.69	0.55	0.35	0.62	0.71	0.76
sd	2.08	2.38	1.86	2.57	2.33	2.10	1.88	2.19	2.48	3.31	2.49	2.00

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
mean	1.99	0.73	0.61	1.65	0.44	0.73	0.75	0.79	0.12	-0.24	1.11	1.17
median	1.76	0.64	0.70	0.87	0.60	1.28	0.66	0.80	0.45	0.77	1.05	1.01
sd	2.35	2.86	1.98	2.94	2.17	2.15	2.13	2.56	2.95	3.98	2.67	1.93

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
mean	2.07	-0.08	0.46	2.17	0.47	-0.63	0.53	1.19	-0.19	-0.47	0.37	2.29
median	1.18	0.19	0.51	1.49	0.53	0.45	0.90	1.16	0.22	1.34	0.99	1.44
sd	2.91	1.79	1.96	3.30	2.55	3.40	3.46	1.28	4.29	6.10	3.59	2.78

FIGURE 8.26: HYCORP - Month of year effect. The four periods since 1926, 1951, 1976 and 2001, each sequence starts in January.

LCAP: Month of Year Effect Chart

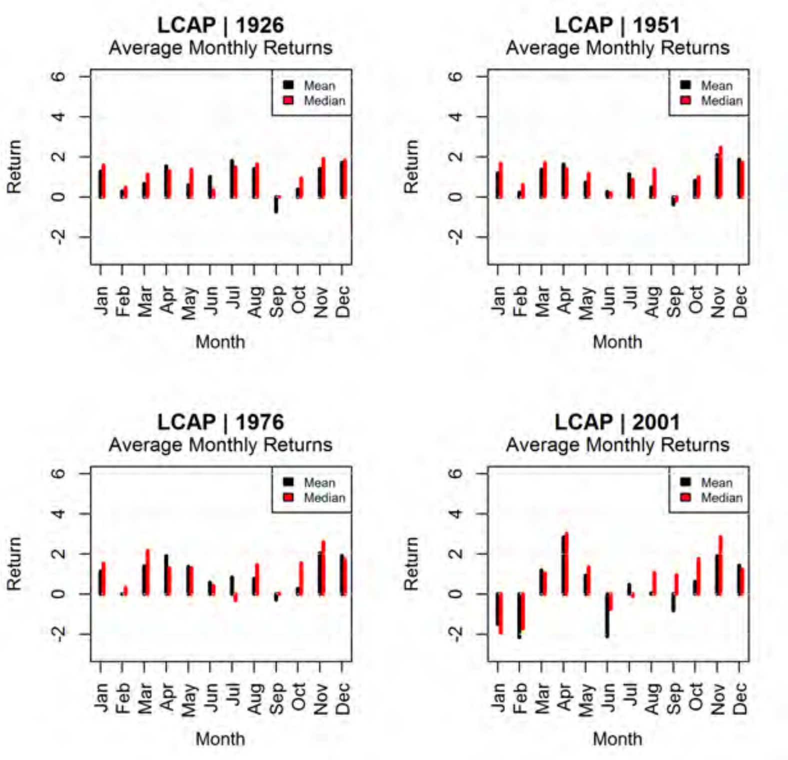

FIGURE 8.27: LCAP - Month of year effect. The four periods since 1926, 1951, 1976 and 2001, each sequence starts in January.

LCAP: Month of Year Effect Statistics

```
        Jan  Feb  Mar  Apr  May  Jun  Jul  Aug   Sep  Oct  Nov  Dec
mean   1.28 0.28 0.66 1.53 0.59 1.00 1.80 1.40 -0.74 0.36 1.41 1.72
median 1.58 0.49 1.12 1.30 1.37 0.33 1.47 1.63  0.00 0.93 1.90 1.83
sd     4.78 4.25 5.12 6.75 5.70 5.38 6.21 6.23  6.03 6.15 5.22 3.64
        Jan  Feb  Mar  Apr  May  Jun  Jul  Aug   Sep  Oct  Nov  Dec
mean   1.18 0.22 1.37 1.63 0.72 0.25 1.14 0.47 -0.37 0.81 2.09 1.87
median 1.66 0.60 1.68 1.41 1.16 0.20 0.86 1.38 -0.18 1.01 2.45 1.72
sd     4.96 3.53 3.44 3.92 3.71 3.52 4.19 4.71  4.46 5.58 4.52 3.19

        Jan  Feb  Mar  Apr  May  Jun   Jul  Aug   Sep  Oct  Nov  Dec
mean   1.11 0.00 1.38 1.90 1.35 0.56  0.83 0.76 -0.29 0.27 2.04 1.92
median 1.51 0.31 2.15 1.30 1.30 0.41 -0.32 1.45  0.02 1.53 2.60 1.72
sd     5.15 4.00 4.02 3.77 3.65 3.45  4.11 5.24  4.57 6.41 4.59 3.32

         Jan   Feb  Mar  Apr  May   Jun   Jul  Aug   Sep  Oct  Nov  Dec
mean   -1.50 -2.15 1.18 2.83 0.93 -2.12  0.46 0.03 -0.81 0.62 1.88 1.43
median -1.95 -1.71 1.04 3.00 1.33 -0.76 -0.11 1.05  0.95 1.75 2.84 1.23
sd      3.91  4.55 4.25 5.06 4.07  3.63  4.77 3.12  6.44 6.89 4.69 3.43
```

FIGURE 8.28: LCAP - Month of year statistics.

SCAP: Month of Year Effect Chart

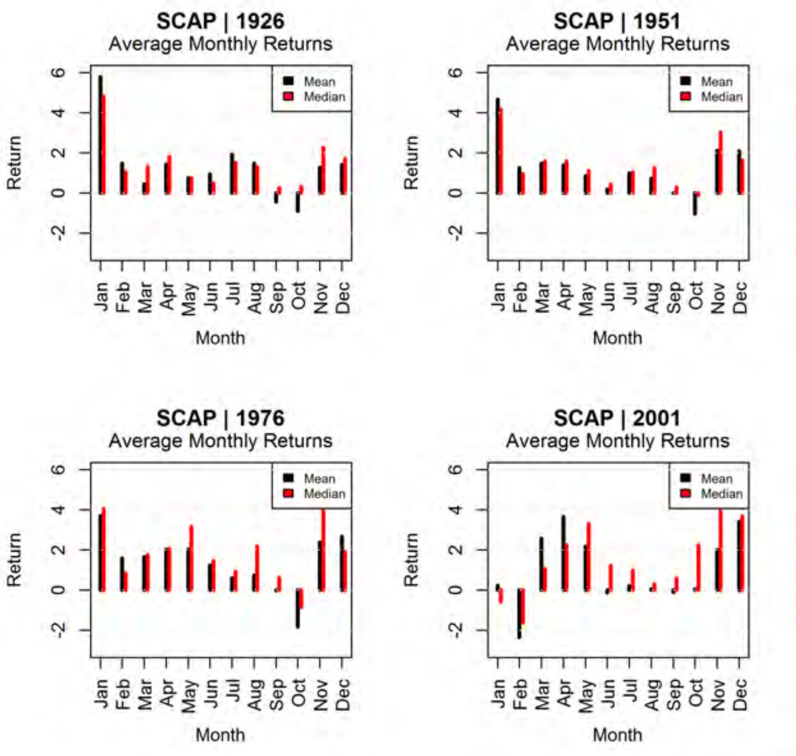

FIGURE 8.29: SCAP - Month of year effect. The four periods since 1926, 1951, 1976 and 2001, each sequence starts in January.

SCAP: Month of Year Effect Statistics

```
        Jan  Feb  Mar  Apr   May  Jun  Jul  Aug   Sep  Oct  Nov  Dec
mean   5.77 1.48 0.44 1.42  0.75 0.95 1.92 1.46 -0.44 -0.9 1.27 1.41
median 4.82 1.06 1.31 1.79  0.75 0.48 1.52 1.30  0.27  0.3 2.25 1.68
sd     8.69 6.54 7.54 9.47 10.05 7.76 8.09 9.86  9.22  8.2 7.14 5.86
        Jan  Feb  Mar  Apr  May  Jun  Jul  Aug   Sep   Oct  Nov  Dec
mean   4.67 1.24 1.48 1.39 0.83 0.18 0.99 0.73 -0.01 -1.04 2.10 2.09
median 4.15 0.94 1.58 1.59 1.10 0.43 1.02 1.26  0.29 -0.10 3.04 1.62
sd     7.38 5.70 4.85 5.71 5.39 5.01 5.58 5.65  4.94  7.30 5.95 4.36

        Jan  Feb  Mar  Apr  May  Jun  Jul  Aug   Sep   Oct  Nov  Dec
mean   3.70 1.59 1.64 2.02 2.02 1.22 0.59 0.73 -0.02 -1.83 2.37 2.65
median 4.04 0.83 1.74 2.07 3.15 1.44 0.92 2.18  0.61 -0.87 4.04 1.94
sd     7.16 6.65 5.81 5.64 5.10 4.70 5.96 6.13  4.93  8.83 5.97 3.88

         Jan   Feb  Mar  Apr  May   Jun  Jul  Aug   Sep  Oct  Nov  Dec
mean    0.21 -2.35 2.56 3.65 2.18 -0.14 0.20 0.04 -0.12 0.05 1.99 3.40
median -0.57 -1.64 1.06 2.25 3.29  1.21 0.97 0.29  0.59 2.21 4.27 3.67
sd      7.75  4.86 5.01 6.91 6.24  5.03 8.21 3.76  7.24 8.64 7.13 4.13
```

FIGURE 8.30: SCAP: Month of year statistics.

CRB: Month of Year Effect Chart

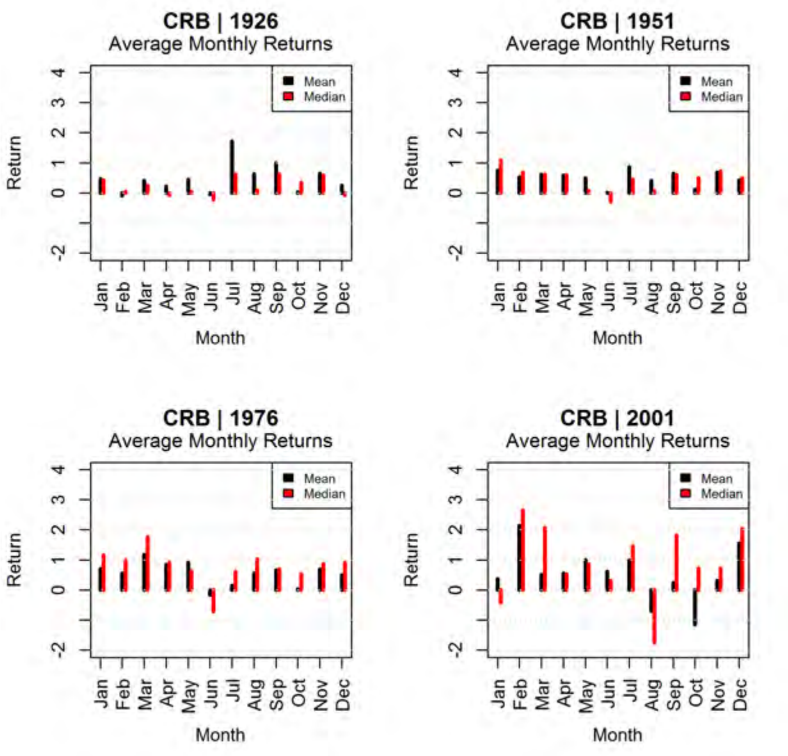

FIGURE 8.31: CRB - Month of year effect. The four periods since 1926, 1951, 1976 and 2001, each sequence starts in January.

CRB: Month of Year Effect Statistics

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
mean	0.46	-0.09	0.41	0.22	0.44	-0.04	1.71	0.63	1.00	0.02	0.64	0.25
median	0.42	0.04	0.24	-0.08	0.04	-0.22	0.61	0.09	0.63	0.35	0.58	-0.07
sd	2.99	3.48	3.88	3.65	4.51	3.08	5.60	4.23	5.22	4.00	3.01	3.35

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
mean	0.75	0.52	0.61	0.59	0.50	0.01	0.86	0.41	0.64	0.11	0.68	0.42
median	1.08	0.69	0.61	0.60	0.07	-0.29	0.45	0.05	0.60	0.50	0.72	0.49
sd	2.95	3.48	3.38	2.31	3.64	2.96	4.69	3.31	3.27	3.88	2.45	3.23

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
mean	0.70	0.55	1.17	0.83	0.91	-0.17	0.15	0.55	0.66	0.03	0.68	0.50
median	1.16	0.98	1.76	0.91	0.63	-0.72	0.61	1.02	0.69	0.53	0.86	0.90
sd	3.10	3.72	3.57	2.55	3.80	3.41	3.94	3.54	3.84	4.72	2.84	3.45

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
mean	0.36	2.14	0.51	0.56	0.99	0.61	0.98	-0.70	0.23	-1.14	0.30	1.55
median	-0.43	2.65	2.06	0.52	0.86	0.31	1.47	-1.75	1.82	0.72	0.72	2.03
sd	4.15	5.45	4.37	3.31	5.54	3.68	4.32	4.31	6.38	8.08	3.88	5.05

FIGURE 8.32: CRB: Month of year statistics.

Part III

Multivariate Analysis

CHAPTER 9

INTRODUCTION

Dependency measures give information on the strengths of correlations of different assets and allow to get insight into the dependence structures in between. They also allow for a more complex ordering and ranking of individual assets, compared to the use of simple univariate metrics like mean performance or risk expressed as variance, value at risk, drawdowns, or recovery times. In this Part we will cover different dependency risk measures: correlation structures, ordering aspects of assets and grouping and clustering of of assets classes.

Correlation refers to any of a broad class of statistical relationships involving dependence. In chapter 10 we will look at the Person's and Kendall's correlations. In chapter 11 we will use the principal component analysis of eigenvectors to order and to rank the assets classes. This can be done by using the orientation and size of the eigenvectors. In chapter 12 we will cover techniques to group assets according to their similaritities. We will report on the use of the hierarchical and k-means clustering to seek for an optimal choice of assets grouping.

Chapter 10

Correlation Structures

In this chapter we present several views on the pairwise correlation structure between individual assets. We display this structure by correlogram plots and correlation image plots for Pearson's product moment correlation coefficient and for Kendall's rank correlation coefficient.

The most common correlation measure between two assets is the *Pearson Correlation.* The measure is calculated from the ratio of the covariance of the two assets and the product of their standard deviations. It measures the sample correlation $\rho_{XY} = cor(X,Y)$ between two random variables X and Y with expected means μ_X and μ_Y and standard deviations σ_X and σ_Y

$$\rho_{XY} = cor(X,Y) = \frac{cov(X,Y)}{\sigma_X \sigma_Y} = \frac{E[(X-\mu_X)(Y-\mu_Y)]}{\sigma_X \sigma_Y}$$

where E denotes the expected value operator, cov means covariance, and cor is used as an alternative notation for Pearson's correlation. The random variables X and Y measure for example the returns of assets X and Y. The Pearson's correlation thus describes the linear relationship between the two assets. Pearson's correlation takes the value $+1$ in the case of a perfect relationship, -1 in the case of a perfect anti-correlation, and becomes 0 if X and Y are uncorrelated, i.e. independent from each other.

Rank correlation coefficients, such as *Spearman's Correlation* or *Kendall's Correlation* coefficients measure the extent to which, as one asset increases, the other variable tends to increase. It is common to regard these rank correlation coefficients as an alternative to Pearson's coefficient, used to make the coefficient less sensitive to non-normality in distributions of the two assets. As in the case of Pearson's correlation a value of $+1$ indicates perfect correlation and a value of -1 perfect anti-correlation. But be aware that they measure a different kind of association. Rank correlation coefficients just relate the extent to which the one variable increases and the other variable tends also to increase or to decrease. A linear relationship is not required.

If we consider many assets we can compute the correlations pairwise and print the results as a square numeric matrix, the correlation matrix. Alternatively, we can visualize the correlation structures in several kind of plots.

The analyses within this chapter were done using the monthly nominal total returns. Exploratory displays for correlation matrices were introduce by Friendly [2002]. He describes a set of techniques based on (i) rendering the value of a correlation to depict its sign and magnitude, and to (ii) reordering the variables in a correlation matrix such that similar variables are positioned adjacently.
Correlogram Plots: These views on the correlation structure show in the lower panel a scatter plot of the different asset pairs and a pie that indicates the correlation value. In the upper panel they show concentration ellipses (circle for no correlation) and lowess smooths of the scatter plots as schematic visual summaries of linear and possibly nonlinear associations.
Correlation Image Plots: This is just a display of the correlation matrix in a pattern of squares given by the pair of assets. The values of the correlations between -1 and $+1$ are then expressed by a color palette to make the strength of the correlations visible. In addition in each square we add the numeric value of the correlation

10.1 PEARSON'S CORRELOGRAM PLOTS

Pearson's Correlograms since 1925

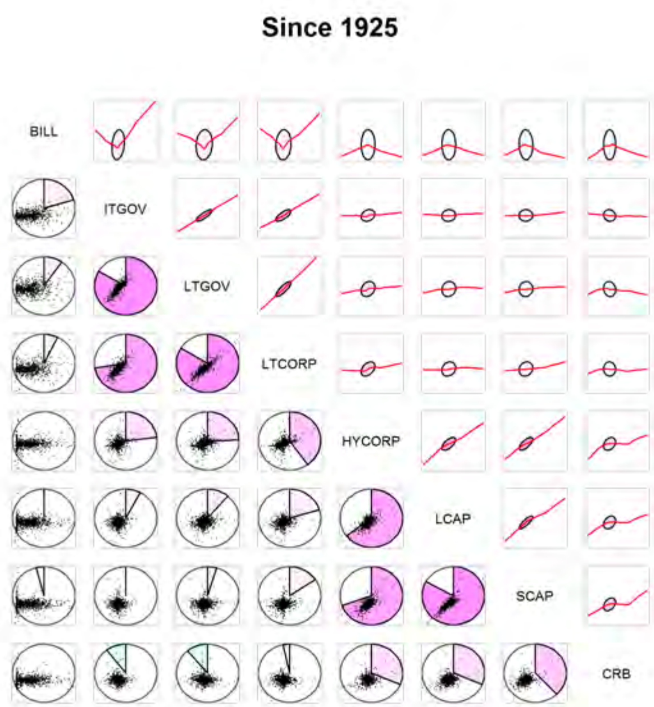

FIGURE 10.1: Pearson's pairwise correlations between asset classes. The data sets start 1925.

	BILL	ITGOV	LTGOV	LTCORP	HYCORP	LCAP	SCAP	CRB
BILL	1.00	0.20	0.10	0.07	0.01	-0.01	-0.04	0.00
ITGOV	0.20	1.00	0.83	0.73	0.23	0.08	0.02	-0.10
LTGOV	0.10	0.83	1.00	0.84	0.25	0.11	0.05	-0.11
LTCORP	0.07	0.73	0.84	1.00	0.40	0.20	0.16	-0.03
HYCORP	0.01	0.23	0.25	0.40	1.00	0.65	0.69	0.31
LCAP	-0.01	0.08	0.11	0.20	0.65	1.00	0.83	0.31
SCAP	-0.04	0.02	0.05	0.16	0.69	0.83	1.00	0.38
CRB	0.00	-0.10	-0.11	-0.03	0.31	0.31	0.38	1.00

FIGURE 10.2: Pearson's pairwise correlation matrix. The data sets start 1925.

Pearson's Correlogram Plots since 1950

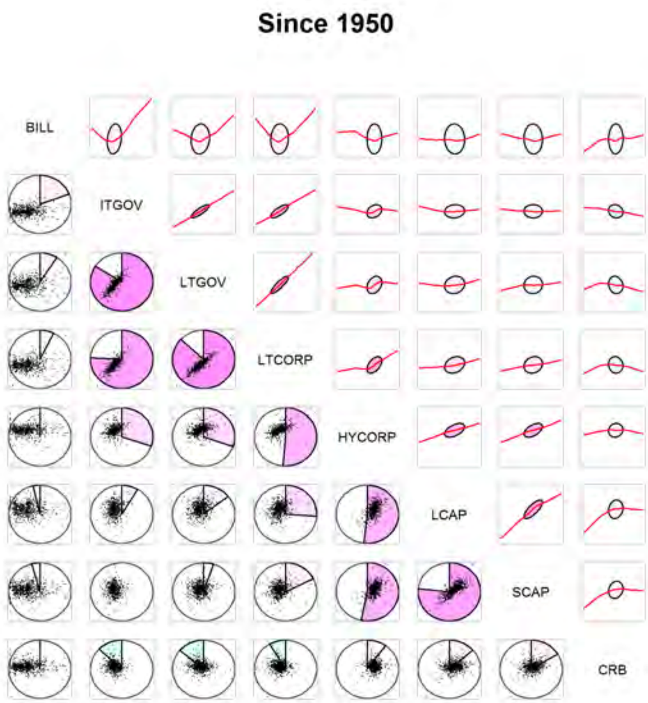

FIGURE 10.3: Pearson's pairwise correlations between asset classes. The data sets start 1950.

	BILL	ITGOV	LTGOV	LTCORP	HYCORP	LCAP	SCAP	CRB
BILL	1.00	0.19	0.09	0.07	0.02	-0.04	-0.04	0.02
ITGOV	0.19	1.00	0.84	0.76	0.30	0.08	0.00	-0.13
LTGOV	0.09	0.84	1.00	0.87	0.30	0.14	0.05	-0.14
LTCORP	0.07	0.76	0.87	1.00	0.51	0.26	0.17	-0.09
HYCORP	0.02	0.30	0.30	0.51	1.00	0.52	0.53	0.10
LCAP	-0.04	0.08	0.14	0.26	0.52	1.00	0.77	0.14
SCAP	-0.04	0.00	0.05	0.17	0.53	0.77	1.00	0.16
CRB	0.02	-0.13	-0.14	-0.09	0.10	0.14	0.16	1.00

FIGURE 10.4: Pearson's pairwise correlation matrix. The data sets start 1950.

Pearson's Correlogram Plots since 1975

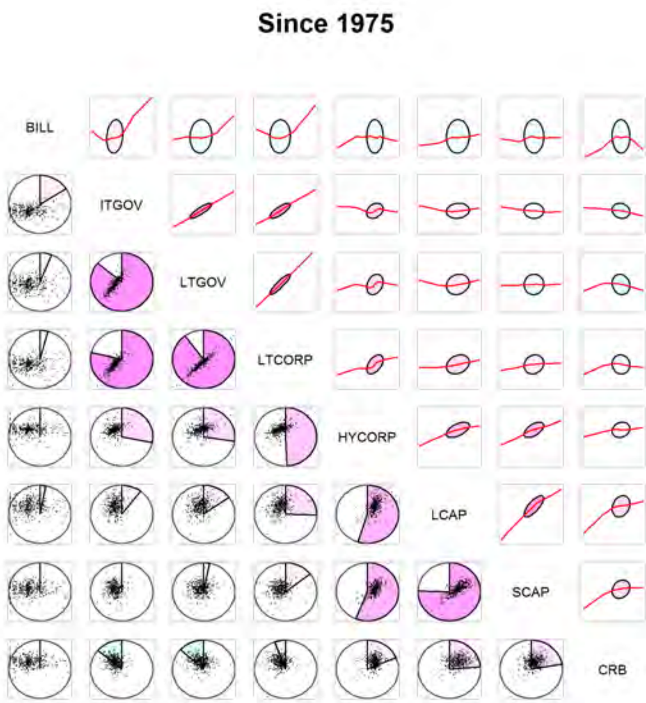

FIGURE 10.5: Pearson's pairwise correlations between asset classes. The data sets start 1975.

	BILL	ITGOV	LTGOV	LTCORP	HYCORP	LCAP	SCAP	CRB
BILL	1.00	0.16	0.06	0.04	0.01	0.03	-0.02	-0.02
ITGOV	0.16	1.00	0.86	0.79	0.28	0.11	-0.02	-0.14
LTGOV	0.06	0.86	1.00	0.90	0.28	0.15	0.03	-0.13
LTCORP	0.04	0.79	0.90	1.00	0.49	0.25	0.14	-0.06
HYCORP	0.01	0.28	0.28	0.49	1.00	0.55	0.56	0.19
LCAP	0.03	0.11	0.15	0.25	0.55	1.00	0.75	0.24
SCAP	-0.02	-0.02	0.03	0.14	0.56	0.75	1.00	0.22
CRB	-0.02	-0.14	-0.13	-0.06	0.19	0.24	0.22	1.00

FIGURE 10.6: Pearson's pairwise correlation matrix. The data sets start 1975.

Pearson's Correlogram Plots since 2000

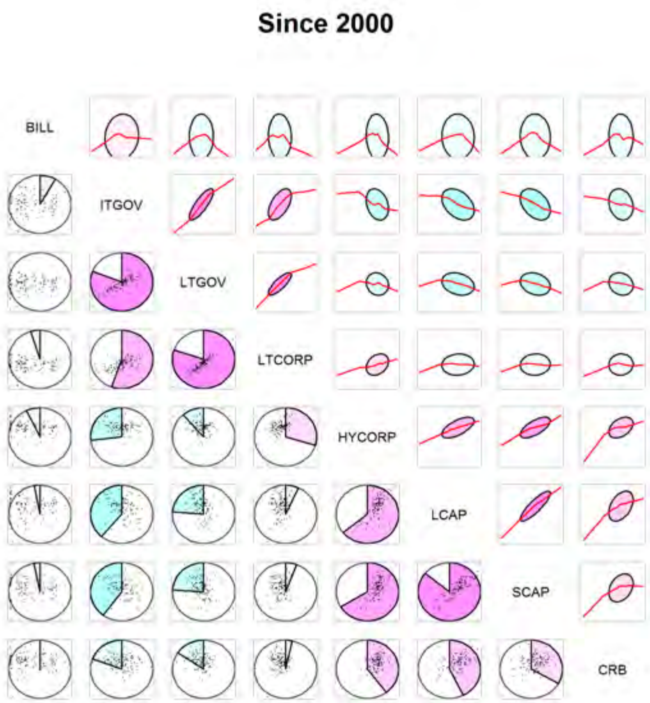

FIGURE 10.7: Pearson's pairwise correlations between asset classes. The data sets start 2000.

	BILL	ITGOV	LTGOV	LTCORP	HYCORP	LCAP	SCAP	CRB
BILL	1.00	0.08	0.00	-0.05	-0.07	-0.03	-0.03	-0.02
ITGOV	0.08	1.00	0.81	0.55	-0.27	-0.39	-0.40	-0.19
LTGOV	0.00	0.81	1.00	0.80	-0.12	-0.24	-0.24	-0.15
LTCORP	-0.05	0.55	0.80	1.00	0.30	0.07	0.06	0.04
HYCORP	-0.07	-0.27	-0.12	0.30	1.00	0.64	0.66	0.40
LCAP	-0.03	-0.39	-0.24	0.07	0.64	1.00	0.86	0.43
SCAP	-0.03	-0.40	-0.24	0.06	0.66	0.86	1.00	0.33
CRB	-0.02	-0.19	-0.15	0.04	0.40	0.43	0.33	1.00

FIGURE 10.8: Pearson's pairwise correlation matrix. The data sets start 2000.

10.2 KENDALL'S RANK CORRELOGRAM PLOTS

Kendall's Rank Correlogram Plots since 1925

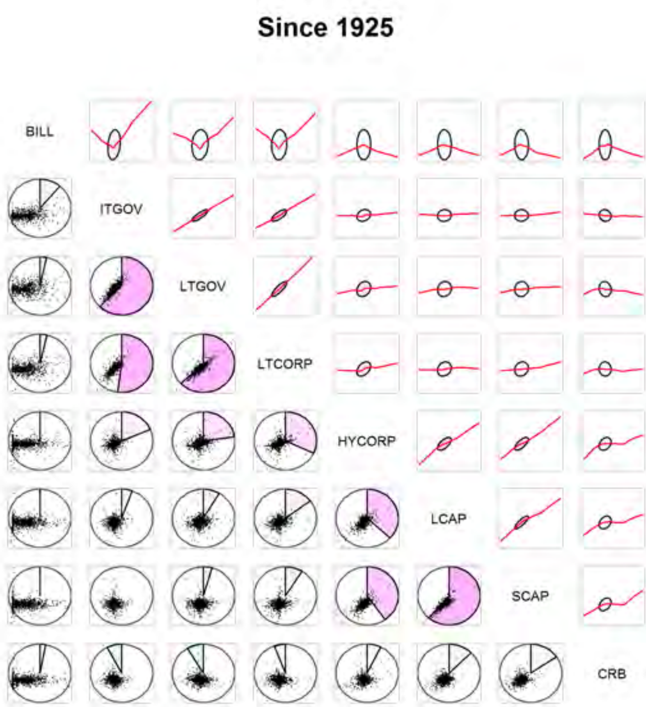

FIGURE 10.9: Kendall's pairwise rank correlations between asset classes. The data sets start 1925.

	BILL	ITGOV	LTGOV	LTCORP	HYCORP	LCAP	SCAP	CRB
BILL	1.00	0.11	0.03	0.04	-0.01	-0.01	-0.02	0.03
ITGOV	0.11	1.00	0.62	0.52	0.18	0.06	0.00	-0.08
LTGOV	0.03	0.62	1.00	0.63	0.23	0.08	0.04	-0.09
LTCORP	0.04	0.52	0.63	1.00	0.32	0.15	0.09	-0.06
HYCORP	-0.01	0.18	0.23	0.32	1.00	0.37	0.40	0.07
LCAP	-0.01	0.06	0.08	0.15	0.37	1.00	0.61	0.13
SCAP	-0.02	0.00	0.04	0.09	0.40	0.61	1.00	0.15
CRB	0.03	-0.08	-0.09	-0.06	0.07	0.13	0.15	1.00

FIGURE 10.10: Kendall's pairwise rank correlation matrix. The data sets start 1925.

Kendall's Rank Correlogram Plots since 1950

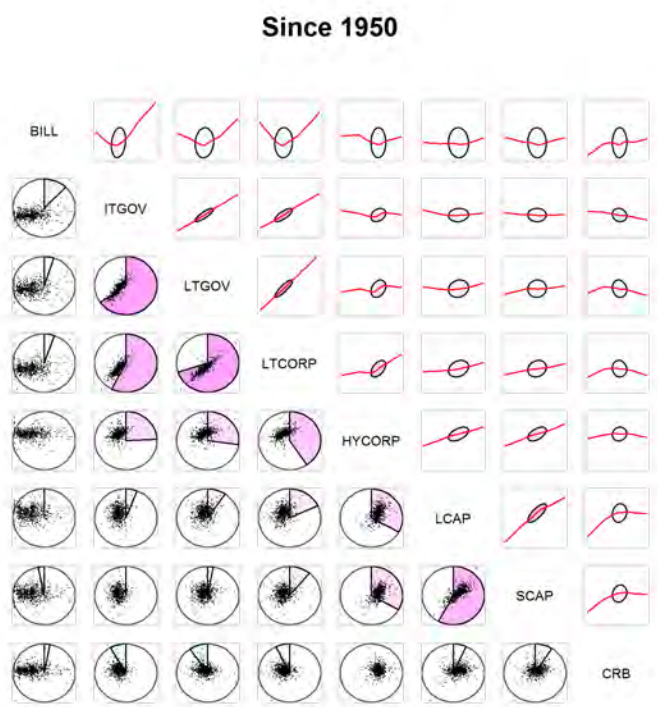

FIGURE 10.11: Kendall's pairwise rank correlations between asset classes. The data sets start 1950.

	BILL	ITGOV	LTGOV	LTCORP	HYCORP	LCAP	SCAP	CRB
BILL	1.00	0.12	0.05	0.05	0.00	-0.02	-0.03	0.03
ITGOV	0.12	1.00	0.65	0.57	0.24	0.06	-0.02	-0.08
LTGOV	0.05	0.65	1.00	0.70	0.27	0.10	0.03	-0.10
LTCORP	0.05	0.57	0.70	1.00	0.41	0.18	0.11	-0.07
HYCORP	0.00	0.24	0.27	0.41	1.00	0.33	0.33	-0.01
LCAP	-0.02	0.06	0.10	0.18	0.33	1.00	0.58	0.07
SCAP	-0.03	-0.02	0.03	0.11	0.33	0.58	1.00	0.09
CRB	0.03	-0.08	-0.10	-0.07	-0.01	0.07	0.09	1.00

FIGURE 10.12: Kendall's pairwise rank correlation matrix. The data sets start 1950.

Kendall's Rank Correlogram Plots since 1975

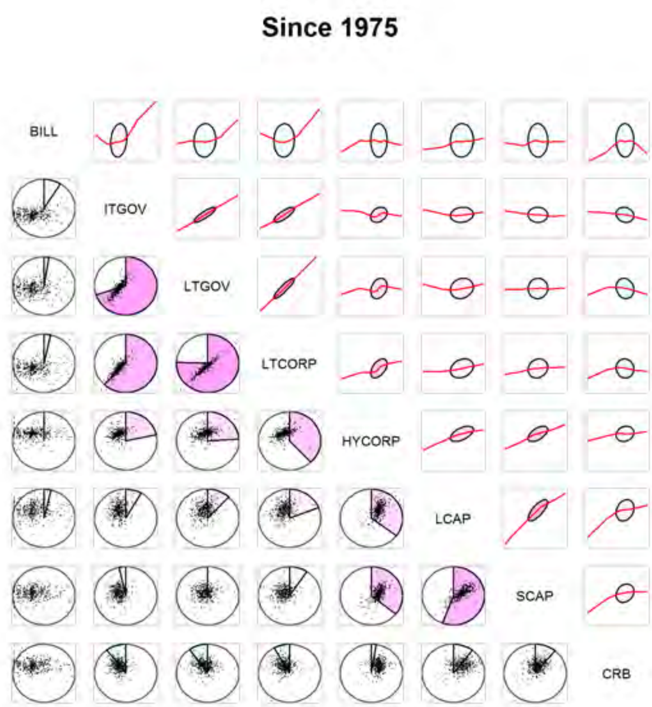

FIGURE 10.13: Kendall's pairwise rank correlations between asset classes. The data sets start 1975.

	BILL	ITGOV	LTGOV	LTCORP	HYCORP	LCAP	SCAP	CRB
BILL	1.00	0.09	0.03	0.04	-0.02	0.04	-0.01	0.00
ITGOV	0.09	1.00	0.70	0.62	0.22	0.08	-0.03	-0.10
LTGOV	0.03	0.70	1.00	0.76	0.24	0.12	0.02	-0.10
LTCORP	0.04	0.62	0.76	1.00	0.38	0.19	0.09	-0.08
HYCORP	-0.02	0.22	0.24	0.38	1.00	0.35	0.36	0.03
LCAP	0.04	0.08	0.12	0.19	0.35	1.00	0.56	0.11
SCAP	-0.01	-0.03	0.02	0.09	0.36	0.56	1.00	0.11
CRB	0.00	-0.10	-0.10	-0.08	0.03	0.11	0.11	1.00

FIGURE 10.14: Kendall's pairwise rank correlation matrix. The data sets start 1975.

Kendall's Rank Correlogram Plots since 2000

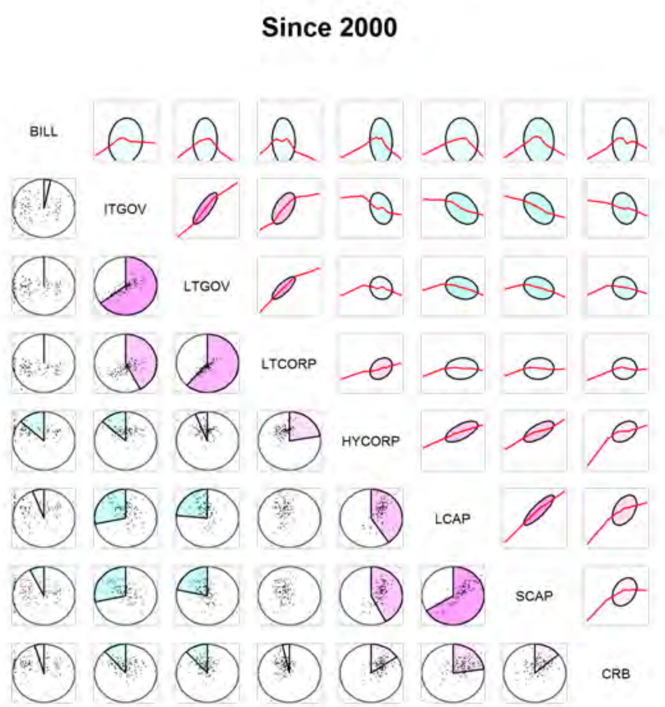

FIGURE 10.15: Kendall's pairwise rank correlations between asset classes. The data sets start 2000.

	BILL	ITGOV	LTGOV	LTCORP	HYCORP	LCAP	SCAP	CRB
BILL	1.00	0.03	0.02	-0.02	-0.13	-0.06	-0.07	-0.05
ITGOV	0.03	1.00	0.65	0.42	-0.14	-0.28	-0.29	-0.11
LTGOV	0.02	0.65	1.00	0.61	-0.06	-0.23	-0.22	-0.12
LTCORP	-0.02	0.42	0.61	1.00	0.22	-0.01	0.00	-0.04
HYCORP	-0.13	-0.14	-0.06	0.22	1.00	0.40	0.42	0.15
LCAP	-0.06	-0.28	-0.23	-0.01	0.40	1.00	0.66	0.23
SCAP	-0.07	-0.29	-0.22	0.00	0.42	0.66	1.00	0.14
CRB	-0.05	-0.11	-0.12	-0.04	0.15	0.23	0.14	1.00

FIGURE 10.16: Kendall's pairwise rank correlation matrix. The data sets start 2000.

10.3 PEARSON'S CORRELATION IMAGE PLOTS

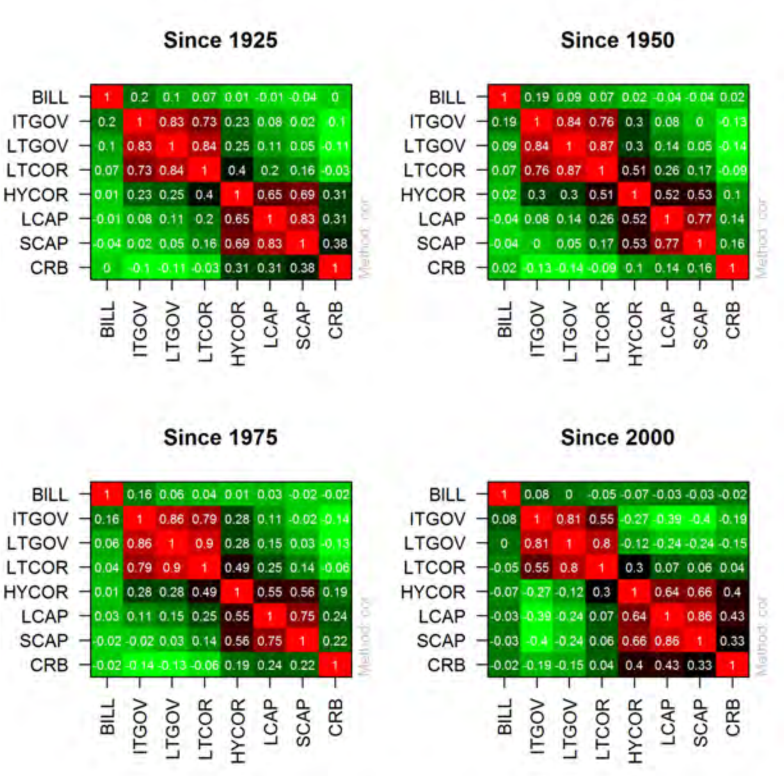

FIGURE 10.17: Pearson's correlation image plot displaying the change in correlations for 4 different horizons starting 1925, 1950, 1975, and 2000, respectively. The numbers on the plaquettes quantify the correlation between two assets ranging from -1 (anti-correlation), 0 (no-correlation) to 1 (full-correlation).

10.4 KENDALL'S RANK CORRELATION IMAGE PLOTS

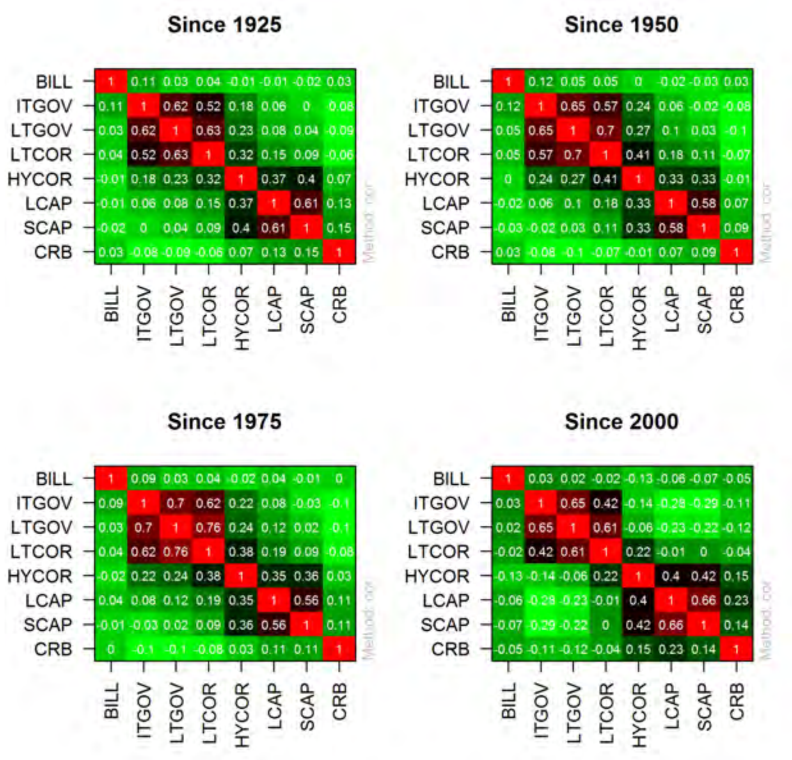

FIGURE 10.18: Kendall's rank correlation image plot displaying the change in correlations for 4 different horizons starting 1925, 1950, 1975, and 2000, respectively. The numbers on the plaquettes quantify the correlation between two assets ranging from -1 (anti-correlation), 0 (no-correlation) to 1 (full-correlation).

Chapter 11

Ordering of Assets

Friendly [2002] addresses the importance of the task of detecting patterns of relations, trends and anomalies through correlations and dependencies in data sets. That also holds in our case where we like to order and rank the assets. Friendly claims that this becomes much easier when "similar" variables are arranged contiguously and ordered in a way that simplifies the pattern of relations among variables. This concept goes back to Friendly and Kwan [2002] where it is called *effect-ordered data display* as an extension to the *main effect ordering* of Cleveland [1993]. This means that our data set of assets, or in a more statistical description, unordered factors or variables, should be always ordered according to what we wish to show.

The principal component (PC) eigenvalue analysis can be used to order or rank assets by analyzing the eigenvalues and eigenvectors of the correlation matrix or a related correlation measure. In the ratio plot derived from the eigenvectors of the two largest eigenvalues similar assets appear close together. This allows for a simple ranking of the assets. First the correlation or another dependence measure is computed. From the resulting matrix the eigenvalues and eigenvectors are computed and the eigenvectors v_1 and v_2 belonging to the two largest eigenvalues are chosen. Then arrows are constructed with start points $(0,0)$ and end points (v_1, v_2) and plotted into a $x-y$ chart.

The analyses within this chapter were done using the monthly nominal total returns. Results are shown using Pearson's and Kendall's rank correlation for different time horizons starting 1925, 1950, 1975 and 2000.

11.1 PEARSON'S CORRELATION

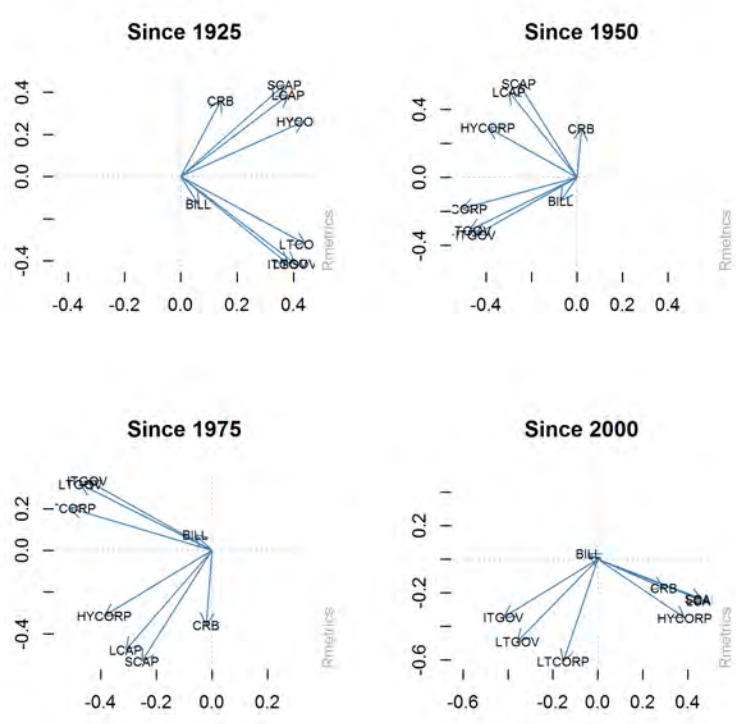

FIGURE 11.1: PCA eigenvector ratio analysis using Pearson's method including the data from 1925, 1950, 1975, and 2000 until now.

11.2 KENDALL'S RANK CORRELATION

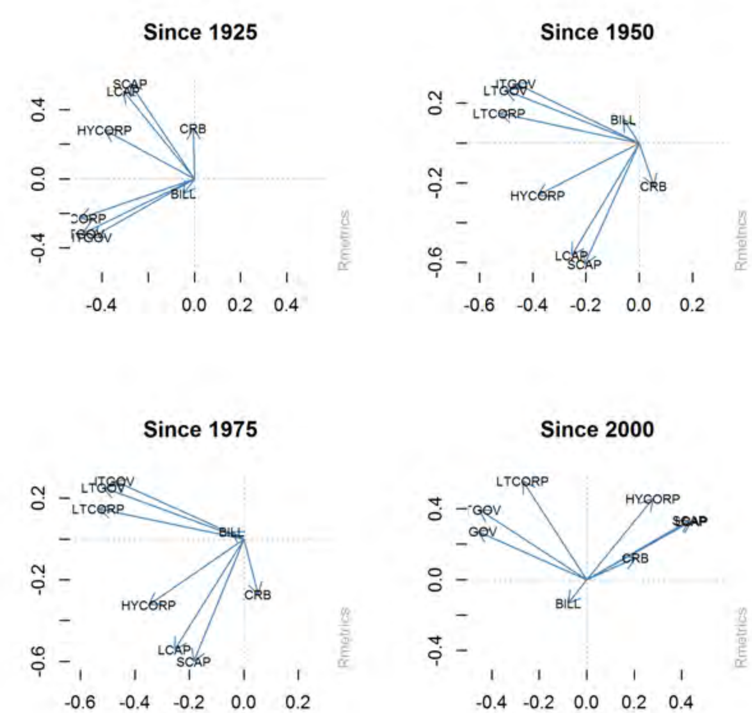

FIGURE 11.2: PCA eigenvector ratio analysis using Kendall's method including the data from 1925, 1950, 1975, and 2000 until now.

CHAPTER 12

GROUPING OF ASSETS

We have already learned how to order and rank similar assets by the Principal Component or "eigen"-analysis of the correlation matrix. Another statistical approach to partition assets in similar groups are clustering approaches.

Hierarchical clustering helps us to build a hierarchy of clusters by merging and splitting groups of similar assets. The merges and splits in groups of assets are determined following a heuristic approach which searches for an optimal choice in the hope to find the best solution. Then the result is usually presented in a dendrogram plot which displays the hierarchy of the assets. We used for the similarity or distance measure the euclidean distance. The "complete" agglomeration method was taken.

k-means clustering helps us to partition *n* observations into *k* clusters. For the similarity or distance measure the nearest mean was used following the algorithm of Hartigan and Wong [1979].

The analyses within this section were done using the monthly nominal total returns. The results for the *Hierarchical clustering* are presented in dendrogram plots which display the hierarchy of the assets for different time horizons starting 1925, 1950, 1975 and 2000. The *k-means clustering* analyses were done using the same time horizons and the results are presented in tables.

12.1 HIERARCHICAL GROUPING OF ASSETS

Dendrogram plots

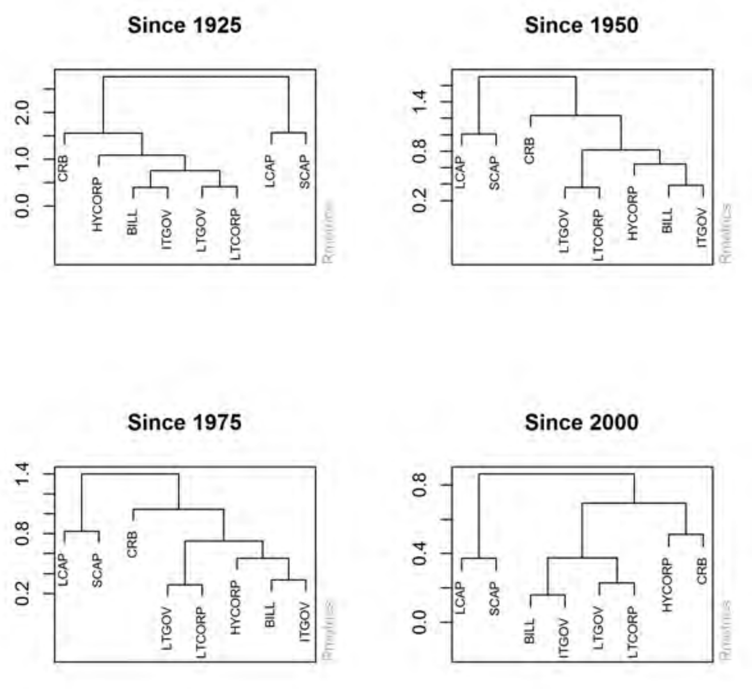

FIGURE 12.1: Dendrogram plot of assets from hierarchical clustering. The following methods were used: Distance Method: "euclidean" and Clustering Method: "complete".

12.2 K-MEANS GROUPING OF ASSETS

k-means grouping tables

	BILL	ITGOV	LTGOV	LTCORP	HYCORP	LCAP	SCAP	CRB
No. of Clusters: 2	2	2	2	2	2	1	1	2
No. of Clusters: 3	1	1	1	1	1	2	2	3
No. of Clusters: 4	2	2	2	2	2	1	3	4
No. of Clusters: 5	1	1	1	1	4	5	3	2
No. of Clusters: 6	4	4	2	2	5	3	1	6

	BILL	ITGOV	LTGOV	LTCORP	HYCORP	LCAP	SCAP	CRB
No. of Clusters: 2	1	1	1	1	1	2	2	1
No. of Clusters: 3	1	1	1	1	1	2	3	1
No. of Clusters: 4	2	3	3	3	2	4	4	1
No. of Clusters: 5	5	4	4	4	5	1	2	3
No. of Clusters: 6	3	3	2	2	6	5	4	1

	BILL	ITGOV	LTGOV	LTCORP	HYCORP	LCAP	SCAP	CRB
No. of Clusters: 2	1	1	1	1	1	2	2	1
No. of Clusters: 3	1	2	2	2	1	3	3	1
No. of Clusters: 4	1	2	2	2	1	4	3	1
No. of Clusters: 5	1	2	2	2	1	4	5	3
No. of Clusters: 6	2	2	4	4	3	6	5	1

	BILL	ITGOV	LTGOV	LTCORP	HYCORP	LCAP	SCAP	CRB
No. of Clusters: 2	2	2	2	2	1	1	1	1
No. of Clusters: 3	1	1	1	1	2	2	2	3
No. of Clusters: 4	2	2	2	2	1	3	3	4
No. of Clusters: 5	5	5	3	3	2	4	4	1
No. of Clusters: 6	3	5	2	2	4	6	6	1

FIGURE 12.2: k-means clustering of assets. The number of groups ranges from two to six.

Part IV

Advanced Analytics

Chapter 13

Introduction

Stability analytics comprehends several concepts and numerical algorithms to better understand the stability of financial and economic time series and portfolios. These new types of measures include the analysis of turning points of financial and economic time series, modeling their volatility, detecting change points and structural breaks in the dynamical process, detecting extremes and outliers and exploring non stationarities by using wavelet multi-resolution techniques. All the analytics we discuss here are retroactive approaches, but they can also be used as leading indicators for changing dynamics. We will illustrate this in many of the displayed graphs.

In chapter 14 we cover turning points and cycles. We will use the turning points to describe the peaks and pits of the assets series. In chapter 15 we provide an estimation of the volatility along the different assets using the GARCH(1,1) model. Change points and structural breaks are studied in chapter 16. In this scope we use a Bayesian estimator to detect structural changes in the time series. An approach to identify extreme values and outliers is the principal component outlier analytics. We apply this approach to the data sets in chapter 17. The last approach used in this Part is the wavelet analytics in chapter 18. It is a new and innovative approach to visualize non stationary behavior in the dynamical process.

Chapter 14

Turning Points and Cycles

In the business cycle analysis of economic indicators, turning points play an important role. They identify the datings and durations of changing economic conditions like expansions and contractions. For financial data we can use similar ideas and concepts to identify increasing (bull) and decreasing (bear) market periods. The method we use is a retrospectrive approach which smoothes the index (or price) series to a given degree. This helps to identify the peaks and/or the pits of the series. From the forecasting point of view, a turning points analysis can help to predict changing market conditions.

Vanhaelen, Dresse and De Mulder [2000] formulate the following general principles for the detection of turning points: (i) The choice of turning points should not be affected by aberrant observations, extremes or outliers, (ii) irregular movements in the series should be excluded and (iii) the application of the method should not delay the identification of the turning points for too long. The authors also suggest additional requirements which may be imposed in accordance with stylized facts of the financial time series. This may concern the succession of peaks and troughs, a minimum length for the phases and cycles and possibly a minimum amplitude for the movements.

In the business cycle literature we find that the most used approach for detecting turning points is the method of G. Bry and C. Boschan [1971]. This approach (i) first involves the detection of extreme and outliers and their replacement by a moving average. (ii) Then an initial set of turning points is identified in a smoothed series, by applying a moving average filter. (iii) The turning points detected at the different stages are checked for the alternation of peaks and troughs, for a minimum span of 15 months between two successive peaks or two successive troughs, and of 5 months between two successive turning points. Finally, (iv) turning points in the first or the last 6 months of the series are rejected, Vanhaelen, Dresse and De Mulder [2000].

Other approaches use filters to smooth the financial time series. Two examples are the HP filter, Hodrick and Prescott [1997] and the CF filter, Christiano and Fitzgerald [1999], which yield smooth cycles, and as a result the Bry-Boschan routine simplifies. Usually the filtered cycles do not require several iteration steps since phase and cycle length-constraints are getting better satisfied by smoothing, Gyomai and Guidetti [2008]. We found out, that one can achieve with spline smoothing the same effects as obtained by filtering. Then we applied the search for peak and trough points.

For all eight asset classes turning point analysis with different smoothing levels were performed. The same was done for blends; which are a weighted sum of two different asset classes. Conservative (25% LCAP and 75% LTGOV), balanced (50% LCAP and 50% LTGOV) and dynamic (75% LCAP and 25% LTGOV) stock/bond blends were considered. The numbers of detected turning points for the different smoothing levels are summarized in a table. For each asset class and blend there is a figure that visualizes the results for the different smoothing levels. The analyses within this section were done using the monthly nominal logarithmic returns.

14.1 TURNING POINTS FREQUENCY

Comparison of Turning Points Frequency

	BILL	ITGOV	LTGOV	LTCORP	HYCORP	LCAP	SCAP	CRB	SBCON	SBBAL	SBDYN
low	2	14	36	32	30	32	43	33	28	34	30
	4	6	24	26	26	26	28	29	16	20	22
	3	0	16	12	18	16	14	25	12	16	16
medium	2	0	8	6	16	12	12	21	6	12	14
	0	0	6	6	8	10	8	12	2	2	10
	0	0	4	2	4	7	7	6	0	2	6
high	2	0	2	0	2	3	4	6	0	2	2

FIGURE 14.1: The table makes a comparison of the number of turning points for bills, bonds, stocks, commodities and the stock/bond blends for different degrees of smoothing levels (low, medium and high). With decreasing smoothing level the number of turning points increases. The number of turning points are significantly different for different asset classes. For bonds we observe less turning points in the time series compared to stocks and the commodity index.

14.2 TURNING POINTS IN ASSETS CLASSES

Turning Points: BILL

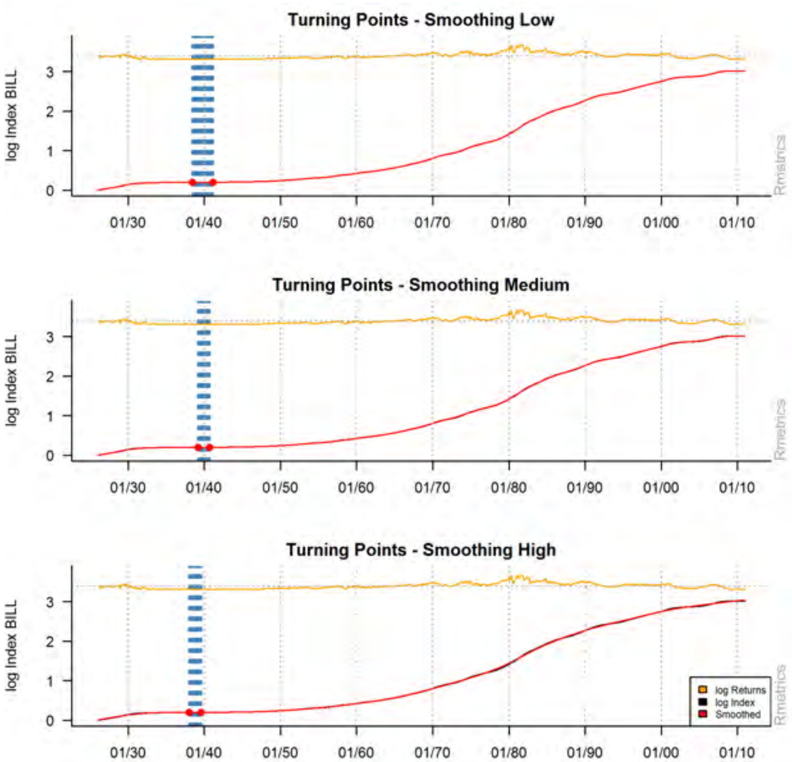

FIGURE 14.2: Turning points of treasury bills. The smoothing factors from up to down are 0.3, 0.5, and 0.7

Turning Points: ITGOV

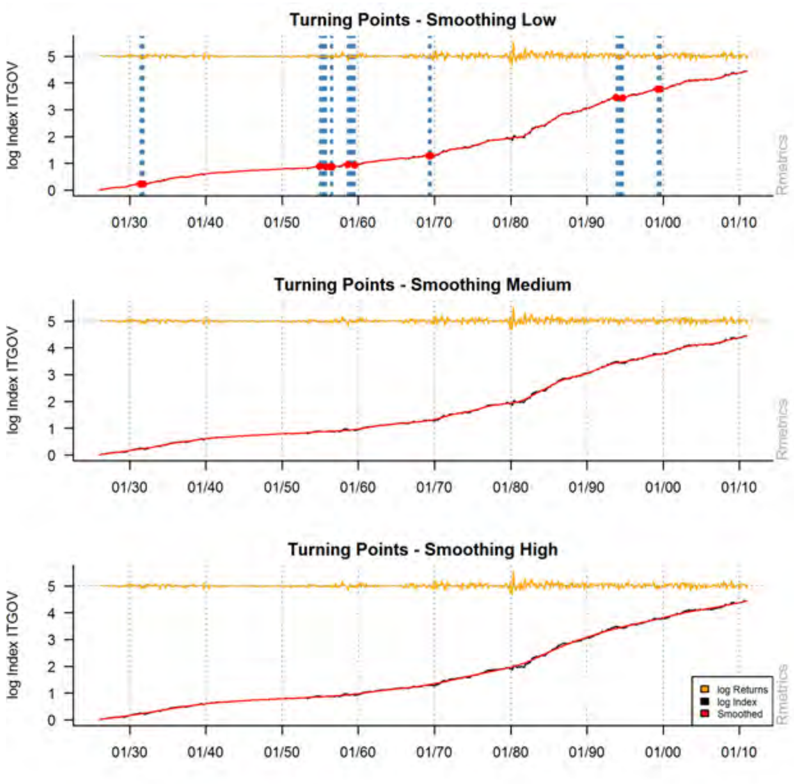

FIGURE 14.3: Turning points of intermediate term government bonds. The smoothing factors from up to down are 0.3, 0.5, and 0.7

Turning Points: LTGOV

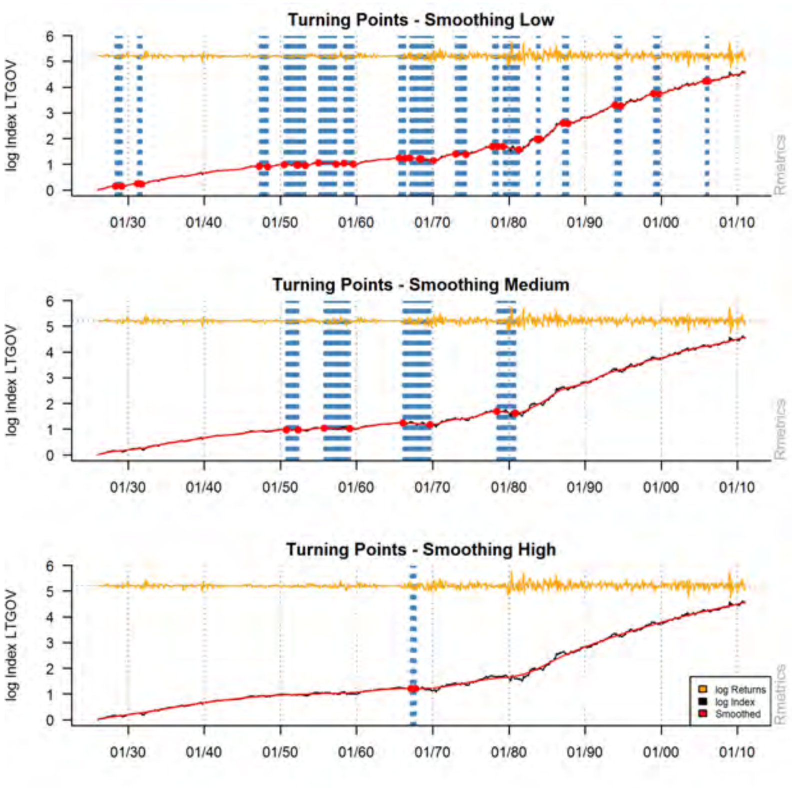

FIGURE 14.4: Turning points of long term government bonds. The smoothing factors from up to down are 0.3, 0.5, and 0.7

Turning Points: LTCORP

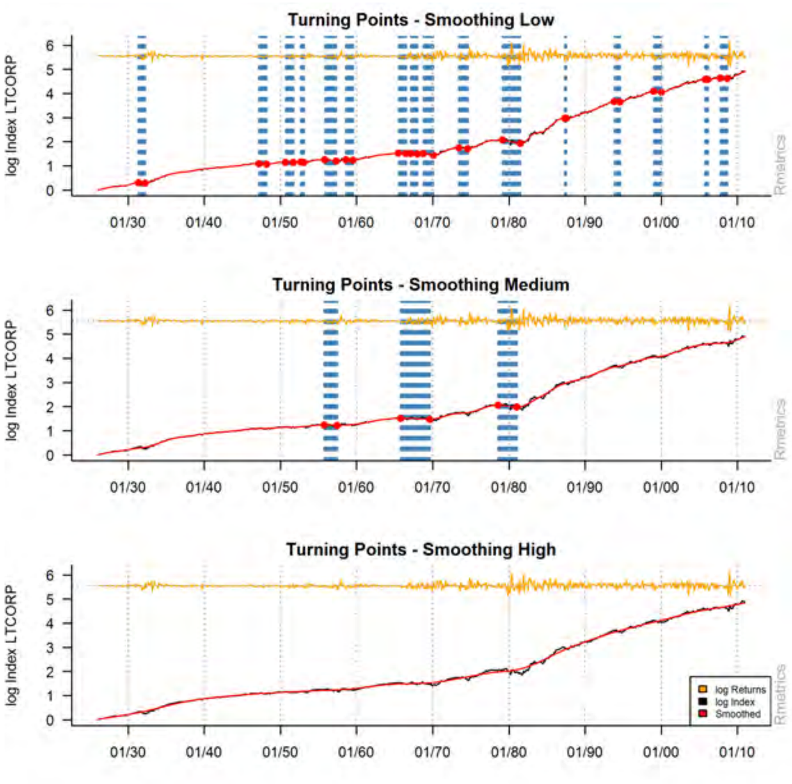

FIGURE 14.5: Turning points of long term corporate bonds. The smoothing factors from up to down are 0.3, 0.5, and 0.7

Turning Points: HYCORP

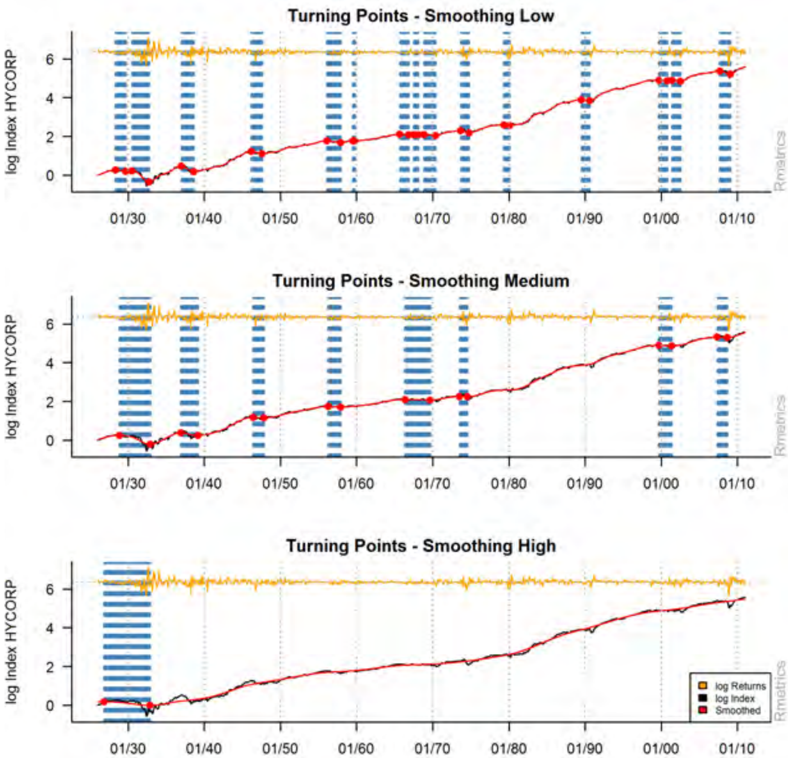

FIGURE 14.6: Turning points of high yield corporate bonds. The smoothing factors from up to down are 0.3, 0.5, and 0.7

Turning Points: LCAP

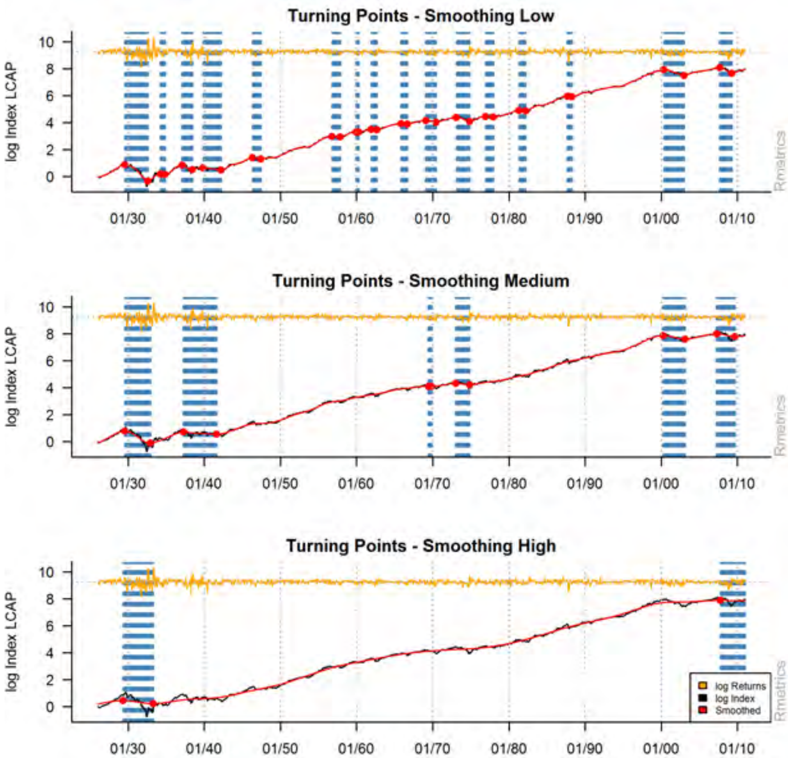

FIGURE 14.7: Turning points of large cap stocks. The smoothing factors from up to down are 0.3, 0.5, and 0.7

Turning Points: SCAP

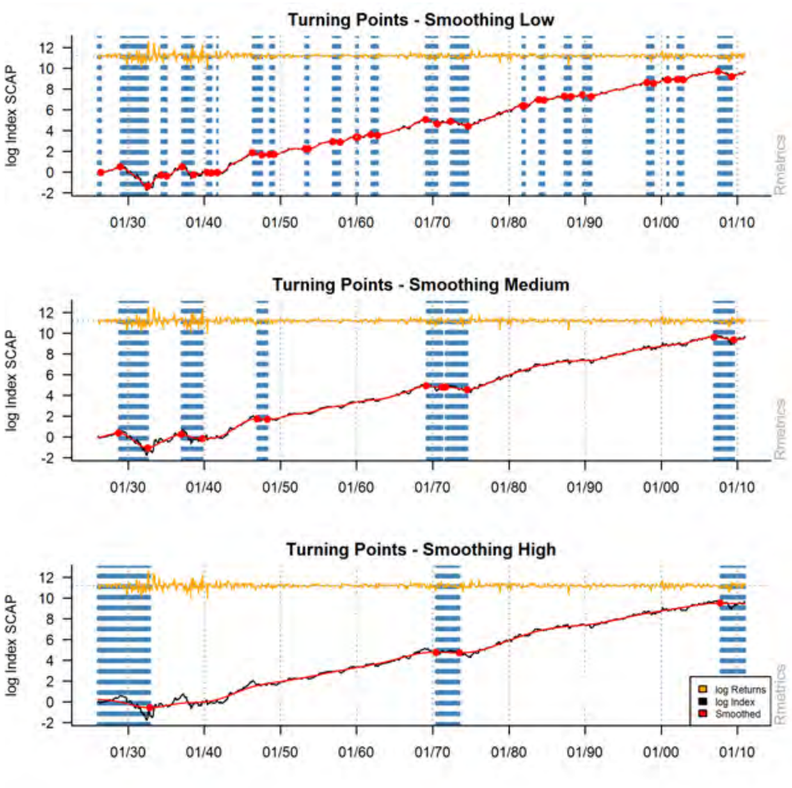

FIGURE 14.8: Turning points of small cap stocks. The smoothing factors from up to down are 0.3, 0.5, and 0.7

Turning Points: CRB

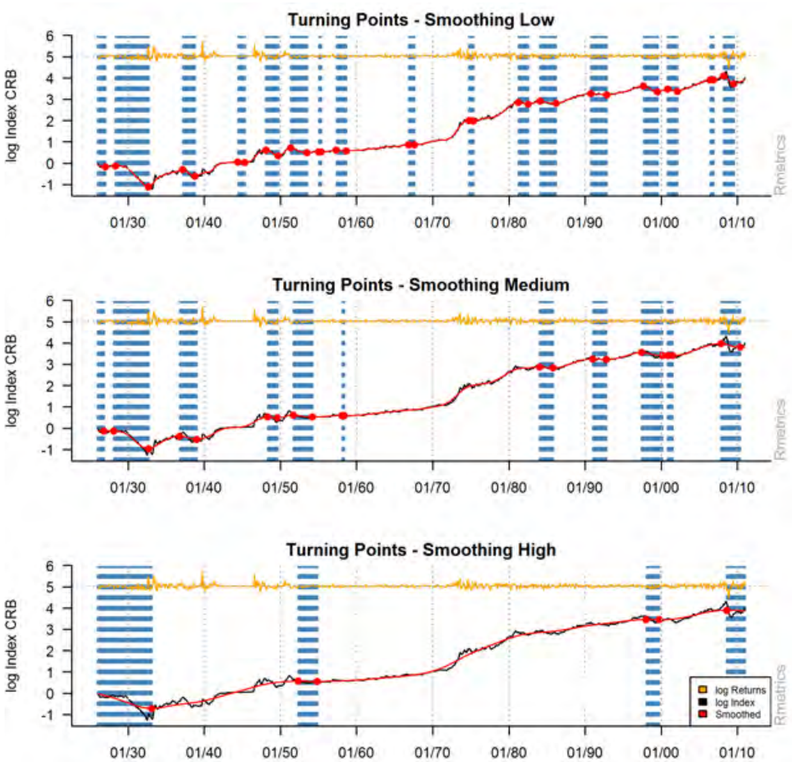

FIGURE 14.9: Turning points of CRB commodity index. The smoothing factors from up to down are 0.3, 0.5, and 0.7

14.3 TURNING POINTS IN STOCK/BOND BLENDS

Turning Points: Conservative Stock/Bond Blend

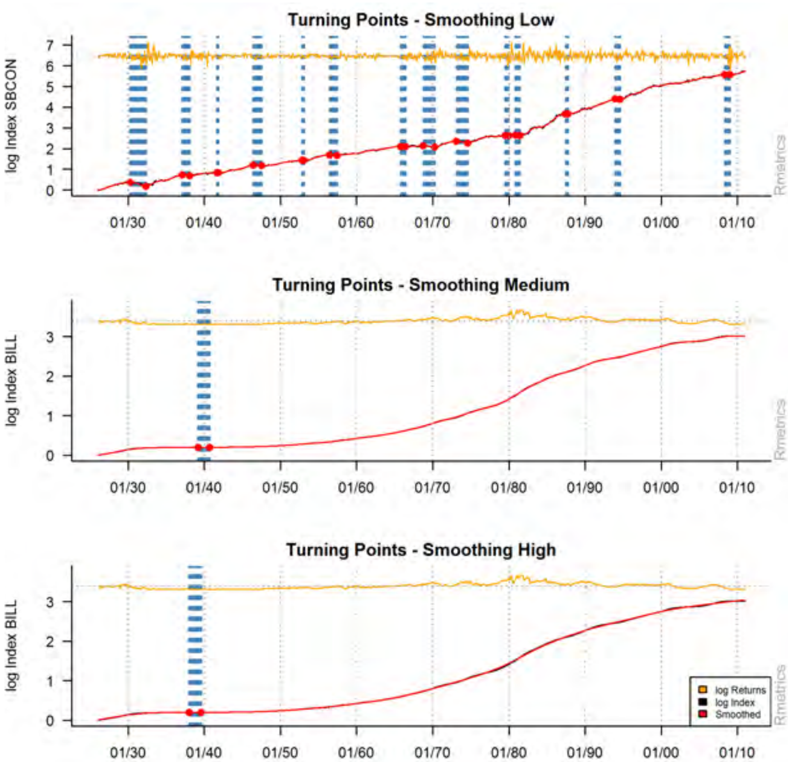

FIGURE 14.10: Turning points of a conservative stock/bond blend. The smoothing factors from up to down are 0.3, 0.5, and 0.7

Turning Points: Balanced Stock/Bond Blend

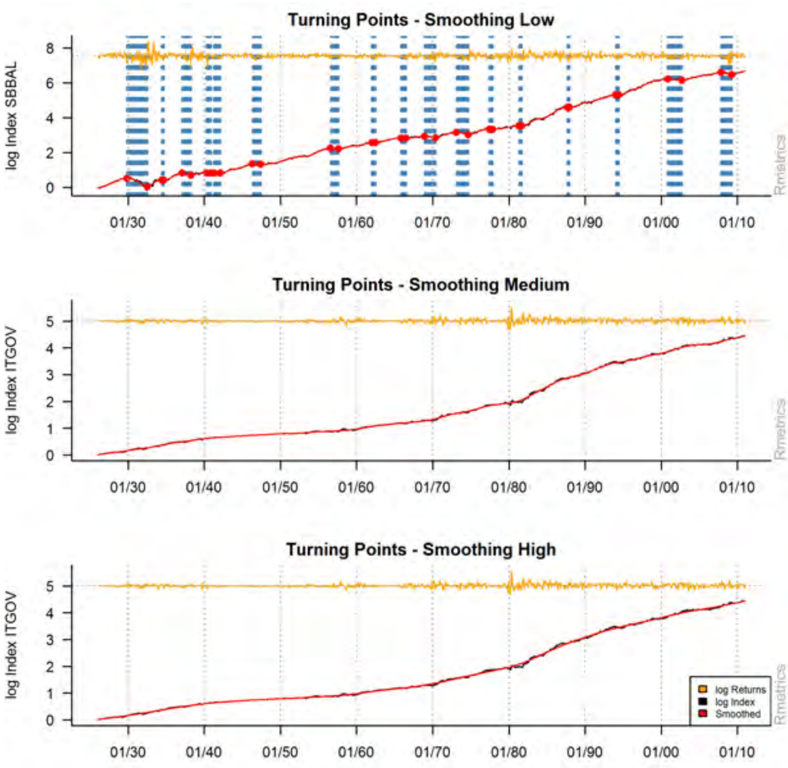

FIGURE 14.11: Turning points of a balanced stock/bond blend. The smoothing factors from up to down are 0.3, 0.5, and 0.7

Turning Points: Dynamic Stock/Bond Blend

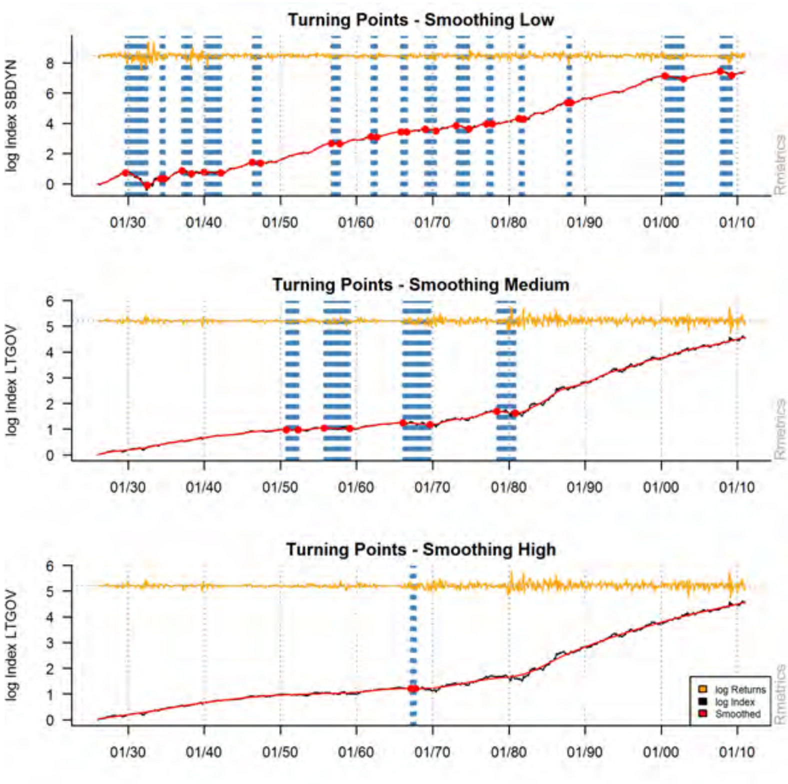

FIGURE 14.12: Turning points of a dynamic stock/bond blend. The smoothing factors from up to down are 0.3, 0.5, and 0.7

Chapter 15

GARCH Volatility Modelling

The estimation of the volatility as a measure for the variability in assets returns is of central importance for risk management and portfolio construction. For the estimation of historical volatility expressed by the variance or standard deviation many estimators have been developed during the last 25 years. This includes the work of Parkinson [1980], Garman and Klass [1980], Rogers and Satchell [1991], Alizahdeh, Brandt and Diebold [2001], and Yang and Zhang [2002] amongst others. We will use the GARCH(1,1) estimator to model the volatility.

The GARCH process, introduced by Bollerslev [1986] models *generalized autoregressive conditional heteroskedasticity*, which means that the volatility is treated as a time-dependent, persistent process. GARCH(1,1) processes follow the formula:

$$\sigma_t^2 = \omega + \alpha(r_{t-1} - \mu)^2 + \beta\sigma_{t-1}^2$$

Here μ, ω, α and β are the estimated model parameters and the persistence is defined as $\alpha + \beta$. The model accounts for volatility clustering, a phenomenon frequently observed in asset returns.

In the remaining of this chapter we will estimate the volatility using the GARCH(1,1) model where we assume normally distributed innovations. For a recent overview on modeling univariate GARCH processes we refer to Zivot [2009] and the references given therein.
For each asset class, as well as the conservative, balanced and dynamic stock/bond blends, a figure shows the 2-σ bands, estimated by using the model parameters from the GARCH(1,1) process. Those parameters are given in form of a list. The analyses within this section were done using the monthly nominal logarithmic returns.

15.1 GARCH MODELLING OF ASSET CLASSES

GARCH(1,1) Modeling: BILL

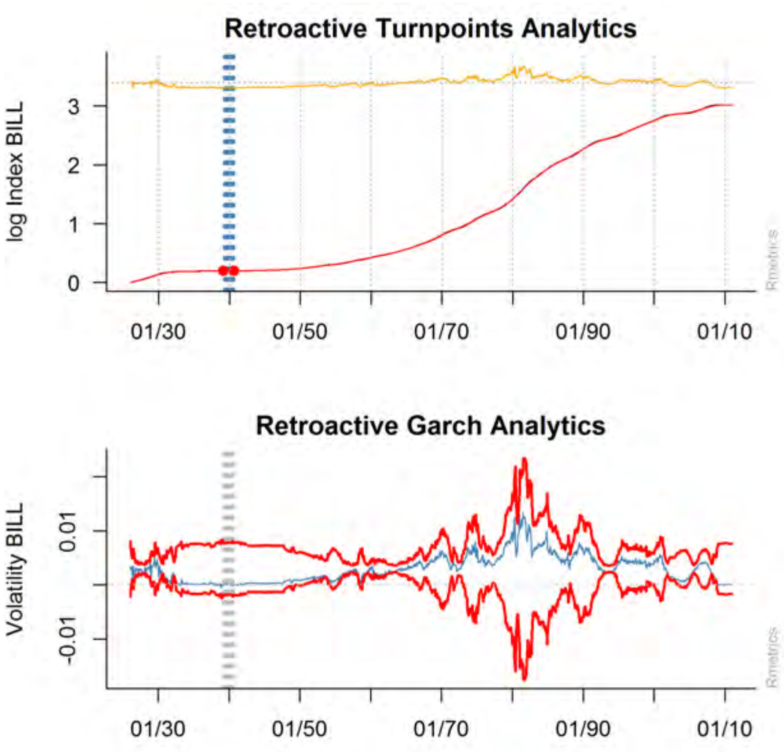

FIGURE 15.1: GARCH(1,1) volatility modeling of treasury bills.

```
      mu   omega alpha1  beta1
  0.2391 0.0005 0.5724 0.4350
```

FIGURE 15.2: GARCH(1,1) model parameters estimated using percentile returns, i.e. $100 \cdot r_t$.

GARCH(1,1) Modeling: ITGOV

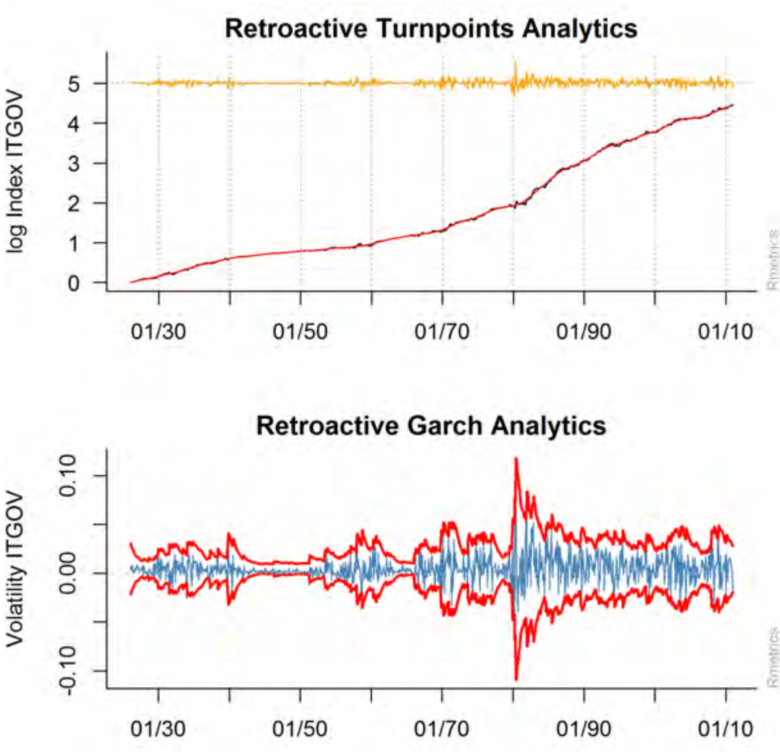

FIGURE 15.3: GARCH(1,1) volatility modeling of intermediate term government bonds.

```
    mu  omega alpha1  beta1
0.2088 0.0075 0.1851 0.8400
```

FIGURE 15.4: GARCH(1,1) model parameters estimated using percentile returns, i.e. $100 \cdot r_t$.

GARCH(1,1) Modeling: LTGOV

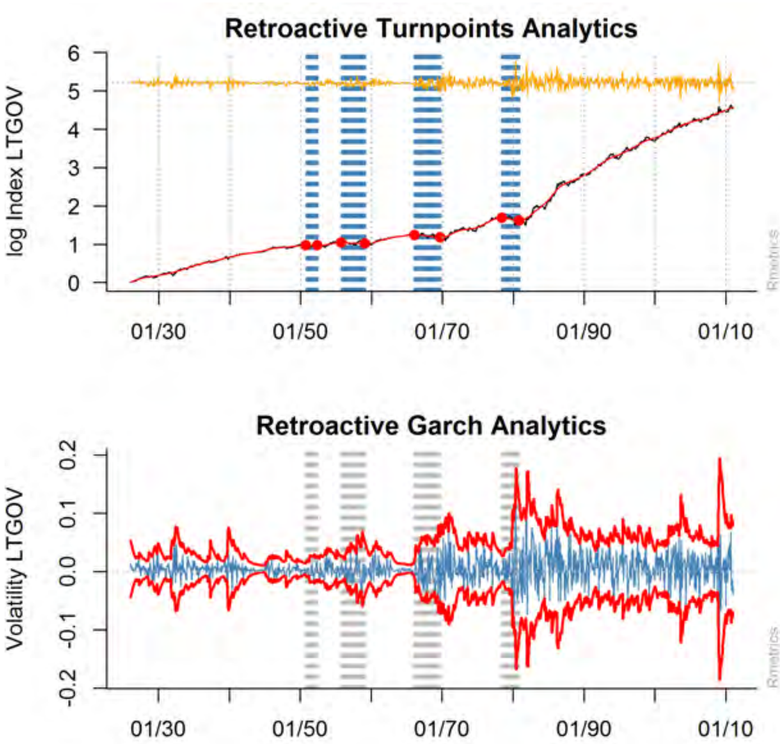

FIGURE 15.5: GARCH(1,1) volatility modeling of long term government bonds.

```
    mu  omega alpha1  beta1
0.2517 0.0153 0.2730 0.7843
```

FIGURE 15.6: GARCH(1,1) model parameters estimated using percentile returns, i.e. $100 \cdot r_t$.

GARCH(1,1) Modeling: LTCORP

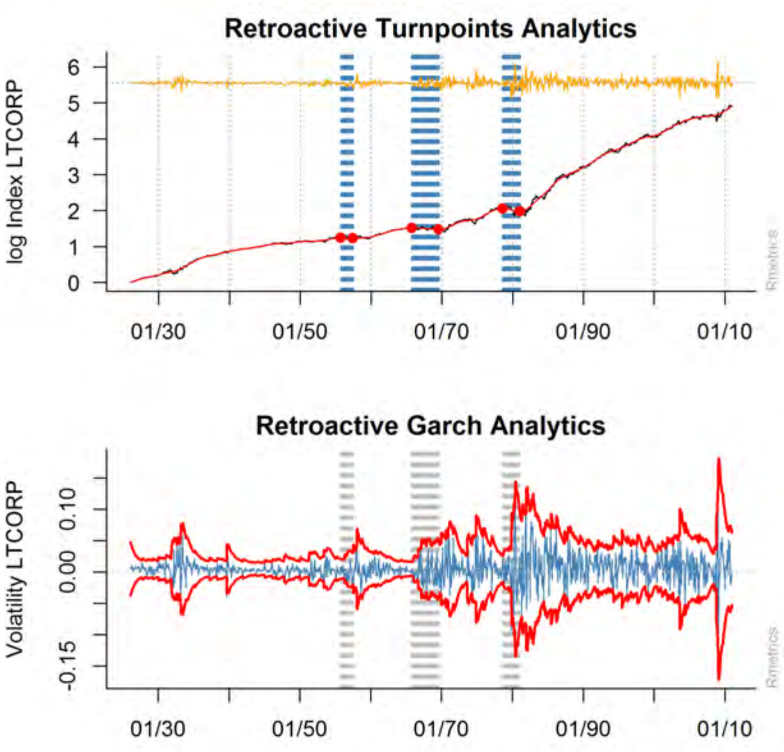

FIGURE 15.7: GARCH(1,1) volatility modeling of long term corporate bonds.

```
    mu  omega alpha1  beta1
0.3029 0.0345 0.1882 0.8326
```

FIGURE 15.8: GARCH(1,1) model parameters estimated using percentile returns, i.e. $100 \cdot r_t$.

GARCH(1,1) Modeling: HYCORP

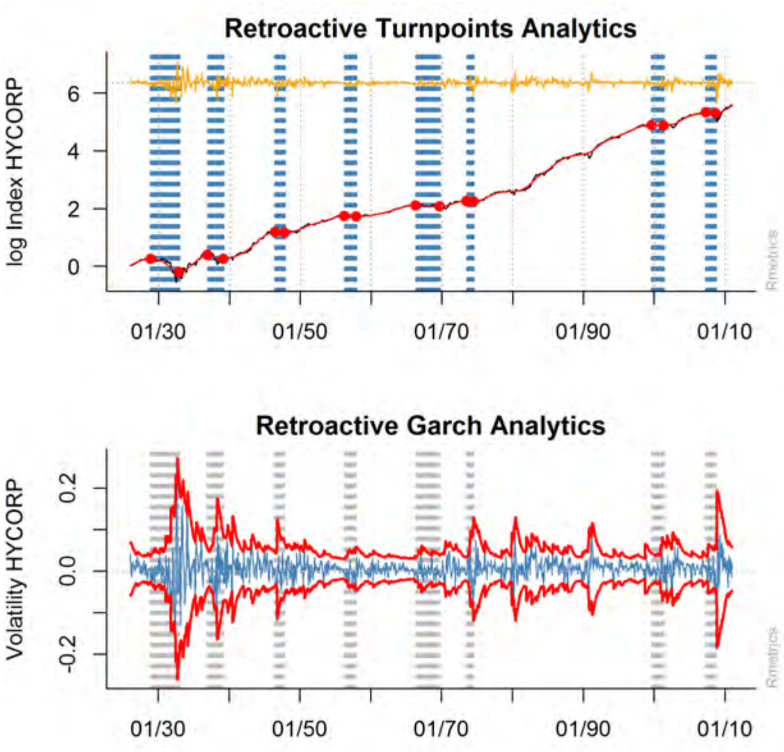

FIGURE 15.9: GARCH(1,1) volatility modeling of high yield corporate bonds.

```
    mu  omega alpha1  beta1
0.5683 0.2547 0.1958 0.7941
```

FIGURE 15.10: GARCH(1,1) model parameters estimated using percentile returns, i.e. $100 \cdot r_t$.

GARCH(1,1) Modeling: LCAP

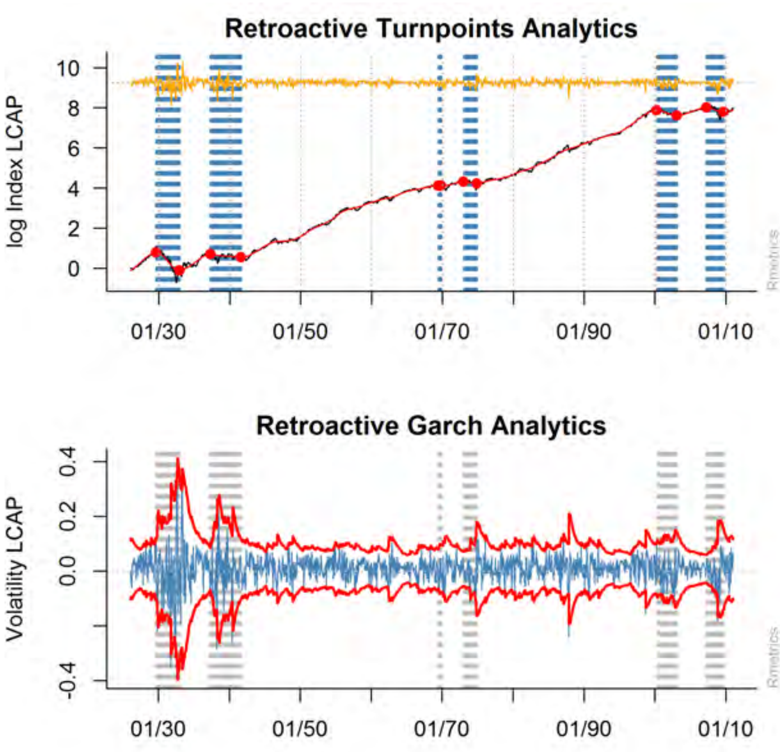

FIGURE 15.11: GARCH(1,1)volatility modeling of large cap stocks.

```
    mu  omega alpha1  beta1
0.9433 0.5955 0.1324 0.8545
```

FIGURE 15.12: GARCH(1,1) model parameters estimated using percentile returns, i.e. $100 \cdot r_t$.

GARCH(1,1) Modeling: SCAP

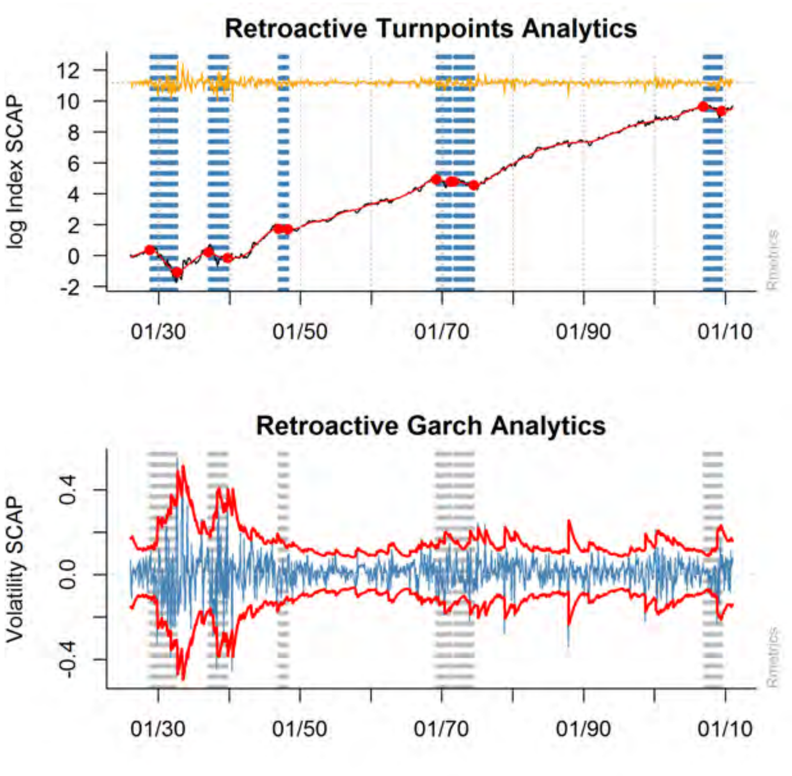

FIGURE 15.13: GARCH(1,1) volatility modeling of small cap stocks.

```
    mu  omega alpha1  beta1
1.0279 1.1122 0.1044 0.8820
```

FIGURE 15.14: GARCH(1,1) model parameters estimated using percentile returns, i.e. $100 \cdot r_t$.

GARCH(1,1) Modeling: CRB

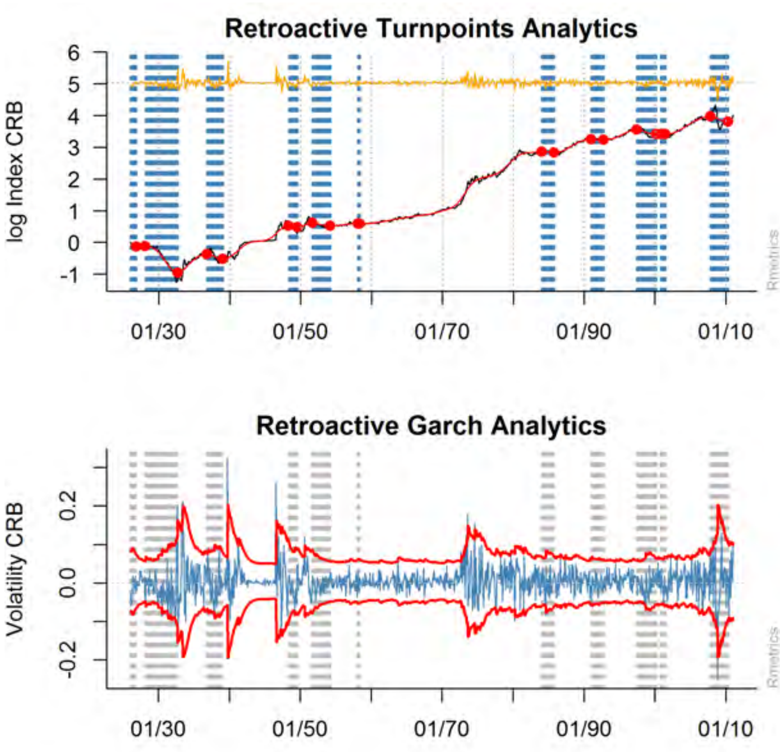

FIGURE 15.15: GARCH(1,1) volatility modeling of commodity index.

```
    mu  omega alpha1  beta1
0.4466 0.6622 0.0879 0.8682
```

FIGURE 15.16: GARCH(1,1) model parameters estimated using percentile returns, i.e. $100 \cdot r_t$.

15.2 GARCH MODELLING OF STOCK/BOND BLENDS

GARCH(1,1): Conservative Stock/Bond Blend

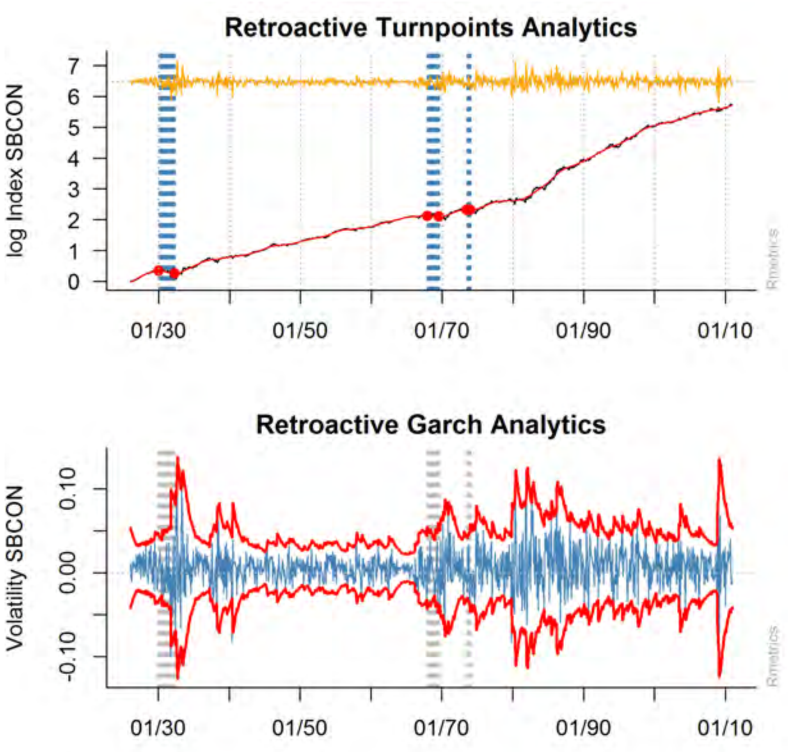

FIGURE 15.17: GARCH(1,1) volatility modeling of a conservative stock/bond blend.

```
      mu   omega alpha1  beta1
  0.5036 0.0695 0.1568 0.8480
```

FIGURE 15.18: GARCH(1,1) model parameters estimated using percentile returns, i.e. $100 \cdot r_t$.

GARCH(1,1): Balanced Stock/Bond Blend

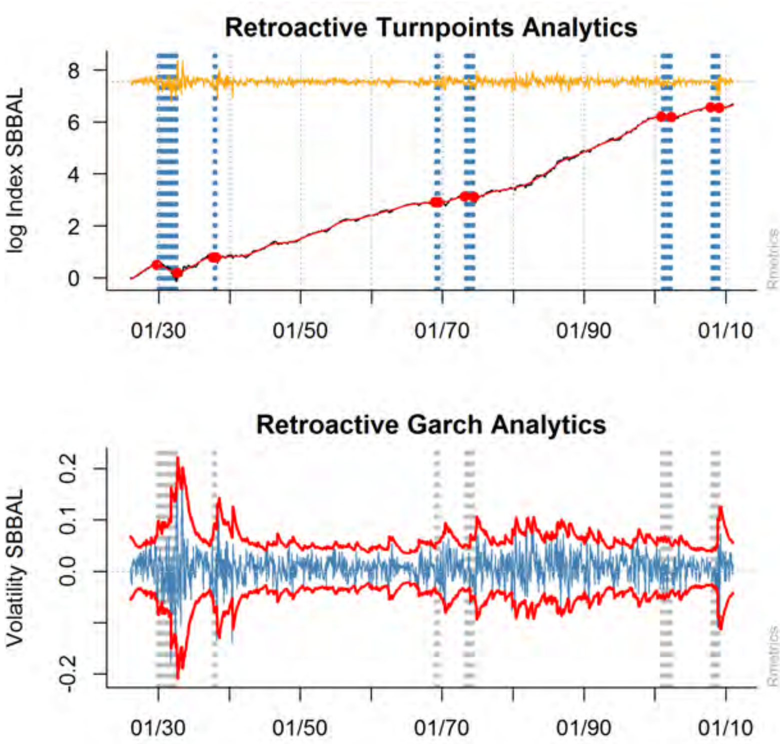

FIGURE 15.19: GARCH(1,1) volatility modeling of a balanced stock/bond blend.

```
    mu  omega alpha1  beta1
0.6891 0.2059 0.1332 0.8519
```

FIGURE 15.20: GARCH(1,1) model parameters estimated using percentile returns, i.e. $100 \cdot r_t$.

GARCH(1,1): Dynamic Stock/Bond Blend

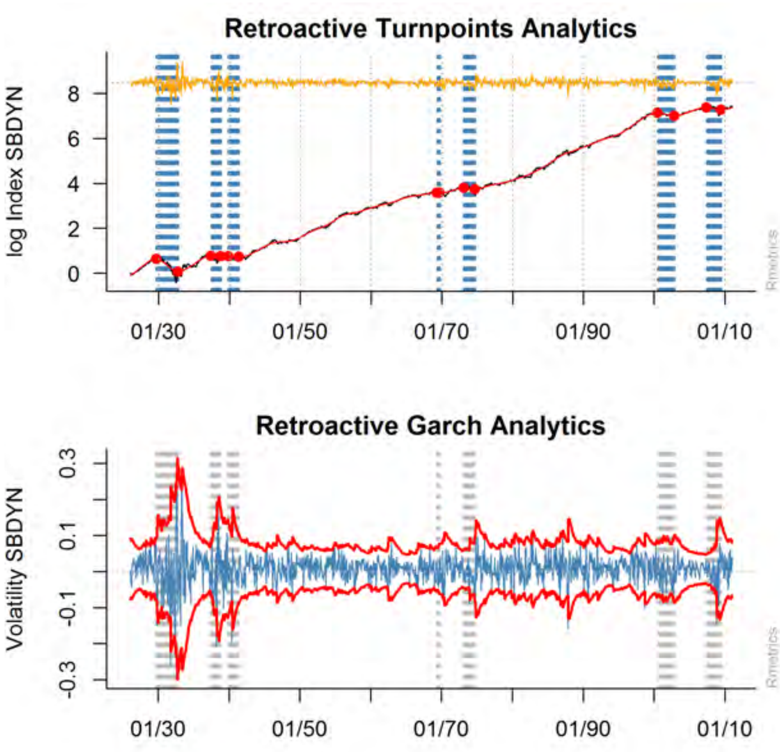

FIGURE 15.21: GARCH(1,1) volatility modeling of a dynamic stock/bond blend.

```
    mu  omega alpha1  beta1
0.8406 0.3564 0.1287 0.8566
```

FIGURE 15.22: GARCH(1,1) model parameters estimated using percentile returns, i.e. $100 \cdot r_t$.

CHAPTER 16

STRUCTURAL BREAKS

The aim of change point analysis is to partition the series into segments that are connected and ordered with increasing time, e.g. {1,2,3} {4,5,6}, {7,8}. This is done by clustering the elements of the series, resulting in a set partition where within each segment the elements have a common parameter. In the detection of change points this parameter is usually the mean. The cuts between the segments are known as change or break points. There are several models for segmentation and partitioning by piecewise variance, correlation or distribution parameters.

Frequentist change point detection methods rely on hypothesis testing. The major frequentist tests are summarized by Zeileis et al. [2007]. The procedures they discuss are concerned with testing or assessing deviations from stability in the classical linear regression model. In these methods the breakpoints are located by minimizing the residual sum of squares of a piecewise linear regression model. The ideas behind the implementation can be found in Zeileis [2003].

Another approach is "Circular Binary Segmentation", see Sen [1975] and Olshen [2004]. Circuar binary segmentaion methods use likelihood ratio statistics to test the null hypothesis of no change point. If the null hypothesis is rejected the series is split and the test is repeated on the subsegments till no additional changes are detected. For further discussions we refer to Olshen [2007].

Here weuse a "Bayesian Change Point Detection" method to identify change points in the mean and variance of the series. The "Bayesian Product Partition Model" was introduced by Barry and Hartigan [1992] allows us to determine the probability of there being a change point at each location in the series. This becomes very advantageous in the stability analysis context as information about which time periods have frequently high change point probabilities and are therefore less stable than when the probability of a change is low. The Bayesian product partition model may also provide estimates of the parameters at each moment in time as well as the posterior distributions of the partitions and the number of change points. This was later extended by Loschi [2005] to give the posterior distribution of the probability that each instant in time is a change point. While frequentist procedures for change point analysis estimate specific locations of change points, the Bayesian procedure offers the probability of a change point at each location in the series. In a retroactive stability study this probability information may be more important than the location of historical changes which are often already known and are easily located. For further discussions, a "Monte Carlo Markov Chain" approximation, and applications we refer to the work of Erdman et al. [2008].

For each asset class, as well as the conservative, balanced and dynamic stock/bond blends, a figure will show the posterior probability for observing a change point. The rainbow colored time series are smoothed curves of the original probability series. The black curve is the difference between the strongest smoothed curve and the weakest smoothed curve which serves as a measure for stability. The analyses within this section were done using the monthly nominal logarithmic returns.

16.1 CHANGE POINTS IN ASSETS CLASSES

Bayesian Change Point Analysis: BILL

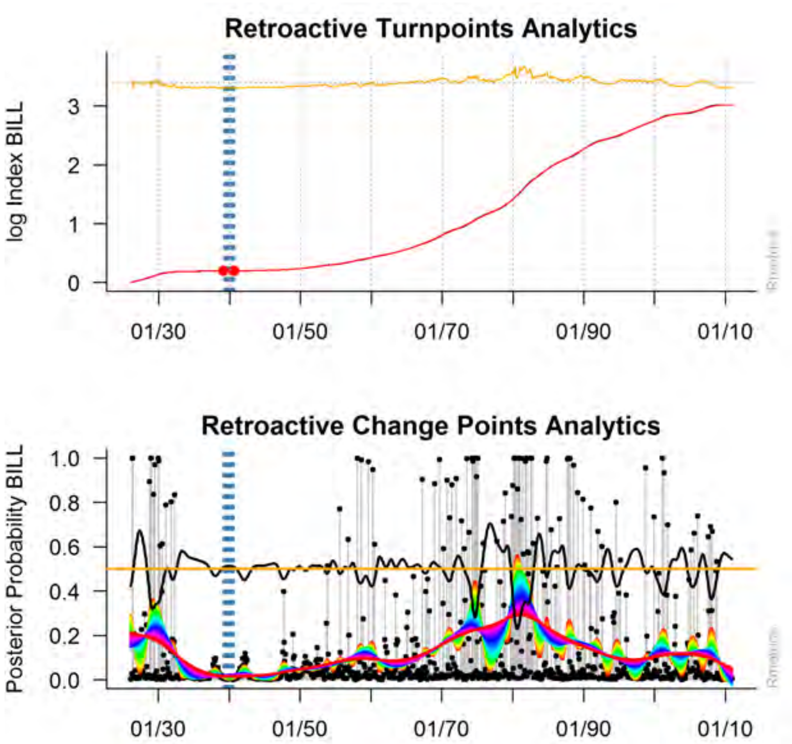

FIGURE 16.1: Bayesian Change Point Analytics: Treasury Bills. The bottom figure shows the posterior probability for each date of having a change point within the time series. The rainbow colored time series are smoothed curves of the original probability series. The black curve is the difference between the strongest smoothed curve and the weakest smoothed curve which serves as a measure for stability.

Bayesian Change Point Analysis: ITGOV

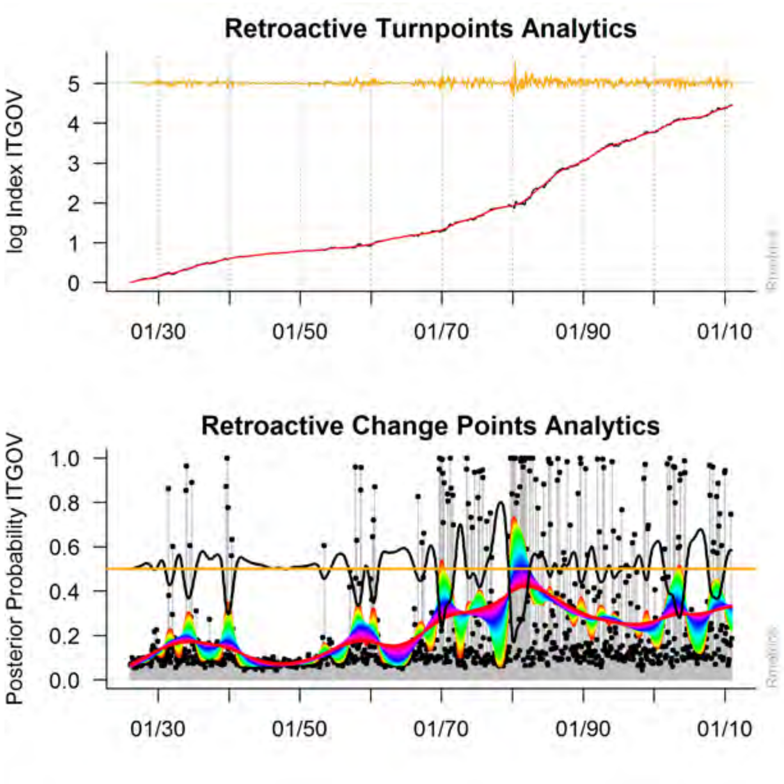

FIGURE 16.2: Bayesian Changepoint Analytics: Intermediate Term Government Bonds

Bayesian Change Point Analysis: LTGOV

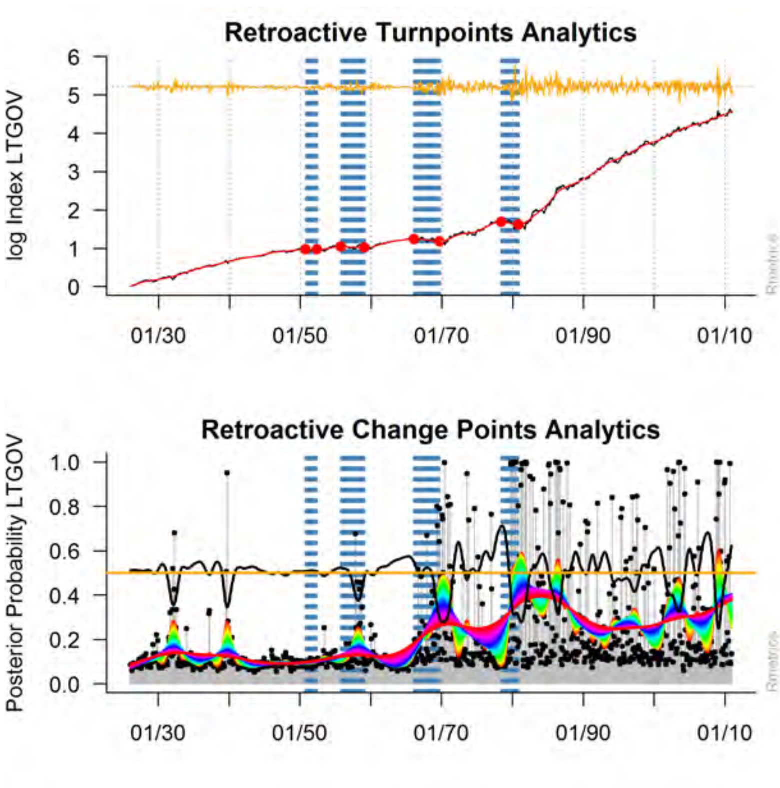

FIGURE 16.3: Bayesian Changepoint Analytics: Long Term Government Bonds

Bayesian Change Point Analysis: LTCORP

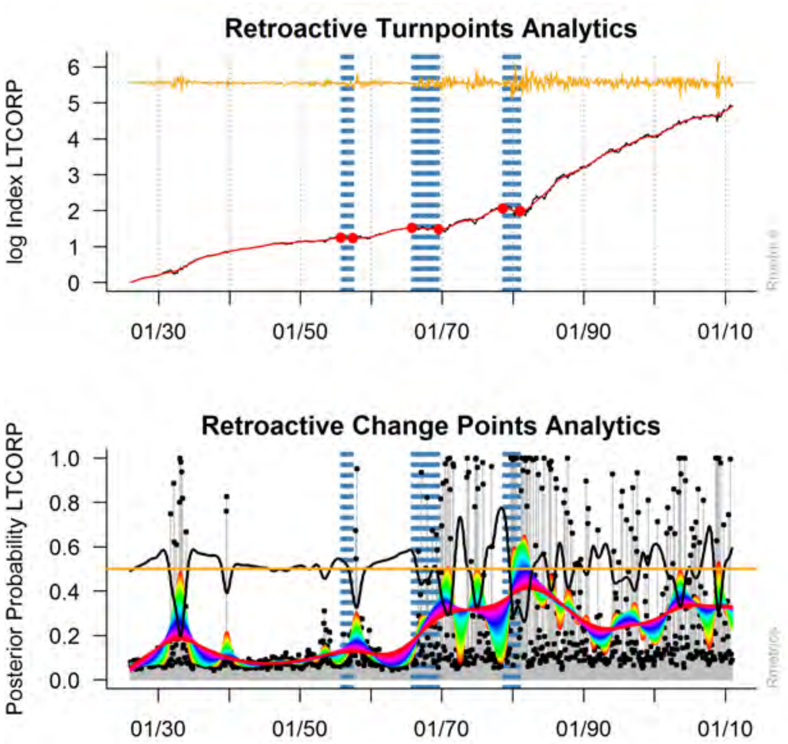

FIGURE 16.4: Bayesian Changepoint Analytics: Long Term Corporate Bonds

Bayesian Change Point Analysis: HYCORP

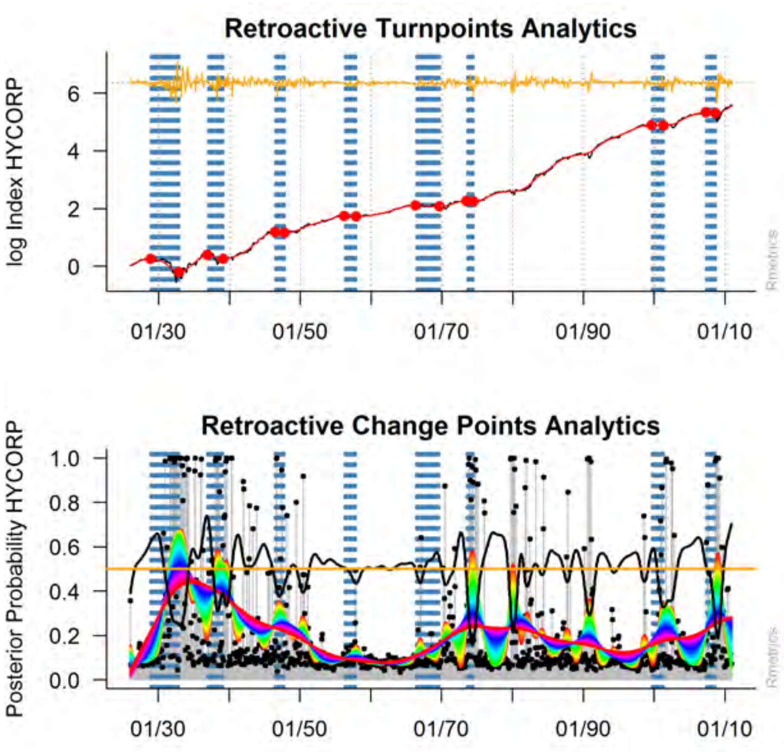

FIGURE 16.5: Bayesian Changepoint Analytics: Domestic High Yield Corporate Bonds

Bayesian Change Point Analysis: LCAP

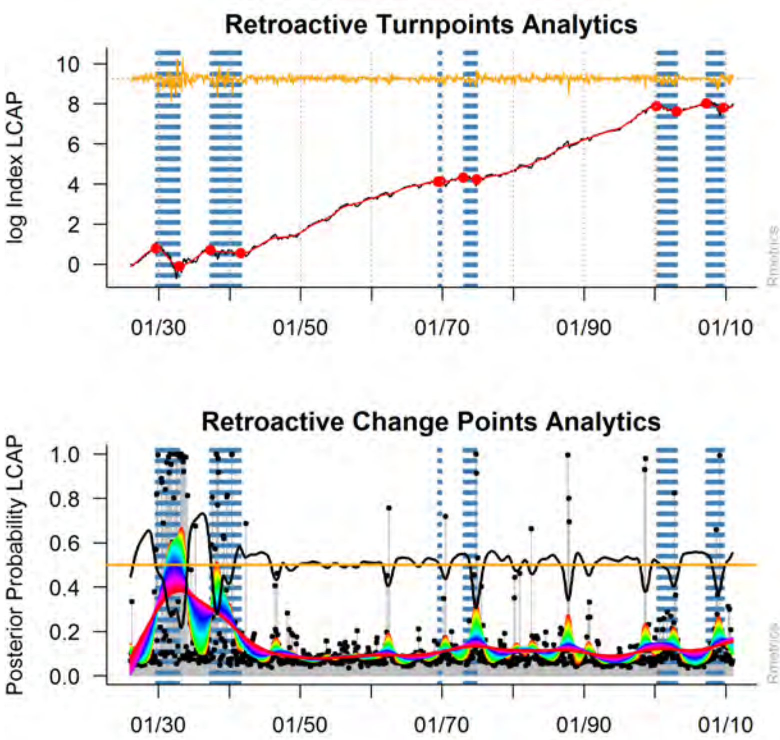

FIGURE 16.6: Bayesian Changepoint Analytics: Large Cap Stocks

Bayesian Change Point Analysis: SCAP

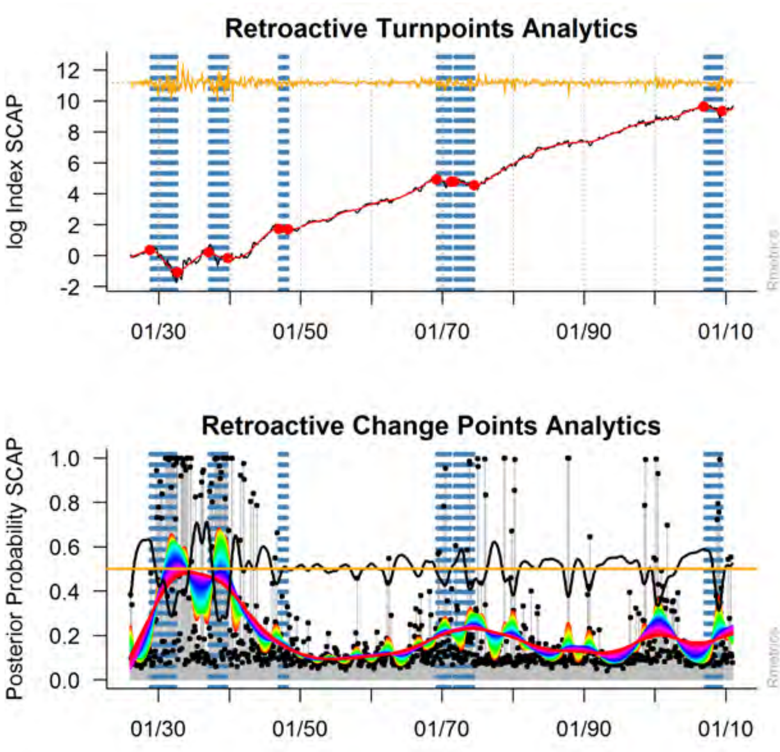

FIGURE 16.7: Bayesian Changepoint Analytics: Small Cap Stocks

Bayesian Change Point Analysis: CRB

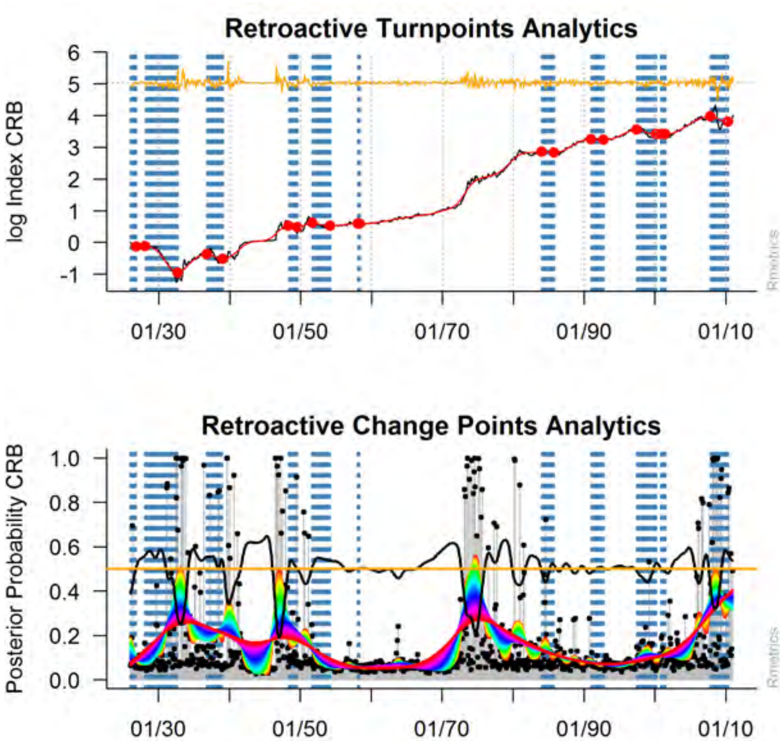

FIGURE 16.8: Bayesian Changepoint Analytics: CRB Commodity Index

16.2 Change Points in Stock/Bond Blends

Change Points: Conservative Stock/Bond Blend

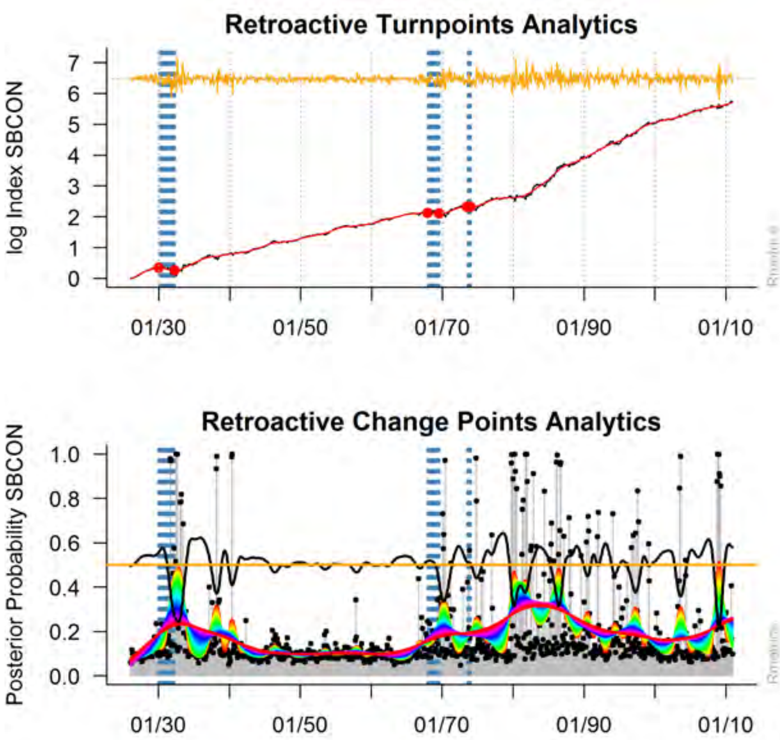

FIGURE 16.9: Bayesian Changepoint Analytics: Dynamic Stock/Bond Blend

Change Points: Balanced Stock/Blond Blend

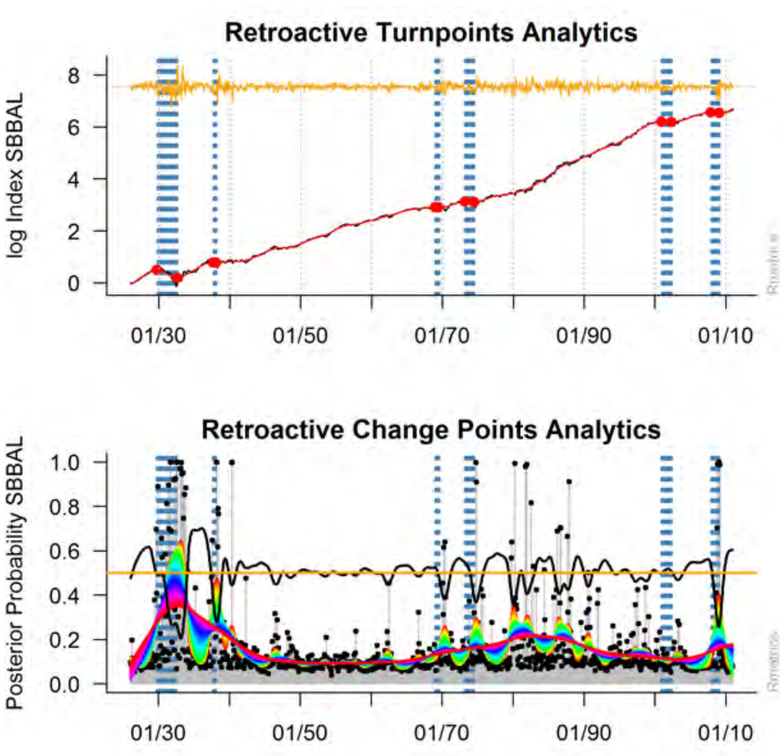

FIGURE 16.10: Bayesian Changepoint Analytics: Balanced Stock/Bond Blend

Change Points: Dynamic Stock/Blond Blend

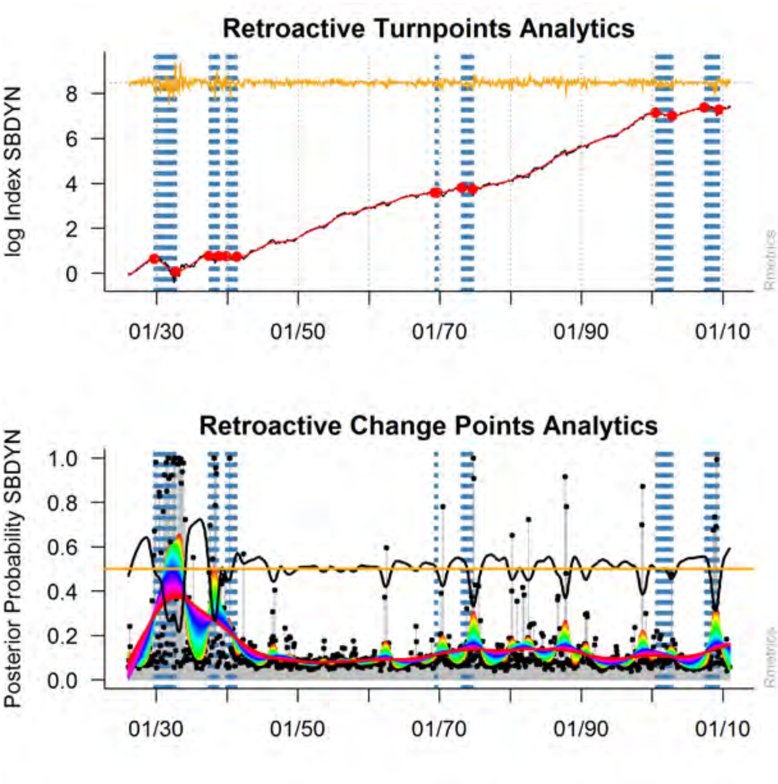

FIGURE 16.11: Bayesian Changepoint Analytics: Dynamic Stock/Bond Blend

CHAPTER 17

EXTREMES AND OUTLIERS

Outlier detection methods are a way of searching for extreme and outlying patterns in the data. Identifying possible outliers and periods with many outlying data points helps to identify periods of instability. The *principal component outlier analytics* can be in this respect. It helps to identify extreme and outlying values. It is a robust method which gives an effective indication when extremes and outliers start to dominate the financial or economic time series process.

There are many different methods of outlier detection. In this study we focus on a principal components (PC) outlier detection method where we use the Mahalanobis distance as the measure of "outlyingness". The method is able to detect outlying points in a way that is both computationally efficient and robust, Filzmoser [2007]. A principal components analysis returns the set of principal components. The PC's are a set of orthogonal variables which each maximize the variance in their respective directions. They are selected such that the first component explains the most variance in the data, the second component the second most and so on. Outlying data points increase the variance of the data. As the PC's are the directions which maximize the amount of variance along each component, it is plausible that the outliers will appear more visible in PC space than in the original data space, Filzmoser [2007].

For each asset class, as well as the conservative, balanced and dynamic stock/bond blends, a figure will show for each date the probability of having an outlier within the time series. The rainbow colored time series are smoothed curves of the original probability series. The black curve is the difference between the strongest smoothed curve and the weakest smoothed curve which serves as a measure for stability. The analyses within this section were done using the monthly nominal logarithmic returns.

17.1 EXTREMES AND OUTLIERS IN ASSET CLASSES

PC Outlier Analysis: BILL

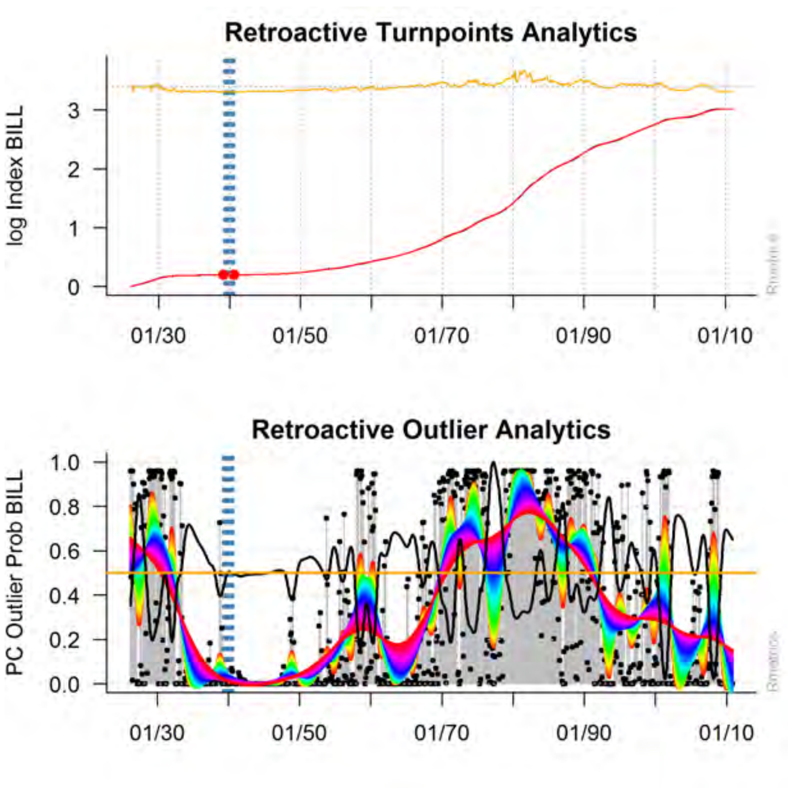

FIGURE 17.1: PC Outlier Analytics: Treasury Bills. The bottom figure shows the posterior probability for each date of having an outlier within the time series. The rainbow colored time series are smoothed curves of the original probability series. The black curve is the difference between the strongest smoothed curve and the weakest smoothed curve which serves as a measure for stability.

PC Outlier Analysis: ITGOV

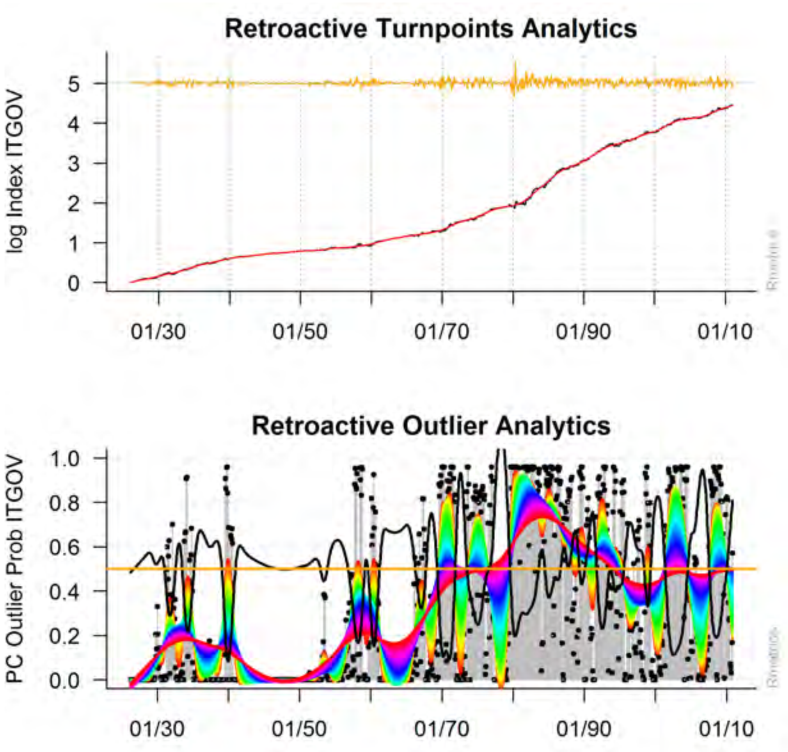

FIGURE 17.2: PC Outlier Analytics: Intermediate Term Government Bonds

PC Outlier Analysis: LTGOV

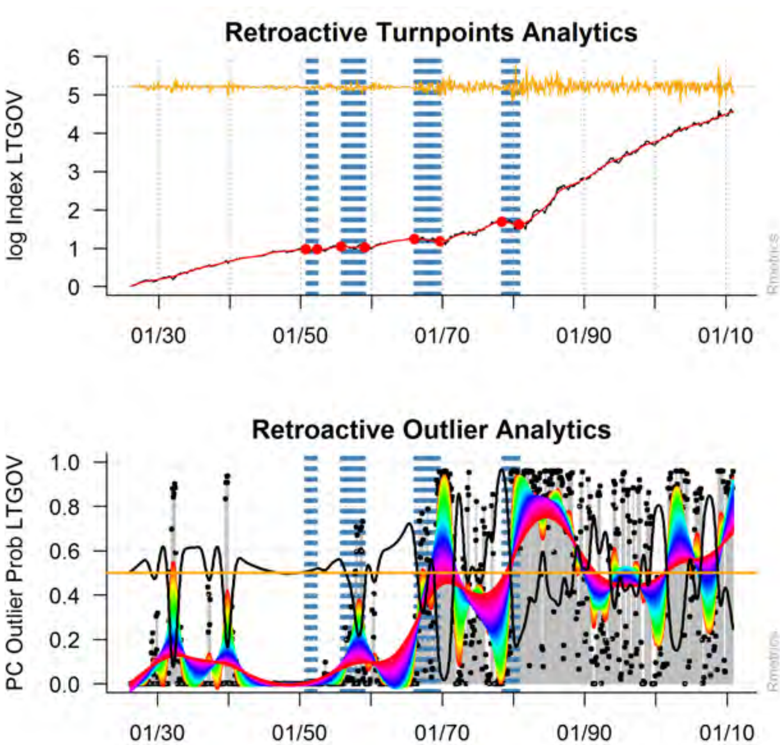

FIGURE 17.3: PC Outlier Analytics: Long Term Government Bonds

PC Outlier Analysis: LTCORP

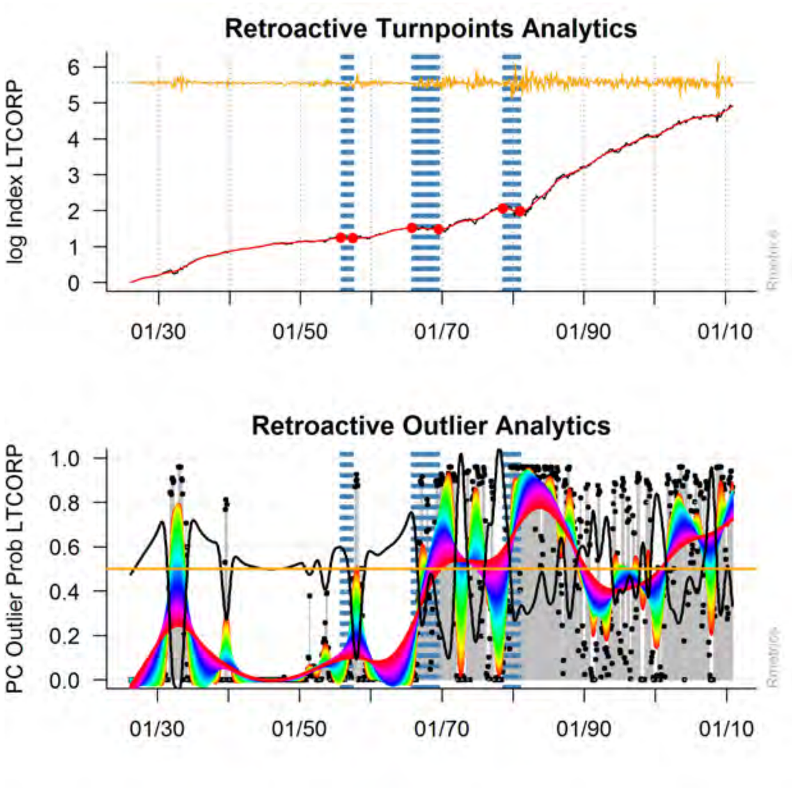

FIGURE 17.4: PC Outlier Analytics: Long Term Corporate Bonds

PC Outlier Analysis: HYCORP

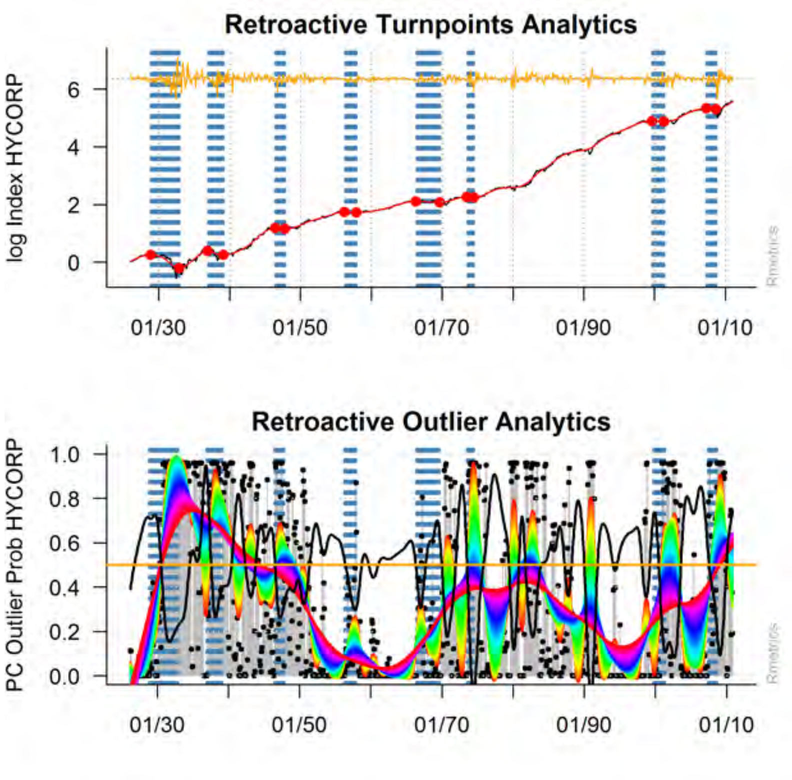

FIGURE 17.5: PC Outlier Analytics: Domestic High Yield Corporate Bonds

PC Outlier Analysis: LCAP

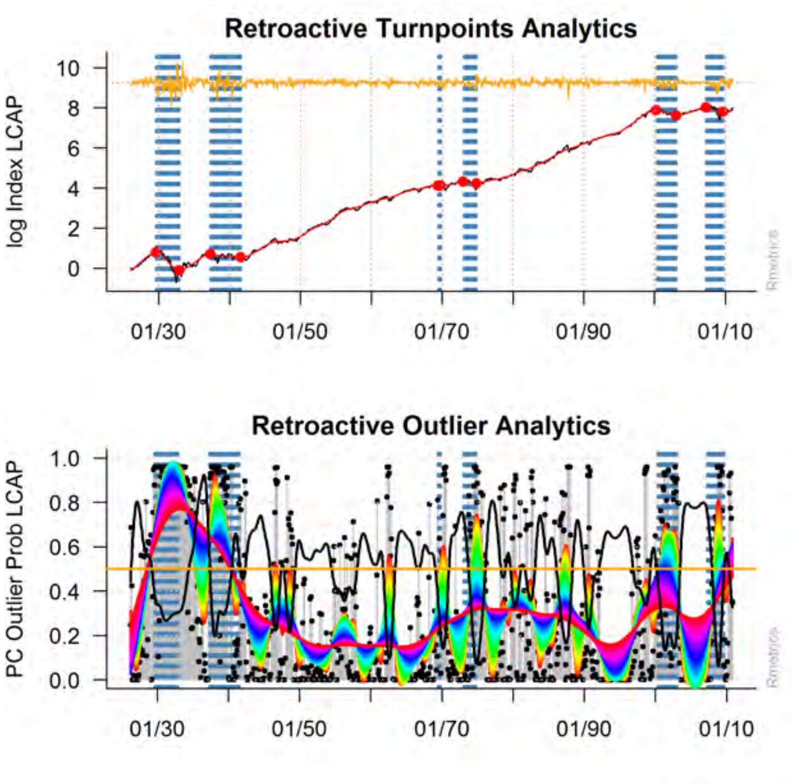

FIGURE 17.6: PC Outlier Analytics: Large Cap Stocks

PC Outlier Analysis: SCAP

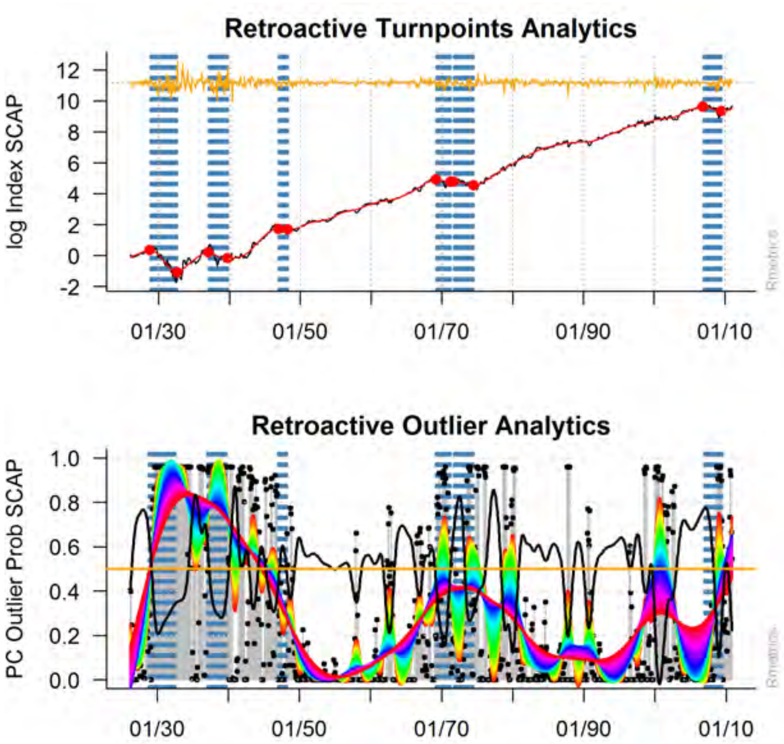

FIGURE 17.7: PC Outlier Analytics: Small Cap Stocks

PC Outlier Analysis: CRB

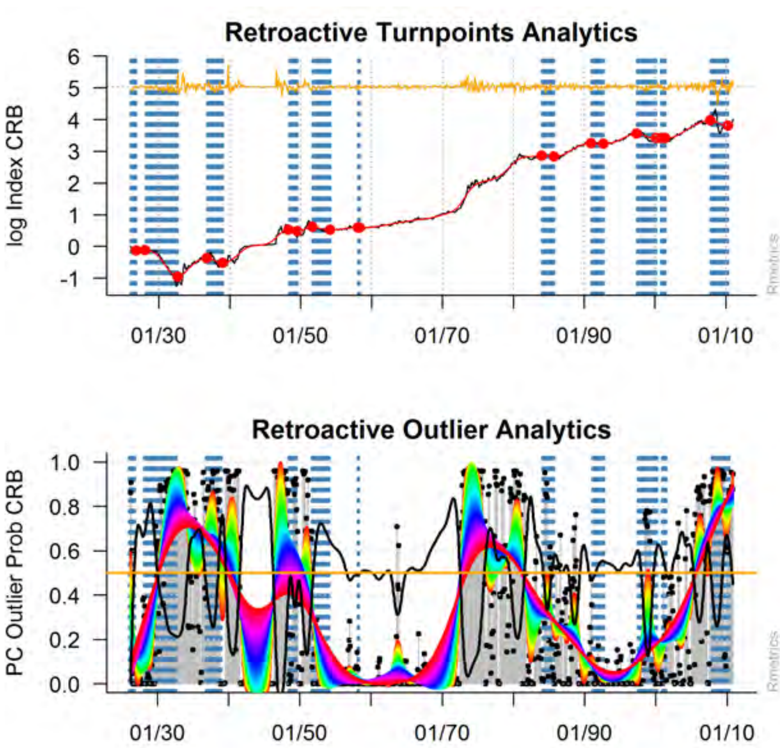

FIGURE 17.8: PC Outlier Analytics: CRB Commodity Index

17.2 EXTREMES AND OUTLIERS IN STOCK/BOND BLENDS

Extremes and Outliers: Conservative Stock/Bond Blend

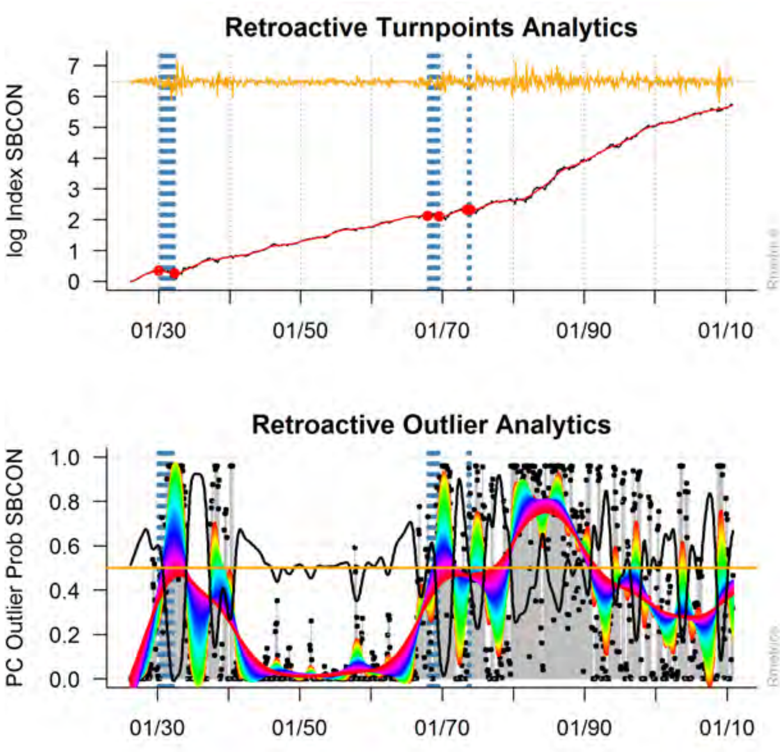

FIGURE 17.9: PC Outlier Analytics: Dynamic Stock/Bond Blend

Extremes and Outliers: Balanced Stock/Bond Blend

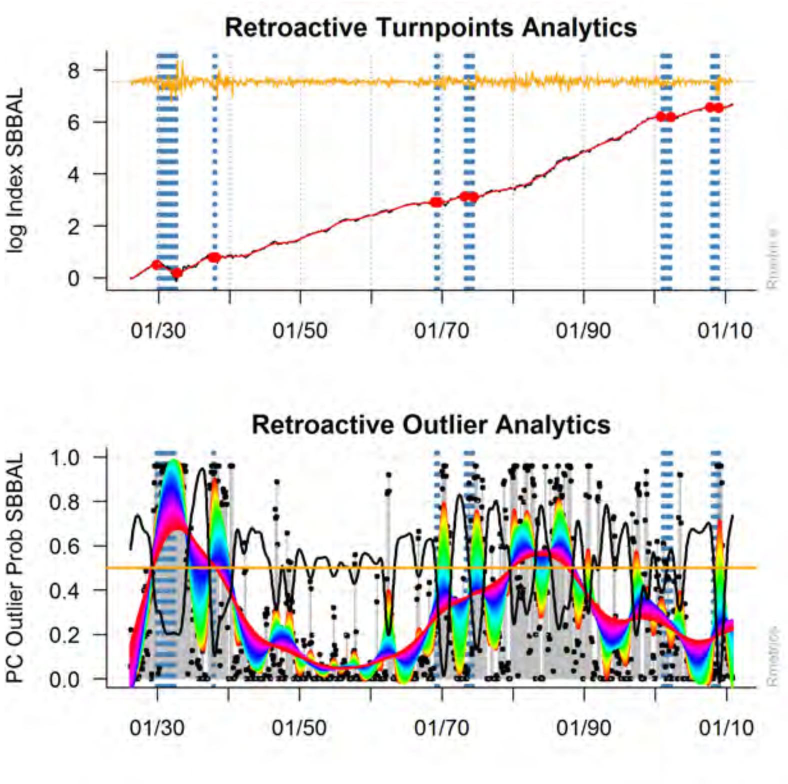

FIGURE 17.10: PC Outlier Analytics: Dynamic Stock/Bond Blend

Extremes and Outliers: Dynamic Stock/Bond Blend

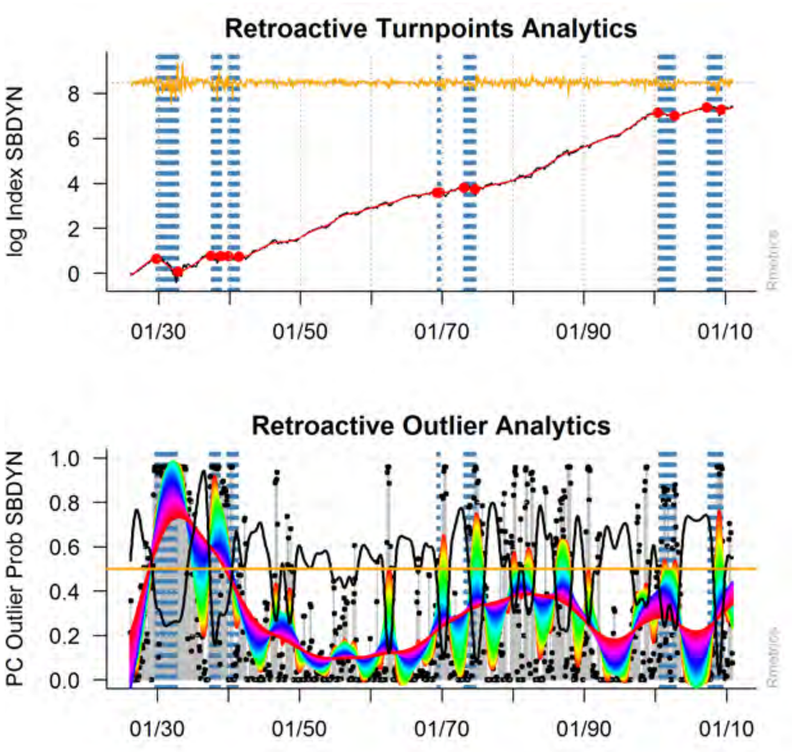

FIGURE 17.11: PC Outlier Analytics: Dynamic Stock/Bond Blend

CHAPTER 18

NON-STATIONARITY ANALYSIS

Stationarity is the fundamental requirement for any time series model. We assume that the distribution of previous observations can be used to forecast future ones. In other terms, the joint probability distribution of the observations is assumed to be stationary over time, i.e. its moments including the mean and volatility are constant. However, we often observe events in financial returns that cannot be described by the distribution of past observations. When such points are observed, the time series is considered to be non-stationary.

Our choice to analyze non-stationary behavior in asset returns is the multi-resolution approach in time/frequency space using wavelet analysis. Wavelet analysis is a widespread technique that offers the user the possibility to decompose a time series into time/frequency space, Torrence [1997]. This allows one to determine the dominant modes of variability in the series and how they vary over time. The signal can be examined in time and frequency simultaneously. The wavelet transform has been found to be particularly useful for series that are aperiodic, noisy, irregular, intermittent and so on, Addison [2002].

Wavelet functions are localized waveforms that can be altered in two ways, either moved in location along the series or altered in frequency - a stretching or squeezing of the wavelet form. At each desired location and frequency (scale) the resulting wavelet is compared to the original series to determine how well it matches. A good match results in a large transform value and vice versa. The transform values for each particular location and scale are plotted on a contour plot. For the Morlet wavelet form, the transform value is a complex number and different possibilities to plot those values are thinkable. We us the square of the absolute value (power spectrum) which has an expectation value that is approximately equal to the variance of the series. By examining the patterns in the wavelet contour plot it is possible to see changes in the frequency and variance of the signal as well as jumps and discontinuities very clearly.

For each asset class, as well as the conservative, balanced and dynamic stock/bond blends a figure will show the transform values of the time series for each location and scale considered. The analyses within this chapter were done using the monthly nominal logarithmic returns.

18.1 WAVELET ANALYSIS OF ASSET CLASSES

Wavelet Analysis: BILL

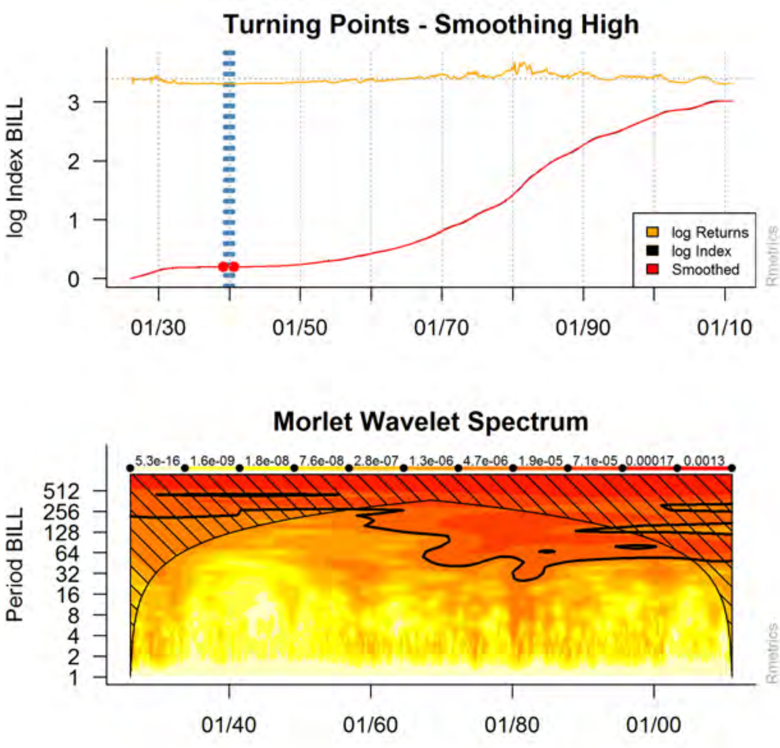

FIGURE 18.1: Wavelet Analytics: Treasury Bills.

Wavelet Analysis: ITGOV

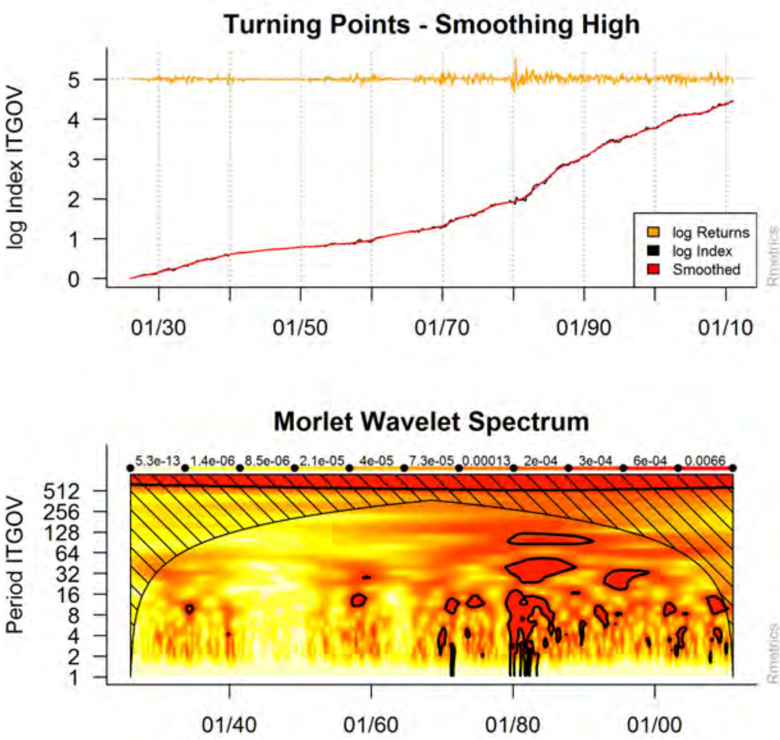

FIGURE 18.2: Wavelet Analytics: Intermediate Term Government Bonds.

Wavelet Analysis: LTGOV

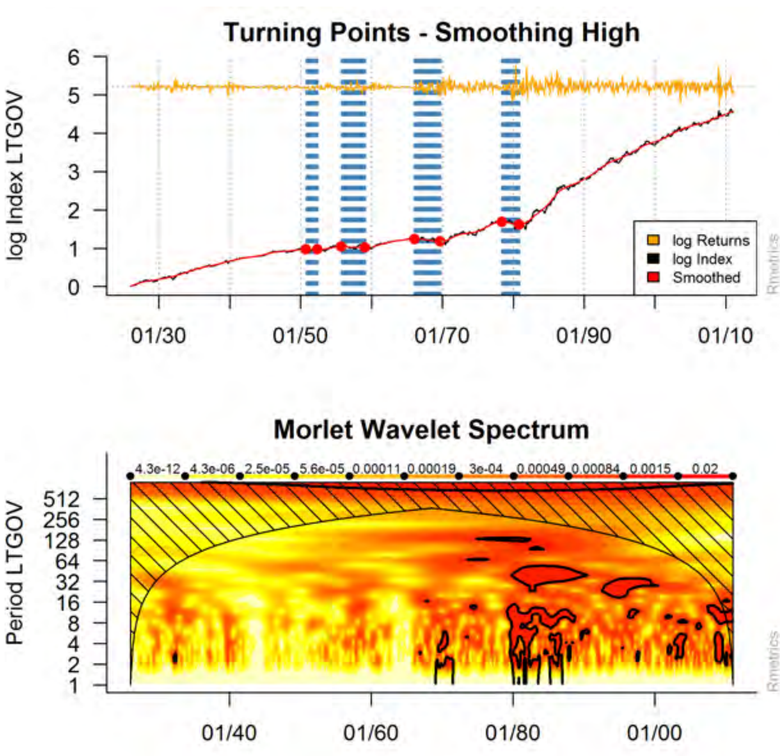

FIGURE 18.3: Wavelet Analytics: Long Term Government Bonds.

Wavelet Analysis: LTCORP

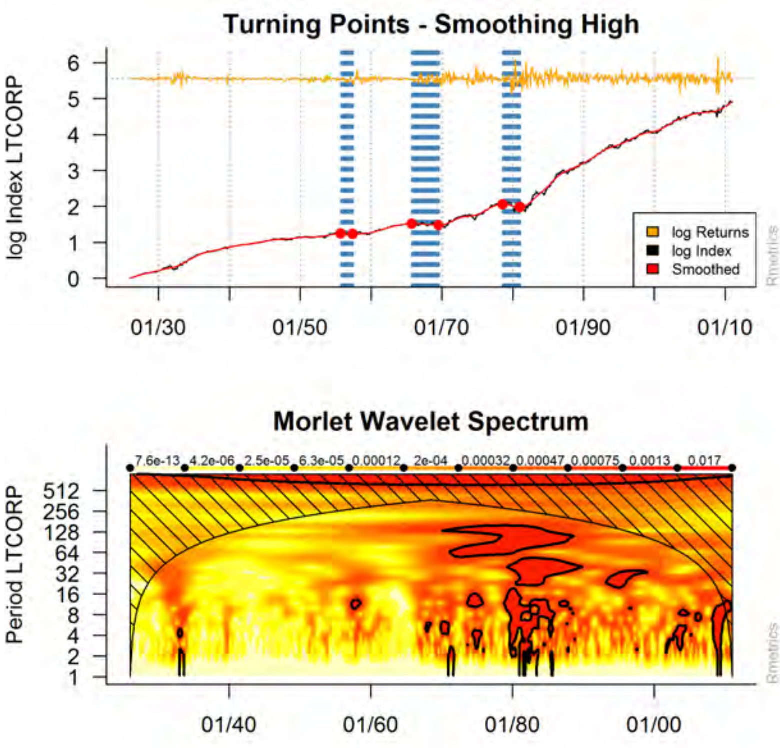

FIGURE 18.4: Wavelet Analytics: Long Term Corporate Bonds.

Wavelet Analysis: HYCORP

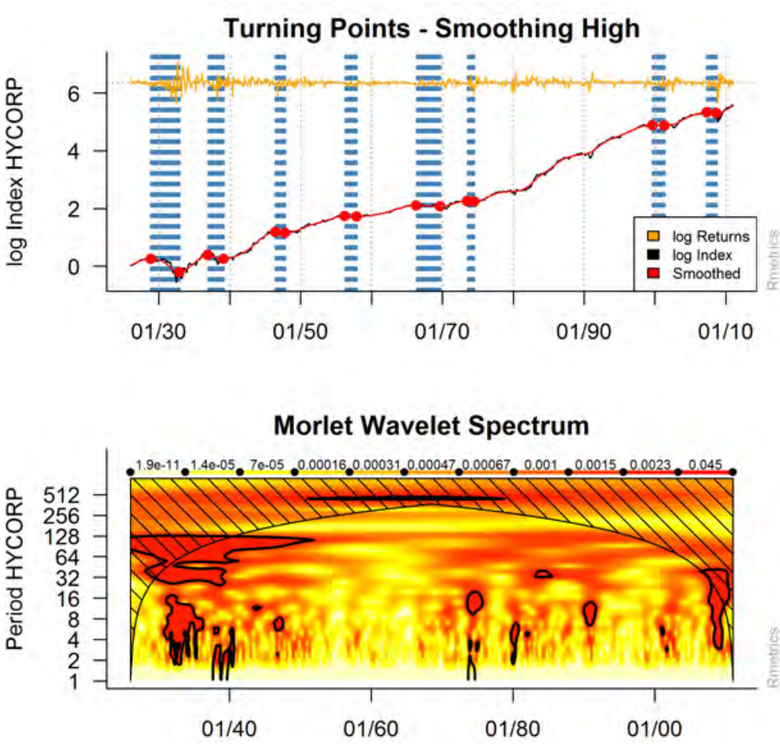

FIGURE 18.5: Wavelet Analytics: Domestic High Yield Corporate Bonds.

Wavelet Analysis: LCAP

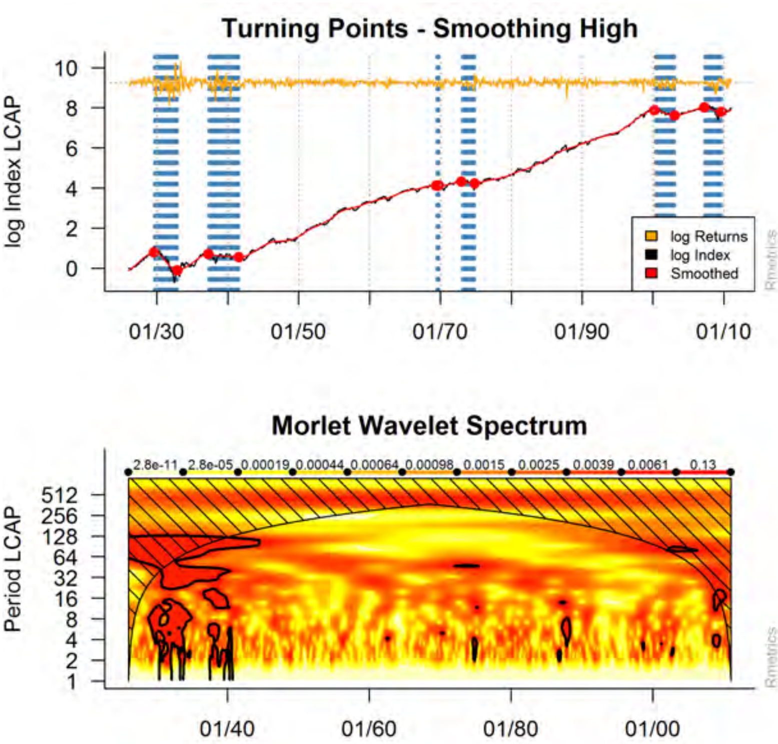

FIGURE 18.6: Wavelet Analytics: Large Cap Stocks.

Wavelet Analysis: SCAP

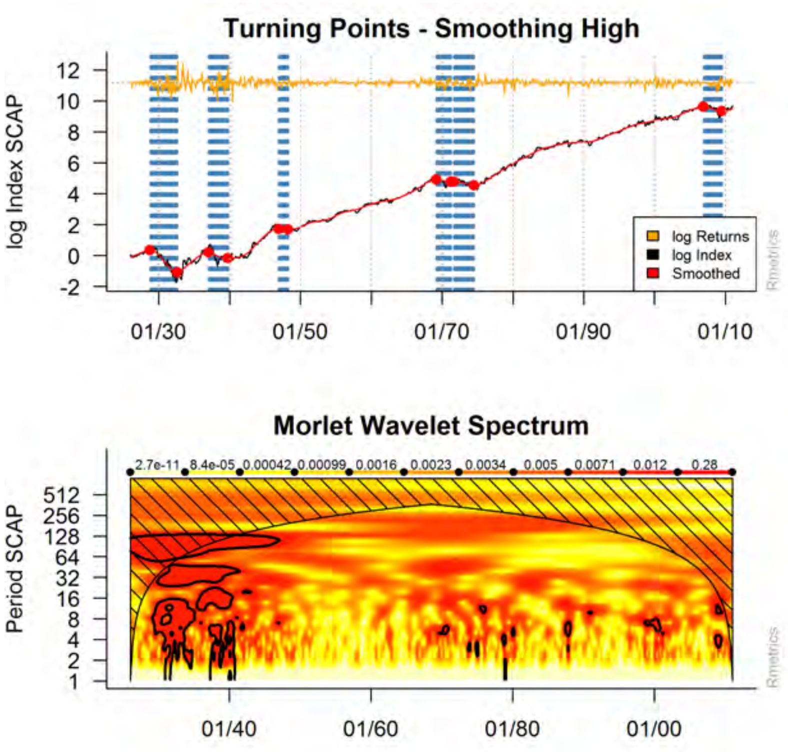

FIGURE 18.7: Wavelet Analytics: Small Cap Stocks.

Wavelet Analysis: CRB

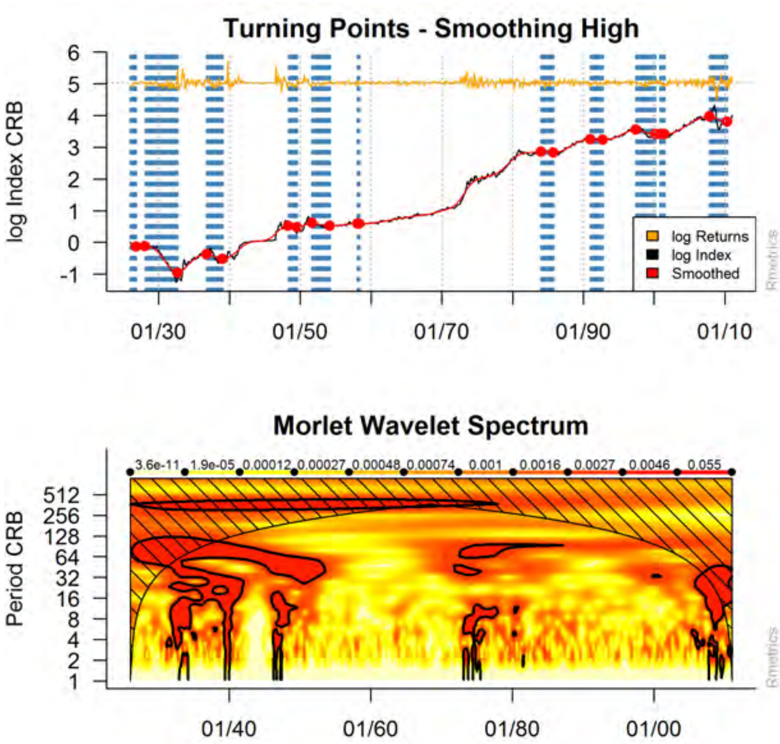

FIGURE 18.8: Wavelet Analytics: CRB Commodity Index.

18.2 WAVELET ANALYSIS OF STOCK/BOND BLENDS

Wavelet Analysis: Conservative Stock/Bond Blend

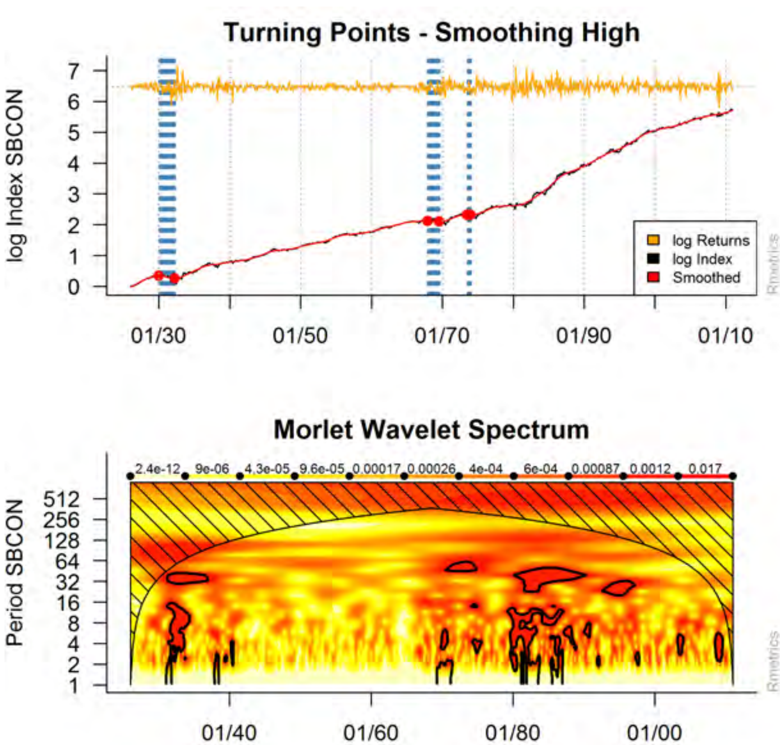

FIGURE 18.9: Wavelet Analytics: Conservative Stock/Bond blend.

Wavelet Analysis: Balanced Stock/Bond Blend

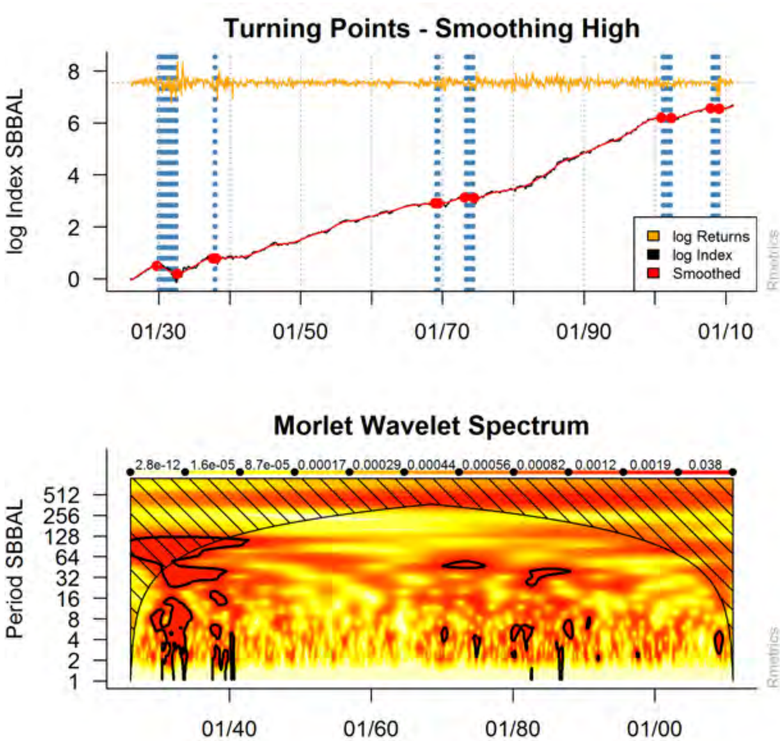

FIGURE 18.10: Wavelet Analytics: Conservative Stock/Bond blend.

Wavelet Analysis: Dynamic Stock/Bond Blend

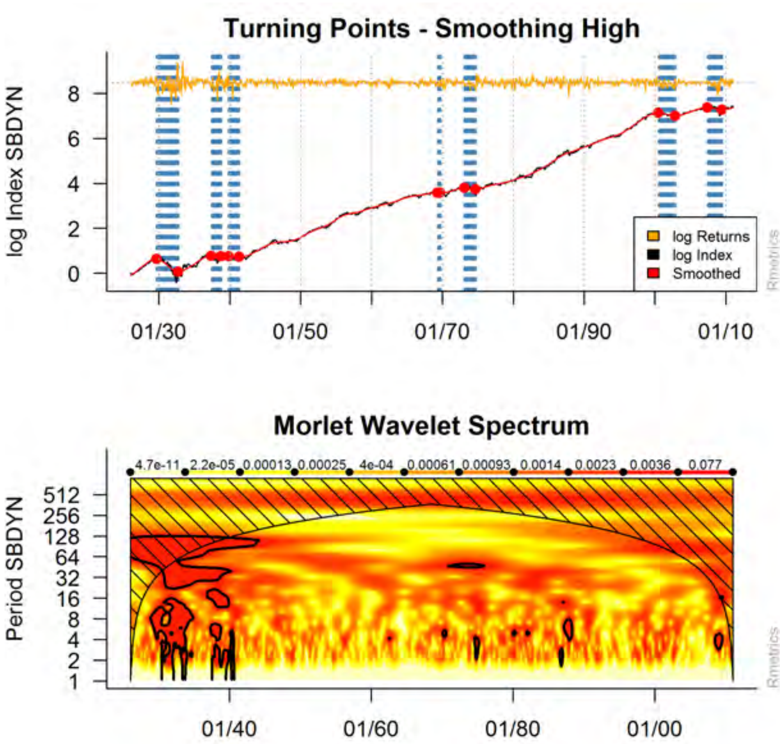

FIGURE 18.11: Wavelet Analytics: Dynamic Stock/Bond blend.

Part V

Portfolio Benchmarking

CHAPTER 19

INTRODUCTION

In this Part we consider three types of portfolios as proxies for portfolio design and construction. Our three benchmark portfolios are composed by combining different asset classes. We consider the following combinations: (i) stock/bond blends, (ii) stock/bond/commodity portfolios and (iii) all asset class portfolios. For the portfolio optimization we use the long only constrained mean - variance portfolio approach of Markovitz [1952].

In chapter 20 we describe bivariate portfolios constructed from two asset classes; the stock/bond blends. In also present in chapter 21 ternary portfolio designs which are constructed from the three asset classes stocks, bonds and commodities. Multivariate portfolios consisting of all our eight asset classes are described in chapter 22. Besides conventional views we introduce new views for portfolios that have more than two assets. In Figure 21 we present portfolio performance and risk maps derived from three asset classes. By using barycentric coordinates those plots offer a unique view on any kind of performance or risk measure characterizing the portfolios. In **??** we present portfolio risk surfaces on top of the feasible set. Those plots can be applied on portfolios consisting of any number of assets and offer a generalized multivariate view on the variability of any kind of performance or risk measure.

CHAPTER 20

BOND AND STOCK BLENDS

Portfolio blends are designed from two assets. As a typical example of those bivariate portfolios we will consider stock/bond blends in this section. Using the long only constrained mean - variance portfolio approach of Markovitz, the weights of the assets, for the first we have W_1 and for the second $W_2 = 1 - W_1$, can easily be evaluated analytically. In this case the feasible set shrinks to a line, which is just the efficient frontier and/or the minimum variance locus.

The efficient frontier is the line that connects the minimum risk portfolio and the portfolio which has the highest return while the minimum variance locus connects the minimum risk portfolio and the portfolio which has the lowest return. In the case where the minimum risk portfolio is itself the portfolio which has the highest or lowest return, the efficient frontier respectively the minimum variance locus vanishes.

In the bivariate case, usually the efficient frontier starts with the pure bond investment and ends with a investment fully in stocks. In between we can explore various portfolio compositions. The following five strategies are special points on the minimum variance locus and the efficient frontier: (i) Fixed Income (100% Bonds - 0% Equities), (ii) Conservative (75% - 25%), (iii) Balanced (50% - 50%), (iv) Dynamic (50% - 75%) and (v) Equity (0% - 100%). Note that the balanced portfolio is the equal weights portfolio (EWP). The portfolios we explore consist of the major two asset classes, bonds and stocks. As a proxy for our binary benchmark we use the blend of long term government bonds from the time series *LTGOV* and large capitalized stocks from the time series *LCAP*.

We optimize portfolios for four different scenarios, i.e. portfolios with datasets starting in 1925, 1950, 1975 and 2000. We display the blends in a risk/reward chart create barplots and tables showing the weights, the weighted returns and the covariance risk budgets of portfolios. The analyses within this section were done using the monthly nominal logarithmic returns.

20.1 BOND/STOCK PORTFOLIO FRONTIER

Bond/Stock Frontier Since 1925

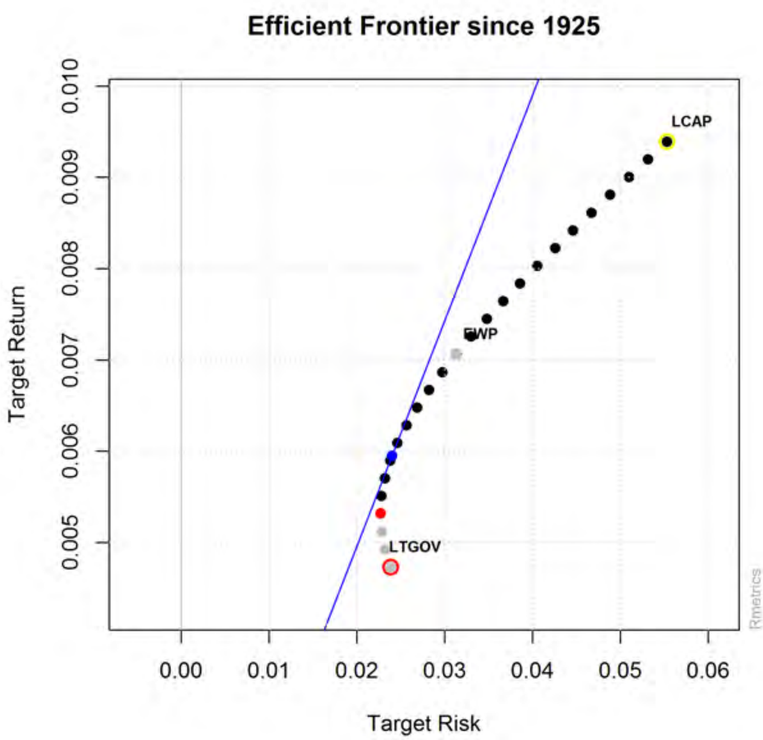

FIGURE 20.1: The red point shows the minimum risk portfolio (MRP). The points above the MRP lie on the efficient frontier (black points), the points below the MRP on the minimum variance locus (gray points). The blue point indicates the tangency portfolio and the brown point the equal weights portfolio (EWP). The asset classes used to build the portfolios are shown as circles.

Bond/Stock Weights Since 1925

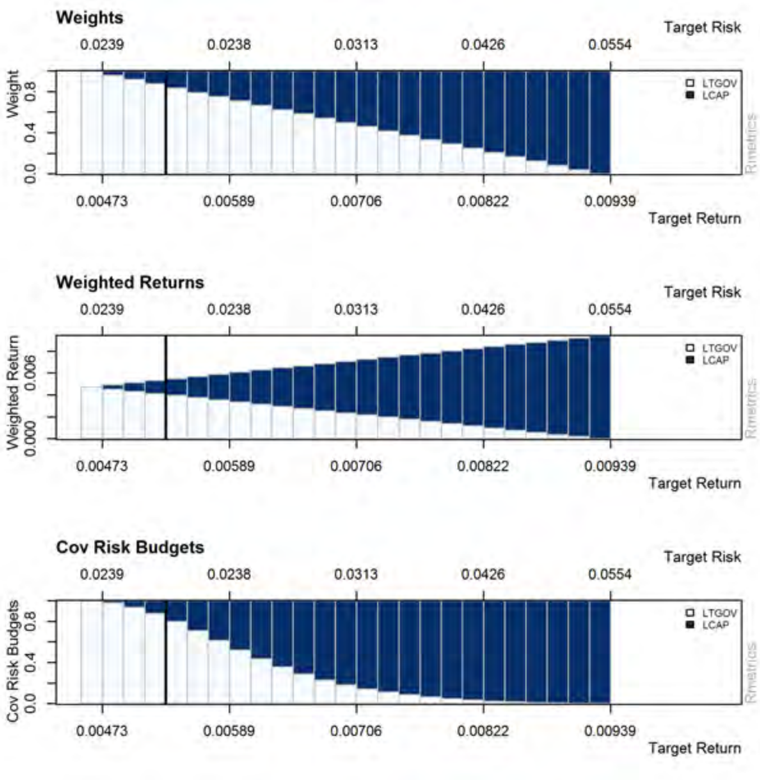

FIGURE 20.2: For every plot the first portfolio on the left side of the black vertical line is the minimum risk portfolio (MRP). The portfolios on the left of the MRP are the portfolios on the minimum variance locus. The portfolios on the right of the MRP are the portfolios on the efficient frontier. The top plot shows the weights for the portfolios. The return (but not the necessarily the risk) along the minimum variance locus / efficient frontier line is steadily increasing which is shown in the middle plot along with the information of how much return is coming form which asset class. The bottom plot shows how much risk is coming form which asset class (in percent).

Bond/Stock Weights Table Since 1925

```
     V1   V2 Target Return
1  1.00 0.00       0.00473
2  0.96 0.04       0.00492
3  0.92 0.08       0.00512
4  0.88 0.12       0.00531
5  0.83 0.17       0.00551
6  0.79 0.21       0.00570
7  0.75 0.25       0.00589
8  0.71 0.29       0.00609
9  0.67 0.33       0.00628
10 0.63 0.37       0.00648
11 0.58 0.42       0.00667
12 0.54 0.46       0.00687
13 0.50 0.50       0.00706
14 0.46 0.54       0.00725
15 0.42 0.58       0.00745
16 0.38 0.62       0.00764
17 0.33 0.67       0.00784
18 0.29 0.71       0.00803
19 0.25 0.75       0.00822
20 0.21 0.79       0.00842
21 0.17 0.83       0.00861
22 0.13 0.87       0.00881
23 0.08 0.92       0.00900
24 0.04 0.96       0.00919
25 0.00 1.00       0.00939
```

FIGURE 20.3: LTGOV and LCAP weights table. Data sets start 1925.

Bond/Stock Blends Cov Risk Budgets since 1925

```
     V1   V2 Target Risk
1  1.00 0.00     0.02387
2  0.98 0.02     0.02325
3  0.94 0.06     0.02286
4  0.88 0.12     0.02273
5  0.80 0.20     0.02285
6  0.71 0.29     0.02322
7  0.61 0.39     0.02382
8  0.52 0.48     0.02465
9  0.43 0.57     0.02568
10 0.36 0.64     0.02689
11 0.29 0.71     0.02825
12 0.23 0.77     0.02974
13 0.18 0.82     0.03135
14 0.14 0.86     0.03305
15 0.11 0.89     0.03484
16 0.09 0.91     0.03670
17 0.06 0.94     0.03862
18 0.05 0.95     0.04059
19 0.03 0.97     0.04261
20 0.02 0.98     0.04466
21 0.02 0.98     0.04675
22 0.01 0.99     0.04887
23 0.01 0.99     0.05101
24 0.00 1.00     0.05318
25 0.00 1.00     0.05536
```

FIGURE 20.4: LTGOV and LCAP portfolio covariance risk budgets. Data sets start 1925.

Bond/Stock Frontier Since 1950

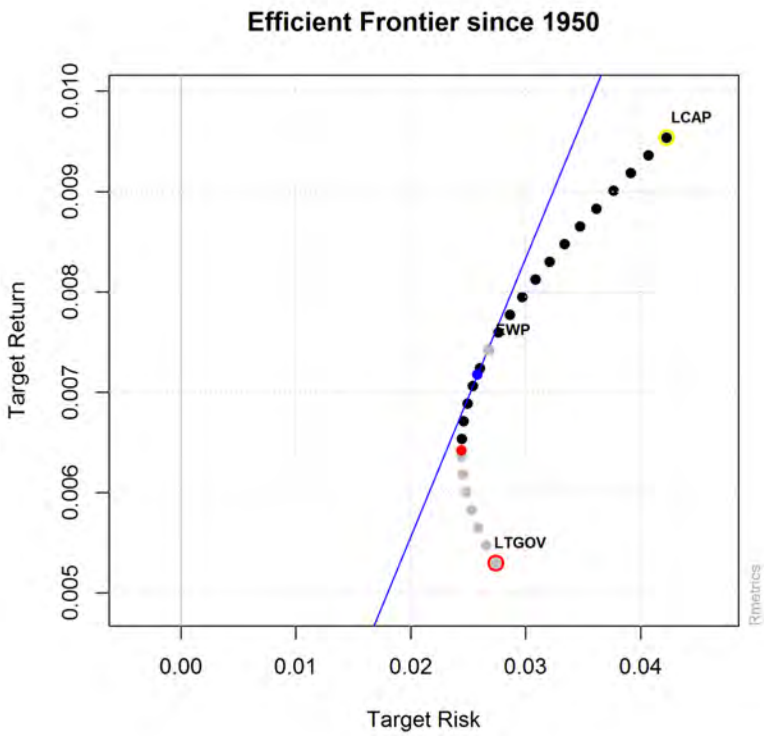

FIGURE 20.5: LTGOV and LCAP portfolio. Data sets start 1950.

Bond/Stock Weights Since 1950

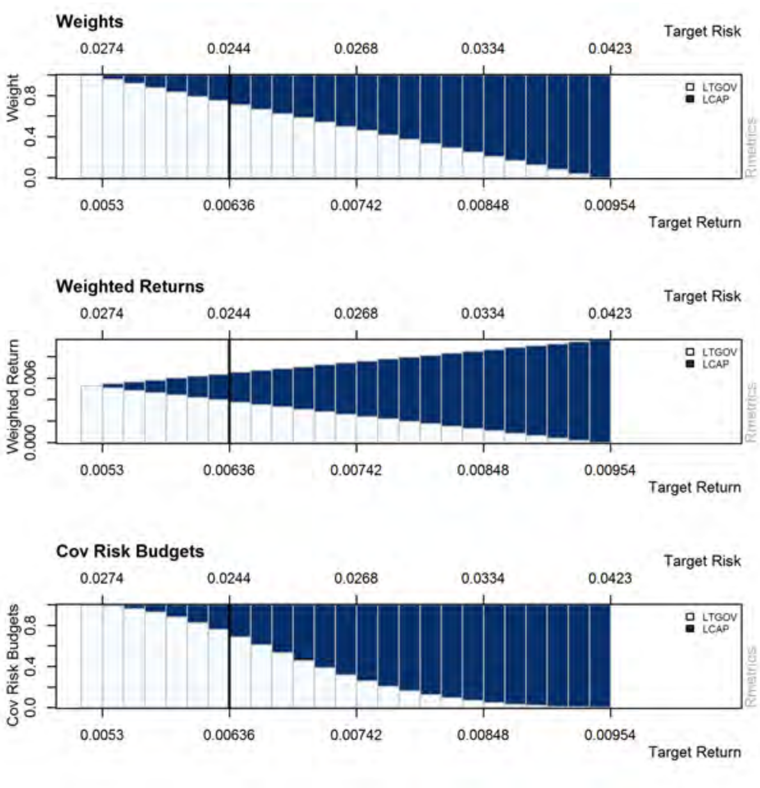

FIGURE 20.6: LTGOV and LCAP portfolio weights plot. Data sets start 1950.

Bond/Stock Weights Table Since 1950

```
     V1   V2 Target Return
1  1.00 0.00       0.00530
2  0.96 0.04       0.00547
3  0.92 0.08       0.00565
4  0.88 0.12       0.00583
5  0.83 0.17       0.00600
6  0.79 0.21       0.00618
7  0.75 0.25       0.00636
8  0.71 0.29       0.00653
9  0.67 0.33       0.00671
10 0.62 0.38       0.00689
11 0.58 0.42       0.00706
12 0.54 0.46       0.00724
13 0.50 0.50       0.00742
14 0.46 0.54       0.00759
15 0.42 0.58       0.00777
16 0.37 0.63       0.00795
17 0.33 0.67       0.00812
18 0.29 0.71       0.00830
19 0.25 0.75       0.00848
20 0.21 0.79       0.00865
21 0.17 0.83       0.00883
22 0.12 0.88       0.00901
23 0.08 0.92       0.00918
24 0.04 0.96       0.00936
25 0.00 1.00       0.00954
```

FIGURE 20.7: LTGOV and LCAP portfolio weights table. Data sets start 1950.

Bond/Stock Blends Cov Risk Budgets since 1950

```
     V1   V2 Target Risk
1  1.00 0.00     0.02741
2  0.99 0.01     0.02657
3  0.96 0.04     0.02586
4  0.93 0.07     0.02529
5  0.88 0.12     0.02485
6  0.83 0.17     0.02456
7  0.76 0.24     0.02443
8  0.69 0.31     0.02445
9  0.61 0.39     0.02463
10 0.53 0.47     0.02496
11 0.46 0.54     0.02544
12 0.39 0.61     0.02605
13 0.32 0.68     0.02680
14 0.26 0.74     0.02766
15 0.21 0.79     0.02863
16 0.16 0.84     0.02970
17 0.13 0.87     0.03086
18 0.10 0.90     0.03209
19 0.07 0.93     0.03339
20 0.05 0.95     0.03476
21 0.03 0.97     0.03618
22 0.02 0.98     0.03764
23 0.01 0.99     0.03915
24 0.00 1.00     0.04070
25 0.00 1.00     0.04228
```

FIGURE 20.8: LTGOV and LCAP portfolio covariance risk budgets. Data sets start 1950.

Bond/Stock Frontier Since 1975

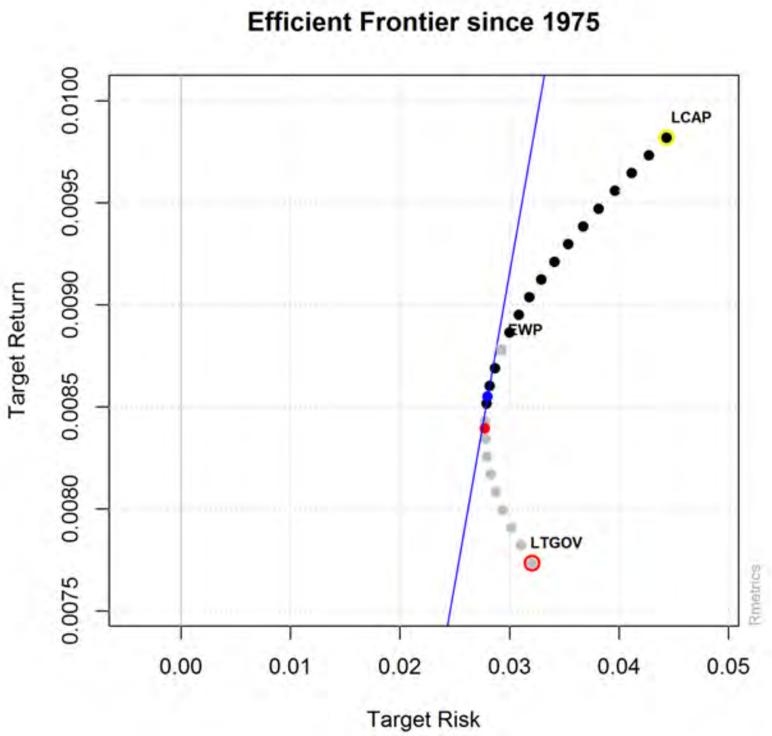

FIGURE 20.9: LTGOV and LCAP portfolio. Data sets start 1975.

Bond/Stock Weights Since 1975

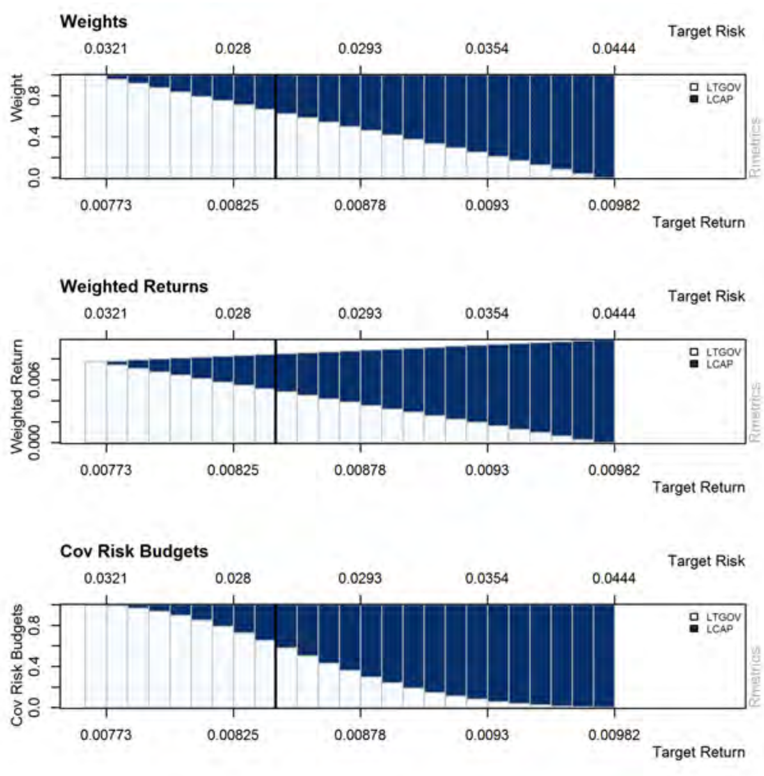

FIGURE 20.10: LTGOV and LCAP portfolio weights plot. Data sets start 1975.

Bond/Stock Weights Table Since 1975

```
     V1   V2 Target Return
1  1.00 0.00       0.00773
2  0.96 0.04       0.00782
3  0.92 0.08       0.00791
4  0.87 0.13       0.00799
5  0.83 0.17       0.00808
6  0.79 0.21       0.00817
7  0.75 0.25       0.00825
8  0.71 0.29       0.00834
9  0.67 0.33       0.00843
10 0.62 0.38       0.00851
11 0.58 0.42       0.00860
12 0.54 0.46       0.00869
13 0.50 0.50       0.00878
14 0.46 0.54       0.00886
15 0.42 0.58       0.00895
16 0.38 0.62       0.00904
17 0.33 0.67       0.00912
18 0.29 0.71       0.00921
19 0.25 0.75       0.00930
20 0.21 0.79       0.00938
21 0.17 0.83       0.00947
22 0.13 0.87       0.00956
23 0.08 0.92       0.00964
24 0.04 0.96       0.00973
25 0.00 1.00       0.00982
```

FIGURE 20.11: LTGOV and LCAP portfolio weights table. Data sets start 1975.

Bond/Stock Blends Cov Risk Budgets since 1975

```
     V1   V2 Target Risk
1  1.00 0.00     0.03207
2  0.99 0.01     0.03107
3  0.97 0.03     0.03018
4  0.94 0.06     0.02941
5  0.90 0.10     0.02878
6  0.85 0.15     0.02830
7  0.79 0.21     0.02797
8  0.73 0.27     0.02779
9  0.65 0.35     0.02777
10 0.58 0.42     0.02791
11 0.51 0.49     0.02822
12 0.43 0.57     0.02867
13 0.36 0.64     0.02927
14 0.30 0.70     0.03000
15 0.24 0.76     0.03087
16 0.19 0.81     0.03185
17 0.15 0.85     0.03293
18 0.11 0.89     0.03412
19 0.08 0.92     0.03539
20 0.06 0.94     0.03673
21 0.04 0.96     0.03815
22 0.03 0.97     0.03963
23 0.01 0.99     0.04116
24 0.01 0.99     0.04274
25 0.00 1.00     0.04437
```

FIGURE 20.12: LTGOV and LCAP portfolio covariance risk budgets. Data sets start 1975.

Bond/Stock Frontier Since 2000

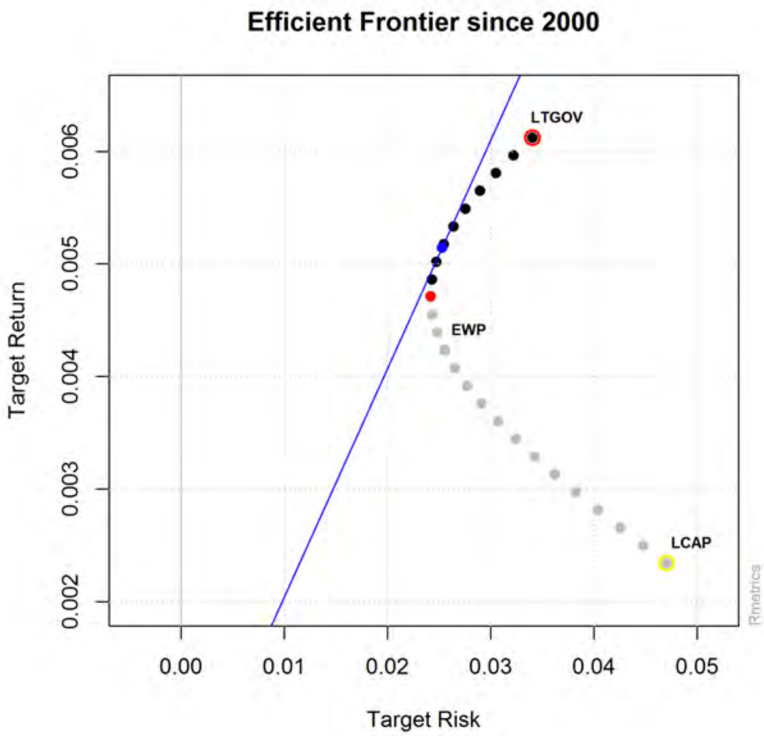

FIGURE 20.13: LTGOV and LCAP portfolio frontier plot. Data sets start 2000.

Bond/Stock Weights Since 2000

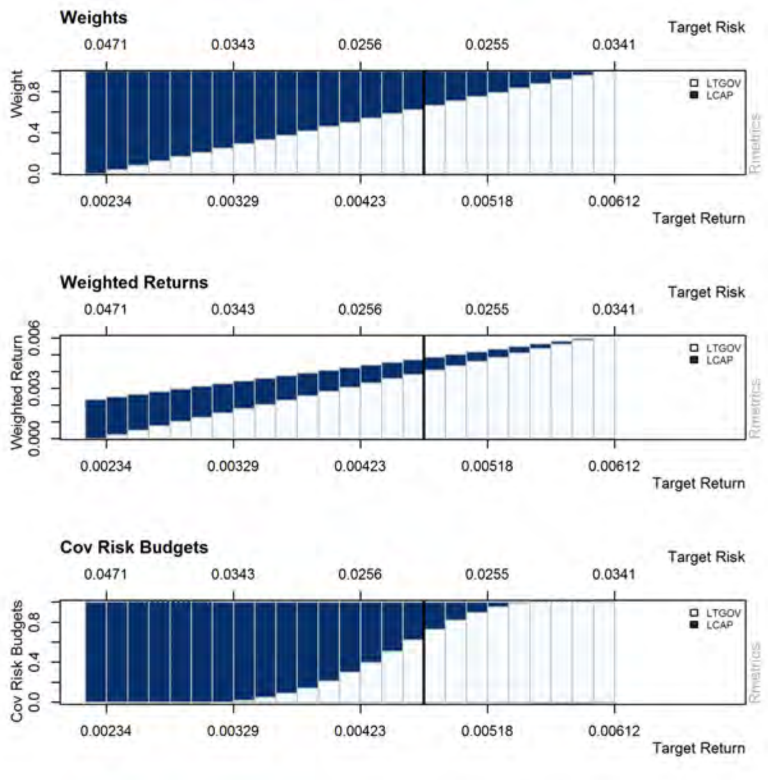

FIGURE 20.14: LTGOV and LCAP portfolio weights plot. Data sets start 2000.

Bond/Stock Weights Table Since 2000

	V1	V2	Target Return
1	0.00	1.00	0.00234
2	0.04	0.96	0.00250
3	0.08	0.92	0.00266
4	0.12	0.88	0.00281
5	0.17	0.83	0.00297
6	0.21	0.79	0.00313
7	0.25	0.75	0.00329
8	0.29	0.71	0.00344
9	0.33	0.67	0.00360
10	0.38	0.62	0.00376
11	0.42	0.58	0.00392
12	0.46	0.54	0.00407
13	0.50	0.50	0.00423
14	0.54	0.46	0.00439
15	0.58	0.42	0.00455
16	0.62	0.38	0.00470
17	0.67	0.33	0.00486
18	0.71	0.29	0.00502
19	0.75	0.25	0.00518
20	0.79	0.21	0.00533
21	0.83	0.17	0.00549
22	0.87	0.13	0.00565
23	0.92	0.08	0.00581
24	0.96	0.04	0.00596
25	1.00	0.00	0.00612

FIGURE 20.15: LTGOV and LCAP portfolio weights table. Data sets start 2000.

Bond/Stock Blends Cov Risk Budgets since 2000

```
      V1   V2 Target Risk
1  0.00 1.00     0.04709
2  0.00 1.00     0.04481
3  0.00 1.00     0.04258
4  0.00 1.00     0.04040
5  0.00 1.00     0.03829
6  0.00 1.00     0.03626
7  0.00 1.00     0.03431
8  0.02 0.98     0.03247
9  0.05 0.95     0.03075
10 0.09 0.91     0.02917
11 0.14 0.86     0.02777
12 0.21 0.79     0.02656
13 0.30 0.70     0.02558
14 0.40 0.60     0.02485
15 0.51 0.49     0.02439
16 0.62 0.38     0.02422
17 0.73 0.27     0.02435
18 0.82 0.18     0.02477
19 0.90 0.10     0.02547
20 0.95 0.05     0.02642
21 0.99 0.01     0.02760
22 1.00 0.00     0.02898
23 1.00 0.00     0.03054
24 1.00 0.00     0.03224
25 1.00 0.00     0.03407
```

FIGURE 20.16: LTGOV and LCAP portfolio covariance risk budgets. Data sets start 2000.

Chapter 21

Ternary Portfolio Design

Ternary portfolios are designed from three assets. Typical examples are combinations of bonds, stocks and a third asset class, for example cash, bills or commodities. Generally the third asset class can be composed from any other financial assets, including real estates, private equities, hedge funds or any other alternative investment. In hierarchical portfolio optimization we can think of combinations of small, medium and large capitalized stocks. In this chapter we will consider ternary portfolios constructed from bonds, stocks and commodities.

The portfolios are optimized using the long only constrained mean - variance portfolio approach of Markovitz which delivers the weights W_1, W_2 and $W_3 = 1 - W_1 - W_2$ for the three assets considered. The problem can only be solved numerically. The feasible set is the area that holds all possible combinations of return and risk that can be achieved through the creation of portfolios. The efficient frontier is the line along the border of the feasible set that connects the minimum risk portfolio and the portfolio which has the highest return. Similarly the minimum variance locus connects the minimum risk portfolio and the portfolio which has the lowest return. In the case where the minimum risk portfolio is itself the portfolio which has the highest or lowest return, the efficient frontier respectively the minimum variance locus vanishes.

As a proxy for bonds we use the long term government bonds from the time series *LTGOV*, for stocks we use the large capitalized stocks from the time series *LCAP* and for commodities we use the commodity index from the time series *CRB*. We optimize portfolios for four different scenarios, i.e. portfolios with datasets starting in 1925, 1950, 1975 and 2000. We display the efficient frontier and/or the minimum variance locus and create plots showing the weights, the weighted returns and the covariance risk budgets of portfolios along that line. Those portfolios are chosen to be equally spaced in their returns. The weights and covariance risk budgets of those portfolios, along with their target return respectively target risk, are additionally listed in tables. The analyses within this chapter were done using the monthly nominal logarithmic returns.

Ternary Portfolio Frontier Since 1925

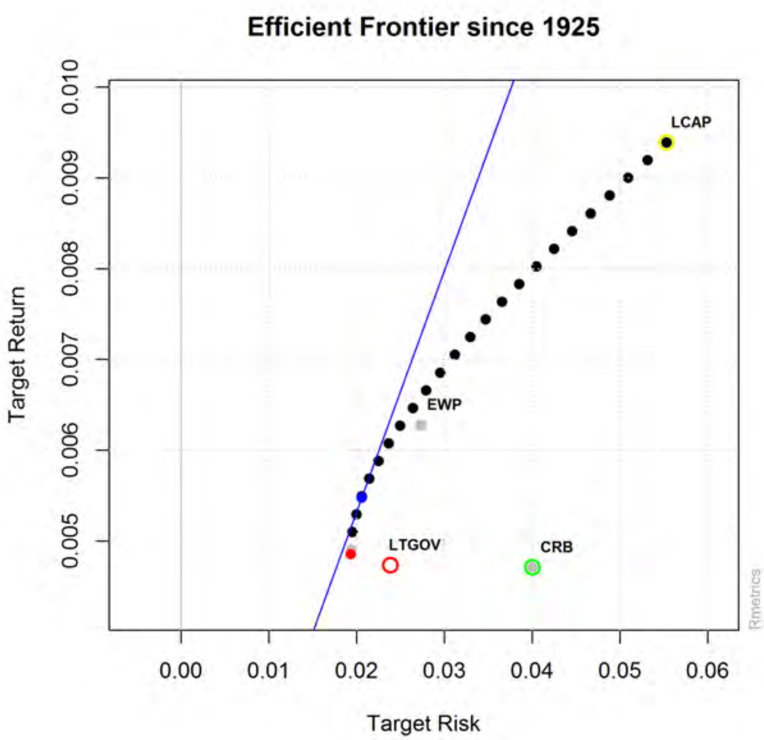

FIGURE 21.1: LTGOV, LCAP, and CRB portfolio. Data sets start 1925. The red point shows the minimum risk portfolio (MRP). The points above the MRP lie on the efficient frontier (black points), the points below the MRP on the minimum variance locus (gray points). The blue point indicates the tangency portfolio and the brown point the equal weights portfolio (EWP). The asset classes used to build the portfolios are shown as circles.

Ternary Portfolio Weights Since 1925

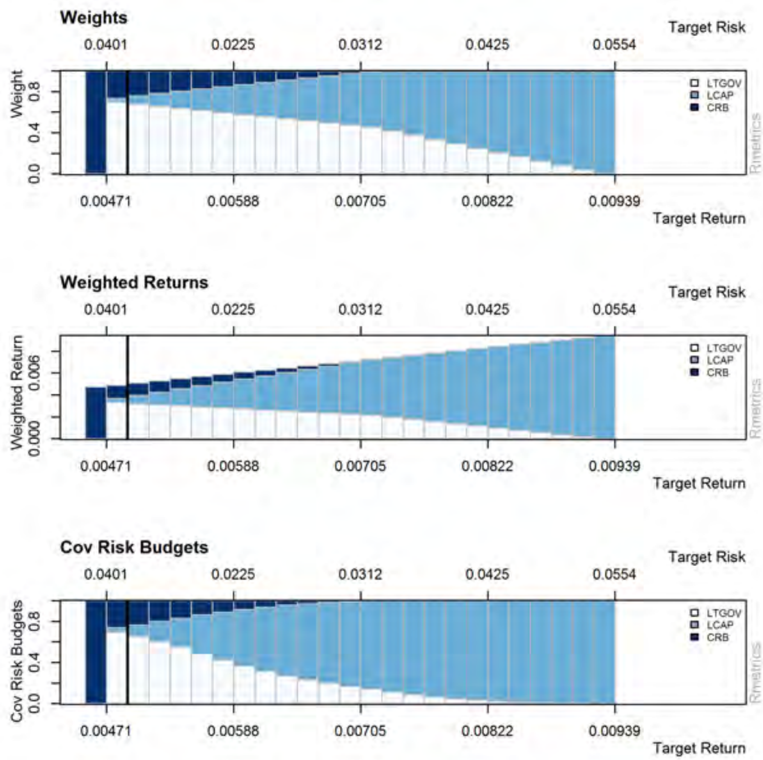

FIGURE 21.2: LTGOV, LCAP, and CRB portfolio weights plot. Data sets start 1925. For every plot the first portfolio on the left side of the black vertical line is the minimum risk portfolio (MRP). The portfolios on the left of the MRP are the portfolios on the minimum variance locus. The portfolios on the right of the MRP are the portfolios on the efficient frontier. The top plot shows the weights for the portfolios. The return (but not the necessarily the risk) along the minimum variance locus / efficient frontier line is steadily increasing which is shown in the middle plot along with the information of how much return is coming form which asset class. The bottom plot shows how much risk is coming form which asset class (in percent).

Ternary Portfolio Weights Table Since 1925

	V1	V2	V3	Target Return
1	0.00	0.00	1.00	0.00471
2	0.70	0.04	0.26	0.00490
3	0.68	0.08	0.24	0.00510
4	0.66	0.12	0.22	0.00529
5	0.64	0.16	0.20	0.00549
6	0.62	0.21	0.18	0.00568
7	0.60	0.25	0.16	0.00588
8	0.58	0.29	0.13	0.00607
9	0.56	0.33	0.11	0.00627
10	0.54	0.37	0.09	0.00646
11	0.52	0.41	0.07	0.00666
12	0.50	0.46	0.05	0.00685
13	0.48	0.50	0.03	0.00705
14	0.46	0.54	0.00	0.00724
15	0.42	0.58	0.00	0.00744
16	0.38	0.62	0.00	0.00763
17	0.33	0.67	0.00	0.00783
18	0.29	0.71	0.00	0.00802
19	0.25	0.75	0.00	0.00822
20	0.21	0.79	0.00	0.00841
21	0.17	0.83	0.00	0.00861
22	0.13	0.87	0.00	0.00880
23	0.08	0.92	0.00	0.00900
24	0.04	0.96	0.00	0.00919
25	0.00	1.00	0.00	0.00939

FIGURE 21.3: LTGOV, LCAP, and CRB portfolio weights table. Data sets start 1925.

Ternary Portfolio Blends Cov Risk Budgets since 1925

```
     V1   V2   V3 Target Risk
1  0.00 0.00 1.00     0.04005
2  0.70 0.04 0.26     0.01940
3  0.66 0.11 0.23     0.01958
4  0.61 0.19 0.20     0.02000
5  0.55 0.28 0.17     0.02064
6  0.49 0.38 0.14     0.02149
7  0.43 0.47 0.11     0.02251
8  0.37 0.55 0.08     0.02369
9  0.32 0.62 0.06     0.02500
10 0.27 0.68 0.04     0.02643
11 0.23 0.74 0.03     0.02795
12 0.20 0.78 0.02     0.02955
13 0.17 0.82 0.01     0.03123
14 0.14 0.86 0.00     0.03296
15 0.11 0.89 0.00     0.03475
16 0.09 0.91 0.00     0.03662
17 0.07 0.93 0.00     0.03854
18 0.05 0.95 0.00     0.04052
19 0.04 0.96 0.00     0.04255
20 0.02 0.98 0.00     0.04461
21 0.02 0.98 0.00     0.04671
22 0.01 0.99 0.00     0.04883
23 0.01 0.99 0.00     0.05099
24 0.00 1.00 0.00     0.05317
25 0.00 1.00 0.00     0.05536
```

FIGURE 21.4: LTGOV, LCAP, and CRB portfolio covariance risk budgets. Data sets start 1925.

Ternary Portfolio Frontier Since 1950

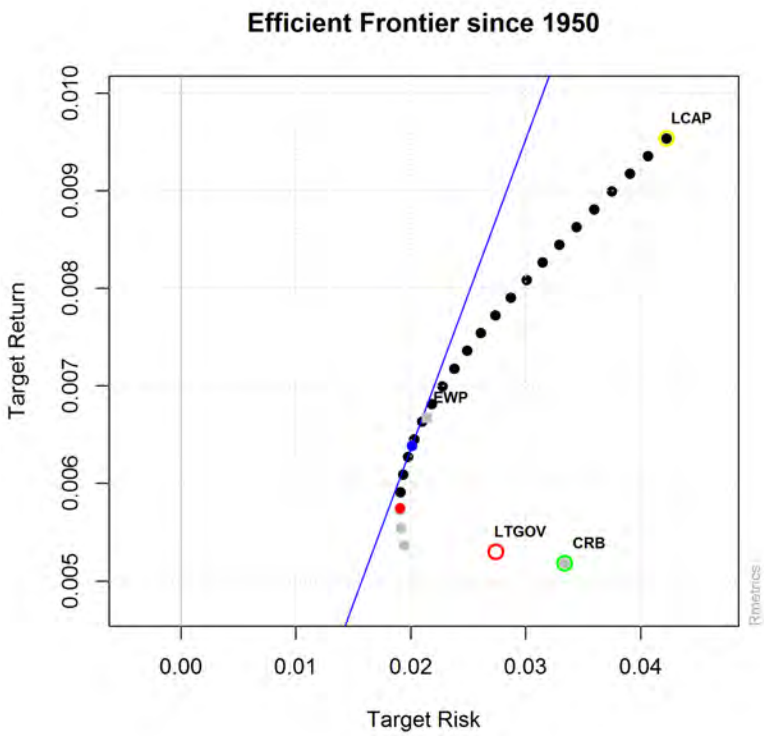

FIGURE 21.5: LTGOV, LCAP, and CRB portfolio frontier. Data sets start 1950.

Ternary Portfolio Weights Since 1950

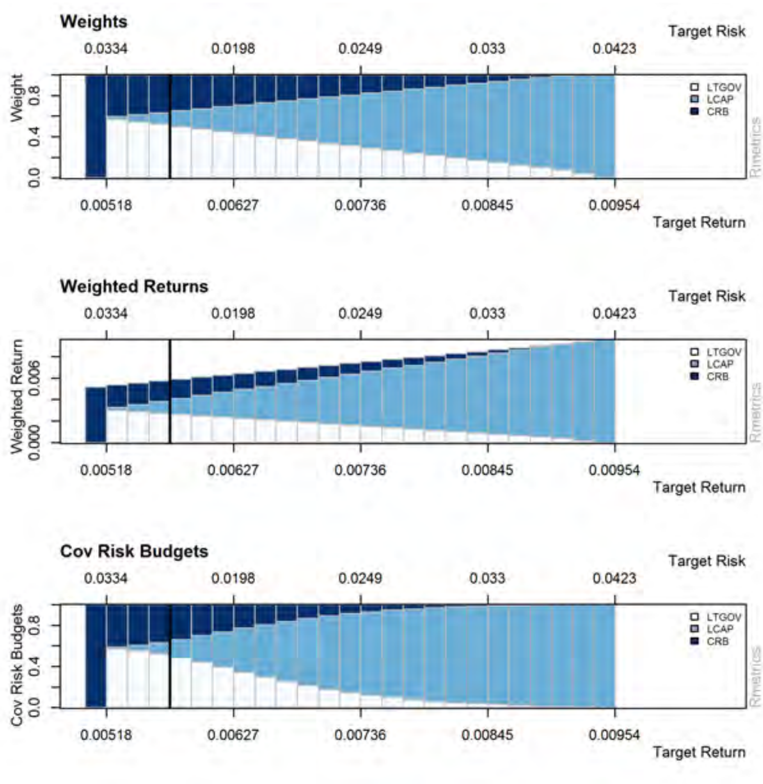

FIGURE 21.6: LTGOV, LCAP, and CRB portfolio weights plot. Data sets start 1950.

Ternary Portfolio Weights Table Since 1950

```
     V1   V2   V3 Target Return
1  0.00 0.00 1.00       0.00518
2  0.57 0.03 0.40       0.00536
3  0.55 0.07 0.39       0.00554
4  0.52 0.11 0.37       0.00572
5  0.50 0.15 0.35       0.00590
6  0.48 0.20 0.33       0.00609
7  0.45 0.24 0.31       0.00627
8  0.43 0.28 0.29       0.00645
9  0.41 0.32 0.27       0.00663
10 0.38 0.36 0.25       0.00681
11 0.36 0.41 0.23       0.00699
12 0.34 0.45 0.21       0.00718
13 0.31 0.49 0.20       0.00736
14 0.29 0.53 0.18       0.00754
15 0.27 0.58 0.16       0.00772
16 0.24 0.62 0.14       0.00790
17 0.22 0.66 0.12       0.00808
18 0.20 0.70 0.10       0.00826
19 0.17 0.75 0.08       0.00845
20 0.15 0.79 0.06       0.00863
21 0.13 0.83 0.04       0.00881
22 0.10 0.87 0.02       0.00899
23 0.08 0.91 0.00       0.00917
24 0.04 0.96 0.00       0.00935
25 0.00 1.00 0.00       0.00954
```

FIGURE 21.7: LTGOV, LCAP, and CRB portfolio weights table. Data sets start 1950.

Ternary Portfolio Blends Cov Risk Budgets since 1950

```
     V1   V2   V3 Target Risk
1  0.00 0.00 1.00     0.03342
2  0.57 0.02 0.41     0.01945
3  0.55 0.05 0.39     0.01918
4  0.52 0.11 0.37     0.01907
5  0.49 0.18 0.34     0.01914
6  0.44 0.26 0.30     0.01938
7  0.39 0.34 0.26     0.01977
8  0.35 0.43 0.23     0.02032
9  0.30 0.51 0.19     0.02102
10 0.25 0.58 0.16     0.02184
11 0.21 0.65 0.13     0.02277
12 0.18 0.71 0.11     0.02380
13 0.15 0.77 0.09     0.02492
14 0.12 0.81 0.07     0.02612
15 0.10 0.85 0.05     0.02739
16 0.08 0.88 0.04     0.02871
17 0.06 0.91 0.03     0.03008
18 0.05 0.93 0.02     0.03150
19 0.04 0.95 0.02     0.03296
20 0.03 0.96 0.01     0.03444
21 0.02 0.97 0.01     0.03596
22 0.02 0.98 0.00     0.03750
23 0.01 0.99 0.00     0.03907
24 0.00 1.00 0.00     0.04066
25 0.00 1.00 0.00     0.04228
```

FIGURE 21.8: LTGOV, LCAP, and CRB portfoliocovariance risk budgets. Data sets start 1950.

Ternary Portfolio Frontier Since 1975

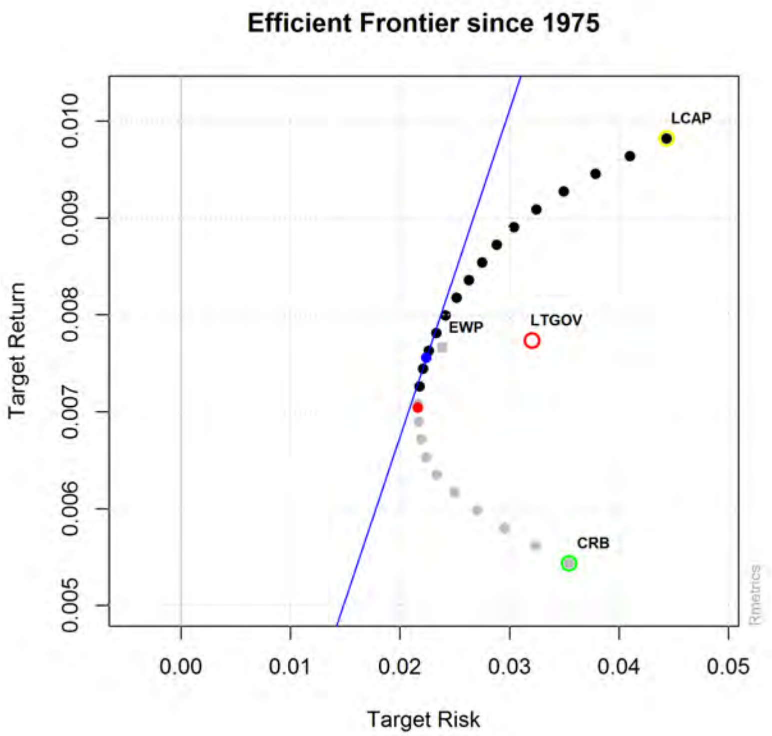

FIGURE 21.9: LTGOV, LCAP, and CRB portfolio frontier. Data sets start 1975.

Ternary Portfolio Weights Since 1975

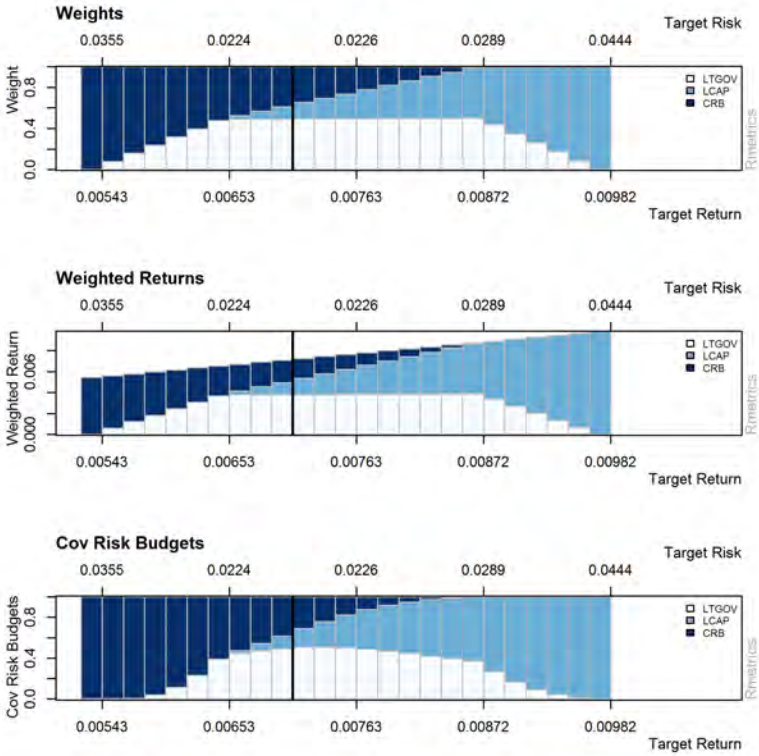

FIGURE 21.10: LTGOV, LCAP, and CRB portfolio weights. Data sets start 1975.

Ternary Portfolio Weights Table Since 1975

	V1	V2	V3	Target Return
1	0.00	0.00	1.00	0.00543
2	0.08	0.00	0.92	0.00562
3	0.16	0.00	0.84	0.00580
4	0.24	0.00	0.76	0.00598
5	0.32	0.00	0.68	0.00617
6	0.40	0.00	0.60	0.00635
7	0.48	0.00	0.52	0.00653
8	0.49	0.03	0.48	0.00671
9	0.49	0.08	0.43	0.00690
10	0.49	0.12	0.39	0.00708
11	0.49	0.16	0.35	0.00726
12	0.49	0.20	0.31	0.00744
13	0.50	0.24	0.26	0.00763
14	0.50	0.28	0.22	0.00781
15	0.50	0.32	0.18	0.00799
16	0.50	0.36	0.14	0.00817
17	0.50	0.40	0.10	0.00836
18	0.50	0.45	0.05	0.00854
19	0.50	0.49	0.01	0.00872
20	0.44	0.56	0.00	0.00890
21	0.35	0.65	0.00	0.00909
22	0.26	0.74	0.00	0.00927
23	0.18	0.82	0.00	0.00945
24	0.09	0.91	0.00	0.00963
25	0.00	1.00	0.00	0.00982

FIGURE 21.11: LTGOV, LCAP, and CRB portfolio weights table. Data sets start 1975.

Ternary Portfolio Blends Cov Risk Budgets since 1975

```
     V1   V2   V3 Target Risk
1  0.00 0.00 1.00     0.03547
2  0.00 0.00 1.00     0.03241
3  0.01 0.00 0.99     0.02958
4  0.04 0.00 0.96     0.02707
5  0.11 0.00 0.89     0.02497
6  0.23 0.00 0.77     0.02337
7  0.39 0.00 0.61     0.02241
8  0.45 0.03 0.53     0.02196
9  0.47 0.07 0.46     0.02170
10 0.50 0.12 0.38     0.02164
11 0.51 0.18 0.31     0.02178
12 0.51 0.25 0.24     0.02212
13 0.50 0.32 0.18     0.02264
14 0.49 0.38 0.13     0.02333
15 0.47 0.45 0.08     0.02419
16 0.45 0.50 0.05     0.02518
17 0.42 0.55 0.03     0.02630
18 0.40 0.59 0.01     0.02753
19 0.37 0.63 0.00     0.02886
20 0.27 0.73 0.00     0.03041
21 0.17 0.83 0.00     0.03248
22 0.09 0.91 0.00     0.03499
23 0.04 0.96 0.00     0.03785
24 0.01 0.99 0.00     0.04100
25 0.00 1.00 0.00     0.04437
```

FIGURE 21.12: LTGOV, LCAP, and CRB portfolio covariance risk budgets. Data sets start 1975.

Ternary Portfolio Frontier Since 2000

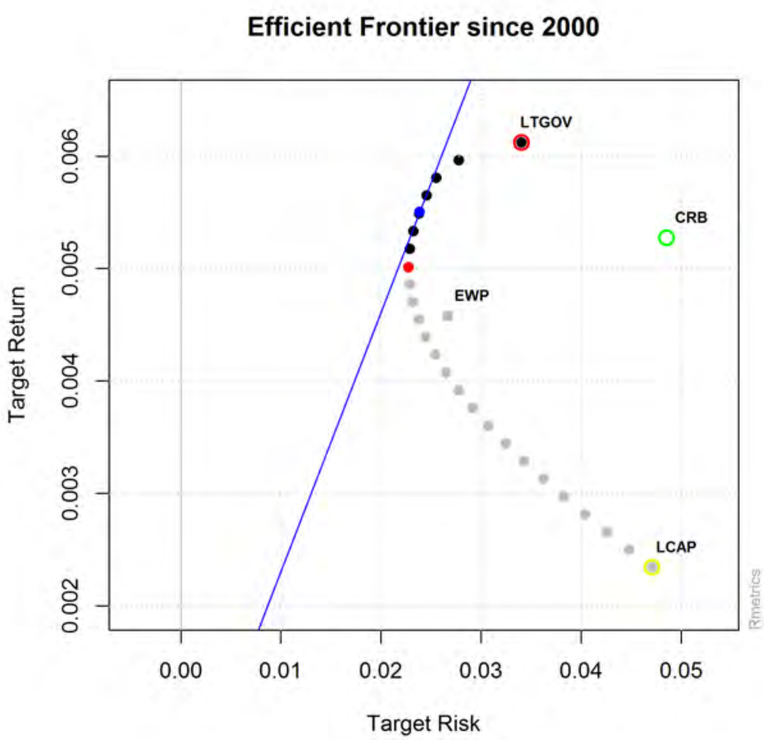

FIGURE 21.13: LTGOV, LCAP, and CRB portfolio frontier. Data sets start 2000.

Ternary Portfolio Weights Since 2000

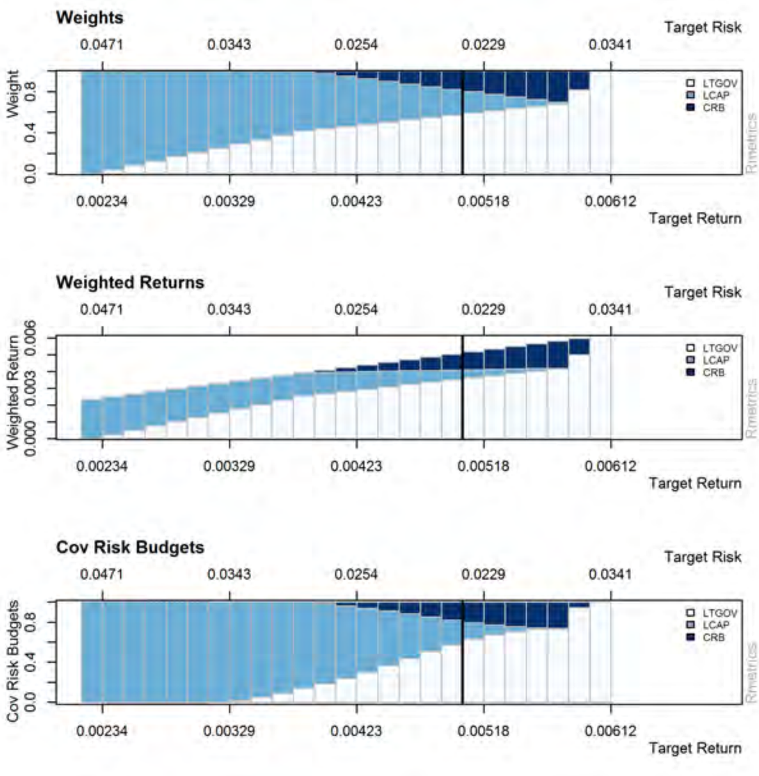

FIGURE 21.14: LTGOV, LCAP, and CRB portfolio weights plot. Data sets start 2000.

Ternary Portfolio Weights Table Since 2000

```
      V1   V2   V3 Target Return
1  0.00 1.00 0.00       0.00234
2  0.04 0.96 0.00       0.00250
3  0.08 0.92 0.00       0.00266
4  0.13 0.87 0.00       0.00281
5  0.17 0.83 0.00       0.00297
6  0.21 0.79 0.00       0.00313
7  0.25 0.75 0.00       0.00329
8  0.29 0.71 0.00       0.00344
9  0.33 0.67 0.00       0.00360
10 0.38 0.62 0.00       0.00376
11 0.42 0.58 0.00       0.00392
12 0.44 0.54 0.02       0.00407
13 0.46 0.49 0.05       0.00423
14 0.48 0.44 0.08       0.00439
15 0.51 0.39 0.10       0.00455
16 0.53 0.35 0.13       0.00470
17 0.55 0.30 0.15       0.00486
18 0.57 0.25 0.18       0.00502
19 0.59 0.20 0.20       0.00518
20 0.61 0.16 0.23       0.00533
21 0.64 0.11 0.25       0.00549
22 0.66 0.06 0.28       0.00565
23 0.68 0.01 0.31       0.00581
24 0.81 0.00 0.19       0.00596
25 1.00 0.00 0.00       0.00612
```

FIGURE 21.15: LTGOV, LCAP, and CRB portfolio weights table. Data sets start 2000.

Ternary Portfolio Blends Cov Risk Budgets since 2000

```
     V1   V2   V3 Target Risk
1  0.00 1.00 0.00     0.04709
2  0.00 1.00 0.00     0.04481
3  0.00 1.00 0.00     0.04258
4  0.00 1.00 0.00     0.04040
5  0.00 1.00 0.00     0.03829
6  0.00 1.00 0.00     0.03626
7  0.00 1.00 0.00     0.03431
8  0.02 0.98 0.00     0.03247
9  0.05 0.95 0.00     0.03075
10 0.09 0.91 0.00     0.02917
11 0.14 0.86 0.00     0.02777
12 0.19 0.80 0.02     0.02653
13 0.24 0.72 0.04     0.02543
14 0.30 0.64 0.06     0.02450
15 0.37 0.54 0.09     0.02376
16 0.44 0.44 0.12     0.02321
17 0.51 0.34 0.15     0.02288
18 0.57 0.25 0.18     0.02278
19 0.63 0.17 0.20     0.02291
20 0.67 0.10 0.23     0.02326
21 0.71 0.05 0.24     0.02382
22 0.73 0.02 0.26     0.02458
23 0.73 0.00 0.26     0.02553
24 0.94 0.00 0.06     0.02781
25 1.00 0.00 0.00     0.03407
```

FIGURE 21.16: LTGOV, LCAP, and CRB portfolio covariance risk budgets. Data sets start 2000.

chapterTernary Risk Maps
Ternary risk maps are barycentric plots on three variables which sum to a constant, usually 1. This is exactly the case for the weights of a fully invested long only constrained portfolio. The barycentric coordinate system allows to display any performance or risk metric on top of a triangle where the triangle describes all feasible portfolios. In the following we consider ternary portfolios which are designed from three asset classes.

A ternary plot describes all possible combinations of the three asset weights as points inside a equilateral triangle where every point represents a different unique portfolio.For each of those points, knowing the weights, we can directly evaluate any performance or risk measure on top of the triangle. Having a look at the plot, the weights of a portfolio can be found by simply following the dashed lines towards each of the three sides. There is only one way such that the found values sum to 1. For more details on how to read and/or construct such a plot we refer to Wikipedia [2011].

We display ternary plots for portfolios composed from bonds, stocks and commodities. The portfolios on top of the equilateral triangle map are evaluated by various risk measures. These are the portfolio return, the covariance risk, the value at risk, the expected shortfall and the portfolios maximum loss. To visualize the results, the values of the risk measure evaluation do have the same color if they lie within the same range. Those ranges are indicated within the plot by the values of the contour lines. We optimize portfolios for one scenario, i.e. portfolios with a dataset starting in 1925. The analyses within this section were done using the monthly nominal logarithmic returns.

21.1 PORTFOLIO RETURN

Bond/Stock/Commodity portfolio

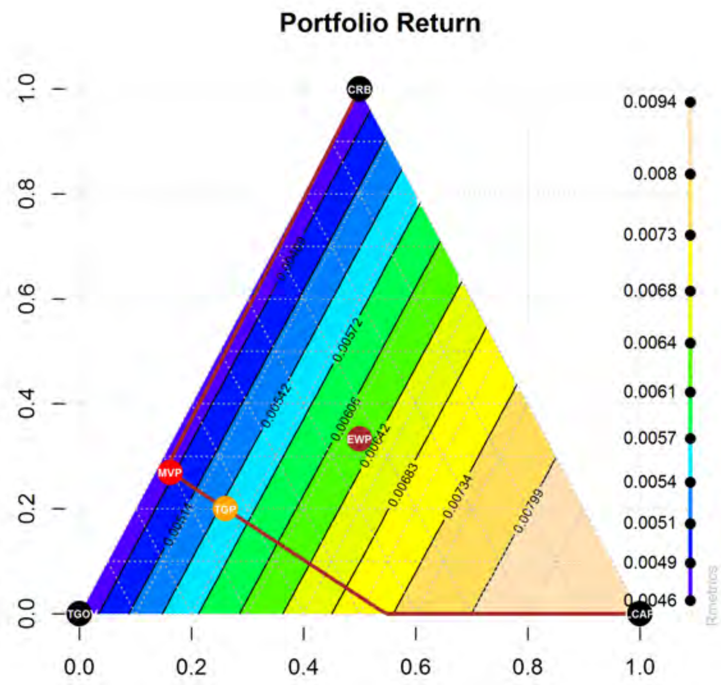

FIGURE 21.17: Ternary map of a portfolio's returns for a long term government bond, large cap stocks, and commoditity assets. The broen curve follows the minimum variance locus and the efficient frontier.

21.2 PORTFOLIO COVARIANCE RISK

Bond/Stock/Commodity portfolio

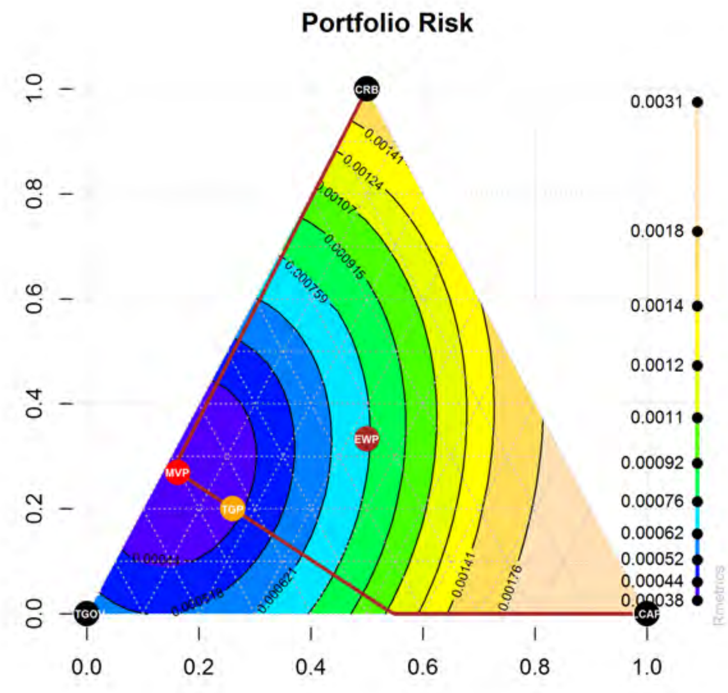

FIGURE 21.18: Ternary map of a portfolio's covariance risk for a long term government bond, large cap stocks, and commodity assets. The broen curve follows the minimum variance locus and the efficient frontier.

21.3 PORTFOLIO VALUE AT RISK

Bond/Stock/Commodity portfolio

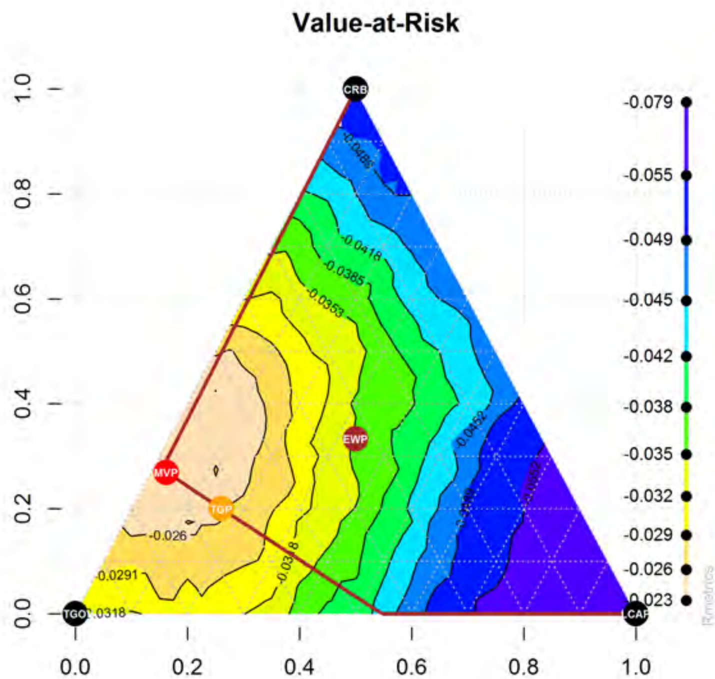

FIGURE 21.19: Ternary map of a portfolio's value at risk for a long term government bond, large cap stocks, and commodity assets. The broen curve follows the minimum variance locus and the efficient frontier.

21.4 PORTFOLIO EXPECTED SHORTFALL RISK

Bond/Stock/Commodity portfolio

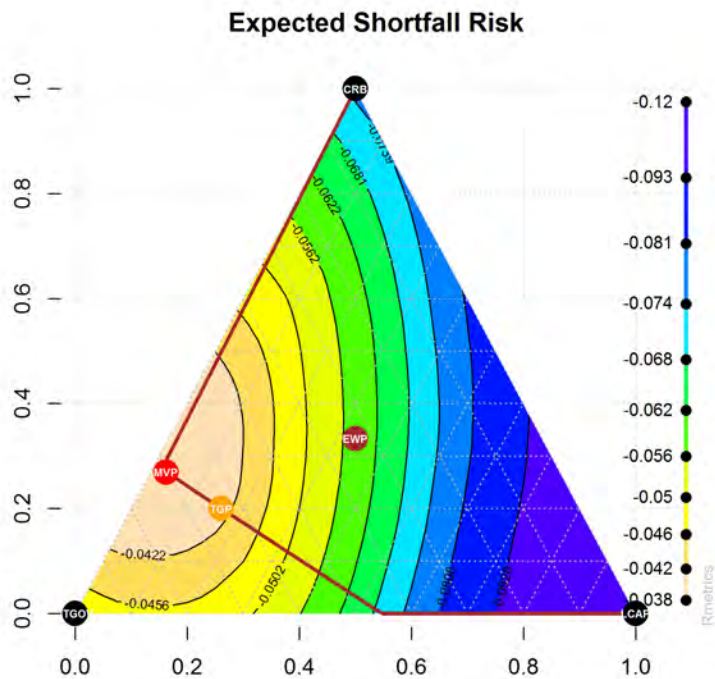

FIGURE 21.20: Ternary map of a portfolio's expected shortfall risk for a long term government bond, large cap stocks, and commodity assets. The broen curve follows the minimum variance locus and the efficient frontier.

21.5 PORTFOLIO MAXIMUM LOSS

Bond/Stock/Commodity portfolio

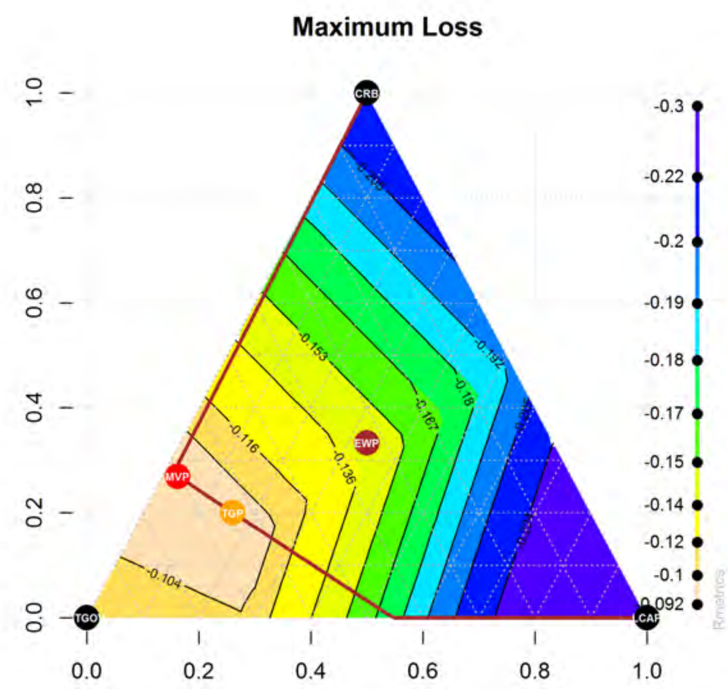

FIGURE 21.21: Ternary map of a portfolio's maximum loss for a long term government bond, large cap stocks, and commodity assets. The broen curve follows the minimum variance locus and the efficient frontier.

CHAPTER 22

MULTI ASSET PORTFOLIOS

Multi asset class portfolios are designed from more than three components. In this chapter we will we consider multi asset portfolios constructed from different types of bills, bonds, stocks and commodities. The portfolios are optimized using the long only constrained mean–variance portfolio approach of Markovitz which delivers a weight W_i for every asset considered.

The feasible set of tje portfolio is the area that holds all possible combinations of return and risk that can be achieved through the composition from different components. The efficient frontier is the line along the border of the feasible set that connects the minimum risk portfolio and the portfolio which has the highest return. Similarly the minimum variance locus connects the minimum risk portfolio and the portfolio which has the lowest return.

The portfolios we explore consist of all of our eight asset classes which are bills, different kinds of bonds and stocks, as well as commodities. As a proxy for bills we use the treasury bills from the time series BILL, for bonds we use the intermediate term government bonds from the time series ITGOV, the long term government bonds from the time series LTGOV, the long term corporate bonds from the time series LTCORP and the high yield corporate bonds from the time series HYCORP, for stocks we use the large cap stocks from the time series LCAP and the small cap stocks from the time series SCAP, and for commodities we use the commodity index from the time series CRB.

We optimize portfolios for four different scenarios, i.e. portfolios with datasets starting in 1925, 1950, 1975 and 2000. We display the efficient frontier and/or the minimum variance locus and create plots showing the weights, the weighted returns and the covariance risk budgets of portfolios along that line. Those portfolios are chosen to be equally spaced in their returns. The weights and covariance risk budgets of those portfolios, along with their target return respectively target risk, are additionally listed in tables. The analyses within this section were done using the monthly nominal logarithmic returns.

22.1 MULTI ASSETS CLASS PORTFOLIO

Multi Assets Class Portfolio

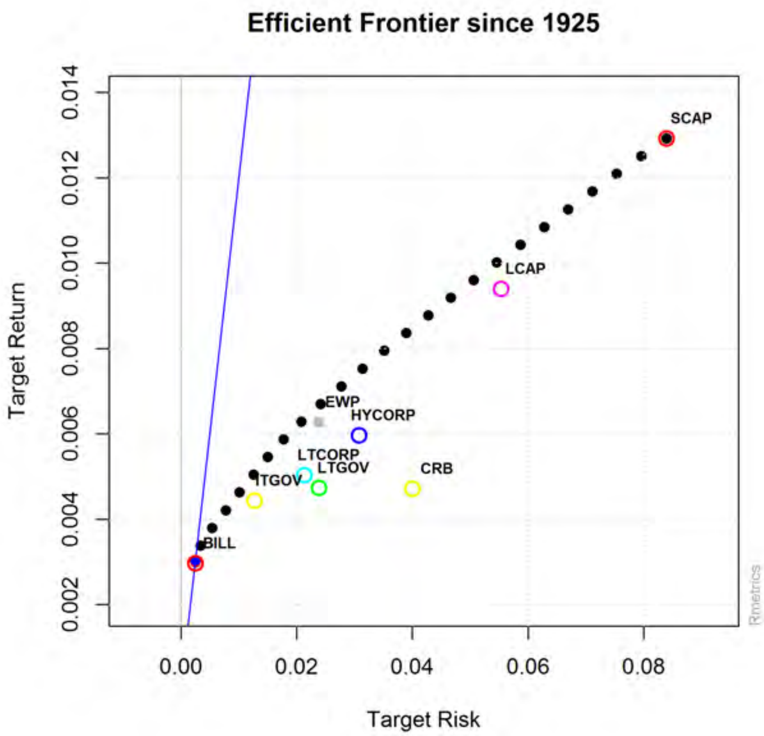

FIGURE 22.1: Multi assets class portfolio. Data set start in 1925. The red point shows the minimum risk portfolio (MRP). The points above the MRP lie on the efficient frontier (black points), the points below the MRP on the minimum variance locus (gray points). The blue point indicates the tangency portfolio and the brown point the equal weights portfolio (EWP). The asset classes used to build the portfolios are shown as circles.

Multi Assets Class Weights

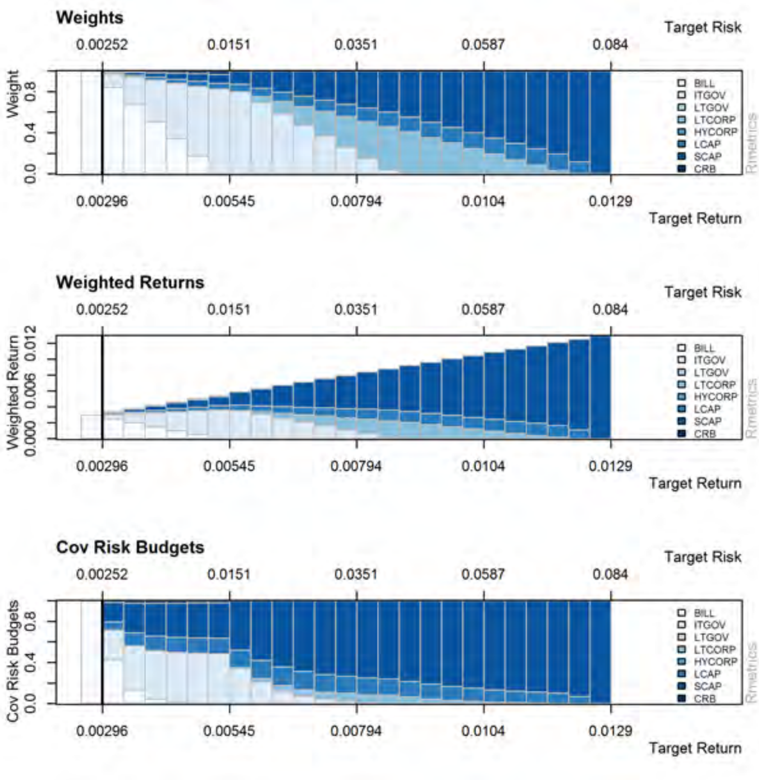

FIGURE 22.2: Multi assets class portfolio. Data set start in 1925. For every plot the first portfolio on the left side of the black vertical line is the minimum risk portfolio (MRP). The portfolios on the left of the MRP are the portfolios on the minimum variance locus. The portfolios on the right of the MRP are the portfolios on the efficient frontier. The top plot shows the weights for the portfolios. The return (but not the necessarily the risk) along the minimum variance locus / efficient frontier line is steadily increasing which is shown in the middle plot along with the information of how much return is coming form which asset class. The bottom plot shows how much risk is coming form which asset class (in percent).

Multi Assets Class Weights Table

	V1	V2	V3	V4	V5	V6	V7	V8	Target Return
1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00296
2	0.84	0.13	0.00	0.00	0.00	0.01	0.02	0.01	0.00338
3	0.67	0.27	0.00	0.00	0.00	0.02	0.03	0.01	0.00379
4	0.51	0.40	0.00	0.00	0.00	0.03	0.04	0.02	0.00421
5	0.34	0.54	0.00	0.00	0.00	0.04	0.06	0.02	0.00462
6	0.17	0.68	0.00	0.00	0.00	0.05	0.07	0.03	0.00504
7	0.01	0.81	0.00	0.00	0.00	0.06	0.09	0.03	0.00545
8	0.00	0.80	0.00	0.00	0.00	0.07	0.13	0.00	0.00586
9	0.00	0.69	0.00	0.06	0.00	0.08	0.17	0.00	0.00628
10	0.00	0.58	0.00	0.12	0.00	0.09	0.20	0.00	0.00669
11	0.00	0.47	0.00	0.18	0.00	0.10	0.24	0.00	0.00711
12	0.00	0.37	0.00	0.24	0.00	0.11	0.28	0.00	0.00752
13	0.00	0.26	0.00	0.30	0.00	0.12	0.32	0.00	0.00794
14	0.00	0.15	0.00	0.36	0.00	0.13	0.36	0.00	0.00835
15	0.00	0.04	0.00	0.42	0.00	0.14	0.40	0.00	0.00877
16	0.00	0.00	0.00	0.41	0.00	0.15	0.45	0.00	0.00918
17	0.00	0.00	0.00	0.35	0.00	0.15	0.50	0.00	0.00960
18	0.00	0.00	0.00	0.30	0.00	0.15	0.55	0.00	0.01001
19	0.00	0.00	0.00	0.25	0.00	0.15	0.60	0.00	0.01043
20	0.00	0.00	0.00	0.19	0.00	0.16	0.65	0.00	0.01084
21	0.00	0.00	0.00	0.14	0.00	0.16	0.70	0.00	0.01126
22	0.00	0.00	0.00	0.09	0.00	0.16	0.75	0.00	0.01167
23	0.00	0.00	0.00	0.03	0.00	0.16	0.80	0.00	0.01209
24	0.00	0.00	0.00	0.00	0.00	0.12	0.88	0.00	0.01250
25	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.01291

FIGURE 22.3: Multi assets class portfolio. Data set start in 1925.

Multi Assets Class Risk Budget Table

	V1	V2	V3	V4	V5	V6	V7	V8	Target Risk
1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00252
2	0.42	0.29	0.00	0.01	0.00	0.08	0.19	0.02	0.00346
3	0.13	0.43	0.00	0.00	0.00	0.13	0.29	0.02	0.00548
4	0.05	0.47	0.00	0.00	0.00	0.14	0.32	0.02	0.00778
5	0.02	0.48	0.00	0.00	0.00	0.15	0.33	0.02	0.01018
6	0.01	0.49	0.00	0.00	0.00	0.15	0.34	0.02	0.01261
7	0.00	0.49	0.00	0.00	0.00	0.15	0.34	0.02	0.01507
8	0.00	0.35	0.00	0.00	0.00	0.17	0.48	0.00	0.01776
9	0.00	0.21	0.00	0.03	0.00	0.18	0.58	0.00	0.02084
10	0.00	0.13	0.00	0.05	0.00	0.18	0.64	0.00	0.02421
11	0.00	0.08	0.00	0.06	0.00	0.17	0.69	0.00	0.02775
12	0.00	0.04	0.00	0.07	0.00	0.17	0.72	0.00	0.03141
13	0.00	0.02	0.00	0.07	0.00	0.17	0.73	0.00	0.03514
14	0.00	0.01	0.00	0.08	0.00	0.16	0.75	0.00	0.03894
15	0.00	0.00	0.00	0.08	0.00	0.16	0.76	0.00	0.04277
16	0.00	0.00	0.00	0.06	0.00	0.15	0.78	0.00	0.04665
17	0.00	0.00	0.00	0.05	0.00	0.14	0.81	0.00	0.05061
18	0.00	0.00	0.00	0.03	0.00	0.13	0.83	0.00	0.05463
19	0.00	0.00	0.00	0.02	0.00	0.13	0.85	0.00	0.05870
20	0.00	0.00	0.00	0.02	0.00	0.12	0.86	0.00	0.06282
21	0.00	0.00	0.00	0.01	0.00	0.11	0.88	0.00	0.06697
22	0.00	0.00	0.00	0.00	0.00	0.11	0.89	0.00	0.07114
23	0.00	0.00	0.00	0.00	0.00	0.10	0.89	0.00	0.07535
24	0.00	0.00	0.00	0.00	0.00	0.07	0.93	0.00	0.07958
25	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.08397

FIGURE 22.4: Multi assets class portfolio covariance risk budgets. Data set start in 1925.

Multi Assets Class Portfolio

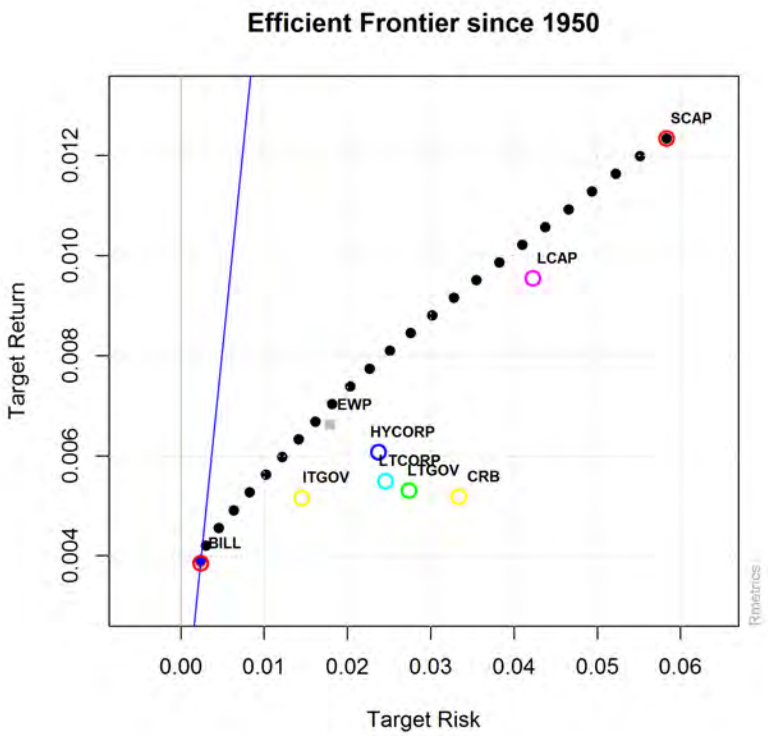

FIGURE 22.5: Multi assets class portfolio frontier. Data set start in 1950.

Multi Assets Class Weights

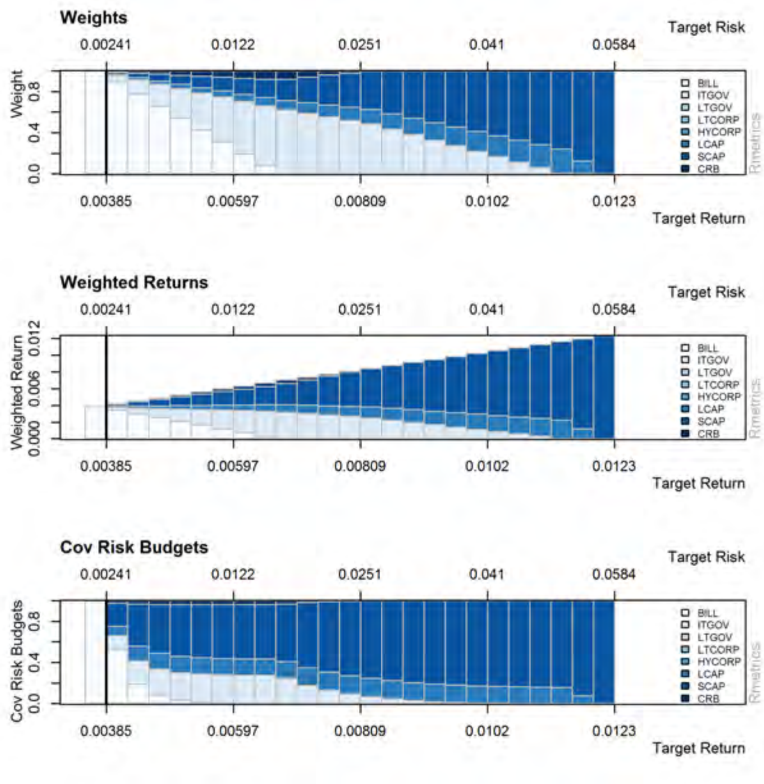

FIGURE 22.6: Multi assets class portfolio weights plot. Data set start in 1950.

Multi Assets Class Weights Table

	V1	V2	V3	V4	V5	V6	V7	V8	Target Return
1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00385
2	0.89	0.06	0.00	0.00	0.00	0.01	0.02	0.01	0.00420
3	0.77	0.14	0.00	0.00	0.00	0.02	0.04	0.02	0.00455
4	0.66	0.21	0.00	0.00	0.00	0.03	0.07	0.03	0.00491
5	0.54	0.29	0.00	0.00	0.00	0.04	0.09	0.04	0.00526
6	0.43	0.36	0.00	0.00	0.00	0.05	0.11	0.05	0.00561
7	0.31	0.44	0.00	0.00	0.00	0.06	0.13	0.06	0.00597
8	0.19	0.51	0.00	0.00	0.00	0.07	0.16	0.07	0.00632
9	0.08	0.58	0.00	0.00	0.00	0.08	0.18	0.08	0.00667
10	0.00	0.62	0.00	0.00	0.00	0.09	0.21	0.08	0.00703
11	0.00	0.59	0.00	0.00	0.00	0.10	0.25	0.06	0.00738
12	0.00	0.56	0.00	0.00	0.00	0.11	0.29	0.04	0.00773
13	0.00	0.52	0.00	0.00	0.00	0.13	0.33	0.02	0.00809
14	0.00	0.49	0.00	0.00	0.00	0.14	0.37	0.00	0.00844
15	0.00	0.43	0.00	0.00	0.00	0.15	0.42	0.00	0.00880
16	0.00	0.38	0.00	0.00	0.00	0.16	0.46	0.00	0.00915
17	0.00	0.33	0.00	0.00	0.00	0.17	0.50	0.00	0.00950
18	0.00	0.27	0.00	0.00	0.00	0.18	0.55	0.00	0.00986
19	0.00	0.22	0.00	0.00	0.00	0.19	0.59	0.00	0.01021
20	0.00	0.17	0.00	0.00	0.00	0.20	0.63	0.00	0.01056
21	0.00	0.11	0.00	0.00	0.00	0.21	0.67	0.00	0.01092
22	0.00	0.06	0.00	0.00	0.00	0.22	0.72	0.00	0.01127
23	0.00	0.01	0.00	0.00	0.00	0.23	0.76	0.00	0.01162
24	0.00	0.00	0.00	0.00	0.00	0.13	0.87	0.00	0.01198
25	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.01233

FIGURE 22.7: Multi assets class portfolio weights table. Data set start in 1950.

Multi Assets Class Risk Budget Table

	V1	V2	V3	V4	V5	V6	V7	V8	Target Risk
1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00241
2	0.53	0.13	0.00	0.00	0.00	0.09	0.22	0.02	0.00304
3	0.19	0.23	0.00	0.00	0.00	0.14	0.41	0.04	0.00456
4	0.08	0.26	0.00	0.00	0.00	0.15	0.47	0.04	0.00637
5	0.04	0.27	0.00	0.00	0.00	0.16	0.50	0.04	0.00827
6	0.02	0.27	0.00	0.00	0.00	0.16	0.51	0.04	0.01022
7	0.01	0.27	0.00	0.00	0.00	0.16	0.52	0.04	0.01220
8	0.00	0.27	0.00	0.00	0.00	0.16	0.52	0.04	0.01418
9	0.00	0.27	0.00	0.00	0.00	0.16	0.53	0.04	0.01618
10	0.00	0.25	0.00	0.00	0.00	0.16	0.56	0.03	0.01819
11	0.00	0.18	0.00	0.00	0.00	0.17	0.63	0.02	0.02036
12	0.00	0.13	0.00	0.00	0.00	0.17	0.69	0.01	0.02268
13	0.00	0.10	0.00	0.00	0.00	0.18	0.72	0.00	0.02510
14	0.00	0.07	0.00	0.00	0.00	0.18	0.75	0.00	0.02761
15	0.00	0.05	0.00	0.00	0.00	0.18	0.78	0.00	0.03019
16	0.00	0.03	0.00	0.00	0.00	0.17	0.80	0.00	0.03283
17	0.00	0.02	0.00	0.00	0.00	0.17	0.81	0.00	0.03552
18	0.00	0.01	0.00	0.00	0.00	0.17	0.82	0.00	0.03825
19	0.00	0.01	0.00	0.00	0.00	0.17	0.83	0.00	0.04101
20	0.00	0.00	0.00	0.00	0.00	0.16	0.83	0.00	0.04379
21	0.00	0.00	0.00	0.00	0.00	0.16	0.84	0.00	0.04660
22	0.00	0.00	0.00	0.00	0.00	0.16	0.84	0.00	0.04942
23	0.00	0.00	0.00	0.00	0.00	0.16	0.84	0.00	0.05225
24	0.00	0.00	0.00	0.00	0.00	0.08	0.92	0.00	0.05519
25	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.05836

FIGURE 22.8: Multi assets class portfolio covariance risk budgets. Data set start in 1950.

Multi Assets Class Portfolio

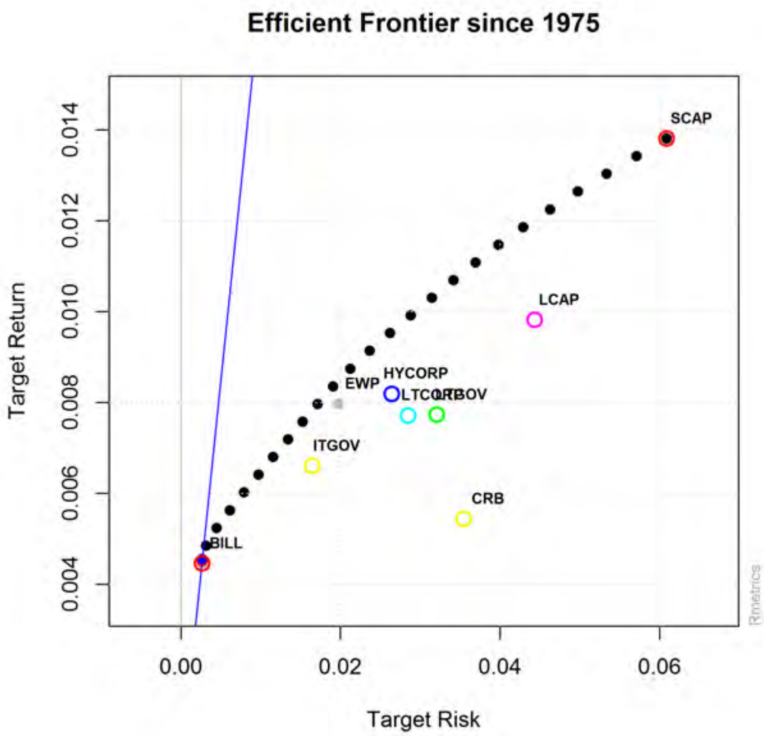

FIGURE 22.9: Multi assets class portfolio frontier. Data set start in 1975.

Multi Assets Class Weights

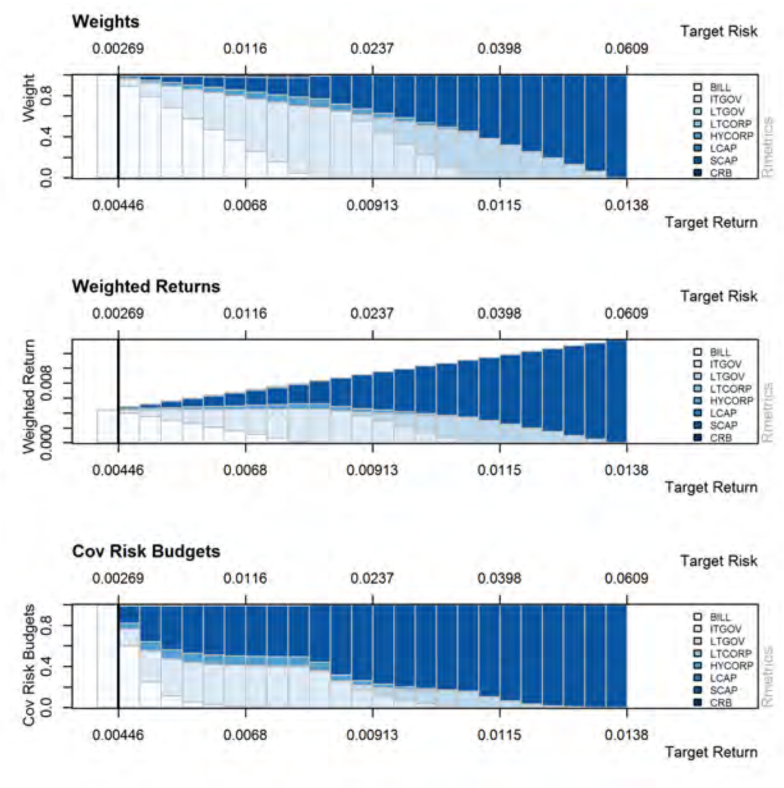

FIGURE 22.10: Multi assets class portfolio weights plot. Data set start in 1975.

Multi Assets Class Weights Table

```
     V1   V2   V3   V4   V5   V6   V7   V8 Target Return
1  1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00       0.00446
2  0.89 0.07 0.00 0.00 0.02 0.00 0.02 0.01       0.00485
3  0.79 0.14 0.00 0.00 0.02 0.00 0.04 0.01       0.00524
4  0.68 0.21 0.00 0.00 0.03 0.00 0.06 0.01       0.00563
5  0.58 0.28 0.00 0.00 0.04 0.00 0.08 0.01       0.00602
6  0.47 0.36 0.00 0.00 0.05 0.00 0.10 0.02       0.00641
7  0.37 0.43 0.00 0.00 0.06 0.00 0.13 0.02       0.00680
8  0.26 0.50 0.00 0.00 0.07 0.00 0.15 0.02       0.00719
9  0.16 0.58 0.00 0.00 0.08 0.00 0.17 0.02       0.00758
10 0.05 0.65 0.00 0.00 0.08 0.00 0.19 0.03       0.00796
11 0.00 0.68 0.00 0.00 0.09 0.00 0.22 0.01       0.00835
12 0.00 0.65 0.00 0.00 0.07 0.00 0.28 0.00       0.00874
13 0.00 0.55 0.07 0.00 0.06 0.00 0.33 0.00       0.00913
14 0.00 0.44 0.14 0.00 0.05 0.00 0.37 0.00       0.00952
15 0.00 0.33 0.22 0.00 0.04 0.00 0.42 0.00       0.00991
16 0.00 0.22 0.29 0.00 0.03 0.00 0.46 0.00       0.01030
17 0.00 0.11 0.36 0.00 0.02 0.00 0.51 0.00       0.01069
18 0.00 0.00 0.44 0.00 0.01 0.00 0.55 0.00       0.01108
19 0.00 0.00 0.38 0.00 0.00 0.00 0.62 0.00       0.01147
20 0.00 0.00 0.32 0.00 0.00 0.00 0.68 0.00       0.01186
21 0.00 0.00 0.26 0.00 0.00 0.00 0.74 0.00       0.01225
22 0.00 0.00 0.19 0.00 0.00 0.00 0.81 0.00       0.01264
23 0.00 0.00 0.13 0.00 0.00 0.00 0.87 0.00       0.01303
24 0.00 0.00 0.06 0.00 0.00 0.00 0.94 0.00       0.01342
25 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00       0.01381
```

FIGURE 22.11: Multi assets class portfolio weights table. Data set start in 1975.

Multi Assets Class Budgets Table

```
     V1   V2   V3   V4   V5   V6   V7   V8 Target Risk
1  1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00     0.00269
2  0.60 0.16 0.00 0.00 0.06 0.00 0.17 0.01     0.00319
3  0.25 0.30 0.00 0.00 0.09 0.00 0.35 0.01     0.00452
4  0.11 0.36 0.00 0.00 0.09 0.00 0.42 0.01     0.00616
5  0.05 0.38 0.00 0.00 0.09 0.00 0.46 0.01     0.00792
6  0.03 0.39 0.00 0.00 0.09 0.00 0.48 0.01     0.00973
7  0.01 0.40 0.00 0.00 0.09 0.00 0.49 0.01     0.01158
8  0.01 0.40 0.00 0.00 0.09 0.00 0.49 0.01     0.01344
9  0.00 0.40 0.00 0.00 0.09 0.00 0.50 0.01     0.01531
10 0.00 0.40 0.00 0.00 0.09 0.00 0.50 0.01     0.01719
11 0.00 0.36 0.00 0.00 0.08 0.00 0.56 0.00     0.01911
12 0.00 0.26 0.00 0.00 0.06 0.00 0.68 0.00     0.02128
13 0.00 0.18 0.04 0.00 0.04 0.00 0.74 0.00     0.02369
14 0.00 0.12 0.08 0.00 0.03 0.00 0.77 0.00     0.02621
15 0.00 0.07 0.10 0.00 0.02 0.00 0.80 0.00     0.02882
16 0.00 0.04 0.13 0.00 0.02 0.00 0.82 0.00     0.03148
17 0.00 0.02 0.14 0.00 0.01 0.00 0.83 0.00     0.03419
18 0.00 0.00 0.16 0.00 0.00 0.00 0.84 0.00     0.03694
19 0.00 0.00 0.10 0.00 0.00 0.00 0.90 0.00     0.03982
20 0.00 0.00 0.06 0.00 0.00 0.00 0.94 0.00     0.04295
21 0.00 0.00 0.04 0.00 0.00 0.00 0.96 0.00     0.04629
22 0.00 0.00 0.02 0.00 0.00 0.00 0.98 0.00     0.04978
23 0.00 0.00 0.01 0.00 0.00 0.00 0.99 0.00     0.05339
24 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00     0.05712
25 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00     0.06092
```

FIGURE 22.12: Multi assets class portfolio covariance risk budgets. Data set start in 1975.

Multi Assets Class Portfolio

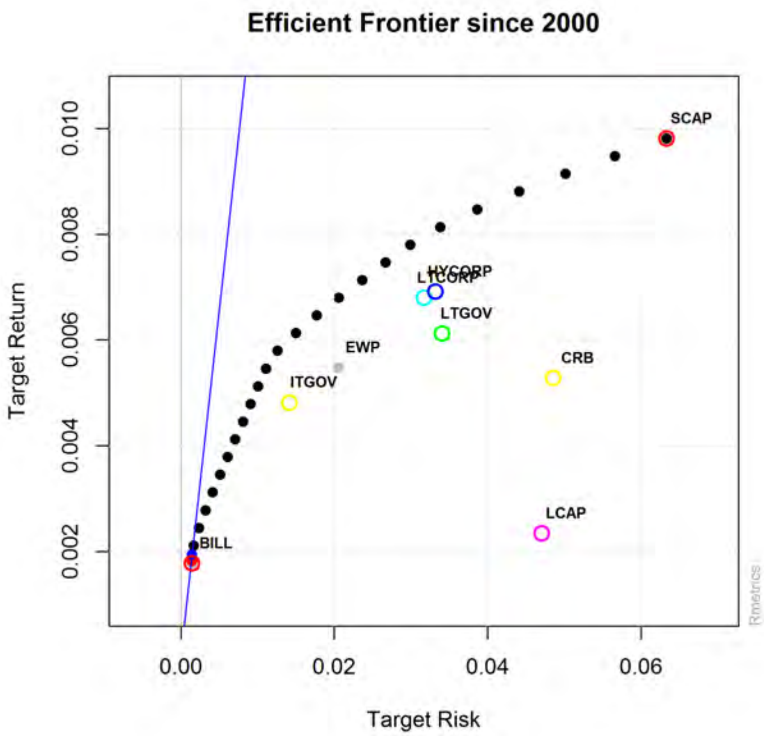

FIGURE 22.13: Multi assets class portfolio. Data set start in 2000.

Multi Assets Class Weights

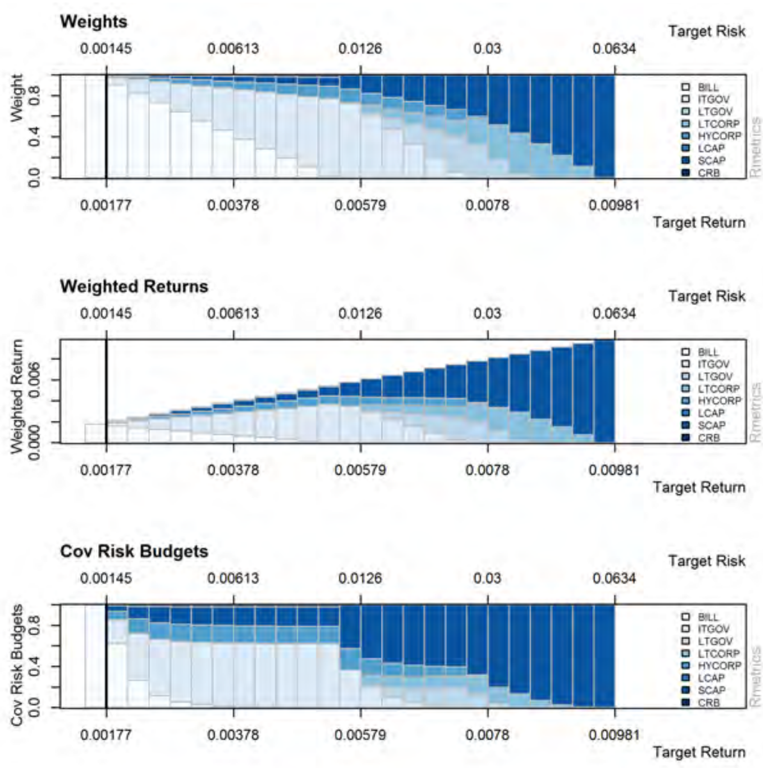

FIGURE 22.14: Multi assets class portfolio weights plot. Data set start in 2000.

Multi Assets Class Weights Table

	V1	V2	V3	V4	V5	V6	V7	V8	Target Return
1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00177
2	0.91	0.07	0.00	0.00	0.02	0.00	0.01	0.00	0.00211
3	0.82	0.13	0.00	0.00	0.03	0.00	0.01	0.00	0.00244
4	0.73	0.20	0.00	0.00	0.04	0.00	0.02	0.01	0.00278
5	0.64	0.27	0.00	0.00	0.05	0.00	0.03	0.01	0.00311
6	0.55	0.34	0.00	0.00	0.06	0.00	0.04	0.01	0.00345
7	0.46	0.41	0.00	0.00	0.07	0.00	0.05	0.01	0.00378
8	0.38	0.48	0.00	0.00	0.08	0.00	0.05	0.01	0.00412
9	0.29	0.55	0.00	0.00	0.09	0.00	0.06	0.02	0.00445
10	0.20	0.62	0.00	0.00	0.10	0.00	0.07	0.02	0.00479
11	0.11	0.68	0.00	0.00	0.11	0.00	0.08	0.02	0.00512
12	0.02	0.75	0.00	0.00	0.12	0.00	0.09	0.02	0.00546
13	0.00	0.73	0.00	0.00	0.13	0.00	0.14	0.00	0.00579
14	0.00	0.61	0.03	0.07	0.11	0.00	0.18	0.00	0.00613
15	0.00	0.47	0.11	0.09	0.11	0.00	0.22	0.00	0.00646
16	0.00	0.33	0.19	0.11	0.11	0.00	0.26	0.00	0.00680
17	0.00	0.19	0.27	0.14	0.11	0.00	0.29	0.00	0.00713
18	0.00	0.05	0.35	0.16	0.11	0.00	0.33	0.00	0.00747
19	0.00	0.00	0.32	0.26	0.02	0.00	0.40	0.00	0.00780
20	0.00	0.00	0.18	0.33	0.00	0.00	0.49	0.00	0.00814
21	0.00	0.00	0.04	0.40	0.00	0.00	0.56	0.00	0.00847
22	0.00	0.00	0.00	0.33	0.00	0.00	0.67	0.00	0.00880
23	0.00	0.00	0.00	0.22	0.00	0.00	0.78	0.00	0.00914
24	0.00	0.00	0.00	0.11	0.00	0.00	0.89	0.00	0.00947
25	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00981

FIGURE 22.15: Multi assets class portfolio weights table. Data set start in 2000.

Multi Assets Class Risk Budget Table

	V1	V2	V3	V4	V5	V6	V7	V8	Target Risk
1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00145
2	0.62	0.23	0.00	0.00	0.08	0.00	0.05	0.01	0.00169
3	0.26	0.46	0.00	0.00	0.14	0.00	0.12	0.02	0.00238
4	0.12	0.55	0.00	0.00	0.16	0.00	0.16	0.02	0.00325
5	0.06	0.59	0.00	0.00	0.17	0.00	0.17	0.02	0.00418
6	0.03	0.60	0.00	0.00	0.17	0.00	0.18	0.02	0.00515
7	0.02	0.61	0.00	0.00	0.17	0.00	0.18	0.02	0.00613
8	0.01	0.62	0.00	0.00	0.17	0.00	0.18	0.02	0.00712
9	0.00	0.62	0.00	0.00	0.17	0.00	0.19	0.02	0.00811
10	0.00	0.62	0.00	0.00	0.17	0.00	0.19	0.02	0.00911
11	0.00	0.62	0.00	0.00	0.17	0.00	0.19	0.02	0.01012
12	0.00	0.62	0.00	0.00	0.17	0.00	0.19	0.02	0.01112
13	0.00	0.36	0.00	0.00	0.21	0.00	0.42	0.00	0.01260
14	0.00	0.20	0.03	0.10	0.16	0.00	0.52	0.00	0.01501
15	0.00	0.10	0.09	0.11	0.13	0.00	0.56	0.00	0.01773
16	0.00	0.05	0.14	0.11	0.11	0.00	0.59	0.00	0.02063
17	0.00	0.02	0.17	0.12	0.10	0.00	0.60	0.00	0.02365
18	0.00	0.00	0.19	0.12	0.08	0.00	0.60	0.00	0.02673
19	0.00	0.00	0.14	0.17	0.01	0.00	0.68	0.00	0.02996
20	0.00	0.00	0.04	0.16	0.00	0.00	0.80	0.00	0.03386
21	0.00	0.00	0.00	0.13	0.00	0.00	0.87	0.00	0.03867
22	0.00	0.00	0.00	0.07	0.00	0.00	0.93	0.00	0.04415
23	0.00	0.00	0.00	0.03	0.00	0.00	0.97	0.00	0.05020
24	0.00	0.00	0.00	0.01	0.00	0.00	0.99	0.00	0.05665
25	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.06338

FIGURE 22.16: Multi assets class portfolio covariance risk budgets. Data set start in 2000.

CHAPTER 23

RISK SURFACES

Risk surfaces are exploratory graphics to display any performance or risk metric on top of the feasible set in the risk versus return plane for a given group of assets. In the following we consider multi asset portfolios which are designed from all our eight asset classes considered in this handbook.

To create a risk surface we first compute the hull of the feasible set within the risk versus return plane. Then we overlay the plane with a quadratic grid of points and choose those points which lie inside the hull. Each grid point belongs to a given return and risk value and represents multiple portfolios. For each of those grid points on the feasible set with given target risk and target return we search for a unique portfolio by optimizing the risk risk measure under investigation.

As examples we consider various types of risk measures. These are the variance of the weights as a measure of diversification, the Sharpe ratio, the maximum value of the covariance risk budgets and the variance of the covariance risk budgets as another measure of diversification. To visualize the results, the values of the risk measure evaluation are displayed as an image plot with optional contour lines. We optimize portfolios for four different scenarios, i.e. portfolios with datasets starting in 1925, 1950, 1975 and 2000. The analyses within this section were done using the monthly nominal logarithmic returns.

23.1 WEIGHTS DIVERSIFICATION

Diversification of weights since 1925

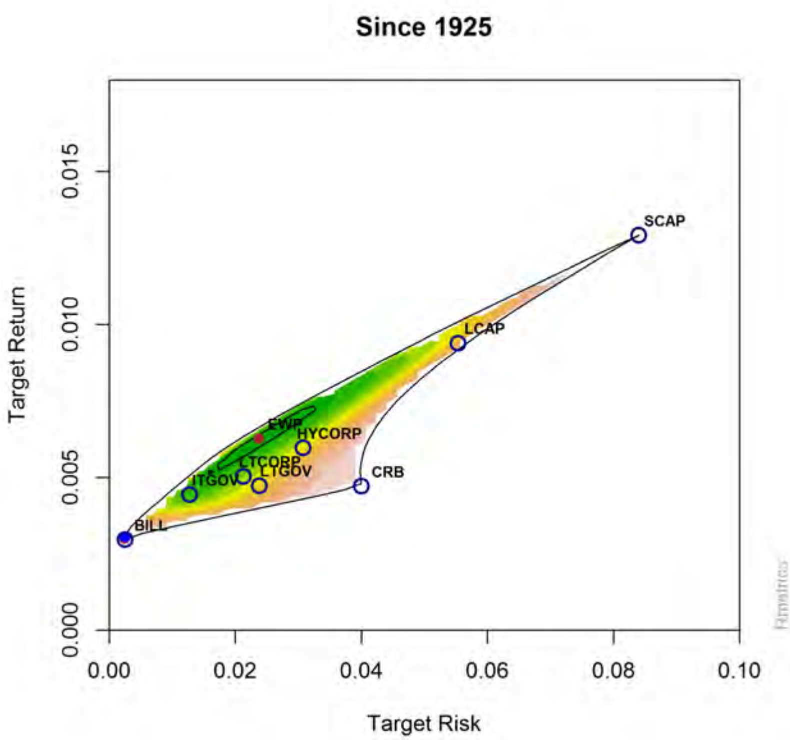

FIGURE 23.1: The solid black line depicts the feasible set. The red point shows the minimum risk portfolio, the blue point the tangency portfolio and the brown point the equal weights portfolio. The asset classes used to build the portfolios are shown as circles. For every portfolio the variance of the weights are plotted on top of the feasible set. The black contour line encircles the 5% lowest values.

Diversification of weights since 1950

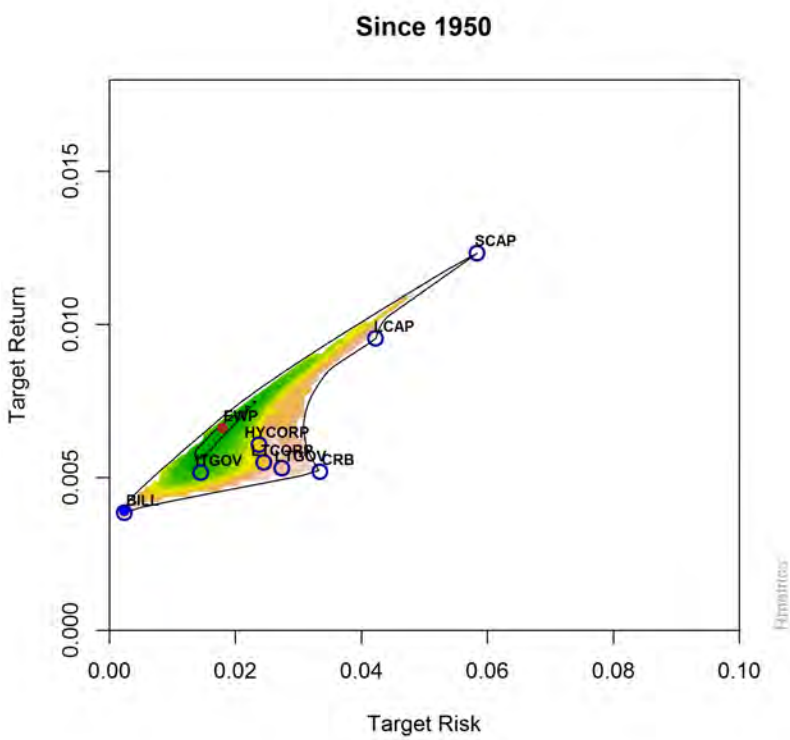

FIGURE 23.2: Risk Surface: Diversification of portfolio weights.

Diversification of weights since 1975

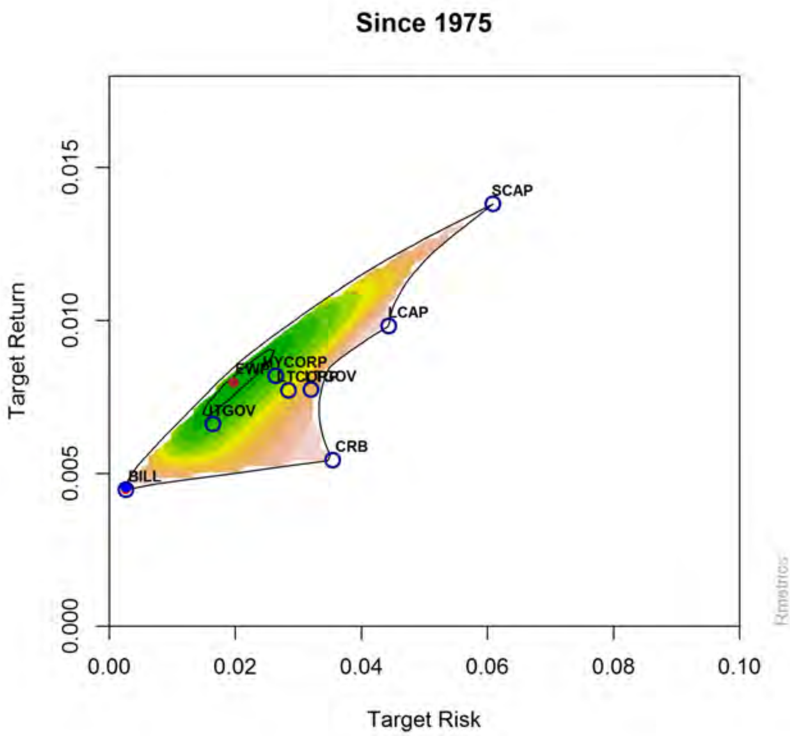

FIGURE 23.3: Risk Surface: Diversification of portfolio weights.

Diversification of weights since 2000

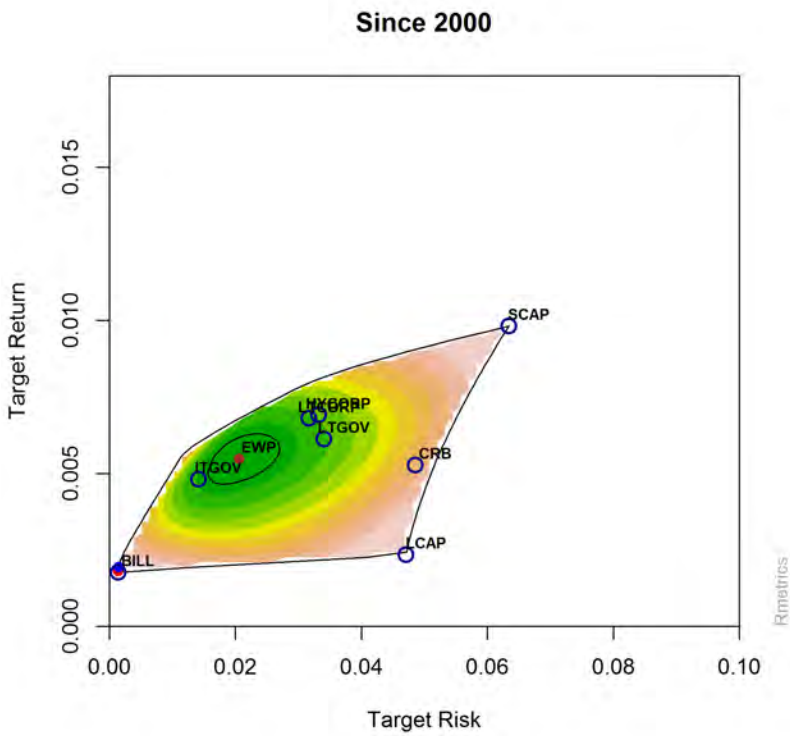

FIGURE 23.4: Risk Surface: Diversification of portfolio weights.

SHARPE RATIO

Sharpe ratio since 1925

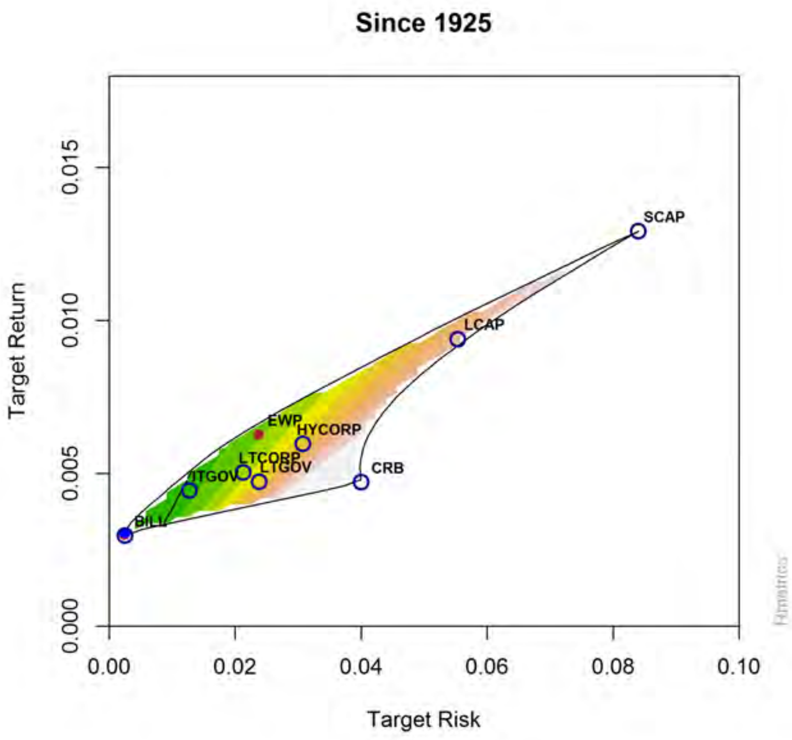

FIGURE 23.5: Risk Surface: Maximum Sharpe ratio. The black contour line encircles the 5% highest values.

Sharpe ratio since 1950

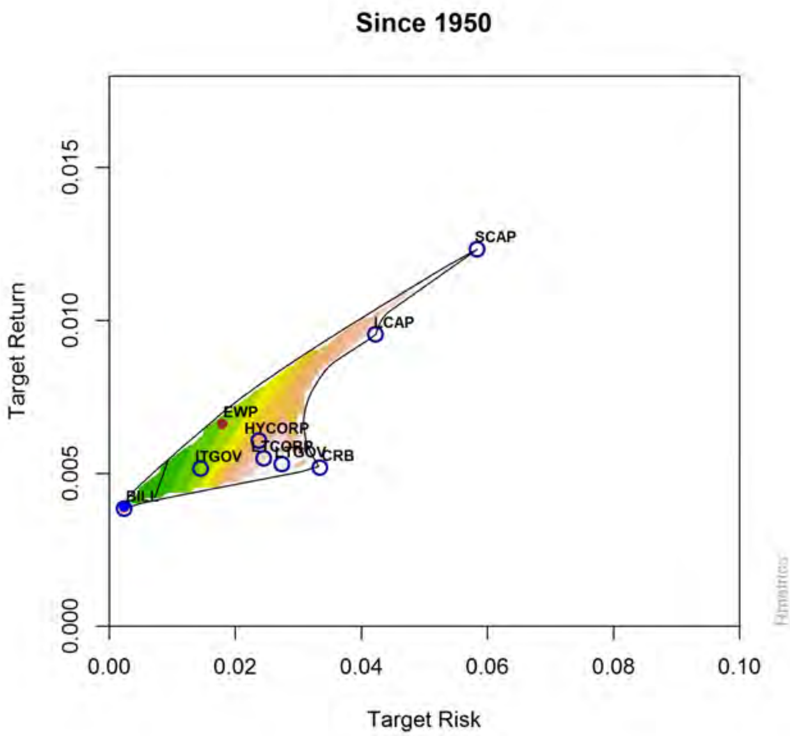

FIGURE 23.6: Risk Surface: Maximum Sharpe ratio.

Sharpe ratio since 1975

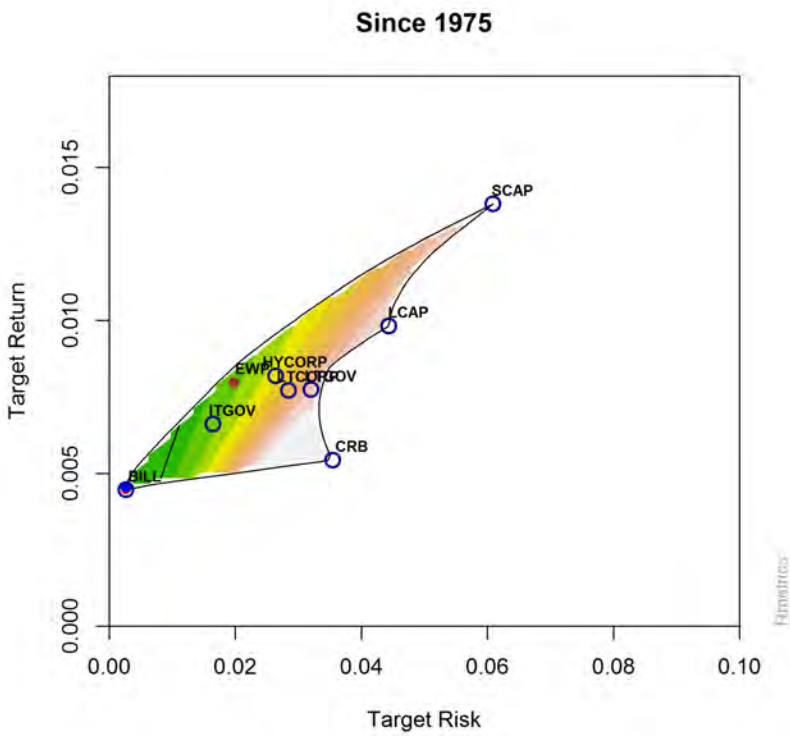

FIGURE 23.7: Risk Surface: Maximum Sharpe ratio.

Sharpe ratio since 2000

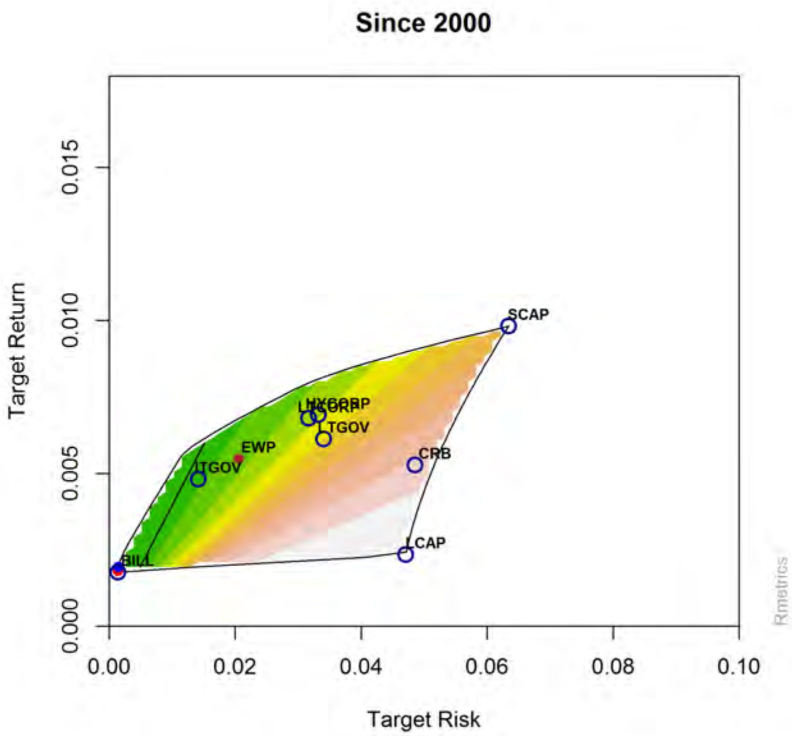

FIGURE 23.8: Risk Surface: Maximum Sharpe ratio.

MAX COVARIANCE RISK BUDGETS

Maximum covariance risk budgets since 1925

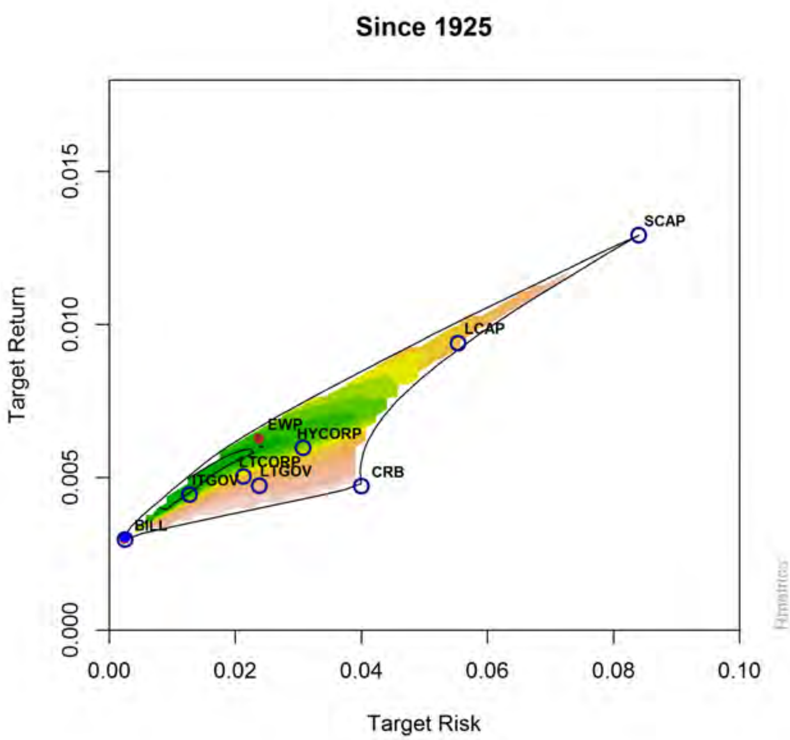

FIGURE 23.9: Risk Surface: Minimize the maximum covariance risk budget. The black contour line encircles the 5% lowest values.

Maximum covariance risk budgets since 1950

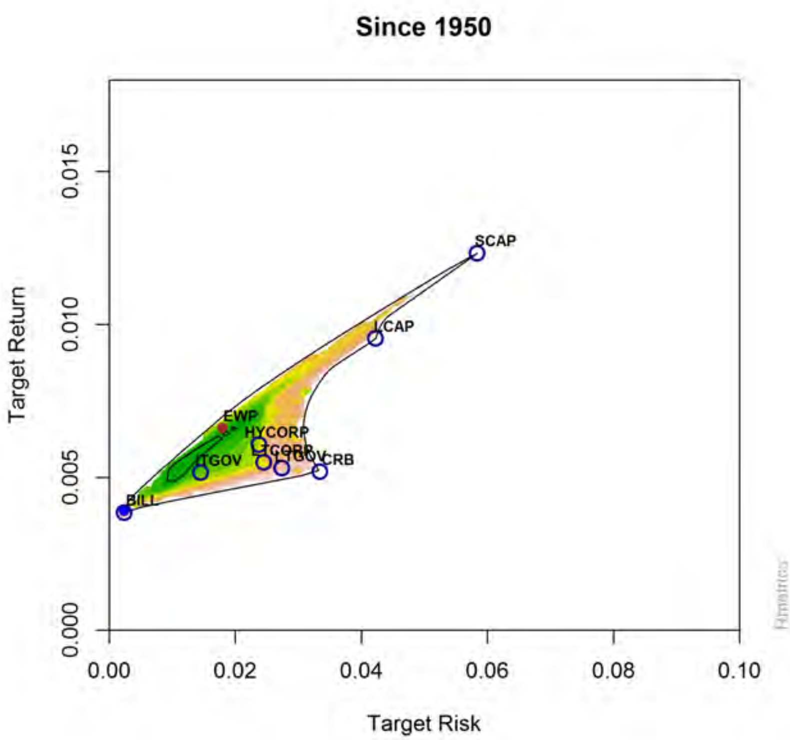

FIGURE 23.10: Risk Surface: Minimize the maximum covariance risk budget.

Maximum covariance risk budgets since 1975

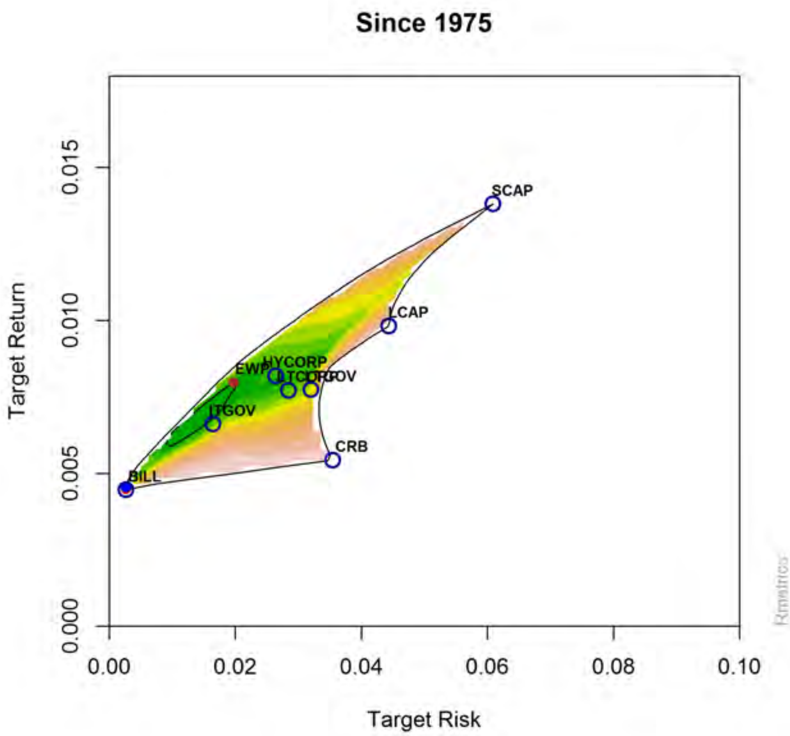

FIGURE 23.11: Risk Surface: Minimize the maximum covariance risk budget.

Maximum covariance risk budgets since 2000

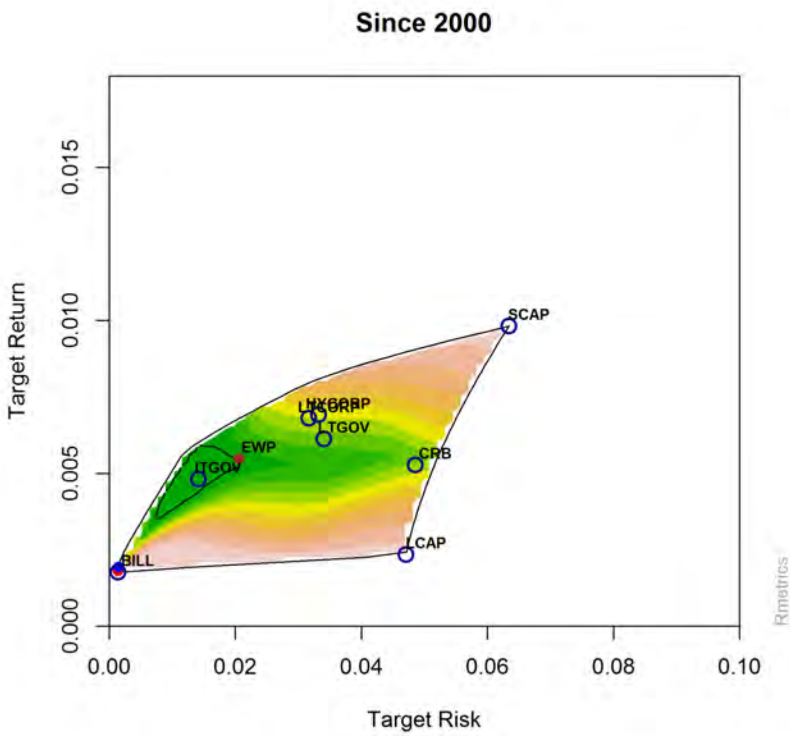

FIGURE 23.12: Risk Surface: Minimize the maximum covariance risk budget.

23.2 VALUE AT RISK DIVERSIFICATION

Diversification of Value at Risk since 1925

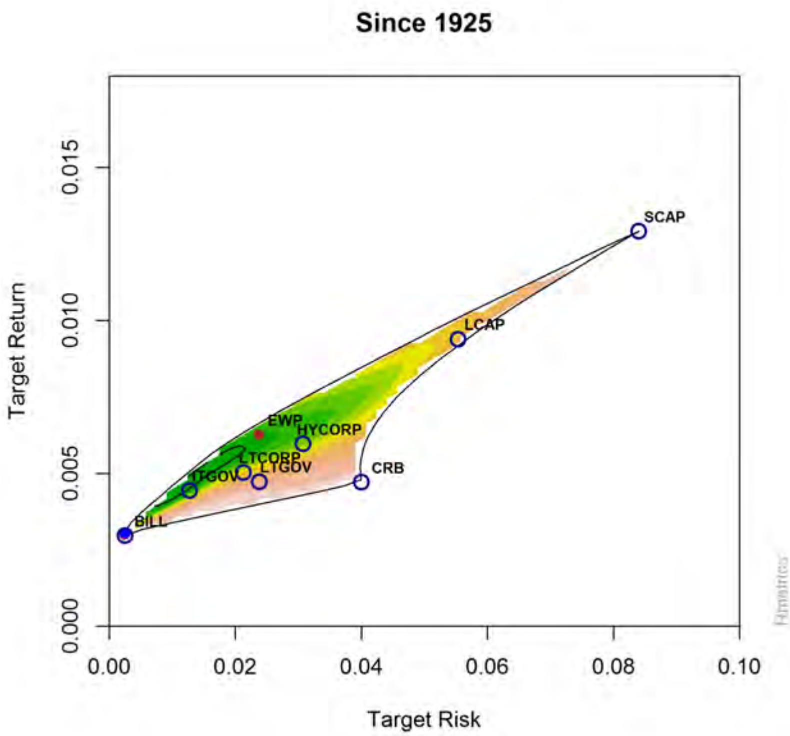

FIGURE 23.13: Risk Surface: Minimize the variance of Value at Risk. The black contour line encircles the 5% lowest values.

Diversification of Value at Risk since 1950

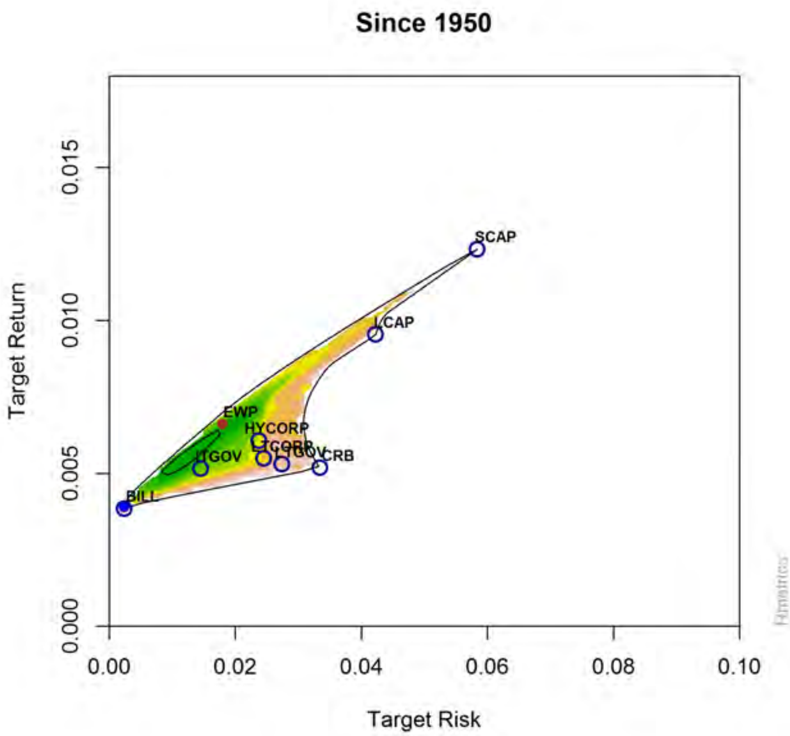

FIGURE 23.14: Risk Surface: Minimize the variance of Value at Risk.

Diversification of Value at Risk 1975

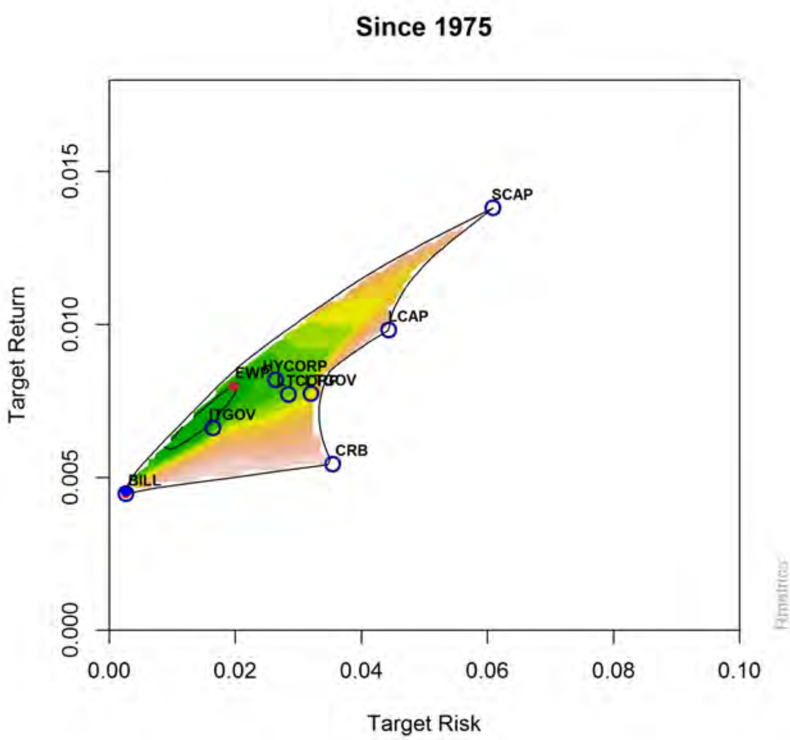

FIGURE 23.15: Risk Surface: Minimize the variance of Value at Risk.

Diversification of Value at Risk since 2000

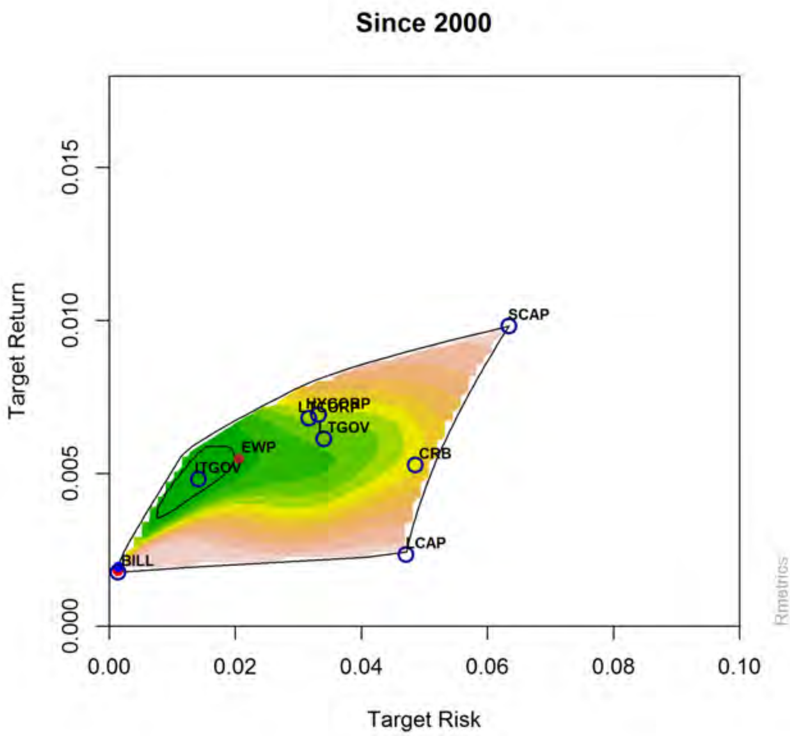

FIGURE 23.16: Risk Surface: Minimize the variance of Value at Risk.

Part VI

Appendix

Appendix A

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