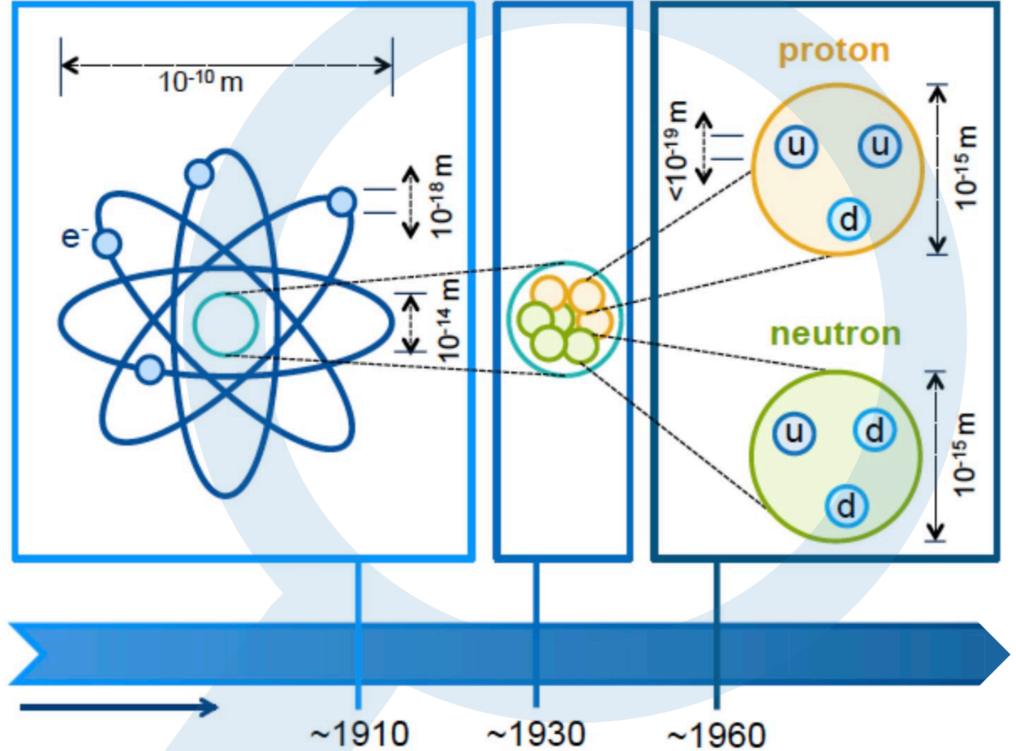


# MAKING PREDICTIONS AT HADRON COLLIDERS

Alexander Huss

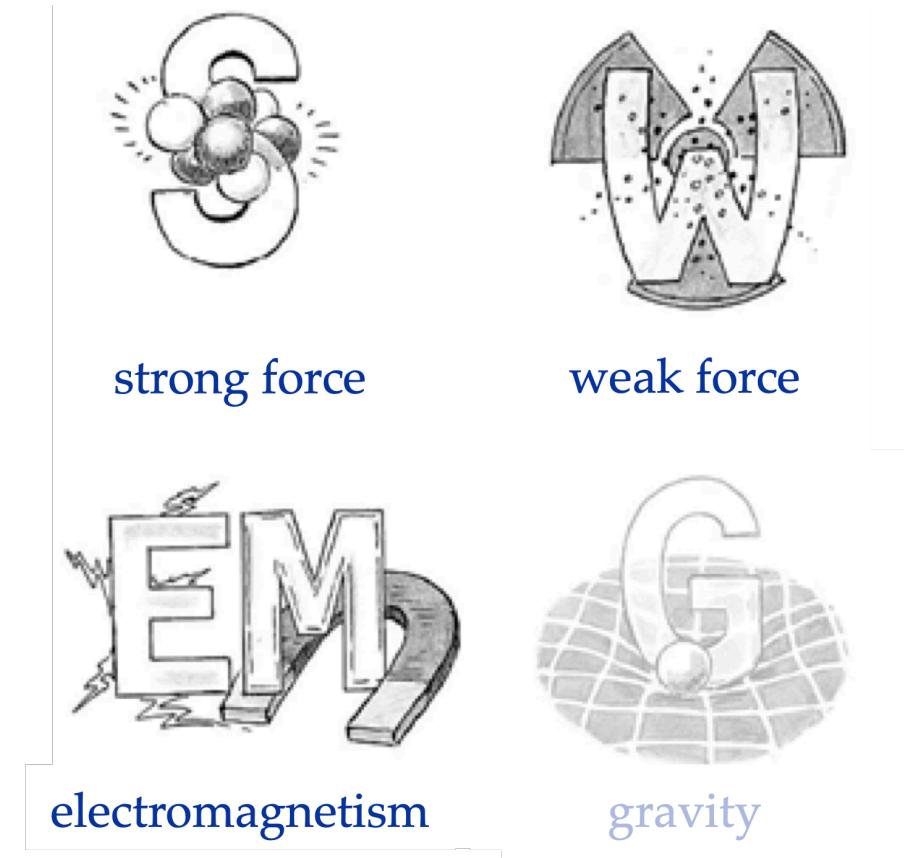
alexander.huss @ cern.ch

What is the  
Universe made of?



Elementary Particles

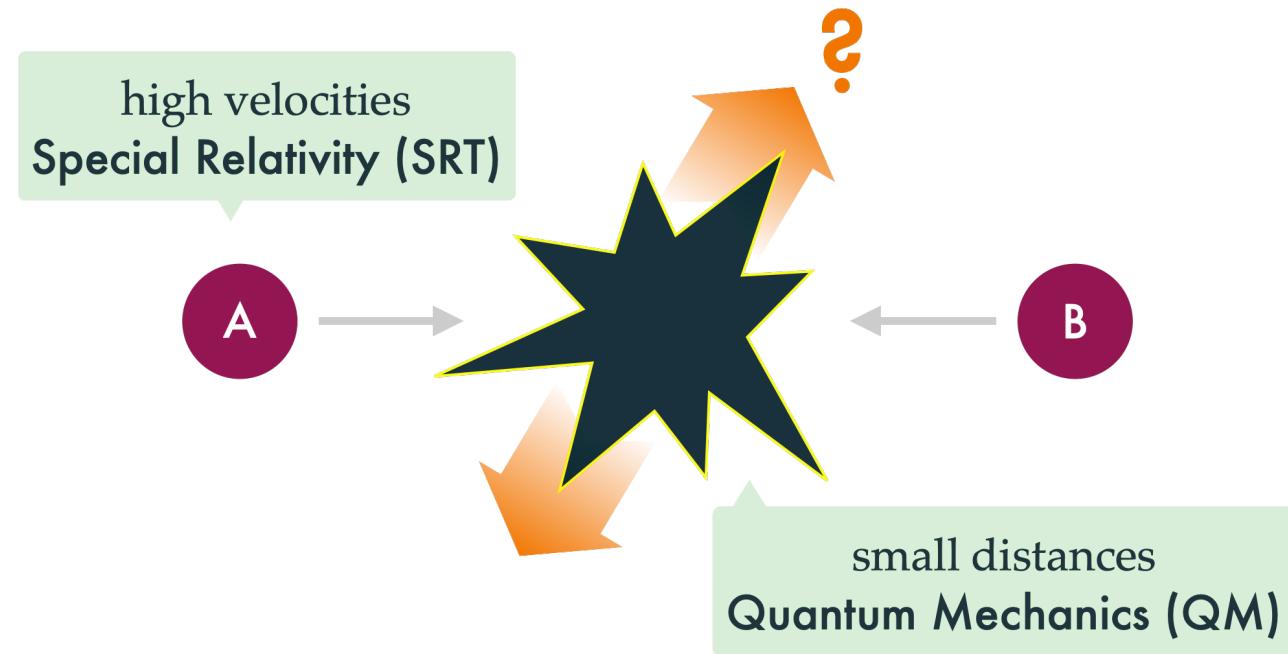
What are the  
Laws of Nature?



Fundamental Forces

# Particle Smashers – Our Microscopes

• <b>LEP</b>	@ CERN	[→ 1999]
↪ $e^+e^-$	90 ↗ 200 GeV	○ 27 km
• <b>HERA</b>	@ DESY	[→ 2008]
↪ $ep$	320 GeV	○ 6.3 km
• <b>Tevatron</b>	@ Fermilab	[→ 2011]
↪ $p\bar{p}$	1 TeV	○ 6.28 km
• <b>LHC</b>	@ CERN	[→ now]
↪ $pp$	7 ↗ 13.6 TeV	○ 27 km
• <b>EIC</b>	@ BNL	[2032 →]
↪ $ep$	20 ↗ 140 GeV	○ 3.8 km
• <b>FCC-ee</b>	@ CERN	[2040? →]
↪ $e^+e^-$	90 ↗ 360 GeV	○ 100 km
• <b>FCC-hh</b>	@ CERN	[2060? →]
↪ $pp$	100 TeV	○ 100 km



# Particle Smashers – Our Microscopes

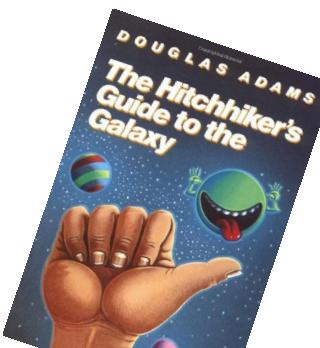
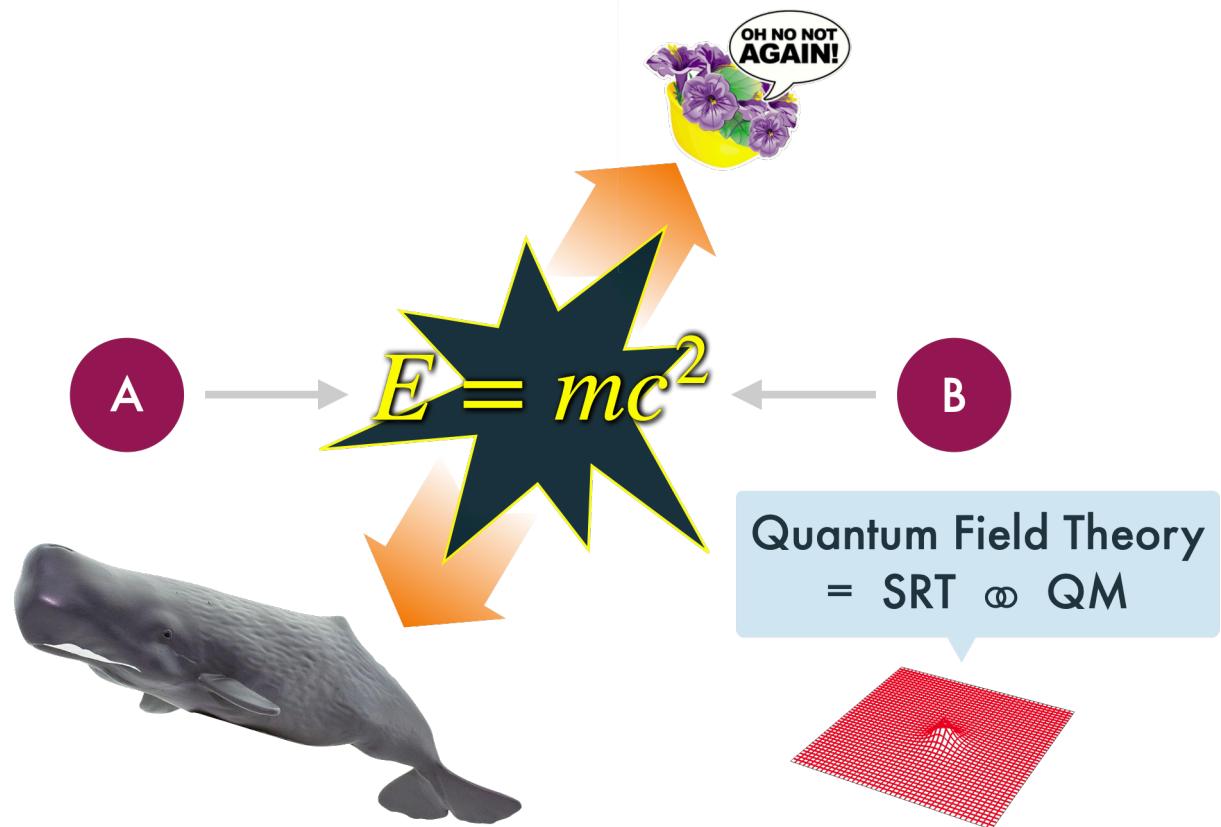
- \* high energies

  - ↳ small distances  $\lambda = \frac{hc}{E}$

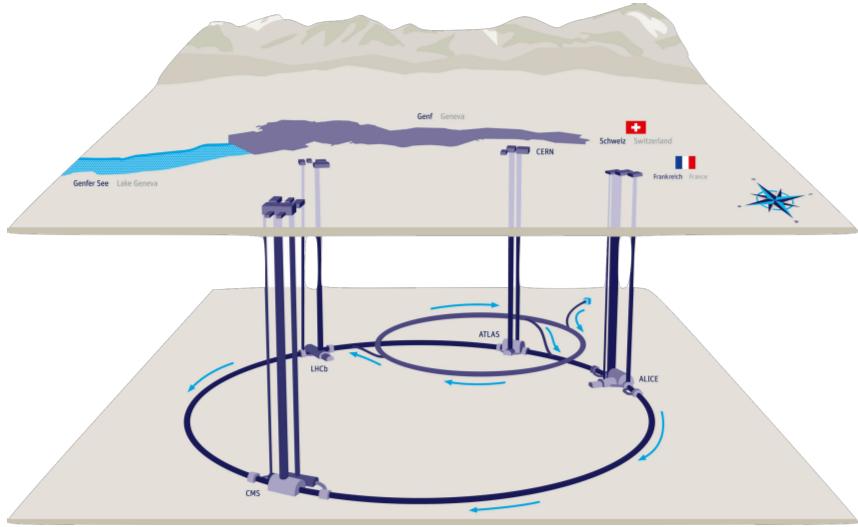
  - ↳ produce heavy (= interesting!) final states

- \* many collisions

  - ↳ access to rare (= interesting!) events & processes



# Particle Smashers – The LHC



\* energy frontier

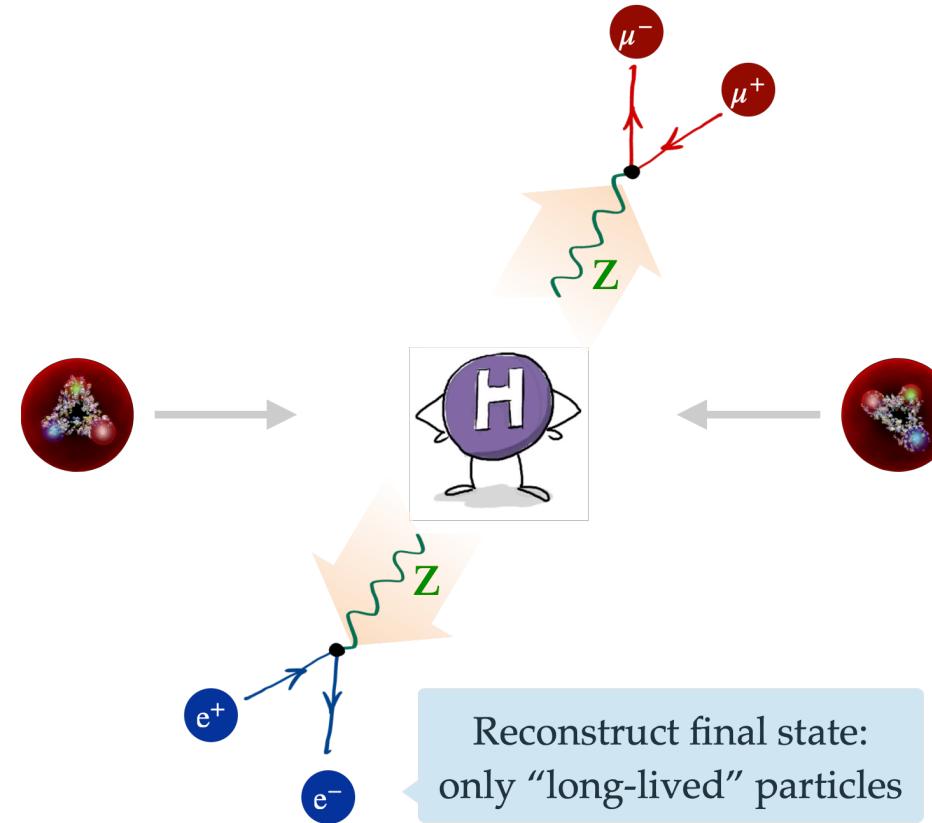
↳ 13.6 TeV collision energy

↳ 1000 x distance to moon (  )

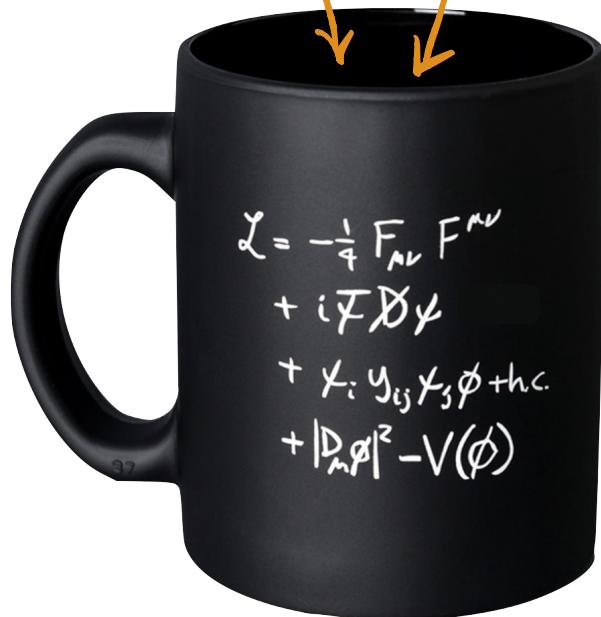
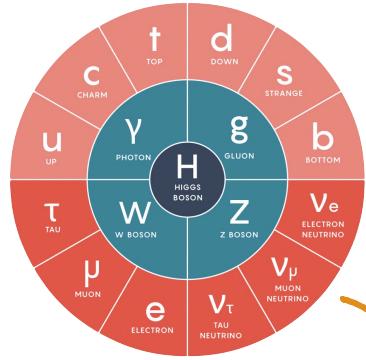
\* intensity frontier

↳  $10^9$  collisions / second

↳ 30 PB/year



# THEORY

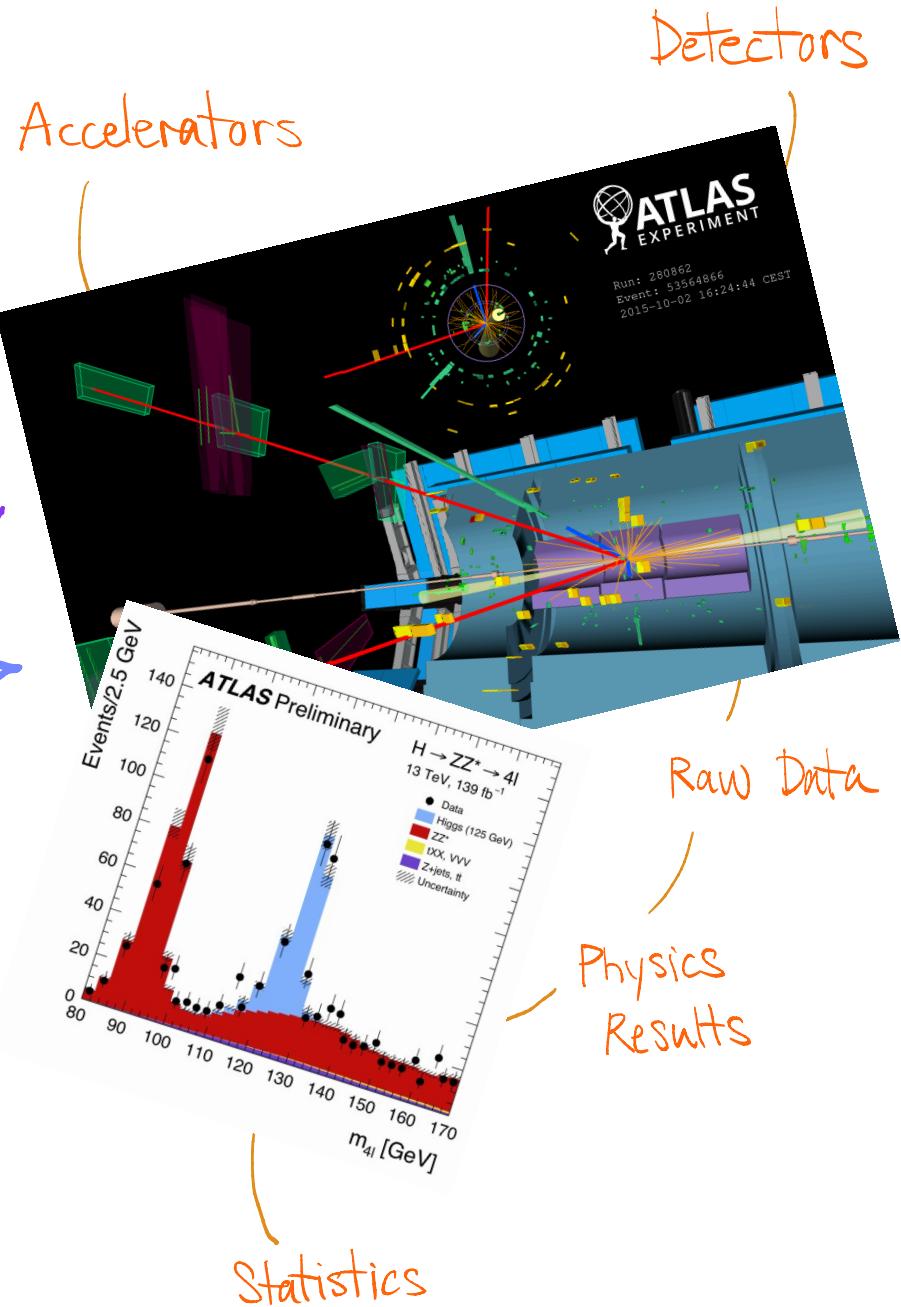


Gauge interactions  
 $SU(3)_c \times SU(2)_L \times U(1)_Y$

# PHENOMENOLOGY

this lecture

[Particle World, Theoretical Concepts, The SM]



# EXPERIMENT

# Repository & Conventions

- \* Notebooks for demonstrations
  - ↳ <https://github.com/aykhuss/Lectures-SSL-MkPred>
- \* Conventions natural units:  $[h] = [c] = 1$  (remember:  $m_{\text{proton}} \sim 1 \text{ GeV}$ )
  - ↳  $[ \text{length} ] = [ \text{time} ] = eV^{-1}$
  - $[ \text{mass} ] = [ \text{energy} ] = [ \text{momentum} ] = eV$
  - ↳ four vectors  $x^\mu = (t, x, y, z)^T$
  - $p^\mu = (E, \underbrace{p_x, p_y, p_z}_\vec{p})^T \rightsquigarrow \text{"on-shell": } p^2 = E^2 - |\vec{p}|^2 \stackrel{!}{=} m^2$ 

invariant mass
  - $\Rightarrow$  energy-momentum conservation:  $(a+b \rightarrow 1+2)$ 
$$\delta^{(4)}(p_1 + p_2 - (p_a + p_b)) = \delta(E_1 + E_2 - (E_a + E_b)) \delta^{(3)}(\vec{p}_1 + \vec{p}_2 - (\vec{p}_a + \vec{p}_b))$$

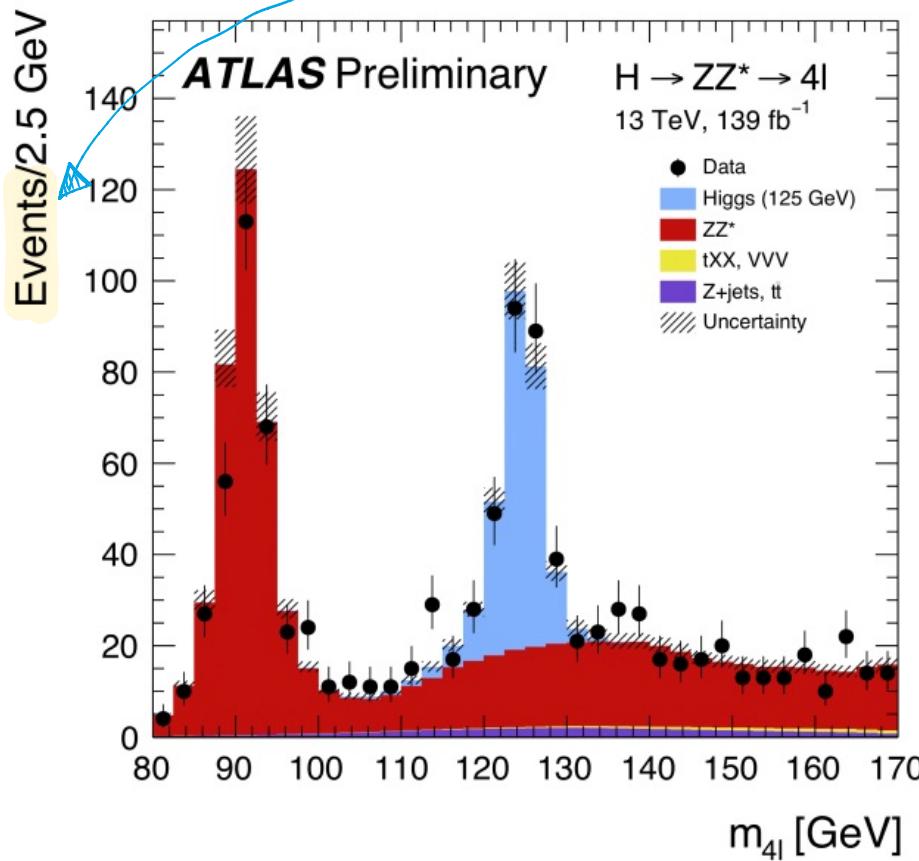
# The Plan

1. Event Rates , Cross Sections & Scattering Amplitudes
2. Warmup: Lepton Collider
3. Hadron Colliders – Parton Distribution Functions
4. The Drell-Yan Process
5. Higher-Order Corrections
6. QCD Jets, Parton Showers & MC Simulations

# Event Rates

We ultimately measure # Events

for a specific process:  $a+b \rightarrow 1+2+\dots+n$



Luminosity  
~# collisions

cross section

$$dN = L d\sigma$$

$$* \sigma_H (13 \text{ TeV}) \approx 50 \text{ pb}$$

$$\int_{\text{Run2}} dt \mathcal{L} \approx 150 \text{ fb}^{-1}$$

$$* \sigma_Z (13 \text{ TeV}) \approx 50 \text{ nb}$$

$$\sigma_{W^\pm} (13 \text{ TeV}) \approx 200 \text{ nb}$$

$$\mathcal{L} (\text{instantaneous}) \approx 0.02 \text{ pb}^{-1} \text{ s}^{-1}$$

$\sim 7 \text{ million}$   
Higgs bosons produced!

$\sim 1000$  Z's

$\sim 4000$  W<sup>+</sup>'s

every  
second!

# Calculating Cross Sections

Fermi's Golden Rule  $a+b \rightarrow 1+2+\dots+n$

$$d\sigma = \frac{1}{F} \underbrace{\langle |M|^2 \rangle}_{\text{flux}} \underbrace{|M|^2}_{\text{amplitude}^2} \underbrace{d\Phi}_{\text{phase space (LIPS)}}$$

$$= \frac{1}{4(P_a \cdot P_b)} = \frac{1}{2E_{cm}^2}$$

$$= \frac{1}{n_a^{\text{d.o.f.}} n_b^{\text{d.o.f.}}} \sum_{\text{d.o.f.}} |M|^2$$

(degrees of freedom)  
spin, colour

$$d\Phi_n(p_1, \dots, p_n; P_a, P_b)$$

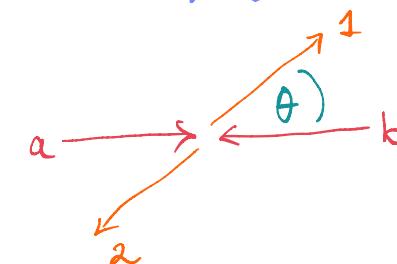
$$= \prod_{i=1}^n \frac{d^4 p_i}{(2\pi)^4} (2\pi) \delta(p_i^2 - m_i^2) \Theta(R^a)$$

$$(2\pi)^4 \delta^{(4)}(p_1 + \dots + p_n - (P_a + P_b))$$

energy-momentum  
conservation

Special case  $a+b \rightarrow 1+2$

$$d\Phi_2 = \frac{d\cos\theta}{16\pi} \quad (\text{massless})$$



# The Scattering Amplitude

→ evaluation of the path integral  
(analogy to QM)

↔ extremely difficult to solve  
except a free theory

↳  $\mathcal{L}$  only has terms with at most two fields  $\Phi$ , e.g.  $\bar{\psi}(x) (i\cancel{\partial} - m) \psi(x)$

\* Feynman Rules for the free theory

incoming

$$f \xrightarrow{p} \bullet = u(p)$$

$$\bar{f} \xleftarrow{p} \bullet = \bar{u}(p)$$

outgoing

$$\bullet \xrightarrow{p} f = \bar{u}(p)$$

$$\bullet \xleftarrow{p} \bar{f} = v(p)$$

propagators

$$\bullet \xrightarrow{p} \bullet = \frac{i}{p - m}$$

$$\bullet \xrightarrow{p} \bullet = \frac{-i g_{\mu\nu}}{p^2 - M_Z^2 + i M_Z \Gamma_Z}$$

$$Z[J] = \int D[\Phi] e^{i \int d^4x [L(\Phi, \partial_\mu \Phi) + J(x) \bar{\Phi}(x)]}$$

$\Phi \in \{\psi, \phi, A_\mu, \dots\}$

boring "scattering"  
 $a+b \rightarrow a+b$

$a$  —————  $a$   
 $b$  —————  $b$

# Perturbation Theory

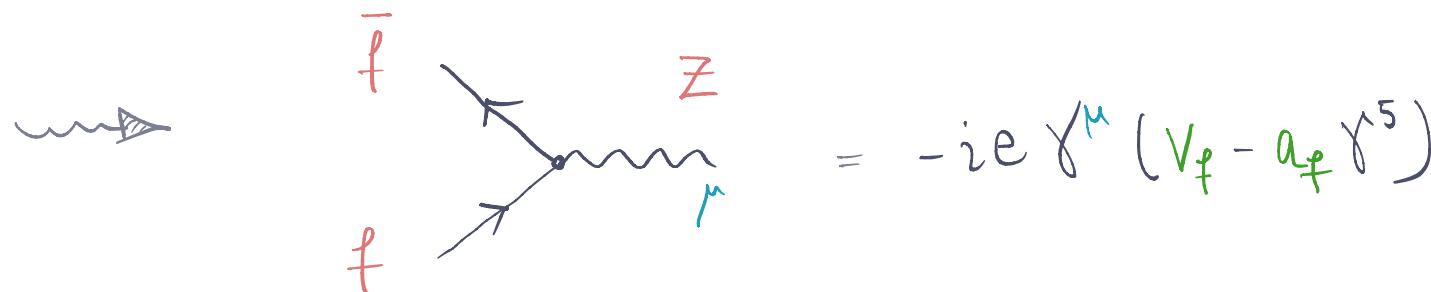
When interaction strength is small ( $\alpha_{em} \sim 1/137$ ,  $\alpha_s(M_Z^2) \sim 0.118$ )

→ compute  $M$  perturbatively by expanding around free theory

\* Feynman rules for interactions - vertices

direct correspondence\* with terms in  $\mathcal{L}$

$$-e Z_\mu \bar{\psi}_f \gamma^\mu (V_f - a_f \gamma^5) \psi_f \in \mathcal{L}$$



\* more subtle than just "dropping" the fields when derivatives ( $\partial_\mu$ ) and/or identical particles

## Warmup: Lepton Collider

Consider the process  $e^+ e^- \rightarrow \mu^+ \mu^-$

at lowest order (tree level). There are two diagrams

What are they?

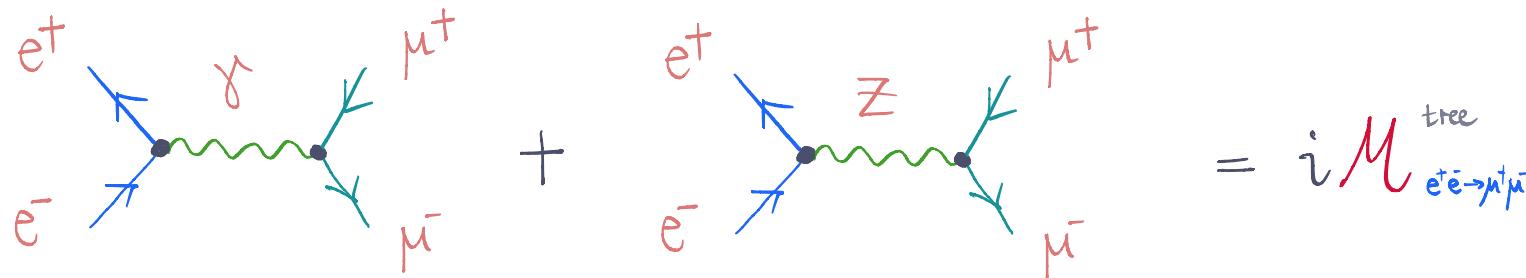
[demo: FeynGame]

# Warmup: Lepton Collider

[demo:  $e^+e^- \rightarrow \mu^+\mu^-$ ]

Consider the process  $e^+e^- \rightarrow \mu^+\mu^-$

at lowest order (tree level). There are two diagrams



$\Rightarrow$  Inserting into Fermi's golden rule  $[S = E_{\text{cm}}^2; p_a \cdot p_b = p_a^\mu p_{b,\mu} = E_{\text{cm}}^2 (1 - \cos\theta)]$

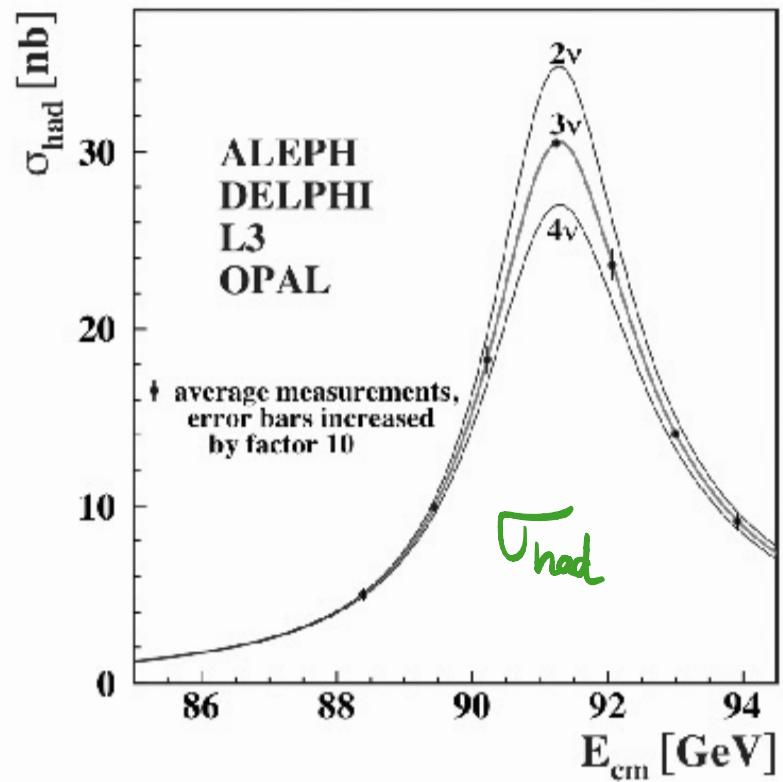
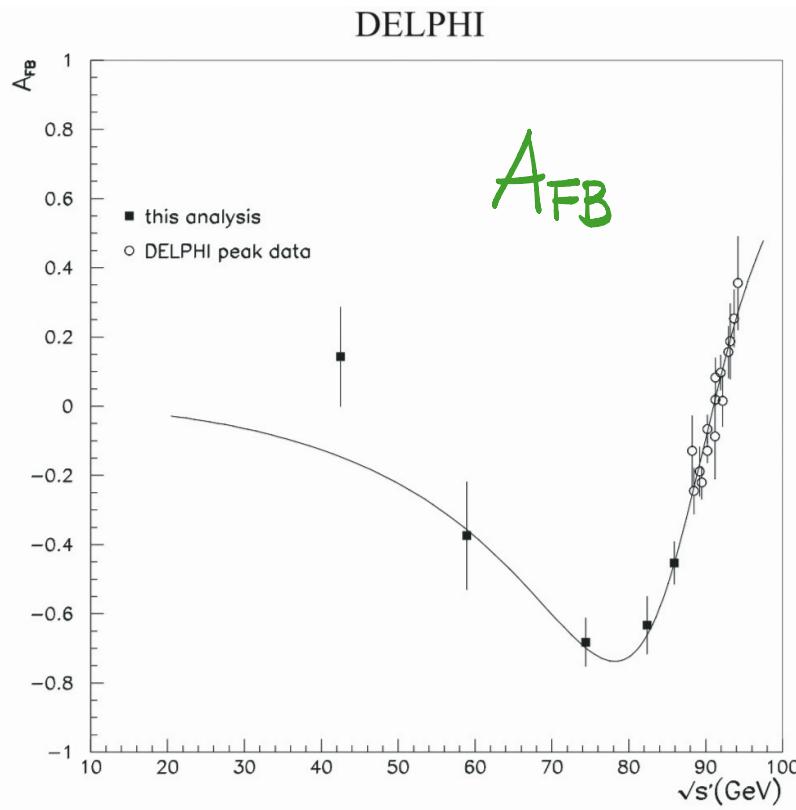
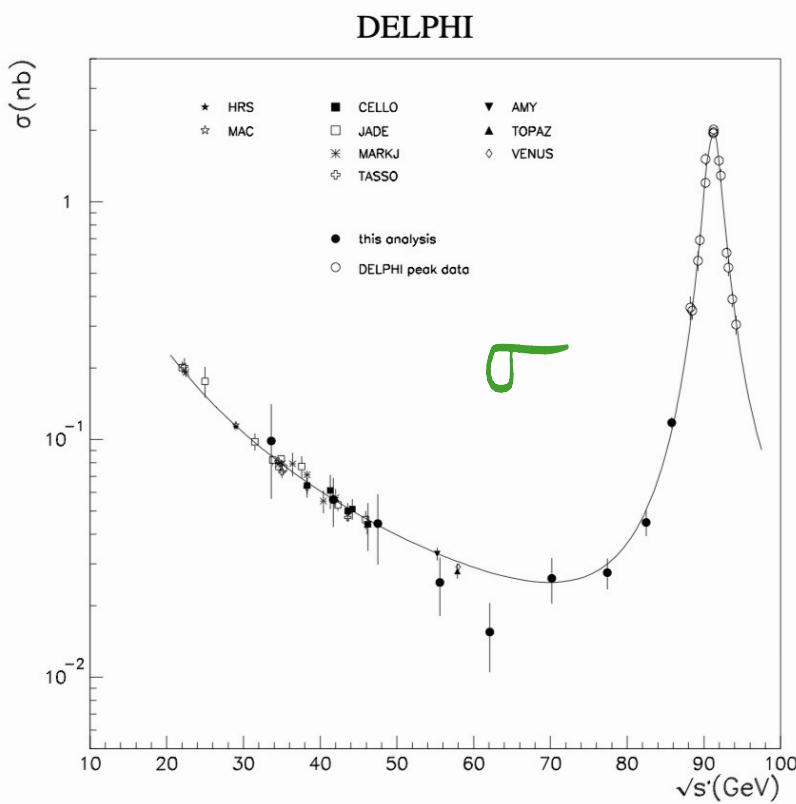
$$\frac{d\sigma}{d\cos\theta} = \frac{\alpha^2 \pi}{2S} \left[ (1 + \cos^2\theta) G_1(s) + 2 \cos\theta G_2(s) \right]$$

interference!

$$G_1(s) = 1 + 2V_L^2 \operatorname{Re} \left\{ \frac{s}{s - M_Z^2 + iM_Z T_Z} \right\} + (V_L^2 + a_L^2) \left| \frac{s}{s - M_Z^2 + iM_Z T_Z} \right|^2$$

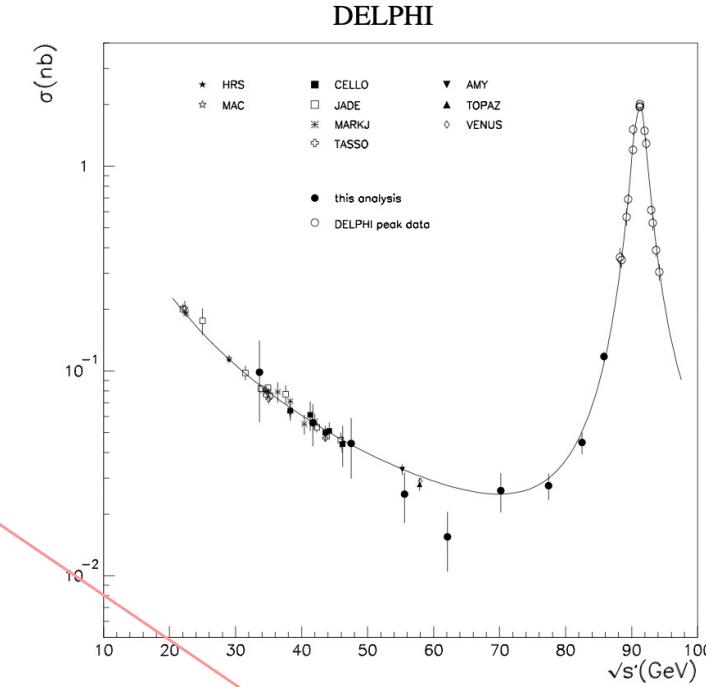
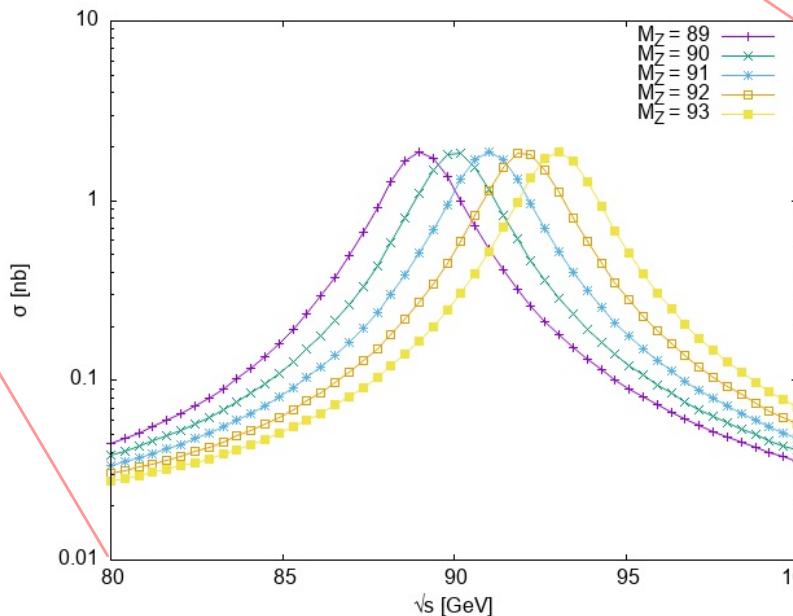
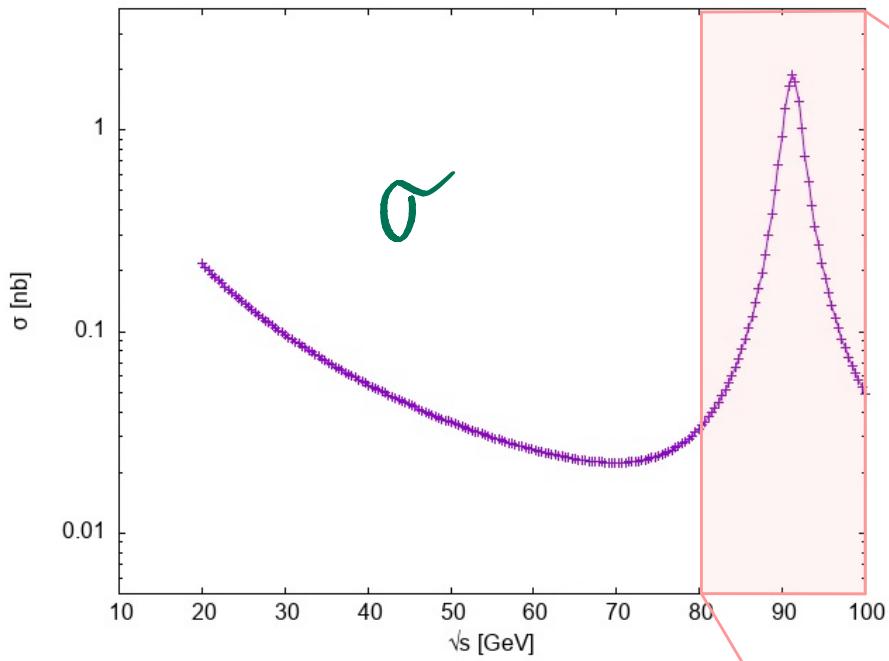
$$G_2(s) = 0 + 2a_L^2 \operatorname{Re} \left\{ \frac{s}{s - M_Z^2 + iM_Z T_Z} \right\} + 4V_L^2 \cdot a_L^2 \left| \frac{s}{s - M_Z^2 + iM_Z T_Z} \right|^2$$

# "Comparison" to Data



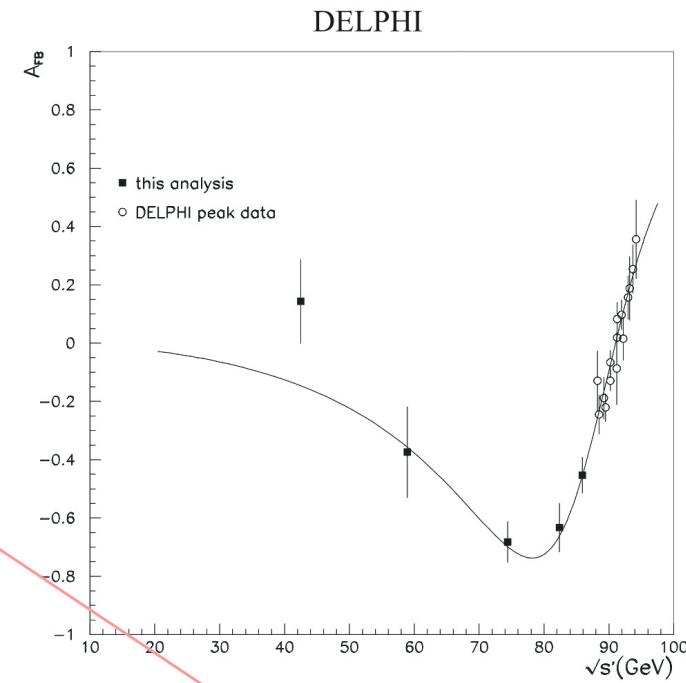
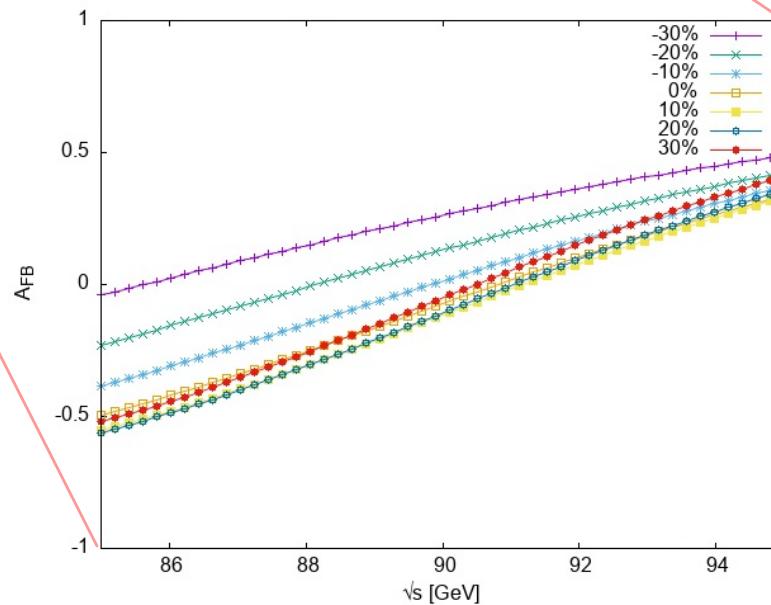
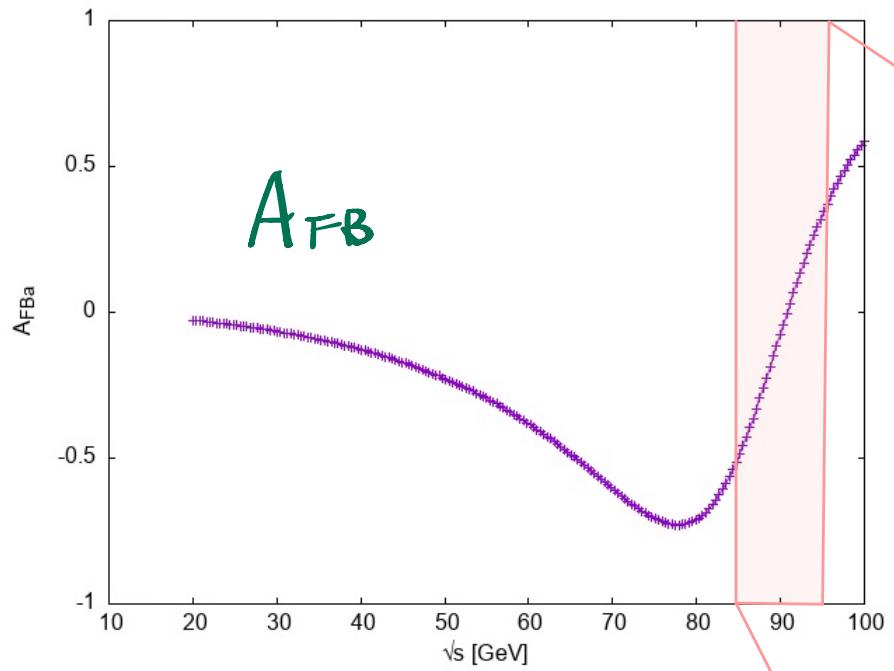
- \* In principle, you now can use the predictions to fit  $M_Z$  &  $\sin^2 \theta_W$  from the data (at leading order)
- \*  $\sigma_{\text{had}}$  is the hadronic cross section: @ LO:  $e^+e^- \rightarrow q\bar{q}$   
→ what changes compared to  $\mu^+\mu^-$ ?

# "Comparison" to Data



$M_Z$   
extraction

# "Comparison" to Data

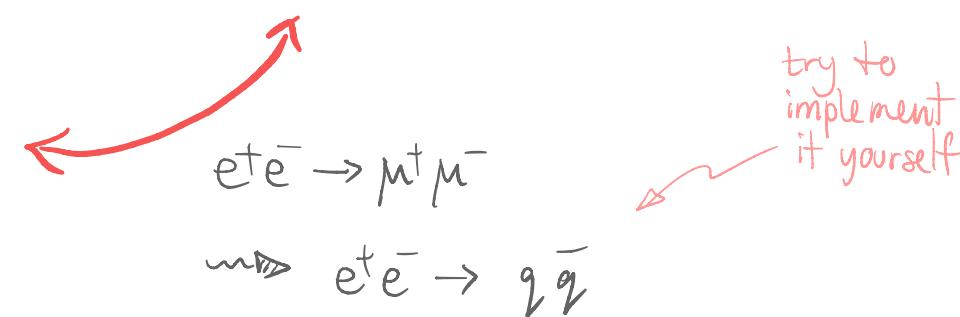
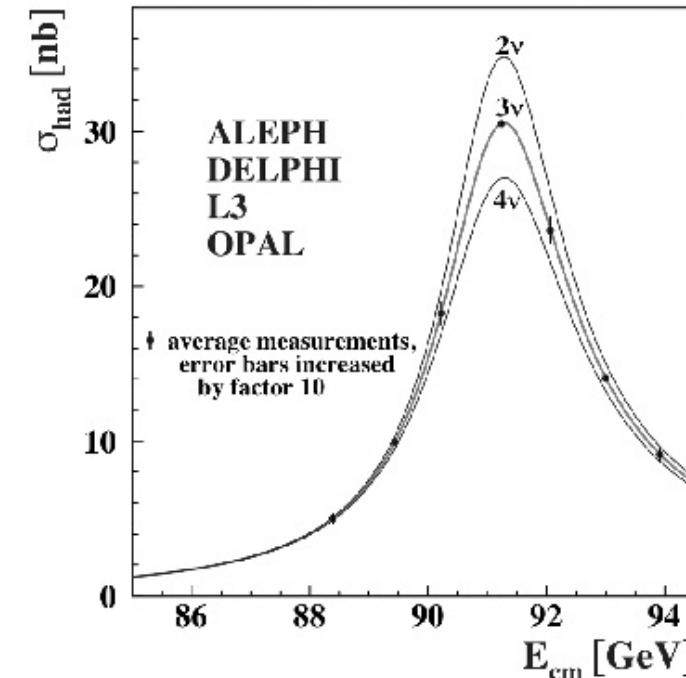
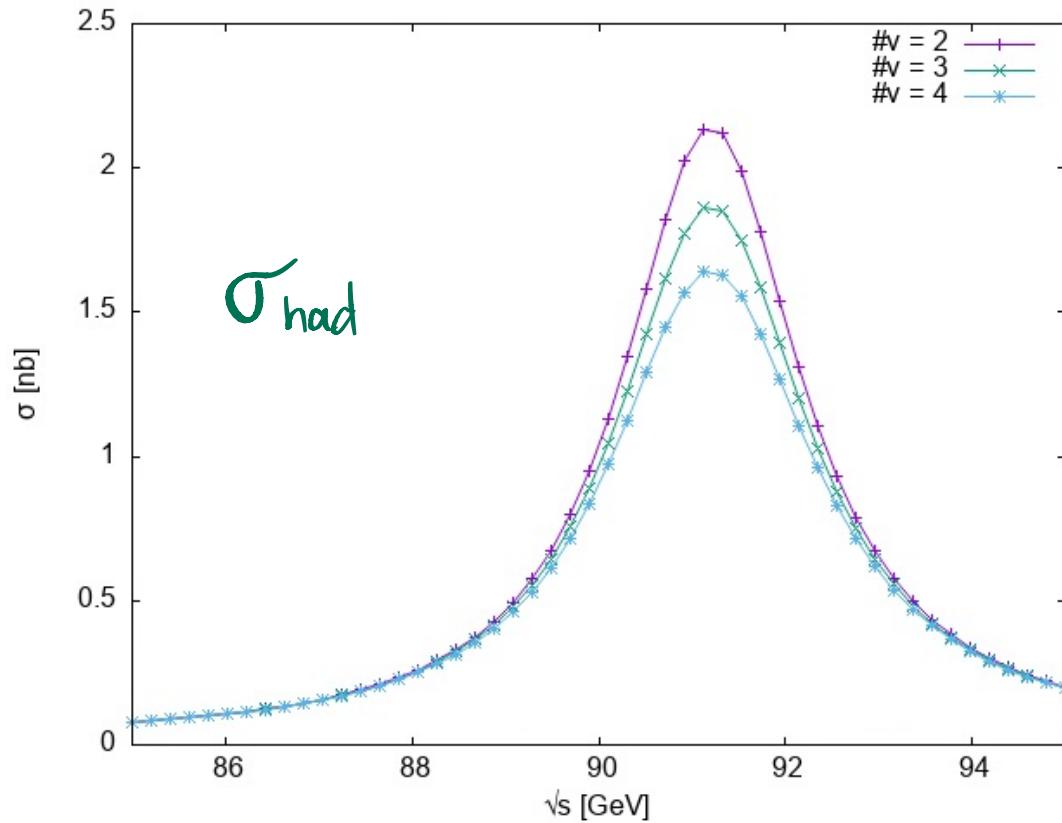


$\sin^2 \theta_W$   
extraction

test:

$$\sin^2 \theta_W = 1 - \frac{M_W^2}{M_Z^2}$$

# "Comparison" to Data



# generations = 3  
(with light neutrinos)

# Hadron Colliders ... ... are "messy"

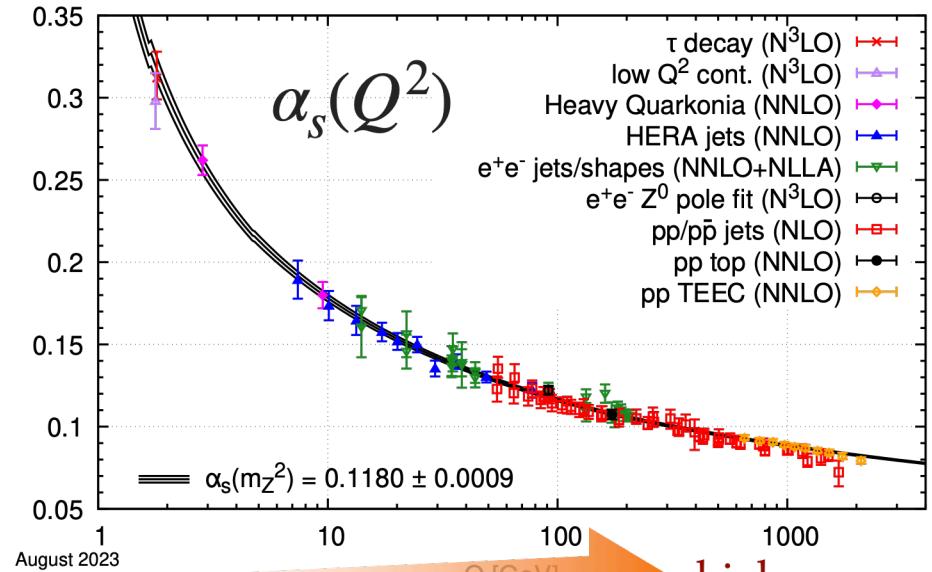
“ Quantum chromodynamics is conceptually simple. Its realization in nature, however, is usually very complex.”

Frank Wilczek

- \* no free quarks & gluons  
→ spray of hadrons ( $\pi^\pm, K^\pm, K^0, p^\pm, n, \dots$ )
- \* colliding objects (p @ LHC) not elementary

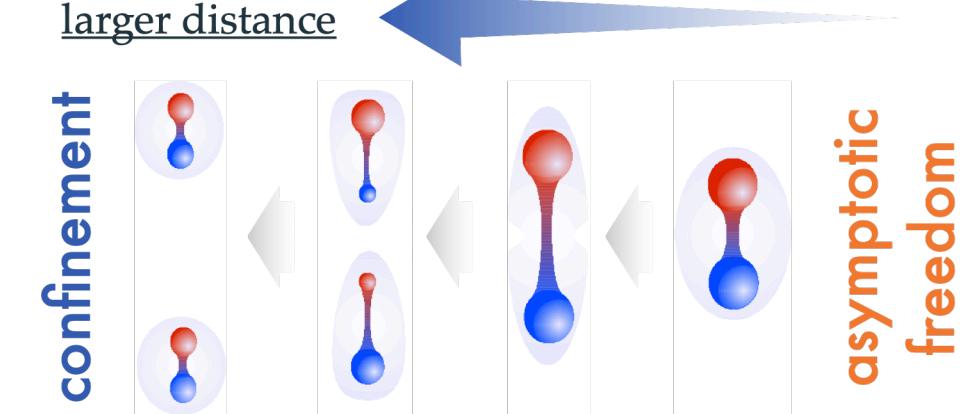
strong interaction ↔ 

Quantum Chromodynamics (QCD)



higher energy

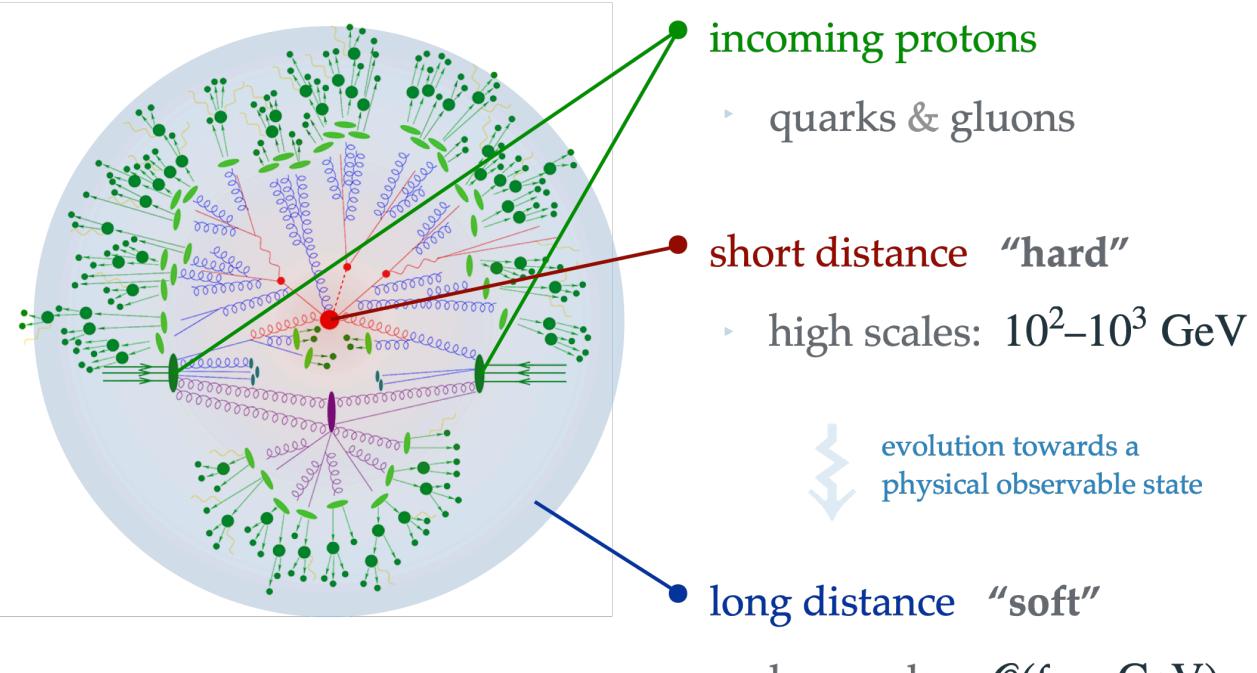
larger distance



# Hadron Colliders ... ... taming the mess

“ Quantum chromodynamics is conceptually simple. Its realization in nature, however, is usually very complex. But not always. ”

Frank Wilczek



## 1. factorization

↳ relevant physics at disparate scales  
(isolates description of proton from rest)

## 2. asymptotic freedom

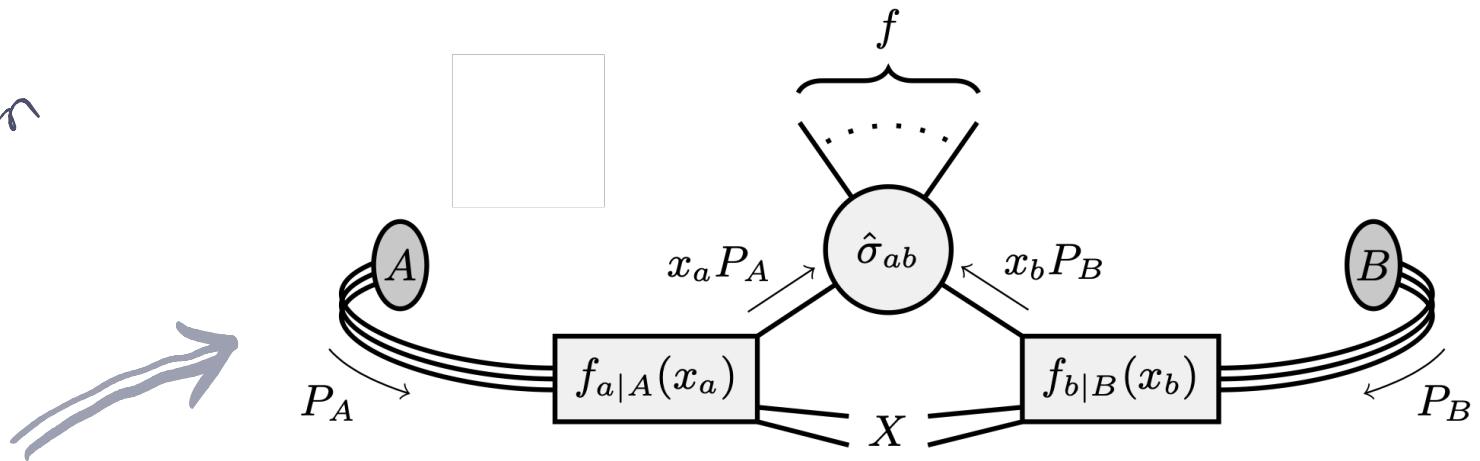
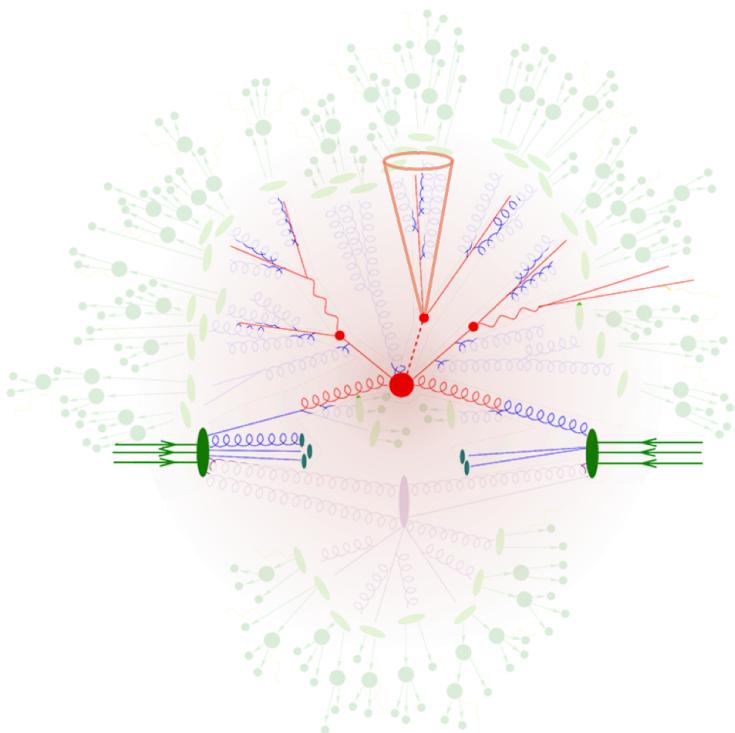
↳ short distance ↳ perturbation theory

$$\sigma = \sigma_{\text{lo}} (1 + \alpha_s c^{(1)} + \alpha_s^2 c^{(2)} + \dots)$$

# Hadron Colliders: The Parton Model

strategy for precision:

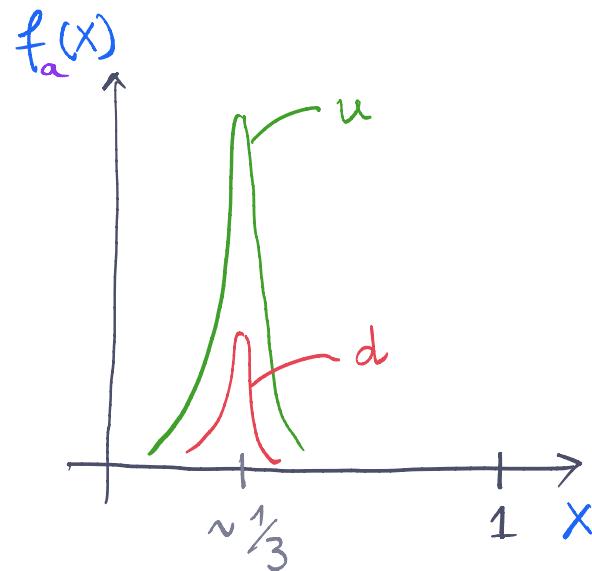
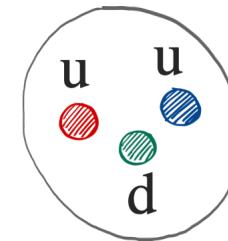
- \* focus on high momentum transfer
- \* clean signatures ( $\ell^\pm$ , jets, ...)



$$\begin{aligned}
 d\sigma_{A+B \rightarrow f} (P_A, P_B) &= \sum_{a,b} \int_0^1 dx_a \int_0^1 dx_b \, f_a(x_a) f_b(x_b) \, d\hat{\sigma}_{a+b \rightarrow f} (x_a P_A, x_b P_B) \\
 &\stackrel{\text{momentum fraction}}{\approx} \text{number density for parton } a \\
 &\quad \text{w/ momentum fraction} \\
 &\quad [x_a, x_a dx_a] \text{ of parent hadron A}
 \end{aligned}$$

# Parton Distribution Functions

\* just free quarks? ( $p \simeq (uud)$ )



$$f_u(x) \sim 2 \delta(x - \frac{1}{3})$$

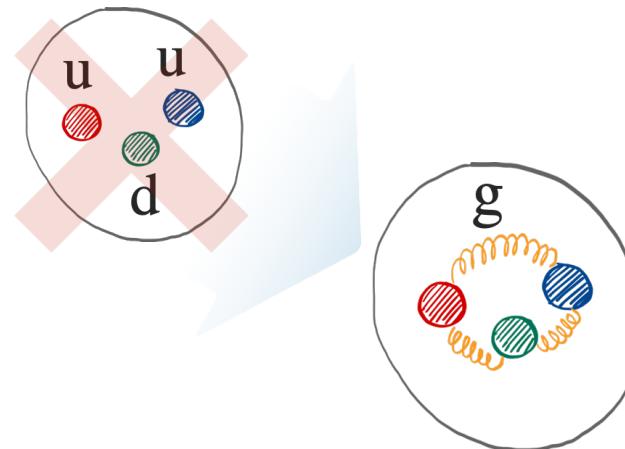
$$f_d(x) \sim 1 \delta(x - \frac{1}{3})$$

$$f_{\text{etc}}(x) \sim \emptyset$$

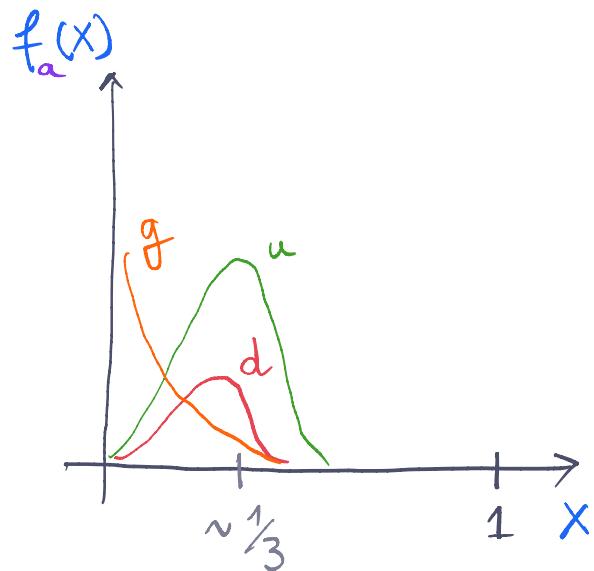
(+ some smearing)

# Parton Distribution Functions

\* just free quarks? ( $p \simeq (uud)$ )



\* bound by gluons?



naive parton model

$\leftrightarrow$  composition of point particles

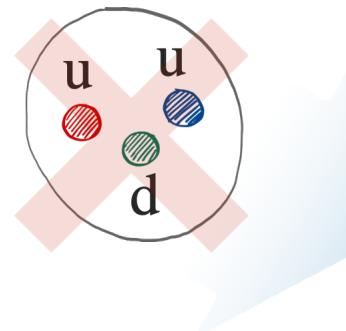
$\mapsto$  zoom in ( $Q^2 \uparrow$ )  $\leftrightarrow$  same composition: scaling

PDFs independent on scale, at which it is probed  
(as long as  $Q^2 \gg m_p^2$ )

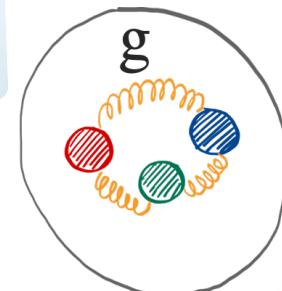
# Parton Distribution Functions

[demo: PDFs]

\* just free quarks? ( $p \simeq (uud)$ )

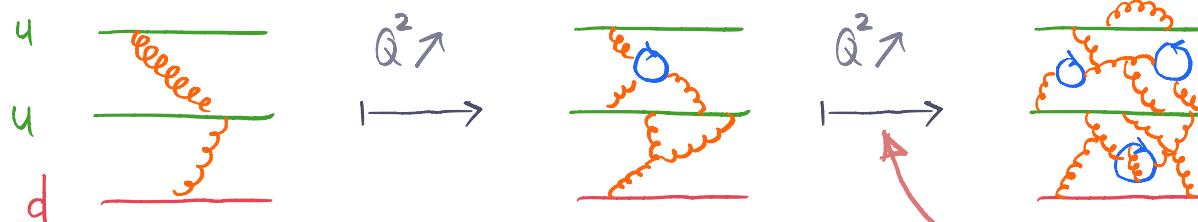


\* bound by gluons?



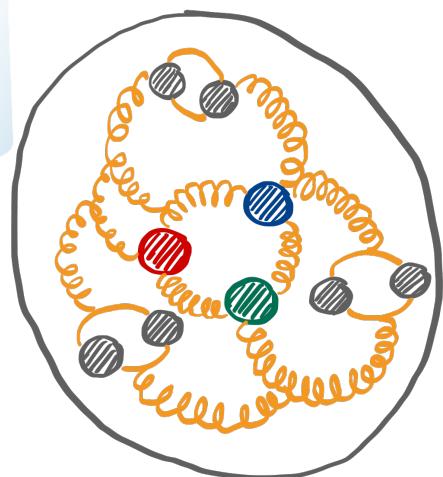
\* QCD - improved parton model

↳ quantum fluctuations  $\rightarrow$  more  $g$  &  $(g\bar{g})$  as we "zoom in" ( $Q^2 \uparrow$ )



$\Rightarrow$  predominantly shifts partons from high- $x$  to low- $x$

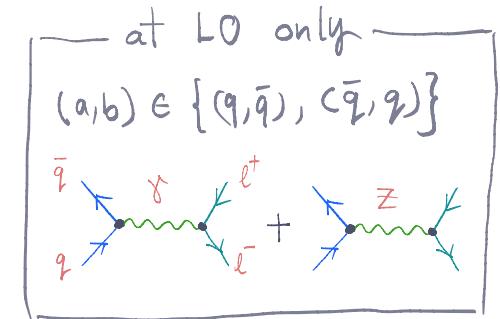
evolution is perturbatively calculable!  
(test of QCD)



# The Drell-Yan Process $P + P \rightarrow l + \bar{l}$

[demo: Drell-Yan]

$$d\sigma_{DY}(P_A, P_B) = \sum_{a,b} \int_0^1 dx_a \int_0^1 dx_b f_a(x_a) f_b(x_b) d\hat{\Gamma}_{a+b \rightarrow l^+ l^-}(x_a P_A, x_b P_B)$$



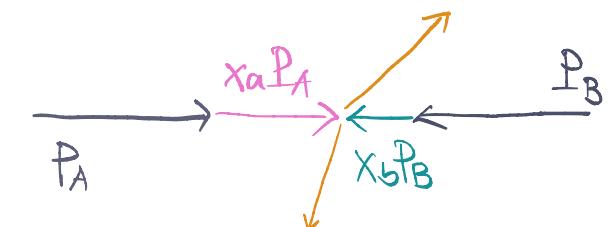
\* Integrate out  $Z \rightarrow l^+ l^-$  decay

\* Observables of intermediate gauge boson  $q^\mu = (P_1 + P_2)^\mu = (P_A + P_B)^\mu$

$$M_{ll} = \sqrt{q^2} ; \quad Y_{ll} = \frac{1}{2} \ln \left( \frac{q^0 + q^3}{q^0 - q^3} \right)$$

rapidity:  $Y \mapsto Y + \frac{1}{2} \ln(\frac{x_a}{x_b})$

$$\Rightarrow \frac{d^2\sigma_{DY}}{dM_{ll} dY_{ll}} = f_a(x_a) f_b(x_b) \frac{2 M_{ll}}{E_{cm}^2} \hat{\Gamma}_{a+b \rightarrow l^+ l^-} \Bigg|_{x_a x_b = \frac{M_{ll}}{E_{cm}} e^{\pm Y_{ll}}}$$



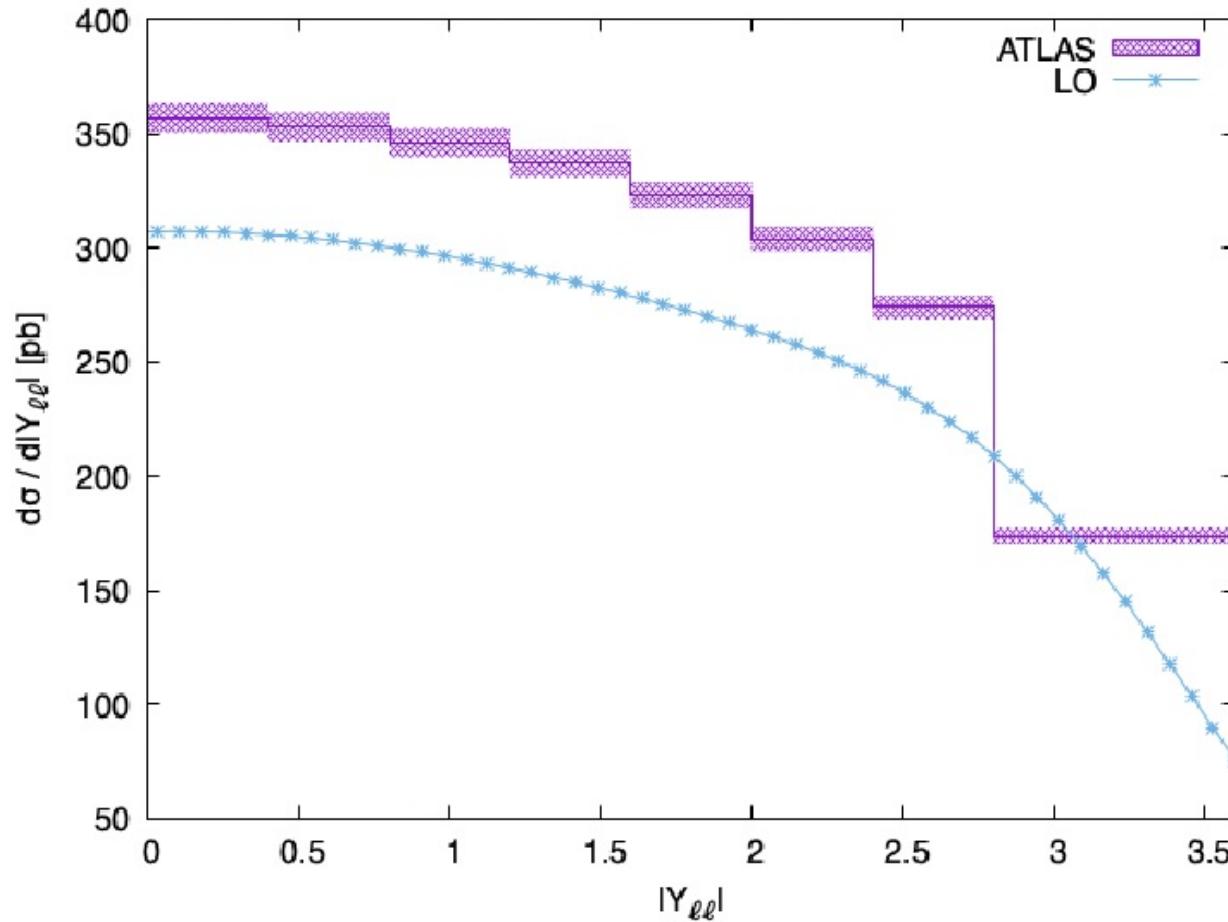
# Drell Yan at Leading Order

ATLAS

$$\sigma_Z = 1055.3 \pm 0.7 \text{ (stat.)} \pm 2.2 \text{ (syst.)} \pm 19.0 \text{ (lumi.) pb}$$

LO :

#total 897.1018242299992 pb



- \* right ballpark  
 $\mathcal{L}_S \sim 0.1$ 
    - ↳  $O(10\%)$  anticipated
    - ↳ often much worse!  
 (extreme:  $gg \rightarrow H$  100%)
  - @ LO only  $q\bar{q}$  annihilation
    - ↳ 50% of protons are  $g$ !
  - \* no error estimate here
    - ↳ no quantitative comp.
- ⇒ need at least NLO!