$$\begin{array}{ccccc}
f(x_{k-1}) & & & & \\
& \searrow & & & \\
& & \frac{f(x_k)-f(x_{k-1})}{h} & & \\
& \nearrow & & \searrow & \\
f(x_k) & & & & \frac{f(x_{k+1})-2f(x_k)+f(x_{k-1})}{2h^2} \\
& \searrow & & \nearrow & \\
& & \frac{f(x_{k+1})-f(x_k)}{h} & & \\
& \nearrow & & & \\
f(x_{k+1}) & & & &
\end{array}$$

D'où

$$L_2(f)(x) = f(x_{k-1}) + \frac{f(x_k) - f(x_{k-1})}{h}(x - x_{k-1}) + \frac{f(x_{k+1}) - 2f(x_k) + f(x_{k-1})}{2h^2}(x - x_{k-1})(x - x_k)$$

$$\int_{x_k}^{x_{k+1}} L_2(f)(t)dt = f(x_{k-1})h + \frac{f(x_{k-1})}{h}\left((\underbrace{x_{k+1} - x_{k-1}}_{=2h})^2 - (\underbrace{x_k - x_{k-1}}_{=h})^2\right) + \frac{f(x_{k+1}) - 2f(x_k) + f(x_{k-1})}{2h^2} \times (c)$$

$$\begin{aligned}
(c) = \int_{x_k}^{x_{k+1}} (x - x_{k-1})(x - x_k)dx &= \int_0^h u(u+h)du \text{ en posant } u = x - x_k \\
&= \frac{1}{3}h^3 + \frac{1}{2}h^3 \\
&= \frac{5}{6}h^3
\end{aligned}$$

$$\begin{aligned}
\Rightarrow \int_{x_k}^{x_{k+1}} L_2(f)(t)dt &= f(x_{k-1})h + f(x_{k-1})\frac{3}{2}h + \frac{5}{12}h(f(x_{k+1}) - 2f(x_k) + f(x_{k-1})) \\
&= \frac{h}{12}(-f_{k-1} + 8f_k + 5f_{k+1})
\end{aligned}$$