Parton Distribution Functions

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1 Introduction

We will call the LHAPDF library from python to evaluate parton distribution functions and play around with them a bit.

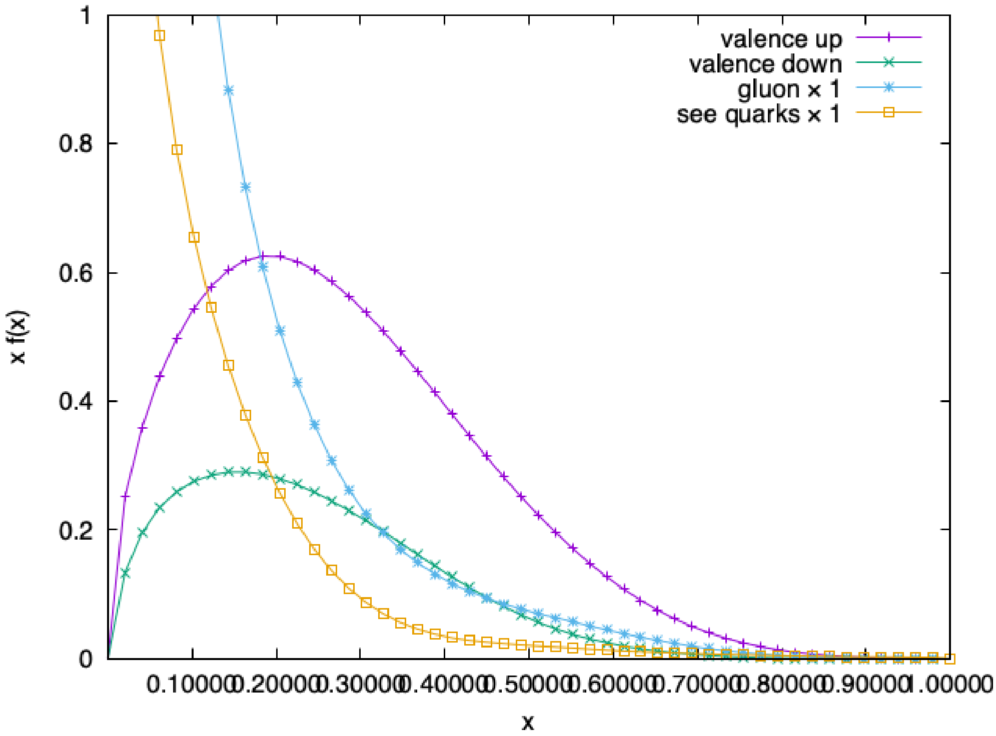
2 Evaluating PDFs

Let's have a look at one specific PDF set `PDF4LHC21_40`

```
import lhapdf
import numpy as np

lhapdf.setVerbosity(0)
pdf = lhapdf.mkPDF("PDF4LHC21_40", 0)
if qlog:
    xs = [x for x in np.logspace(-5, 0, 50)]
else:
    xs = [x for x in np.linspace(0, 1, 50)]
res = np.empty([len(xs),6])
for ix, x in enumerate(xs):
    fac = 1. #1./x
    res[ix,0] = x
    res[ix,1] = q
    res[ix,2] = fac*(pdf.xfxQ(2, x, q) - pdf.xfxQ(-2, x, q))  # valence up-quark
    res[ix,3] = fac*(pdf.xfxQ(1, x, q) - pdf.xfxQ(-1, x, q))  # valence down-quark
    res[ix,4] = fac*(pdf.xfxQ(0, x, q))  # gluon (or `21`)
```

```
    res[ix,5] = fac*(
         2.*pdf.xfxQ(-2, x, q)
        +2.*pdf.xfxQ(-1, x, q)
        +pdf.xfxQ(3, x, q)+pdf.xfxQ(-3, x, q)
        +pdf.xfxQ(4, x, q)+pdf.xfxQ(-4, x, q)
        +pdf.xfxQ(5, x, q)+pdf.xfxQ(-5, x, q)
    )  # sum over all see quarks
return res.tolist()
```



- Change the scale in the figure from 10 to 100 GeV or even to 1 TeV; adjust the scaling factor as necessary. How does the distributions change? Does it correspond to the expectation of the QCD improved parton model?

3 The quantum numbers of the proton

The quantum numbers of the proton are determined by the valence quarks, p=(uud). Given that PDFs are *number densities* for the constituent partons, the following flavour sum rules hold:

$$\int_0^1 \mathrm{d}x \Big(f_{\mathrm{u|p}}(x) - f_{\bar{\mathrm{u}}\mathrm{|p}}(x) \Big) = 2, \qquad \int_0^1 \mathrm{d}x \Big(f_{\mathrm{d|p}}(x) - f_{\bar{\mathrm{d}}\mathrm{|p}}(x) \Big) = 1, \tag{1}$$

$$\int_0^1 \mathrm{d}x \Big(f_{\mathrm{q|p}}(x) - f_{\bar{\mathrm{q}}\mathrm{|p}}(x) \Big) = 0 \quad \forall q \notin \{\mathrm{u},\, \mathrm{d}\} \tag{2}$$

Let's see if these hold for one of the global PDF sets:

```
import lhapdf
import math
import scipy
lhapdf.setVerbosity(0)
pdf = lhapdf.mkPDF("PDF4LHC21_40", 0)
res = list()
for id,label in [(1,"d"),(2,"u"),(3,"s"),(4,"c"),(5,"b")]:
    int_res = scipy.integrate.quad(lambda x : (pdf.xfxQ(id, x, q)-pdf.xfxQ(-id, x, q))/x, 1e-6, 1, limit=100, epsrel=1e-
    #print(int_res)
    res.append( (label, int_res[0]) )
return res
```

d	0.98672744228762 53
u	1.9924213767549757
s	0.003708032343248529
c	-0.00018756880668405516
b	-8.819432616746102e-05

4 Momentum sum rules

The parton a carries a momentum fraction x_a of the parent hadron, $p_a^\mu = x_a\, P^\mu$. Therefore, the momentum density associated with that parton is given by $x_a\, f_{a|H}(x_a)$. Since the sum over all parton momenta must sum back up to the parent hadron one, the PDF sets satisfy a *momentum sum rule* (in $\overline{\text{MS}}$):

$$\sum_a \int_0^1 \mathrm{d}x_a\; x_a\, f_{a|H}(x_a) = 1 \tag{3}$$

Let's see how the momenta are distributed across different flavours

```
import lhapdf
import math
import scipy
lhapdf.setVerbosity(0)
pdf = lhapdf.mkPDF("PDF4LHC21_40", 0)
res = list()
int_sum = [0.,0.]
for id,label in [(-5,"bb"),(-4,"cb"),(-3,"sb"),(-2,"ub"),(-1,"db"),(0,"g"),(1,"d"),(2,"u"),(3,"s"),(4,"c"),(5,"b")]:
    int_res = scipy.integrate.quad(lambda x : pdf.xfxQ(id, x, q), 1e-6, 1, limit=100, epsrel=1e-3)
    int_sum[0] += int_res[0]
    int_sum[1] += int_res[1]**2
    #print(int_res)
    res.append( (label, "{:.0f}%".format(int_res[0]*100.)) )
res.append( ("SUM", "{:.0f}%".format(int_sum[0]*100.)) )
return res
```

So the gluon actually carries almost 50% of the proton's momentum! The up quark, with $\sim$ 20%, has the second largest contribution, followed by the down-quark $\sim$half the size of the up (which makes sense as p=(uud)).

bb	1%
cb	2%
sb	3%
ub	4%
db	4%
g	47%
d	11%
u	22%
s	3%
c	2%
b	1%
SUM	100%

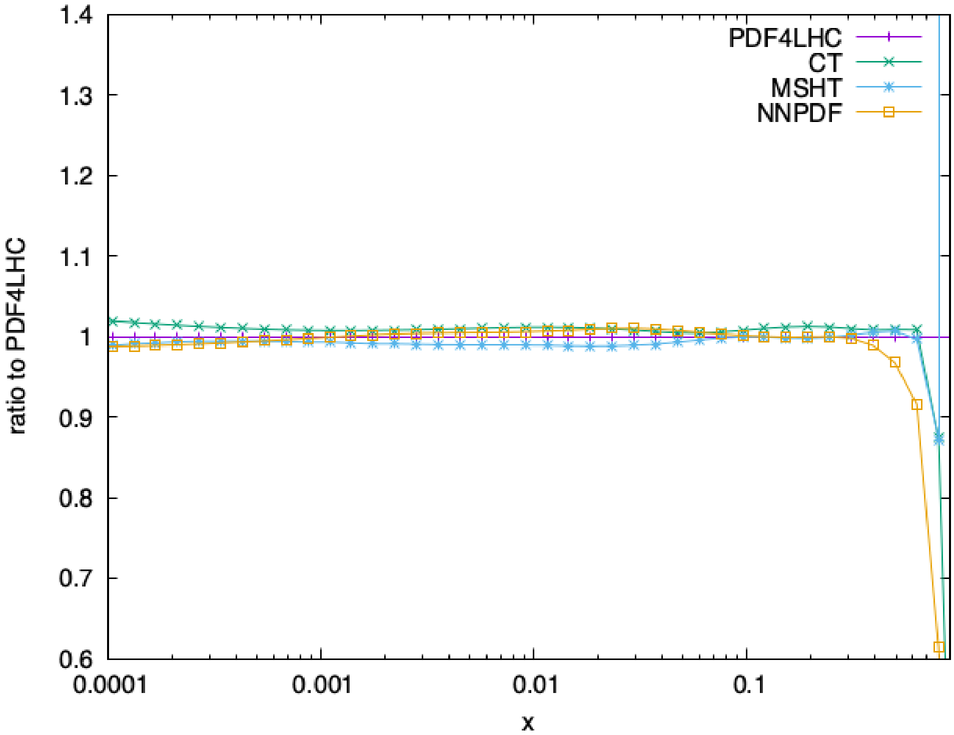
- Vary the scale and see how the momentum composition of the proton changes. How robust are the numbers?

5 Comparison of different PDF sets

In order to get a sense on how well these PDFs are constrained, it is useful to compare the PDF sets as determined by different groups.

```
import lhapdf
import numpy as np

lhapdf.setVerbosity(0)
pdfs = [ lhapdf.mkPDF(pdf_name, 0) for pdf_name in ["PDF4LHC21_40", "CT18NNLO", "MSHT20nnlo_as118", "NNPDF40_nnlo_as_011
if qlog:
    xs = [x for x in np.logspace(-5, 0, 50)]
else:
    xs = [x for x in np.linspace(0, 1, 50)]
res = np.empty([len(xs),2+len(pdfs)])
for ix, x in enumerate(xs):
    res[ix,0] = x
    res[ix,1] = q
    for ip, pdf in enumerate(pdfs):
        res[ix,2+ip] = pdf.xfxQ(a, x, q)
return res.tolist()
```



Overall, the PDFs are determined at the level of a few percent in the intermediate \$x\$-range.

- Try changing the parton type in the above plot.
- The PDF sets also come with error sets to determine the error of the PDF sets. Try to adjust the above script to add error bands for the indivudial PDFs.