Optimization Techniques

Assignment.

Allwin Dsouza
18 IEE166

① Reliability Problem:

Req. design for the electric drive device has 3 stages and the final device should have the 3 necessary parts in the final stage

At stage -1 we have 3 options,

ie. Selecting one of the three parts.

The max cost is \$10,000
→ part selected
S_1^1 → stage

$S_1^1 = \prod_{i=1} r_i = 0.6$; $C_1^1 = \$1$ (x1000)

$S_1^2 = 0.8$ $C_1^2 = \$2$
$S_1^3 = 0.9$ $C_1^3 = \$3$

we have 3 ordered pair for stage -1

$S_1 = \{(0.6, 1), (0.8, 2), (0.9, 3)\}$

Now, each end of S_1 section has two options. The reliability will be multiplied & the costs will be added.

~~For~~

Scanned with CamScanner

Allwin D'souza
18IEE166

For $(0.6, 1)$ selection is the first stage the possible stage-2 selections are

$(0.6 \times 0.8, 1+5)$ $(0.6 \times 0.9 + 6)$

$\rightarrow (0.48, 6)$ $(0.54, 7)$

Simply for $(0.8, 2)$ selection,

part-1 part-3

$\rightarrow (0.56, 5)$, $(0.72, 8)$

And for $(0.9, 3)$ selections,

$\rightarrow (0.63, 6)$, $(0.72, 8)$

part-1 part-2

Now we consider only the following four selections and remove other since their costs order^n is not satisfied (Bellman's rule)

$\rightarrow (0.56, 5)$ $(0.72, 8)$ $(0.63, 6)$, $(0.72, 8)$

Now, starting with $(0.56, 5)$ we have

$\rightarrow (0.56 \times 0.5, 5+5) = (0.504, 10)$

for $(0.72, 8)$ we have

$\rightarrow (0.72 \times 0.5, 8+2) = (0.36, 10)$

Allwin Dsouza
18 IEE 106

For $(0.63, 6)$ we have
$\rightarrow (0.63 \times 0.7, 6+4) = (0.441, 10)$

for $(0.72, 8)$ we have
$\rightarrow (0.72 \times 0.5, 8+2) = (0.36, 10)$

Hence the most optimised way is to select ~~S2~~ x_2 in stage 1,
x_1 in stage 2,
x_3 in stage 3.

Scanned with CamScanner

Allwin Dsouza
18IEE106

Sol (2) Multiplicative Constraint :-

Let $y_1 \cdot y_2 \cdot y_3 \cdots y_n = d \quad \forall\ x_j \geq 0$ for $j = 1, 2, \cdots n$

Then let $f_n(d)$ denote the minimum attainable sum $y_1 + y_2 + \cdots + y_n$ when the quantity 'd' is factorized into n factors, where $\boxed{y_i = r(x_j)\ \forall\ j}$

For $n = 1$, 'd' is factorized into one factor only
So $f_1(d) = \min\{y_1\} = d \quad y_1 = d$

For $n = 2$ 'd' is factorized into two factors y_1 & y_2
If $y_1 = x$ then $y_2 = d/x$.

then, $f_2(d) = \min\{y_1 + y_2\} = \min\{x + d/x\}$

$= \min\{x + f_1(d/x)\}$

For $n = 3$, d is factorized into three factors $y_1, y_2, \& y_3$

If $y_1 = x$ & $y_2 y_3 = d/x$, then

$f_3(d) = \min\{y_1 + y_2 + y_3\} = \min\{x + f_2(d)\}$

proceeding likewise, the recurrence relation for $n = i$ becomes.

$$f_i(d) = \min_{0 < x \leq d}\{x + f_{i-1}(d/x)\}$$

Now we proceed to solve this function equation as follows:

$f_1(d) = d$; $f_2(d) = \min\limits_{0 < x \leq d}\{x + d/x\} = \sqrt{d} + d/\sqrt{d}$

Allwin Dsouza
181EE106

$$d^{1/3} + 2\sqrt{(d/d^{1/3})} = 3d^{1/3}$$

by induction hypothesis, for $n = M$ we got
$f_M(d) = Md^{1/M}$

This result can be proved for $n = M+1$, as follows.

$$f_M(d) = \min_{0 < x \le d} \{ x + f_m(d/x) \}$$

$$= (M+1)d^{1/(M+1)}$$

Therefore the result which is assumed to be true for $n = M$ is forced to be $n = M+1$, Hence the result is true for all values of n.

This the req. optimal policy is

$(d^{1/n}, d^{1/n}, \ldots d^{1/n})$ with $\boxed{f_n(d) = nd^{1/n}}$

Allwin D'souza
181EE106

(3)

x_1 \ x_2	0	1	2	3	4	5	6	7	8	9	10
0	0	-4	0	12	32	60	96	140	192	252	320
1	-3	-7	-3	9	29	57	93	137	189	249	317
2	-4	-8	-4	8	28	56	92	136	188	296	316
3	-3	-7	-3	9	29	57	93	137	189	249	317
4	0	-4	0	12	32	60	96	140	192	252	320
5	5	1	5	17	37	65	101	145	197	257	327

$z = -8$ at $x_1 = 2$ $x_2 = 1$

P4

$$z = 4x_1 + 14x_2$$

$$r_1 = 4x_1 \qquad r_2 = 14x_2$$

$$x_1 \le \frac{21}{2} \quad \& \quad x \le \frac{21}{7} \qquad x_1 = 3$$

$$x_2 = 3$$

$x_1 = 0, 1, 2, 3$

$x_2 = 0, 1, 2, 3$

x	0	1	2	3
$r_1(x)$	0	4	8	12
$r_2(x)$	0	14	28	42

x_2 \ x_1	0	1	2	3
0	0	14	28	42 Infeasible
1	4	18	32	46
2	8	22	36	50
3	12	26	40	54

Allwin Dsouza
18 IEE106

$z = 42$ at $x_1 = 0$ & $x_2 = 3$

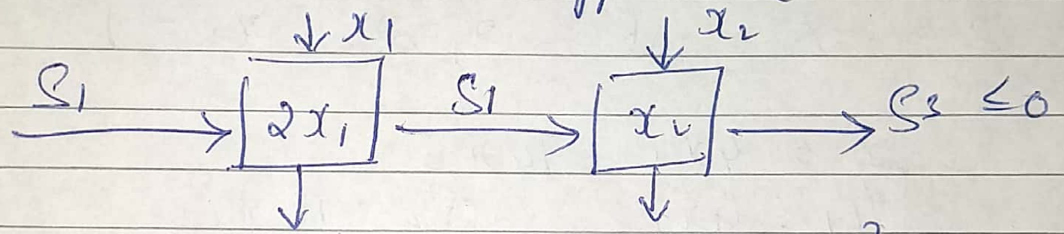
P_3 $\quad \min f(x) = x_1^2 + 4x_2^2 \cdot 2^7 - 4x_1 - 8x_2$

st $\quad 2x_1 + x_2 \le 10$

$x_1, \; x_2 \ge 0$ integers

$\min f(x) = (x_1^2 - 4x_1) + (4x_2^2 - 8x_2)$

Forward Tabulatn approach



$r_1 = x_1^2 - 4x_1 \qquad r_2 = 4x_2^2 - 8x_2$

Range of x_1 $\qquad 0 \le x_1 \le \frac{10}{2} \qquad 0 \le x_1 \le 5$

Range of x_2 $\qquad 0 \le x \le 10$

x	0	1	2	3	4	5	6	7	8	9	10
$r_1(x)$	0	-3	-4	-3	0	5	–	–	–	–	–
$r_2(x)$	0	-4	0	12	32	60	96	140	192	252	320

$$f_1(s_1) = \min_{0 \le x_1 \le 5} (r_1(x_1, s_1)) = 0 \text{ for } x_1 = 0$$

& $x_1 = 4$

$$f_2(s_2) = \min_{0 \le x_2 \le 10} (r_2(x_2, S_2) + f_1(x_1, s_1))$$

Allwin Dsouza
18IEE106

P4 Given,

$$\text{Max}, \; z = 4x_1 + 14x^2$$
$$2x_1 + 7x_2 \leq 21$$
$$7x_1 + 2x_2 \leq 21$$
$$x_1, x_2 \geq 0$$

Solving with integer variables (Tabular approach)

$$z = 4x_1 + 14x_2$$
$$r_1 = 4x_1 \quad \& \quad r_2 = 14x_2$$

$$\Rightarrow \; x_1 \leq \frac{21}{2} \quad \& \quad x_1 \leq \frac{21}{2}$$

$$x_{1u} = \min\left\{\frac{21}{2}, \frac{21}{7}\right\} = 3$$
$$x_{2u} = \min\left\{\frac{21}{7}, \frac{21}{2}\right\} = 3$$

$$\Rightarrow \; x_1 = 0, 1, 2, 3$$
$$x_2 = 0, 1, 2, 3$$

x	0	1	2	3
$r_1(x)$	0	4	8	12
$r_2(x)$	0	14	28	42

For finding z_{max}, tabular is given as

Allwin Dsouza
18IEE106

x_1 \ x_2	0	1	2	3
0	0	14	28	42
1	4	18	32	46
2	8	22	36	50
3	12	26	40	54

This will be in infeasible region since violates constraint condition for given x_1 & x_2

Therefore from table, $Z_{max} = 42$
at $x_1 = 0$ &
$x_2 = 3$

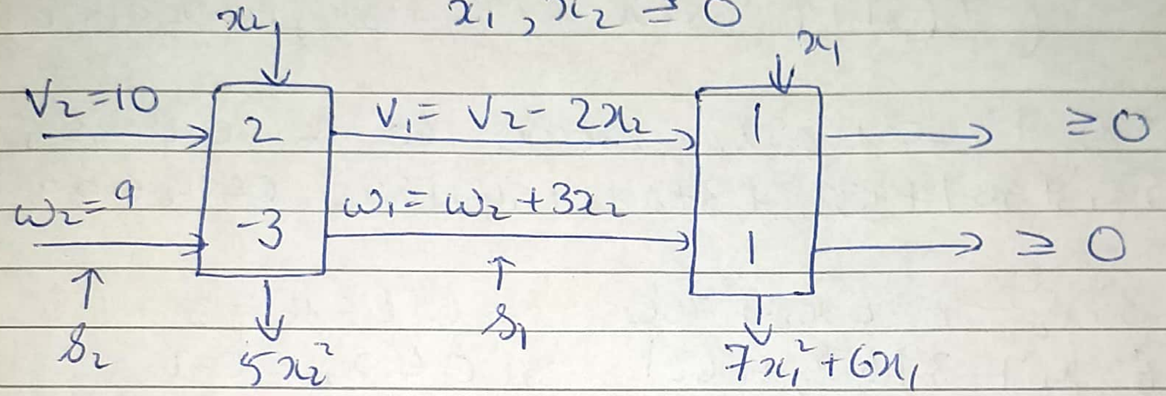
Allwin D'Souza
18IEE106

(5) Max, $z = 7x_1^2 + 6x_1 + 5x_2^2$

S.t, $x_1 + 2x_2 \le 10$

$x_1 - 3x_2 \le 9$

$x_1, x_2 \ge 0$

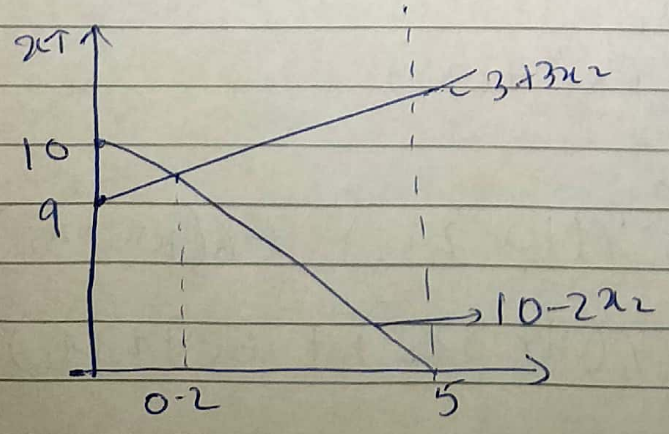


Backward approach:

$$f_1(v_1, w_1) = \max_{0 \le x_1 \le x_m} (7x_1^2 + 6x_1)$$

where $x_m = \min \begin{Bmatrix} V_2 - 2x_2 \\ w_2 + 3x_2 \end{Bmatrix}$

$= \min \begin{Bmatrix} 10 - 2x_2 \\ 9 + 3x_2 \end{Bmatrix} \ge 0$



$\Rightarrow x_1 = \begin{cases} 10 - 2x_2 & 0.2 \le x_2 \le 5 \\ 9 + 3x_2 & 0 \le x_2 \le 0.2 \end{cases}$

$10 - 2x_2 = 9 + 3x_2$

$\Rightarrow x_2 = 0.2$

Allwin Dsouza
181EE106

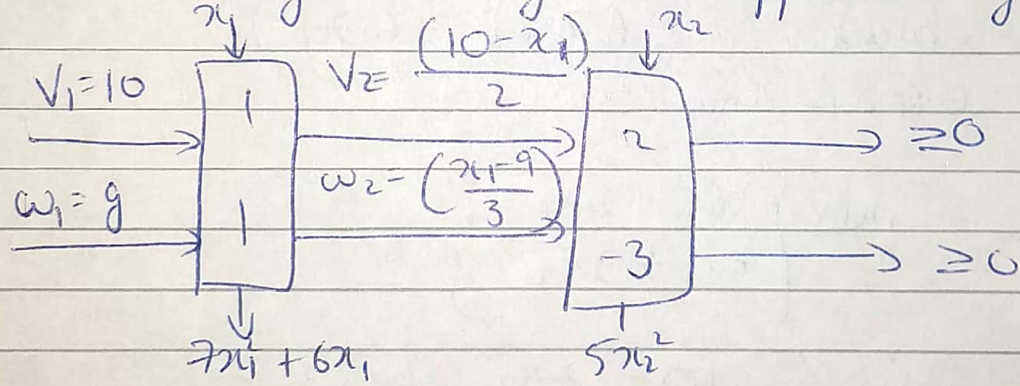
$$f_2(x_2 \mid v_1, w_1) = 5x_2^2 + \begin{cases} 7(10-2x_2)^2 + 6(10-2x_2) & 0.2 \le x_2 \le 5 \\ 7(9+3x_2)^2 + 6(9+3x_2) & 0 \le x_2 \le 0.2 \end{cases}$$

$$5x_2^2 + \begin{cases} 7(100 + 4x_2^2 - 40x_2) + 60 - 12x_2 ; & 0.2 \le x_2 \le 5 \\ 7(81 + 9x_2^2 + 54x_2) + 54 + 18x ; & 0 \le x_2 \le 0.2 \end{cases}$$

$$= \max \begin{cases} 5x_2^2 + 700 + 28x_2^2 - 280x_2 + 60 - 12 & 0.2 \le x_2 \le 5 \\ 5x_2^2 + 567 + 63x_2^2 + 378x_2 + 54 + 18x_2 & 0 \le x_2 \le 0.2 \end{cases}$$

$$= \max \begin{cases} 33x_2^2 - 282x_2 + 760 & ; 0.2 \le x_2 \le 5 \\ 68x_2^2 + 396x_2 + 621 & ; 0 \le x_2 \le 0.2 \end{cases}$$

2) Checking with forward approach for alternate nomenclature.



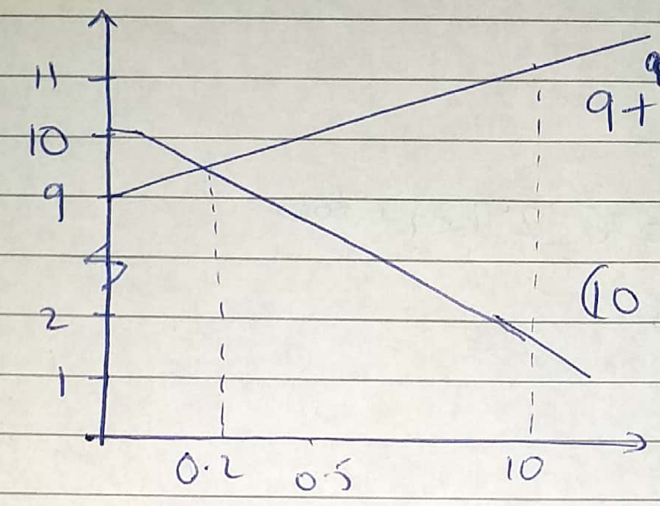
$$f_1(v_1, w_1) = 7x_1^2 + 6x_1$$

$$= \max_{0 \le x_1 \le x} (7x_1^2 + 6x_1)$$

$$= \max_{0 \le x_2 \le x_{2u}} \begin{cases} 7(10-2x_2)^2 + 6(10-2x_2) \\ 7(9+3x_2)^2 + 6(9+3x_2) \end{cases}$$

Allwin D'souza
18IEE106

$$x_{2u} = \min \begin{cases} 10-2x_2 & 0.2 \le x_2 \le 10 \\ 9+3x_2 & 0 \le x_2 \le 0.2 \end{cases}$$



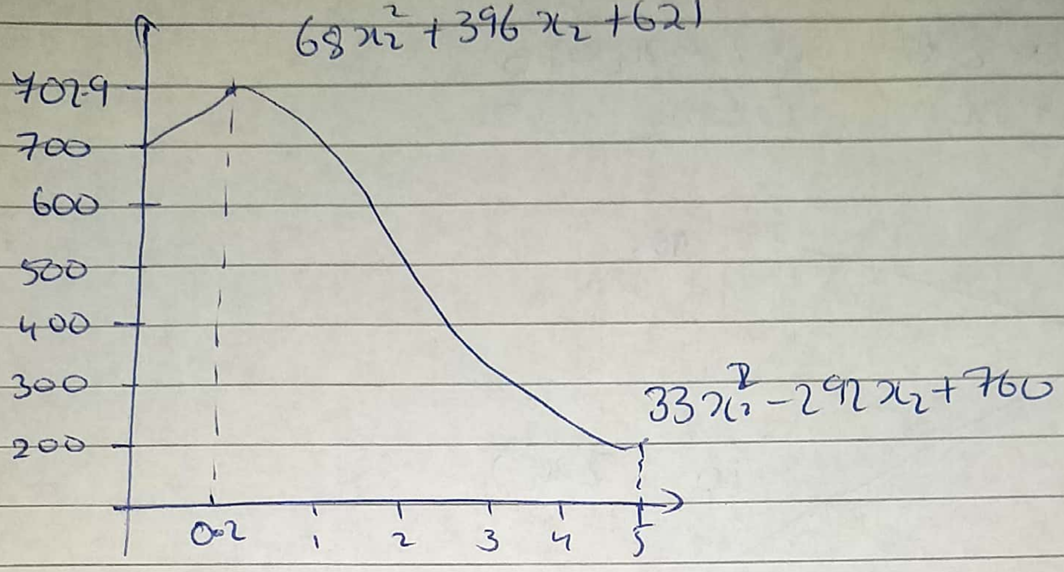
$$10-2x_2 = 9+3x_2$$
$$x_2 = 0.2$$

$$f_1(V_1, \omega_1) = \max \begin{cases} 7(10-2x_2)^2 + 6(10-2x_2) & 0.2 \le x_2 \le 10 \\ 7(9+3x_2)^2 + 6(9+3x_2) & 0 \le x_2 \le 0.2 \end{cases}$$

$$f_2(x_2 | V_1, \omega_1) = 5x_1^2 + \begin{cases} 7(10-2x_2)^2 + 6(10-2x_2) & 0.2 \le x_2 \le 10 \\ 7(9+3x_2)^2 + 6(9+3x_2) & 0 \le x_2 \le 0.2 \end{cases}$$

$$= \max \begin{cases} 33x_2^2 - 292x_2 + 760, & 0.2 \le x_2 \le 5 \\ 68x_2^2 + 396x_2 + 621, & 0 \le x_2 \le 0.2 \end{cases}$$

Allwin Dsouza
18 IEE106



Max is at $x_2 = 0.2$

$x_1 = 9 + 3(0.2)$

$x_1 = 9.6$

$x_1 = 9.6$

$x_2 = 0.2$

$Z_{max} = 702.9$

Hence verified

Conclusion:

Forward approach for alternative method is same as backward approach of initial nomenclature.

Allwin Dsouza
181EE106

3) Solving using backward approach for alternate nomenclature.

$$F_2(V_2, \omega_2) = \max_{0 \le x_2 \le x_{2u}} 5x_2^2$$

$$= \max_{0 \le x_1 \le x_{1u}} \begin{cases} 5\left(\frac{10-x_1}{2}\right)^2 \\ 5\left(\frac{x_1-9}{3}\right)^2 \end{cases}$$

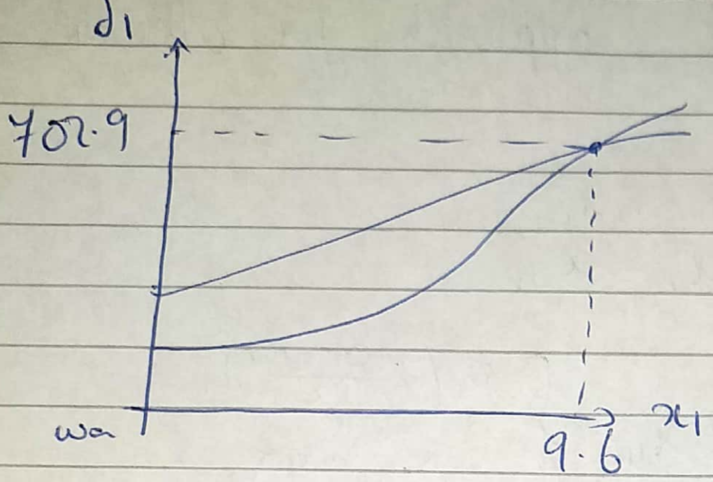
$$x_{1u} = \min \begin{cases} \frac{10-x_1}{2} & 9.6 \le x_1 \le 10 \\ \frac{x_1-9}{3} & 0 \le x_1 \le 9.6 \end{cases}$$

$$f_2(V_2, \omega_2) = \max \begin{cases} 5\left(\frac{10-x_1}{2}\right)^2 & 9.6 \le x_1 \le 10 \\ 5\left(\frac{x_1-9}{3}\right)^2 & 0 \le x_1 \le 9.6 \end{cases}$$

$$f_1(x_1 | V_2, \omega_2) = 7x_1^2 + 6x_1 + \begin{cases} 5\left(\frac{10-x_1}{2}\right)^2 & 9.6 \le x_1 \le 10 \\ 5\left(\frac{x_1-9}{3}\right)^2 & 0 \le x_1 \le 9.6 \end{cases}$$

$$= \max \begin{cases} \frac{(19x_1^2 - 88x_1 + 500)}{2}, & 9.6 \le x_1 \le 10 \\ \frac{(26x_1^2 - 72x_1 + 405)}{3}, & 0 \le x_1 \le 9.6 \end{cases}$$

Allwin Dsouza
18 IEE106



$\therefore$ Max is at $x_1 = 9.6$

$$x_2 = \frac{10 - 9.6}{2} = 0.2$$

$x_1 = 9.6$
$x_2 = 0.2$

$Z_{max} = 702.9$

Hence Verified.

Conclusion: Backwards approach for alternate nomenclature is same as forward approach of initial nomenclature.