

Parton Distribution Functions

Alexander Huss

July 20, 2025

Contents

1	Introduction	1
2	Evaluating PDFs	1
3	The quantum numbers of the proton	2
4	Momentum sum rules	3
5	Comparison of PDF sets	4

1 Introduction

We will call the LHAPDF library from python to evaluate parton distribution functions and play around with them a bit.

2 Evaluating PDFs

Let's have a look at one specific PDF set PDF4LHC21_40

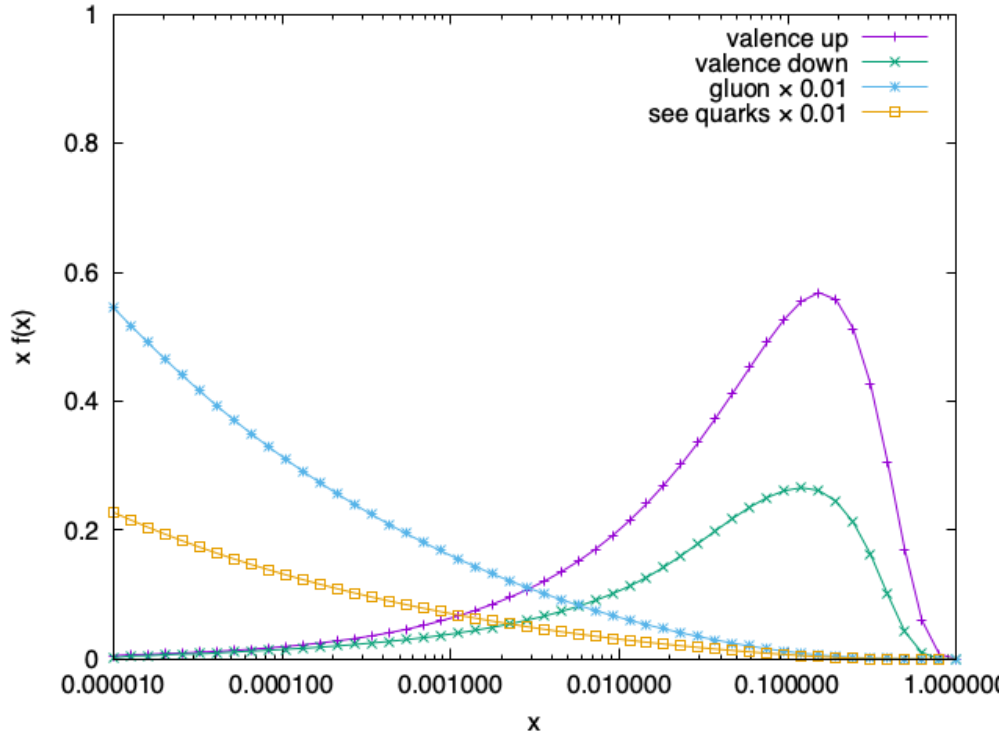
```
import lhpdf
import numpy as np

lhpdf.setVerbosity(0)
pdf = lhpdf.mkPDF("PDF4LHC21_40", 0)
if qlog:
    xs = [x for x in np.logspace(-5, 0, 50)]
else:
    xs = [x for x in np.linspace(0, 1, 50)]
res = np.empty([len(xs), 6])
for ix, x in enumerate(xs):
    fac = 1. #1./x
    res[ix,0] = x
    res[ix,1] = q
    res[ix,2] = fac*(pdf.xfxQ(2, x, q) - pdf.xfxQ(-2, x, q)) # valence up-quark
    res[ix,3] = fac*(pdf.xfxQ(1, x, q) - pdf.xfxQ(-1, x, q)) # valence down-quark
    res[ix,4] = fac*(pdf.xfxQ(0, x, q)) # gluon (or `21`)
```

```

res[ix,5] = fac*(
    2.*pdf.xfxQ(-2, x, q)
    +2.*pdf.xfxQ(-1, x, q)
    +pdf.xfxQ(3, x, q)+pdf.xfxQ(-3, x, q)
    +pdf.xfxQ(4, x, q)+pdf.xfxQ(-4, x, q)
    +pdf.xfxQ(5, x, q)+pdf.xfxQ(-5, x, q)
) # sum over all sea quarks
return res.tolist()

```



- Change the scale in the figure from 10 to 100 GeV or even to 1 TeV; adjust the scaling factor as necessary. How does the distributions change? Does it correspond to the expectation of the QCD improved parton model?

3 The quantum numbers of the proton

The quantum numbers of the proton are determined by the valence quarks, $p=(uud)$. Given that PDFs are *number densities* for the constituent partons, the following flavour sum rules hold:

$$\int_0^1 dx \left(f_{u|p}(x) - f_{\bar{u}|p}(x) \right) = 2, \quad \int_0^1 dx \left(f_{d|p}(x) - f_{\bar{d}|p}(x) \right) = 1, \quad (1)$$

$$\int_0^1 dx \left(f_{q|p}(x) - f_{\bar{q}|p}(x) \right) = 0 \quad \forall q \notin \{u, d\} \quad (2)$$

Let's see if these hold for one of the global PDF sets:

```
import lhpdf
import math
import scipy
lhpdf.setVerbosity(0)
pdf = lhpdf.mkPDF("PDF4LHC21_40", 0)
res = list()
for id,label in [(1,"d"),(2,"u"),(3,"s"),(4,"c"),(5,"b")]:
    int_res = scipy.integrate.quad(lambda x : (pdf.xfxQ(id, x, q)-pdf.xfxQ(-id, x, q))/x, 1e-6, 1, limit=100, epsrel=1e-3)
    #print(int_res)
    res.append( (label, int_res[0]) )
return res
```

d	0.9867274422876253
u	1.9924213767549757
s	0.003708032343248529
c	-0.00018756880668405516
b	-8.819432616746102e-05

4 Momentum sum rules

The parton a carries a momentum fraction x_a of the parent hadron, $p_a^\mu = x_a P^\mu$. Therefore, the momentum density associated with that parton is given by $x_a f_{a|H}(x_a)$. Since the sum over all parton momenta must sum back up to the parent hadron one, the PDF sets satisfy a *momentum sum rule* (in $\overline{\text{MS}}$):

$$\sum_a \int_0^1 dx_a x_a f_{a|H}(x_a) = 1 \quad (3)$$

Let's see how the momenta are distributed across different flavours

```
import lhpdf
import math
import scipy
lhpdf.setVerbosity(0)
pdf = lhpdf.mkPDF("PDF4LHC21_40", 0)
res = list()
int_sum = [0.,0.]
for id,label in [(-5,"bb"),(-4,"cb"),(-3,"sb"),(-2,"ub"),(-1,"db"),(0,"g"),(1,"d"),(2,"u"),(3,"s"),(4,"c"),(5,"b")]:
    int_res = scipy.integrate.quad(lambda x : pdf.xfxQ(id, x, q), 1e-6, 1, limit=100, epsrel=1e-3)
    int_sum[0] += int_res[0]
    int_sum[1] += int_res[1]**2
    #print(int_res)
    res.append( (label, "{:.0f}%".format(int_res[0]*100.)) )
res.append( ("SUM", "{:.0f}%".format(int_sum[0]*100.)) )
return res
```

So the gluon actually carries almost 50% of the proton's momentum! The up quark, with $\sim 20\%$, has the second largest contribution, followed by the down-quark \sim half the size of the up (which makes sense as $p=(uud)$).

bb	1%
cb	2%
sb	3%
ub	4%
db	4%
g	47%
d	11%
u	22%
s	3%
c	2%
b	1%
SUM	100%

- Vary the scale and see how the momentum composition of the proton changes. How robust are the numbers?

5 Comparison of PDF sets

todo