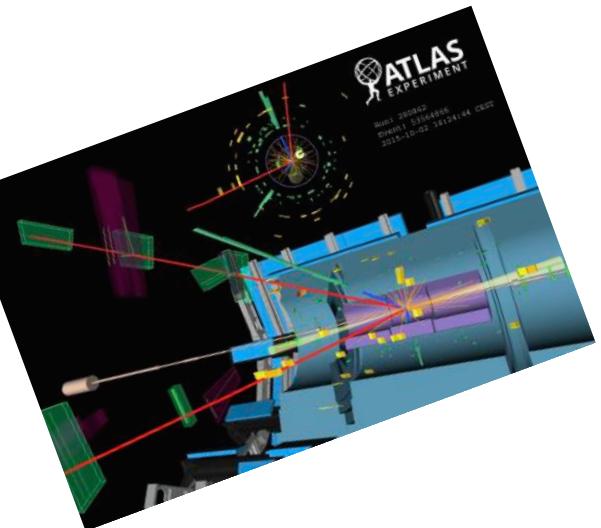


MAKING PREDICTIONS AT HADRON COLLIDERS

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Previously on "Making Predictions"



Event Rates

$$N = L \underbrace{\sigma}_{\text{Cross Sections}}$$

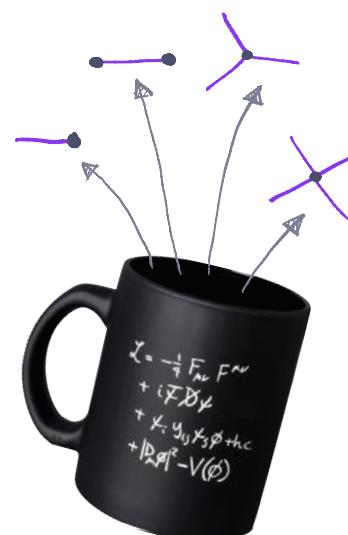
Cross Sections

$$d\sigma_{2 \rightarrow n} = \frac{1}{F} \underbrace{\langle |M|^2 \rangle}_{\text{Scattering Amplitudes}} d\Omega_n$$

Scattering Amplitudes



Feynman diagrams
& rules



* another important case Decay Rates ($\tau = 1/\Gamma$)

$$d\Gamma_{1 \rightarrow n} = \frac{1}{2M} \langle |M|^2 \rangle d\Omega_n$$

The Drell-Yan process

We saw that a leading order (LO) prediction in QCD
is not sufficient for precision phenomenology

want to include
higher order(s) !
→ diagrams with loops

$$\mathcal{M}_2 = \text{Diagram } 0 + \text{Diagram } 1 + \dots + \text{Diagram } n + \dots$$

$\mathcal{O}(\alpha) \leftrightarrow \mathcal{M}_2^{(0)}$ $\mathcal{O}(\alpha_s \alpha) \leftrightarrow \mathcal{M}_2^{(1)}$ $\mathcal{O}(\alpha_s^2 \alpha) \leftrightarrow \mathcal{M}_2^{(2)}$

$$\Rightarrow |\mathcal{M}_2|^2 = |\mathcal{M}_2^{(0)}|^2 + 2 \operatorname{Re} \{ (\mathcal{M}_2^{(0)})^* \mathcal{M}_2^{(1)} \} + |\mathcal{M}_2^{(1)}|^2 + 2 \operatorname{Re} \{ (\mathcal{M}_2^{(0)})^* \mathcal{M}_2^{(2)} \} + \dots$$

$\mathcal{O}(\alpha^2)$

$\mathcal{O}(\alpha_s \alpha^2)$

$\mathcal{O}(\alpha_s^2 \alpha^2)$

"virtual corrections"

Divergences in Loop Diagrams

QM \rightarrow sum over all possible intermediate configurations

\hookrightarrow integration over unconstrained loop momentum: $\int \frac{d^4 k}{(2\pi)^4}$



\hookrightarrow results are plagued by divergences

1. ultraviolet (UV) \leftrightarrow large loop momentum

\Rightarrow treated by renormalization $\alpha_s(\mu_R^2)$ ✓

2. infrared (IR) \leftrightarrow { small energy (soft) and/or
small angle (collinear) }

\Rightarrow only cancels after adding "real corrections"

technically, a different process but cannot be distinguished when unresolved

$$|M_3|^2 = \left| \text{diagram} + \dots \right|^2 \leftrightarrow \mathcal{O}(\alpha_s \alpha^2)$$

Cancellation of Divergences

Consider this sub-diagram that appears both in the virtual & real

* performing the integration in $D = 4 - 2\epsilon$ dimensions,
we expose the divergences as poles in ϵ

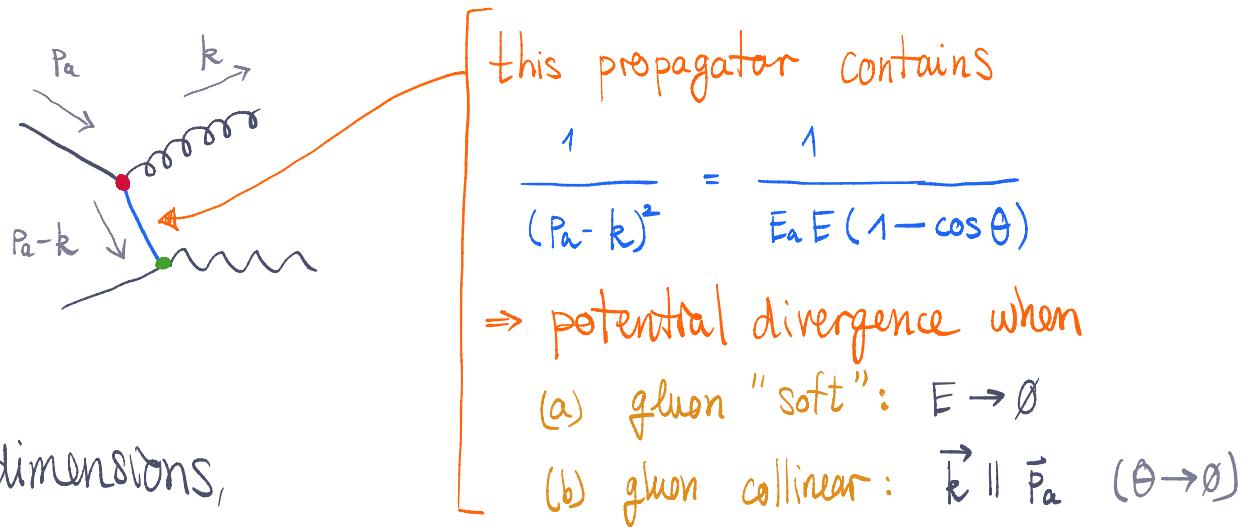
↳ "virtual" (loop) corrections

$$\hat{\sigma}_{\text{LO}}(P_a, P_b) \frac{\alpha_s}{2\pi} C_F \frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)} \left\{ \frac{1}{\epsilon^2} + \frac{3}{2} \frac{1}{\epsilon} + \text{finite} \right\}$$

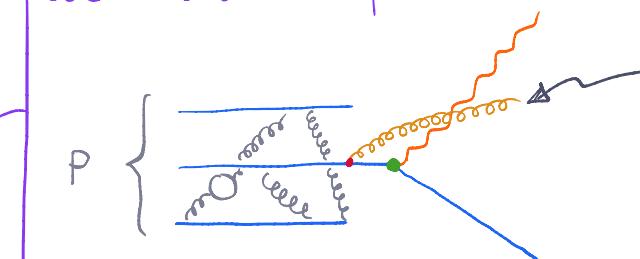
↳ "real" emission corrections

$$\hat{\sigma}_{\text{LO}}(P_a, P_b) \frac{\alpha_s}{2\pi} C_F \frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)} \left\{ -\frac{1}{\epsilon^2} - \frac{3}{2} \frac{1}{\epsilon} + \text{finite} \right\}$$

$$+ \int_0^1 dz_a \hat{\sigma}_{\text{LO}}(z_a P_a, P_b) \frac{\alpha_s}{2\pi} C_F \frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)} \left\{ -\frac{1}{\epsilon} - \ln(\mu_F^2 / Q^2) \right\} P_{qg}(z_a) + \begin{pmatrix} z_a \leftrightarrow z_b \\ q_{\text{in}} \leftrightarrow g_{\text{in}} \end{pmatrix}$$



absorbed as part of "NLO PDFs"

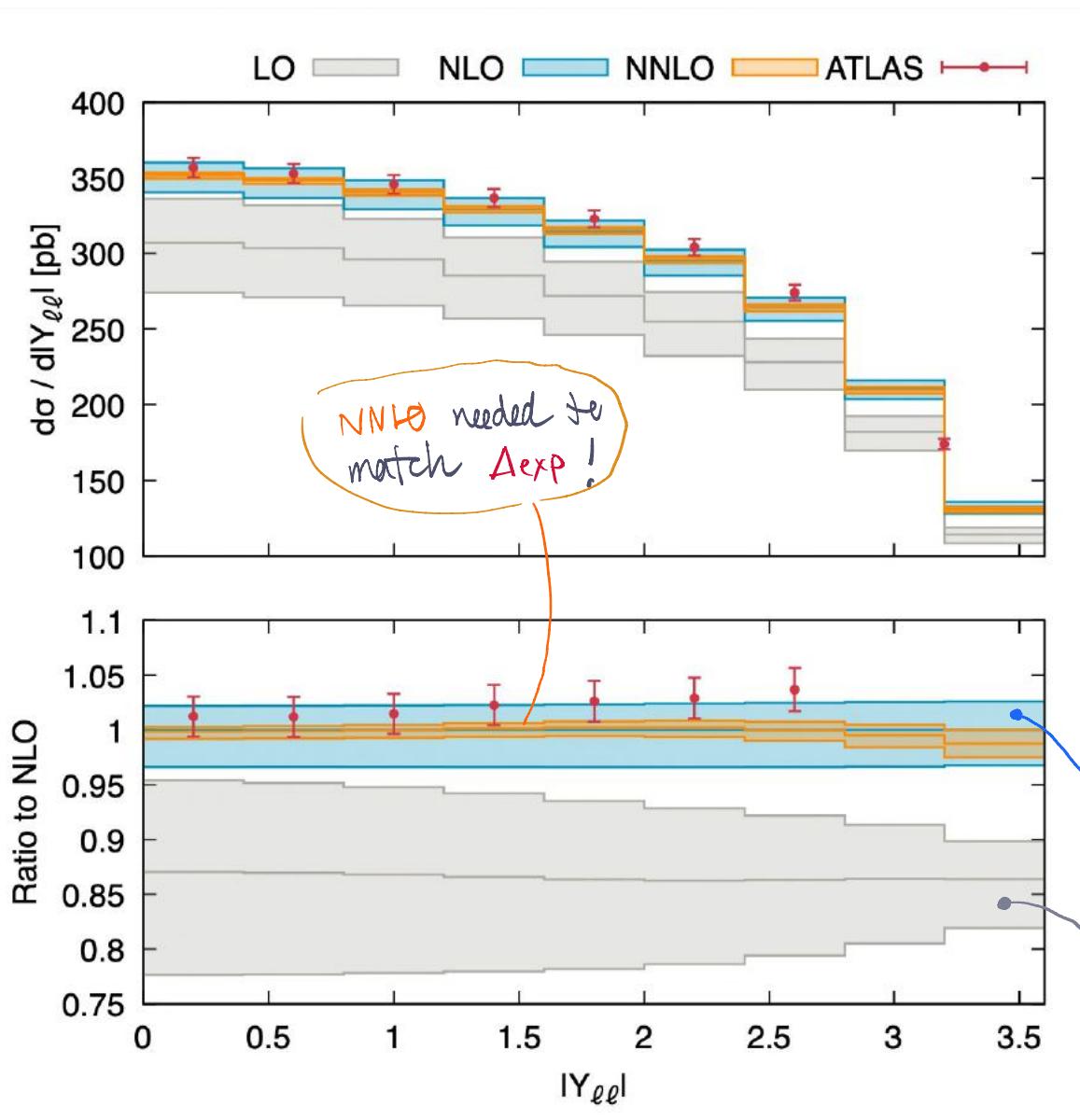


part of the proton
if its scale $< \mu_F$

$$f_a(x) \mapsto f_a(x, \mu_F^2)$$

universal \leftrightarrow PDF evolution

The Drell-Yan process at higher orders

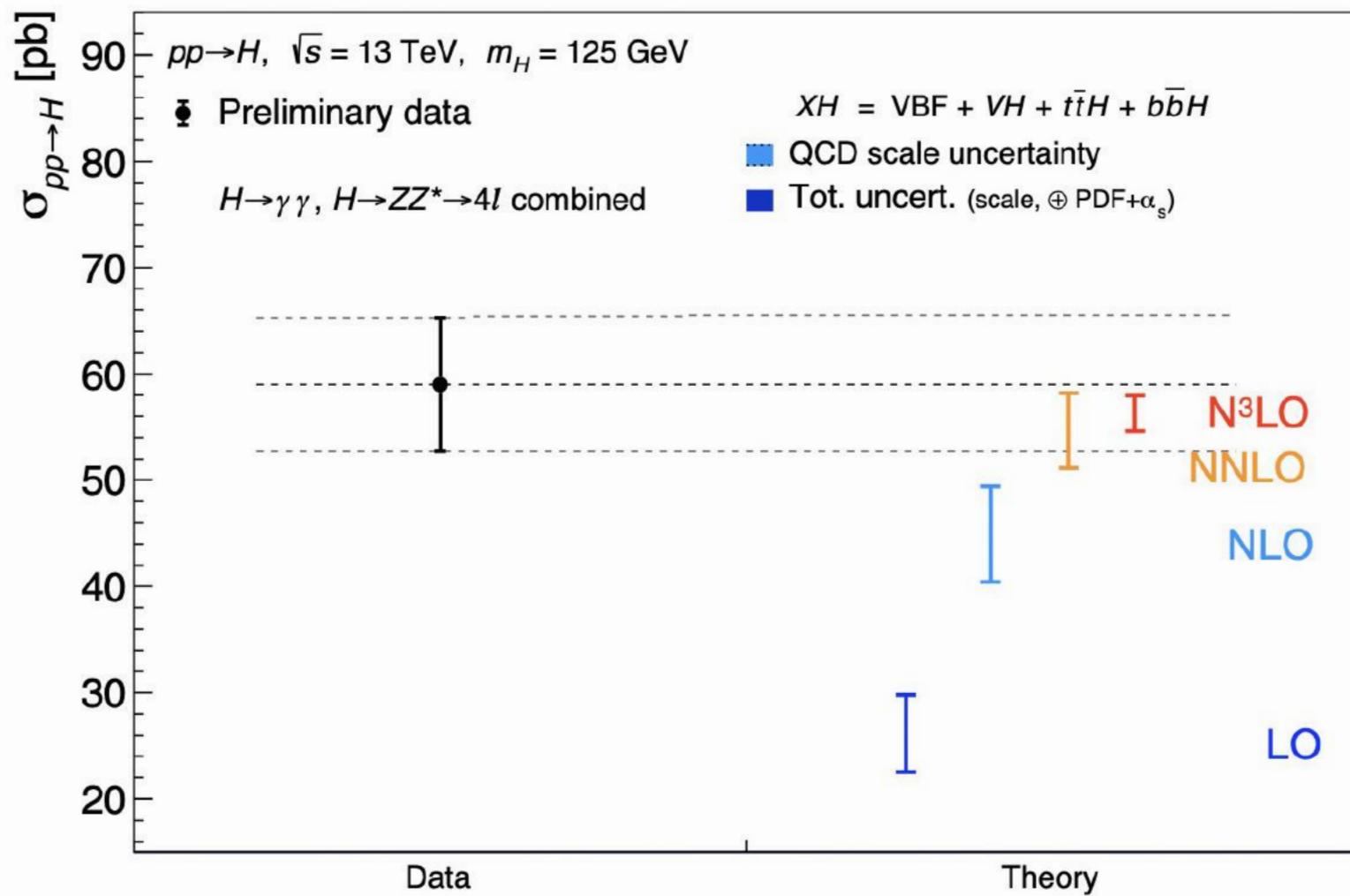


Theory uncertainties

- * missing higher orders
arbitrary scales μ_R & μ_F in our calculation
 \leftrightarrow Variation induces terms beyond the order we computed $\times [\frac{1}{2}, 2]$
- * parametric ($\alpha_s(M_Z), \dots$) (not included)
 $\curvearrowleft \mathcal{O}(1\%)$
- * PDF uncertainties (not included)
 $\curvearrowleft \mathcal{O}(1\%)$

on repository
last lecture

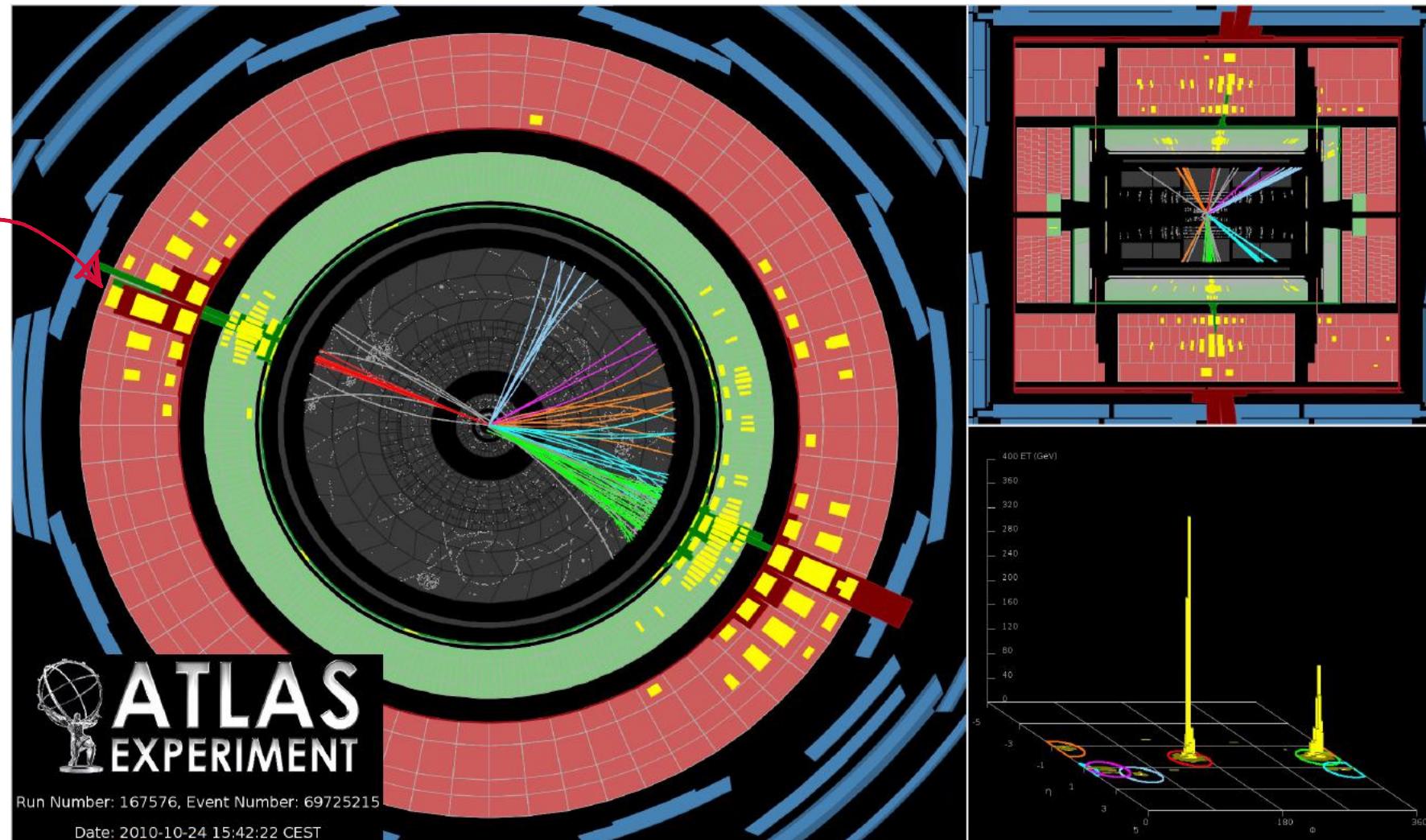
The state of the art in Fixed Order



- * Leading order (LO)
basically any process
- * Next-to-leading order (NLO)
most processes
& automation
(up to $2 \rightarrow 8$)
- * NNLO
 $2 \rightarrow 2$ done,
steady progress for $2 \rightarrow 3$
- * N³LO
limited to simple $2 \rightarrow 1$

Events at hadron colliders look more complex

[demo: diagtams]



Why? Any chance to compute this with what we did so far?

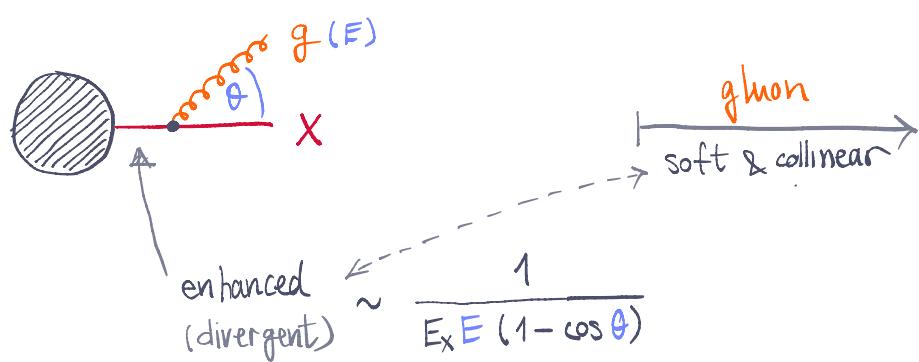
Nope...

- 2->2 gluon scattering has 4 diagrams
- 2->3 gluon scattering has 25 diagrams
- 2->4 gluon scattering has 220 diagrams
- 2->5 gluon scattering has 2485 diagrams
- 2->6 gluon scattering has 34300 diagrams
- 2->7 gluon scattering has 559405 diagrams
- 2->8 gluon scattering has 10525900 diagrams
- 2->9 gluon scattering has 224449225 diagrams
- 2->10 gluon scattering has 5348843500 diagrams
- 2->11 gluon scattering has 140880765025 diagrams

10^M !

- * Even with more efficient recursive approaches,
not the multiplicities you'd want to tackle $\# \text{dim}(\Phi_n) = 3n - 4$
- ↳ define observables that "map back" the physics
to fewer initiating objects } JETS
- ↳ identify the relevant physics &
model the full complexity approximately } Parton Showers

The QCD emission pattern

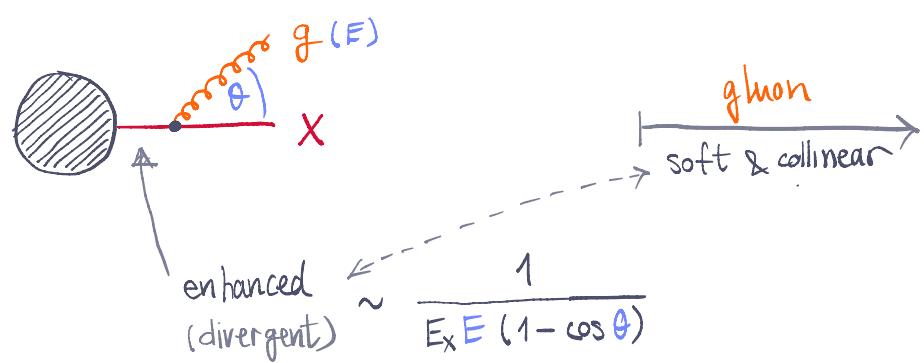


A diagram showing a shaded circular vertex labeled 'X' emitting a gluon (curly orange line). A dashed arrow labeled $d\omega_{x \rightarrow x+g}$ points to the right, indicating the differential solid angle.

$\boxed{2 \frac{\alpha_s}{\pi} C_X \frac{dE}{E} \frac{d\theta}{\theta}}$

$\nabla \left\{ \begin{array}{l} = C_F = 4/3 \text{ if } X = q \\ = C_A = 3 \text{ if } X = g \end{array} \right.$

The QCD emission pattern



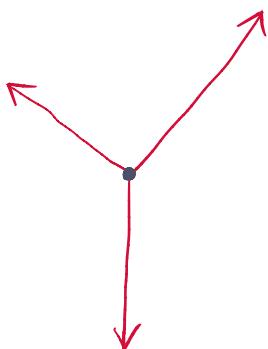
X \times $d\omega_{X \rightarrow X+g}$

$$2 \frac{\alpha_s}{\pi} C_X \frac{dE}{E} \frac{d\theta}{\theta}$$

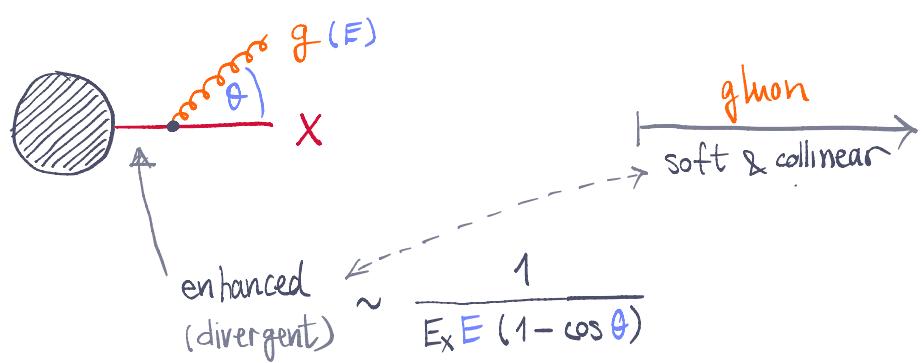
$\nabla \left\{ \begin{array}{l} = C_F = \frac{4}{3} \text{ if } X = q \\ = C_A = 3 \text{ if } X = g \end{array} \right.$

\Rightarrow jets are an emergent feature of QCD

1. high energetic partons
 \hookrightarrow hard scattering



The QCD emission pattern



A diagram illustrating the factorization of the emission pattern. A shaded circle labeled X emits a gluon (orange wavy line) with differential solid angle $d\omega_{X \rightarrow X+g}$. This is multiplied by a factor $2 \frac{\alpha_s}{\pi} C_X \frac{dE}{E} \frac{d\theta}{\theta}$.

$\boxed{2 \frac{\alpha_s}{\pi} C_X \frac{dE}{E} \frac{d\theta}{\theta}}$

$\nabla \left\{ \begin{array}{l} = C_F = \frac{4}{3} \text{ if } X = q \\ = C_A = 3 \text{ if } X = g \end{array} \right.$

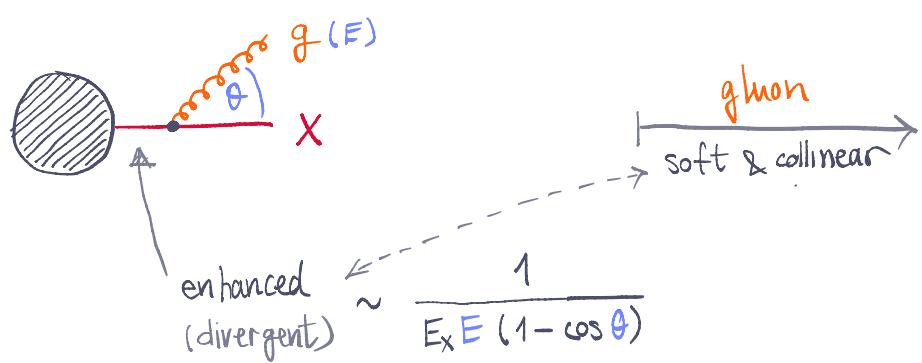
\Rightarrow jets are an emergent feature of QCD

1. high energetic partons
 \hookrightarrow hard scattering

2. asymp. freedom & $d\omega$
 \hookrightarrow pert. parton shower



The QCD emission pattern



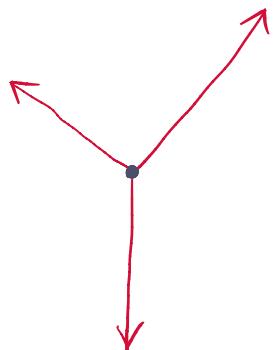
$\times d\omega_{x \rightarrow x+g}$

$$2 \frac{\alpha_s}{\pi} C_x \frac{dE}{E} \frac{d\theta}{\theta}$$

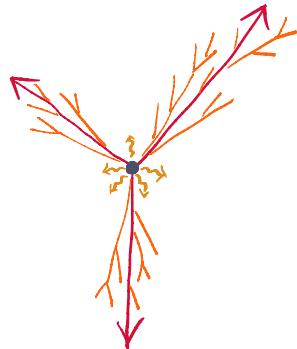
$\nabla \left\{ \begin{array}{l} = C_F = \frac{4}{3} \text{ if } X = g \\ = C_A = 3 \text{ if } X = f \end{array} \right.$

\Rightarrow jets are an emergent feature of QCD

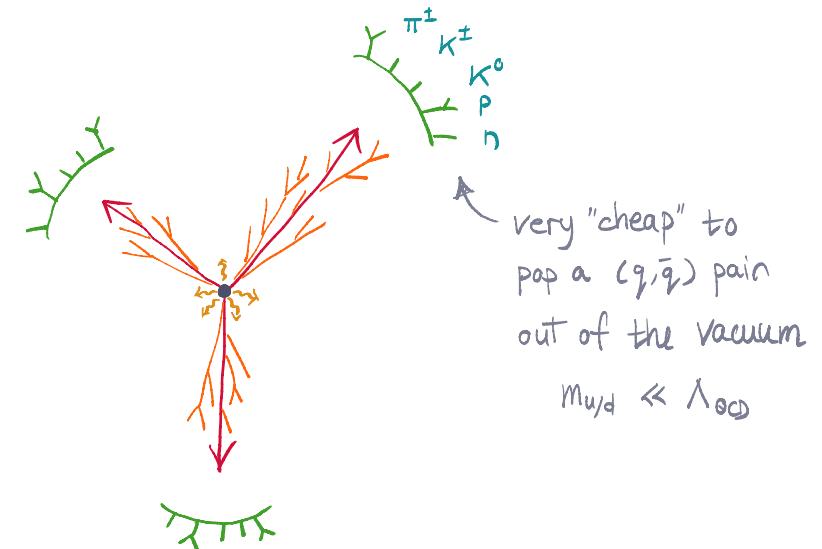
1. high energetic partons
 ↳ hard scattering



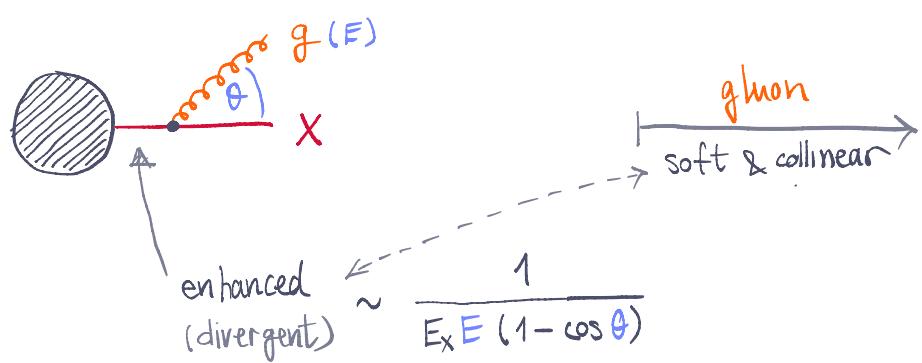
2. asymp. freedom & $d\omega$
 ↳ pert. parton shower



3. hadronization



The QCD emission pattern



A diagram showing a shaded circular source labeled 'X' emitting a gluon with differential cross-section $d\omega_{X \rightarrow X+g}$.

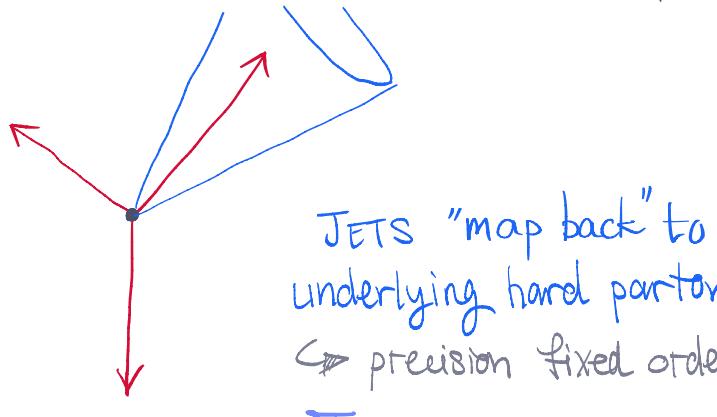
$$2 \frac{\alpha_s}{\pi} C_X \frac{dE}{E} \frac{d\theta}{\theta}$$

$$\boxed{X} \begin{cases} = C_F = \frac{4}{3} & \text{if } X = g \\ = C_A = 3 & \text{if } X = f \end{cases}$$

\Rightarrow jets are an emergent feature of QCD

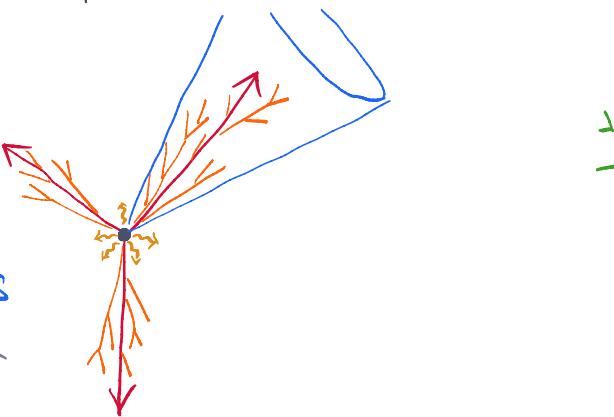
1. high energetic partons

\hookrightarrow hard scattering

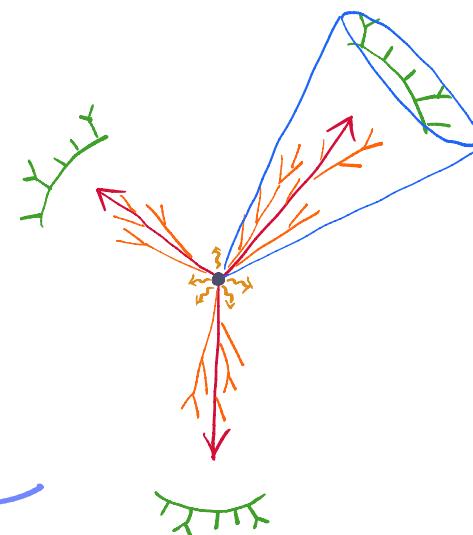


2. asymp. freedom & $d\omega$

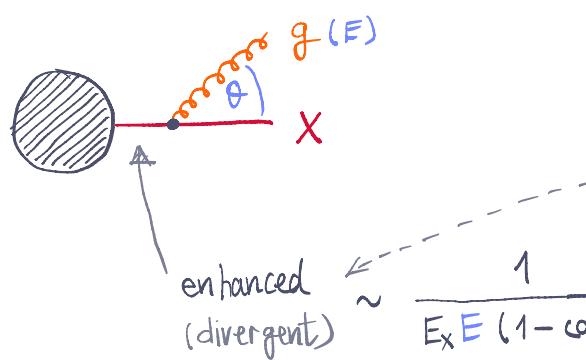
\hookrightarrow pert. parton shower



3. hadronization



The QCD emission pattern



gluon

soft & collinear

$X \times d\omega_{x \rightarrow x+g}$

$\boxed{2 \frac{\alpha_s}{\pi} C_x \frac{dE}{E} \frac{d\theta}{\theta}}$

$\nabla \left\{ \begin{array}{l} = C_F = \frac{4}{3} \text{ if } X = g \\ = C_A = 3 \text{ if } X = g \end{array} \right.$

\Rightarrow emission factorizes!

Integral over E & θ diverges $\Rightarrow \left\{ \begin{array}{l} \text{introduce a scale } q^2 > Q_0^2 \\ \Leftrightarrow \text{emission "resolved"} \end{array} \right.$

$$\Rightarrow P_X \simeq \frac{\alpha_s C_F}{2\pi} \ln^2 \left(\frac{Q^2}{Q_0^2} \right) + \mathcal{O}(\alpha_s \ln Q^2, \alpha_s^2)$$

probability to emit
a resolved gluon

potentially a very large log \Rightarrow will want to "resum" these to all orders

$Q_0 = \Lambda_{QCD} = 0.2 \text{ GeV}$

$Q = 100 \text{ GeV}$

$\left[\ln(\dots) = \mathcal{O}(10) \right]$

Parton Showers

- * We wish to account for an **arbitrary** number of emissions ordered in our resolution variable $Q^2 > q_1^2 > q_2^2 > \dots > Q_0^2$ (strong ordering)
- * current scale $q_n^2 \Rightarrow$ probability to have next emission @ q_{n+1}^2 ?

$$\underbrace{\left(\begin{array}{l} \text{probability of having} \\ \text{no emissions } q_n^2 \mapsto q_{n+1}^2 \end{array} \right)}_{\Delta(q_n^2, q_{n+1}^2)} \times \underbrace{\left(\begin{array}{l} \text{emission} \\ @ q_{n+1}^2 \end{array} \right)}_{\frac{d\omega_{x \rightarrow x+g}}{dq^2} \Big|_{q^2 = q_{n+1}^2}}$$

[demo: toy shower]

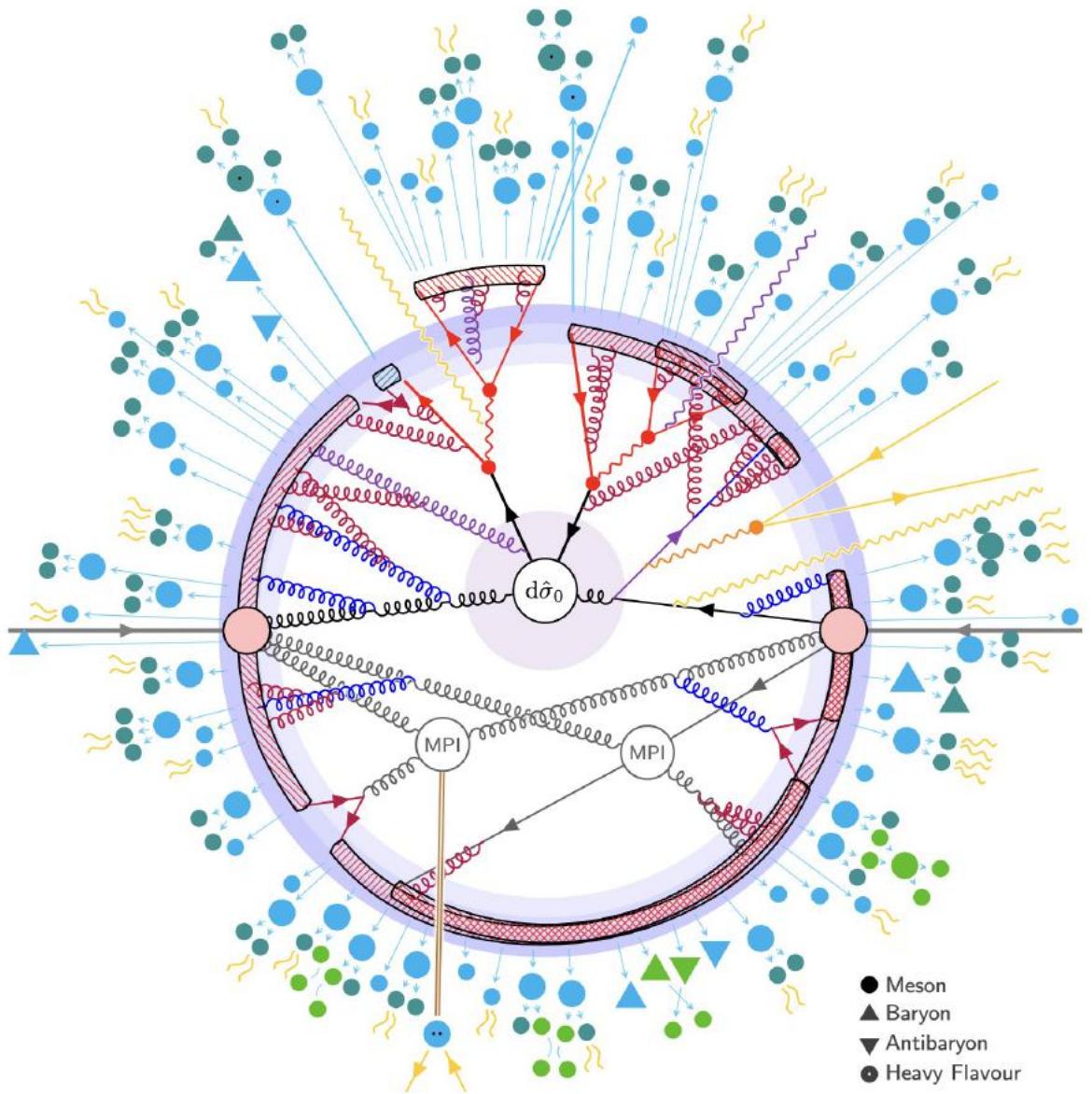
$$\Delta(q_n^2, q_{n+1}^2)$$

Sudakov form factor

$$\hookrightarrow \frac{d\Delta(Q^2, q^2)}{dq^2} = \Delta(Q^2, q^2) \frac{d\omega}{dq^2}$$

$$\left[\Delta(Q^2, q^2 - dq^2) = \Delta(Q^2, q^2) \Delta(q^2, q^2 - dq^2) = \Delta(Q^2, q^2) \left(1 - \frac{d\omega}{dq^2}\right) \right]$$

Full Event Generator

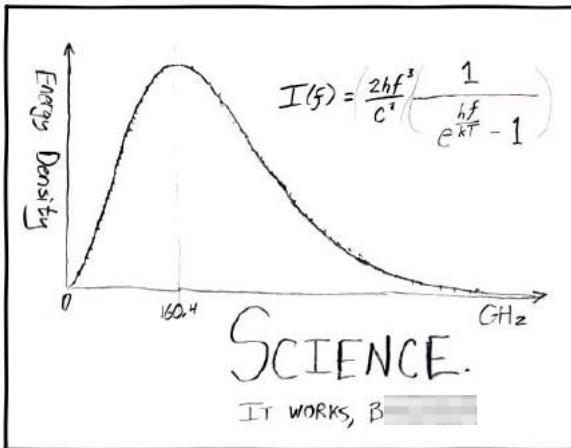


- Hard Interaction
- Resonance Decays
- MECs, Matching & Merging
- FSR
- ISR*
- QED
- Weak Showers
- Hard Onium
- Multiparton Interactions
- Beam Remnants*
- Strings
- Ministrings / Clusters
- Colour Reconnections
- String Interactions
- Bose-Einstein & Fermi-Dirac
- Primary Hadrons
- Secondary Hadrons
- Hadronic Reinteractions
- (*: incoming lines are crossed)

+ pile-up

Conclusions

- * covered basic ingredients that goes into hadron-collider predictions
 - ↳ a key idea: separation of scales ("factorization")
- * Moment of comparing your predictions to data always exciting
 - ↳ learn to play with the tools ; break them (often interesting physics)
- * Hope was able to lower the fear of entry for some of you, as it is sometimes perceived as very technical
 - ↳ pushing frontiers in precision can become arbitrary complex
 - new ideas needed (maybe one of you?)



Thank you for your
attention & participation!