Multivariate non-parametric efficiency test notes

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Let's start with the definition of the application: We want to check whether a forecast error is correlated with past (=known, available) information, including its own past. We assume non-Gaussian innovations, otherwise we would just have a case for various standard regression-based tests (possibly with corrections for heteroskedasticity). But with non-Gaussianity and the fact that the sample size in the time dimension is typically limited this opens the field for non-parametric approaches, like in C&G. (Or also C& Dufour 1995).

The simple case just considers one candidate information variable, say the first lag. The variable to be tested is then the product $e_t e_{t-1}$. Under the null of no correlation this variable would be centered around zero, which can be tested with a sign or signed rank test (Wilcoxon, which also assumes symmetry).

Extension to two lags: Now we have two information variables e_{t-1} and e_{t-2}. Strictly speaking this is *not* a standard paired two-sample test setup, because the location difference between $e_t e_{t-1}$ and $e_t e_{t-2}$ is not perfectly informative – even if the difference were zero the null hypothesis of no correlation could be violated at each lag. However, it is clear that if a location difference between the products is indeed found (significantly), then this would also reject the null that each location is zero. This means that the plain matched-sample test would have no power against one direction included in the alternative, namely if all information variables share the same correlation with the forecast errors. While the exact location equality (under H1) seems quite unlikely (set of measure zero), the power of the test might already be noticeably affected in a neighborhood of that point.

Extension to three or more lags: With more information variables we have in principle a matched K-sample situation. The standard Friedman test cannot explicitly test for non-correlation (zero location), either. But as in the two-lag case above, it is clear that if the test rejects homogeneity of correlation (homogeneity of locations of $e_t e_{t-1} ... e_t e_{t-p}$) then this implies rejection of global non-correlation.

A slightly different approach would be to introduce an artificial constant variable that fixes the location that is to be tested. In our case this would be an all-zero variable, and the K-sample setup would be converted into an effective (K+1)-sample setup. In general the inclusion of a constant helper variable is a non-standard method and this is probably the reason why nobody has done it so far. (?? Don't know if that's true.)

Some problems with the Friedman test to keep in mind:

1. Apparently presupposes homogeneous distributions (apart from the location). This is a problem if we want to include an aux variable of all-zeros.[1]

2. For variables with different dispersions across the "blocks" there's the Quade test as a refinement of the Friedman test.[2] That the different period-t observations in the forecast context might have inherently different variances sounds plausible.

3. The inclusion of an aux all-zero variable affects the effective K parameter of the test without adding an "actual" variable, so this might render the test conservative.

[1] OTOH this doesn't really seem to be a necessary assumption, might just affect the power (because only the ordinal information is used in Friedman). Remains to be seen.

[2] In order to make sense of the dispersions some metric/cardinal measurement is assumed here, but for forecast errors this is of course not a problem.

1 Proposed test setups

All this leads us to the following list of tests that could be compared,[3] where K denotes the number of information variable candidates (either own lags of forecast errors, or other available information), and $\tilde{K}$ denotes the effective number of "groups" in the test. An asterisk "*" marks those tests that have no power against the situation that the non-correlation is violated, but equally across all information variables (equal non-zero location).

1. $K = 2$: plain sign test* (on the difference)
2. $K = 2$: plain signed-rank test* (on the difference)
3. $K \geq 3$: plain Friedman test*
4. $K \geq 3$: plain Quade test*
5. $K \geq 2$: Friedman test including an auxiliary zero (constant) variable ($\tilde{K} = K + 1$)
6. $K \geq 2$: Quade test including an auxiliary zero (constant) variable ($\tilde{K} = K + 1$)
7. any K: all $K - 1$ possible pairwise C&G sign tests with a Bonferroni correction (as in C & Dufour?)
8. any K: all $K - 1$ possible pairwise C&G signed-rank tests with a Bonferroni correction (as in C & Dufour?)

[3] Non-parametric, that is. Further competition comes from the more or less standard parametric tests.