Economics Letters 35 (1991) 285–290
North-Holland

Over-rejections in rational expectations models

A non-parametric approach to the Mankiw–Shapiro problem *

Bryan Campbell
Concordia University, Montréal, Que. H3C 3J7, Canada

Jean-Marie Dufour
C.R.D.E., Université de Montréal, Que. H3C 3J7, Canada

Received 13 August 1990
Accepted 2 October 1990

Standard tests for rationality may not have the correct size if there is feedback from innovations to future values of the regressors. It is shown that nonparametric tests reject at their nominal level and display good power in a variety of specifications of a model involving feedback.

1. Introduction

The hypothesis that two time series are stochastically independent given the past is a frequent implication of economic theory. In particular, under usual stochastic assumptions, the claim that expectations are rational implies that prediction errors have mean zero and are independent of the information available when the predictions were formed. Yet standard testing procedure which attempt to assess the rationality of expectations may reject much too often, even with fairly large samples. The problem largely originates in feedback from disturbances that may affect future values of the regressors, even though the disturbance and the regressors are contemporaneously uncorrelated. For a simple linear regression, striking evidence of this difficulty was presented recently by Mankiw and Shapiro (1986, in what follows referred to as MS) who found by Monte Carlo techniques that the true level of the t-test may be considerably greater than its nominal level in a fairly simple model. The same authors also determined a revised set of critical values for the correct application of the t-statistic in the particular model studied by them. These critical values, however,

* This work was supported by the Centre de recherche et développement en économique (C.R.D.E.) of the Université de Montréal, the Natural Sciences and Engineering Research Council of Canada, the Social Sciences and Humanities Research Council of Canada and the Fonds FCAR (Government of Quebec). We thank Marcel Dagenais, Eric Ghysels, Marc Hallin and Pierre Perron for helpful comments and discussions.

are difficult to use in practice because they depend on unknown nuisance parameters; this problem is emphasized by Galbraith, Dolado and Banerjee (1987). For related results, see also Banerjee and Dolado (1987, 1988), Banerjee, Dolado and Glabraith (1990) and Mankiw and Shapiro (1985).

In this paper, we introduce non-parametric analogues of the t-test, based on sign statistics and Wilcoxon signed-rank statistics, that are applicable in the context of an important variant of the MS framework, and we compare their behavior with the t-test both under the null hypothesis and the alternative. The sign tests are *probably exact* in this case, irrespective of the presence of feedback even if the disturbances are non-normal or heteroskedastic (with heteroskedasticity of unknown form). With regard to the Wilcoxon statistics we could not prove exactness analytically in the model studied here, but we present simulation results indicating that Wilcoxon tests reject at their nominal levels and are robust to various changes in the nature of the disturbances. Further the non-parametric tests suggested display good power relative to the t-test applied using either asymptotic or size-corrected critical values. Indeed, in the presence of feedback or non-normality, the powers of the sign and Wilcoxon tests can be considerably superior to that of the t-test. The methods described in this paper are an extension of Dufour (1981) where various non-parametric statistics are proposed to test independence against serial correlation alternative in a time series.

In section 2, after describing the model, we introduce the relevant test statistics and derive the exact distribution of the sign statistics in this context. In section 3, we present Monte Carlo results on the level and power of the tests proposed. Section 4 contains a few concluding remarks.

2. The model and four nonparametric statistics

We consider the following variant of the two-equation MS model:

$$Y_t = \beta_1 (X_{t-1} - \mu) + \epsilon_t, \quad t = 1, \ldots, T, \tag{1}$$

$$X_t = \theta_0 + \theta_1 X_{t-1} + \eta_t, \quad t = 1, \ldots, T. \tag{2}$$

where the vectors $(\epsilon_1, \eta_1)', \ldots, (\epsilon_T, \eta_T)'$ are independent with absolutely continuous (not necessarily identical) distribution, ϵ_t and η_t have median zero (for all t), $\epsilon_1, \ldots, \epsilon_T$ are independent of X_0, and β_1, μ, θ_0 and θ_1 are fixed coefficients. Clearly, if ϵ_t and η_t have symmetric distributions (e.g., normal distributions), they also have mean zero. μ may be interpreted as a centering constant for X_t, e.g. the mean of X_t when X_t is weakly stationary. However, the tests proposed below remain valid when μ is an arbitrary constant. ϵ_t and η_t may not be independent. For example, MS assume that ϵ_t and η_t have finite second moments with $\mathrm{Corr}(\epsilon_s, \eta_t) = \rho\ \epsilon \delta_{st}$ for all s and t (where $\delta_{st} = 1$ if $s = t$, and $\delta_{st} = 0$ if $s \neq t$). In this case, the coefficient ρ determines the importance of feedback; when $\rho = 0$, there is no feedback from the disturbances ϵ_t to the explanatory variable. We study the problem of testing $\mathrm{H}_0 : \beta_1 = 0$.

Note that eq. (1) has no intercept when $\beta_1 = 0$, so that the mean (or the median) of Y_t is zero under the null hypothesis. In several important cases, such as the one where Y_t is a forecast error under rational expectations, this is a reasonable assumption. To test whether such errors are independent of past movements of X_t, it is standard procedure to apply the t-test associated with the slope coefficient of the regression model

$$Y_t = \alpha_0 + \alpha_1 X_{t-1} + \epsilon_t. \tag{3}$$

However, with (ϵ_t, η_t) identically and normally distributed, MS found that this approach rejects much too often the null hypothesis even for sample sizes as large as 200, especially when ρ and θ_1 take values close to 1. This result leads to the problem of finding procedures with the correct size despite the presence of feedback. For this purpose, we suggest nonparametric analogues of the t-test that are applicable in the context of the model given by (1) and (2).

The statistics proposed make use of centered values of X_t where the mean (or median) of X_t is estimated from the observations available only up to time t (instead an estimate based on the complete sample). It turns out that this repeated estimation procedure is important in obtaining the exactness of the sign tests. Let $\bar{X}_t = (\sum_{s=0}^{t} X_s)/(t+1)$ and $\tilde{X}_t = \text{med}\{X_0, X_1, \ldots, X_t\}$ denote the sample mean and median of X_t based on the observations up to time t. Let also $u(z) = 1$ if $z \geq 0$, and $u(z) = 0$ if $z < 0$. We consider the following four statistics:

$$S_1 = \sum_{t=1}^{T} u\left[Y_t(X_{t-1} - \bar{X}_{t-1})\right], \qquad S_2 = \sum_{t=1}^{T} u\left[Y_t(X_{t-1} - \tilde{X}_{t-1})\right], \tag{4}$$

$$W_1 = \sum_{t=1}^{T} u\left[Y_t(X_{t-1} - \bar{X}_{t-1})\right] R_{1t}^{+}, \qquad W_2 = \sum_{t=1}^{T} u\left[Y_t(X_{t-1} - \tilde{X}_{t-1})\right] R_{2t}^{+}. \tag{5}$$

With $Z_{1t} = Y_t(X_{t-1} - \bar{X}_{t-1})$, R_{1t}^{+} is the rank of $|Z_{1t}|$ when $|Z_{11}|, |Z_{12}|, \ldots, |Z_{1T}|$ are placed in ascending order. R_{2t}^{+} is defined analogously from $Z_{2t} = Y_t(X_{t-1} - \tilde{X}_{t-1})$. S_1 and S_2 are the sign statistics based on the series Z_{1t} and Z_{2t} respectively, while W_1 and W_2 are the corresponding Wilcoxon signed-rank statistics. The distributions of such statistics are typically established under the assumption that the observations are mutually independent; see Lehmann (1975) or Pratt and Gibbons (1981). Here it is clear that $Z_{11}, \ldots, Z_{1T}$ are not independent, and similarly for $Z_{21}, \ldots, Z_{2T}$. Despite this difficulty, we can establish the following result for sign statistics.

Theorem 1. Let the model given by (1) *and* (2) *hold, let $g_t = g_t(I_t)$ be a measurable function of the information vector $I_t = (X_0, X_1, \ldots, X_t, Y_1, \ldots, Y_t)$, where $I_0 \equiv X_0$, and let $S_g \equiv \sum_{t=1}^{T} u(Y_t g_{t-1})$. Then, when $\beta_1 = 0$, S_g follows a binomial distribution with number of trials T and probability of success* 0.5.

Proof. When $\beta_1 = 0$, we have $Y_t = \epsilon_t$, $t = 1, \ldots, T$. Let $s_t = u(\epsilon_t g_{t-1})$ and consider the characteristic function of S_g when $\beta_1 = 0$:

$$\phi_g(\tau) = \mathrm{E}\left[\exp(i\tau S_g)\right] = \mathrm{E}\left[\prod_{t=1}^{T} \exp(i\tau s_t)\right],$$

where $\tau \in \mathbb{R}$ and $i = \sqrt{-1}$. Conditional on the information vector $I_{T-1} = (X_0, X_1, \ldots, X_{T-1}, \epsilon_1, \ldots, \epsilon_{T-1})$, the variables $s_1, \ldots, s_{T-1}$, g_{T-1} are fixed and s_T follows the uniform Bernoulli distribution on $\{0, 1\}$, i.e. $P[s_T = 0] = P[s_T = 1] = 0.5$. We thus have

$$\phi_g(\tau) = \tfrac{1}{2}[1 + \exp(i\tau)]\,\mathrm{E}\left[\prod_{t=1}^{T-1} \exp(i\tau s_t)\right].$$

This argument can be repeated recursively and we find

$$\phi_g(\tau) = \left\{\left[\tfrac{1}{2}\right]\left[1 + \exp(i\tau)\right]\right\}^{T},$$

which is the characteristic function of the binomial distribution with number of trials T and probability of success 0.5. □

This theorem motivates the introduction of the variables S_1 and S_2. Even though the variables $Z_t = Y_t g_{t-1}$, $t = 1,\dots, T$, are not generally independent, we can conclude from Theorem 1 that the distribution of S_g is the same as in the case where the Z_t's are independent with continuous distributions having median zero. In particular, it is easy to see that this result applies to S_1 and S_2 in (4). We see here the importance of estimating the mean (or median) of X_t recursively. Note also that ϵ_t and η_t need not have finite second moments.

If the variables $Z_{11},\dots, Z_{1T}$ were independent with continuous distributions symmetric about zero, W_1 would be distributed like a weighted sum $W_0 = \sum_{t=1}^{T}$ tB$_t$ of T independent uniform Bernoulli variables $B_1,\dots, B_T$ on $\{0, 1\}$, and similarly for W_2. W_0 has been extensively tabled; see, for example, Wilcoxon, Katti and Wilcox (1970). It should be noted that the normal approximation with $E(W_0) = T(T+1)/4$ and $\mathrm{Var}(W_0) = T(T+1)/(2T+1)/24$ works well even for small values of T; see Lehmann (1975) for a general discussion of W_0. However, we do not have an analytical result like Theorem 1 for the Wilcoxon statistics W_1 and W_2. For this reason. We will study their distributions by Monte Carlo methods.

3. Simulation results

For each of the following experiments, data were generated from model (1)–(2) by setting $\eta_t = \rho\epsilon_t + \sqrt{1-\rho^2}\,\eta_t'$, $\theta_0 = \mu = 0$ and $X_0 = \eta_0'/\sqrt{1-\theta_1^2}$, where ϵ_t and η_t' are independent with the same distribution either normal, $t(3)$ or Cauchy. Each experiment comprises 1000 replications. We

Table 1
Mankiw–Shapiro Model: Normal disturbances. [a]

β_1	t-test		S_1	S_2	W_1	W_2
	Asymptotic	Corrected [b]				
$(\rho, \theta_1) = (0, 0.9)$						
0.00	5.2	n.a.	3.8	3.7	5.3	5.2
0.07	30.5	n.a.	11.1	13.8	20.7	19.1
0.10	53.3	n.a.	18.5	23.1	34.6	33.2
$(\rho, \theta_1) = (0.9, 0.9)$						
0.00	8.1	4.9	3.4	3.5	4.0	3.4
0.07	19.8	13.5	15.3	17.7	21.8	21.4
0.10	36.0	28.2	22.3	26.8	34.7	33.1
$(\rho, \theta_1) = (0.9, 0.99)$						
0.00	18.4	4.7	2.9	3.6	5.0	4.9
0.07	29.6	18.2	32.6	32.9	43.0	41.8
0.10	52.5	37.5	43.5	43.6	56.3	53.6

[a] $T = 100$ and $\mu = 10$. Entries represent percentages of rejections.
[b] Empirical critical points are used in power calculations when $\rho \neq 0$.

Table 2
Mankiw–Shapiro Model: Non-classical disturbances. [a]

β_1	t-test		S_1	S_2	W_1	W_2
	Asymptotic	Corrected [b]				
Break heteroskedasticity [c]						
0.00	12.4	4.1	3.5	4.3	4.6	4.6
0.07	23.9	11.0	14.2	17.2	19.6	21.2
0.10	37.8	21.4	22.9	27.0	31.5	32.2
$t(3)$ distribution [d]						
0.00	8.2	n.a.	3.3	3.2	4.6	4.8
0.07	18.2	n.a.	21.4	23.4	31.9	32.0
0.10	35.1	n.a.	35.6	39.0	48.9	49.5
Cauchy distribution [e]						
0.00	5.2	n.a.	3.0	3.4	5.0	3.9
0.07	12.3	n.a.	57.8	70.2	70.8	75.4
0.10	28.8	n.a.	64.7	77.6	78.9	85.0

[a] $\mu = 10$ and $T = 100$. Entries represent percentage rejections.
[b] Empirical critical points are used in power calculations when $\rho \neq 0$.
[c] Before $t = 50$ the disturbances are N(0, 1); afterwards they are N(0, 16); $\rho = \theta_1 = 0.9$.
[d] Student's t-distribution with 3 degrees of freedom; $\rho = \theta_1 = 0.9$.
[e] $\rho = 1$ and $\theta_1 = 0.9$.

present here results for the sample size $T = 100$. More complete simulation results are available in Campbell (1990).

Table 1 presents the results based on normal errors with $E(\epsilon_t^2) = E(\eta_t^2) = 1$. The t-test is applied using both the asymptotic critical point for the 5% level and the exact 5% points (determined empirically using 10000 replications); the exact critical points depend on ρ and θ_1. The results for W_1 and W_2 use the normal approximation. Because the distribution of the sign statistics is discrete, the critical value for the sign tests was set at the closest point that yields a level less than or equal to 5%; in this case, this gives a level of 3.52%. Three specifications of the MS model are considered: $(\rho, \theta_1) = (0, 0.9)$, $(0.9, 0.9)$ and $(0.9, 0.99)$. With $\rho = 0$, all is well for the t-test: the level is correct and it is more powerful than the nonparametric tests. With $\rho = 0.9$ and $\theta_1 = 0.9$, the results when $\beta_1 = 0$ confirm the MS conclusion that the t-test rejects too often when ρ and θ_1 are both close to 1. Here the nonparametric tests reject at their nominal levels under the null, and the powers of S_2 and W_2 generally exceed the power of the t-test applied with the correct critical points. As θ_1 approaches 1, the over-rejection problem of the asymptotic t-test becomes more acute: Under the third specification in table 1, the t-test rejects at more than three times its nominal level. But the non-parametric tests continue to reject at their nominal levels and, in this specification, can be considerably more powerful than the size-corrected t-test.

The results of table 2 highlight the behavior of the non-parametric tests for three cases where the disturbances ϵ_t are not normal homoskedastic (non-classical disturbances): Normal distribution with the standard error jumping from 1 to 4 half-way through the sample, Student's t-distribution with 3 degrees of freedom, and Cauchy distribution. Consider first the impact of a jump in the variability of the disturbances (break heteroskedasticity). The asymptotic t-test then rejects overly. The sign tests are exact irrespective of the form of the heteroscedasticity as long as the conditions outlined in (1) and (2) are met. The Wilcoxon tests appear reliable as well. Further, the non-parametric tests are more powerful than the size-corrected t-test. With Cauchy and $t(3)$ disturbances, the asymptotic t-test has approximately the right level (so that we do not consider size-corrected tests), but the non-parametric tests significantly outperform the t-test from the point of view of power. The conventional wisdom that non-parametric test perform well in the presence of outliers is thus corroborated. Notice as well that the performance of the non-parametric tests is improved considerably when the median, instead of the mean, is used to center the X_t's.

4. Concluding remarks

In certain contexts, such as in a consideration of the claim that forecasts are rational, the null hypothesis may reduce considerably the complexity of the model and allow the application of traditional non-parametric statistical methods. The presence of non-normal and/or heteroskedastic disturbances can undermine the reliability or the efficiency of parametric tests. In addition,even though the disturbances may be independent of the regressors, feedback can jeopardize the reliability of usual testing procedures. The model studied by MS is one such example. This paper has highlighted the potential usefulness of non-parametric testing procedures in such a context. It is well-known that sign tests and signed-rank tests are robust to problems of non-normality and heteroskedasticity. Our results suggest that they can also be valid despite the presence of feedback, such as the one studied by Mankiw and Shapiro (1986). The sign tests, in particular, are exact under very weak conditions allowing the presence of feedback. Further, the power of the non-parametric tests can be considerably superior to that of the parametric t-tests, especially in the presence of non-normality of feedback.

Even though the simulations in this paper have focused on a simple stochastic set-up, it should be stressed that the nonparametric procedures developed are hardly tied to the Mankiw–Shapiro model. As suggested by the theorem, sign-type statistics work reliably in any environment where the disturbances are independent of the past. The applicability of the Wilcoxon statistic merits further investigation.

References

Banerjee, Anindya and Juan Dolado, 1987, Do we reject rational expectations models too often? Interpreting evidence using Nagar expansions, Economics Letters 24, 27–32.

Banerjee, Anindya and Juan Dolado, 1988, Tests of the life-cycle permanent income hypothesis in the presence of random walks: Asymptotic theory and small sample interpretation, Oxford Economic Papers 40, 610–633.

Banerjee, Anindya, Juan Dolado and John W. Galbraith, 1990, Orthogonality tests with de-trended data: Interpreting Monte Carlo results using Nagar expansions, Economics Letters 32, 19–24.

Campbell, Bryan, 1990, Nonparametric tests of independence with econometric applications, Mimeo. (Concordia University, Montréal, Que.)

Dufour, Jean-Marie, 1981, Rank tests for serial dependence, Journal of Time Series Analysis 2, 117–128.

Galbraith, John W., Juan Dolado and Anindya Banerjee, 1987, Rejections of orthogonality in rational expectations models: Further Monte Carlo evidence for an extended set of regressors, Economics Letters 25, 243–247.

Lehmann, Eric L., 1975, Nonparametrics: Statistical methods based on ranks (Holden Day, San Fransisco, CA).

Mankiw, N. Gregory and Matthew D. Shapiro, 1985, Trends, random walks, and tests of the permanent income hypothesis, Journal of Monetary Economics 16, 165–174.

Mankiw, N. Gregory and Matthew D. Shapiro, 1986, Do we reject too often? Small sample properties of rational expectation models, Economics Letters 20, 243–247.

Pratt, John W. and Jean D. Gibbons, 1981, Concepts of nonparametric theory (Springer-Verlag, New York).

Wilcoxon, Frank S., S.K. Katti and Roberta Wilcox, 1970, Critical values and probability levels for the Wilcoxon rank sum test and the Wilcoxon signed rank test, in: H.L. Harter and D.B. Owen, eds., Selected tables in mathematical statistics (Institute of Mathematical Statistics, Providence, RI).