

The IMK is an institute of the Hans-Böckler-Stiftung

WORKING PAPER

No. 190 · March 2018 · Hans-Böckler-Stiftung

FEP – THE FORECAST EVALUATION PACKAGE FOR GRETL

Based on FEP version 2.1, March 2018

Artur Tarassow*, Sven Schreiber**

ABSTRACT

The FEP function package for the *gretl* program is a collection of functions for computing different types of forecast evaluation statistics as well as tests. For ease of use a common scripting interface framework is provided, which is flexible enough to accommodate future additions. Most of the functionality is also accessible through a graphical dialog window interface within gretl. This documentation explains the usage and capabilities as well as providing some econometric background where necessary. An illustration with expert forecasts of euro area growth is also provided.

University of Hamburg, artur.tarassow@gmail.com.

^{**} IMK Düsseldorf and Free University Berlin.

FEP – the forecast evaluation package for *gretl*

Artur Tarassow and Sven Schreiber*

based on FEP version 2.1, March 2018

Abstract

The FEP function package for the *gretl* program is a collection of functions for computing different types of forecast evaluation statistics as well as tests. For ease of use a common scripting interface framework is provided, which is flexible enough to accommodate future additions. Most of the functionality is also accessible through a graphical dialog window interface within gretl. This documentation explains the usage and capabilities as well as providing some econometric background where necessary. An illustration with expert forecasts of euro area growth is also provided.

Contents

I	Intr	oduction	2		
2	The	applyFCtests() function	2		
3	Forecast descriptive statistics				
	3.1	Calculate the (forecast error) loss	4		
	3.2	Draw some loss functions	4		
	3.3	Tools for binary outcomes	4		
		3.3.1 The Kuipers Score (KS)	4		
		3.3.2 Probability scores – doPS() and probscore()	8		
4	Eva	luation of individual forecasts	8		
	4.1	$\label{lem:mincer-Zarnowitz} \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \$	8		
	4.2	Holden-Peel test of forecast efficiency – doHPtest()	9		
	4.3	Campbell-Ghysels efficiency tests – doCGtest()	9		
	4.4	Elliott/Komunjer/Timmermann test (asymmetric loss) – doEKTtest()	12		
	4.5	Pesaran-Timmermann test of market timing – doPTtest()	13		
	4.6	Diebold-Lopez direction-of-change test – doDLtest()	15		

https://sites.google.com/site/arturtarassow;

Schreiber: IMK Düsseldorf and Free University Berlin, econ.svens.de.

^{*}Tarassow: University of Hamburg, artur.tarassow@gmail.com,

5	Eva	luation and comparison of multiple forecasts	15
	5.1	Diebold-Mariano test – doDMtest()	15
6	Mei	nu-driven (GUI) usage	17
7	Illu	strative example	18
	7.1	Tests of unbiasedness	19
	7.2	Tests of efficiency	20
	7.3	Tests of (a)symmetric loss and forecast rationality	21
	7.4	Forecast comparison	22
	7.5	Directional forecast	23
8	Sun	nmary and outlook	25
A	Dat	a	27

1 Introduction

The FEP function package is a collection of gretl functions for computing different types of forecast evaluation statistics as well as tests. *Gretl* is an open-source econometrics and statistics program that offers both a command line and a graphical menu-driven interface. Its core functions are implemented in C, but it also comprises a versatile scripting and matrix language named *hansl* which can be used to develop contributed function packages easily. FEP is such a package written in hansl. More information on gretl can be found online at http://gretl.sf.net. The FEP package currently comprises the functions listed in Table 1. In Section 6 we explain the easy usage from gretl's menu-driven interface (GUI), but until then we focus on the scripting way of performing the analysis. A function package can be downloaded and installed simply by invoking the install command:

install FEP

Then, in each work session, as with all function packages the FEP package needs to be loaded by:

include FEP.gfn

2 The applyFCtests() function

This is a convenient top-level function which calls the other test functions in the background as needed. The standard workflow is to define the needed gretl bundle¹ with your input data and choices, and then call this function with the appro-

¹A "bundle" is a flexible gretl datatype which is basically a collection of other gretl objects. See Section 10.7 of the gretl user guide (as of November 2017).

	Table 1: Included functions				
#	Function	Code	Description		
1	applyFCtests()		Top-level function		
2	doDMtest()	DM	Diebold-Mariano regression based test of forecast		
			accuracy		
3	doMZtest()	MZ	Mincer-Zarnowitz regression based test of optimal		
			(point) forecasts		
4	doHPtest()	HP	Holden-Peel extension of the Mincer-Zarnowitz		
			regression based test		
5	doCGtest()	CG	Campbell-Ghysels sign or signed rank (Wilcoxon		
			type) tests for unbiasedness or efficiency of forecasts		
6	<pre>doEKTtest_series()</pre>	EKT	Elliott, Komunjer & Timmermann test of forecast		
			rationality under flexible loss		
7	doDLtest()	DL	Diebold-Lopez direction of change test		
8	doKS()	KS	Computes the (Hanssen-) Kuipers Score for forecasts		
			of binary outcomes		
9	doPTtest()	PT	Pesaran-Timmermann test of market timing		
10	DrawLoss()		draws a single loss function and its associated		
			confidence interval		
11	DrawLoss2()		draws two loss functions and their associated		
			confidence intervals		
12	getLoss()		calculate the (forecast error) loss series		
13	doPS() / probscore()	PS	calculate the log and quadratic probability scores for		
			forecasts of binary outcomes		

T-1-1- 1. I. -1. . 1 - 1 (... - C. . . .

priate string code. Here's an example with the Mincer-Zarnowitz test:

```
bundle b
series b.y = growth # assuming 'growth' exists in current workfile
series b.fc = myforecast # ditto
applyFCtests(&b, 'MZ'')
```

The first three lines set up the bundle and put some relevant data in it.² Note that the names "y" and "fc" need to match exactly and are case sensitive. See Table 2 for the possible and/or necessary bundle elements and their names, which depends on the test that you wish to perform. It is possible to store additional elements in the bundle that are not used by a certain test. Therefore you can set up the bundle once and then pass it around as an input to various tests.

In the last line some things are noteworthy: The applyFCtests() function does not have any return value, so the function call stands for itself without any assignment to a variable. The results of the tests are instead added to the bundle ("b" here) that is passed to the function. To actually enable the function to change the input bundle we need to pass it in so-called pointer form, which just means to

 $^{^2}$ If you're obsessed with saving lines of code, you might instead use something like the following: bundle b = defbundle("y",growth, "fc",myforecast).

prepend the ampersand character: "&b".³ Finally, a string code must be specified to indicate which test should be run, where the possible codes are given in Table 1. There appear also some supplementary functions in that table which do not have a string code; those functions have to be called directly and cannot be accessed through applyFCtests(). On the other hand, several string codes may be included in a single call to applyFCtests(), separated by spaces.

If the function is called like this, then there will typically be some printed output showing the test results. The details depend on the respective background function, see the corresponding documentation below. The other possibility to access the results is to inspect the new members that are added to the bundle. Again, see the documentation below to learn which new members each function adds to the bundle.

3 Forecast descriptive statistics

3.1 Calculate the (forecast error) loss

The function getLoss() helps to calculate the forecast losses (disutilities) implied by the given forecast errors, according to various loss function assumptions such as lin-lin, square, linex, double linex. See Table 3.

3.2 Draw some loss functions

Draw a single (DrawLoss) or two (DrawLoss2) loss functions and their associated confidence intervals. See Table 4.

3.3 Tools for binary outcomes

3.3.1 The Kuipers Score (KS)

The (Hanssen-) Kuipers Score (KS) is used to evaluate binary outcomes. Let $f = \{0,1\}$ be a forecast of the binary variable $y = \{0,1\}$. The KS is defined as the difference between the probability of detection (POD) and the probability of false detection (POFD).⁴ The POD is the proportion of times where y = 1 is correctly predicted. The POFD is the proportion of times where y = 1 is wrongly predicted. Thus, KS is defined as KS = POD - POFD. See Table 5 for the function interface and elements.

³See "Function programming details", Section 13.4 of the gretl user guide (as of November 2017).

⁴It seems that this terminology is not universal. Sometimes the POD might be called hit rate, whereas usually the hit rate denotes something else. Similarly with the POFD and the false alarm rate.

	Tab	ole 2: Bund	lle membe	ers neede	d for each	function			
	CG	DL	DM	EKT*	HP	MZ	KS	PT	PS
		series	inputs ([]	: optional)					
Realizations	y		у	у	y	y			
Forecast values	fc		f1, f2	fc	fc	fc			
Forecast errors	(E)			(E)					
(replaces y, fc)									
Binary indicator		yup					yup	yup	yup
Binary FC of yup		fcup					fcup	fcup	
FC of yup prob.									pfc
Further tested	[CGX]								
			list inpu	ıts					
Exog. regressors					Z				
Instruments				\mathbf{z}					
	scala	r inputs (m	ostly integ	ger / all are	optional)				
FC horizon			[fhor]						
Loss type			[loss]	[loss]					
Lag (efficiency	[k]								
test)									
Bootstrap iter.					[nboot]	[nboot]			
Robust switch					[robust]	[robust]		[robust]	
Initial shape				[a0]					
Verbosity	[verb]	[verb]	[verb]	[verb]	[verb]	[verb]	[verb]	[verb]	
		strii	ıg input (c	ptional)					
Loss drawing				[lossdra	w]				

Notes:

robust can be 0 (default) or 1 (use HAC/robust SE);
 loss can be 1 (U-shape, default) or 2 (V-shape);
 verb can be 0 (no details, default) or 1 (print details);
 lossdraw can be "no" (default), "display", or consist of path + filename;
 fcup is binary, not a probability;
 k can be 0 (default, no test on lags) or a positive (not too large) integer; note that only a single lag is tested;
 a0 can be between 0 and 1 (non-integer, default 0.5).
 *: EKT refers to the doEKTtest_series() variant. For the matrix-based variant doEK-

^{*:} EKT refers to the doEKTtest_series() variant. For the matrix-based variant doEK-Ttest() see the source code.

Table 3: Forecast error loss				
Function	Function getLoss(matrix fce, string LF[null], matrix param[null], matrix realiz[null])			
Description Return type	Calculate the (for matrix	recast error) loss		
Function arguments	matrix <i>fce</i>	T by k matrix of forecast errors; if <i>realiz</i> is given, <i>fce</i> is understood as the forecasts themselves		
	string <i>LF</i> (optional)	specify loss function: "ll" (or "linlin"), "qq" (or "quadquad"), "sq" (or "square", default), "abs", "linex" (Varian), "dlinex" (or "dle") for double linex (Granger)		
	matrix <i>param</i> (optional)	loss function parameter(s), default 0.5; param must be a 1-element or 2-element vector		
	matrix <i>realiz</i> (optional)	matrix of realizations		
Output Returns a matrix with for dimension as the input r		with forecast error losses, of the same (T x k) input matrix.		
Reference Varian [1975], Granger [1999]		anger [1999]		

Notes: About input format for *param*: For gretl versions until 2017a scalar values must explicitly be provided as a pseudo matrix (e.g $\{0.4\}$) while for later versions, gretl accepts a scalar as a 1x1 matrix.

	Table 4: Draw losses
Function	<pre>DrawLoss(int p, scalar aT, scalar V, string fpath[null])</pre>
Description Return type	Draw (forecast error) loss none (void)
Function arguments	int p : loss function type, $1 = \text{lin-lin}$, $2 = \text{quad-quad}$ scalar aT : loss function shape parameter $\alpha \in [0, 1]$ scalar V : estimated variance of aT string $fpath$ (optional): null = display figure (default) or provide complete "path+file name" to store figure
Output	displays plot (or saves to file path)
Function Description Return type	DrawLoss2(int p, scalar aT1, scalar V1, scalar aT2, scalar V2, string fpath[null]) Draw two (forecast error) losses none (void)
Function arguments	int p : loss function type, $1 = \text{lin-lin}$, $2 = \text{quad-quad}$ scalar $aT1$: loss function shape parameter $\alpha \in [0, 1]$ scalar $V1$: estimated variance of $aT1$ scalar $aT2$: loss function shape parameter $\alpha \in [0, 1]$ scalar $V2$: estimated variance of $aT2$ string $fpath$ (optional): null = display figure (default) or provide complete "path+file name" to store figure

Table 5: Kuipers Score

Function Description Return type	doKS(bundle *b) compute the Kuipers Score none (void)		
arguments series yup binary-valued se		el bundle. This bundle includes as members: binary-valued series of actuals that takes the value of unity for ups, otherwise zero	
	series fcup	binary-valued forecast that takes the value of unity for ups, otherwise zero	
	scalar verb	0 = no printout, 1 = print details	
Output	The following new scalar <i>KSstat</i> : matrix <i>KSmat</i> :	elements are stored into the model bundle: Kuipers score matrix holding all classifications	
Reference	Pesaran [2015, p. 396]		

Table 6: Probability scores

Function Description	probscore(matrix y, matrix Pr) computes the log (LPS) and quadratic (QPS) probability scores
Return type	matrix
Function arguments	matrix <i>y</i> : binary-valued vector of actuals that takes the value of unity for ups and otherwise zero matrix <i>Pr</i> : vector of forecast probabilities
Output	2-element (1 by 2) vector with QPS, LPS

Table 7: Probability scores wrapper

Function Description Return type	doPS(bundle *b) computes the log (LPS) and quadratic (QPS) probability scores none (void)		
Function arguments	Pointer to the moseries <i>yup</i> series <i>pfc</i>	del bundle. This bundle includes as members: binary-valued series of actuals that takes the value of unity for ups, otherwise zero probability forecast (between 0 and 1) that yup takes the value of unity	
Output	The following new scalar <i>qps</i> scalar <i>lps</i>	w elements are stored into the model bundle: quadratic probability score log probability score	

3.3.2 Probability scores – doPS() and probscore()

The probscore() function computes forecast accuracy statistics used for probability forecasts. The observed outcome is still binary, but in contrast to the Kuipers score the forecast here is a continuous probability. See Table 6. The doPS() function is just a thin wrapper around probscore that is harmonized with the evaluation test functions in the package; see Table 7.

4 Evaluation of individual forecasts

4.1 Mincer-Zarnowitz test of forecast unbiasedness – doMZtest()

Define the h-step ahead forecast made in t as $f_{t+h|t}$ and the actual outcome as y_{t+h} . Mincer&Zarnowitz suggest to run the following regression:

$$y_{t+h} = \beta_0 + \beta_1 f_{t+h|t} + u_{t+h}$$

Unbiasedness is viewed as a joint test of $\beta_0 = 0$ and $\beta_1 = 1$. (H0: Forecast is unbiased and efficient. H1: Forecast is biased and/or inefficient.) Usually the corresponding test statistics is compared with asymptotic critical values from the F-

Table 8: Mincer-Zarnowitz test		
Function doMZtest(bundle *b) Description computes the Mincer-Zarnowitz test regression of unbiase none (void)		
Function arguments	Pointer to the mod series <i>y</i> series <i>fc</i> scalar <i>robust</i>	del bundle. This bundle includes as members: actual observations h-step ahead forecast 0 = no robust SEs (default), 1 = compute HAC SEs
	scalar nboot	0 = no bootstrap (default), or number of bootstrap iterations 0 = no printout (default), 1 = print details
		1
Output	The following nev scalar <i>MZstat</i> scalar <i>MZpval</i>	v elements are stored into the model bundle: test statistics p-value
Reference Mincer and Zarnowitz [1969]		witz [1969]

distribution.

However, remaining serial correlation in u_{t+h} yields inefficient estimates. Also the small sample properties are unknown. To account for these two potential sources of inefficiency, the user can compute HAC robust standard errors as well as bootstrap p-values. An intercept will be automatically inserted.

See Table 8 for a synopsis.

4.2 Holden-Peel test of forecast efficiency – doHPtest()

The Mincer-Zarnowitz regression can be extended to include another regressor (or a whole vector of additional regressors), Z_t , such that (assuming for simplicity that Z_t is a scalar value)

$$y_{t+h} = \beta_0 + \beta_1 f_{t+h|t} + \beta_2 Z_t + u_{t+h}$$

The hypothesis to test for a strong form of unbiasedness involves the null $\beta_0 = 0$, $\beta_1 = 1$ and $\beta_2 = 0$. An intercept will be automatically inserted.

Again the user can compute HAC robust standard errors and/or bootstrap p-values; see Table 9.

4.3 Campbell-Ghysels efficiency tests – doCGtest()

Here we understand "efficiency" in a broad sense, comprising unbiasedness for example.

Let us define the one-period forecast errors as $e_{1t} = y_{t+1} - f_{t+1|t}$. An indicator function indicates whether the forecast error is positive or negative such that

	Table 9: Holden-Peel test			
Function Description	doHPtest(bundle *b) computes the Holden-Peel variant of the Mincer-Zarnowitz test regression of unbiasedness			
Return type	none (void)			
		del bundle. This bundle includes as members: actual observations		
O	series <i>fc</i>	h-step ahead forecast		
	$\operatorname{list} z$	gretl list of additional control variables		
	scalar robust	0 = no robust SEs (default), 1 = compute HAC SEs		
	scalar <i>nboot</i>	0 = no bootstrap (default), or number of bootstrap iterations		
	scalar <i>verb</i>	0 = no printout (default), 1 = print details		
Output	The following new scalar <i>HPstat</i> scalar <i>HPpval</i>	w elements are stored into the model bundle: test statistics p-value		
Reference	erence Holden and Peel [1990]			

u(z) = 1 if $z \ge 0$ and u(z) = 0 otherwise. The test statistics of the sign test of unbiasedness of forecast errors is

$$S = \sum_{t=1}^{T} u(e_{1t}),$$

where *T* is the number of available forecast errors. While the signed rank test (see below) is defined as

$$W = \sum_{t=1}^{T} u(e_{1t}) R_{1t}^{+}$$

with R_{1t}^+ referring to the rank of each forecast error when $|e_{1t}|,...,|e_{1T}|$ are placed in ascending order. The forecast errors are independent with zero median. The sign statistic S is binomially distributed with Bi(T, 0.5). Under the additional assumption of symmetrically distributed forecast errors around zero, the test statistics W follows a Wilcoxon signed rank distribution. Note that this test is performed once you set k = 0 (or leave it out, as this is the default; see Table 10).

This test idea can also be employed to test for serial correlation in the forecast errors. Construct the product series $Z_{1t}^k = e_{1t}e_{1(t-k)}$ with $k \ge 1$, and compute the statistics:

$$S_k = \sum_{t=k+1}^{T} u(Z_{1t}^k)$$
 and $W_k = \sum_{t=k+1}^{T} u(Z_{1t}^k) R_{2t}^+$

where R_{2t}^+ is the signed rank of the product Z_{1t}^k , t = 1, ..., T. These location tests were proposed by Dufour [1981]. Serial correlation in the forecast errors will move

Table 10: Campbell-Ghysels sign and signed rank

Function Description Return type	doCGtest(bundle *b) Campbell-Ghysels sign and signed rank (Wilcoxon-type) tests for unbiasedness or efficiency of forecasts none (void)		
Function arguments	Pointer to the model bundle. This bundle includes as members: series y actual observations series fc forecast (or series E forecast error) scalar $verb$ $0 = no printout (default), 1 = print details series CGX variable for orthogonality (see also k) (optional) scalar k set k > 0 to test efficiency with respect to information at lag k (own lag of forecast error if X is absent), else k = 0 (or omit) to test for unbiasedness$		
Output	The following new elements are stored into the model bundle: scalar test statistic CGSIGNstat and CGWILCstat scalar p-value CGSIGNpval and CGWILCpval		
Reference	Campbell and Ghysels [1995], Campbell and Ghysels [1997], Dufour [1981]		

Notes: The calculation of CGWILCpval, the p-value of the Wilcoxon-type signed rank tests, depends on the WSRpvalue function in the special package "extra" for gretl (\geq v0.41). Please install that package manually from the gretl package repository if the automatic loading fails.

the centre of their product away from zero. The sign statistic S_k is binomially distributed with Bi(T - k, 0.5). The test statistics W_k follows a Wilcoxon signed rank distribution of size T - k. Note, both tests on serial correlation require to set the option k > 0, and the necessary product series Z_{1t}^k will be constructed automatically.

Lastly, one can use this framework to assess whether the forecast has made efficient use of the available information represented by the series X in t. For this, one needs to construct the product series $Z_t^k = e_{ht}X_{t-k}^c$ with k > 0 based on the recursively re-centered series $X_t^c = X_t - \text{median}(X_1, X_2, \dots, X_t)$. This way of recentering requires, however, that the series X_t is stationary and has no trend.

⁵"Recursive" in the sense that for the calculation of the median in each period t only realizations up to t are used, not all observations from the forecast evaluation sample. Otherwise the centering would contradict the real-time information flow and would not be feasible for actual forecasting.

The sign and signed rank statistics are given by

$$S_{ok} = \sum_{t=k+1}^{T} u(Z_t^k)$$
 and $W_{ok} = \sum_{t=k+1}^{T} u(Z_t^k) R_{1t}^+$,

noting that the ranks used refer to the forecast errors, not to Z_t^k , to obtain a statistic with a known and valid distribution; see Campbell and Ghysels [1995, pp. 3] or Campbell and Ghysels [1997, p. 560] for a discussion.⁶ This orthogonality test can be achieved by passing the series as CGX in the bundle. See Table 10.

4.4 Elliott, Komunjer, and Timmermann test with asymmetric loss – doEK-Ttest()⁷

This is a test for forecast rationality that allows for asymmetric loss. As a side product the parameters of a general class of loss functions can be estimated.

Elliott et al. [2005] propose a flexible class of loss functions, the so-called EKT loss function:

$$L(e) = [\alpha + (1 - 2\alpha)\mathbf{I}(e < 0)] |e|^p, \quad \alpha \in [0, 1]$$

where **I** is an indicator function which equals one if the forecast error e = y - f < 0, and otherwise zero. Asymmetric loss is given for $\alpha \neq 0.5$, and where values exceeding 0.5 indicate greater aversion to positive forecast errors. Setting p = 1 results in a lin-lin loss function with possibly different slopes under asymmetric loss. The quadratic loss function is given for the special setting p = 2 and $\alpha = 0.5$. Asymmetric quadratic loss functions are produced under p = 2 and $\alpha \neq 0.5$.

The unknown loss function parameter α is estimated by means of a linear instrumental variable (IV) approach. The computation of the linear IV estimator $\hat{\alpha}$ is done iteratively. Using the same notation as in Elliott et al. [2005] we set the initial weighting matrix S to be the identity matrix I_d , where d refers to the number of instruments used. Based on the initial S one can compute the initial $\hat{\alpha}_1$ which in turn can be used to update the precision matrix $S^{-1} = S^{-1}(\hat{\alpha}_1)$. These steps are repeated until some convergence criteria is met.

The user can use the framework to (i) test for symmetric loss, and (ii) for rationality. As $\hat{\alpha}$ is asymptotically normal and its variance is identical to the one obtained by the standard GMM estimator, one can test the null of symmetry, H_0 :

⁶When X_t is strictly exogenous without any feedback occurring from the forecast errors to future realizations of X the ranks of Z_t^k could also be used, and this would then indeed equal a Wilcoxon signed rank test on Z_t^k . This variant is not implemented, however, because the assumption appears very strong and hardly relevant in practice.

⁷For historical reasons and backward compatibility the doEKTtest function up to FEP version 2.1 has operated on matrices. The preferred interface now is based on a bundle and series as described in this document. The doEKTtest_series function has fulfilled that role since version 2.0. Starting with version 2.2 the doEKTtest function will follow the preferred interface (be an alias for doEKTtest_series), and a new wrapper function doEKTtest_matrix will provide the old access for those who need it.

Table 11: Elliott, Komunjei	r and Timmermann test
-----------------------------	-----------------------

Function		s(bundle *b) <doekttest(bundle *b)="" from="" v2.2=""></doekttest(bundle>	
Description		and Timmermann test for forecast	
Bescription	rationality that allows for asymmetric loss		
Return type	none (void)		
Function	Pointer to the model bundle. This bundle includes as members:		
arguments	series y	actual observations	
_	series fc	h-step ahead forecast	
	(or series E	forecast error)	
	$\operatorname{list} z$	instruments	
	scalar a0	initial value of shape parameter alpha	
		(between 0 and 1, default 0.5)	
	scalar <i>loss</i>	Loss function type: 1 = U-shape (default), 2	
		= V-shape	
	scalar <i>verb</i>	0 = no printout (default), 1 = print details	
	string lossdraw	"no" = no draw (default), "display" =	
		immediate plot, "Path+filename"	
Output	The following new elements are stored into the model bundle:		
	scalar <i>alpha</i>	estimated shape parameter	
	scalar V	estimated variance of alpha	
	scalar SymTest	test statistics of test for symmetric loss	
	1	function	
	scalar SymPval	p-value of test for symmetric loss function	
	scalar <i>RatTest</i>	test statistics of test for rationality	
	1 D (D	conditional on estimated alpha	
	scalar <i>RatPv</i>	p-value of test for rationality conditional on estimated <i>alpha</i>	
	scalar RatTest05	test statistics of test for rationality	
		conditional on symmetry (as if alpha = 0.5)	
	scalar RatPv05	p-value of test for rationality conditional	
		conditional on symmetry (as if alpha = 0.5)	
Reference	Elliott et al. [2005]		

 $\alpha = 0.5$, against the alternative, $H_0: \alpha \neq 0.5$, by a t-test.

For a given p_0 the user can test for rationality if the number of instruments d > 1. Testing for over-identification by means of a J-type test provides a joint test for rationality of the forecasts. The EKT approach allows to test for rationality (i) either for an unknown α or (ii) by imposing the restriction of symmetry $\alpha = 0.5$.

See Table 11 for the synopsis, and note that the instruments *z* should not contain an intercept, it will be added automatically.

4.5 Pesaran-Timmermann test of market timing – doPTtest()

While the Kuipers score just computes the difference between the hit rate, H, and the false alarm rate, F, it is not a proper statistical test. Pesaran and Timmermann

Table 12: Pesaran-Timmermann test of market timing

Function Description Return type	doPTtest(bundle *b) Pesaran-Timmermann test of market timing based on binary outcomes none (void)		
Function	Pointer to the model bundle. This bundle includes as members:		
arguments	series <i>yup</i> : binary-valued series of actuals that takes		
	the value of unity for ups, otherwise zero		
	series fcup:	binary-valued forecast that takes the value	
		of unity for ups otherwise zero	
	scalar robust:	0 = correlation-based PT test, 1 =	
		regression-based with HAC robust SEs	
	scalar verb:	0 = no printout, 1 = print details	
Output	The following new elements are stored into the model bundle:		
•	scalar <i>PTstat</i>	test statistic	
	scalar <i>PTpval</i>	p-value	
Reference	Pesaran and Timmermann [1992], Pesaran [2015, p. 398]		

[1992, PT] have proposed a simple test on market timing using binary outcomes. The basic idea is to test whether predicted ups are independent of actual ups or not.

Let $f = \{0,1\}$ be a forecast of the binary variable $Y = \{0,1\}$. Denote the corresponding time series of binary predictions as x_t and actual realizations of "ups" (or unity values) as y_t . How the forecaster obtains the forecasts x_t –e.g. through a model of a latent variable in the background– is irrelevant here.

The PT test statistic can be approximated by the t-ratio of the β_1 coefficient of the following OLS regression

$$y_t = \beta_0 + \beta_1 x_t + u_t.$$

Under the null hypothesis that predictions and realizations are independently distributed, the restriction $\beta_1 = 0$ holds. The test statistic follows asymptotically a standard normal distribution. A rejection of the null hypothesis indicates predictive failure. Serial correlation in the errors, u_t , are likely to occur but can be dealt with by using Bartlett weights to compute HAC standard errors.

A second –non-regression based– approach to compute the test instead is to rely on the correlation coefficient between forecasts and predictions, ρ . The test statistics is computed by $\rho \times \sqrt{T}$ where T refers to the number of forecasts available. The test statistic also follows a standard normal distribution asymptotically and cannot be robustified.

See Table 12 for a synopsis.

Table 13: Diebold-Lopez test

		•	
Function	<pre>doDLtest(bundle *b)</pre>		
Description		Diebold-Lopez test for predictive accuracy of a	
	direction-of-change forecast		
Return type	none (void)		
Function	Pointer to the model bundle. This bundle includes as members:		
arguments	series <i>yup</i>	binary-valued series of actuals that takes	
		the value of unity for ups, otherwise zero	
	series fcup	binary-valued forecast that takes the value	
		of unity for ups otherwise zero	
	scalar <i>verb</i>	0 = no printout (default), 1 = print details	
Output	The following new elements are stored into the model bundle:		
	scalar DLstat	test statistic	
	scalar DLpval	p-value	
	scalar <i>DLinfo</i>	additional info value	
Reference	Diebold and Lopez [1996]		

4.6 Diebold-Lopez direction-of-change test – doDLtest()

Performs the Diebold-Lopez direction of change test which is in principle just Pearson's χ - square test. H0: The direction-of-change forecast has no value meaning that the forecasts and realizations are independent. H1: The direction-of-change forecast has some value. (The side output of *DLinfo* is closely related to the Kuipers score, namely by adding unity to it.)

See Table 13 for a synopsis.

5 Evaluation and comparison of multiple forecasts

5.1 Diebold-Mariano test – doDMtest()

First, note that there also exists a dedicated contributed gretl function package called "DiebMar.gfn" by Giulio Palomba. It only partly overlaps with the features of the doDMtest() function, so it might be useful for you, too.

The Diebold-Mariano (DM) test of equivalent expected loss takes explicitly into account the underlying loss function as well as sampling variation in the average losses.

We define the h-step ahead forecast error and its associated loss function by $e_{t+h|t}$ and $L(e_{t+h|t})$, respectively. The loss differential of two non-nested forecasts for observation t+h is given by $d_{12,t+h}=L(e_{1,t+h|t})-L(e_{2,t+h|t})$. Specifically, one can test the null hypothesis of equal predictive accuracy

$$H_0: E(d_{12,t+h}) = 0$$

Table 14: Diebold-Mariano test			
Function Description	doDMtest(bundle *b) Diebold-Mariano test for predictive accuracy, computation regression-based with HAC robust SEs		
Return type	none (void)		
Function arguments	Pointer to the model bundle. This bundle includes as members: series <i>y</i> realized values		
	series f1	forecast values of model 1	
	series $f2$	forecast values of model 2	
	scalar <i>fhor</i>	forecast horizon (default 1)	
	scalar <i>loss</i>	Loss function type: 1 = U-shape (default), 2	
		= V-shape	
	scalar verb	0 = print no details (default), 1 = print details	
Output	The following new elements are stored into the model bundle:		
•	series L loss differentials		
	scalar DMstat	test statistics (not small-sample adjusted)	
	scalar DMpvaln	p-value based on standard normal	
	scalar DMpvalt	p-value based on t-distribution	
	scalar DMpvalssc	p-value using small-sample correction	
Reference	Diebold and Mariano [1995], Harvey et al. [1997]		

against either an one-sided alternative or against a two-sided alternative. Under the null, the test statistics is $DM = \bar{d}_{12,t+h}/\widehat{\sigma}_{\bar{d}_{12,t+h}} \to N(0,1)$ where $\bar{d}_{12,t+h} = F^{-1}\sum_{j=1}^F d_{12,j,t+h}$ refers to the sample mean loss differential and $\widehat{\sigma}_{\bar{d}_{12,t+h}}$ is a consistent estimate of its standard deviation based on F forecasts available.

However, due to serial correlation in the forecast errors, we compute alternatively the following regression-based version using OLS combined with HAC robust standard errors:

$$d_{12,t+h|t} = \beta + u_t$$

where u_t is an *i.i.d.* zero-mean error term. The null of equal forecast accuracy between the two point forecasts is defined as $\beta = 0$. The test statistics, t_{DM} , follows asymptotically a standard normal distribution.

Harvey et al. [1997] have suggested the following small-sample corrected degress-of-freedom adjusted t-statistics

$$t_{HLN} = [1 + F^{-1}(1 - 2 \times h) + F^{-2}h \times (h - 1)]^{0.5} \times t_{DM}$$

where F and h refer to the number of forecasts and the h-step forecast horizon. See Table 14 for a synopsis.

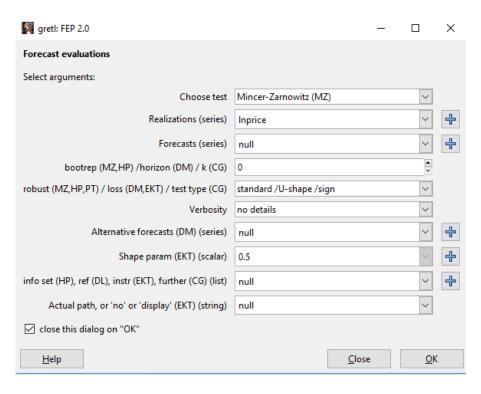


Figure 1: Screenshot of the FEP graphical interface (under Windows 10)

6 Menu-driven (GUI) usage

To install the FEP package using the GUI one follows the usual steps for contributed function packages: Open the function package list window for example via the menu File / Function packages / On server, then find the FEP entry and click the install button (or mouse right-click and choose Installation in the context pop-up menu).

For the precise meaning of the inputs to the respective functions please see the function documentation above, but ideally, using this package from gretl's menu interface should be mostly self-explanatory. From the menus you can only execute one test at a time.

Figure 1 shows the layout of the central window where choices and specifications are entered. In order to keep the number of argument fields in this window within reasonable bounds, some fields have different overloaded meanings depending on which test is chosen. For example, the penultimate entry field expects a gretl list, and this input is relevant for the HP, DL, EKT, and CG variants.⁸

The output from executing the function from the GUI is presented to the user in a new window, mostly simply with the printed test result. At the top of that output there is a toolbar including "save" and "bundle" icons. The save button allows you to save first the textual output, and secondly the whole produced bundle to

⁸This will be transferred internally to the various function arguments "z", "ref", or "CGX", respectively.

the current gretl session. The bundle button in turn gives you direct access to the various produced bundle members which can also be saved.⁹

Note that only the actual test functions are choosable from the GUI, not the helper functions from Section 3.

7 Illustrative example

In the following an applied example using survey-based forecasts for annual growth of euro area (EA) real GDP is presented. Forecasts are averages based on surveys among professional forecasters, fc_t , conducted by the Economist magazine and obtained through the Macrobond database. The realizations, y_t , are from the AMECO database provided by the European Commission. Both series' values are reported in the data appendix A, and Figure 2 depicts the forecast errors, $e_t = y_t - fc_t$. For the following code to work we assume that a gretl annual time-series datafile is loaded with the series named y and fc.

In the following the main (but not all) functions of the FEP package are applied. Assuming the package is already installed on the local machine, at the beginning of the session the package must be loaded with:

include FEP.gfn

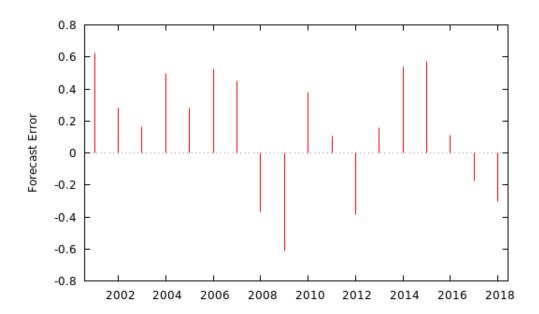


Figure 2: Forecast error for euro area growth of real GDP

⁹The concrete images of those icons will depend on your operating system and/or your desktop theme, but they are the first two buttons from the left.

7.1 Tests of unbiasedness

We run the parametric Mincer-Zarnowitz test on forecast unbiasedness using bootstrap p-values (999 iterations, *iid*) and HAC standard errors, see Section 4.1. First an empty bundle is defined, then the series and parameters are added:

```
bundle b = null
series b.y = y  # realizations
series b.fc = fc # forecasts
b.nboot = 999
b.robust = 1 # 1= HAC robust VCV
b.verb = 1  # 1= print details
applyFCtests(&b, "MZ") # call the top-level function
```

Yields as output:

As can be seen, the null hypothesis cannot be rejected at standard levels which indicates that the forecast errors are unbiased. (Please bear in mind that your exact numerical values may differ slightly due to different initializations of the bootstrap. To avoid this you would have to give gretl's random number generator a seed: set seed <whatever>)

A non-parametric alternative test approach for forecast unbiasedness was proposed by Campbell and Ghysels, as discussed in Section 4.3. Assuming that the previous code has been executed and thus b, b.y, b.fc and b.verb are already defined, the CG test can be performed as follows:¹⁰

```
applyFCtests(&b, "CG")
printf "P-value Wilcoxon-type rank sign test = %.3f\n", b.CGWILCpval
printf "P-value sign test = %.3f\n", b.CGSIGNpval
```

Yields as output:

¹⁰Formatting the number printout as %.3f in the printf command is already slightly advanced, rounding to three decimal digits. A straightforward alternative that usually also works fine for floating point numbers is %g.

These nonparametric tests do not provide strong evidence against unbiasedness, either. However, if a researcher assumed an underlying asymmetric distribution the only adequate test would be the nonparametric sign test, which might be considered borderline significant here.¹¹

7.2 Tests of efficiency

The Holden-Peel test is a parametric test of efficiency, see Section 4.2. We simply use lagged forecasts, fc_{t-1} , as the conditional variable. The HP test with bootstrap HAC standard errors is easily executed by:

```
list b.z = fc(-1) # use lagged forecast as conditional variable applyFCtests(&b, "HP")
```

Yields as output:

Hence, this parametric test result does not indicate any issue with forecast efficiency conditional on fc_{t-1} .

A non-parametric version is also provided by the CG test approach which can be used as follows. Note that here $\,k$ already specifies the lag to be applied to CGX, so the series for CGX should be given contemporaneously (here: fc), not with a lag (not: fc(-1)); otherwise errors due to missing values may occur.

¹¹The empirical skewness in this sample is -0.6. A skewed distribution would obviously imply non-Gaussianity; the Shapiro-Wilk test of the null of a normal distribution yields a p-value of 0.096, the Doornik-Hansen test in turn produces 0.152, thus somewhat borderline again.

```
b.k = 1
series b.CGX = fc
applyFCtests(&b, "CG")
printf "P-value Wilcoxon-type rank sign test = %.3f\n", b.CGWILCpval
printf "P-value sign test = %.3f\n", b.CGSIGNpval
```

Yields as output:

Thus, the non-parametric test results broadly agree with the parametric ones, although we might add that for lag 2 the result is quite different. (The reader is invited to verify this statement using the FEP package and the data in the appendix.)

7.3 Tests of (a)symmetric loss and forecast rationality

The framework proposed by Elliott et al. [2005] extends parts of the previous analysis to the case of asymmetric loss functions, cf. Section 4.4. We will use lagged forecast errors as an additional instrument apart from an intercept for illustration, and assume a quad-quad loss function. (Strictly speaking, the explicit specification of b.loss is redundant, since the quadratic U-shape is the default, see Table 2.)

Yields as output:

```
Test stat. =
            -2.41
P-value =
          0.0158
************
**************
Rationality Test
Estim. alpha =
             0.236
J-statistics =
              1.46
          0.227
P-value =
**************
*************
Rationality Test
Alpha fixed to 0.5
J-statistics =
              7.28
P-value =
          0.0262
*************
```

The null of symmetric loss can be rejected at the 5% level, and after imposing symmetry anyway one would indeed reject the null of forecast rationality at the 5% level. However, under the estimated asymmetric loss function one cannot reject the null of forecast rationality. Here the suggested conclusion is that the forecasts may be regarded as "rational", but only if one departs from the assumption of a symmetric forecast error loss function.

The estimated $\hat{\alpha}$ is 0.236 which indicates greater aversion to negative forecast errors. This is perhaps not surprising considering the sequence of positive forecast errors in the first part of the sample. Apart from the reported test results, a plot of the estimated loss function is returned (see Figure 3). The upper and lower lines represent a confidence band due to the estimation uncertainty of the asymmetry parameter $\hat{\alpha}$.

7.4 Forecast comparison

The Diebold-Mariano approach tests for equal predictive accuracy of two competing forecasts (Section 5.1). We compare the current forecast for simplicity with the pre-year (naive) forecast assuming linear loss. The test is called by:

```
b.f1 = fc  # FC 1 series
b.f2 = y(-1) # FC 2 series: naive pre-year realization
b.loss = 2  # 1="U-shape", 2="V-shape"
applyFCtests(&b, "DM")
```

Yields as output:

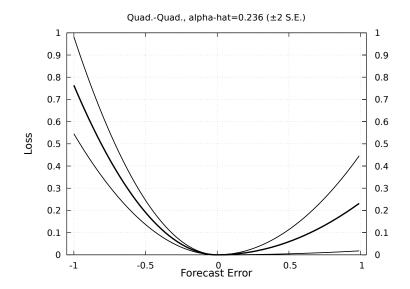


Figure 3: Estimated loss function using the EKT approach

The null hypothesis can be rejected at the 5% significance level for all three versions computed. The negative test statistic (-2.56) indicates that the "real" survey-based forecast is more accurate compared to the naive one.

7.5 Directional forecast

Apart from point forecasts, directional forecasts may be of interest. The *Kuipers score* (KS) is a widely applied simple statistic to summarize directional forecasts, see Section 3.3.1. The following lines first compute the dummy variables yup and fcup which take the value of one if the period-change in realized (or forecast) values is positive (otherwise zero), and then execute the KS computation.

```
series b.yup = (diff(y) > 0)
series b.fcup = (diff(fc) > 0)
applyFCtests(&b, "KS") # could also use doKS(&b)
```

Yields as output:

Accordingly, the forecasts have a perfect hit rate of 1 meaning that all "ups" are correctly predicted. The false alarm rate is only 10% such that the KS statistic is 0.9 which indicates a good directional forecast performance.

Finally, the *Diebold-Lopez* test (Section 4.6) tests whether a directional forecast is significantly different from a coin flip. For reasons of backwards compatibility with older FEP versions the bundle for the DL test currently must not contain other non-binary y and fc series, and therefore we cannot re-use the previously defined bundle. Instead we build a new bundle here.

The positive test statistic of 1.88 indicates that the actual forecast outperforms a pure coin flip which is statistically confirmed by the p-value.

```
bundle bDL = null
bDL.verb = 1
series bDL.yup = (diff(y) > 0)
series bDL.fcup = (diff(fc) > 0)
applyFCtests(&bDL, "DL")
```

Yields as output:

In this context the Pesaran-Timmermann test described in Section 4.5 may be seen as an alternative test for directional forecasts.

8 Summary and outlook

FEP makes it easy to analyze and evaluate forecasts and the associated forecast errors (ex post). It already offers many popular tests that have been proposed in the literature. The plan is to extend the package continually and to further broaden its scope. For example, tools to analyze density forecasts would be useful. Another interesting area for inclusion could be recent developments in the literature concerning the comparison of multiple forecasts. The bundle-based framework of FEP would allow these extensions while maintaining a coherent user interface.

References

- B. Campbell and E. Ghysels. Federal budget projections: A nonparametric assessment of bias and efficiency. *The Review of Economics and Statistics*, 77(1):17 31, 1995.
- Bryan Campbell and Eric Ghysels. An Empirical Analysis of the Canadian Budget Process. *Canadian Journal of Economics*, 30(3):553–76, August 1997. URL https://ideas.repec.org/a/cje/issued/v30y1997i3p553-76.html.
- Francis X Diebold and Jose A. Lopez. Forecast evaluation and combination. In G. S. Maddala and C. R. Rao, editors, *Handbook of Statistics*, volume 14 of *Statistical Methods in Finance*, pages 241–268. 1996.
- Francis X. Diebold and Roberto S. Mariano. Comparing predictive accuracy. *Journal of Business and Economic Statistics*, 20(1):134–144, 1995.
- Jean-Marie Dufour. Rank tests for serial dependence. *Journal of Time Series Analysis*, 2(3):117–128, 1981. ISSN 1467-9892. doi: 10.1111/j.1467-9892.1981.tb00317.x. URL http://dx.doi.org/10.1111/j.1467-9892.1981.tb00317.x.
- Graham Elliott, Ivana Komunjer, and Allan Timmermann. Estimation and testing of forecast rationality under flexible loss. *Review of Economic Studies*, 72:1107–1125, 2005.
- C.W.J. Granger. Outline of forecast theory using generalized cost functions. *Spanish Economic Review*, 1:161–173, 1999.
- David Harvey, Stephen Leybourne, and Paul Newbold. Testing the equality of prediction mean squared errors. *International Journal of Forecasting*, 13(2):281–291, 1997. URL https://ideas.repec.org/a/eee/intfor/v13y1997i2p281-291.html.
- K Holden and David Peel. On Testing for Unbiasedness and Efficiency of Forecasts. The Manchester School of Economic & Social Studies, 58(2):120–27, 1990. URL http: //EconPapers.repec.org/RePEc:bla:manch2:v:58:y:1990:i:2:p:120-27.

- J. Mincer and V. Zarnowitz. The evaluation of economic forecasts. In J. Mincer, editor, *Economic Forecasts and Expectations: Analysis of Forecasting Behavior and Performance*, pages 1 46. NBER, NBER, 1969.
- M. Hashem Pesaran. *Time Series and Panel Data Econometrics*. Number 9780198759980 in OUP Catalogue. Oxford University Press, 2015. ISBN ARRAY(0x79444480). URL https://ideas.repec.org/b/oxp/obooks/9780198759980.html.
- M Hashem Pesaran and Allan Timmermann. A Simple Nonparametric Test of Predictive Performance. *Journal of Business & Economic Statistics*, 10(4):561–65, 1992. URL https://ideas.repec.org/a/bes/jnlbes/v10y1992i4p561-65.html.
- H.R. Varian. A Bayesian approach to real estate assessment. In S.E. Fienberg and A. Zellner, editors, *Studies in Bayesian Econometrics and Statistics in Honor of Leonard J. Savage*, pages 195–208. North-Holland, Amsterdam, 1975.

A Data

For replication purposes we reproduce the data used in the illustration in Section 7. For the definitions see the text; source Macrobond and AMECO.

	fc	у
2000		
2001	1.5	2.123350
2002	0.7	0.980211
2003	0.5	0.661216
2004	1.8	2.299620
2005	1.4	1.679013
2006	2.7	3.228410
2007	2.6	3.049257
2008	0.8	0.429603
2009	-3.9	-4.514502
2010	1.7	2.081707
2011	1.5	1.605639
2012	-0.5	-0.886516
2013	-0.4	-0.240252
2014	0.8	1.336558
2015	1.5	2.070988
2016	1.7	1.811255
2017	2.4	2.227502
2018	2.4	2.099555

Impressum

Publisher: Hans-Böckler-Stiftung, Hans-Böckler-Str. 39, 40476 Düsseldorf, Germany **Phone:** +49-211-7778-331, IMK@boeckler.de, http://www.imk-boeckler.de

IMK Working Paper is an online publication series available at:

http://www.boeckler.de/imk_5016.htm

ISSN: 1861-2199

The views expressed in this paper do not necessarily reflect those of the IMK or the Hans-Böckler-Foundation.

All rights reserved. Reproduction for educational and non-commercial purposes is permitted provided that the source is acknowledged.