Making Predictions at Hadron Colliders

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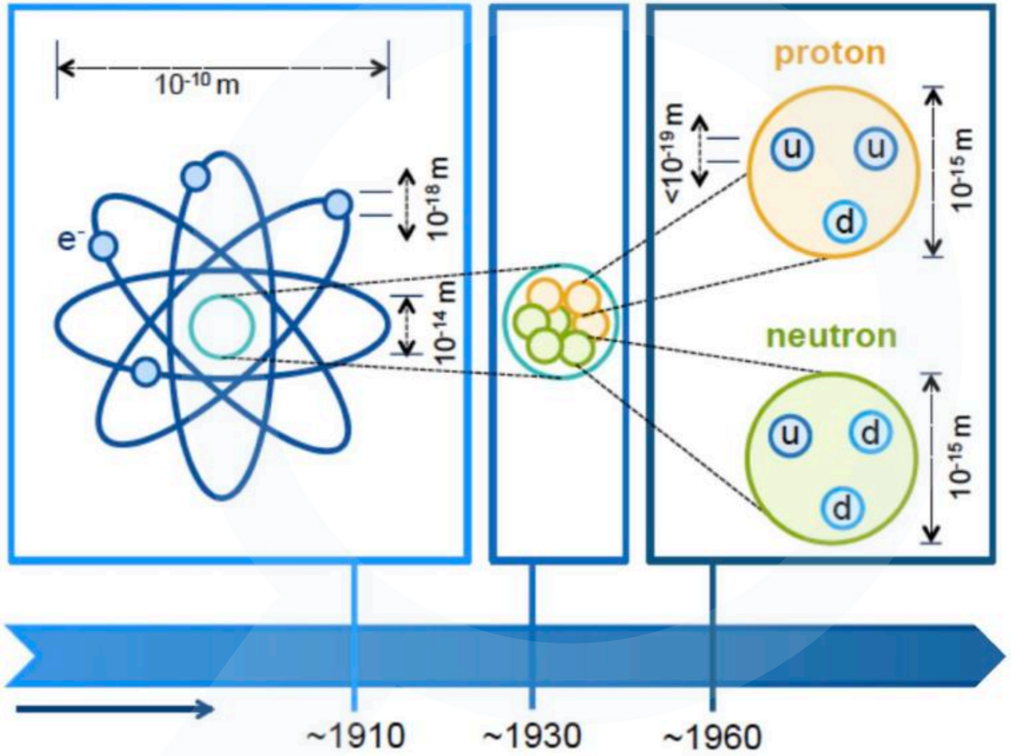
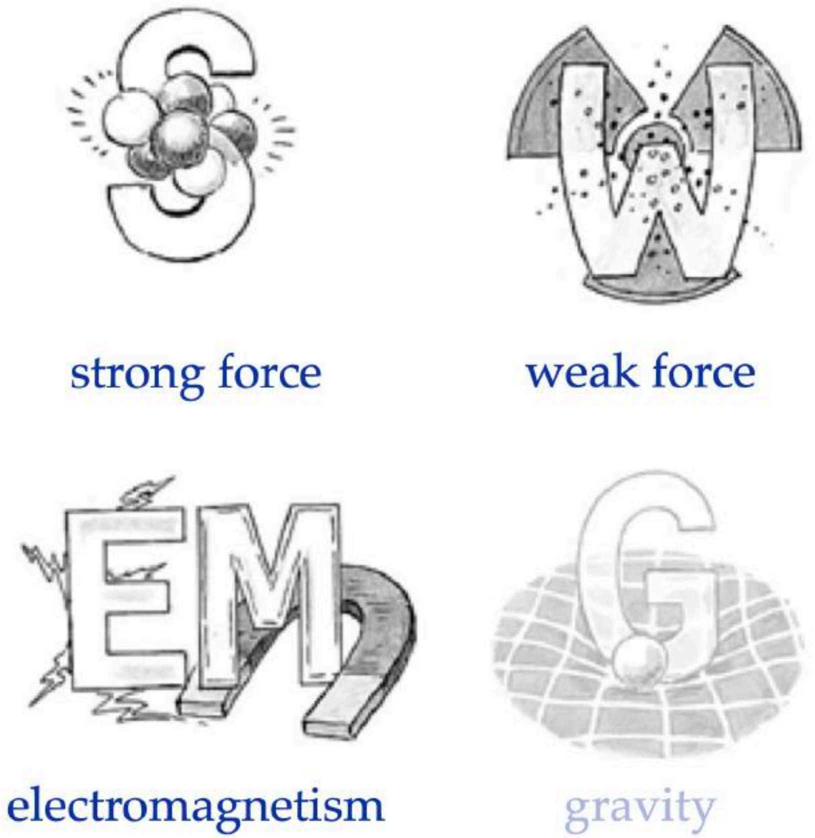
CERN Summer Student Lectures 2024

What is the Universe made of?

What are the Laws of Nature?





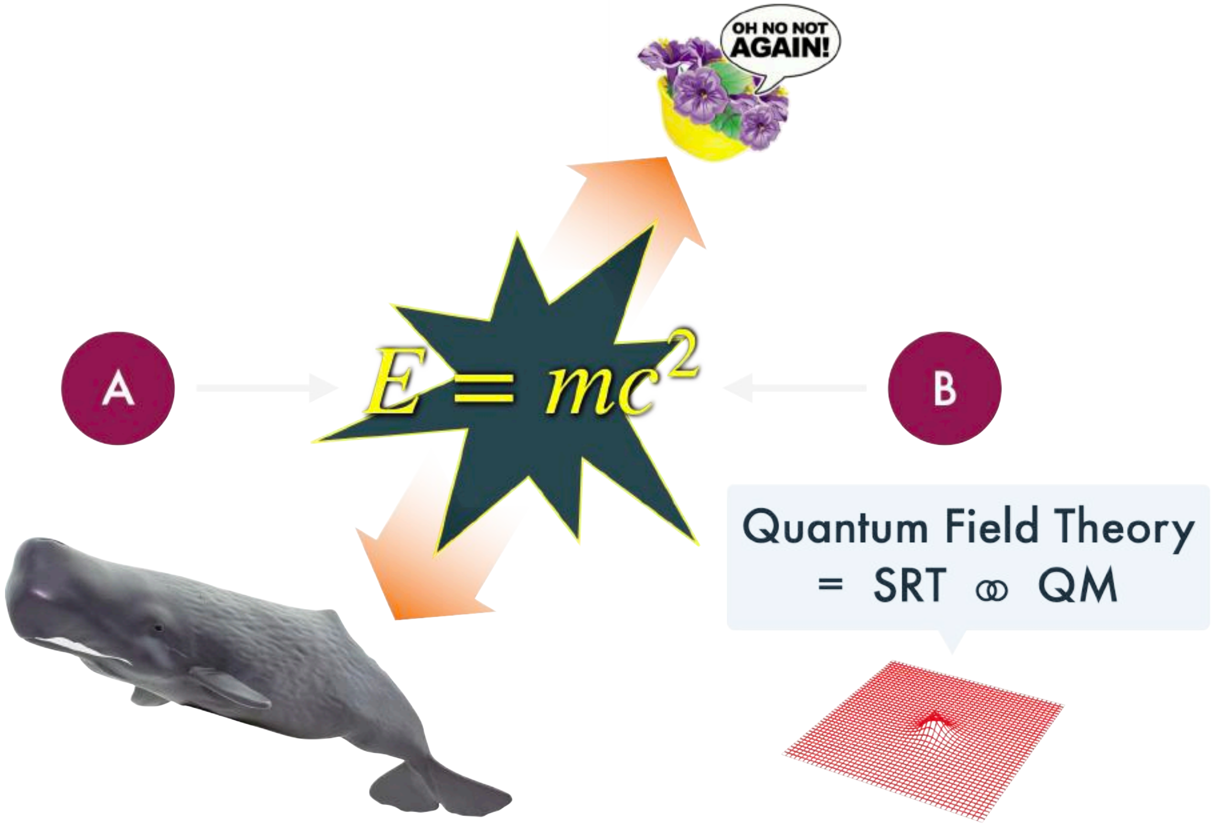
Elementary Particles ⟷ Fundamental Forces

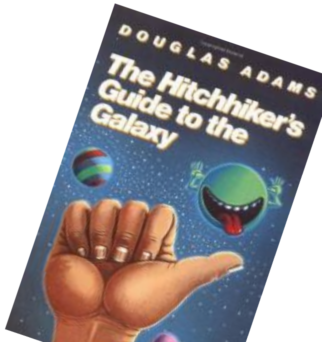
Particle Smashers — Our Microscopes

LEP	@ CERN	[→ 1999]
↪ e^+e^-	90 ↗ 200 GeV	◯27 km
HERA	@ DESY	[→ 2008]
↪ ep	320 GeV	◯6.3 km
Tevatron	@ Fermilab	[→ 2011]
↪ $p\bar{p}$	1 TeV	◯6.28 km
LHC	@ CERN	[→ now]
↪ pp	7 ↗ 13.6 TeV	◯27 km
EIC	@ BNL	[2032 →]
↪ ep	20 ↗ 140 GeV	◯3.8 km
FCC-ee	@ CERN	[2040?→]
↪ e^+e^-	90 ↗ 360 GeV	◯100 km
FCC-hh	@ CERN	[2060?→]
↪ pp	100 TeV	◯100 km

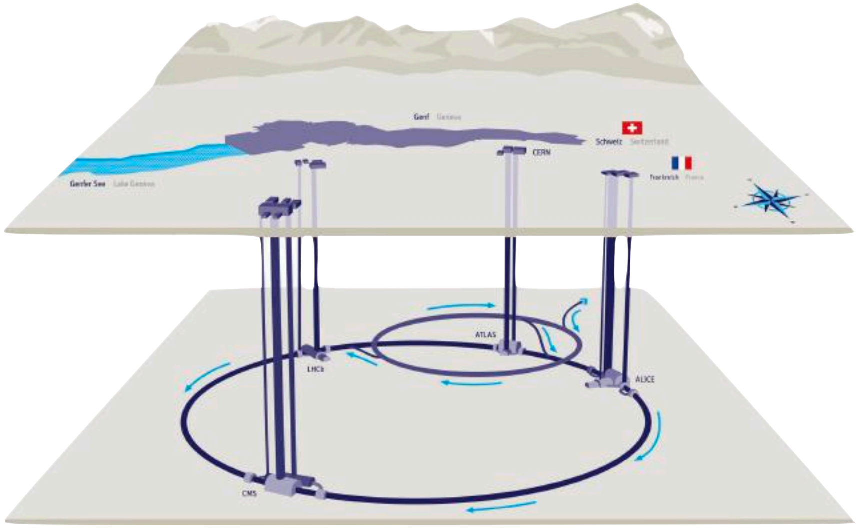
Particle Smashers — Our Microscopes

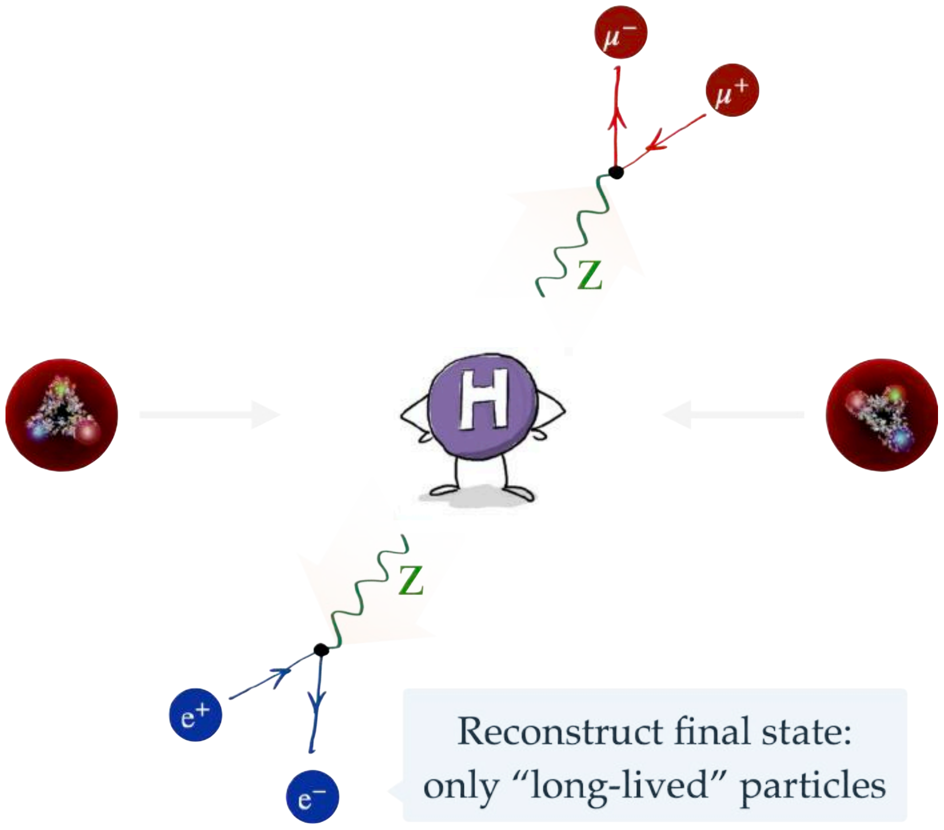
* high energies
  - ↳ small distances $\lambda = \frac{hc}{E}$
  - ↳ produce heavy (= interesting!) final states
* many collisions
  - ↳ access to rare (= interesting!) events & processes



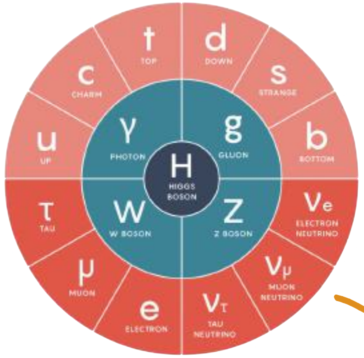


Particle Smashers – The LHC





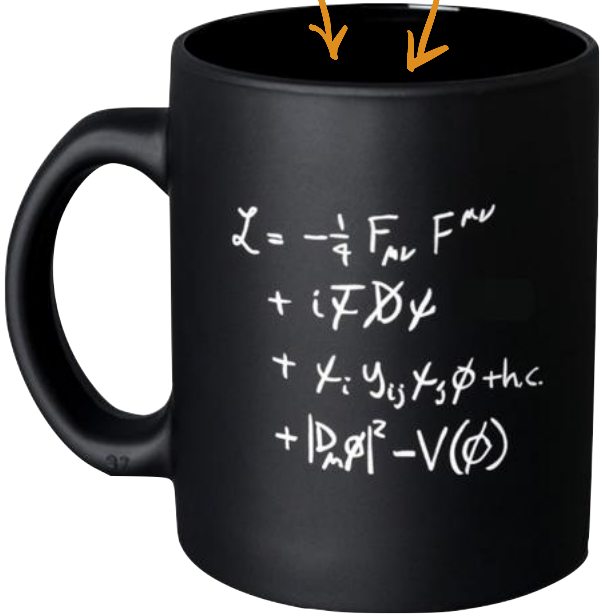
* energy frontier
  - ↳ 13.6 TeV collision energy
  - ↳ 1000 x distance to moon ()
* intensity frontier
  - ↳ 10^9 collisions / second
  - ↳ 30 PB / year



Gauge interactions

$SU(3)_c \times SU(2)_L \times U(1)_Y$

THEORY

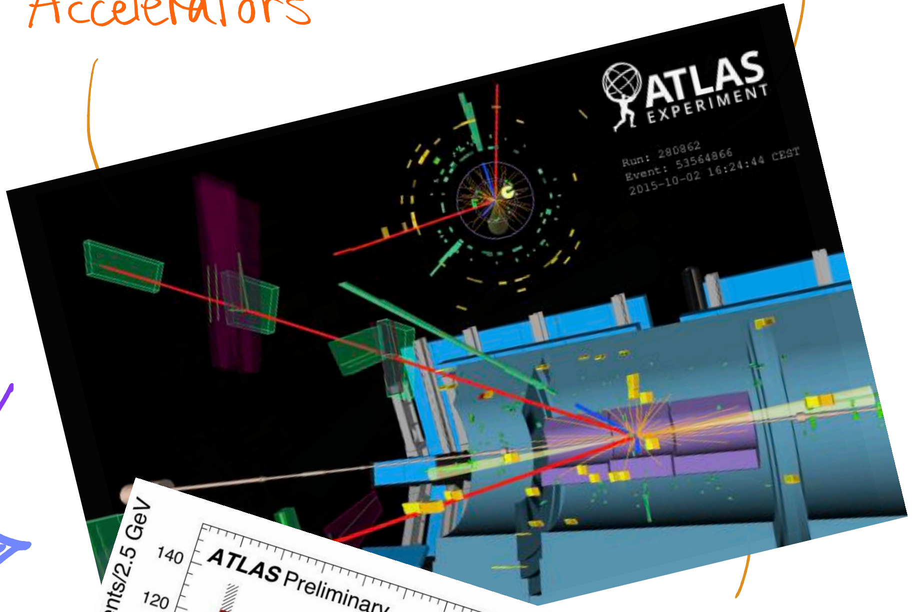


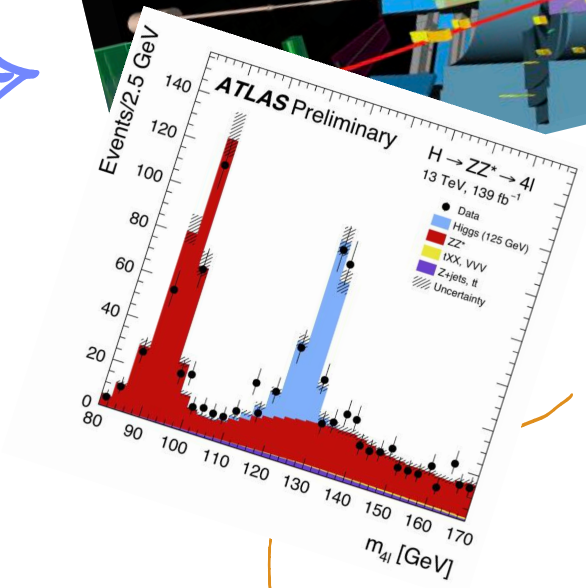
PHENOMENOLOGY

this lecture

Accelerators

Detectors





Raw Data

Physics Results

Statistics

EXPERIMENT

$QFT = QM \otimes SR$

[Particle World, Theoretical Concepts, The SM]

Repository & Conventions

* Notebooks for demonstrations

  ⇒ https://github.com/aykhuss/Lectures-SSL-MkPred

"Making Predictions"

* Conventions natural units: $[\hbar] = [c] = 1$ (remember: $m_{\text{proton}} \sim 1\,\text{GeV}$)

  ⇒ $[\text{length}] = [\text{time}] = \text{eV}^{-1}$

  $[\text{mass}] = [\text{energy}] = [\text{momentum}] = \text{eV}$

  ⇒ four vectors $x^\mu = (t, x, y, z)^T$

  $p^\mu = (E, \underbrace{p_x, p_y, p_z}_{\vec{p}})^T$ ⇝ "on-shell": $p^2 = E^2 - |\vec{p}|^2 \overset{!}{=} m^2$

  invariant mass

⇒ energy-momentum conservation: $(a + b \to 1 + 2)$

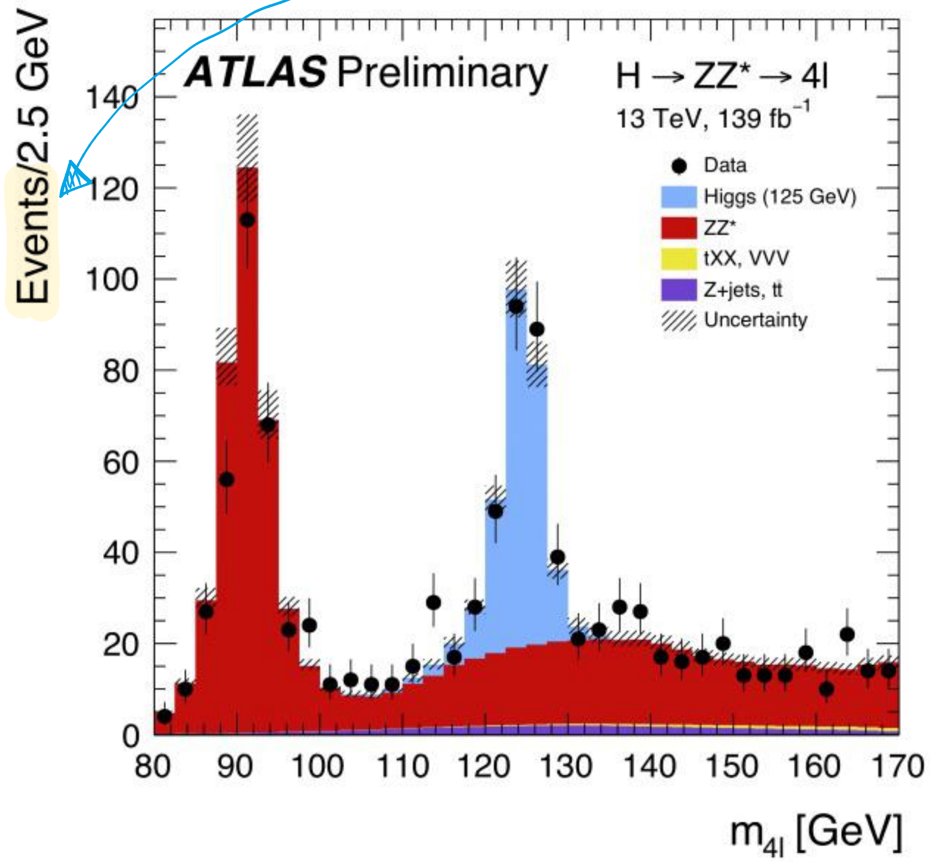
$$\delta^{(4)}(p_1 + p_2 - (p_a + p_b)) = \delta(E_1 + E_2 - (E_a + E_b))\,\delta^{(3)}(\vec{p}_1 + \vec{p}_2 - (\vec{p}_a + \vec{p}_a))$$

The Plan

1. Event Rates, Cross Sections & Scattering Amplitudes
2. Warmup: Lepton Collider
3. Hadron Colliders — Parton Distribution Functions
4. The Drell-Yan Process
5. Higher-Order Corrections
6. QCD Jets, Parton Showers & MC Simulations

Event Rates

We ultimately measure # Events
for a specific process: $a+b \to 1+2+\ldots+n$



Luminosity ~ # collisions

cross section

$$dN = \mathcal{L}\, d\sigma$$

* σ_H (13 TeV) $\approx 50\,\text{pb}$

$\int_{\text{Run2}} dt\, \mathcal{L} \approx 150\,\text{fb}^{-1}$

~ 7 million Higgs bosons produced!

* σ_Z (13 TeV) $\simeq 50\,\text{nb}$

$\sigma_{W^\pm}$ (13 TeV) $\simeq 200\,\text{nb}$

$\mathcal{L}$ (instantaneous) $\approx 0.02\,\text{pb}^{-1}\text{s}^{-1}$

~ 1000 Z's
~ 4000 $W^\pm$'s
every second!

Calculating Cross Sections

Fermi's Golden Rule $\quad a+b \to 1+2+\dots+n$

$$d\sigma = \underbrace{\frac{1}{F}}_{\text{flux}} \; \underbrace{\langle|\mathcal{M}|^2\rangle}_{|\text{amplitude}|^2} \; \underbrace{d\Phi}_{\text{phase space}}$$

(LIPS)

$$= \frac{1}{4(p_a\cdot p_b)} = \frac{1}{2E_{cm}^2}$$

$$= \frac{1}{n_a^{d.o.f.}\, n_b^{d.o.f.}} \sum_{d.o.f.} |\mathcal{M}|^2$$

(degrees of freedom
spin, colour)

$$d\Phi_n(p_1,\dots,p_n;\, p_a+p_b)$$

"on shell"

$E_i > 0$

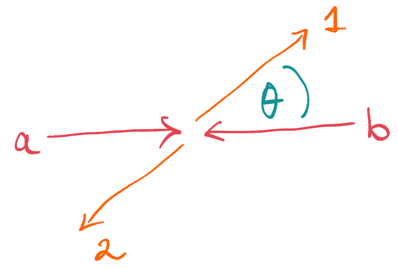
$$= \prod_{i=1}^{n} \frac{d^4 p_i}{(2\pi)^4}\,(2\pi)\,\delta(p_i^2 - m_i^2)\,\Theta(p_i^0)$$

$$(2\pi)^4\,\delta^{(4)}(p_1+\dots+p_n-(p_a+p_b))$$

energy-momentum
conservation

Special case $\quad a+b \to 1+2$

$$d\Phi_2 = \frac{d\cos\theta}{16\pi} \quad \text{(massless)}$$



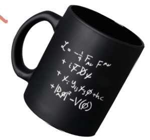
The Scattering Amplitude

$\leftrightarrow$ evaluation of the path integral

(analogy to QM)

$$Z[J] = \int \mathcal{D}[\Phi]\; e^{i\int d^4x\,[\mathcal{L}(\Phi,\partial_\mu\Phi) + J(x)\Phi(x)]}$$

$\Phi \in \{\psi, \phi, A_\mu, \dots\}$



$\hookrightarrow$ extremely difficult to solve except a free theory

$\hookrightarrow$ $\mathcal{L}$ only has terms with at most two fields Φ, e.g. $\bar\psi(x)\,(i\not\partial - m)\,\psi(x)$

* Feynman rules for the free theory

incoming

f  p $= u(p)$

$\bar f$  p $= \bar v(p)$

outgoing

 p, f $= \bar u(p)$

 p, $\bar f$ $= v(p)$

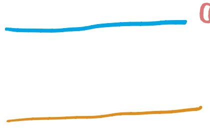
propagators

 p, f $= \dfrac{i}{\not p - m}$

 p, Z $= \dfrac{-i\,g_{\mu\nu}}{p^2 - M_Z^2 + i M_Z \Gamma_Z}$

boring "scattering"

$a+b \to a+b$

a  a

b b

Perturbation Theory

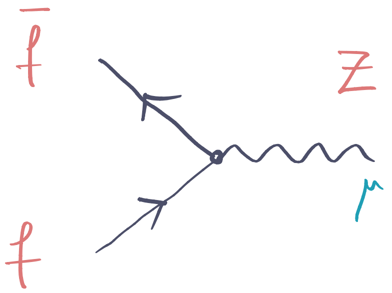
When interaction strength is small ($\alpha_{em} \sim 1/137$, $\alpha_s(M_Z^2) \sim 0.118$)

$\Rightarrow$ compute $\mathcal{M}$ perturbatively by expanding around free theory

* Feynman rules for interactions - vertices

direct correspondence* with terms in $\mathcal{L}$

$$-e\, Z_\mu\, \bar{\psi}_f\, \gamma^\mu (v_f - a_f \gamma^5)\, \psi_f \in \mathcal{L}$$

$\rightsquigarrow$ 

$$= -ie\, \gamma^\mu (v_f - a_f \gamma^5)$$

* more subtle than just "dropping" the fields when derivatives (∂_μ) and/or identical particles

Warmup: Lepton Collider

Consider the process $e^+ e^- \to \mu^+ \mu^-$

at lowest order (tree level). There are two diagrams

What are they?

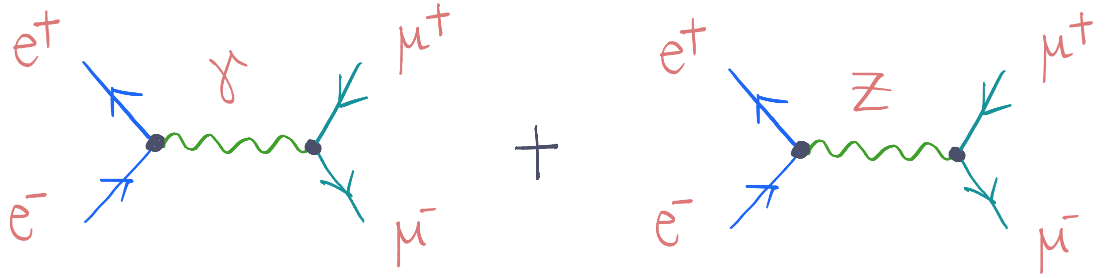
[demo: FeynGame]

Warmup: Lepton Collider

[demo: $e^+e^- \to \mu^+\mu^-$]

Consider the process $e^+ e^- \to \mu^+ \mu^-$

at lowest order (tree level). There are two diagrams



$= i\mathcal{M}^{\text{tree}}_{e^+e^-\to\mu^+\mu^-}$

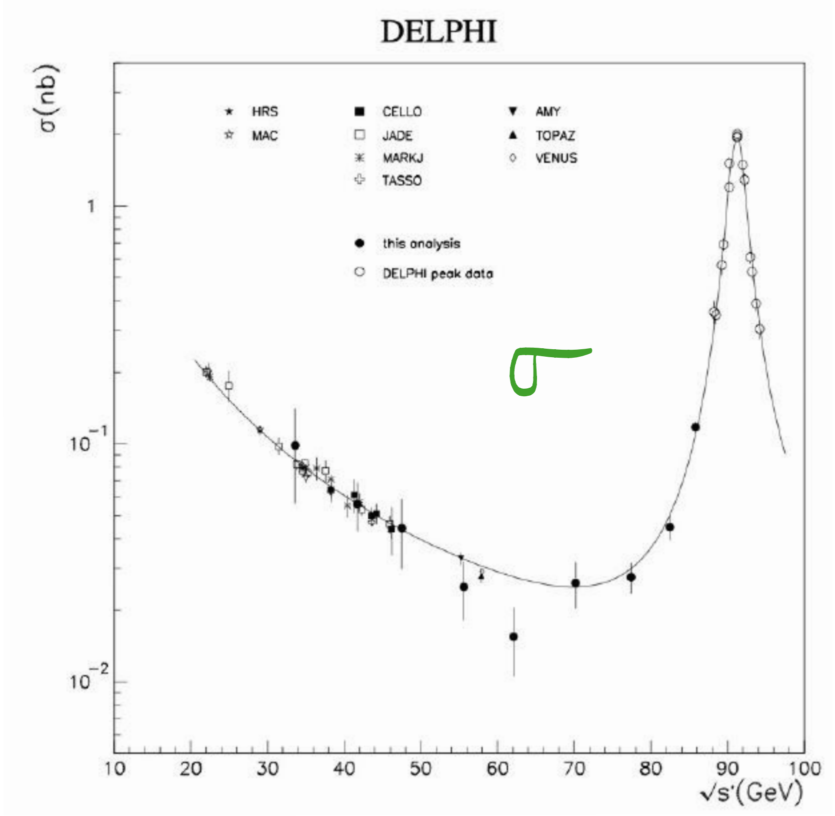
$\Rightarrow$ Inserting into Fermi's golden rule $[s = E_{CM}^2 \;;\; p_a \cdot p_1 = p_a^\mu p_{1,\mu} = E_{cm}^2 (1-\cos\theta)]$

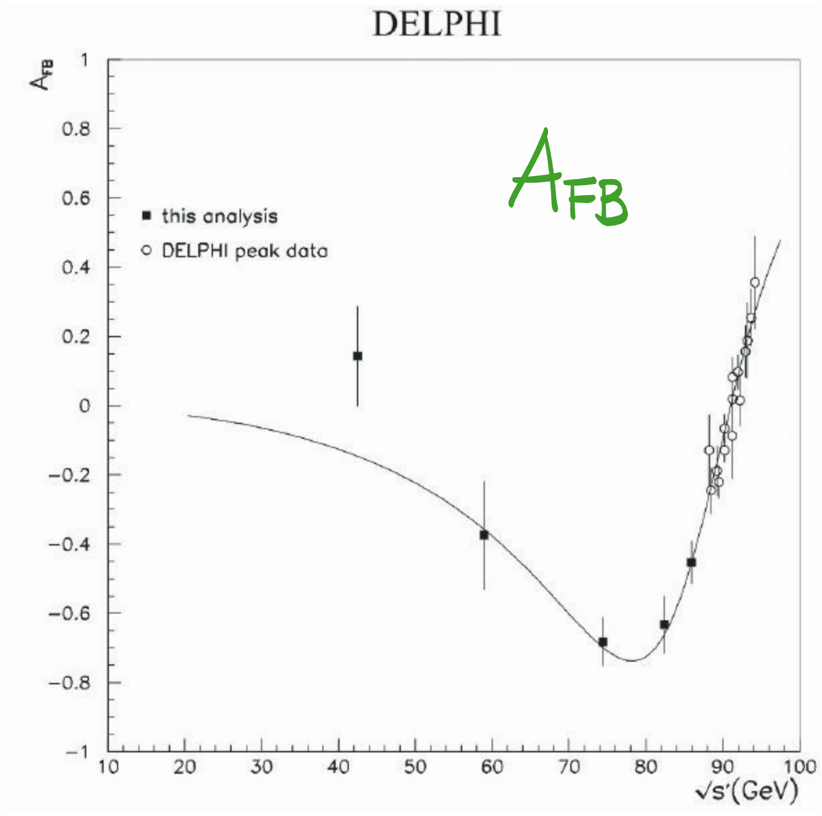
$$\frac{d\sigma}{d\cos\theta} = \frac{\alpha^2 \pi}{2s}\left[(1+\cos^2\theta)\, G_1(s) + 2\cos\theta\, G_2(s)\right]$$

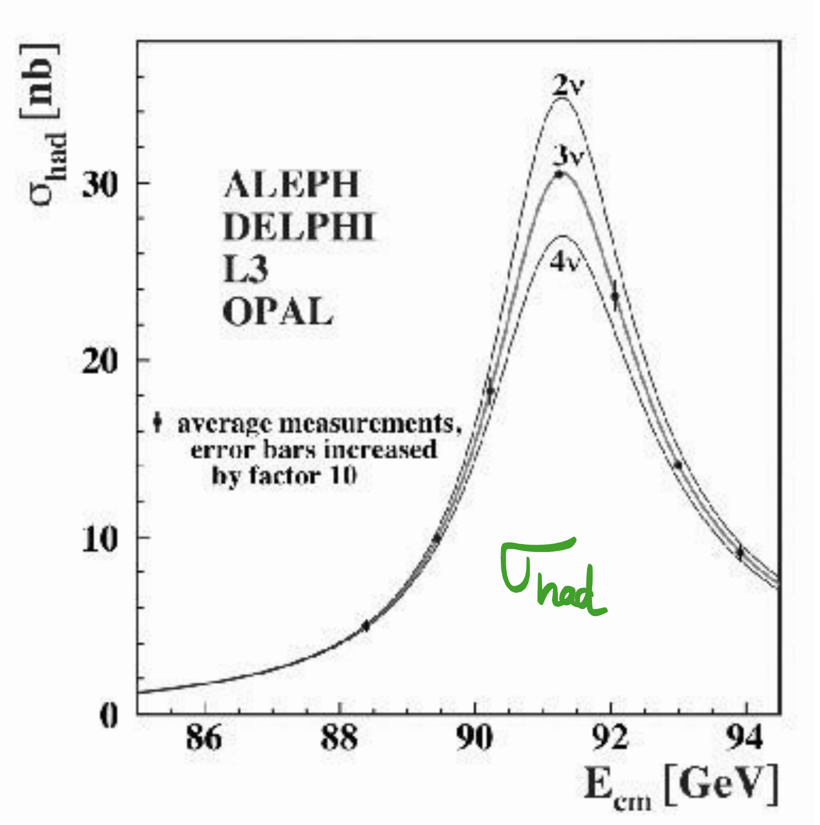
$$G_1(s) = 1 + 2v_\ell^2 \,\mathrm{Re}\left\{\frac{s}{s-M_Z^2+iM_Z\Gamma_Z}\right\} + (v_\ell^2+a_\ell^2)\left|\frac{s}{s-M_Z^2+iM_Z\Gamma_Z}\right|^2$$

$$G_2(s) = 0 + 2a_\ell^2 \,\mathrm{Re}\left\{\frac{s}{s-M_Z^2+iM_Z\Gamma_Z}\right\} + 4v_\ell^2\cdot a_\ell^2\left|\frac{s}{s-M_Z^2+iM_Z\Gamma_Z}\right|^2$$

"Comparison" to Data







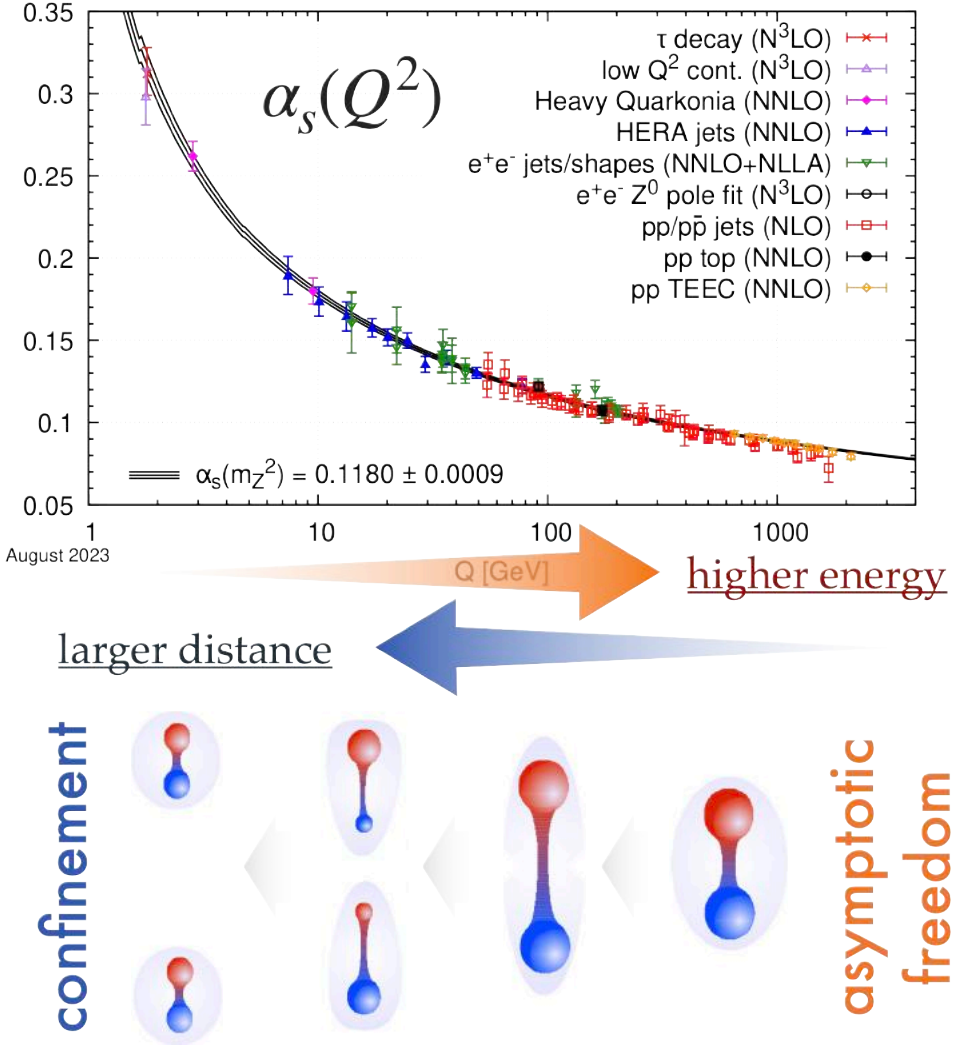
* In principle, you now can use the predictions to fit M_Z & $\sin^2\theta_W$ from the data (at leading order)
* σ_{had} is the hadronic cross section: @ LO: $e^+e^- \to q\bar{q}$
  $\Rightarrow$ what changes compared to $\mu^+\mu^-$?

Hadron Colliders ...

... are "messy"

strong interaction ⇝ 

- Quantum Chromodynamics (QCD)



> Quantum chromodynamics is conceptually simple. Its realization in nature, however, is usually very complex.
>
> Frank Wilczek

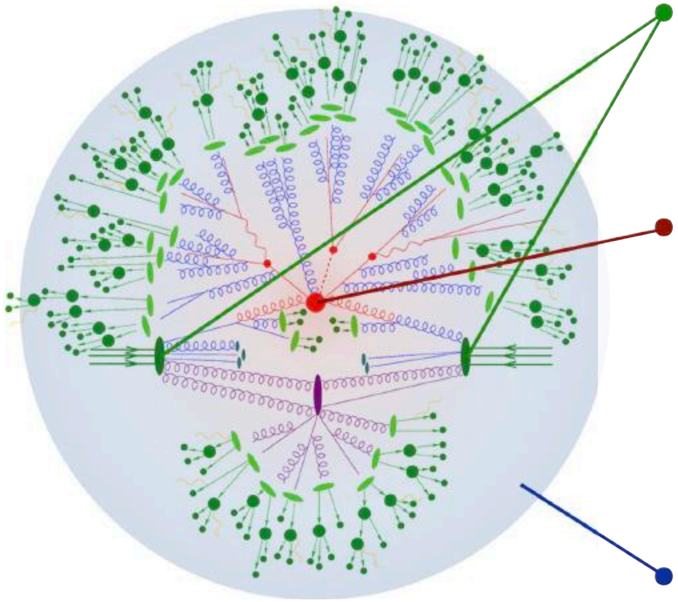
* no free quarks & gluons
  ⇒ spray of hadrons ($\pi^\pm, K^\pm, K^0, p^\pm, n, \ldots$)

* colliding objects (p @ LHC) <u>not</u> elementary

Hadron Colliders ...

... taming the mess

> "Quantum chromodynamics is conceptually simple. Its realization in nature, however, is usually very complex. But not always."
>
> Frank Wilczek



incoming protons

- quarks & gluons

short distance "hard"

- high scales: 10^2–10^3 GeV

evolution towards a physical observable state

long distance "soft"

- low scales: $\mathcal{O}$(few GeV)

1. factorization

↪ relevant physics at disparate scales (isolates description of proton from rest)

2. asymptotic freedom

↪ short distance ⟷ perturbation theory

$$\sigma = \sigma_{LO}\left(1 + \alpha_s\, c^{(1)} + \alpha_s^2\, c^{(2)} + \ldots\right)$$

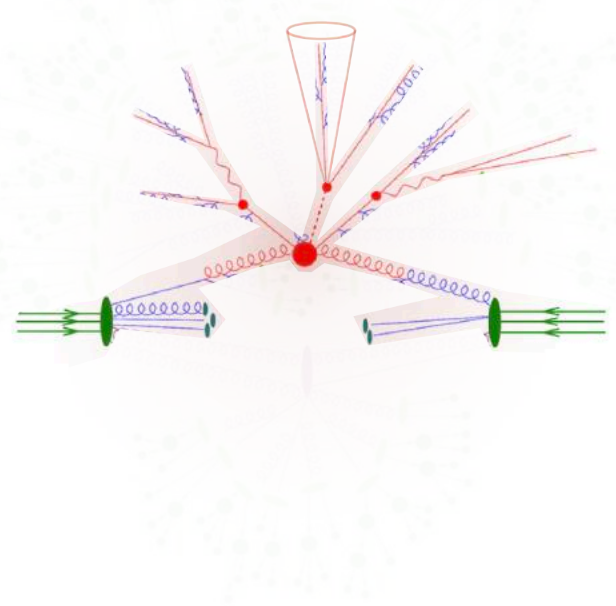
⇒ strategy: focus on high-mom transfer

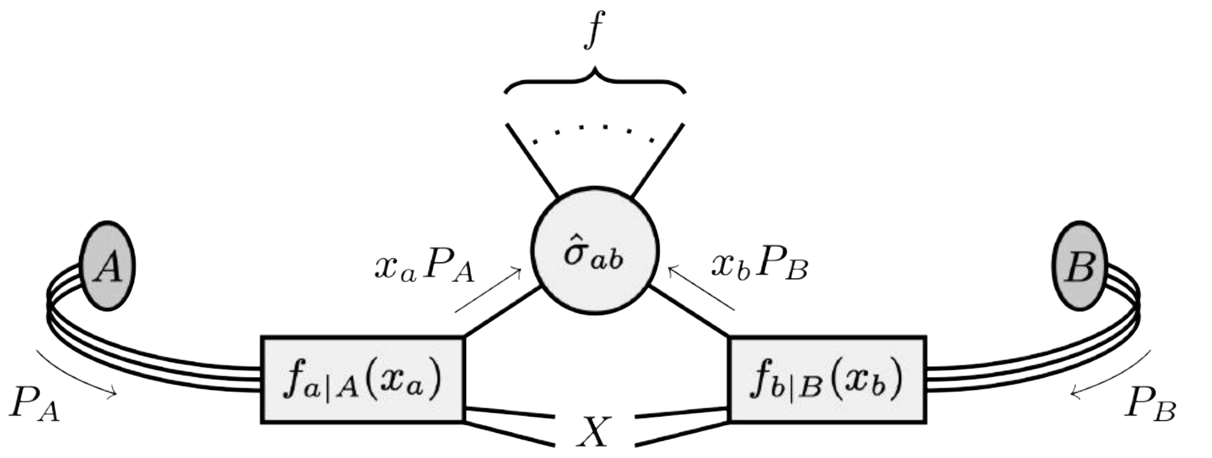
clean signature ($\ell^\pm, \gamma, Z, \ldots$) objects jets

Hadron Colliders: The Parton Model

strategy for precision:

* focus on high momentum transfer
* clean signatures ($\ell^{\pm}$, jets, ...)





momentum fraction

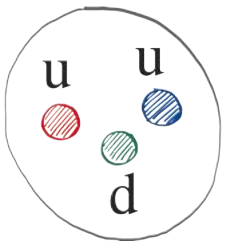
$$d\sigma_{A+B\to f}(P_A, P_B) = \sum_{a,b} \int_0^1 dx_a \int_0^1 dx_b \; f_a(x_a)\, f_b(x_b)\; d\hat{\sigma}_{a+b\to f}(x_a P_A, x_b P_B)$$

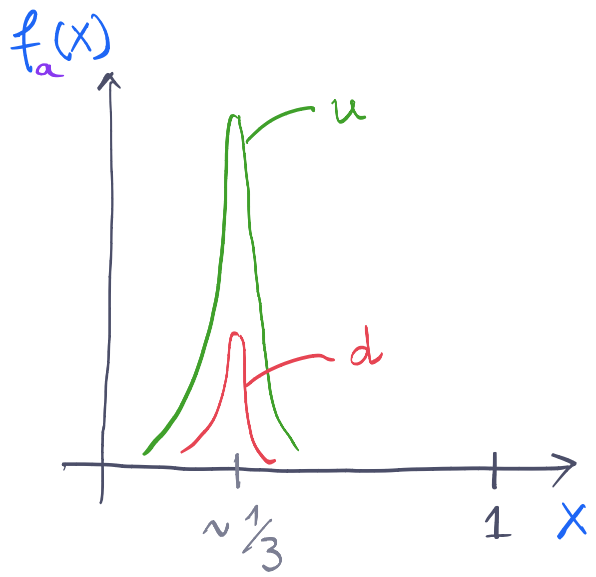
parton distribution function (PDF)

$\hat{=}$ number density for parton a w/ momentum fraction $[x_a, x_a+dx_a]$ of parent hadron A

Parton Distribution Functions

* just free quarks? ($p \simeq (uud)$)





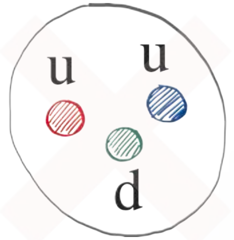
$$f_u(x) \sim 2\,\delta(x - 1/3)$$

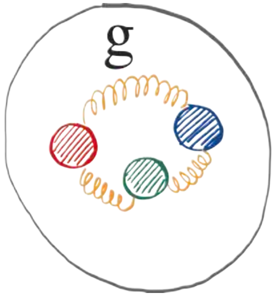
$$f_d(x) \sim 1\,\delta(x - 1/3)$$

$$f_{etc}(x) \sim \emptyset$$

(+ some smearing)

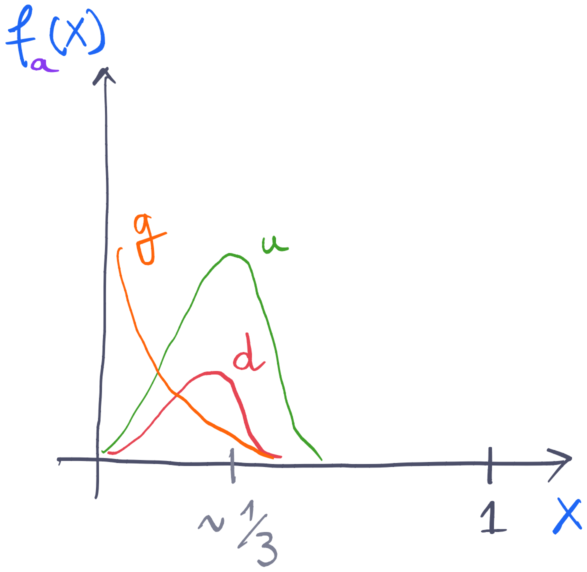
Parton Distribution Functions





* just free quarks? ($p \simeq (uud)$)

* bound by gluons?



naive parton model

⇔ composition of point particles

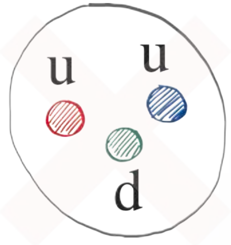
⇝ zoom in ($Q^2 \nearrow$) ↔ same composition: scaling

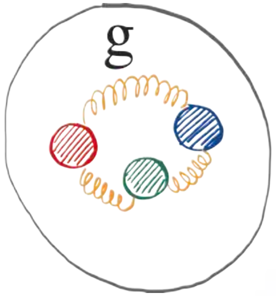
PDFs independent on scale, at which it is probed

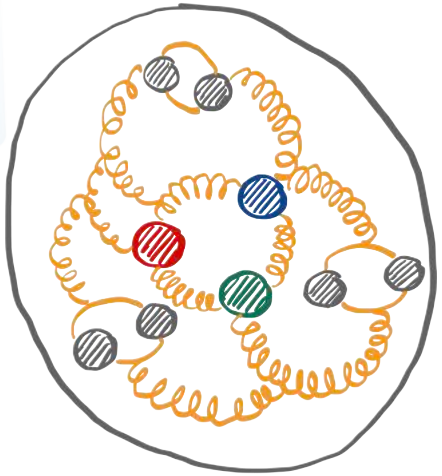
(as long as $Q^2 \gg m_p^2$)

Parton Distribution Functions

[demo: PDFs]



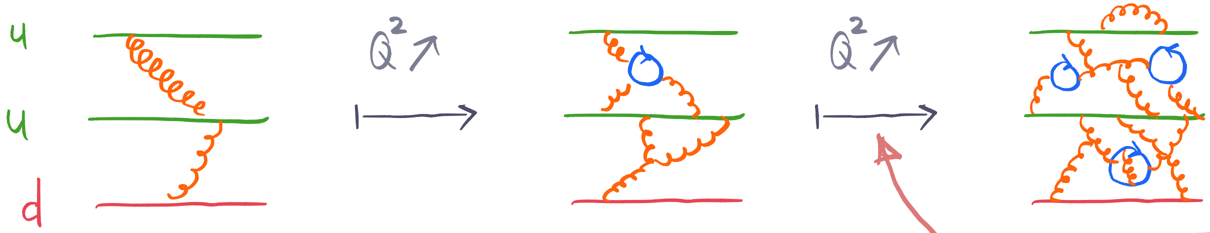




* just free quarks? ($p \simeq (uud)$)

* bound by gluons?

* QCD-improved parton model
  ↪ quantum fluctuations → more g & $(q\bar{q})$ as we "zoom in" ($Q^2 \nearrow$)



⇒ predominantly shifts partons from high-x to low-x

evolution is perturbatively calculable!
(test of QCD)

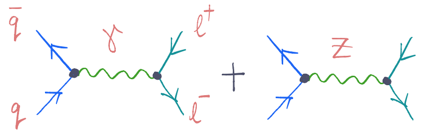
The Drell-Yan Process $p + p \longrightarrow \ell + \bar{\ell}$

[demo: Drell-Yan]

$$d\sigma_{DY}(P_A, P_B) = \sum_{a,b} \int_0^1 dx_a \int_0^1 dx_b \; f_a(x_a)\, f_b(x_b)\; d\hat{\sigma}_{a+b \to \ell^+\ell^-}(x_a P_A, x_b P_B)$$

at LO only

$(a,b) \in \{(q,\bar{q}), (\bar{q},q)\}$



* Integrate out $Z \to \ell^+\ell^-$ decay
* Observables of intermediate gauge boson $q^\mu = (p_1+p_2)^\mu = (p_a+p_b)^\mu$

$$M_{\ell\ell} = \sqrt{q^2} \quad ; \quad Y_{\ell\ell} = \frac{1}{2} \ln\left(\frac{q^0+q^3}{q^0-q^3}\right)$$

rapidity: $Y \longmapsto Y + \frac{1}{2}\ln(x_a/x_b)$

$$\Rightarrow \boxed{\frac{d^2\sigma_{DY}}{dM_{\ell\ell}\, dY_{\ell\ell}} = f_a(x_a)\, f_b(x_b)\, \frac{2 M_{\ell\ell}}{E_{cm}^2}\, \hat{\sigma}_{a+b\to\ell^+\ell^-} \Big|_{x_{a/b} = \frac{M_{\ell\ell}}{E_{cm}} e^{\pm Y_{\ell\ell}}}}$$