

Correctness of Bellman-Ford's Algorithm:

This proof is made by Contradiction :

show $d[v] = \delta(s,v)$ after $|V|-1$ passes

If $\text{shortest distance}[v] > \text{shortest}[u] + w$ then we can't update that $\text{shortest}[v] = \text{shortest}[u] + w$

– Lemma: $d[v] \geq \delta(s,v)$ always • Initially true • Let v be first vertex for which $d[v] < \delta(s,v)$

• Let u be the vertex that caused $d[v]$ to change: $d[v] = d[u] + w(u,v)$

• If $d[v] < \delta(s,v)$ then

$$\delta(s,v) \leq \delta(s,u) + w(u,v)$$

$$\delta(s,u) + w(u,v) \leq d[u] + w(u,v)$$

Therefore, $d[v] < d[u] + w(u,v)$. Contradiction.