MAKING PREDICTIONS AT HADRON COLLIDERS

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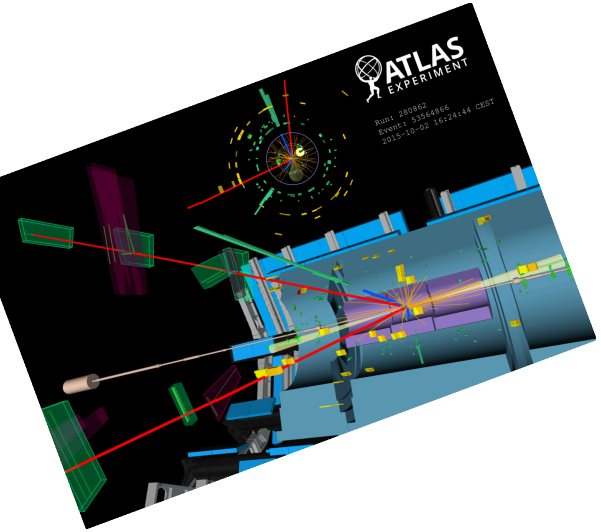
CERN Summer Student Lectures

Previously on "Making Predictions"



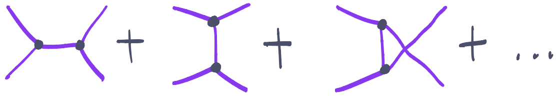
Event Rates

$$N = L\underbrace{\sigma}$$

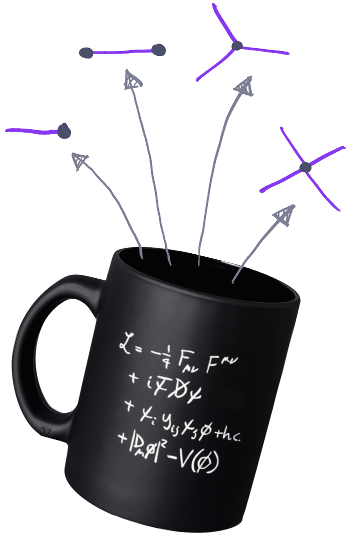
Cross Sections

$$d\sigma_{2\to n} = \frac{1}{F}\underbrace{\langle|\mathcal{M}|^2\rangle}\, d\Phi_n$$

Scattering Amplitudes



Feynman diagrams & rules $\longleftrightarrow$ $\mathcal{L}$



* another important case Decay Rates $(\tau = 1/\Gamma)$

$$d\Gamma_{1\to n} = \frac{1}{2M}\langle|\mathcal{M}|^2\rangle\, d\Phi_n$$

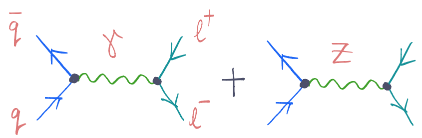
The Drell-Yan Process $p+p \to \ell+\bar{\ell}$

[demo: Drell-Yan]

$$d\sigma_{DY}(P_A,P_B) = \sum_{a,b} \int_0^1 dx_a \int_0^1 dx_b \; f_a(x_a)\, f_b(x_b)\; d\hat{\sigma}_{a+b\to\ell^+\ell^-}(x_a P_A, x_b P_B)$$

at LO only

$(a,b) \in \{(q,\bar{q}), (\bar{q},q)\}$

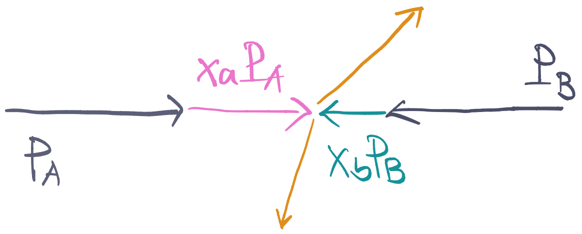


* Integrate out $Z \to \ell^+\ell^-$ decay
* Observables of intermediate gauge boson $\quad q^\mu = (p_1+p_2)^\mu = (p_a+p_b)^\mu$

$$M_{\ell\ell} = \sqrt{q^2} \quad ; \quad Y_{\ell\ell} = \frac{1}{2}\ln\left(\frac{q^0+q^3}{q^0-q^3}\right)$$

rapidity: $Y \longmapsto Y + \frac{1}{2}\ln\left(x_a/x_b\right)$

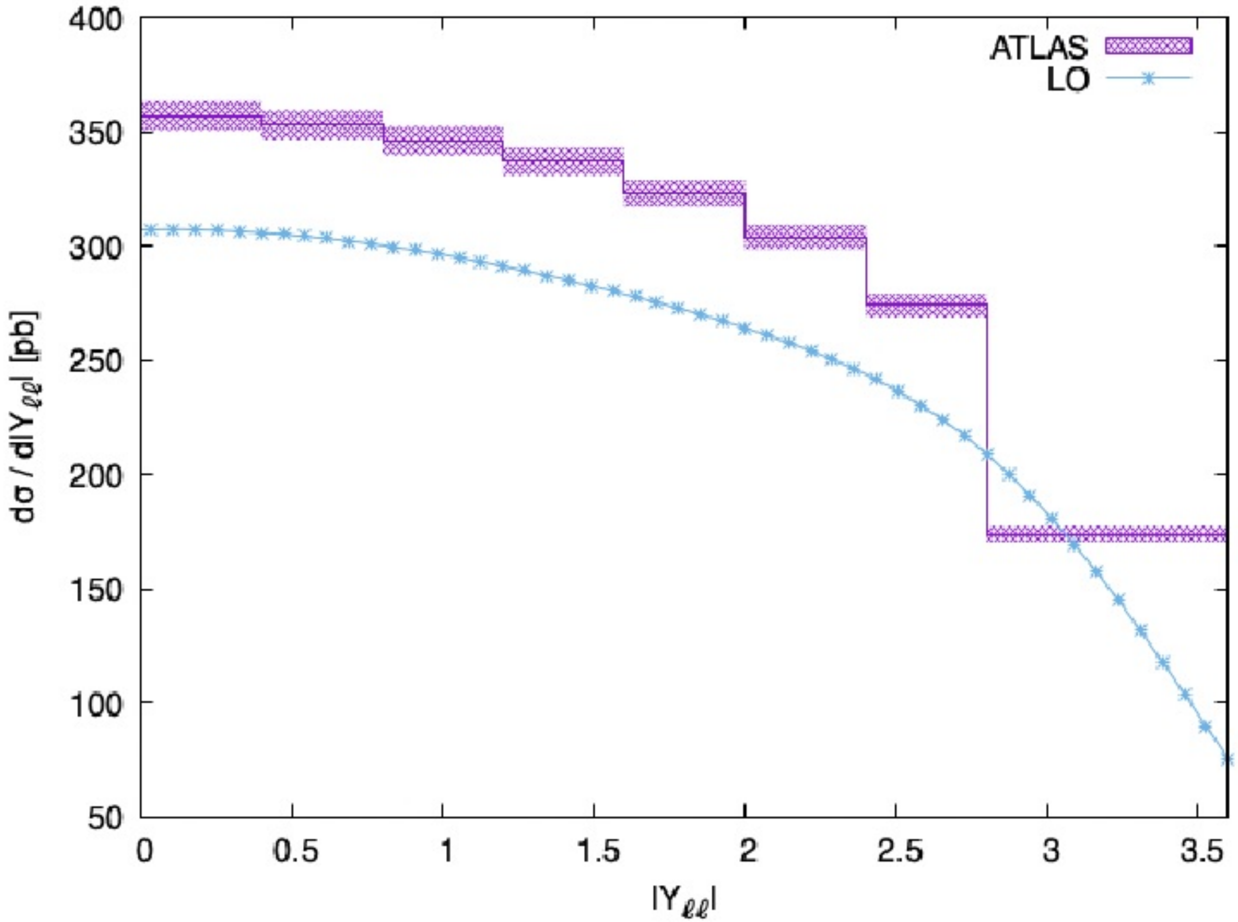
$$\Rightarrow \boxed{\frac{d^2\sigma_{DY}}{dM_{\ell\ell}\, dY_{\ell\ell}} = f_a(x_a)\, f_b(x_b)\, \frac{2M_{\ell\ell}}{E_{cm}^2}\, \hat{\sigma}_{a+b\to\ell^+\ell^-}\Big|_{x_{a/b} = \frac{M_{\ell\ell}}{E_{cm}} e^{\pm Y_{\ell\ell}}}}$$



Drell Yan at Leading Order

ATLAS $\sigma_Z = 1055.3 \ \pm 0.7\,(\text{stat.}) \ \pm 2.2\,(\text{syst.}) \ \pm 19.0\,(\text{lumi.})\,\text{pb}$

LO: #total 897.1018242299992 pb

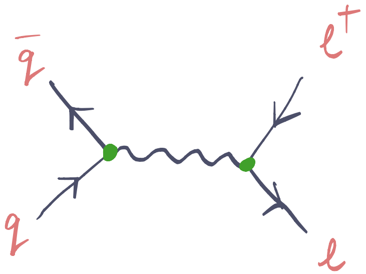


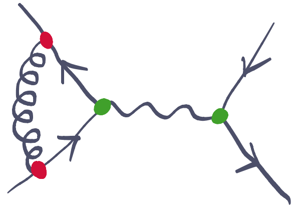
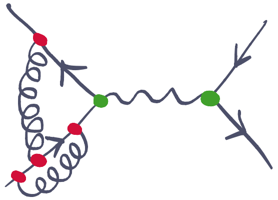
* right ballpark
  $\alpha_s \sim 0.1$
  ↳ $\mathcal{O}(10\%)$ anticipated
  ↳ often much worse!
  (extreme: $gg \to H$ 100%)
* @ LO only $q\bar{q}$ annihilation
  ↳ 50% of protons are g!
* no error estimate here
  ↳ no quantitative comp.

$\Rightarrow$ need at least NLO!

The Drell-Yan process

We saw that a leading order (LO) prediction in QCD is not sufficient for precision phenomenology } want to include higher order(s)! $\Rightarrow$ diagrams with loops

$\mathcal{M}_2 =$ 

$+$  $+ \dots +$  $+ \dots$

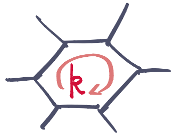
$\mathcal{O}(\alpha) \leftrightarrow \mathcal{M}_2^{(0)}$ $\quad$ $\mathcal{O}(\alpha_s \alpha) \leftrightarrow \mathcal{M}_2^{(1)}$ $\quad$ $\mathcal{O}(\alpha_s^2 \alpha) \leftrightarrow \mathcal{M}_2^{(2)}$

$$\Rightarrow |\mathcal{M}_2|^2 = \underset{\mathcal{O}(\alpha^2)}{|\mathcal{M}_2^{(0)}|^2} + \underset{\mathcal{O}(\alpha_s \alpha^2)}{2\,\mathrm{Re}\{(\mathcal{M}_2^{(0)})^* \mathcal{M}_2^{(1)}\}} + \underbrace{|\mathcal{M}_2^{(1)}|^2 + 2\,\mathrm{Re}\{(\mathcal{M}_2^{(0)})^* \mathcal{M}_2^{(2)}\}}_{\mathcal{O}(\alpha_s^2 \alpha^2)} + \dots$$

"virtual corrections"

Divergences in Loop Diagrams

QM $\rightsquigarrow$ sum over all possible intermediate configurations

↪ integration over unconstrained loop momentum: $\int \frac{d^4k}{(2\pi)^4}$ 

↪ results are plagued by divergences

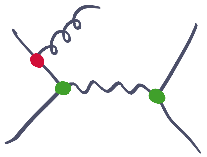
1. ultraviolet (UV) $\longleftrightarrow$ large loop momentum

   $\Rightarrow$ treated by renormalization $\alpha_s(\mu_R^2)$

2. infrared (IR) $\longleftrightarrow$ $\begin{cases} \text{small energy (soft) and/or} \\ \text{small angle (collinear)} \end{cases}$

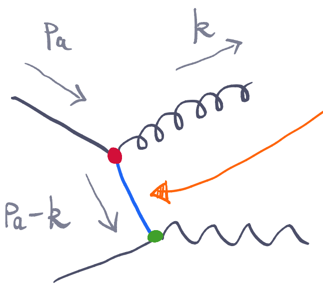
   $\Rightarrow$ only cancels after adding "real corrections"

technically, a different process but cannot be distinguished when unresolved

$$|\mathcal{M}_3|^2 = \Big|$$  $$+ \dots \Big|^2 \longleftrightarrow \mathcal{O}(\alpha_s \alpha^2)$$

Cancellation of Divergences

Consider this sub-diagram that appears both in the virtual & real



this propagator contains

$$\frac{1}{(p_a-k)^2} = \frac{1}{E_a E\,(1-\cos\theta)}$$

$\Rightarrow$ potential divergence when

(a) gluon "soft": $E \to \emptyset$

(b) gluon collinear: $\vec{k} \parallel \vec{p}_a$ $(\theta \to \emptyset)$

* performing the integration in $D = 4 - 2\epsilon$ dimensions, we expose the divergences as poles in ϵ

$\hookrightarrow$ "virtual" (loop) corrections

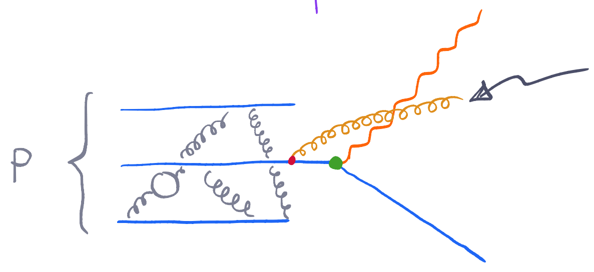
$$\hat{\sigma}_{LO}(p_a, p_b)\,\frac{\alpha_s}{2\pi}\,C_F\,\frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}\left\{\frac{1}{\epsilon^2} + \frac{3}{2}\frac{1}{\epsilon} + \text{finite}\right\}$$

$\hookrightarrow$ "real" emission corrections

$$\hat{\sigma}_{LO}(p_a, p_b)\,\frac{\alpha_s}{2\pi}\,C_F\,\frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}\left\{-\frac{1}{\epsilon^2} - \frac{3}{2}\frac{1}{\epsilon} + \text{finite}\right\}$$

$$+ \int_0^1 dz_a\,\hat{\sigma}_{LO}(z_a p_a, p_b)\,\frac{\alpha_s}{2\pi}\,C_F\,\frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}\left\{-\frac{1}{\epsilon} - \ln\left(\mu_F^2/Q^2\right)\right\} P_{qq}(z_a) + \begin{pmatrix} z_a \leftrightarrow z_b \\ q_{in} \leftrightarrow g_{in} \end{pmatrix}$$

absorbed as part of "NLO PDFs"

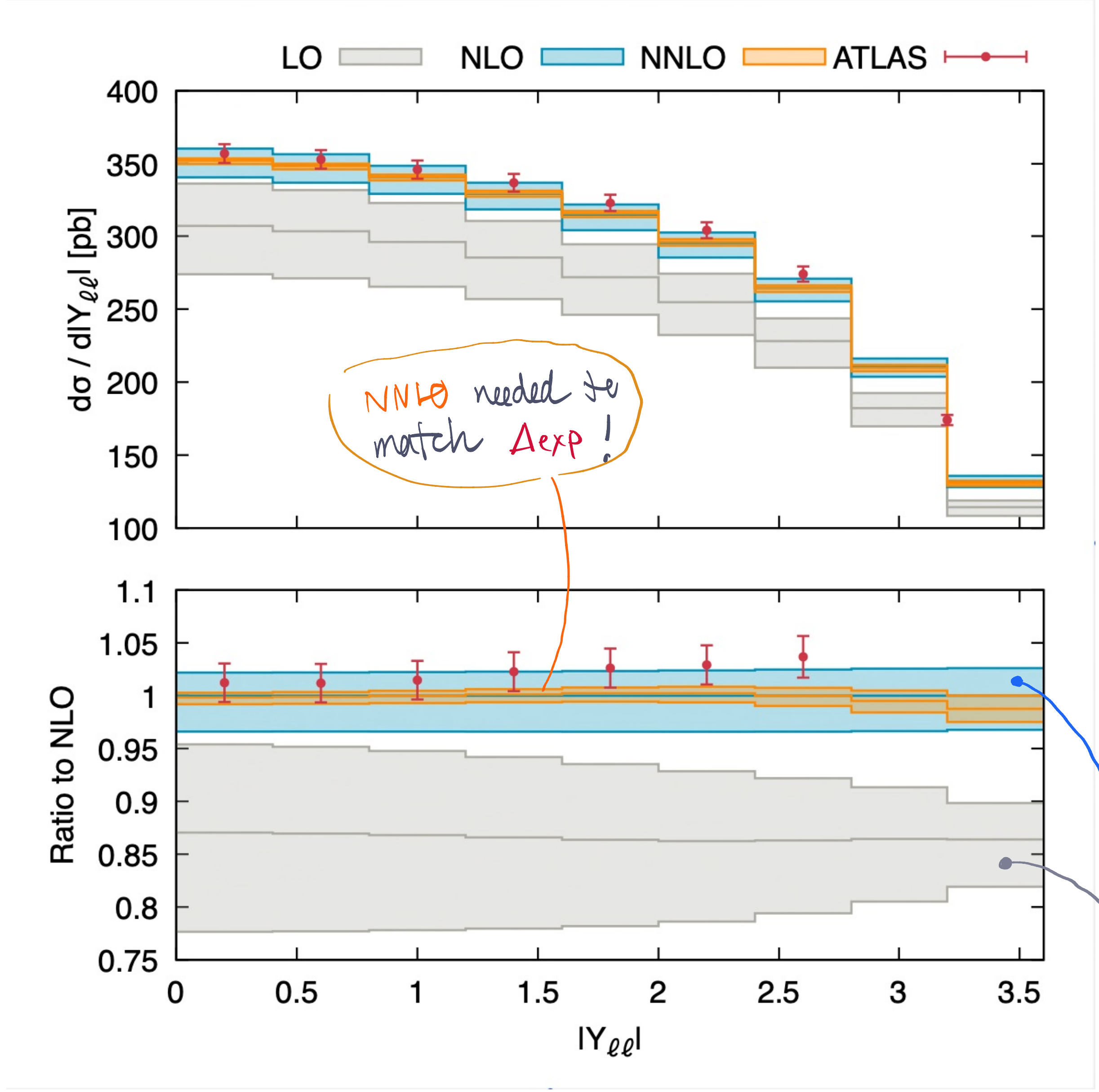


part of the proton if its scale $< \mu_F$

$f_a(x) \longmapsto f_a(x, \mu_F^2)$

universal $\leftrightarrow$ PDF evolution

The Drell-Yan process at higher orders



on repository

last lecture

Theory uncertainties

* missing higher orders

  arbitrary scales μ_R & μ_F in our calculation

  $\longleftrightarrow$ variation induces terms beyond the order we computed $\times\, [1/2, 2]$

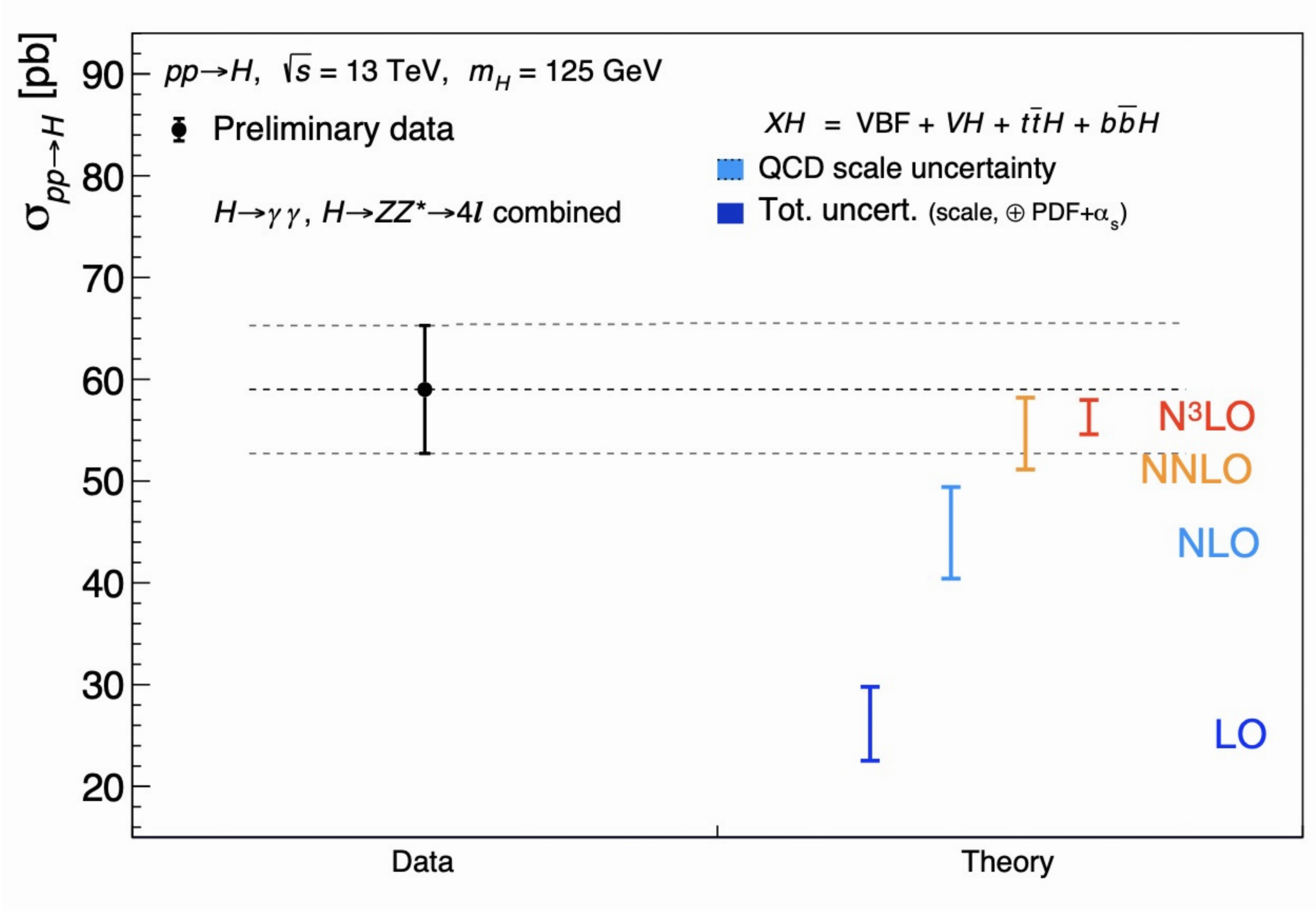
* parametric ($\alpha_s(M_Z)$, ...) (not included)

  $\nwarrow$ $\mathcal{O}(1\%)$

* PDF uncertainties (not included)

  $\nwarrow$ $\mathcal{O}(1\%)$

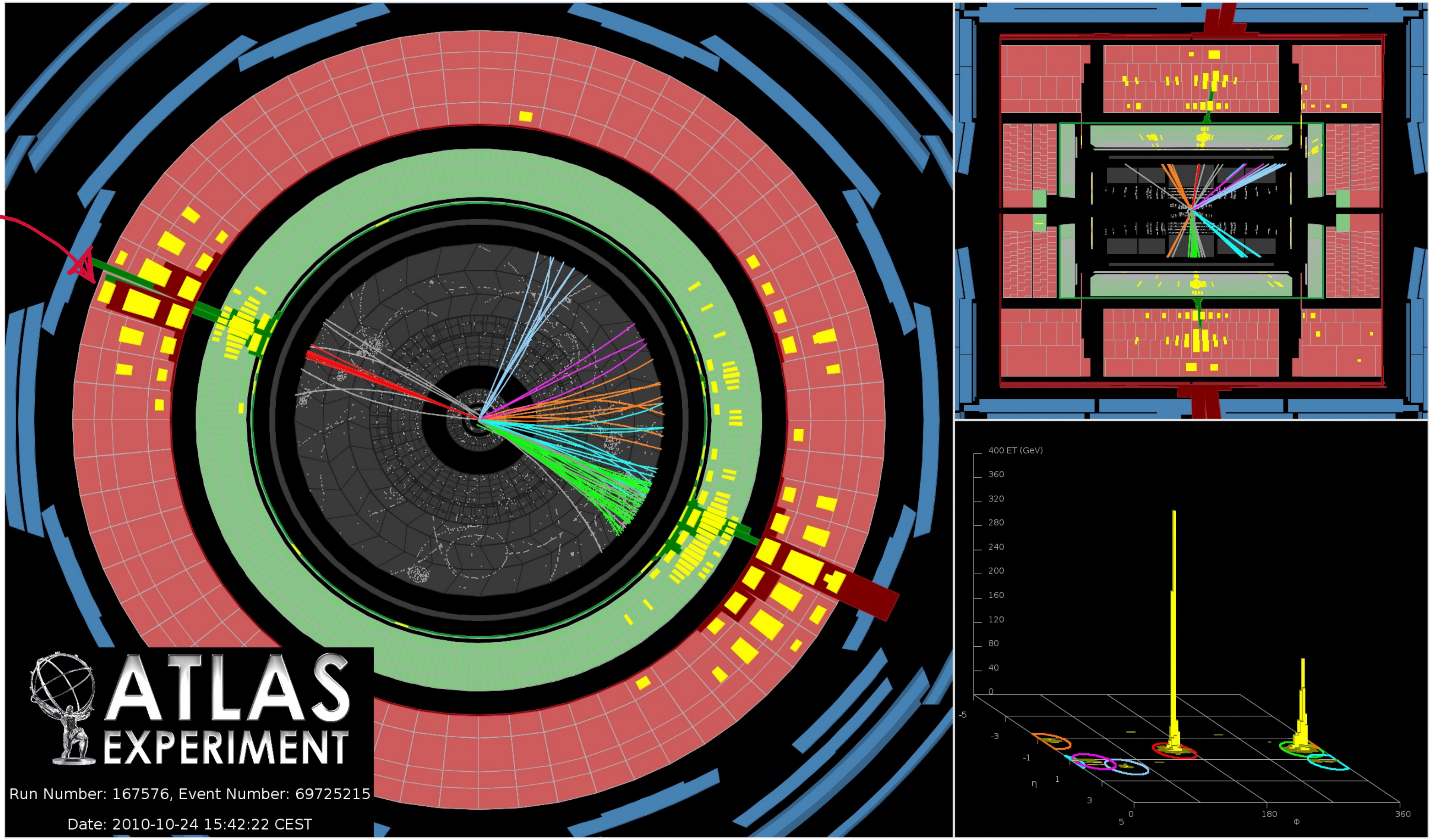
The state of the art in Fixed Order



* Leading order (LO)
  basically any process
* Next-to-leading order (NLO)
  most processes
  & automation
  (up to $2 \mapsto 8$)
* NNLO
  $2 \to 2$ done,
  steady progress for $2 \to 3$
* N^3LO
  limited to simple $2 \to 1$

Events at hadron colliders look more complex [demo: diagrams]

O(50) tracks



Why? Any chance to compute this with what we did so far?

Nope ...

```
2->2 gluon scattering has 4 diagrams
2->3 gluon scattering has 25 diagrams
2->4 gluon scattering has 220 diagrams
2->5 gluon scattering has 2485 diagrams
2->6 gluon scattering has 34300 diagrams
2->7 gluon scattering has 559405 diagrams
2->8 gluon scattering has 10525900 diagrams
2->9 gluon scattering has 224449225 diagrams
2->10 gluon scattering has 5348843500 diagrams
2->11 gluon scattering has 140880765025 diagrams
```

10^{11} !

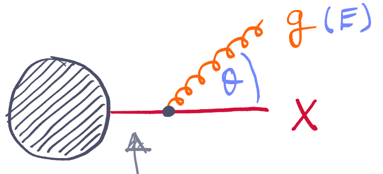
* Even with more efficient recursive approaches, not the multiplicities you'd want to tackle

$\# \dim(\Phi_n) = 3n-4$

↳ define observables that "map back" the physics to fewer initiating objects } JETS

↳ identify the relevant physics & model the full complexity approximately } Parton Showers

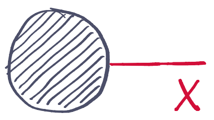
The QCD emission pattern



gluon

soft & collinear

enhanced (divergent) $\sim \dfrac{1}{E_X\, E\, (1-\cos\theta)}$



$\times \; d\omega_{X\to X+g}$

$$\boxed{2\,\frac{\alpha_s}{\pi}\, C_X\, \frac{dE}{E}\, \frac{d\theta}{\theta}}$$

$$C_X \begin{cases} = C_F = 4/3 & \text{if } X = q \\ = C_A = 3 & \text{if } X = g \end{cases}$$

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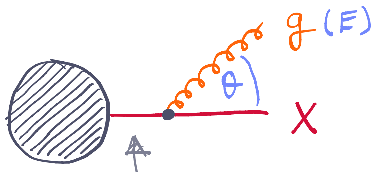

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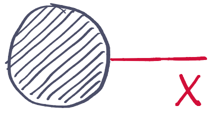

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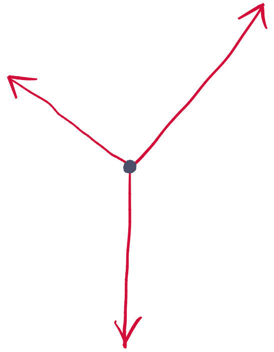
$\Rightarrow$ jets are an emergent feature of QCD

1. high energetic partons
   $\hookrightarrow$ hard scattering



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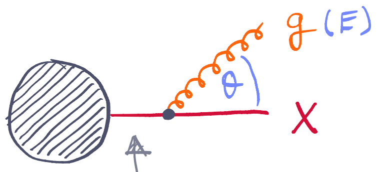

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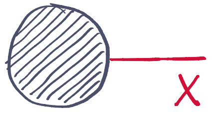

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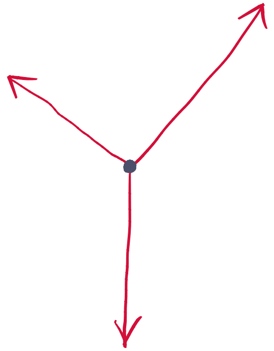
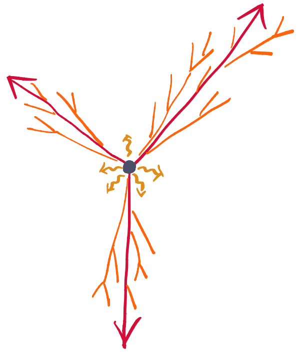
⇒ <u>jets are an emergent feature of QCD</u>

1. high energetic partons
   ↪ hard scattering

2. asymp. freedom & $d\omega$
   ↪ pert. parton shower





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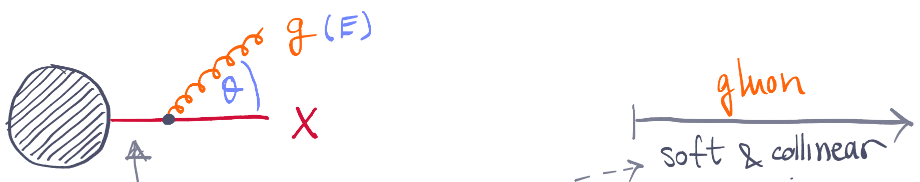

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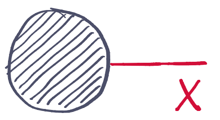

$\times \; d\omega_{X \to X+g}$

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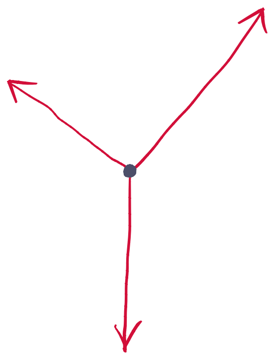

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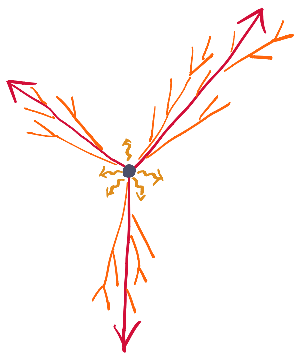
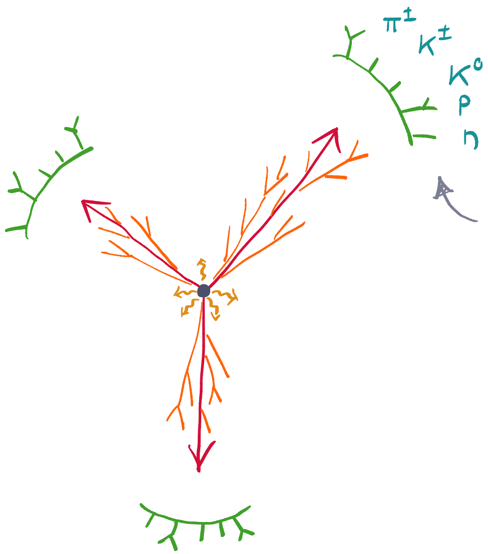


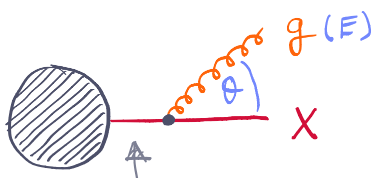
3. hadronization



very "cheap" to pop a $(q,\bar{q})$ pair out of the vacuum

$m_{u/d} \ll \Lambda_{QCD}$

The QCD emission pattern



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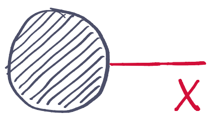

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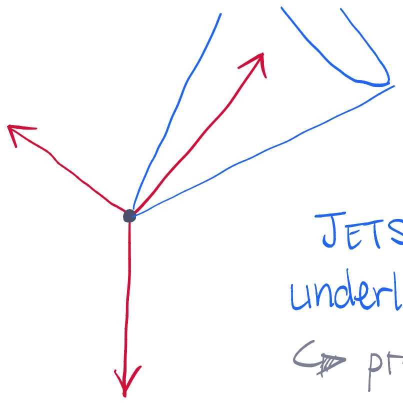
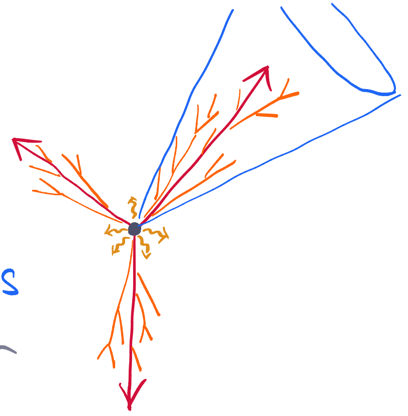
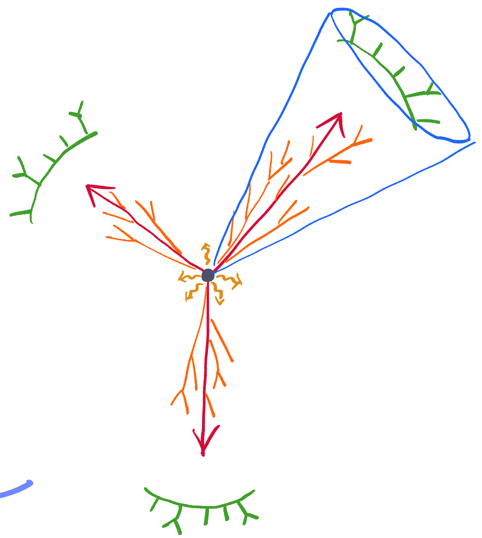
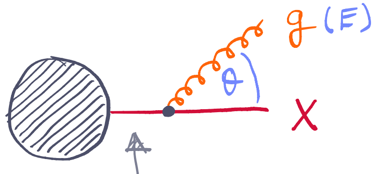


JETS "map back" to underlying hard partons
↪ precision fixed order





The QCD emission pattern



gluon: soft & collinear

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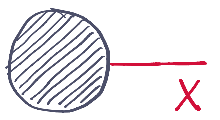

$\times \; d\omega_{X\to X+g}$

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$$C_X \begin{cases} = C_F = 4/3 & \text{if } X = q \\ = C_A = 3 & \text{if } X = g \end{cases}$$

$\Rightarrow$ <u>emission factorizes!</u>

Integral over E & θ diverges $\Rightarrow$ $\left\{ \begin{array}{l} \text{introduce a scale } q^2 > Q_0^2 \\ \leftrightarrow \text{ emission "resolved"} \end{array} \right.$

$$\Rightarrow \; P_X \simeq \frac{\alpha_s C_F}{2\pi}\, \underbrace{\ln^2\!\left(Q^2/Q_0^2\right)}_{} + \mathcal{O}\!\left(\alpha_s \ln Q^2,\ \alpha_s^2\right)$$

probability to emit a resolved gluon

potentially a very large log $\rightsquigarrow$ will want to "resum" these to all orders

$$\left[\; \left.\begin{array}{l} Q_0 = \Lambda_{QCD} = 0.2 \text{ GeV} \\ Q = 100 \text{ GeV} \end{array}\right\} \Rightarrow \ln(\ldots) = \mathcal{O}(10) \;\right]$$

Parton Showers

* We wish to account for an arbitrary number of emissions ordered in our resolution variable $Q^2 > q_1^2 > q_2^2 > \ldots > Q_0^2$ (stong ordering)
* current scale $q_n^2 \rightsquigarrow$ probability to have next emission @ q_{n+1}^2 ?

$$\underbrace{\left(\text{probability of having } \underline{\text{no emissions}}\ q_n^2 \mapsto q_{n+1}^2\right)}_{\Delta(q_n^2,\, q_{n+1}^2)} \times \underbrace{\left(\text{emission @ } q_{n+1}^2\right)}_{\left.\frac{d\omega_{x\to x+g}}{dq^2}\right|_{q^2=q_{n+1}^2}}$$

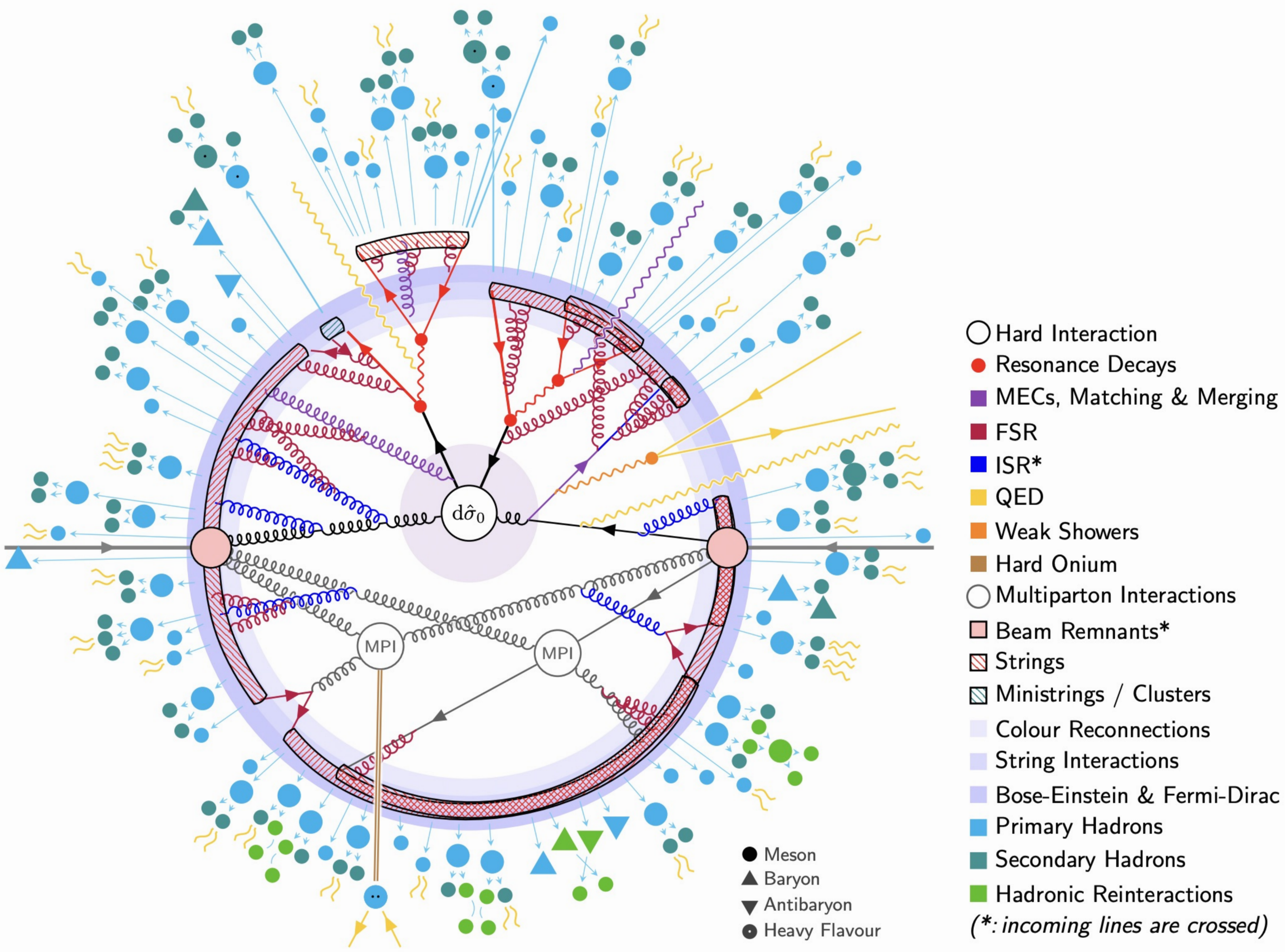
[demo: toy shower]

Sudakov formfactor

$$\hookrightarrow \quad \frac{d\Delta(Q^2,q^2)}{dq^2} = \Delta(Q^2,q^2)\,\frac{d\omega}{dq^2}$$

$$\left[\Delta(Q^2,q^2-dq^2) = \Delta(Q^2,q^2)\,\Delta(q^2,q^2-dq^2) = \Delta(Q^2,q^2)\left(1-\frac{d\omega}{dq^2}\right)\right]$$

Full Event Generator



+ pile-up

Conclusions

* covered basic ingredients that goes into hadron-collider predictions
  ↳ a key idea: seperation of scales ("factorization")

* Moment of comparing your predictions to data always exciting
  ↳ learn to play with the tools; break them (often interesting physics)

* Hope was able to lower the fear of entry for some of you,
  as it is sometimes perceived as very technical
  ↳ pushing frontiers in precision can become arbitrary complex
  new ideas needed (maybe one of you?)

Thank you for your attention & participation!