Campbell & Ghysels

Let's define the one-period forecast errors as $e_{1t} = y_{t+1} - f_{t+1|t}$. An indicator function indicates whether the forecast error is positive or negative such that $u(z) = 1$ if $z \geq 0$ and $u(z) = 0$ otherwise. The test statistics of the sign test on unbiasedness of forecast errors is

$$S = \sum_{t=1}^{T} u(e_{1t})$$

while the ranks sign test is defined as

$$W = \sum_{t=1}^{T} u(e_{1t}) R_{1t}^{+}$$

with R_{1t}^{+} referring to the rank of each forecast error when $|e_{1t}|$,..., $|e_{1T}|$ are placed in ascending order and T is the no. of available forecast errors. The forecast errors are independent with zero median. The sign statistic S is binomially distributed with $Bin(T, 0.5)$. Under the additional assumption of symmetrically distributed forecast errors around the zero-centre, the test statistics W follows a Wilcoxon signed rank distribution. Note, that this test is called once you set $\texttt{k} = 0$.

This test idea can also be employed to test for serial correlation in the forecast errors. Construct the product series $Z_{1\mathrm{t}}^{k} = e_{1t} e_{1(t-k)}$ with $k \geq 1$, and compute the statistics:

$$S_k = \sum_{t=k+1}^{T} u(Z_{1t}^{k}) \quad \text{and} \quad W_k = \sum_{t=k+1}^{T} u(Z_{1t}^{k}) R_{2t}^{+}$$

where R_{2t}^{+} is the signed rank of the product Z_{1t}^{k}, $t = 1, \ldots, T$. These location tests were proposed by Dufour (1981). Serial correlation in the forecast errors will move the centre of their product away from zero. The sign statistic S_k is binomially distributed with $Bin(T-k, 0.5)$. The test statistics W_k follows a Wilcoxon signed rank distribution of size $T-k$. Note, both tests on serial correlation require to set the option $\texttt{k} > 0$, and the necessary product series Z_{1t}^{k} will be constructed automatically.

Lastly, one can use this framework to assess whether the forecast has made efficient use of the available information represented by the series X in t. For this, one needs to construct the product series $Z_t^k = e_{1t} X_{t-k}^c$ with $k \geq 0$ based on the re-centered series $X_t^c = X_t - \text{median}(X_1, X_2, \ldots, X_t)$. For details see Campbell&Ghysels (1997, p.560). Even though this test is not explicitly incorporated (yet!), the user can easily apply it herself using both the doCGWILCtest() and doCGRANKtest() functions.