Correctness of Floyd-warshall's Algorithm:

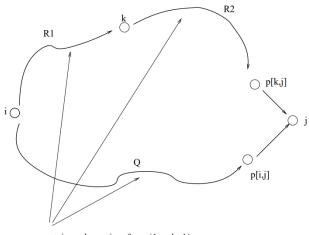
This proof is made by induction:

Suppose that prior to iteration k it holds that for i, $j \in v$ y[I, j] contains length of the shortest path Q from i to j in G containing only vertices in the vertex set $\{1, ..., k-1\}$, and that p[I, j] contains the immediate predecessor of j on Q. This is obviously true after the initialization.

In iteration k, the length of Q is compared to the length of a path R composed of two subpaths, R1 and R2. R1 is an i, k path with "intermediate vertices" only in $\{1, ..., k-1\}$, and R1 is a k, j path with "intermediate vertices" only in $\{1, ..., k-1\}$. The shorter of these two is chosen.

The shortest path from i to j in G containing only vertices in the vertex set {1, ..., k} either

- a) Path does not contain k and hence is the one found in iteration k-1 or
- b) contains k and then can be decomposed into an i, k followed by a k, j path, each of which has been found in iteration k-1. Hence the update ensures the correctness of the induction hypothesis after iteration k.



contains only vertices from {1,..., k-1}