

Correctness of Floyd-warshall's Algorithm:

This proof is made by induction:

Suppose that prior to iteration k it holds that for $i, j \in V$ $[i, j]$ contains length of the shortest path Q from i to j in G containing only vertices in the vertex set $\{1, \dots, k-1\}$, and that $p[i, j]$ contains the immediate predecessor of j on Q . This is obviously true after the initialization.

In iteration k , the length of Q is compared to the length of a path R composed of two subpaths, R_1 and R_2 . R_1 is an i, k path with "intermediate vertices" only in $\{1, \dots, k-1\}$, and R_2 is a k, j path with "intermediate vertices" only in $\{1, \dots, k-1\}$. The shorter of these two is chosen.

The shortest path from i to j in G containing only vertices in the vertex set $\{1, \dots, k\}$ either

- a) Path does not contain k - and hence is the one found in iteration $k-1$ - or
- b) contains k - and then can be decomposed into an i, k followed by a k, j path, each of which has been found in iteration $k-1$. Hence the update ensures the correctness of the induction hypothesis after iteration k .

