Pricing green financial products

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Hedging weather risk

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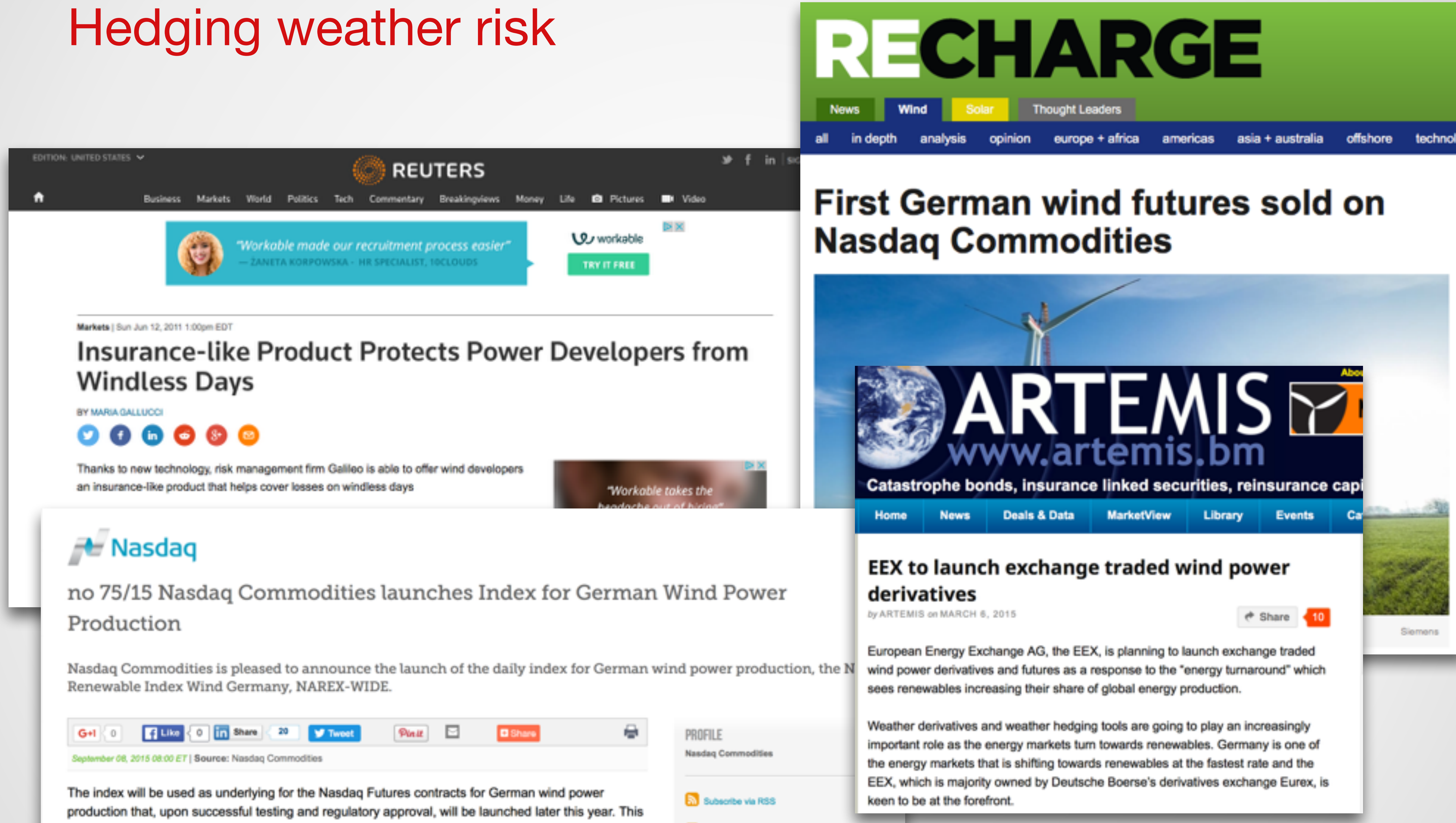

REUTERS

Markets | Sun Jun 12, 2011 1:00pm EDT

Insurance-like Product Protects Power Developers from Windless Days

BY MARIA GALLUCCI

Thanks to new technology, risk management firm Galileo is able to offer wind developers an insurance-like product that helps cover losses on windless days

RECHARGE

News | Wind | Solar | Thought Leaders

First German wind futures sold on Nasdaq Commodities

Nasdaq

no 75/15 Nasdaq Commodities launches Index for German Wind Power Production

Nasdaq Commodities is pleased to announce the launch of the daily index for German wind power production, the N… Renewable Index Wind Germany, NAREX-WIDE.

September 08, 2015 08:00 ET | Source: Nasdaq Commodities

The index will be used as underlying for the Nasdaq Futures contracts for German wind power production that, upon successful testing and regulatory approval, will be launched later this year. This …

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Catastrophe bonds, insurance linked securities, reinsurance capi…

EEX to launch exchange traded wind power derivatives

by ARTEMIS on MARCH 6, 2015

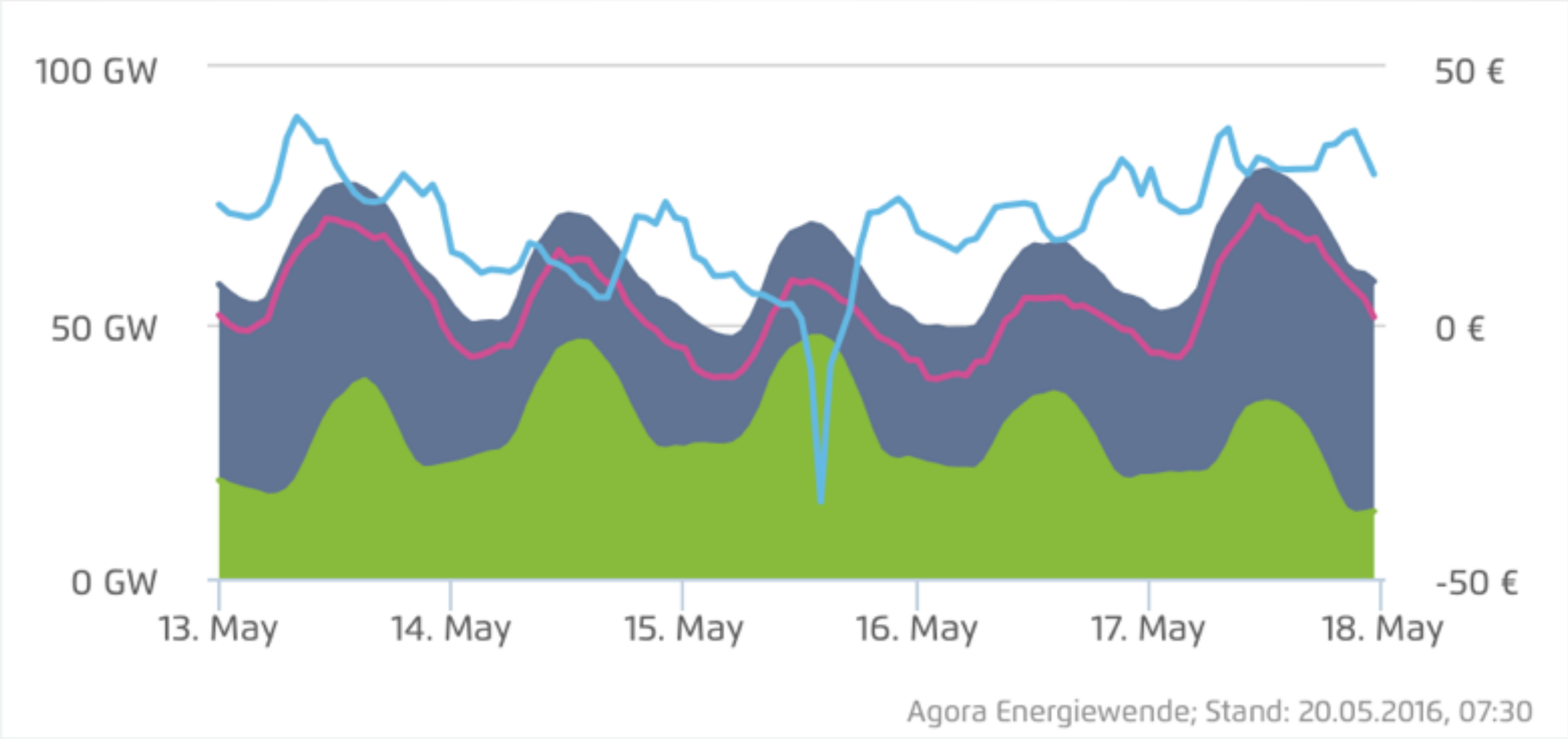
European Energy Exchange AG, the EEX, is planning to launch exchange traded wind power derivatives and futures as a response to the "energy turnaround" which sees renewables increasing their share of global energy production.

Weather derivatives and weather hedging tools are going to play an increasingly important role as the energy markets turn towards renewables. Germany is one of the energy markets that is shifting towards renewables at the fastest rate and the EEX, which is majority owned by Deutsche Boerse's derivatives exchange Eurex, is keen to be at the forefront.

Weather risk in energy production

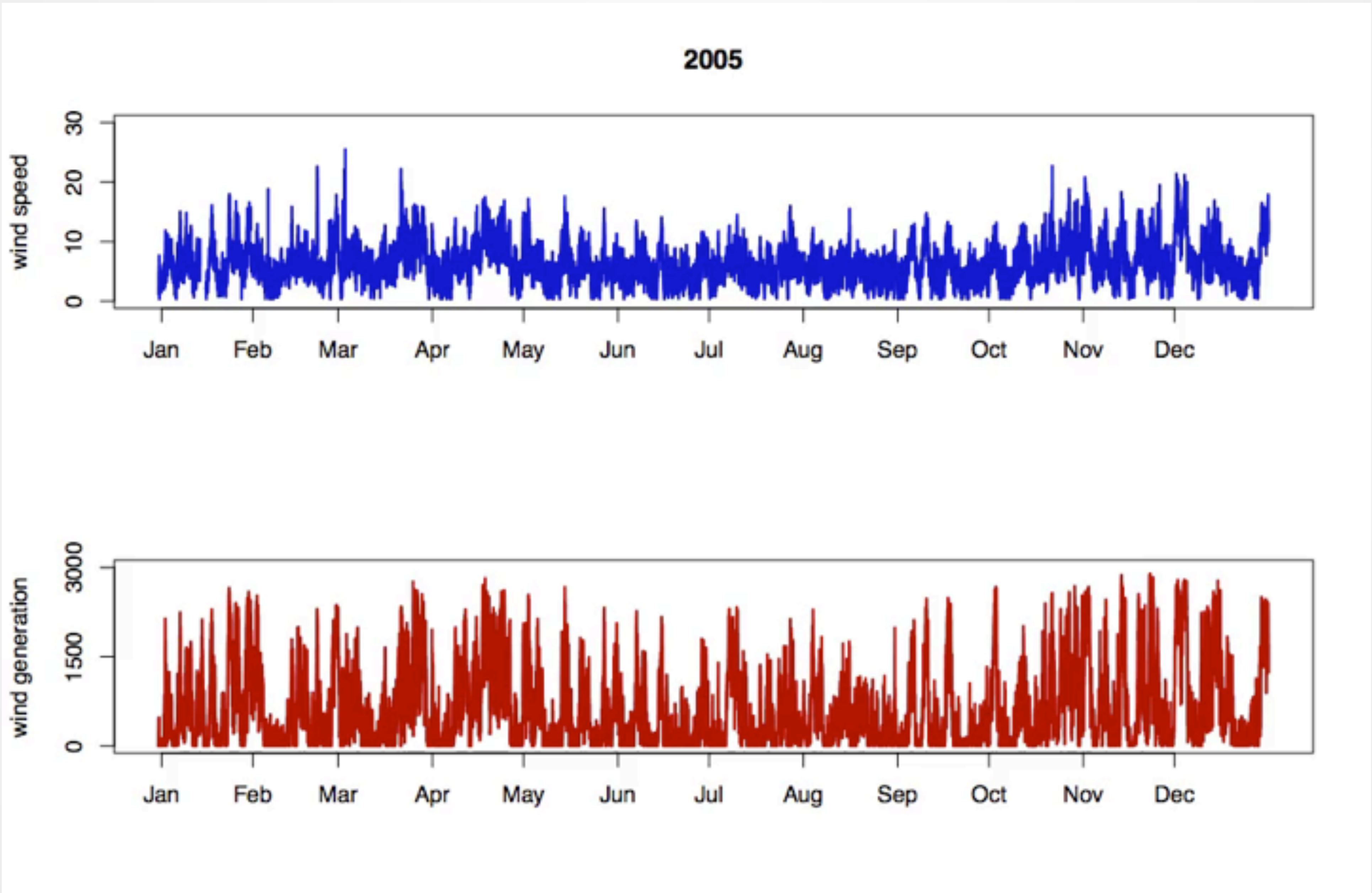
- Renewables become dominating energy source
- Energy output highly dependent on weather conditions
  - “Lack of wind” phenomenon
  - Sudden wind speed changes: strong power drop
  - Unsteady wind direction: zero wind power
  - Cloud coverage over one solar panel affects the panel output of an entire farm excessively

Pentecost 2016 in Germany: renewables cover over 80% of electricity demand - prices drop below zero



Electricity price; electricity demand; conventional power generation; renewable power generation

Having wind ≠ producing energy



Green financial products

Hedge weather related risk exposures on energy production due to climate volatility

- Payments based on weather related measurements
- Applicable to any renewable energy project
  - biomass, wave, tidal, hydropower, solar energy
  - hedging against “lack of wind”
  - prices below zero (post-feed-in)
- Underlying: indices on wind speed, sun duration, cloud coverage, etc.

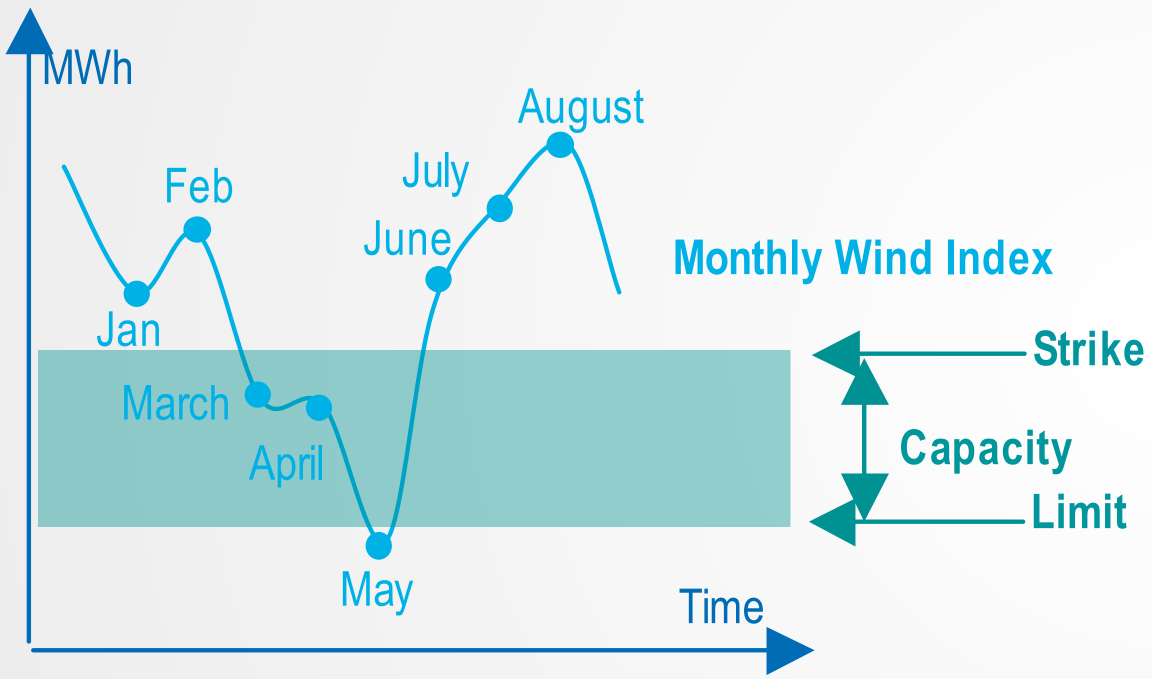
Wind futures at European Energy eXchange (EEX)

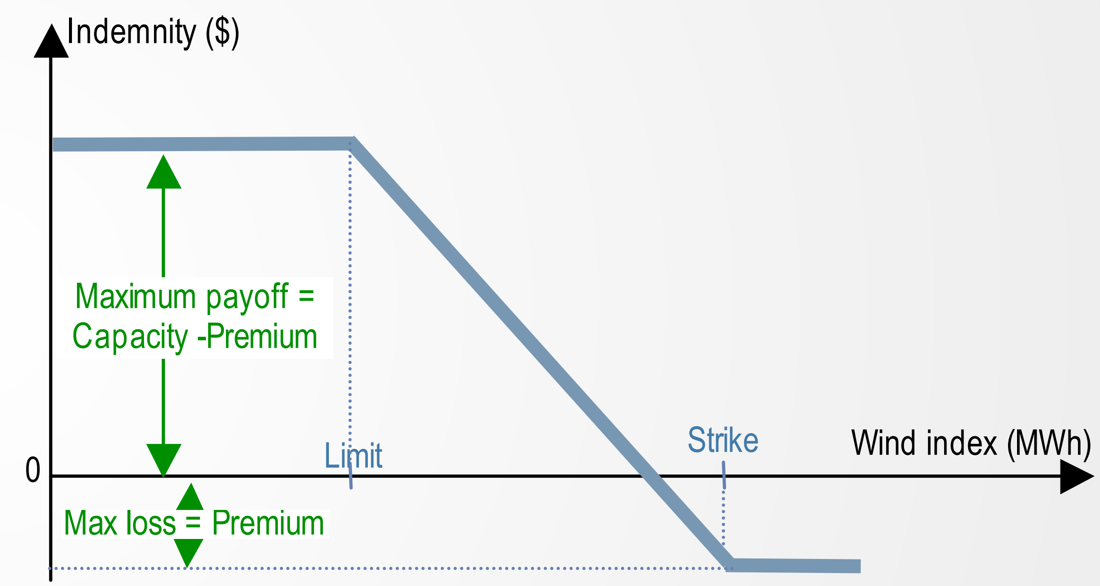
- Contract: A contract settling against the expected on-site power production of future delivery periods
- Underlying: Average wind load factor per contract period; alternatives:
  - wind speed, wind direction, wind duration indices

Example: Cumulative Average Wind Speed index:

$$CAWS(\tau_1;\tau_2) = \int_{\tau_1}^{\tau_2} W(s)ds$$

Wind futures mechanism





Left: Wind futures mechanism; Right: payoff profile

Weather Derivatives at Chicago Mercantile Exchange

CME products

- $\text{HDD}(\tau_1, \tau_2) = \int_{\tau_1}^{\tau_2} \max(18^\circ\text{C} - T_t, 0)dt$
- $\text{CDD}(\tau_1, \tau_2) = \int_{\tau_1}^{\tau_2} \max(T_t - 18^\circ\text{C}, 0)dt$
- $\text{CAT}(\tau_1, \tau_2) = \int_{\tau_1}^{\tau_2} T_t dt$, where $T_t = \frac{T_{t,max} + T_{t,min}}{2}$
- $\text{AAT}(\tau_1, \tau_2) = \int_{\tau_1}^{\tau_2} \widetilde{T}_t dt$, where $\widetilde{T}_t = \frac{1}{24} \int_1^{24} T_{t_i} dt_i$ and T_{t_i} denotes the temperature of hour t_i, (also referred to as C24AT index).

Research questions

- How to model weather dynamics?
- How to estimate market price of risk?
- How to create Gaussian stochastic drivers?

The FEB Four Algorithm

Econometrics

$$T_t$$
$$\downarrow$$
$$X_t = T_t - \Lambda_t$$
$$\downarrow$$
$$X_{t+p} = a^\top X_t + \sigma_t \varepsilon_t$$
$$\downarrow$$
$$\hat{\varepsilon}_t = \frac{\hat{X}_t}{\hat{\sigma}_t} \sim \mathbf{N}(0,1)$$

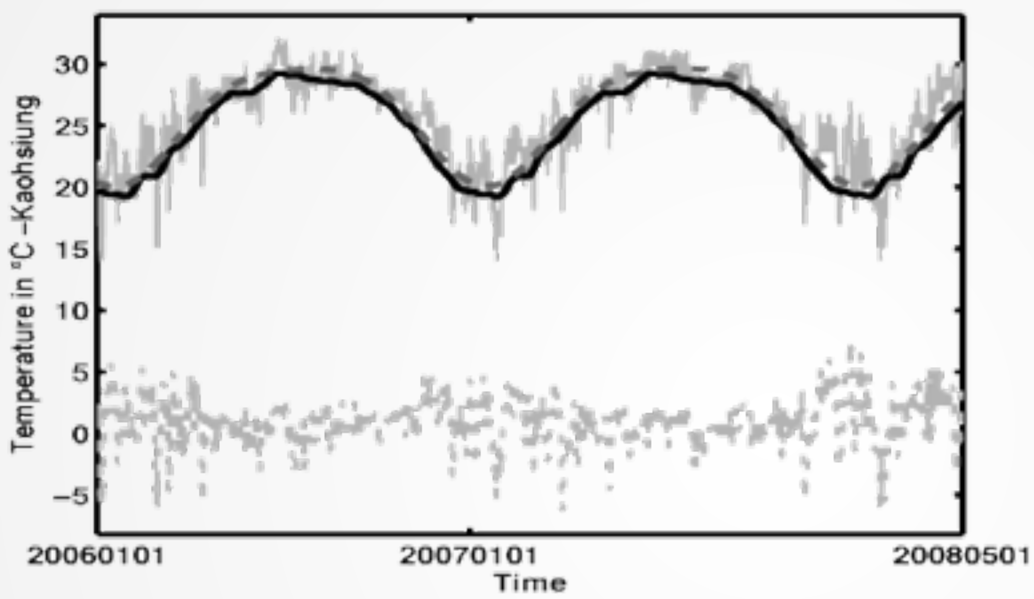
Fin. Mathematics.

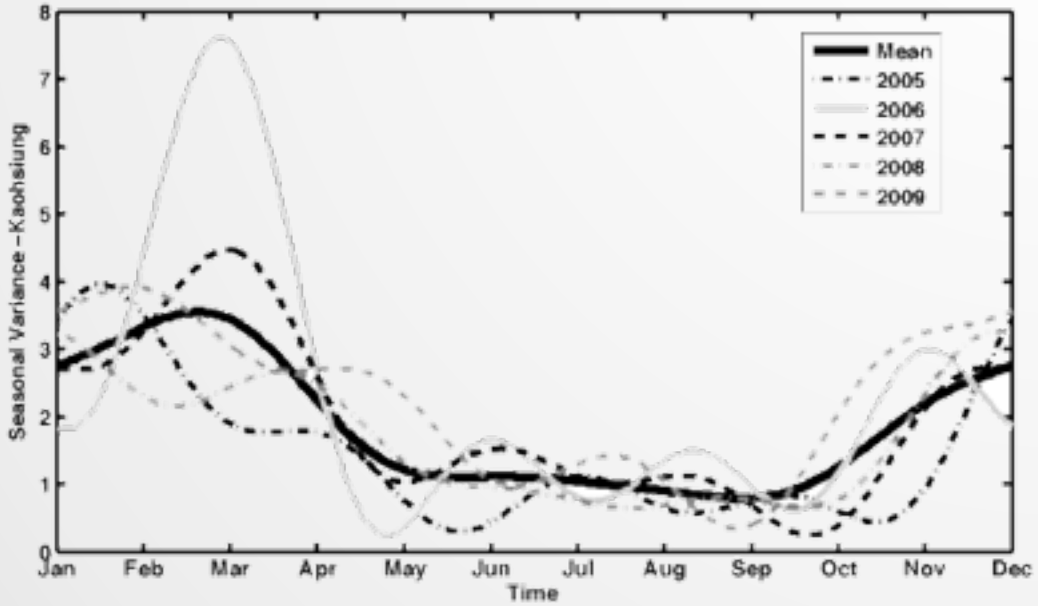
$$CAR(3)$$
$$\downarrow$$
$$F_{CAT(t,\tau_1,\tau_2)}$$
$$\downarrow$$
$$MPR$$

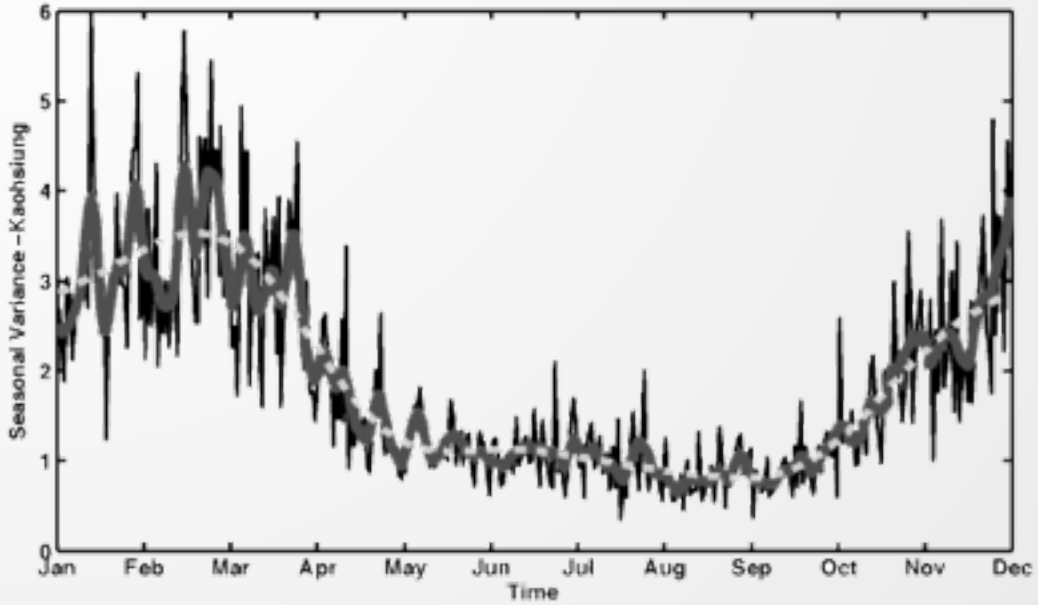
Outline

1. Motivation
2. Econometric methods and normality
3. Stochastic pricing of WD
4. Summary

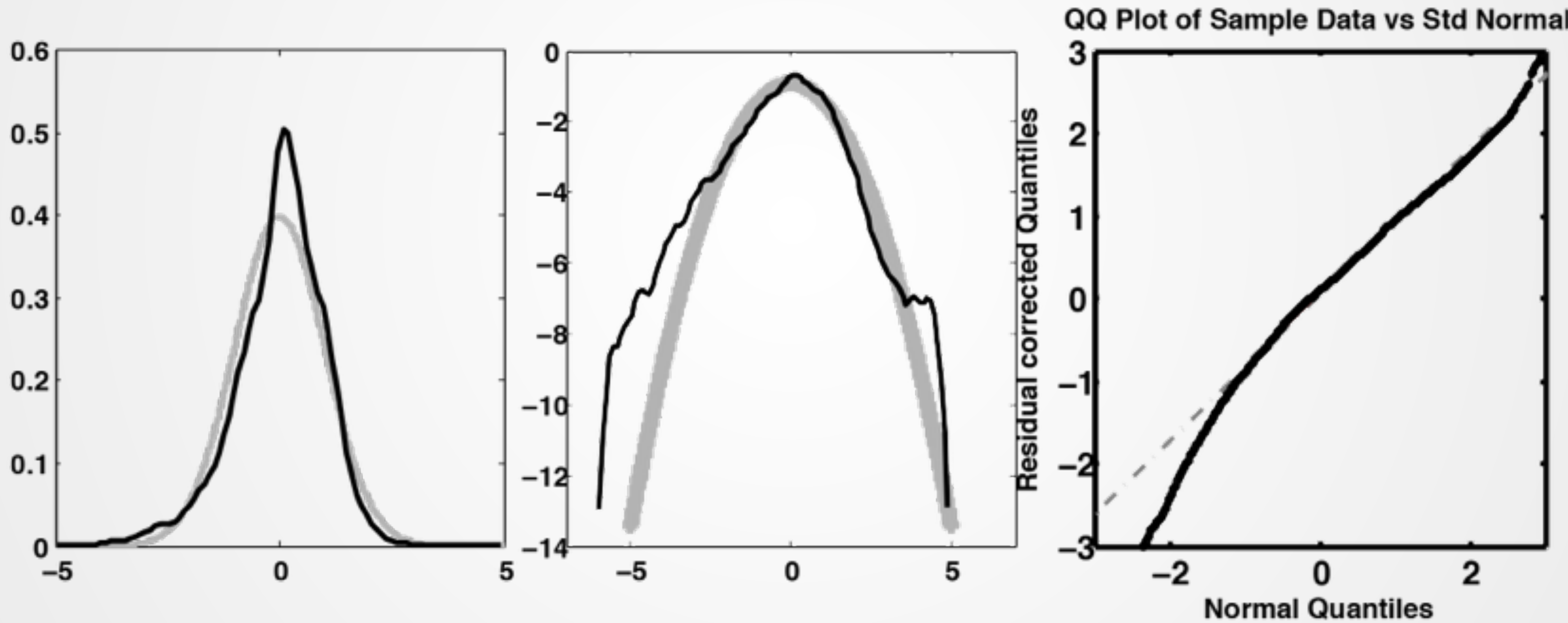
Stylised facts in temperature



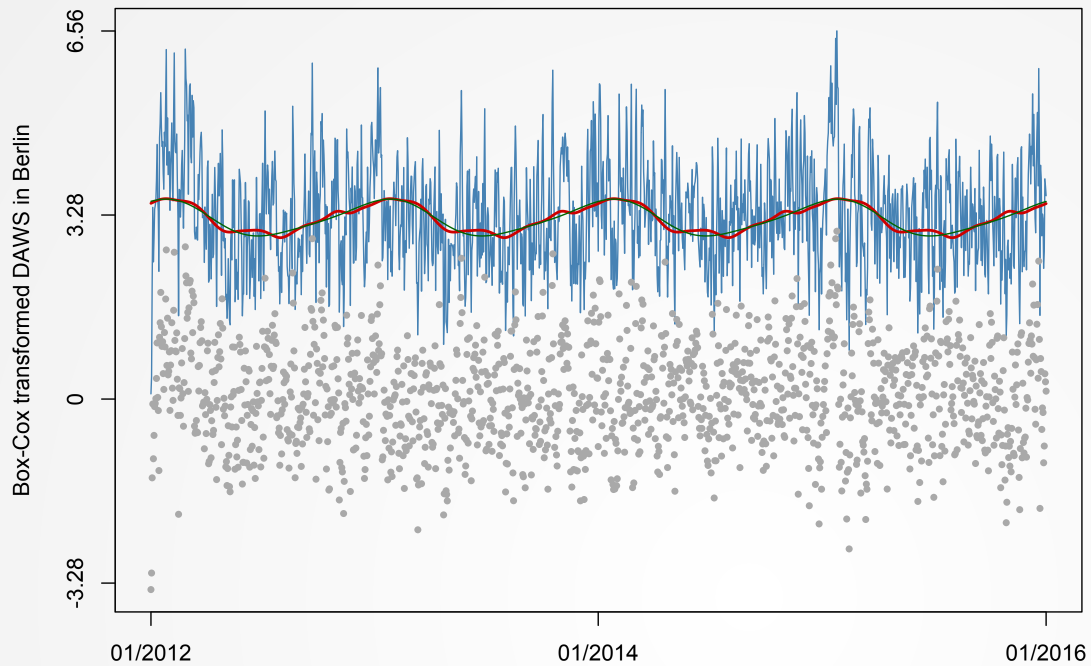




Stylised facts in temperature



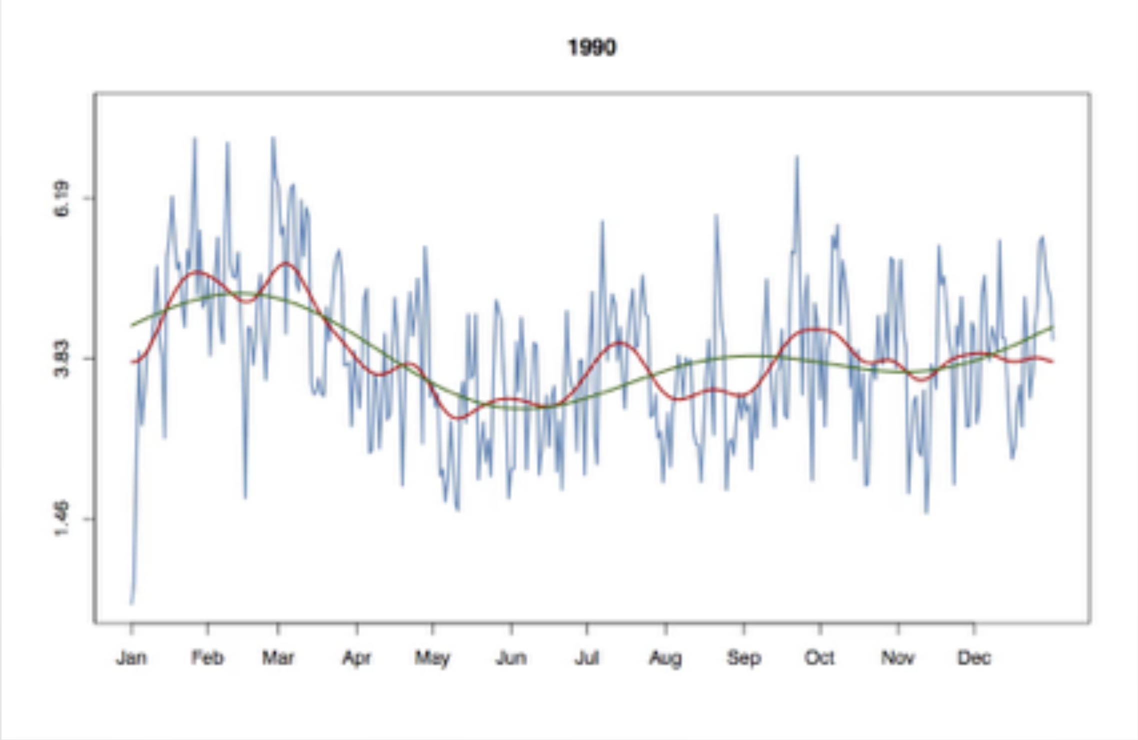
Stylised facts in wind

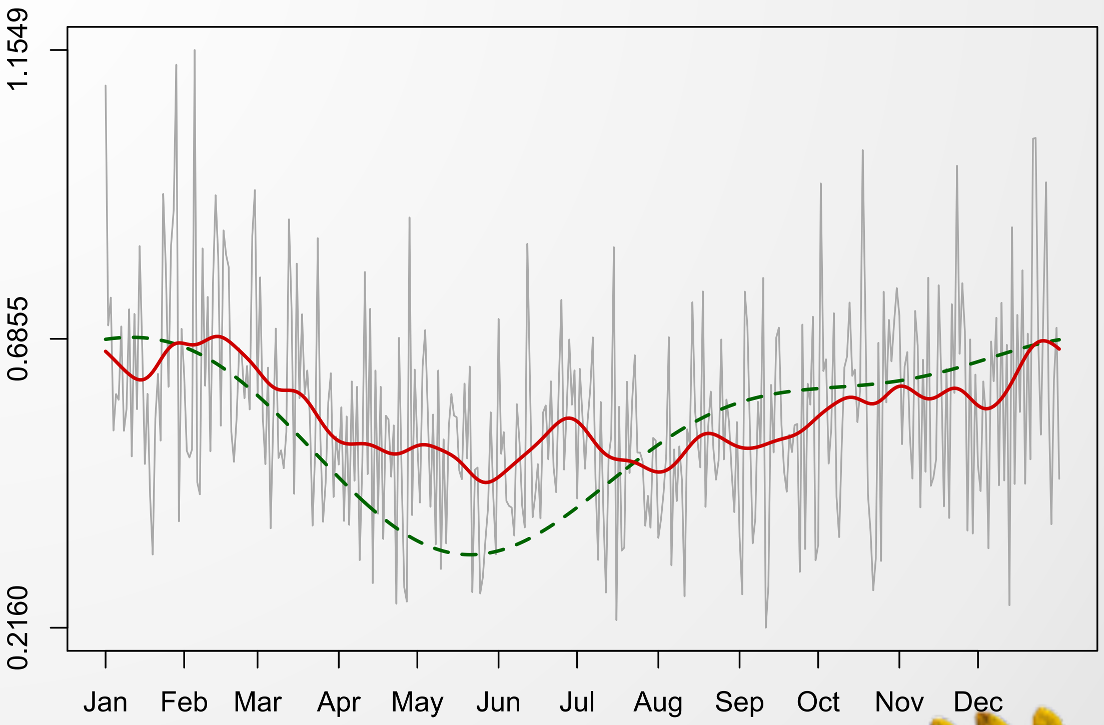


Box-Cox transformation

$$\widetilde{W}_t = \frac{W_t^{\lambda} - 1}{\lambda}$$

$$\widehat{\lambda}_{norm} = 0.375$$





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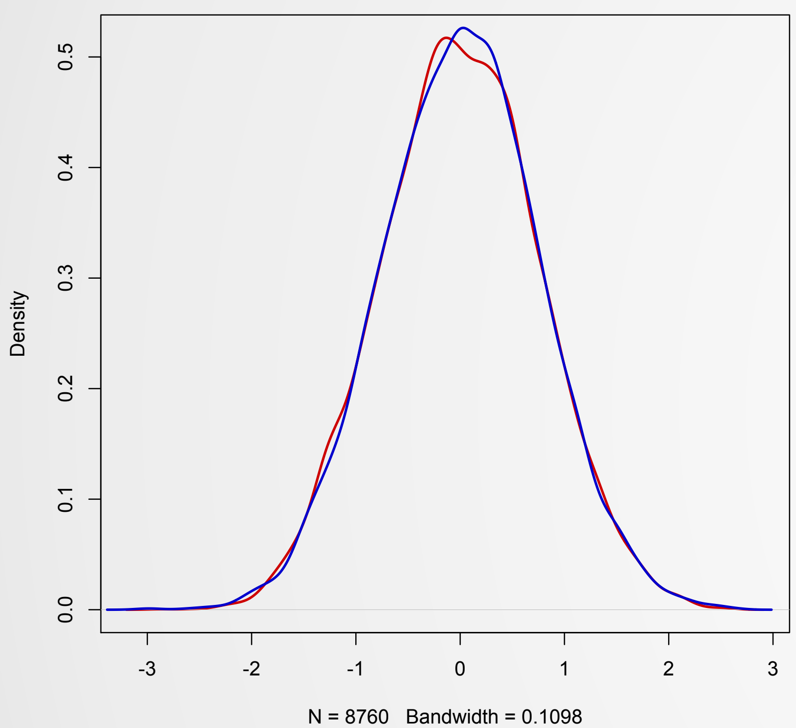
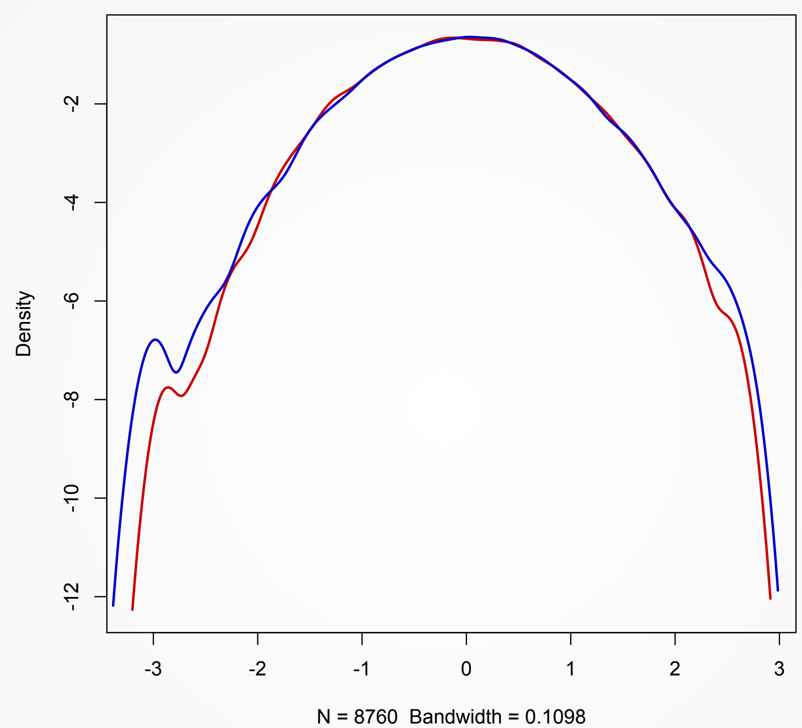
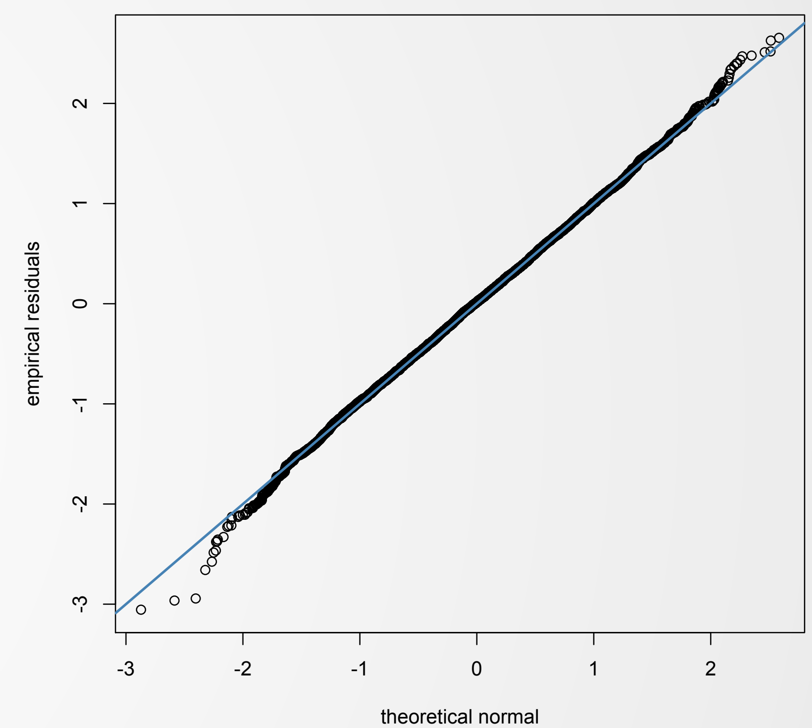
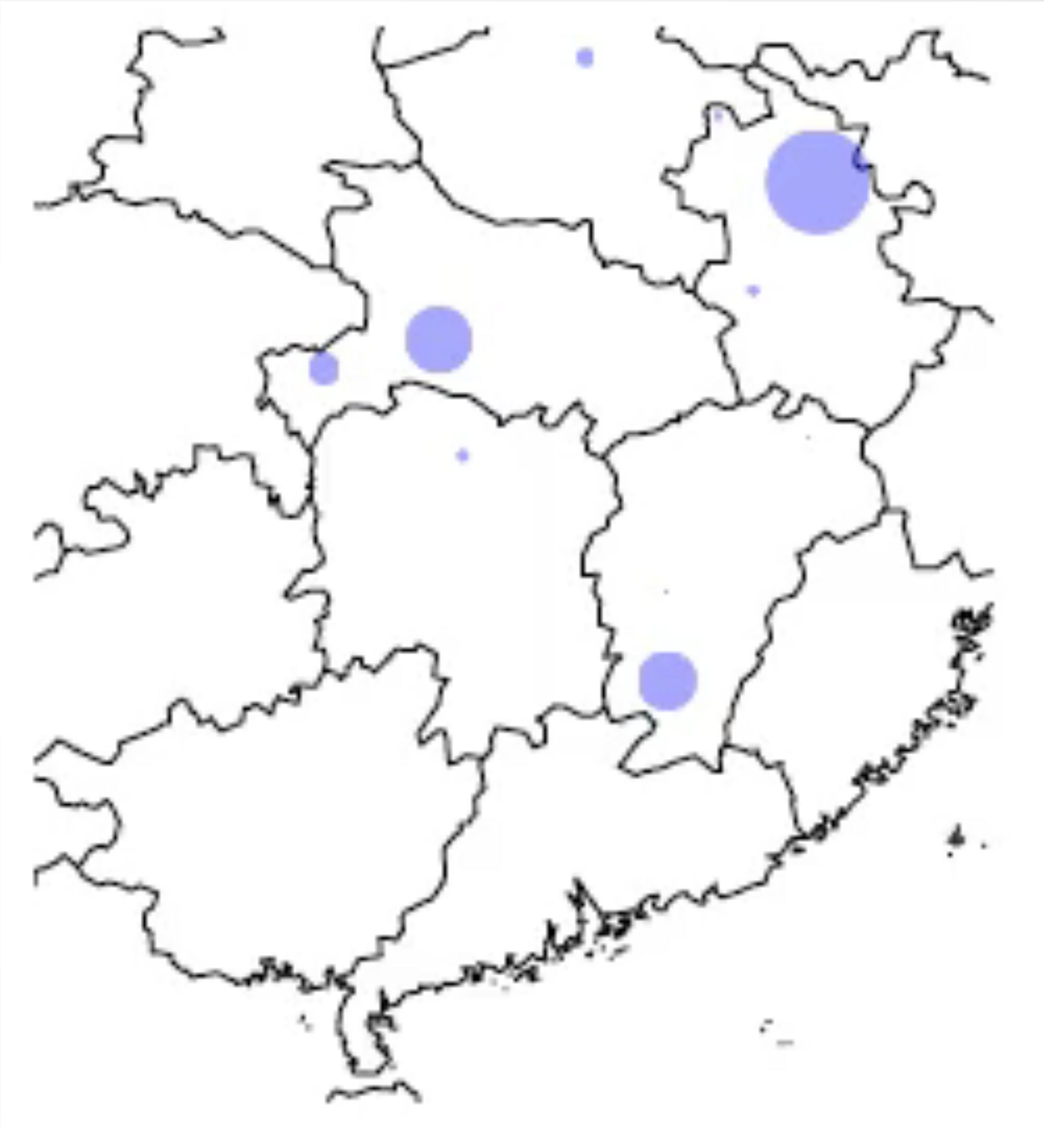






PGFPwindforpricing

Other areas: pricing rain



Härdle and Osipenko (2016)

Pricing Chinese rain

ChinesRain

Weather Derivatives

CME products

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Stochastic pricing

Ornstein-Uhlenbeck process $\mathbf{X}_t \in \mathbb{R}^p$:

$$d\mathbf{X}_t = \mathbf{A}\mathbf{X}_t dt + \mathbf{e}_p \sigma_t dB_t$$

$\mathbf{e}_k$: kth unit vector in $\mathbb{R}^p$ for $k = 1, ...p$, $\sigma_t > 0$, A: $(p \times p)$-matrix

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & \dots & \dots & 0 & 1 \\ -\alpha_p & -\alpha_{p-1} & \dots & & -\alpha_1 \end{pmatrix}$$

Proof

CAT Futures

For $0 \leq t \leq \tau_1 < \tau_2$:

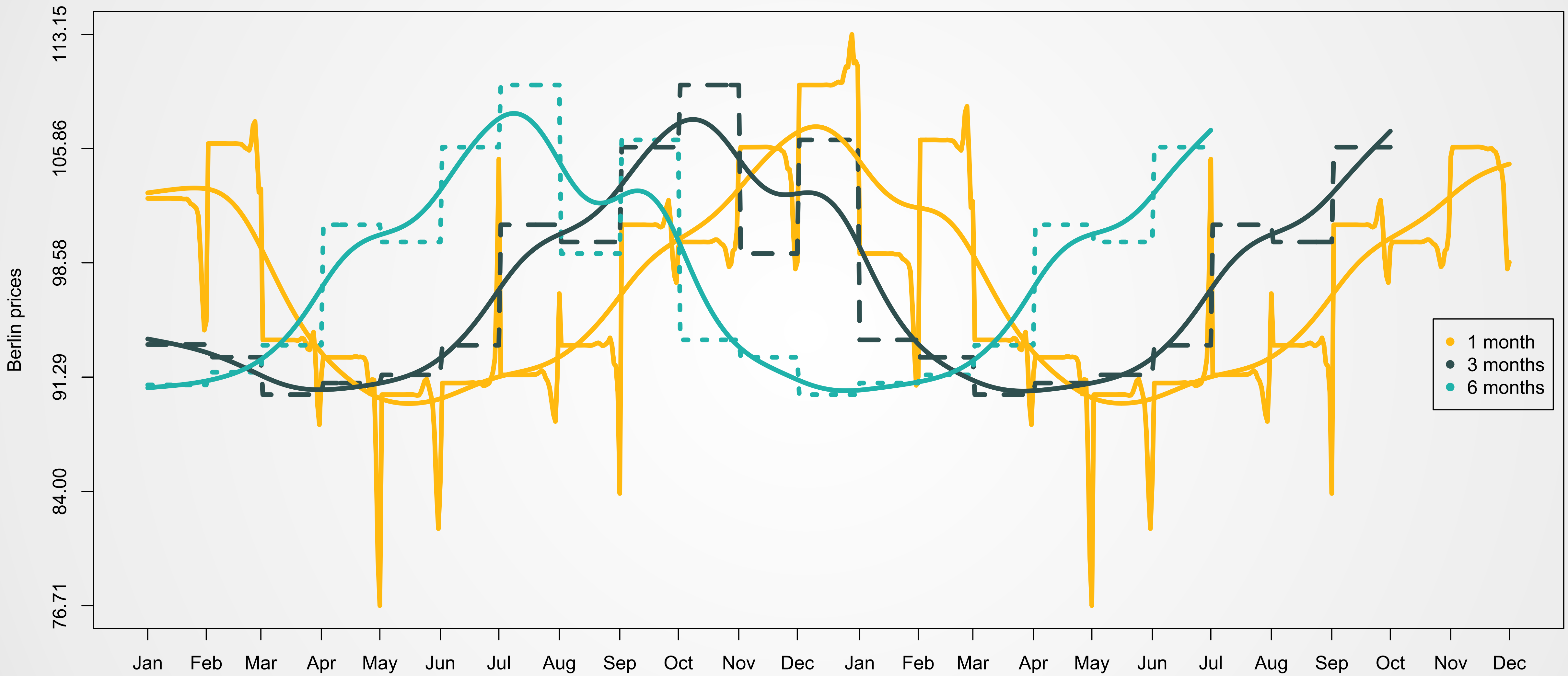
$$\begin{aligned} F_{CAT(t,\tau_1,\tau_2)} &= \mathsf{E}^{Q_\lambda}\left[\int_{\tau_1}^{\tau_2} T_s ds | \mathcal{F}_t\right] \\ &= \int_{\tau_1}^{\tau_2} \Lambda_u du + \mathbf{a}_{t,\tau_1,\tau_2}\mathbf{X}_t + \int_t^{\tau_1} \lambda_u \sigma_u \mathbf{a}_{t,\tau_1,\tau_2}\mathbf{e}_p du \\ &+ \int_{\tau_1}^{\tau_2} \lambda_u \sigma_u \mathbf{e}_1^\top \mathbf{A}^{-1}\left[\exp\left\{\mathbf{A}(\tau_2 - u)\right\} - I_p\right]\mathbf{e}_p du \qquad (2) \end{aligned}$$

with $\mathbf{a}_{t,\tau_1,\tau_2} = \mathbf{e}_1^\top \mathbf{A}^{-1}\left[\exp\left\{\mathbf{A}(\tau_2 - t)\right\} - \exp\left\{\mathbf{A}(\tau_1 - t)\right\}\right]$, $I_p : p \times p$ identity matrix

Benth et al. (2007)

fCAWSpricing

CAWS Futures



Estimated wind future prices for contract length of 1, 3 and 6 months, in Berlin 2012-2013

fCAWSpricing

Conclusion & further research

-  - procedure performs well & applicable to many subjects
- MPR estimation in wind energy: Nasdaq, EEX prices required
- Adaptive optimality for bandwidth selection and transformation
- Extend to spatial wind pricing

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$\mathbf{X}_t$ can be written as a Continuous-time AR(p) (CAR(p)):

For $p = 1$,

$$dX_{1t} = -\alpha_1 X_{1t} dt + \sigma_t dB_t$$

For $p = 2$,

$$\begin{aligned} X_{1(t+2)} &\approx (2-\alpha_1)X_{1(t+1)} \\ &+ (\alpha_1 - \alpha_2 - 1)X_{1t} + \sigma_t(B_{t-1} - B_t) \end{aligned}$$

For $p = 3$,

$$\begin{aligned} X_{1(t+3)} &\approx (3-\alpha_1)X_{1(t+2)} + (2\alpha_1 - \alpha_2 - 3)X_{1(t+1)} \\ &+ (-\alpha_1 + \alpha_2 - \alpha_3 + 1)X_{1t} + \sigma_t(B_{t-1} - B_t) \end{aligned}$$

Proof $CAR(3) \approx AR(3)$

Let

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\alpha_3 & -\alpha_2 & -\alpha_1 \end{pmatrix}$$

- use $B_{t+1} - B_t = \varepsilon_t$
- assume a time step of length one $dt = 1$
- substitute iteratively into X_1 dynamics

Proof $CAR(3) \approx AR(3)$:

$$
\begin{aligned}
X_{1(t+1)} - X_{1(t)} &= X_{2(t)} dt \\
X_{2(t+1)} - X_{2(t)} &= X_{3(t)} dt \\
X_{3(t+1)} - X_{3(t)} &= -\alpha_1 X_{1(t)} dt - \alpha_2 X_{2(t)} dt - \alpha_3 X_{3(t)} dt + \sigma_t \varepsilon_t \\
X_{1(t+2)} - X_{1(t+1)} &= X_{2(t+1)} dt \\
X_{2(t+2)} - X_{2(t+1)} &= X_{3(t+1)} dt \\
X_{3(t+2)} - X_{3(t+1)} &= -\alpha_1 X_{1(t+1)} dt - \alpha_2 X_{2(t+1)} dt \\
&\quad -\alpha_3 X_{3(t+1)} dt + \sigma_{t+1} \varepsilon_{t+1} \\
X_{1(t+3)} - X_{1(t+2)} &= X_{2(t+2)} dt \\
X_{2(t+3)} - X_{2(t+2)} &= X_{3(t+2)} dt \\
X_{3(t+3)} - X_{3(t+2)} &= -\alpha_1 X_{1(t+2)} dt - \alpha_2 X_{2(t+2)} dt \\
&\quad -\alpha_3 X_{3(t+2)} dt + \sigma_{t+2} \varepsilon_{t+2}
\end{aligned}
$$

Return

Seasonal variance: LLE - mirroring observations

To avoid the boundary problem, use mirrored observations:

Assume $h_K < 365/2$, then the observations look like $\hat{\varepsilon}^2_{-364}, \hat{\varepsilon}^2_{-363}, \ldots, \hat{\varepsilon}^2_0, \hat{\varepsilon}^2_1, \ldots, \hat{\varepsilon}^2_{730}$, where

$$
\begin{aligned}
\hat{\varepsilon}^2_t &\stackrel{\text{def}}{=} \hat{\varepsilon}^2_{365+t}, -364 \le t \le 0 \\
\hat{\varepsilon}^2_t &\stackrel{\text{def}}{=} \hat{\varepsilon}^2_{t-365}, 366 \le t \le 730
\end{aligned}
$$