Making Predictions at Hadron Colliders

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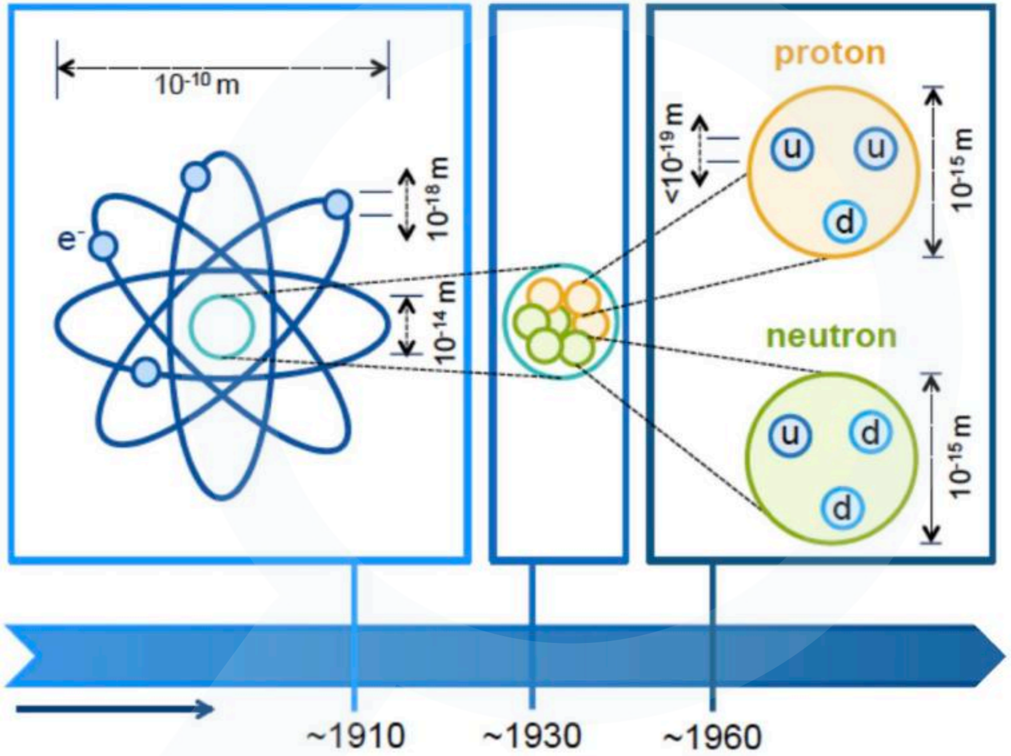
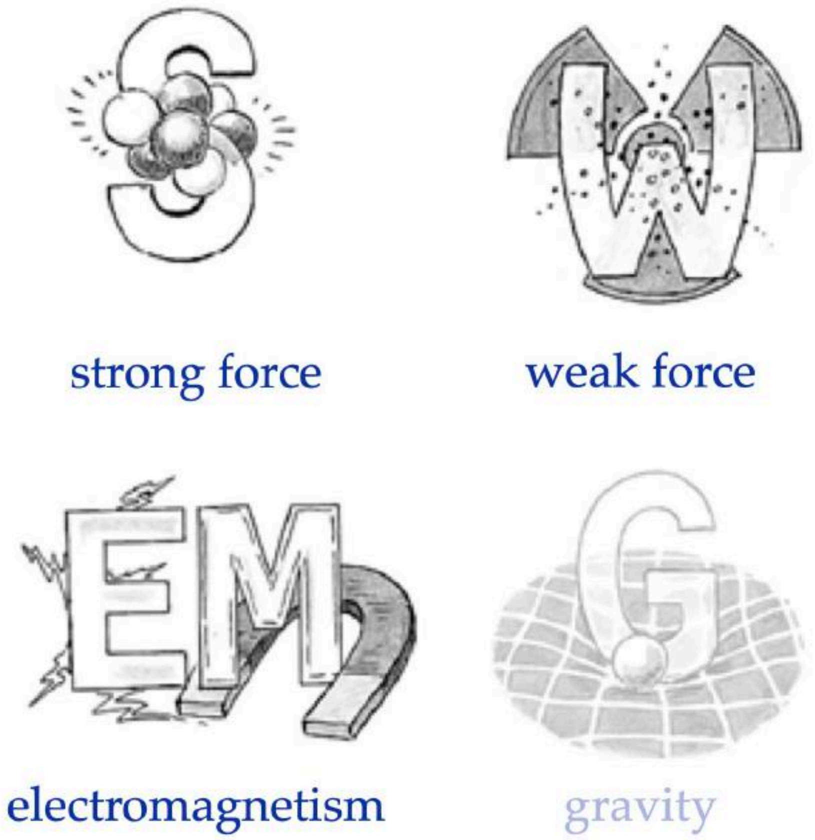
CERN Summer Student Lectures 2024

What is the Universe made of?

What are the Laws of Nature?

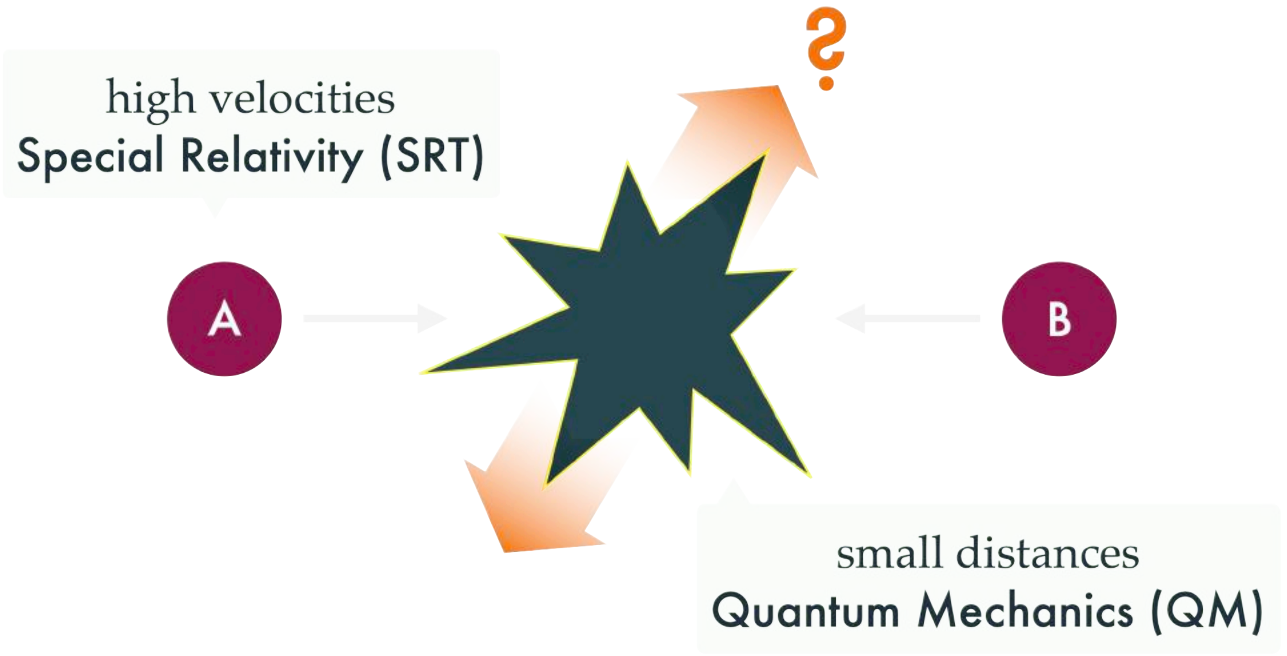




Elementary Particles ⟷ Fundamental Forces

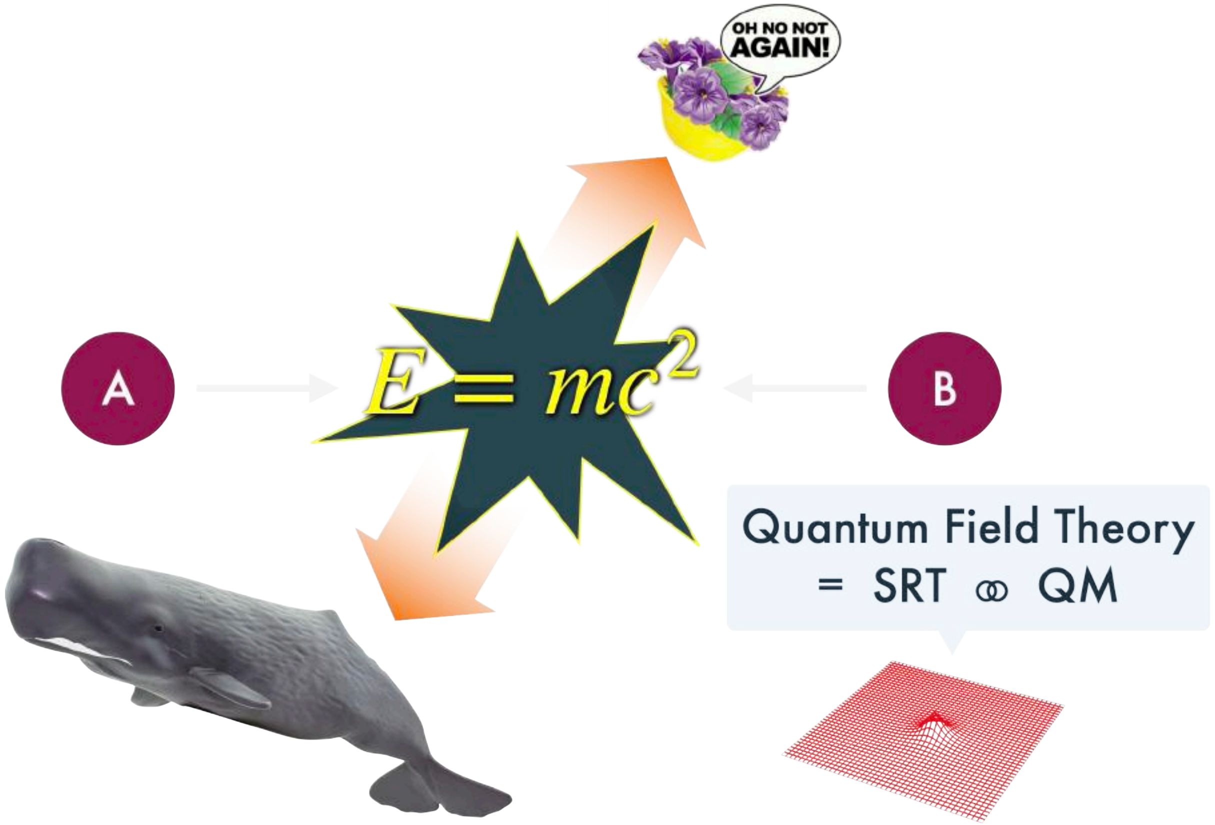
Particle Smashers — Our Microscopes

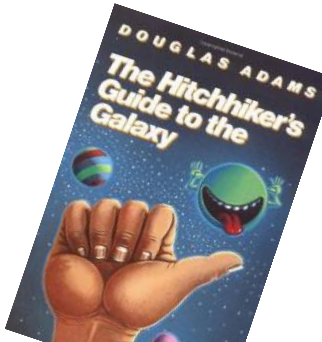
- **LEP** @ CERN [→ 1999]
  ↪ e^+e^- | 90 ↗ 200 GeV | ◯27 km
- **HERA** @ DESY [→ 2008]
  ↪ ep | 320 GeV | ◯6.3 km
- **Tevatron** @ Fermilab [→ 2011]
  ↪ $p\bar{p}$ | 1 TeV | ◯6.28 km
- **LHC** @ CERN [→ now]
  ↪ pp | 7 ↗ 13.6 TeV | ◯27 km
- **EIC** @ BNL [2032 →]
  ↪ ep | 20 ↗ 140 GeV | ◯3.8 km
- **FCC-ee** @ CERN [2040?→]
  ↪ e^+e^- | 90 ↗ 360 GeV | ◯100 km
- **FCC-hh** @ CERN [2060?→]
  ↪ pp | 100 TeV | ◯100 km



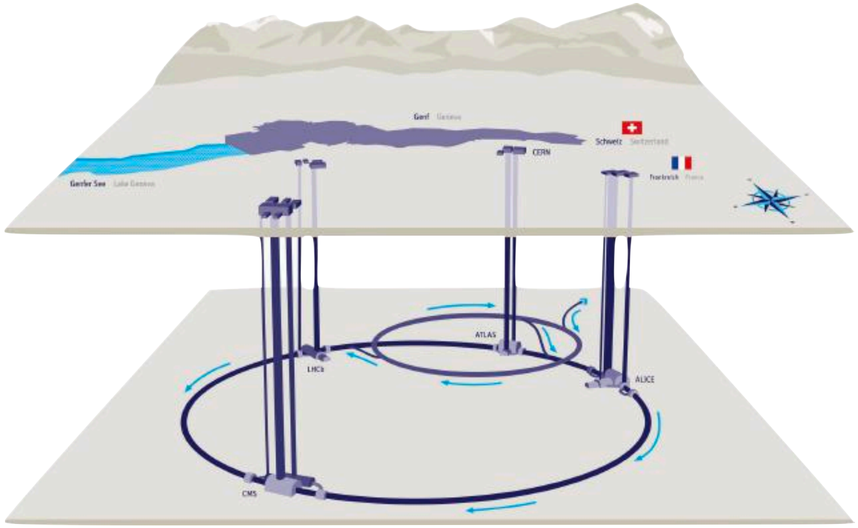
Particle Smashers — Our Microscopes

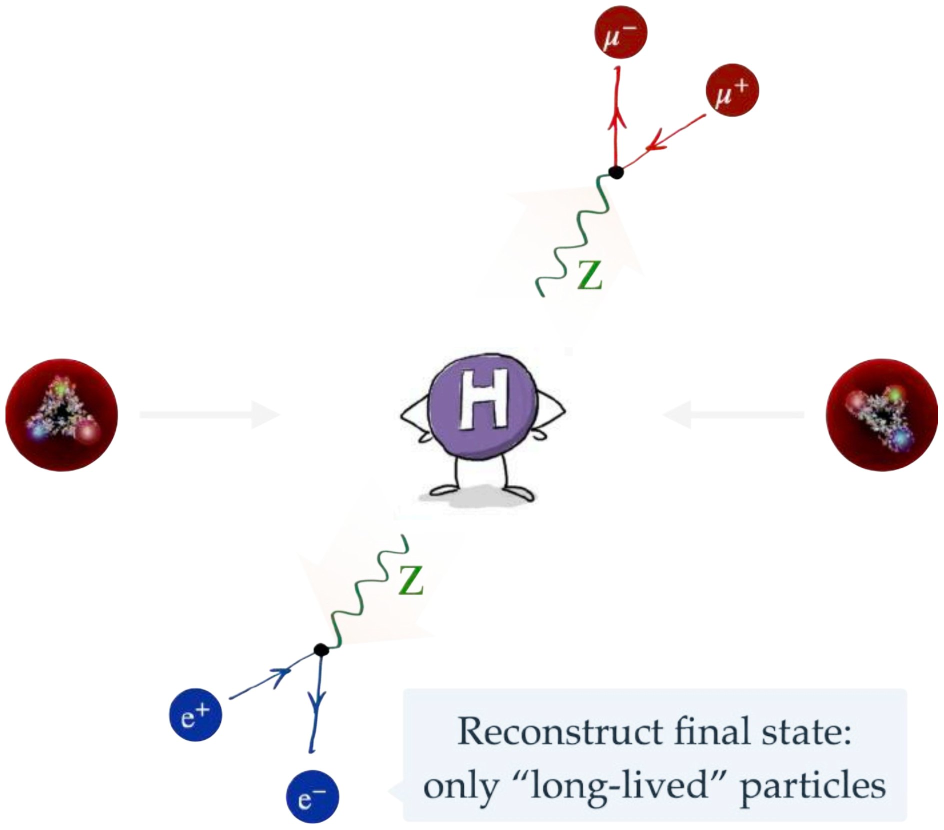
* high energies
  - ↪ small distances $\lambda = \frac{hc}{E}$
  - ↪ produce heavy (= interesting!) final states
* many collisions
  - ↪ access to rare (= interesting!) events & processes





Particle Smashers — The LHC

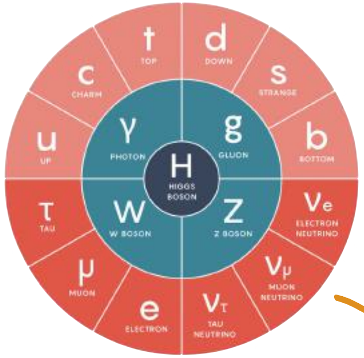




* energy frontier
  - ⇒ 13.6 TeV collision energy
  - ⇒ 1000 x distance to moon ()

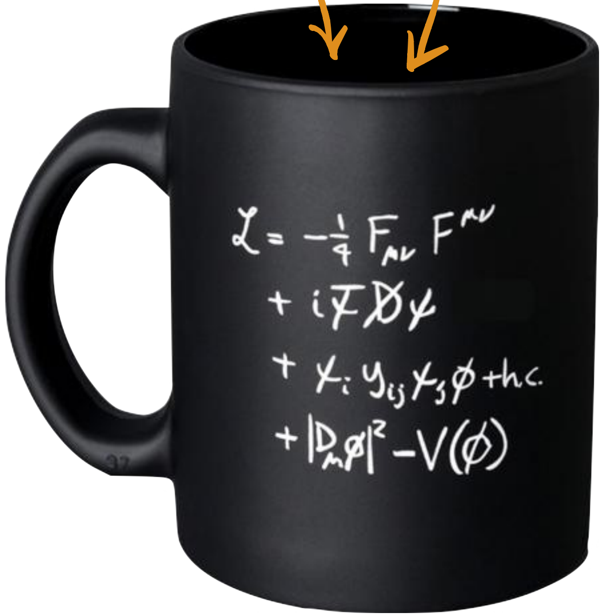
* intensity frontier
  - ⇒ 10^9 collisions / second
  - ⇒ 30 PB / year

THEORY



Gauge interactions

$$\boxed{SU(3)_c \times SU(2)_L \times U(1)_Y}$$



PHENOMENOLOGY

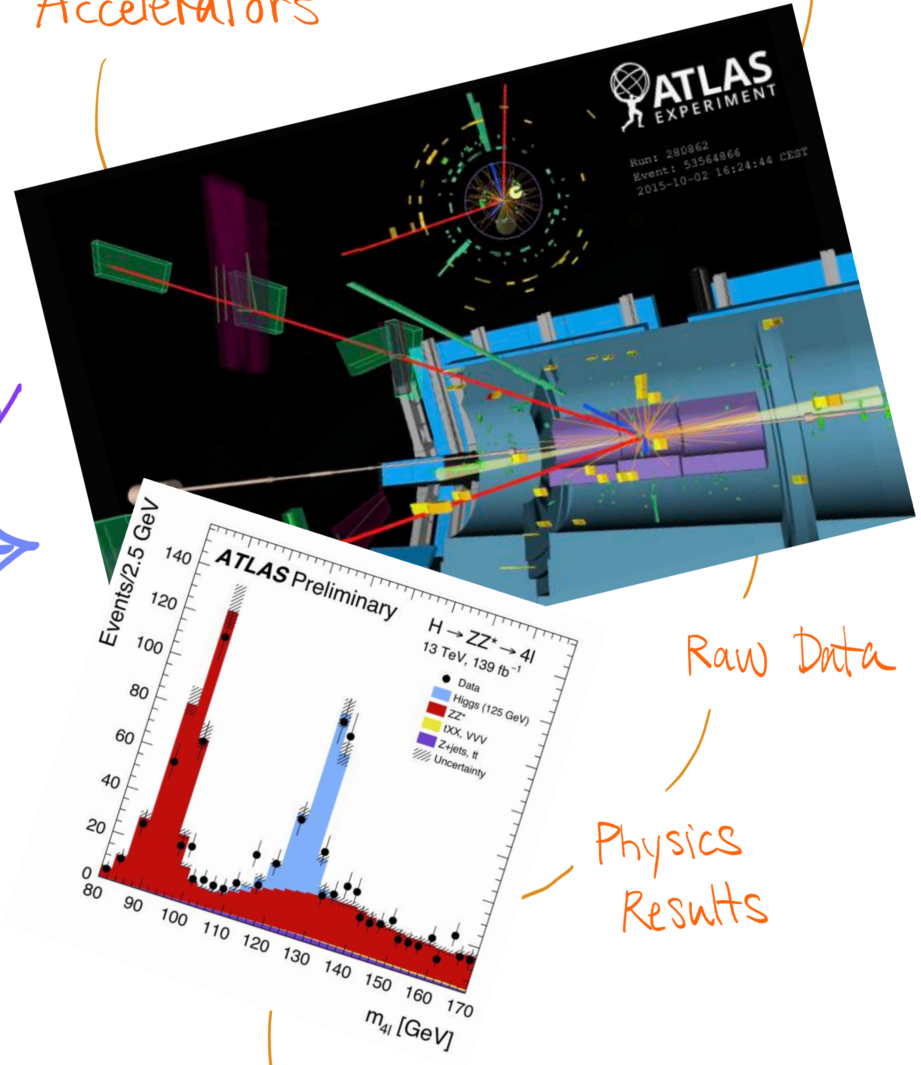
this lecture

$$QFT = QM \otimes SR$$

[Particle World, Theoretical Concepts, The SM]

Accelerators

Detectors



Raw Data

Physics Results

Statistics

EXPERIMENT

Repository & Conventions

* Notebooks for demonstrations

  ⟶ https://github.com/aykhuss/Lectures-SSL-MkPred

  "Making Predictions"

* Conventions natural units: $[\hbar] = [c] = 1$ (remember: $m_{\text{proton}} \sim 1\,\text{GeV}$)

  ⟶ $[\text{length}] = [\text{time}] = \text{eV}^{-1}$

  $[\text{mass}] = [\text{energy}] = [\text{momentum}] = \text{eV}$

  ⟶ four vectors $x^\mu = (t, x, y, z)^T$

  $p^\mu = (E, \underbrace{p_x, p_y, p_z}_{\vec{p}})^T$ ⟿ "on-shell": $p^2 = E^2 - |\vec{p}|^2 \overset{!}{=} m^2$

  invariant mass

$\Rightarrow$ energy-momentum conservation: $(a + b \to 1 + 2)$

$$\delta^{(4)}(p_1 + p_2 - (p_a + p_b)) = \delta(E_1 + E_2 - (E_a + E_b))\, \delta^{(3)}(\vec{p}_1 + \vec{p}_2 - (\vec{p}_a + \vec{p}_a))$$

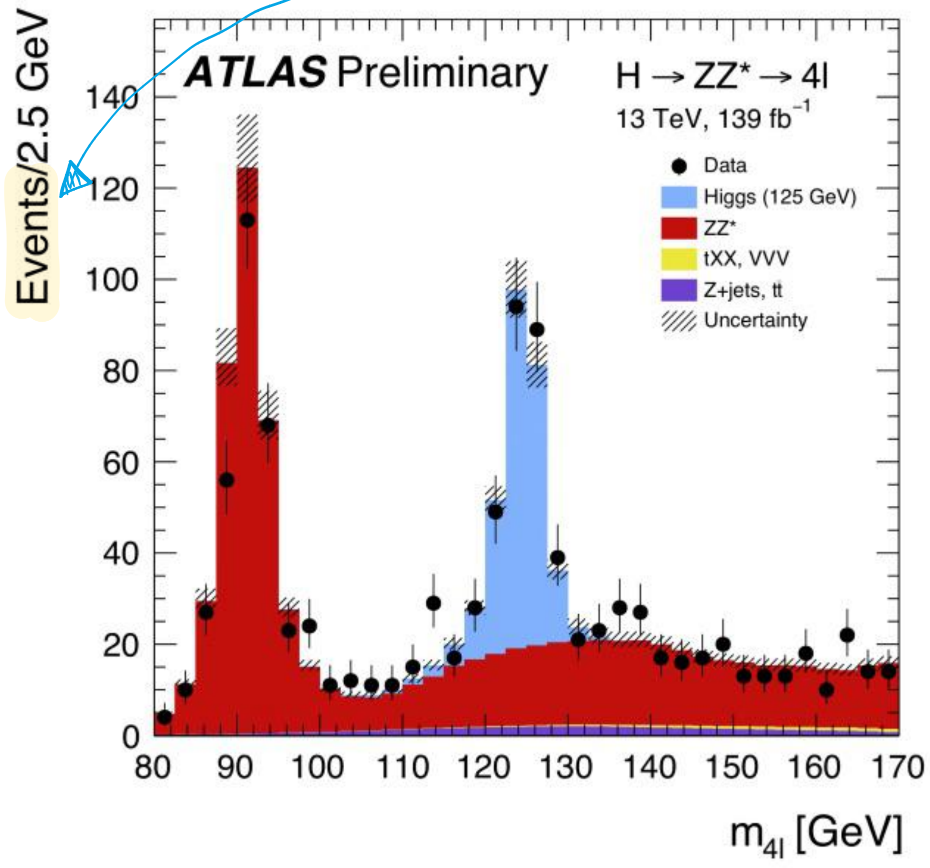
The Plan

1. Event Rates, Cross Sections & Scattering Amplitudes
2. Warmup: Lepton Collider
3. Hadron Colliders — Parton Distribution Functions
4. The Drell-Yan Process
5. Higher-Order Corrections
6. QCD Jets, Parton Showers & MC Simulations

Event Rates

We ultimately measure # Events

for a specific process: $a+b \to 1+2+\dots+n$



Luminosity $\sim$ # collisions

cross section

$$dN = \mathcal{L}\, d\sigma$$

* $\sigma_H\ (13\,\text{TeV}) \approx 50\ \text{pb}$

  $\int_{\text{Run2}} dt\, \mathcal{L} \approx 150\ \text{fb}^{-1}$

  $\sim$ 7 million Higgs bosons produced!

* $\sigma_Z\ (13\,\text{TeV}) \simeq 50\ \text{nb}$

  $\sigma_{W^\pm}\ (13\,\text{TeV}) \simeq 200\ \text{nb}$

  $\mathcal{L}$ (instantaneous) $\simeq 0.02\ \text{pb}^{-1}\,\text{s}^{-1}$

  $\sim$ 1000 Z's

  $\sim$ 4000 $W^\pm$'s

  every second!

Calculating Cross Sections

Fermi's Golden Rule $\quad a+b \to 1+2+\dots+n$

$$d\sigma = \underbrace{\frac{1}{F}}_{\text{flux}} \underbrace{\langle |\mathcal{M}|^2 \rangle}_{|\text{amplitude}|^2} \underbrace{d\Phi}_{\substack{\text{phase space} \\ \text{(LIPS)}}}$$

$$= \frac{1}{4(p_a \cdot p_b)} = \frac{1}{2E_{cm}^2}$$

$$= \frac{1}{n_a^{d.o.f.}\, n_b^{d.o.f.}} \sum_{d.o.f.} |\mathcal{M}|^2$$

(degrees of freedom spin, colour)

$$d\Phi_n(p_1, \dots, p_n; p_a + p_b)$$

$$= \prod_{i=1}^{n} \frac{d^4 p_i}{(2\pi)^4} (2\pi)\, \delta(p_i^2 - m_i^2)\, \Theta(p_i^0)$$

$$(2\pi)^4\, \delta^{(4)}(p_1 + \dots + p_n - (p_a + p_b))$$

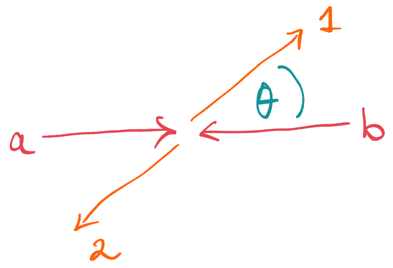
"on shell"

$E_i > 0$

energy-momentum conservation

Special case $a+b \to 1+2$

$$d\Phi_2 = \frac{d\cos\theta}{16\pi} \quad (\text{massless})$$



The Scattering Amplitude

$\leftrightarrow$ evaluation of the path integral

(analogy to QM)

$$Z[J] = \int \mathcal{D}[\Phi]\, e^{i\int d^4x \left[\mathcal{L}(\Phi,\partial_\mu\Phi) + J(x)\Phi(x)\right]}$$

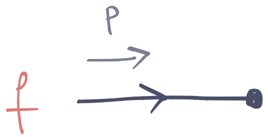
$\Phi \in \{\psi, \phi, A_\mu, \dots\}$

$\hookrightarrow$ extremely difficult to solve except a free theory

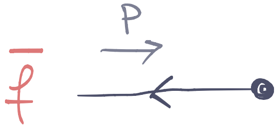
$\hookrightarrow$ $\mathcal{L}$ only has terms with at most two fields Φ, e.g. $\bar\psi(x)\,(i\not\partial - m)\,\psi(x)$

* Feynman rules for the free theory

incoming

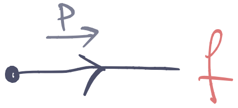


$= u(p)$

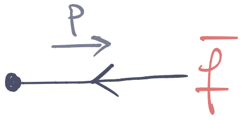


$= \bar v(p)$

outgoing

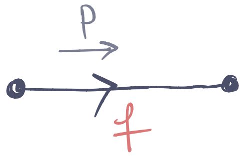


$= \bar u(p)$

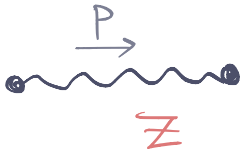


$= v(p)$

propagators



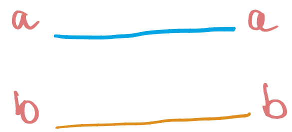
$$= \frac{i}{\not p - m}$$



$$= \frac{-i\, g_{\mu\nu}}{p^2 - M_Z^2 + i M_Z \Gamma_Z}$$

boring "scattering"

$a+b \to a+b$



Perturbation Theory

When interaction strength is small $\left(\alpha_{em} \sim 1/137\ ,\ \alpha_s(M_Z^2) \sim 0.118\right)$

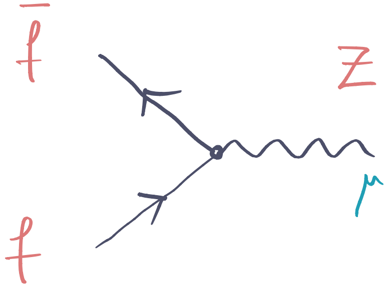
$\Rightarrow$ compute $\mathcal{M}$ perturbatively by expanding around free theory

* Feynman rules for interactions - vertices

direct correspondence* with terms in $\mathcal{L}$

$$-e\, Z_\mu\, \bar{\psi}_f\, \gamma^\mu (v_f - a_f \gamma^5)\, \psi_f \in \mathcal{L}$$

$\rightsquigarrow$



$$= -ie\, \gamma^\mu (v_f - a_f \gamma^5)$$

* more subtle than just "dropping" the fields when derivatives (∂_μ) and/or identical particles

Warmup: Lepton Collider

Consider the process $e^+ e^- \to \mu^+ \mu^-$

at lowest order (tree level). There are two diagrams

What are they?

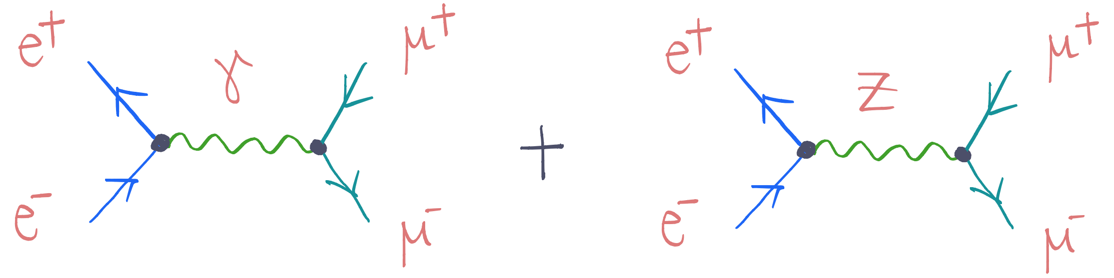
[demo: FeynGame]

Warmup: Lepton Collider

[demo: $e^+e^- \to \mu^+\mu^-$]

Consider the process $e^+ e^- \to \mu^+ \mu^-$

at lowest order (tree level). There are two diagrams



$= i\mathcal{M}^{\text{tree}}_{e^+e^-\to\mu^+\mu^-}$

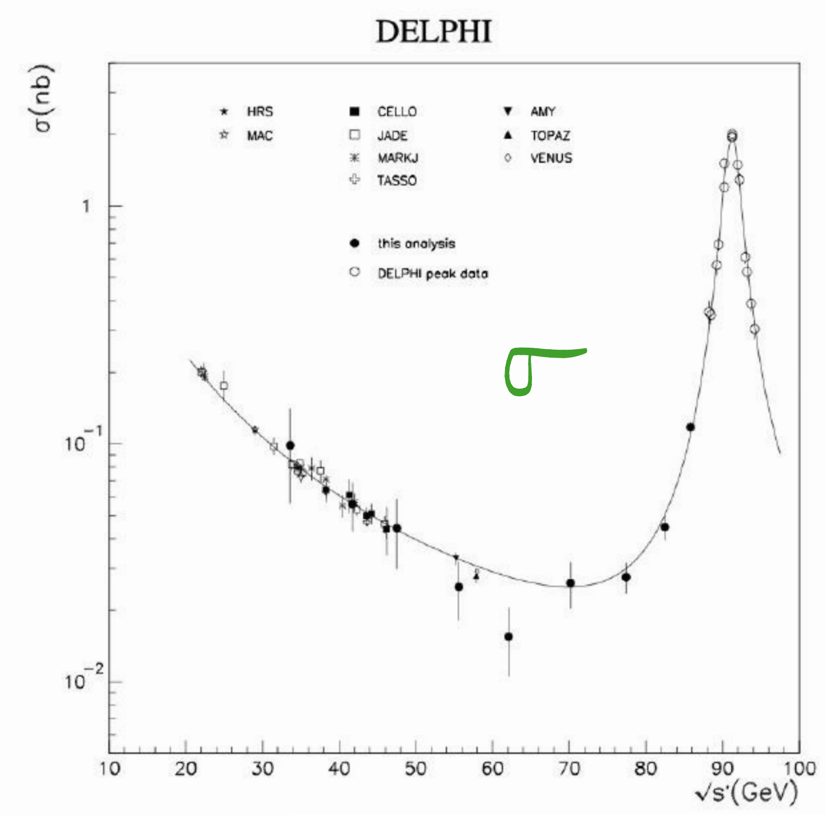
$\Rightarrow$ Inserting into Fermi's golden rule $[\, s = E_{CM}^2 \,;\; p_a \cdot p_1 = p_a^\mu \, p_{1,\mu} = E_{cm}^2 (1-\cos\theta) \,]$

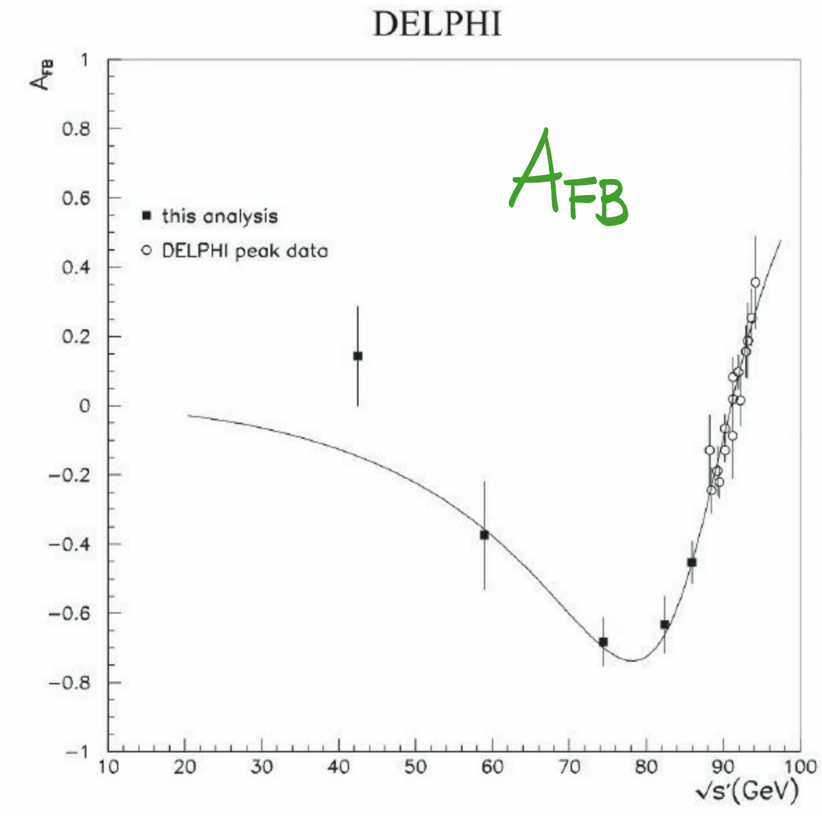
$$\frac{d\sigma}{d\cos\theta} = \frac{\alpha^2 \pi}{2s} \left[(1+\cos^2\theta)\, G_1(s) + 2\cos\theta \, G_2(s) \right]$$

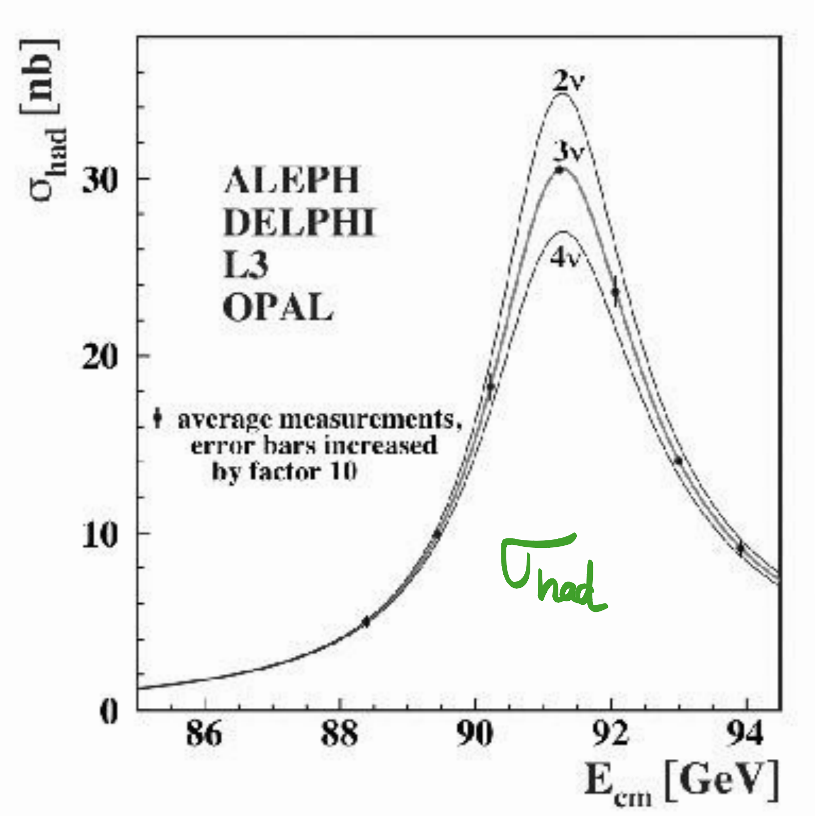
$$G_1(s) = 1 + 2 v_\ell^2 \, \mathrm{Re}\left\{ \frac{s}{s - M_Z^2 + i M_Z \Gamma_Z} \right\} + (v_\ell^2 + a_\ell^2) \left| \frac{s}{s - M_Z^2 + i M_Z \Gamma_Z} \right|^2$$

$$G_2(s) = 0 + 2 a_\ell^2 \, \mathrm{Re}\left\{ \frac{s}{s - M_Z^2 + i M_Z \Gamma_Z} \right\} + 4 v_\ell^2 \cdot a_\ell^2 \left| \frac{s}{s - M_Z^2 + i M_Z \Gamma_Z} \right|^2$$

"Comparison" to Data







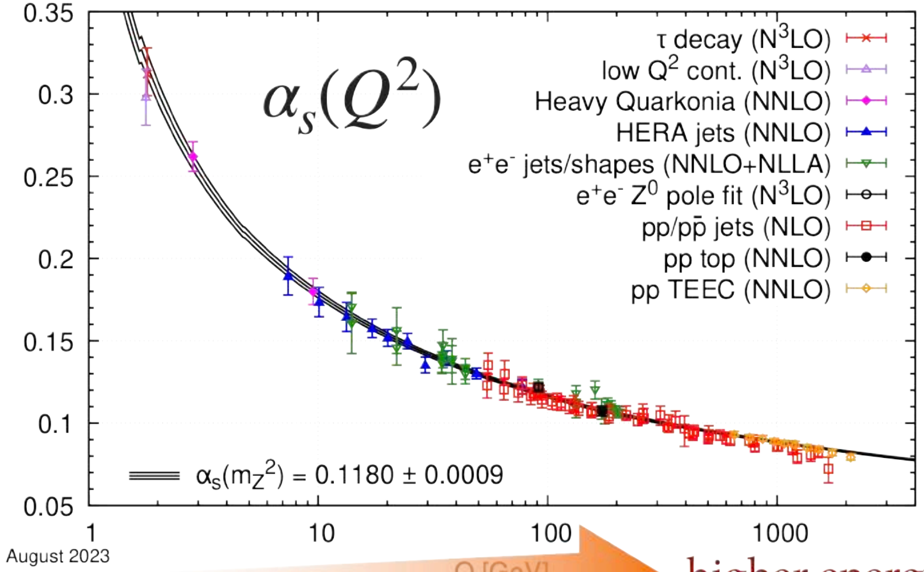
* In principle, you now can use the predictions to fit M_Z & $\sin^2\theta_W$ from the data (at leading order)
* σ_{had} is the hadronic cross section: @ LO: $e^+e^- \to q\bar{q}$
  ⇒ what changes compared to $\mu^+\mu^-$?

Hadron Colliders ...

... are "messy"

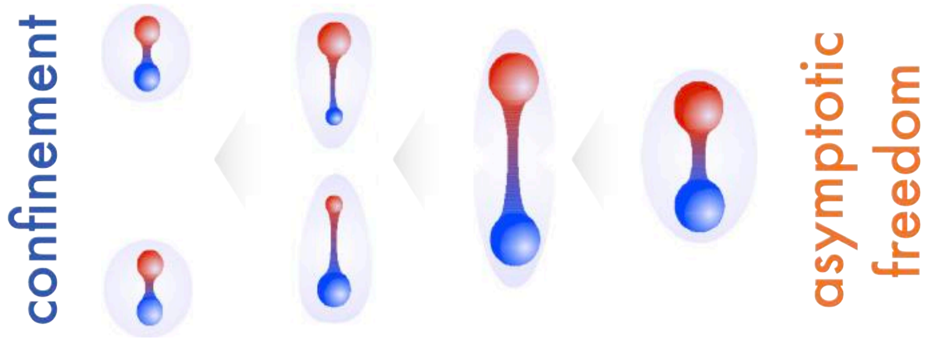
strong interaction ⟿ 

- Quantum Chromodynamics (QCD)



higher energy

larger distance



> "Quantum chromodynamics is conceptually simple. Its realization in nature, however, is usually very complex."
>
> Frank Wilczek

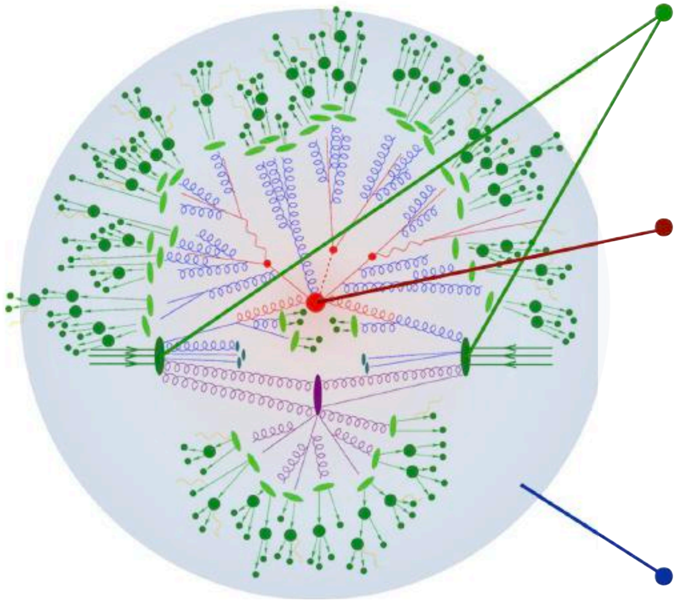
* no free quarks & gluons
  ⇝ spray of hadrons ($\pi^\pm, K^\pm, K^0, p^\pm, n, \ldots$)

* colliding objects (p @ LHC) <u>not</u> elementary

Hadron Colliders ...

... taming the mess

> “Quantum chromodynamics is conceptually simple. Its realization in nature, however, is usually very complex. But not always.”
>
> Frank Wilczek



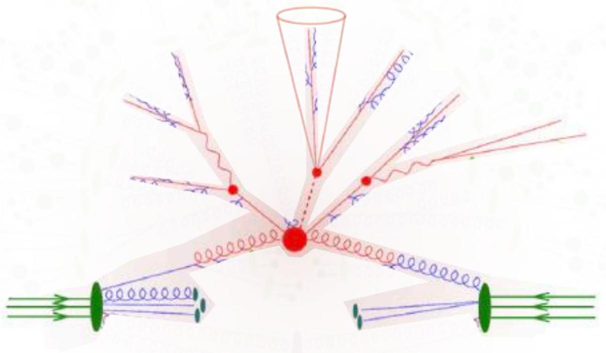
1. factorization
   - ⟶ relevant physics at disparate scales (isolates description of proton from rest)
2. asymptotic freedom
   - ⟶ short distance ⟷ perturbation theory

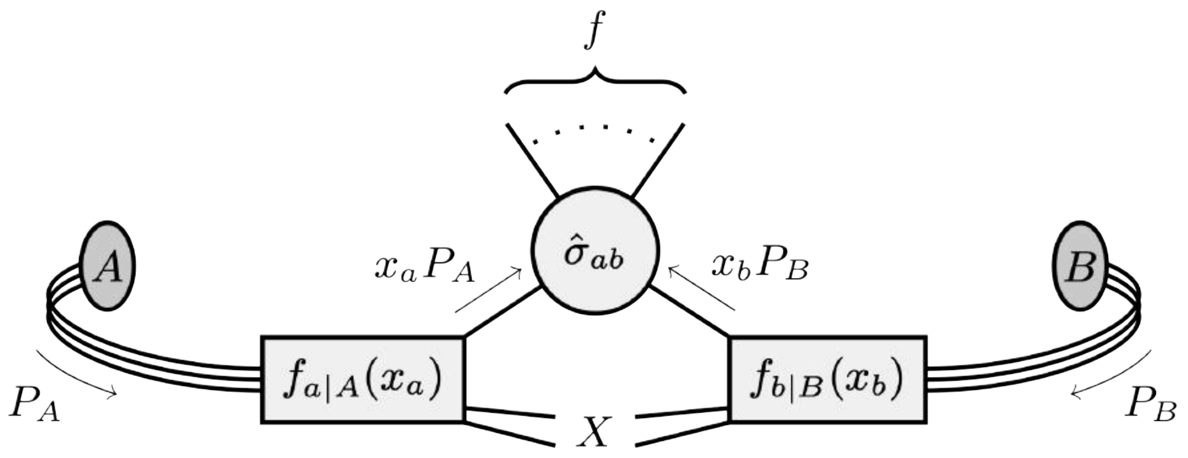
$$\sigma = \sigma_{LO} \left(1 + \alpha_s \, c^{(1)} + \alpha_s^2 \, c^{(2)} + \ldots \right)$$

Hadron Colliders: The Parton Model

strategy for precision:

* focus on high momentum transfer
* clean signatures ($\ell^\pm$, jets, ...)





$$d\sigma_{A+B\to f}(P_A, P_B)$$

$$= \sum_{a,b} \int_0^1 dx_a \int_0^1 dx_b \; f_a(x_a)\, f_b(x_b)\, d\hat{\sigma}_{a+b\to f}(x_a P_A, x_b P_B)$$

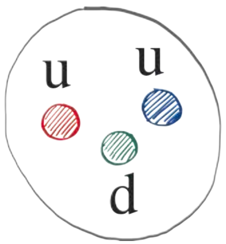
momentum fraction

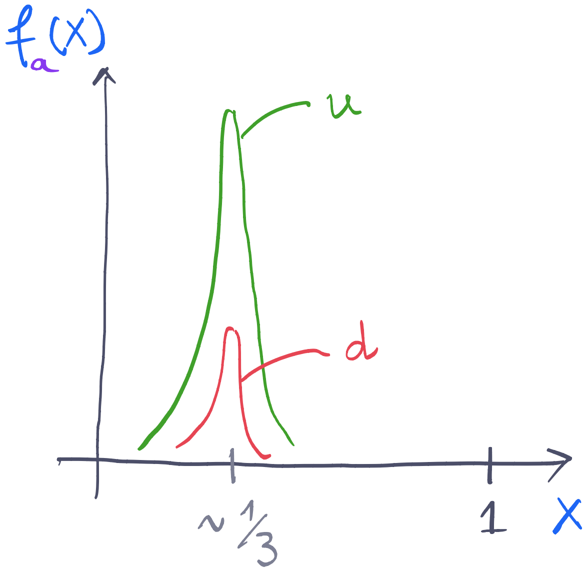
parton distribution function (PDF)

$\hat{=}$ number density for parton a w/ momentum fraction $[x_a, x_a+dx_a]$ of parent hadron A

Parton Distribution Functions

* just free quarks? ($p \simeq (uud)$)





$$f_u(x) \sim 2\,\delta(x - 1/3)$$

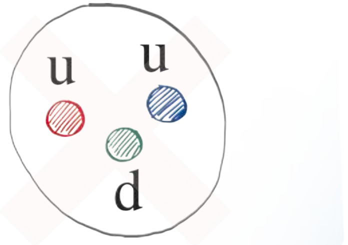
$$f_d(x) \sim 1\,\delta(x - 1/3)$$

$$f_{etc}(x) \sim \emptyset$$

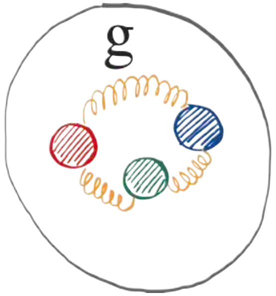
(+ some smearing)

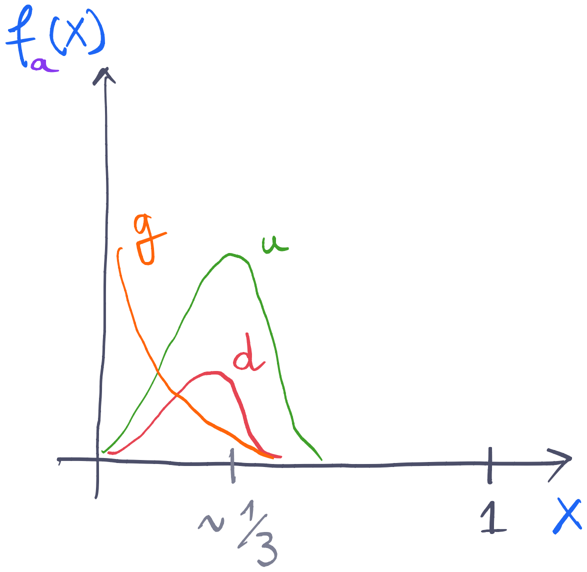
Parton Distribution Functions

* just free quarks? ($p \simeq (uud)$)



* bound by gluons?





naive parton model

$\Leftrightarrow$ composition of point particles

$\rightsquigarrow$ zoom in ($Q^2 \nearrow$) $\leftrightarrow$ same composition: scaling

PDFs independent on scale, at which it is probed

(as long as $Q^2 \gg m_p^2$)

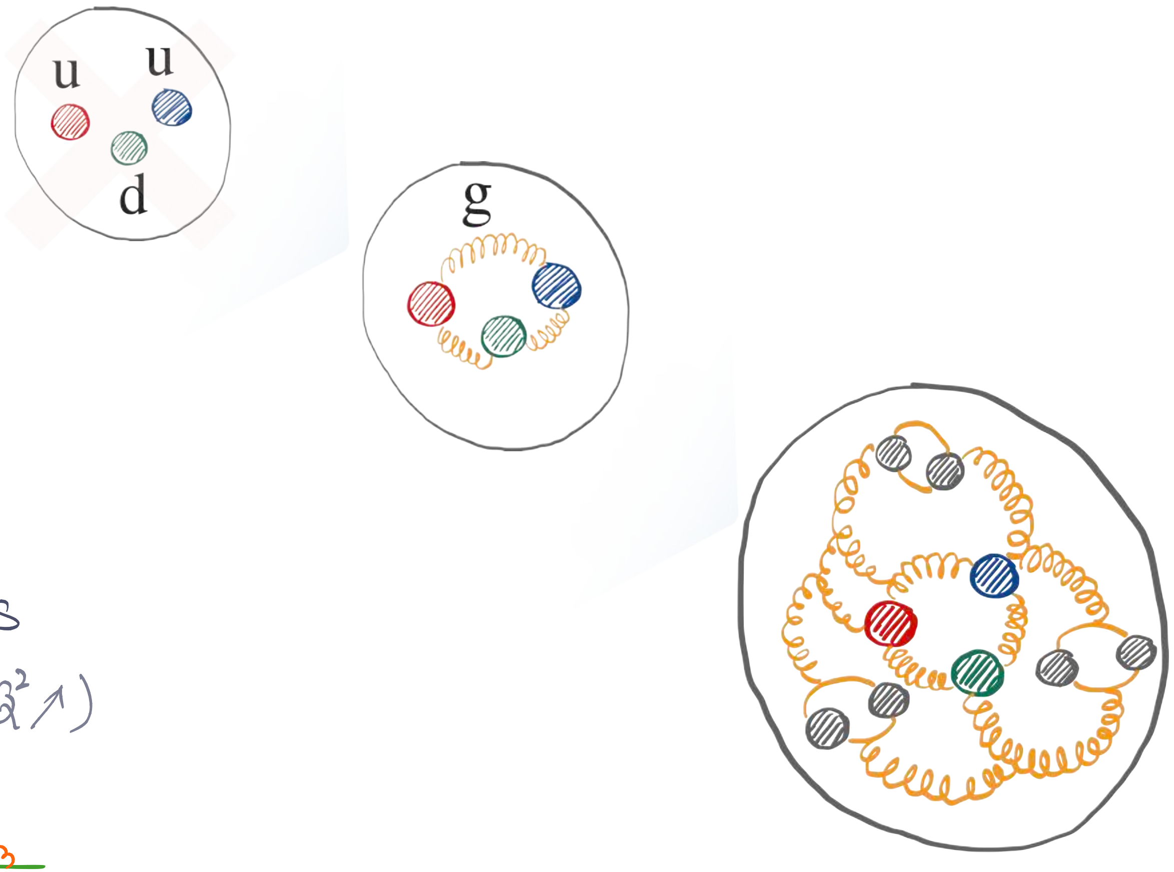
Parton Distribution Functions

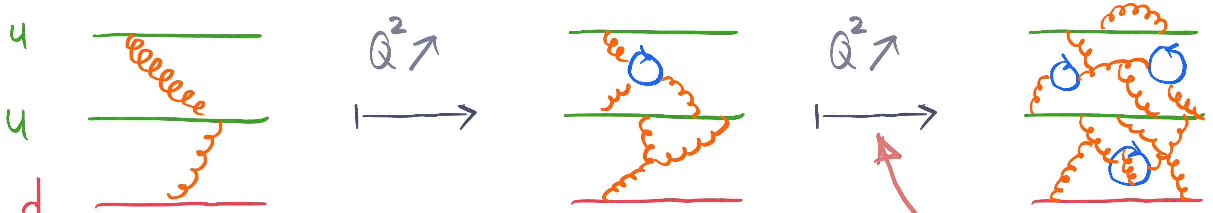
[demo: PDFs]

* just free quarks? ($p \simeq (uud)$)

* bound by gluons?

* QCD-improved parton model
  ↳ quantum fluctuations → more g & ($q\bar{q}$) as we "zoom in" ($Q^2 \nearrow$)





⇒ predominantly shifts partons from high-x to low-x

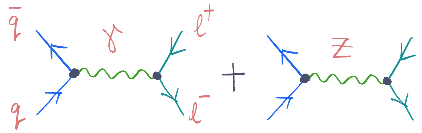
evolution is perturbatively calculable!
(test of QCD)

The Drell-Yan Process $p+p \to \ell + \bar{\ell}$

[demo: Drell-Yan]

at LO only

$(a,b) \in \{(q,\bar{q}), (\bar{q},q)\}$



$$d\sigma_{DY}(P_A, P_B) = \sum_{a,b} \int_0^1 dx_a \int_0^1 dx_b \; f_a(x_a)\, f_b(x_b)\; d\hat{\sigma}_{a+b \to \ell^+\ell^-}(x_a P_A, x_b P_B)$$

* Integrate out $Z \to \ell^+\ell^-$ decay
* Observables of intermediate gauge boson $q^\mu = (p_1+p_2)^\mu = (p_a+p_b)^\mu$

$$M_{\ell\ell} = \sqrt{q^2} \quad ; \quad Y_{\ell\ell} = \frac{1}{2} \ln\left(\frac{q^0+q^3}{q^0-q^3}\right)$$

rapidity: $Y \longmapsto Y + \frac{1}{2}\ln(x_a/x_b)$

$$\Rightarrow \boxed{\frac{d^2\sigma_{DY}}{dM_{\ell\ell}\, dY_{\ell\ell}} = f_a(x_a)\, f_b(x_b)\, \frac{2M_{\ell\ell}}{E_{cm}^2}\, \hat{\sigma}_{a+b\to\ell^+\ell^-}\Big|_{x_{a/b} = \frac{M_{\ell\ell}}{E_{cm}} e^{\pm Y_{\ell\ell}}}}$$