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TSP is NP:

We are given a solution of the problem. That is given a tour on a graph. We need to verify, if the tour contanis all the ~~edg~~ vertices, and the cost of the tour is at most K.

We can easily verify that. check all the vertices in the original graph to all the vertices in the TSP tour and see if any one is missing. If not the tour contains all vertices.

Now, add up all the costs of the tour edges. Thus we get the cost of tour. We can verify in constant time if this cost $\leq K$.

So, we can verify the TSP in polynomial time, thus it is NP problem.

TSP is NP complete:

→ As we know, Hamiltonian cycle problem (HCP) is NP-complete, if we reduce HCP to TSP, we conclude that TSP "is as hard" as HCP. Meaning, TSP is also NP-complete.

Reducing ~~TSP t~~ HCP to TSP:

we take an instance of HCP: $G(V, E)$

here we have vertices V and set of edges E. Note that, necessarily this is not a complete graph.

we re-construct this graph to form an instance of TSP such that we make a complete graph of V vertices, and the cost is defined: (G')

$$c(i,j) = \begin{cases} 0, & \text{if } (i,j) \in E \\ 1, & \text{if } (i,j) \notin E \end{cases}$$

Now, as the HCP edges has cost of zero, we add them all, we get a total cost of zero, and we have found a tour that covers all vertices. So, the version of TSP here is: $(G', c, 0)$.

Thus HCP reduces to TSP.

Reducing TSP to HCP:

Now, we have a complete graph $G' = (V, E')$ where the cost function is the same as defined before.

Now, this graph forms a TSP of cost at most zero.

As in TSP, we need to cover all vertices, the tour we get has all the vertices.

Now, as the cost of this tour is at most 0, each of the edges must have a weight of zero.

we formed the graph in a way that, only the edges falling in HCP has cost zero, so we are actually getting the Hamiltonian cycle at the end from this version $(G', c, 0)$ of TSP.

Thus the TSP can be reduced to HCP.

finally, we conclude, TSP is NP-complete.