

Pricing green financial products

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Motzen Conference 2016, July 7-9

Hedging weather risk



Hedging weather risk

EDITION: UNITED STATES

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"Workable made our recruitment process easier"
— ZANETA KORPOWSKA - HR SPECIALIST, 10CLOUDS

TRY IT FREE

Markets | Sun Jun 12, 2011 1:00pm EDT

Insurance-like Product Protects Power Developers from Windless Days

BY MARIA GALLUCCI

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Thanks to new technology, risk management firm Galileo is able to offer wind developers an insurance-like product that helps cover losses on windless days

"Workable takes the headache out of business"

Nasdaq

no 75/15 Nasdaq Commodities launches Index for German Wind Power Production

Nasdaq Commodities is pleased to announce the launch of the daily index for German wind power production, the N Renewable Index Wind Germany, NAREX-WIDE.

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September 08, 2015 08:00 ET | Source: Nasdaq Commodities

The index will be used as underlying for the Nasdaq Futures contracts for German wind power production that, upon successful testing and regulatory approval, will be launched later this year. This will allow investors to hedge their exposure to the volatility of wind power production.

PROFILE

Nasdaq Commodities

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First German wind futures sold on Nasdaq Commodities



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Catastrophe bonds, insurance linked securities, reinsurance capital

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EEX to launch exchange traded wind power derivatives

by ARTEMIS on MARCH 6, 2015

Share 10

European Energy Exchange AG, the EEX, is planning to launch exchange traded wind power derivatives and futures as a response to the "energy turnaround" which sees renewables increasing their share of global energy production.

Weather derivatives and weather hedging tools are going to play an increasingly important role as the energy markets turn towards renewables. Germany is one of the energy markets that is shifting towards renewables at the fastest rate and the EEX, which is majority owned by Deutsche Boerse's derivatives exchange Eurex, is keen to be at the forefront.

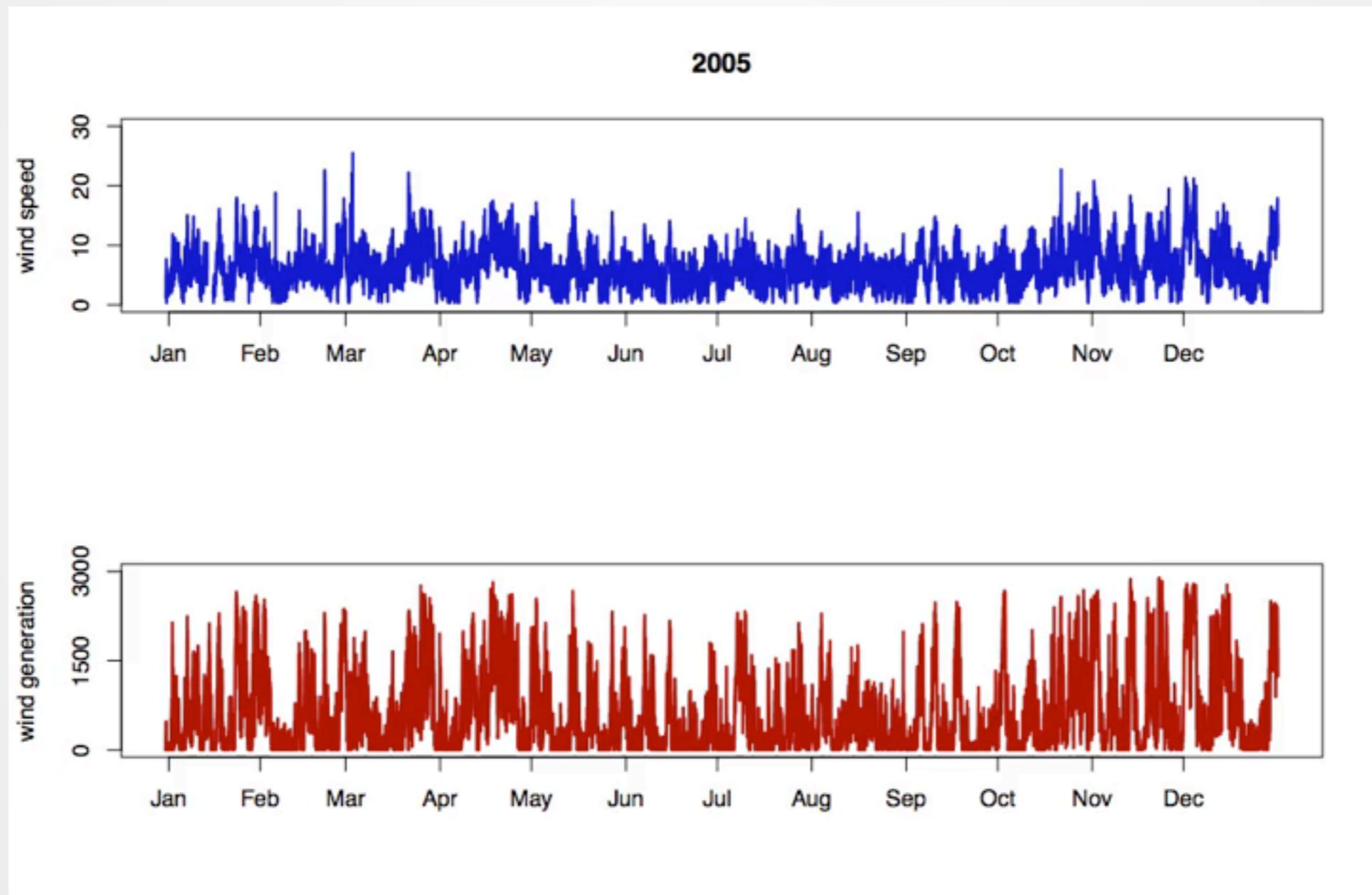


Weather risk in energy production

- Renewables become dominating energy source
- Energy output highly dependent on weather conditions
 - “Lack of wind” phenomenon
 - Sudden wind speed changes: strong power drop
 - Unsteady wind direction: zero wind power
 - Cloud coverage over one solar panel affects the panel output of an entire farm excessively



Having wind ≠ producing energy



Green financial products

Hedge weather related risk exposures on energy production due to climate volatility

- Payments based on weather related measurements
- Applicable to any renewable energy project
 - biomass, wave, tidal, hydropower, solar energy
 - hedging against “lack of wind”
- Underlying: indices on wind speed, sun duration, cloud coverage, etc.



Wind futures at European Energy eXchange (EEX)

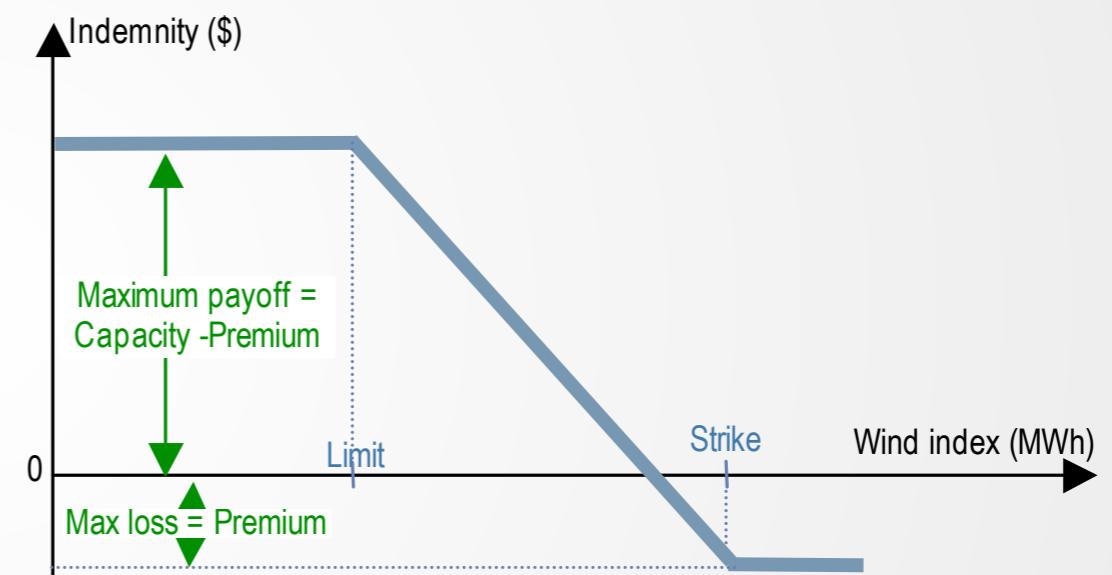
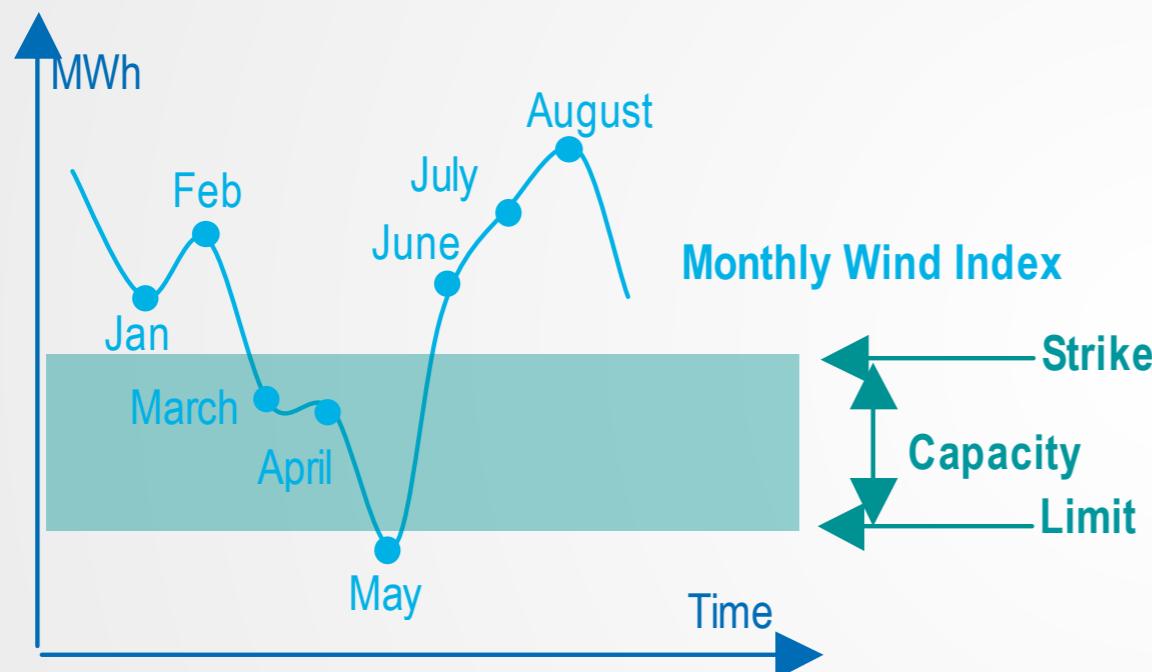
- **Contract:** A contract settling against the expected on-site power production of future delivery periods
- **Underlying:** Average wind load factor per contract period;
alternatives:
 - wind speed, wind direction, wind duration indices

Example: Cumulative Average Wind Speed index:

$$CAWS(\tau_1; \tau_2) = \int_{\tau_1}^{\tau_2} W(s) ds$$



Wind futures mechanism



Left: Wind futures mechanism; Right: payoff profile



Weather Derivatives at Chicago Mercantile Exchange

CME products

- $\text{HDD}(\tau_1, \tau_2) = \int_{\tau_1}^{\tau_2} \max(18^\circ\text{C} - T_t, 0) dt$
- $\text{CDD}(\tau_1, \tau_2) = \int_{\tau_1}^{\tau_2} \max(T_t - 18^\circ\text{C}, 0) dt$
- $\text{CAT}(\tau_1, \tau_2) = \int_{\tau_1}^{\tau_2} T_t dt$, where $T_t = \frac{T_{t,\max} + T_{t,\min}}{2}$
- $\text{AAT}(\tau_1, \tau_2) = \int_{\tau_1}^{\tau_2} \tilde{T}_t dt$, where $\tilde{T}_t = \frac{1}{24} \int_1^{24} T_{t,i} dt_i$ and $T_{t,i}$ denotes the temperature of hour t_i , (also referred to as C24AT index).



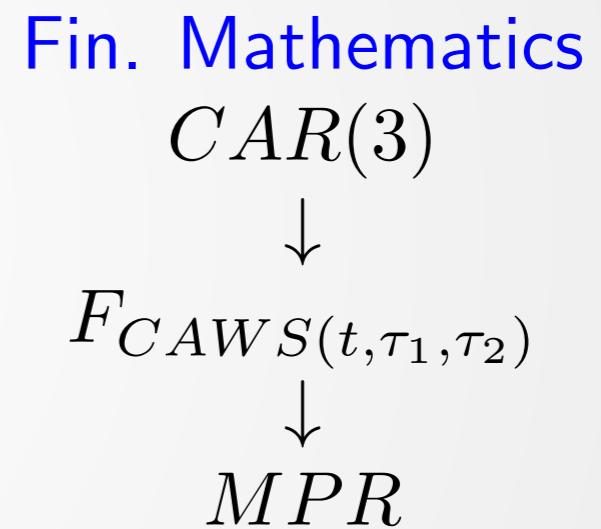
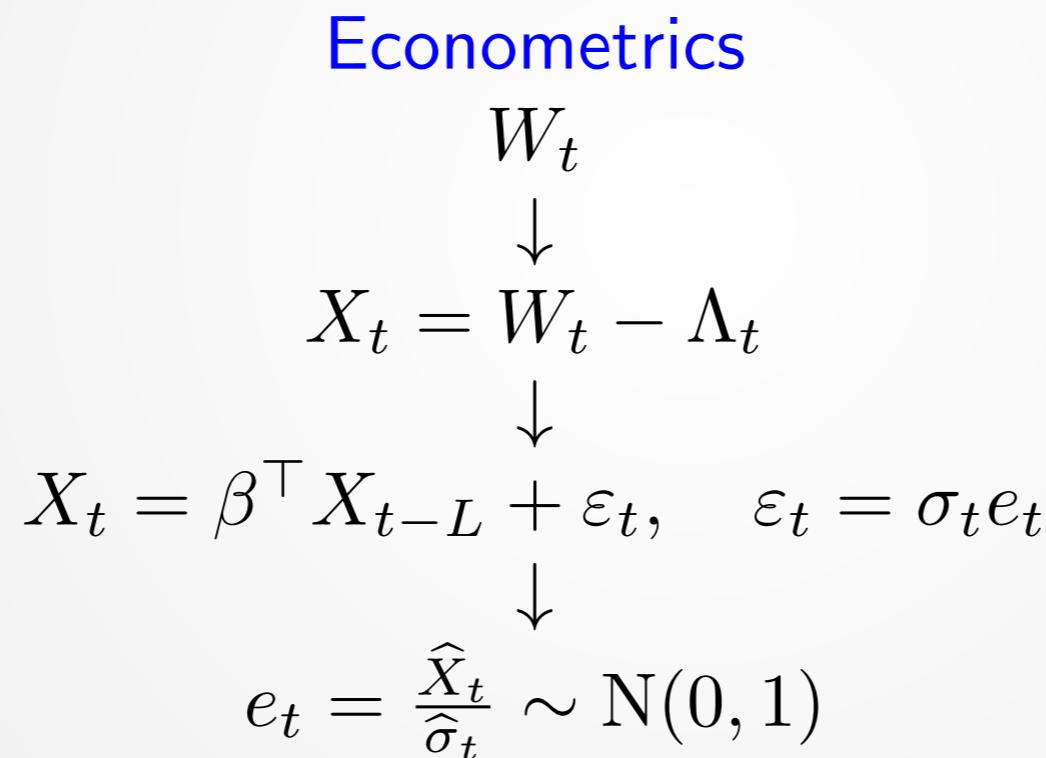
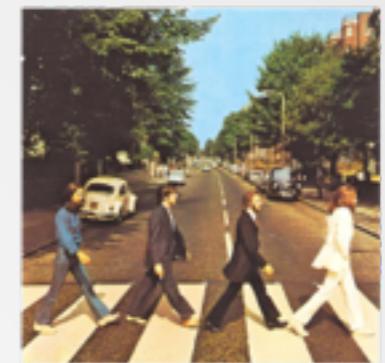
Research questions

- How to model weather dynamics?
- How to estimate market price of risk?
- How to create Gaussian stochastic drivers?





The FEB Four Algorithm



Benth et al. (2007), Härdle et al. (2011)

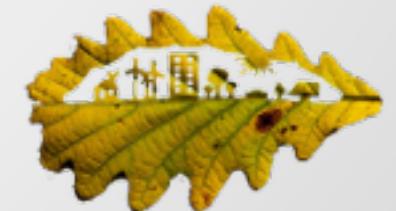
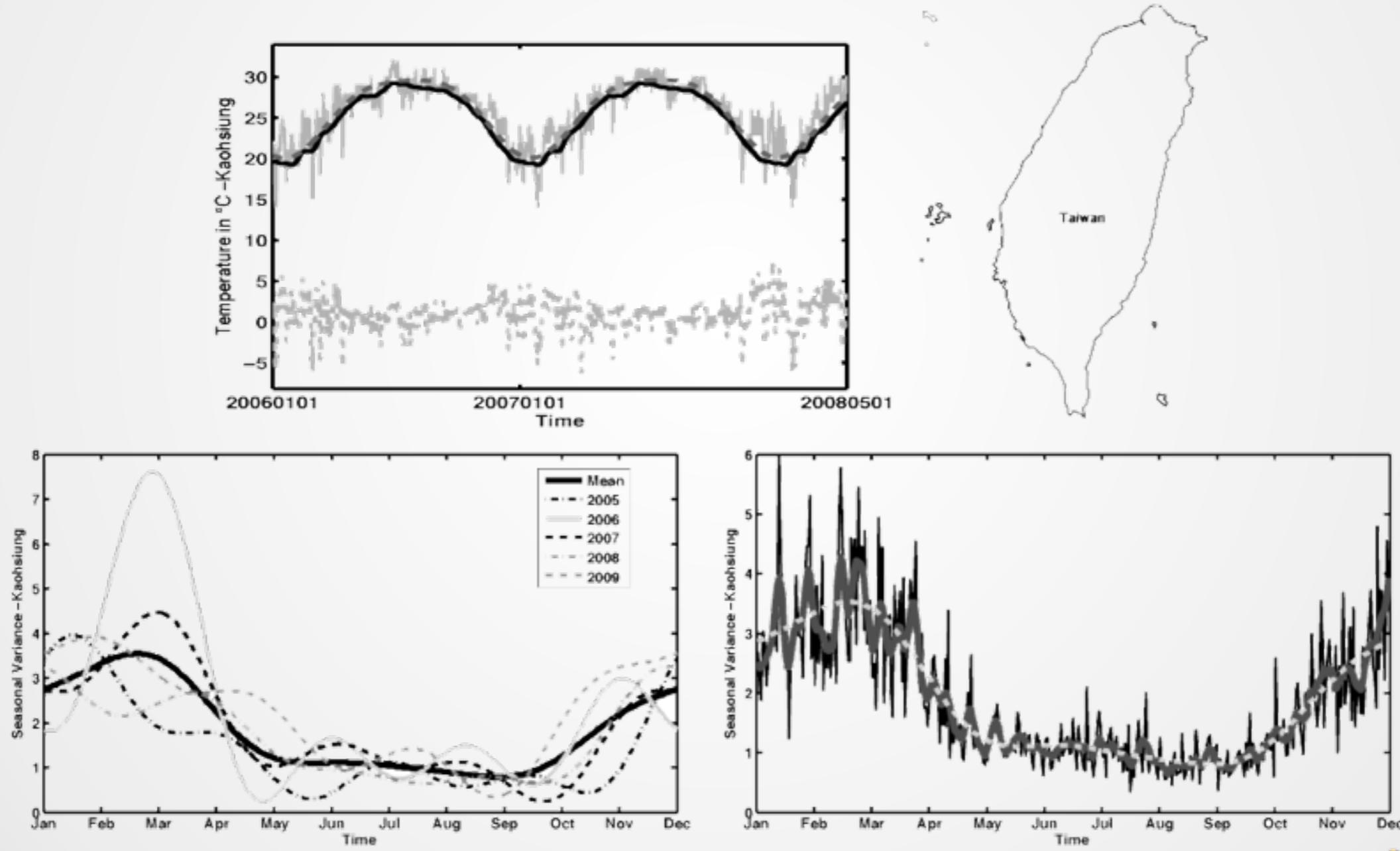


Outline

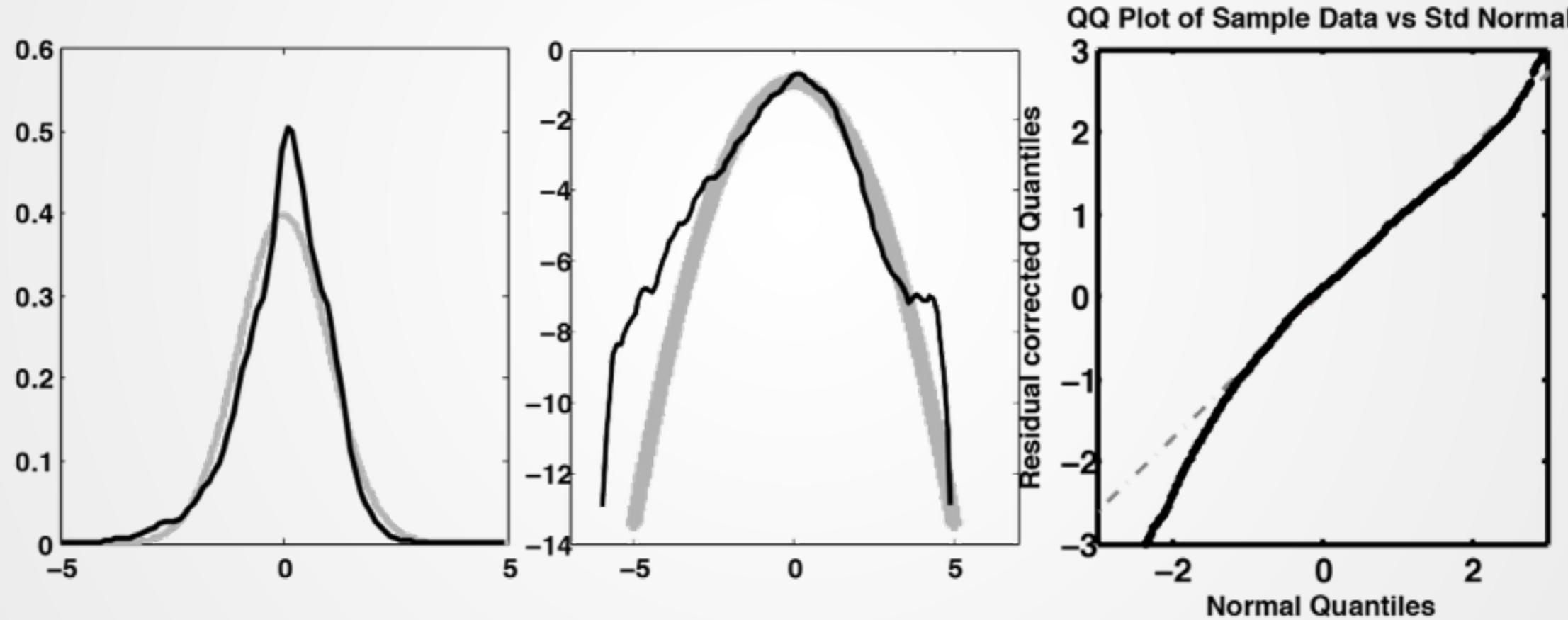
1. Motivation
2. Econometric methods and normality
3. Stochastic pricing of WD
4. Summary



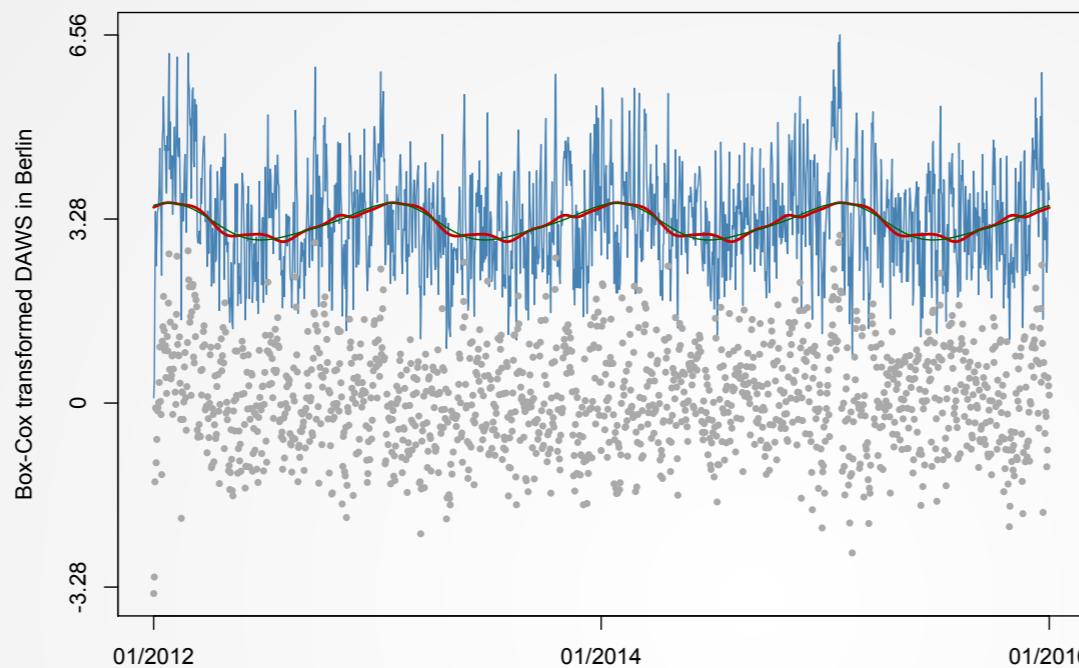
Stylised facts in temperature



Stylised facts in temperature



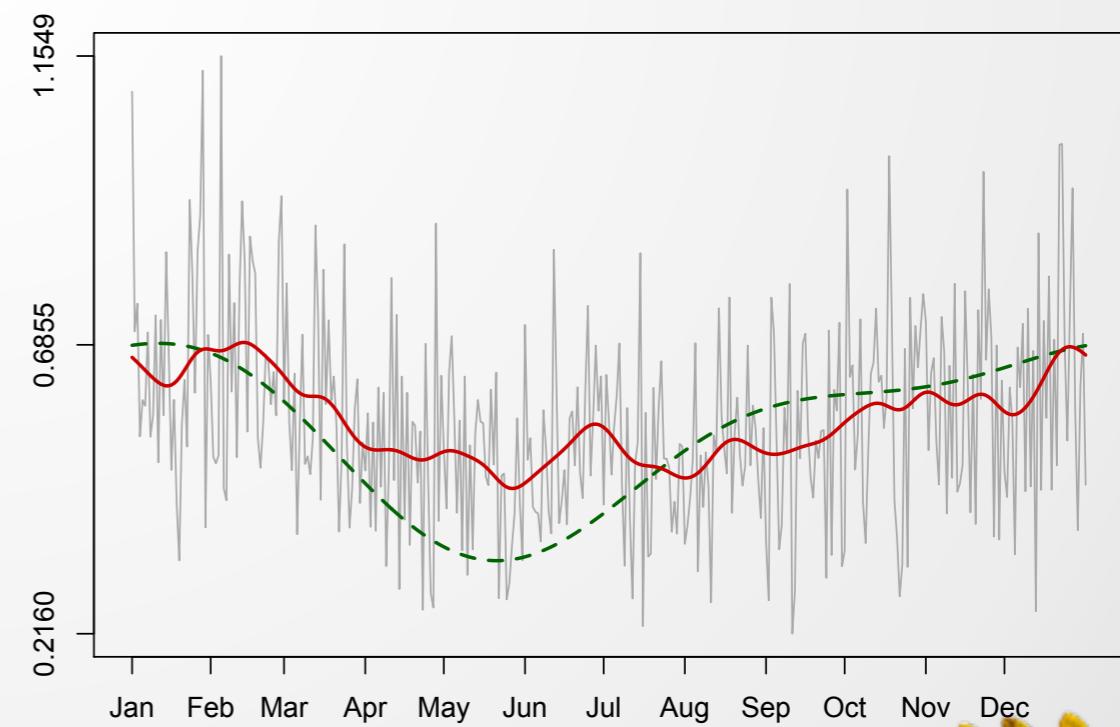
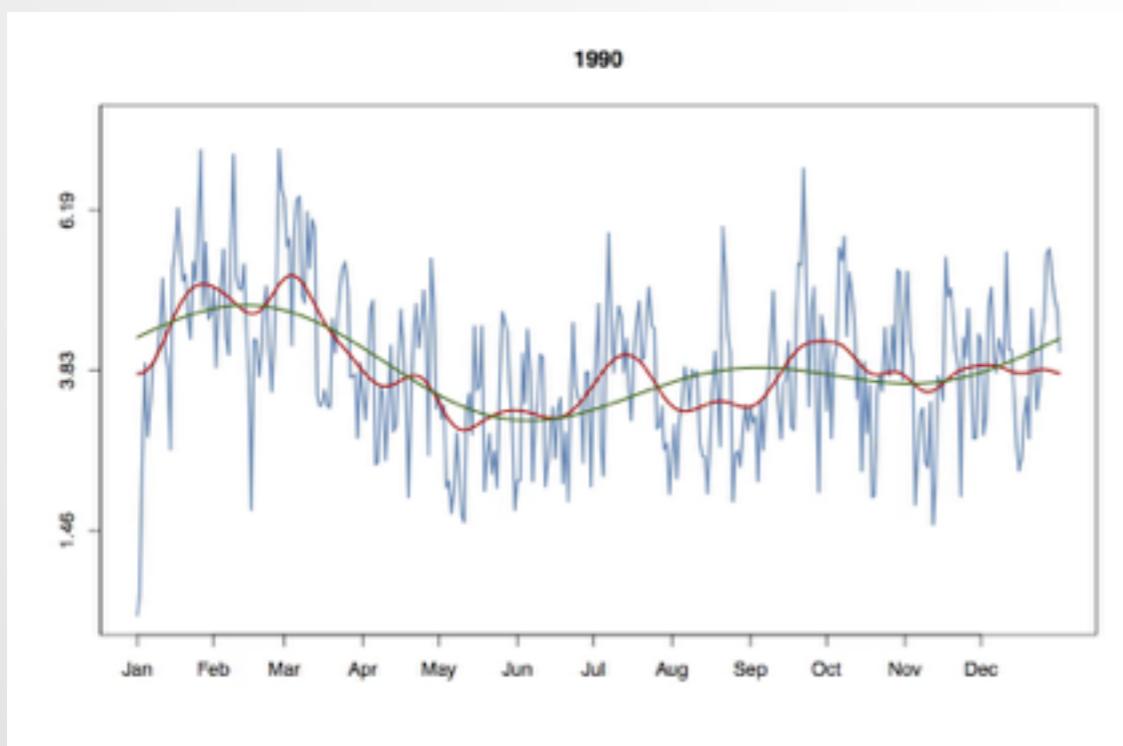
Stylised facts in wind



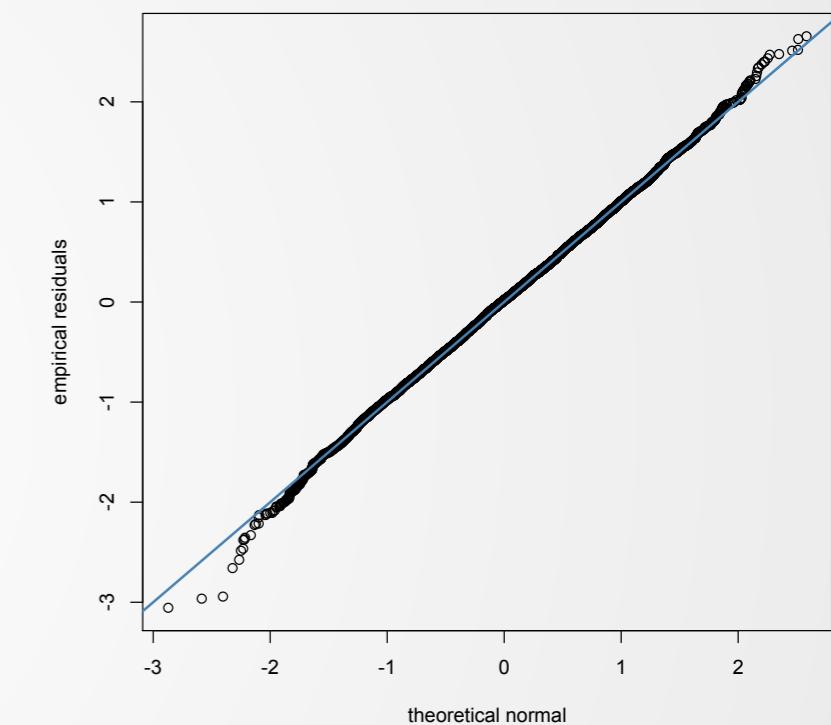
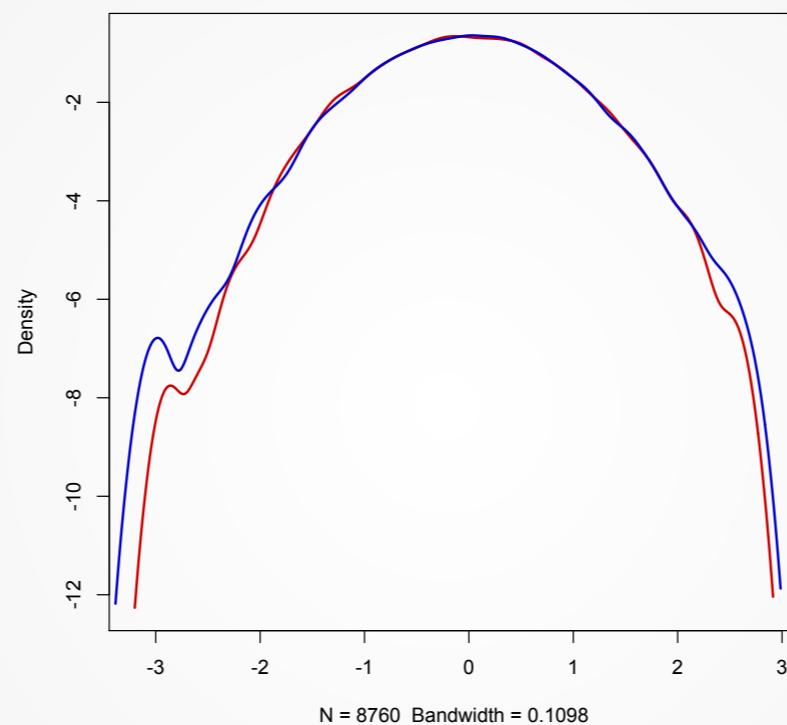
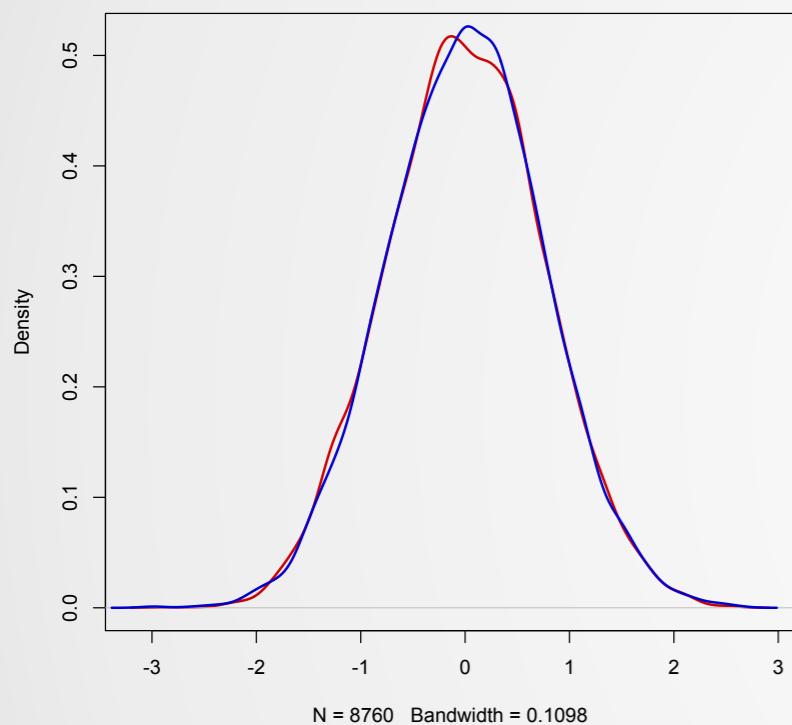
Box-Cox transformation

$$\tilde{W}_t = \frac{W_t^\lambda - 1}{\lambda}$$

$$\hat{\lambda}_{norm} = 0.375$$



Stylised facts in wind



 PGFPwiforpricing



Gaussian stochastic drivers?

Common methods for estimation of $\hat{\sigma}_t$

- Truncated Fourier Series
 - Local Linear Smoothing
- do not reach normality

Example: Berlin

	TFS	LL
Jarque-Bera-test	10.591	10.413
p-value	0.005	0.005
AD-test	0.761	0.585
p-value	0.048	0.052

Use: Inter Expectile Factor Range

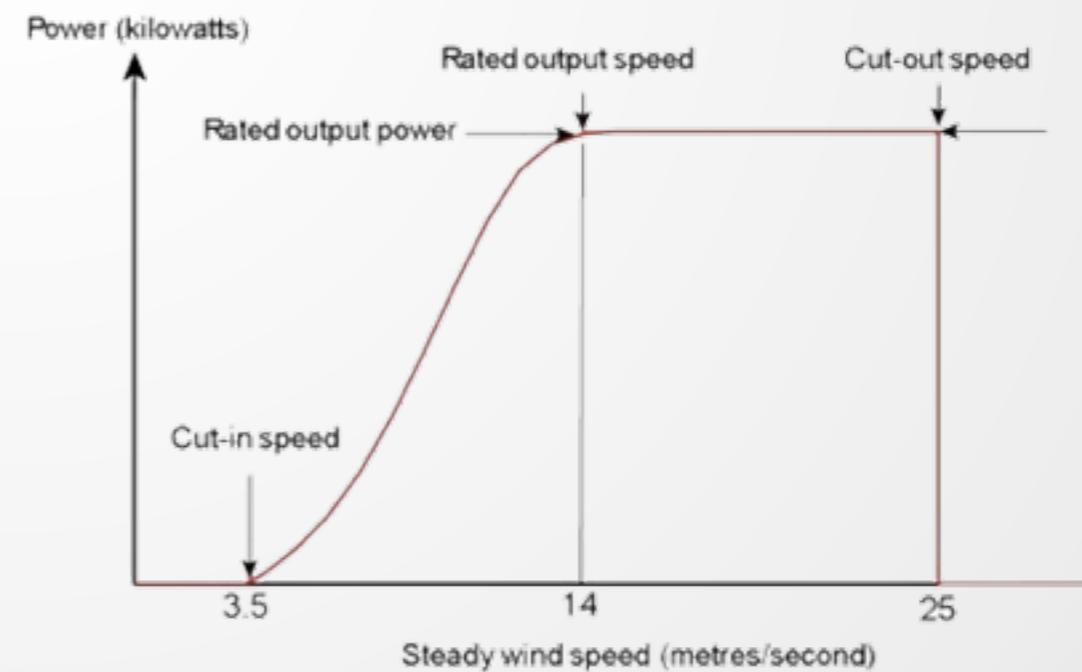


Have a closer look at the tail dynamics

Typically: $\mu_t + \sigma_t$

Non-Gaussian world: tail event variability related to cut-in and cut-out speed

Employ FASTEC methodology
(FActorisable Sparse Tail Event Curves)



Wind index data

- Wind speed data scattered over a region
- Wind speed at turbine height (extrapolation methods)
- MERRA hourly reanalysis data for Germany
 - ▶ Latitude: ϕ : 61 levels at $0.125^\circ \approx 10\text{km}$ resolution
 - ▶ Longitude: λ : 75 levels at $0.125^\circ \approx 10\text{km}$ resolution
 - ▶ Time: 8760 hours p.a.
 - ▶ Period: 1990-2014: 25 years



Measures of tail events

- Loss function, Breckling and Chambers (1988)

$$\rho_{\tau,\gamma}(u) = |\tau - \mathbf{I}\{u < 0\}| |u|^\gamma, \quad \gamma \geq 1$$

$$z_\tau = \arg \min_{\theta} \mathbb{E} \rho_{\tau,\gamma}(Y - \theta)$$

- ▶ Quantile - ALD location estimate

$$q_\tau = \arg \min_{\theta} \mathbb{E} \rho_{\tau,1}(Y - \theta)$$

- ▶ Expectile - AND location estimate

$$e_\tau = \arg \min_{\theta} \mathbb{E} \rho_{\tau,2}(Y - \theta)$$



FASTECE construction

- Data: $\{(\mathbf{X}_i, \mathbf{Y}_i)\}_{i=1}^n$ in \mathbb{R}^{p+m} i.i.d.
- Linear model for τ -expectile curve of Y_j ,
 $j = 1, \dots, m, 0 < \tau < 1$:

$$e_j(\tau | \mathbf{X}_i) = \mathbf{X}_i^\top \Gamma_{*j}(\tau),$$

where coefficients for j response: $\Gamma_{*j}(\tau) \in \mathbb{R}^p$

- Sparse factorisation: $f_k^\tau(\mathbf{X}_i) = \varphi_k^\top(\tau) \mathbf{X}_i$ factors

$$e_j(\tau | \mathbf{X}_i) = \sum_{k=1}^r \psi_{j,k}(\tau) f_k^\tau(\mathbf{X}_i),$$

where r : number of factors;

$$\Gamma_{*j}(\tau) = (\sum_{k=1}^r \psi_{j,k}(\tau) \varphi_{k,1}(\tau), \dots, \sum_{k=1}^r \psi_{j,k}(\tau) \varphi_{k,p}(\tau))$$



FASTECE_with_Expectiles



Multivariate Expectile Regression with penalised loss

$$\hat{\Gamma}_\lambda(\tau) = \arg \min_{\Gamma \in \mathbb{R}^{p \times m}} \left\{ (mn)^{-1} \sum_{i=1}^n \sum_{j=1}^m \rho_\tau(Y_{ij} - \mathbf{X}_i^\top \Gamma_{*j}) + \lambda \|\Gamma\|_* \right\},$$

$\|\Gamma\|_* = \sum_{j=1}^{\min(p,m)} \sigma_j(\Gamma)$ nuclear norm of Γ

\mathbf{X}_i : B-splines

\mathbf{Y}_i : wind power density; $(n \times m)$ -matrix

Γ : factor matrix

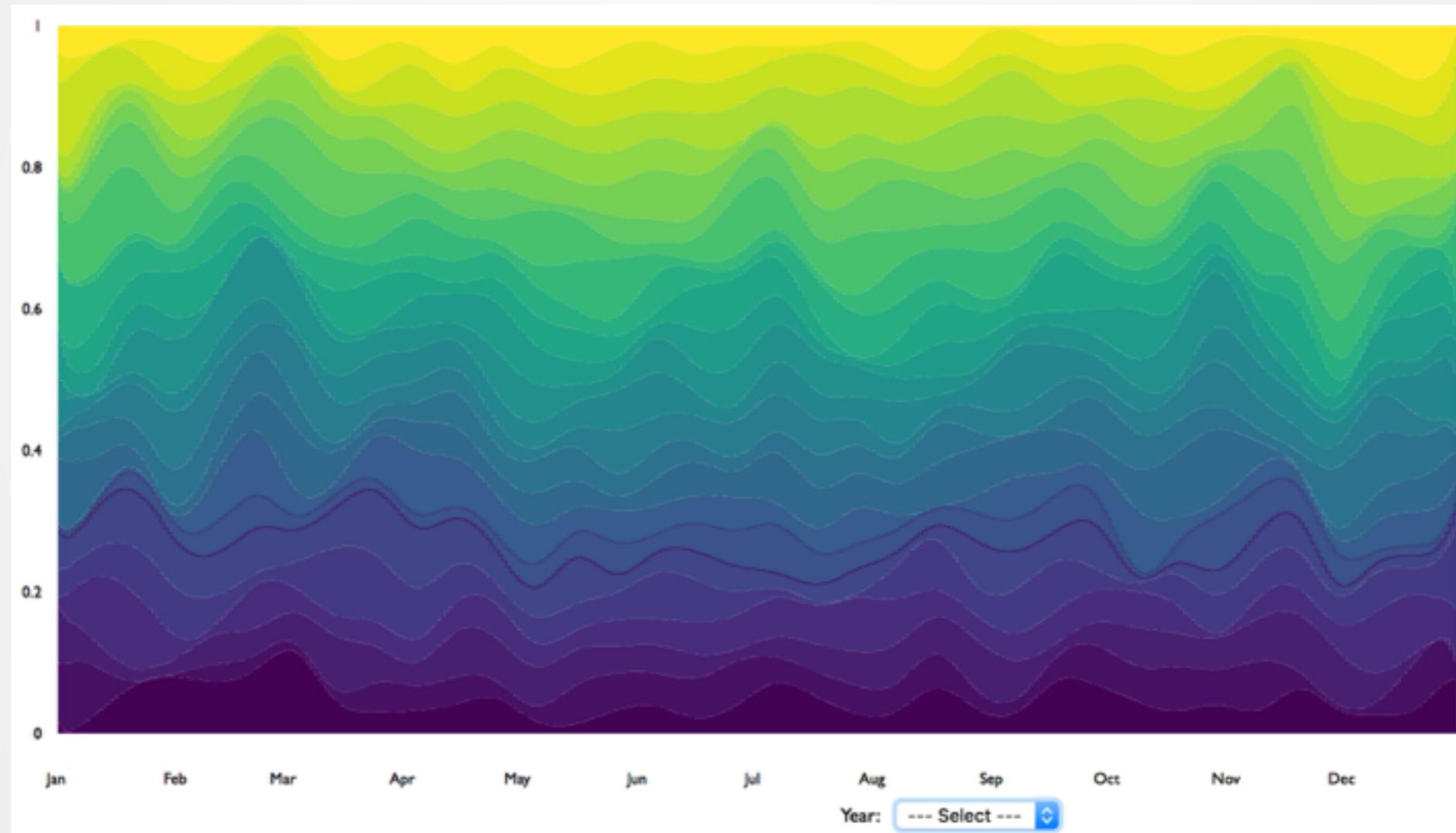
λ : penalisation parameter [► Optimal \$\lambda\$](#)

[► FISTA algorithm](#)



Inter Expectile Factor Range

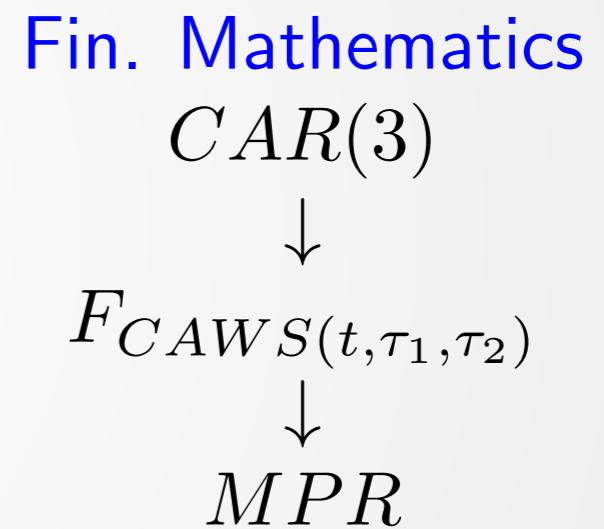
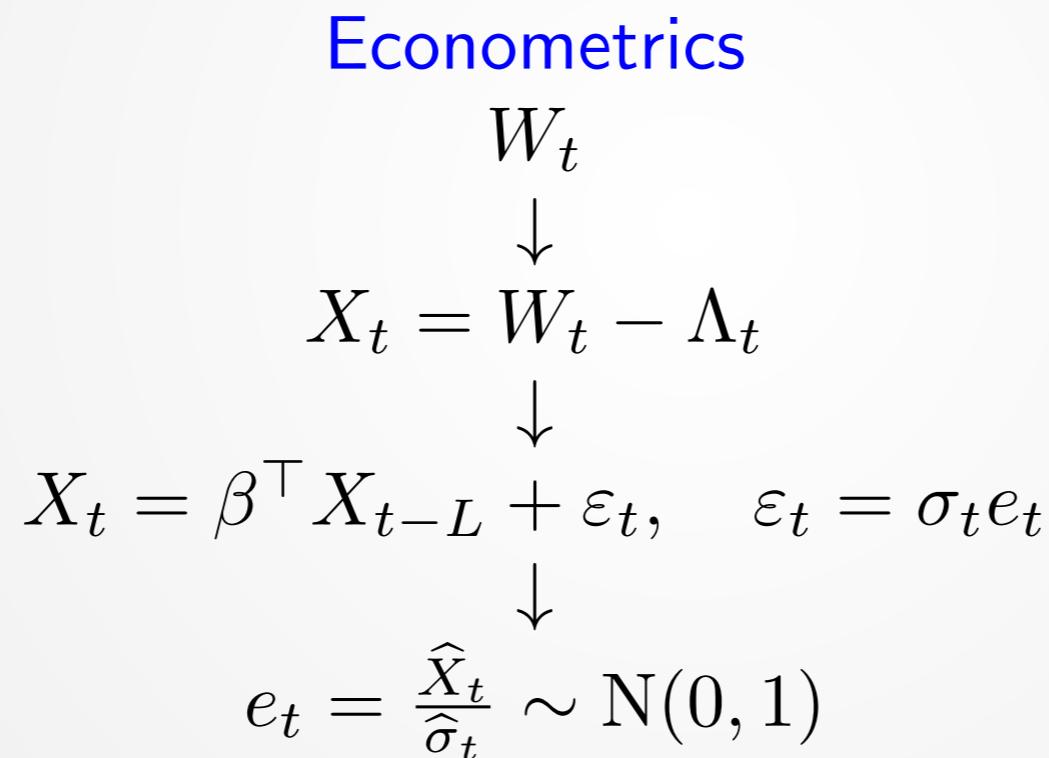
$$\text{IEFR} = |\varphi_1^\top(\tau = 0.99)X_i - \varphi_1^\top(\tau = 0.25)X_i|$$



 PGFPIEFRstreamgraph



The FEB Four Algorithm



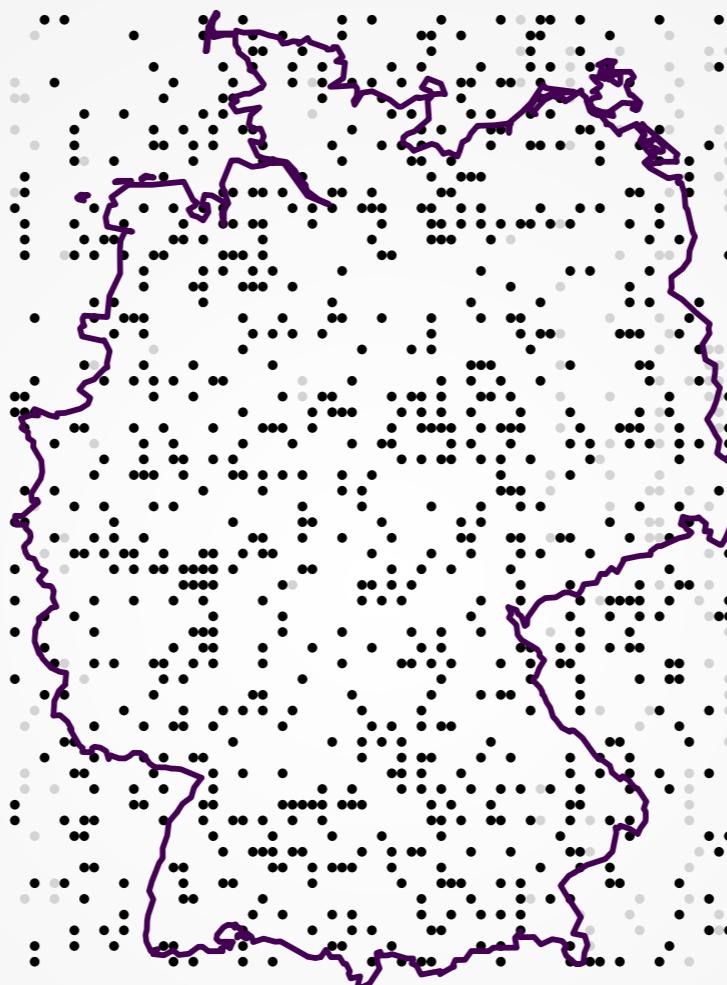
Benth et al. (2007), Härdle et al. (2011)



Normalisation with spread of factors



$\widehat{\sigma}_t$ employing LL



$\widehat{\sigma}_t$ employing IEFR

Normality $\alpha > 0.1$

1000 random sites
in across Germany,
tested with
Anderson Darling,
(Jarque Bera)

LL: 42% (57%) not
rejected

IEFR: 73% (84%)
not rejected



Stochastic pricing

Ornstein-Uhlenbeck process $\mathbf{X}_t \in \mathbb{R}^p$:

$$d\mathbf{X}_t = \mathbf{A}\mathbf{X}_t dt + \mathbf{e}_p \sigma_t dB_t$$

\mathbf{e}_k : k th unit vector in \mathbb{R}^p for $k = 1, \dots, p$, $\sigma_t > 0$, \mathbf{A} : $(p \times p)$ -matrix

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & \dots & \dots & 0 & 1 \\ -\alpha_p & -\alpha_{p-1} & \dots & & -\alpha_1 \end{pmatrix}$$

Proof



CAWS Futures

For $0 \leq t \leq \tau_1 < \tau_2$:

$$\begin{aligned}
 F_{CAWS(t, \tau_1, \tau_2)} &= \mathbb{E}^{Q_\lambda} \left[\int_{\tau_1}^{\tau_2} W_s ds | \mathcal{F}_t \right] \\
 &= \int_{\tau_1}^{\tau_2} \Lambda_u du + \mathbf{a}_{t, \tau_1, \tau_2} \mathbf{X}_t + \int_t^{\tau_1} \lambda_u \sigma_u \mathbf{a}_{t, \tau_1, \tau_2} \mathbf{e}_p du \\
 &\quad + \int_{\tau_1}^{\tau_2} \lambda_u \sigma_u \mathbf{e}_1^\top \mathbf{A}^{-1} [\exp \{\mathbf{A}(\tau_2 - u)\} - I_p] \mathbf{e}_p du \quad (2)
 \end{aligned}$$

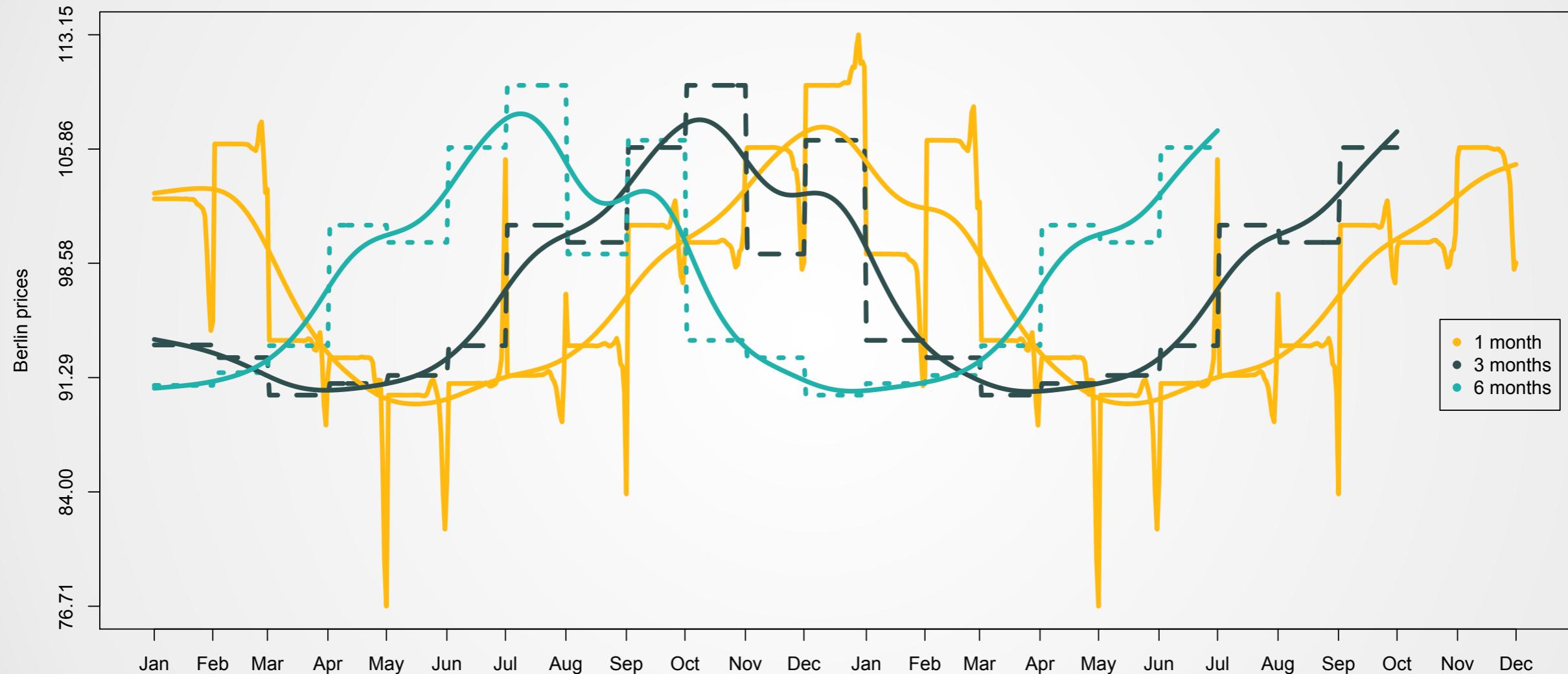
with $\mathbf{a}_{t, \tau_1, \tau_2} = \mathbf{e}_1^\top \mathbf{A}^{-1} [\exp \{\mathbf{A}(\tau_2 - t)\} - \exp \{\mathbf{A}(\tau_1 - t)\}]$, $I_p : p \times p$
identity matrix

Benth et al. (2007)

 fCAWSpricing



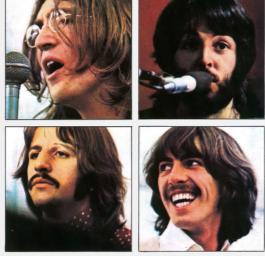
CAWS Futures



Estimated wind future prices for contract length of 1, 3 and 6 months, in Berlin 2012-2013



Conclusion & further research

- IEFR increases percentage of non-rejection in residual normalisation
-  - procedure performs well & applicable to many subjects
- MPR estimation in wind energy: Nasdaq, EEX prices required
- Adaptive optimality for bandwidth selection and transformation
- Extend to spatial wind pricing



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X_t can be written as a Continuous-time AR(p) (CAR(p)):

For $p = 1$,

$$dX_{1t} = -\alpha_1 X_{1t} dt + \sigma_t dB_t$$

For $p = 2$,

$$\begin{aligned} X_{1(t+2)} &\approx (2 - \alpha_1)X_{1(t+1)} \\ &+ (\alpha_1 - \alpha_2 - 1)X_{1t} + \sigma_t(B_{t-1} - B_t) \end{aligned}$$

For $p = 3$,

$$\begin{aligned} X_{1(t+3)} &\approx (3 - \alpha_1)X_{1(t+2)} + (2\alpha_1 - \alpha_2 - 3)X_{1(t+1)} \\ &+ (-\alpha_1 + \alpha_2 - \alpha_3 + 1)X_{1t} + \sigma_t(B_{t-1} - B_t) \end{aligned}$$



Proof $CAR(3) \approx AR(3)$

Let

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\alpha_3 & -\alpha_2 & -\alpha_1 \end{pmatrix}$$

- use $B_{t+1} - B_t = \varepsilon_t$
- assume a time step of length one $dt = 1$
- substitute iteratively into X_1 dynamics



Proof $CAR(3) \approx AR(3)$:

$$\begin{aligned}
 X_{1(t+1)} - X_{1(t)} &= X_{2(t)} dt \\
 X_{2(t+1)} - X_{2(t)} &= X_{3(t)} dt \\
 X_{3(t+1)} - X_{3(t)} &= -\alpha_1 X_{1(t)} dt - \alpha_2 X_{2(t)} dt - \alpha_3 X_{3(t)} dt + \sigma_t \varepsilon_t \\
 X_{1(t+2)} - X_{1(t+1)} &= X_{2(t+1)} dt \\
 X_{2(t+2)} - X_{2(t+1)} &= X_{3(t+1)} dt \\
 X_{3(t+2)} - X_{3(t+1)} &= -\alpha_1 X_{1(t+1)} dt - \alpha_2 X_{2(t+1)} dt \\
 &\quad - \alpha_3 X_{3(t+1)} dt + \sigma_{t+1} \varepsilon_{t+1} \\
 X_{1(t+3)} - X_{1(t+2)} &= X_{2(t+2)} dt \\
 X_{2(t+3)} - X_{2(t+2)} &= X_{3(t+2)} dt \\
 X_{3(t+3)} - X_{3(t+2)} &= -\alpha_1 X_{1(t+2)} dt - \alpha_2 X_{2(t+2)} dt \\
 &\quad - \alpha_3 X_{3(t+2)} dt + \sigma_{t+2} \varepsilon_{t+2}
 \end{aligned}$$

[Return](#)



Seasonal variance: LLE - mirroring observations

To avoid the boundary problem, use mirrored observations:

Assume $h_K < 365/2$, then the observations look like

$\hat{\varepsilon}_{-364}^2, \hat{\varepsilon}_{-363}^2, \dots, \hat{\varepsilon}_0^2, \hat{\varepsilon}_1^2, \dots, \hat{\varepsilon}_{730}^2$, where

$$\hat{\varepsilon}_t^2 \stackrel{\text{def}}{=} \hat{\varepsilon}_{365+t}^2, -364 \leq t \leq 0$$

$$\hat{\varepsilon}_t^2 \stackrel{\text{def}}{=} \hat{\varepsilon}_{t-365}^2, 366 \leq t \leq 730$$



Fast Iterative Shrinkage-Thresholding Algorithm

- Objective: $\min_{\Gamma} \left\{ F(\Gamma) \stackrel{\text{def}}{=} g(\Gamma) + h(\Gamma) \right\}$
- g : smooth convex function with Lipschitz continuous gradient

$$\|\nabla g(\Gamma_1) - \nabla g(\Gamma_2)\|_F \leq L_{\nabla g} \|\Gamma_1 - \Gamma_2\|_F, \quad \forall \Gamma_1, \Gamma_2$$

where $L_{\nabla g} = 2(mn)^{-1} \max(\tau, 1 - \tau) \|X\|_F^2$ is the Lipschitz constant of ∇g

- h : continuous convex function, possibly nonsmooth
- $|F(\Gamma_t) - F(\Gamma^*)| \leq \frac{2L_{\nabla g} \|\Gamma_0 - \Gamma^*\|_F^2}{(t+1)^2}$



FISTA Algorithm

- 1 Initialise: $\Gamma_0 = 0, \Omega_1 = 0$, step size $\delta_1 = 1$
- 2 For $t = 1, 2, \dots, T$
 - ▶ $\Gamma_t = \arg \min_{\Gamma} \left[\frac{g(\Gamma)}{L_{\nabla g}} + \frac{1}{2} \left\| \Gamma - \left\{ \Omega_t - \frac{1}{L_{\nabla g}} \nabla g(\Omega_t) \right\} \right\|^2 \right]$
 - ▶ when penalising nuclear norm $\Gamma_t = \mathbf{P} \left(\mathbf{R} - \frac{\lambda}{L_{\nabla g}} \mathbf{I}_{p \times m} \right) \mathbf{Q}^\top$, and $\Omega_t - \frac{1}{L_{\nabla g}} \nabla g(\Omega_t) = \mathbf{P} \mathbf{R} \mathbf{Q}^\top$ with ALS-SVD (Hastie et al. (2014)) or sparse SVD (Yang et al. (2011))
 - ▶ $\delta_{t+1} = \frac{1 + \sqrt{1 + 4\delta_t^2}}{2}$
 - ▶ $\Omega_{t+1} = \Gamma_t + \frac{\delta_{t+1}}{t+1} (\Gamma_t - \Gamma_{t-1})$
- 3 $\widehat{\Gamma} = \Gamma_T$

► [Return](#)



Assumptions

- $\{(\mathbf{X}_i, \mathbf{Y}_i)\}_{i=1}^n \in \mathbb{R}^{p+m}$ are i.i.d., $\{X_i\}_{i=1}^n \in \mathbb{R}^p \sim \mathcal{N}(0, \Sigma)$
- Conditional on \mathbf{X}_i , $u_{ij} = \{Y_{ij} - \mathbf{X}_i^\top \Gamma_{\cdot j}\}_{j=1}^m$ are cross-sectional independent over j
- u_{i1}, \dots, u_{im} are sub-gaussian: $\exists C > 0$ s.t.
 $P(|u_{ij}| > s) \leq \exp\{1 - (\frac{s}{C})^2\}, j \in \{1, \dots, m\}$
- $K_u \stackrel{\text{def}}{=} \max_{1 \leq j \leq m} \|U_{ij}\|_{\psi_2} = \max_{1 \leq j \leq m} \sup_{p \leq 1} p^{-\frac{1}{2}} (\mathbb{E} |u_{ij}|^p)^{\frac{1}{p}}$



Optimal choice of tuning parameter λ

- Under the assumptions of sample setting, selecting

$$\lambda = \frac{2}{m} S \max(\tau, 1 - \tau) \sqrt{K_u^2 \|\Sigma\| \frac{p + m}{n}}, \quad \text{for } n \leq 2 \min(m, p)$$

any optimal solution $\hat{\Gamma}_\lambda$ satisfies error bound conditions
(Huang et al. (2016))

- Then

$$\begin{aligned} P \left\{ \|\nabla g(\Gamma)\| \leq m^{-1} S \max(\tau, 1 - \tau) \sqrt{K_u^2 \|\Sigma\| \frac{p + m}{n}} \right\} \\ \leq 1 - 3 \cdot 8^{-(p+m)} - 4 \exp(-n/2), \end{aligned}$$

where S is an absolute constant.

► Return

