

# Ragax: Ragalur Expressions

Using derivatives to validate Indian Classical Music

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2 May 2018

Amsterdam Functional Programming Meetup

## **Derivatives** are **Intuitive** and **Extendable**

- implement matcher
- invent operators
- listen to music

# Regular Expressions

$a(a|b)^*$

ab ✓

aabbba ✓

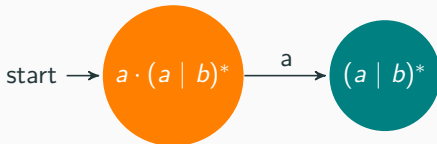
ac ✗

ba ✗

$a \cdot (a \mid b)^*$

# What is a Brzozowski Derivative

The Brzozowski derivative (1964) [1] of an expression is the expression that is left to match after the given character has been matched.



$$\begin{aligned}\partial_a a \cdot b \cdot c &= b \cdot c \\ \partial_a (a \cdot b \mid a \cdot c) &= (b \mid c) \\ \partial_c c &= \epsilon \\ \partial_a a^* &= a^*\end{aligned}$$

# Basic Operators

|               |               |
|---------------|---------------|
| empty set     | $\emptyset$   |
| empty string  | $\varepsilon$ |
| character     | $a$           |
| concatenation | $r \cdot s$   |
| zero or more  | $r^*$         |
| logical or    | $r \mid s$    |

# Basic Operators

```
data Regex = EmptySet
  | EmptyString
  | Character Char
  | Concat Regex Regex
  | ZeroOrMore Regex
  | Or Regex Regex
```

Does the expression match the empty string.

$$\nu(\emptyset) = \text{false}$$

$$\nu(\varepsilon) = \text{true}$$

$$\nu(a) = \text{false}$$

$$\nu(r \cdot s) = \nu(r) \text{ and } \nu(s)$$

$$\nu(r^*) = \text{true}$$

$$\nu(r \mid s) = \nu(r) \text{ or } \nu(s)$$

Does the expression match the empty string.

```
nullable :: Regex -> Bool
nullable EmptySet = False
nullable EmptyString = True
nullable Character{} = False
nullable (Concat a b) = nullable a && nullable b
nullable ZeroOrMore{} = True
nullable (Or a b) = nullable a || nullable b
```



# Nullable Examples

$$\nu(a \cdot b \cdot c) = \text{X}$$

$$\nu(\varepsilon) = \checkmark$$

$$\nu(a \mid b) = \text{X}$$

$$\nu(\varepsilon \mid a) = \checkmark$$

$$\nu(a \cdot \varepsilon) = \text{X}$$

$$\nu((a \cdot b)^*) = \checkmark$$

$$\nu(c \cdot (a \cdot b)^*) = \text{X}$$

# Derivative Rules

$$\begin{aligned}\partial_a \emptyset &= \emptyset \\ \partial_a \epsilon &= \emptyset \\ \partial_a a &= \epsilon \\ \partial_a b &= \emptyset && \text{for } b \neq a \\ \partial_a(r \cdot s) &= \partial_a r \cdot s && \text{not}(\nu(r)) \\ \partial_a(r \cdot s) &= \partial_a r \cdot s \mid \partial_a s && \nu(r) \\ \partial_a(r^*) &= \partial_a r \cdot r^* \\ \partial_a(r \mid s) &= \partial_a r \mid \partial_a s\end{aligned}$$

# Derivative Rules

```
deriv :: Regex -> Char -> Regex
deriv EmptyString _ = EmptySet
deriv EmptySet _ = EmptySet
deriv (Character a) c = if a == c
    then EmptyString else EmptySet
deriv (Concat r s) c =
    let left = deriv r c
        right = deriv s c
    in if nullable r
        then Or (Concat left s) right
        else Concat left s
deriv (ZeroOrMore r) c =
    Concat (deriv r c) (ZeroOrMore r)
deriv (Or r s) c =
    Or (deriv r c) (deriv s c)
```

# Our regular expression matcher

$$\begin{aligned} \nu(\text{foldl}(\partial, r, \text{str})) & \quad \text{where} \\ \text{foldl}(\partial, r, \text{str}) &= r \quad \text{if } \text{str} == "" \\ &= \text{foldl}(\partial, \partial_{s[0]}(r), s[1 :]) \quad \text{otherwise} \end{aligned}$$

```
match :: Regex -> String -> Bool
```

```
match r str = nullable (foldl deriv r str)
```

```
func matches(r *expr, str string) bool {  
    for _, c := range str {  
        r = deriv(r, c)  
    }  
    return nullable(r)  
}
```

# Simplification

$$\emptyset \cdot r \approx \emptyset$$

$$r \cdot \emptyset \approx \emptyset$$

$$\varepsilon \cdot r \approx r$$

$$r \cdot \varepsilon \approx r$$

$$r \mid r \approx r$$

$$\emptyset \mid r \approx r$$

$$(r^*)^* \approx r^*$$

$$\varepsilon^* \approx \varepsilon$$

$$\emptyset^* \approx \varepsilon$$

## Example: Matching a sequence of notes

Using a regex we can validate the C Major Pentatonic Scale.

$$c \cdot (c|d|e|g|a)^*$$

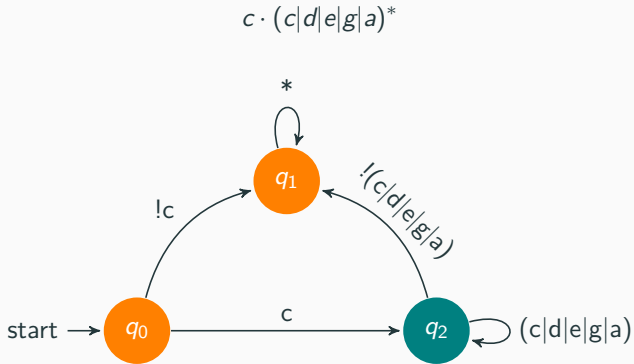
ceg ✓

$$\begin{aligned}\partial_c c \cdot (c|d|e|g|a)^* &= \varepsilon \cdot (c|d|e|g|a)^* \\ \partial_e \varepsilon \cdot (c|d|e|g|a)^* &= (\emptyset \cdot (c|d|e|g|a)^*) \mid (\emptyset|\emptyset|\varepsilon|\emptyset|\emptyset) \cdot (c|d|e|g|a)^* \\ &= (c|d|e|g|a)^* \\ \partial_g (c|d|e|g|a)^* &= (\emptyset|\emptyset|\emptyset|\varepsilon|\emptyset) \cdot (c|d|e|g|a)^* \\ &= (c|d|e|g|a)^*\end{aligned}$$

$$\nu((c|d|e|g|a)^*) = \checkmark$$

**Questions?**

# Deterministic Finite Automata



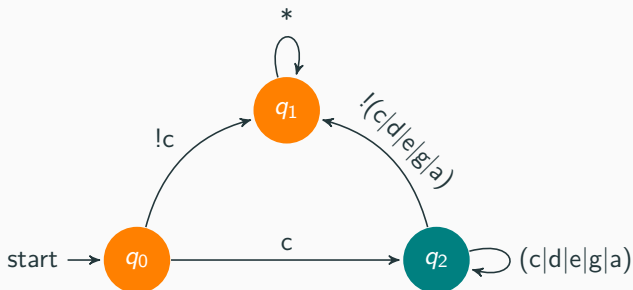


# Memoization and Simplification

$$q_0 = c \cdot (c|d|e|g|a)^*$$

$$q_1 = \emptyset$$

$$q_2 = (c|d|e|g|a)^*$$



- Memoizing deriv = transition function
- Memoizing nullable = accept function
- Simplification = minimization [4]

# Recursive Regular Expressions

$$((a \cdot b)^* \mid c)$$

Two new operations:

- Define a reference:  $\#myref = (a \cdot b)^*$
- Use a reference:  $(@myref \mid c)$

$$\begin{aligned}\partial_a @q &= \partial_a \#q \\ \nu(@q) &= \nu(\#q)\end{aligned}$$

# Ragas - Indian Classical Music

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<https://youtu.be/iElMWziZ62A?t=136>

Ragas are indian version of western scales [5]:

- Stricter
- Next note depends on current note.
- Notes named differently and relative to root note.

|         |   |    |   |    |   |   |    |   |    |   |    |   |
|---------|---|----|---|----|---|---|----|---|----|---|----|---|
| Raga    | S | r  | R | g  | G | m | M  | P | d  | D | n  | N |
| Western | c | c♯ | d | d♯ | e | f | f♯ | g | g♯ | a | a♯ | b |

## Example Raga

- Raag Bhupali (a type of Pentatonic scale)
- Ascent: S R G P D S'
- Descent: S' D P G R S
  
- Western Pentatonic scale
- Ascent: c d e g a c<sup>1</sup>
- Descent: c<sup>1</sup> a g e d c

[http://raag-hindustani.com/22\\_files/  
ArohBhupali.mp3](http://raag-hindustani.com/22_files/ArohBhupali.mp3)

**Questions?**



# A Grammar for a Raga

- Raag Bhupali (a type of Pentatonic scale)
- Ascent: S R G P D S'
- Descent: S' D P G R S

$$\#S = (S \cdot (@R \mid @D))^*$$

$$\#R = R \cdot (@G \mid \varepsilon)$$

$$\#G = G \cdot (@P \mid @R)$$

$$\#P = P \cdot (@D \mid @G)$$

$$\#D = D \cdot (\varepsilon \mid @P)$$

**Demo**

# Context Free Grammars

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# Left Recursive Raga

$$\begin{aligned}\#S &= (S \cdot (@R \mid @D))^* = @S \cdot (S \cdot (@R \mid @D)) \mid \varepsilon \\ \#R &= R \cdot (@G \mid \varepsilon) \\ \#G &= G \cdot (@P \mid @R) \\ \#P &= P \cdot (@D \mid @G) \\ \#D &= D \cdot (\varepsilon \mid @P)\end{aligned}$$

nullable and derivative each have infinite recursion.

$$\nu(\#S) = (\nu(@S) \text{ and } \nu(S \cdot (@R \mid @D))) \text{ or } \nu(\varepsilon)$$

This has been solved using [3] functional concepts:

- laziness - The Brake
- memoization - The Handbrake
- least fixed point - The Gas

Strict:

```
func strictPlus(a, b int) int {  
    return a + b  
}
```

Lazy:

```
func lazyPlus(a, b int) func() int {  
    return func() int {  
        return a + b  
    }  
}
```

$\lambda \implies$  laziness

$$\begin{aligned}\partial_a(r|s) &= \partial_a r \mid \partial_a s &= \lambda(\partial_a r) \mid \lambda(\partial_a s) \\ \partial_a(r^*) &= \partial_a r \cdot r^* &= \lambda(\partial_a r) \cdot r^* \\ \partial_a(r \cdot s) &= \partial_a r \cdot s \mid j(r) \cdot \partial_a s &= \lambda(\lambda(\partial_a r) \cdot s) \mid \lambda(\lambda(j(r)) \cdot \lambda(\partial_a s))\end{aligned}$$

$$\begin{aligned}j(r) &= \epsilon && \text{if } \nu(r) \\ &= \emptyset && \text{otherwise}\end{aligned}$$

$$\partial_n \# S = \lambda(\partial_n(@S \cdot (S \cdot (@R \mid @D)))) \mid \lambda(\partial_n \epsilon)$$

# Memoization - The Handbrake

Nullable is called:

$$\begin{aligned}\nu(\partial_n \# S) &= \nu(\lambda(\partial_n(@S \cdot (S \cdot (@R \mid @D)))) \mid \lambda(\partial_n \varepsilon)) \\ &= \nu(\lambda(\partial_n(@S \cdot (S \cdot (@R \mid @D)))) \mid \nu(\lambda(\partial_n \varepsilon)))\end{aligned}$$

Lazy function is executed:

$$\begin{aligned}\lambda(\partial_n(@S \cdot (S \cdot (@R \mid @D)))) &= \partial_n(@S \cdot (S \cdot (@R \mid @D))) \\ &= \lambda(\lambda(\partial_n @S) \cdot \lambda((S \cdot (@R \mid @D)))) \mid \\ &\quad \lambda(\lambda(j(@S)) \cdot \lambda(\partial_n(S \cdot (@R \mid @D)))) \\ \lambda(\partial_n @S) &= \partial_n @S\end{aligned}$$

Infinite recursion:

$$\partial_n @S = \partial_n \# S$$

Memoizing closes the loop:

$$\partial_n @S = \lambda(\partial_n(@S \cdot (S \cdot (@R \mid @D)))) \mid \lambda(\partial_n \varepsilon)$$



# Memoization

```
func memoize(eval func(a) b) func(a) b {  
    mem := make(map[a]b)  
    return func(input a) b {  
        if output, ok := mem[input]; ok {  
            return output  
        }  
        output := eval(a)  
        mem[input] = output  
        return output  
    }  
}
```

$$f(x) = x^2$$

$$f(0) = 0^2$$

$$f(1) = 1^2$$

fixed points =  $\{0, 1\}$

least fixed point = 0

# Least Fixed Point of Derivative

$$\partial_a r = r$$

$$\partial_a \emptyset = \emptyset$$

$$\partial_a a^* = a^*$$

fixed points =  $\{\emptyset, a^*\}$

least fixed point =  $\emptyset$

# Least Fixed Point - The Gas

Nullable is relentless:

$$\begin{aligned}\nu(\lambda(\partial_n \# S)) &= \dots \\ \nu(\lambda(\partial_n(@S \cdot (S \cdot (@R \mid @D)))) \mid \nu(\lambda(\partial_n \varepsilon))) &= \dots \\ \nu(\lambda(\lambda(\partial_n @S) \cdot \lambda(\dots))) &= \dots \\ \nu(\lambda(\partial_n @S)) &= \nu(\text{fix}) \\ &= \nu(\emptyset) \\ &= \text{false} \\ \nu(\lambda(\lambda(\partial_n @S) \cdot \lambda(\dots))) &= \text{false} \ \& \ \text{false} \\ \nu(\lambda(\partial_n(@S \cdot (S \cdot (@R \mid @D)))) \mid \nu(\lambda(\partial_n \varepsilon))) &= \text{false} \mid \text{false} \\ \nu(\lambda(\partial_n \# S)) &= \text{false}\end{aligned}$$

<http://awalterschulze.github.io/ragax/>

# Yacc is Dead

Yacc, Antlr, Flex, Bison, etc. perform better.

But derivatives:

- more intuitive than LR and LALR parsers;
- only use functional techniques;
- recognize generalized Context Free Grammars, not just a subset.

# Trees

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<http://relaxng.org/> [2] - RELAX NG is a schema language for XML, like XSchema and DTD.

Derivatives used for Implementation and Specification.

Polymorphic Regular Expressions: Characters  $\Rightarrow$  XMLNodes.

New Operators:

$$\partial_a(r \&\& s) = (\partial_a r \&\& s) \mid (\partial_a s \&\& r)$$

$$\nu(r \&\& s) = \nu(r) \text{ and } \nu(s)$$

$$\emptyset \&\& r \approx \emptyset$$

$$\varepsilon \&\& r \approx r$$

$$\partial_a!(r) = !(\partial_a r)$$

$$\nu(!(r)) = \text{not}(\nu(r))$$

$$(r)? \approx r \mid \varepsilon$$



# TreeNode

```
data Expr = ...
    TreeNode String Expr
    ...

deriv :: Expr -> Tree -> Expr
deriv (TreeNode nameExpr childExpr)
    (Node name children) =
    if nameExpr == name then
        let childDeriv = foldl deriv childExpr children
        in if nullable childDeriv
            then Empty
            else EmptySet
    else EmptySet

nullable (TreeNode _ _) = False
```

<https://youtu.be/SvjSP2xYZm8>

<https://katydid.github.io>

# Katydid: Relapse

Relapse: Tree Validation Language.

JSON, Protobufs, Reflected Go Structures and XML

Go, Haskell + Cross language testsuite

New Operators:

$$\begin{aligned}\partial_a(r \& s) &= (\partial_a r \& \partial_a s) \\ \nu(r \& s) &= \nu(r) \text{ and } \nu(s) \\ \emptyset \& r &\approx \emptyset \\ r \& r &\approx r \\ * &\approx !(\emptyset) \\ .r &\approx * \cdot r \cdot *\end{aligned}$$

<https://github.com/katydid/katydid-haskell>

<http://katydid.github.io/play/>

<http://katydid.github.io/tour/>

# References



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