

- Suppose that, in addition to edge capacities, a flow network has **vertex capacities**. That is each vertex has a limit  $l$  on how much flow can pass through. Show how to transform a flow network with vertex capacities into an equivalent flow network  $G_0$  without vertex capacities, such that a maximum flow in  $G_0$  has the same value as a maximum flow in  $G$ . How many vertices and edges does  $G_0$  have?
- (true/false with explanation) Let  $G$  be a weighted directed graph with exactly one source  $s$  and exactly one sink  $t$ . Let  $(A,B)$  be a maximum cut in  $G$ , that is,  $A$  and  $B$  are disjoint sets whose union is  $V$ ,  $s \in A$ ,  $t \in B$ , and the sum of the weights of all edges from  $A$  to  $B$  is the maximum for any two such sets. Now let  $H$  be the weighted directed graph obtained by adding 1 to the weight of each edge in  $G$ . Then  $(A,B)$  must still be a maximum cut in  $H$ .
- Find the max-flow in the given graph and the min-cut.

