Md. Jehadul Karim 1805057

Ans: Let, TSP := Traveling salesman Problem

we have to show that,

i) TSP $\in$ NP

and ii) TSP $\in$ NP-hard

For checking a tour for credibility, we check that the tour contains each vertex once. Then we sum the total cost of the edges and finally check if the cost is minimum. This can be verified in polynomial ~~thir~~ time. So,

TSP $\in$ NP [(i) Proved]

We prove the second one using the knowledge that Hamiltonian cycle is NP-complete. We will show that,

Hamiltonian Cycle $\leq_p$ TSP

by reducing HCP to TSP which yeilds TSP is atleast hard as HCP.

For this, we change our given graph $G(V, E)$ for HCP to an complete graph $G'(V, E')$.

We define a cost function to be,

$$c(i,j) = \begin{cases} 1 ; & \text{if } (i,j) \in E \\ \infty ; & \text{otherwise} \end{cases}$$

If there exists a hamiltonian cycle, the min cost of TSP will be n (as cycle will contain n edges, each having cost of 1). Thus, if graph G has Hamiltonian cycle, then graph G' has traveling salesman cost of n, only using the edges with cost 1.

So, we have proven that G has hamiltonian cycle iff G' has a tour of cost ~~at~~ at most n.

So, TSP $\in$ NP-hard. (atleast as hard as hamiltonian cycle problem)

$\therefore$ TSP $\in$ NP-Complete.