

## Correctness of the Ford–Fulkerson Algorithm

Claim: The Ford–Fulkerson Algorithm gives a maximum flow.

Proof:

We must show that the algorithm always stops, and that when it stops, the output is indeed a maximum flow.

We will consider the case of integer capacities.

Each iteration increases the throughput of the flow by an integer. The sum of the capacities on the edges out of  $s$  is finite. The output is a maximum flow.

Upon termination: There are no flow augmenting paths in the companion graph, so: Edges from  $X$  to  $X_c$  are full and edges from  $X_c$  to  $X$  are empty. The capacity of  $[X, X_c]$  equals the throughput of the flow.

Conclusion: The flow is a max flow and the st-cut is a min cut.