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Connectivity Analysis of Device-to-device Cellular Network Using Stochastic Geometry

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Abstract

Device-to-device (D2D) communication enables direct communication between nearby mobiles that improves spectrum utilization, overall throughput and energy efficiency. The thesis considers the performance of the D2D communication, i.e. connectivity with the aid of mobile users acting as signal relays, and compares with direct cellular link. Using stochastic geometry, a model of independent homogeneous Poisson point processes is made with the incorporation of contact distance probability, Rayleigh fading and a noise-limited environment. As a result, a connectivity performance is obtained. Also, in order to represent the achieved result, a series of numerical experiments were carried out and are displayed in the thesis.

Contents

1 Thesis introduction

A constant need to increase the network capacity for meeting the growing demands of the subscribers has led to the evolution of cellular communication networks from the first generation (1G) to the fifth generation (5G). An emerging facilitator of the upcoming high rate demanding next generation networks (NGNs) is a device-to-device (D2D) communication. Device-to-device (D2D) communication enables direct communication between nearby mobiles that improves spectrum utilization, overall throughput and energy efficiency.

The D2D communication usually is categorized based on the spectrum in which the communication occurs. In this sense, 2 major categories are derived: the communication that occurs in the cellular spectrum (inband D2D) and the one that exploits unlicensed spectrum (outband D2D). According to [7], the inband D2D can be further decomposed into underlay and overlay categories: in the underlay D2D communications cellular and D2D communications share the same radio resources; while in the overlay ones a portion of cellular radio resources is given specifically for D2D links. The features of the inband D2D communications are its advantages – both underlay and overlay kinds of this type of D2D communications improve the spectrum efficiency. But along with it, as D2D links are formed in the same spectrum as the cellular links, the key disadvantage is the interference caused by D2D and cellular users to each other.

On the other hand, using unlicensed D2D spectrum (outband D2D communication) eliminates the interference issue. But introduces another type of challenge - using unlicensed spectrum requires a special interface: outband D2D may suffer from the uncontrolled nature of unlicensed spectrum. Now a brief overview of the literature on D2D communications will be given, according to the [7]: each studied will be reviewed it terms of the problem addressed, the introduced solution and the a portion of statistical results. In [8] interference in the uplink spectrum is studied. The proposed interference mitigation methods are: 1) D2D users read the resource block allocation information from the control channel, thus resource blocks being used by cellular users are not disturbed; 2) Broadcast the expected interference from D2D communication on cellular resource block to all D2D users, thus in order to keep the interference from D2D users to uplink transmission below a certain threshold D2D users can adjust their transmission power and resource block selection; 3) numerical simulations showed the improvement of the system throughput by 41%.

In [9] Han-Kobayashi rate splitting techniques are proposed to use to improve the throughput of D2D communications. As a message is divided into private (decoded only by the addressee) and public (decoded by any receiver of the message) parts, this technique along with the 'best-effort successive interference cancellation' algorithm cancel the interference from the public part. Numerical simulations with D2D pair positioned close to each other and far from the base station, showed a 650% increase of the cell throughput. A capacity of cellular networks is addressed in [10]. A joint D2D communication and network coding scheme are proposed. The model considers cooperative networks, where D2D communication is used to exchange uplink messages among cellular users before the messages are transmitted to the base station. The idea is to group the users with complementary characteristics to enhance the performance of network coding. Numerical simulations show a 34% increase in the network capacity. It's worth noting that multi-antenna capability contributes to interference reduction from a base station and to 30% increase of D2D users. In terms of **power efficiency**, the work [11] introduces an algorithm for power allocation and mode selection in D2D communication. This algorithm considers power efficiency of users in cellular and D2D modes (as a function of transmission rate and power consumption). Based on the measurements of each user, it switches to the mode with the highest power efficiency. The drawback of such algorithm is exhaustive search for all possible combinations of modes for all devices. As opposed to [12] scheme, the simulation indicate an up to 100% gain. In [13] the user spectrum utility (combination of users' data rates, power expenditure and bandwidth) is considered. Optimal transmission power for cellular and D2D modes are obtained, then with the evolutionary game mode selection is simulated (base station collects the users' decisions on mode selection and broadcasts the information to all users to aid the future mode selections). Numerical results show a 33% increase in cellular mode and an up to 500% - in D2D mode. The authors of [14] propose a way of enlarging cellular coverage. A node within the base station service range depending on the conditions and traffic can be associated with a relay node. Close to each other nodes are grouped. Base stations serve the groups according to 'Round-Robin' scheduling policy mitigating the interference. Monte-Carlo simulations show an improvement from 150% to 300% in terms of throughput for cell edge users. The study on using D2D communication as a disaster relief solution is done in [1]. The authors consider an interference included environment, and aim to derive a tractable way to calculate performance a D2D enabled network with some damaged cellular areas. Chain relaying is adopted in order to extend the cellular coverage. Monte-Carlo simulation indicate the damage equivalent to 50-70% of the capacity of the network can be absorbed by D2D nodes and still maintain its nominal service.

Content distribution is considered as a way of D2D communication usage: in [15] a location-aware scheme is proposed - it keeps track of the location of every user and his requests. This way, a base station doesn't require the content re-transmission that has already been transmitted to a nearby user. Thus a reduction in transmission delay and a increase of the network capacity can be achieved. Though high control overhead and battery drain issues might need an additional consideration. As of the **overlaying inband D2D**, a work of [16] suggest an incremental relay mode for D2D communication. D2D transmitters multicast to both D2D receiver and a base station, and in case of D2D reception failute, the base station retransmits a copy to the D2D receiver. The scheme reduces the outage probability of D2D transmissions. Numerical simulations indicate a 40% improvement of the cell throughput compared to underlay mode. Another example of multicast usage in overlay D2D communication is [17]. The study aims to improve the performance of multicast transmissions. In case of incorrect data reception, a multicast group is proposed to use D2D communication to enhance multicast performance (those members of the group who couldn't decode the message will get the decoded message from other members of the group who managed to decode it). Novelty of this study is in order to maximize the spectral efficiency, a dynamic change of the number of transmitters is allowed. Numerical simulations show a 90% decrease of spectrum resource consumption compared to the one retransmitter scenatio.

Now, the **outband D2D**. In [18] and [19] D2D communications is considered to improve throughput and energy efficiency of cellular networks. The idea is to form clusters of cellular users within a WiFi communication range. Each cluster then has a cluster head (based upon the highest cellular channel quality), responsible for distributing the content from the base station, as well as forwarding the date from other cluster members to the base station. Thus, spectral and energy efficiency are increased. Numerical simulations show a 50% improvement of throughput and 30% - of energy efficiency, compared to 'Round-Robin' schedulers. An almost perfect fairness is also achieved. Another study, [20], addresses the video transmission scenario in cellular networks using D2D communications. The idea is to use the property of asynchronous content reuse (by combining D2D communication and video caching on the devices). Under assumption of fixed data rate and no power control over D2D link, the purpose is to maximize per-user throughput constrained to the outage probability. Numerical simulations indicate the proposed method achieve at least 10000% gain compared to conventional broadcasting methods and a 1000% gain over the coded broadcasting methods.

Contribution

This study considers a D2D communication enabled cellular network with no inband or outband specification. The environment assumes noise-limited situation with signal fading effect. Thus, 2 sets of points are considered - base stations and mobile users, with initial parameters of density, transmission power, signal propagation exponent and the service threshold. The purpose of the model is to estimate the performance of such network and compare it with conventional cellular-only model by means of numerical experiments. The study incorporates stochastic geometry to derive analytical results for scenarios without fading effect and involving it.

Paper structure.

The work starts off with the list of the main definitions of tools used later in the text; next section inroduces the problem formalization and performance evaluation characteristics. Afterwards, scenarios without fading effect are described: 1) cellular link; 2) single D2D relay link; 3) miltiple D2D relay link. The description involves the derivation of the analytical result for performance estimation. Next section describes scenarios involving fading effect: 1) cellular link; 2) multiple D2D relay link; The analytical formulas are derived as well. The numerical experiments are described in section 6: the parameters list used for experimenting and graphical representation of the results. Section 7 concludes the study. Appendix includes the plots for scenarios representation, necessary proofs of the derivation process.

2 Main definitions

Symbol	Meaning
D2D	Device-to-device
Φ_u	Poisson point process of mobile users
λ_u	Mobile users denisty
Φ_b	Poisson point process of base stations
λ_b	base stations density
r	Distance between the receiver and its serving base station
R_1	Radius of the search disk of a base station
R_2	Radius of the search disk of a mobile user
β_u	Propagation exponent mobile user-mobile user
β_b	Propagation exponent base station-mobile user
P_u	Transmission power of a mobile user
P_b	Transmission power of a base station
Θ_u	Service threshold at a mobile user (Downlink)
Θ_b	Service threshold at a base station (Uplink)
N_u	Mobile user thermal noise
N_b	Base station thermal noise

Poisson point process.
The Poisson point process on $\mathbb{R}^d$ with intensity measure Λ is a point process s.t.

1. for every compact set $B \subset \mathbb{R}^d$, N(B) has a Poisson distribution with mean Λ. If Λ admits a density λ, we may write:

$$\mathbb{P}(N(B) = k) = \frac{\left(\int_B \lambda(x)dx\right)^k}{k!} e^{-\int_B \lambda(x)dx};$$

2. if $B_1, B_2, ..., B_m$ are disjoint compact sets, then $N(B_1), N(B_2), ..., N(B_m)$ are independent.

The homogeneous PPP is a special case where $\Lambda(B) = \lambda|B|$

Signal-to-Noise Ratio

$$SNR = \frac{S}{N},$$

where S – serving signal, carrying the necessary information; and N – thermal noise of the operating unit.

Discrete Case.
Suppose that X is a random variable for the experiment, taking values in a set S. Suppose that X has a discrete distribution with probability mass function $g(x) = \mathbb{P}(X = x)$.
If E is an event in the experiment and A is a subset of S then

$$\mathbb{P}(E, X \in A) = \sum_{x \in A} g(x)\mathbb{P}(E|X = x) \tag{1}$$

Continuous Case.
Based on the characterization of the discrete case, define the conditional probability in a continuous case.
Suppose that X has a continuous distribution on $S \subseteq \mathbb{R}^n$, with probability density function g. Assume that $g(x) > 0$ for $x \in S$.
For any measurable subset A of S:

$$\mathbb{P}(E, X \in A) = \int_A g(x)\mathbb{P}(E|X = x)dx, x \in S \tag{2}$$

Conditional expected value
For any event $A \in \mathcal{A} \subseteq \mathcal{B}$, define the indicator function:

$$\mathbb{1}_A \omega = \begin{cases} 1 & : \omega \in A \\ 0 & : \omega \notin A \end{cases}$$

which is a random variable. Then the conditional probability given $\mathcal{B}$ is a function $\mathbb{P}(\cdot|\mathcal{B}) : \mathcal{A} \times \Omega \to (0, 1)$, such that $\mathbb{P}(A|\mathcal{B})$ is the conditional expectation of the indicator function of A:

$$\mathbb{P}(A|\mathcal{B}) = \mathbb{E}(\mathbb{1}_A|\mathcal{B})$$

In other words, $\mathbb{P}(A|\mathcal{B})$ is a $\mathcal{B}$-measurable function satisfying

$$\int_B \mathbb{P}(A|\mathcal{B})(\omega)dP(\omega) = \mathbb{P}(A \cap B), \forall A \in \mathcal{A}, B \in \mathcal{B} \tag{3}$$

or,

$$\int_B \mathbb{E}(\mathbb{1}_A|\mathcal{B})(\omega)dP(\omega) = \mathbb{P}(A \cap B) \tag{4}$$

Distance to the nearest base station

Since each user communicates with the closest base station, the probability density function of d can be derived using a simple fact that the null probability of a PPP in an area A is $e^{-\lambda|A|}$:

$$\mathbb{P}[d > R] = e^{-\lambda\pi R^2}$$

Therefore,

$$\mathbb{P}[d \leq R] = 1 - e^{-\lambda\pi R^2},$$

and

$$f_d(r) = 2\lambda\pi r e^{-\lambda\pi r^2}$$

3 Problem Statement and Performance characteristics

3.1 System model

The study considers 2 independent homogeneous stationary Poisson point processes with given densities:

1. (Φ_b, λ_b) - set of base stations;
2. (Φ_u, λ_u) - set of mobile users.

Respective initial parameters (a transmission power P, a signal propagation exponent β, a thermal noise value N and a service threshold Θ) are also given:

1. $(P_b, \beta_b, N_b, \Theta_b)$ - parameters of the base stations;
2. $(P_u, \beta_u, N_u, \Theta_u)$ - parameters of the mobile users.

The random channel effects include only a Rayleigh fading with a mean 1. The environment is noise-limited. The receiver is located at the origin o of $\mathbb{R}^2$. The location of the nearest base station is denoted as B_{nst}. The distance between the receiver and its nearest serving base station is denoted as d_b. The distance between 2 mobile users located near each other is denoted as d_u. The objective is to calculate the performance of the link between the receiver and it nearest serving base station.

3.2 Performance evaluation

The main performance characteristics in this model is Signal-to-Noise ratio (SNR):

$$SNR = \frac{S}{N},$$

representing the signal with the necessary message (S) and the thermal noise of the transmitter (N). For scenarios without fading the SNR would look like:

$$SNR_{nf} = \frac{S}{N} = \frac{P \times d^{-\beta}}{N},$$

where P is a transmission power of a node, $d^{-\beta}$ – a path-loss function, comprised of the distance between the nodes d and the signal propagation exponent β.
For scenarios involving the fading effect, a random variable h is incorporated that follows an exponential distribution with mean $1/\mu$ $\Big(h \sim \exp(\mu)\Big)$:

$$SNR_f = \frac{P \times h \times d^{-\beta}}{N}$$

The purpose of the thesis is to consider the following probabilities:

$$p_{nf} = \mathbb{P}\Big[SNR_{nf} \geq \Theta\Big] \tag{5}$$

$$p_f = \mathbb{P}\Big[SNR_f \geq \Theta\Big] \tag{6}$$

4 Considered cases under "non-fading channel" assumption

Let's investigate (5) a bit deeper in terms of the link between the receiver and its nearest serving base station:

$$p_{nf}^{cel} = \mathbb{P}[SNR_{nf}^{cel} \geq \Theta_b] = \mathbb{P}\left[\frac{P_b d_b^{-\beta_b}}{N_b} \geq \Theta_b\right] = \mathbb{P}\left[d_b \leq \Big(\frac{P_b}{N_b\Theta_b}\Big)^{1/\beta_b}\right]$$
$$= \mathbb{P}\left[d_b \leq R_1\right] \tag{7}$$

In terms of a link between 2 nodes:

$$p_{nf}^{d2d} = \mathbb{P}[SNR_{nf}^{d2d} \geq \Theta_u] = P\left[d_u \leq \Big(\frac{P_u}{N_u\Theta_u}\Big)^{1/\beta_u}\right]$$
$$= \mathbb{P}\left[d_u \leq R_2\right] \tag{8}$$

So, the consideration for the scenarios without fading will be based on the distance between the receiver and its nearest serving base station:

1. cellular link $\Big(d_b \leq R_1\Big)$;
2. cellular link or a single D2D relay link $\Big(R_1 + R_2 \geq d_b > R_1\Big)$

4.1 Case 1: Direct cellular link

See Appendix B, picture 1 for the illustration.

Theorem 1. The probability of getting a successful cellular link would be expressed as:

$$p_{nf}^{cel} = 1 - e^{-\lambda_b \pi R_1^2}$$

Proof: See Appendix A.

4.2 Case 2: Connectivity by means of a single device-to-device relay link

See Appendix B, picture 2 for the illustration

Theorem 2. The probability of getting a successful single D2D relay link would be expressed as:

$$p_{nf}^{s-hop} = \int_{(R_1;R_1+R_2]} 2\lambda_b \pi r e^{-\lambda_b \pi r^2}\Big(1 - e^{-\lambda_u |D(r)|}\Big)dr$$

where

$$|D(r)| = R_2^2 \cos^{-1}\Big(\frac{r^2+R_2^2-R_1^2}{2rR_2}\Big) + R_1^2 \cos^{-1}\Big(\frac{r^2+R_1^2-R_2^2}{2rR_1}\Big)$$

$$-\frac{1}{2}\sqrt{(-r+R_2+R_1)(r+R_2-R_1)(r-R_2+R_1)(r+R_1+R_2)}$$

Proof: See Appendix A.

Optimal value of PDF of the distance between the receiver and its serving base station.
As it's stated in the *Appendix A, proof of Theorem 2*, the PDF of r.v. d_b has the following expression:

$$f_{d_b}(r) = 2\lambda_b \pi r \exp\Big(-\lambda_b \pi r^2\Big)$$

We calculate its derivative:

$$f'_{d_b}(r) = 2\lambda_b \pi \exp\Big(-\lambda_b \pi r^2\Big)\Big(1 - 2\lambda_b \pi r^2\Big)$$

Let's find its critical point:

$$f'_{d_b}(r) = 0$$

$$r = \sqrt{\frac{1}{2\lambda_b \pi}}$$

Then,

$$f_{d_b}\Big(\sqrt{\frac{1}{2\lambda_b \pi}}\Big) = \sqrt{\frac{2\pi\lambda_b}{e}}\text{ , or } \approx 1.52\sqrt{\lambda_b}$$

5 Considered cases under "channel with fading" assumption

Let's investigate (6) a bit deeper:

$$p_f^{cel} = \mathbb{P}[SNR_f^{cel} \geq \Theta_b] = \mathbb{P}\left[\frac{P_b h d_b^{-\beta_b}}{N_b} \geq \Theta_b\right]$$

$$= \mathbb{P}\left[h \geq \frac{N_b \Theta_b d_b^{-\beta_b}}{P_b}\right] = 1 - \mathbb{P}\left[h < \frac{N_b \Theta_b d_b^{-\beta_b}}{P_b}\right]$$

$$= \exp(-\frac{\mu N_b \Theta_b d_b^{-\beta_b}}{P_b})$$

this is in terms of a link between the receiver and its nearest serving base station, and:

$$p_f^{d2d} = \mathbb{P}[SNR_f^{d2d} \geq \Theta_u] = \mathbb{P}\left[\frac{P_u h d_u^{-\beta_u}}{N_u} \geq \Theta_u\right]$$

$$= \mathbb{P}\left[h \geq \frac{N_u \Theta_u d_u^{-\beta_u}}{P_u}\right] = 1 - \mathbb{P}\left[h < \frac{N_u \Theta_u d_u^{-\beta_u}}{P_u}\right]$$

$$= \exp(-\frac{\mu N_u \Theta_u d_u^{-\beta_u}}{P_u})$$

in terms of a link between 2 mobile users.
The consideration is based upon the performance between the nodes(receiver, base stations and other mobile users).

5.1 Direct cellular link

Theorem 4. The probability of the cellular link with fading effect in the channel would be expressed as:

$$p_c = 2\lambda_b \pi \int_{r>0} e^{-\frac{\mu \Theta_b N_b r^{\beta_b}}{P_b}} r e^{\lambda_b \pi r^2} dr$$

Proof: See Appendix A.

6 Numerical Experiments

Simulations were conducted for cases:

1. direct cellular connection (without fading);
2. direct cellular connection (with fading);
3. direct cellular connection or single D2D relay link (without fading);

The following data was used as a baseline sample for experiments:

Symbol	Simulation value
P_u	23 dBm
P_b	46 dBm+14 dBi
λ_u	5×10^{-5}
λ_b	10^{-6}
N_u	-105 dBm
N_b	-99 dBm
Θ_u	10 dB
Θ_b	5 dB
β_u	3.68
β_b	3.52

The value of the precision used during the numerical integration is 10^{-3}. The following graph displays the performance in the considered cases.

6.1 Direct cellular connection without fading

№	$\lambda_b(\text{km}^{-2})$	$P_b(\text{W})$	β_b	$N_b(\text{W})$	$\Theta_b(\text{dB})$	$R_1(\text{km})$	$\mathbb{P}$
1	1.131	1000	3.52	1×10^{-7}	3.16	499	1.000
2	2×10^{-6}	1000	3.52	1×10^{-7}	3.16	499	0.792
3	1×10^{-7}	1000	3.52	1×10^{-7}	3.16	499	0.076
4	1.131	1000	2.7	1×10^{-7}	3.16	3300	1.000
5	3×10^{-8}	1000	2.7	1×10^{-7}	3.16	3300	0.642
6	8×10^{-9}	1000	2.7	1×10^{-7}	3.16	3300	0.240
7	0.001	1000	6	1×10^{-7}	3.16	38	0.990
8	2×10^{-4}	1000	6	1×10^{-7}	3.16	38	0.602
9	8×10^{-5}	1000	6	1×10^{-7}	3.16	38	0.309

Analysis: Here we tested three sets of different densities on three values of the environment with $\beta = 3.52$ denoting a shadowed urban cellular radio, $\beta = 2.7$ denoting an ordinary urban area cellular radio and $\beta = 6$ denoting obstructed in building. Naturally, decreasing density results in a less service probability; also, as the environment gets less space, a less distance between the receiver and its nearest serving base station is necessary to connect to successfully.

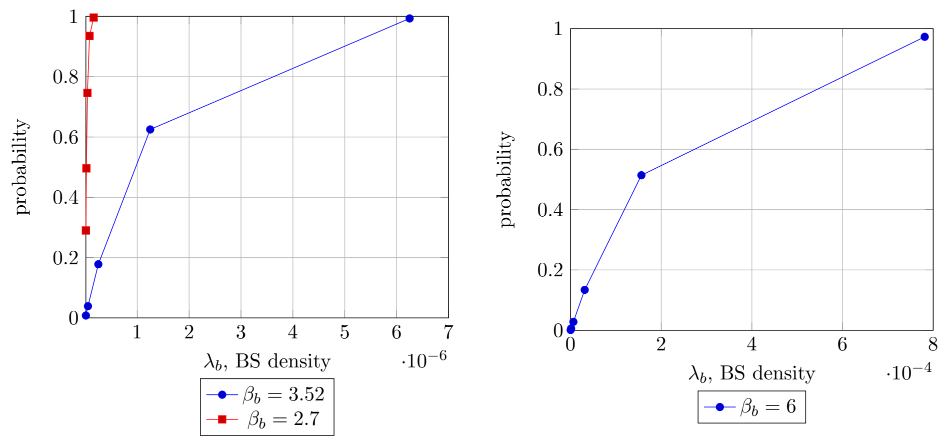


Figure 1: (a) and (b)

Analysis: Figure (a) demonstrates a difference between urban ($\beta = 3.52$) and open ($\beta = 2.7$) environment. Naturally, the connection is much more rapid in the open environment even if the density of base station is very low. The (b) figure indicates the demand of a more dense base stations network to operate normal in the environment obstructed with builidings.

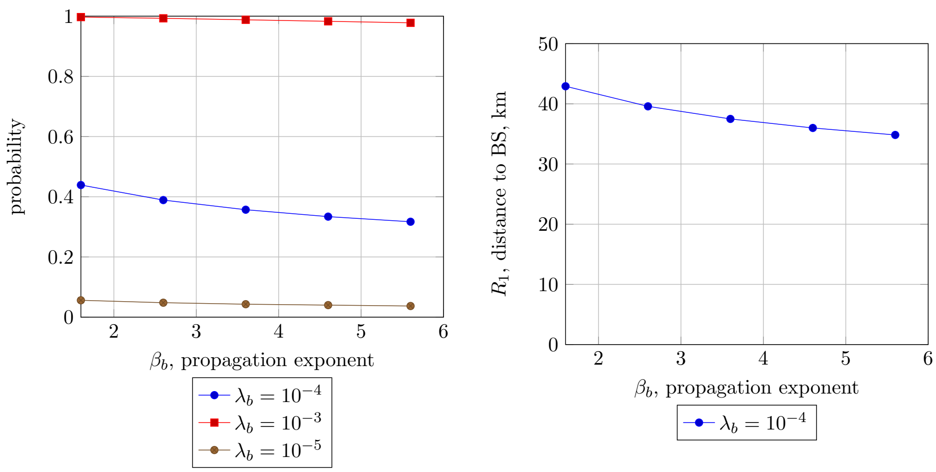


Figure 2: (a) and (b)

Analysis: A similar approach as the previous experiment, but here we display how a gradual obstruction level affects the connectivity for 3 different densities of base stations. The more dense the base stations are, the better the connectivity remains.

6.2 Direct cellular connection with fading

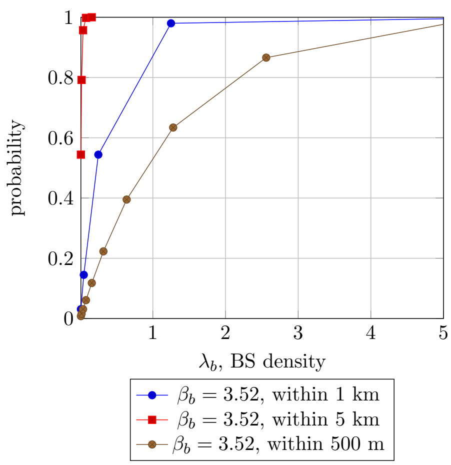


Figure 3:

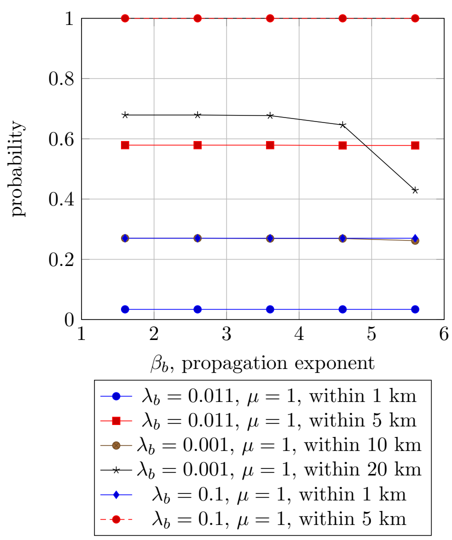


Figure 4:

Analysis: Figure 3 demonstrates the experiment of connectivity in bad urban macrocell environment ($\beta = 3.52$) with channel suffering fading effects.

6.3 Direct cellular connection or single D2D relay link without fading

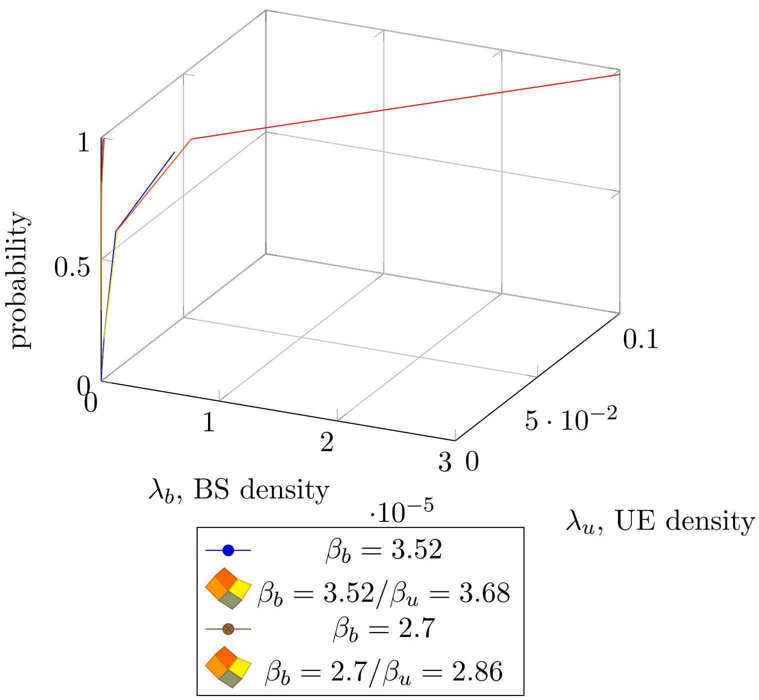

probability
1
0.5
0
0
1
2
3
λ_b, BS density
$\cdot 10^{-5}$
0
$5 \cdot 10^{-2}$
0.1
λ_u, UE density
$\beta_b = 3.52$
$\beta_b = 3.52/\beta_u = 3.68$
$\beta_b = 2.7$
$\beta_b = 2.7/\beta_u = 2.86$

Figure 5:

$\beta_b = 3.52$:

λ_b	$\mathbb{P}$
0.00000001	0.008
0.00000005	0.039
0.00000025	0.178
0.00000125	0.625
0.00000625	0.993

$\beta_b = 3.52/\beta_u = 3.68$:

λ_b	λ_u	$\mathbb{P}$
0.00000001	0.000001	0.008
0.00000005	0.000010	0.039
0.00000025	0.000100	0.178
0.00000125	0.001000	0.626
0.00000625	0.010000	0.993
0.00003125	0.100000	1.000

$\beta_b = 2.7$:

λ_b	$\mathbb{P}$
0.00000001	0.290
0.00000002	0.496
0.00000004	0.746
0.00000008	0.935
0.00000016	0.996

$\beta_b = 2.7/\beta_u = 2.86$:

λ_b	λ_u	$\mathbb{P}$
0.00000001	0.000001	0.290
0.00000005	0.000010	0.819
0.00000025	0.000100	1.000

Analysis: Testing the single D2D relay link in urban and open environments and comparing the performance with the respective one for direct cellular connectivity indicates an almost identical result, meaning in these types of environment both types network are equally good.

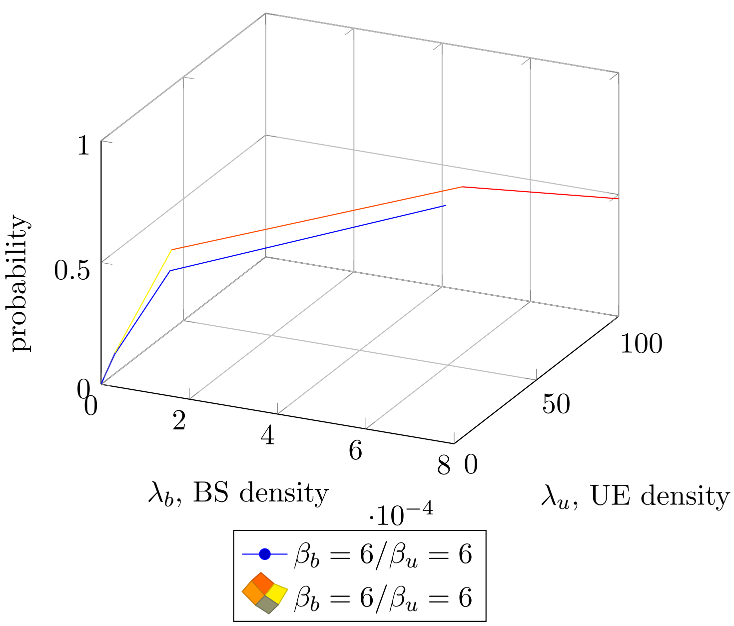

probability
1
0.5
0
0
2
4
6
8
λ_b, BS density
$\cdot 10^{-4}$
0
50
100
λ_u, UE density
$\beta_b = 6/\beta_u = 6$
$\beta_b = 6/\beta_u = 6$

Figure 6:

$\beta_b = 6$:

λ_b	$\mathbb{P}$
0.00000025	0.001
0.00000125	0.006
0.00000625	0.028
0.00003125	0.134
0.00015625	0.514
0.00078125	0.973

$\beta_b = 6/\beta_u = 6$:

λ_b	λ_u	$\mathbb{P}$
0.00000001	0.000001	0.000
0.00000005	0.000010	0.000
0.00000025	0.000100	0.001
0.00000125	0.001000	0.006
0.00000625	0.010000	0.029
0.00003125	0.100000	0.139
0.00015625	1.000000	0.595
0.00078125	10.000000	0.997
0.00390625	100.000000	1.000

Analysis: Testing an environment with serious signal obstructions indicates a better performance in case of D2D relay link: starting from BS density of 10^{-6} and mobile user density of 10^{-2} and forth the performance gets more reliable.

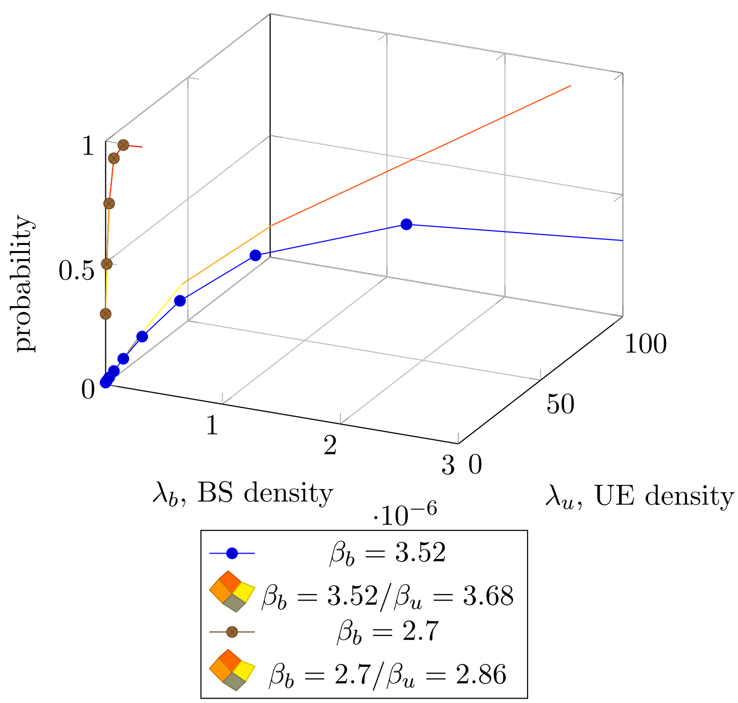


Figure 7:

$\beta_b = 3.52$:

λ_b	$\mathbb{P}$
0.00000001	0.008
0.00000002	0.016
0.00000004	0.031
0.00000008	0.061
0.00000016	0.118
0.00000032	0.222
0.00000064	0.395
0.00000128	0.634
0.00000256	0.866
0.00000512	0.982

$\beta_b = 3.52/\beta_u = 3.68$:

λ_b	λ_u	$\mathbb{P}$
0.00000001	0.000001	0.008
0.00000002	0.000010	0.016
0.00000004	0.000100	0.031
0.00000008	0.001000	0.061
0.00000016	0.010000	0.119
0.00000032	0.100000	0.239
0.00000064	1.000000	0.457
0.00000128	10.000000	0.705
0.00000256	100.000000	0.913

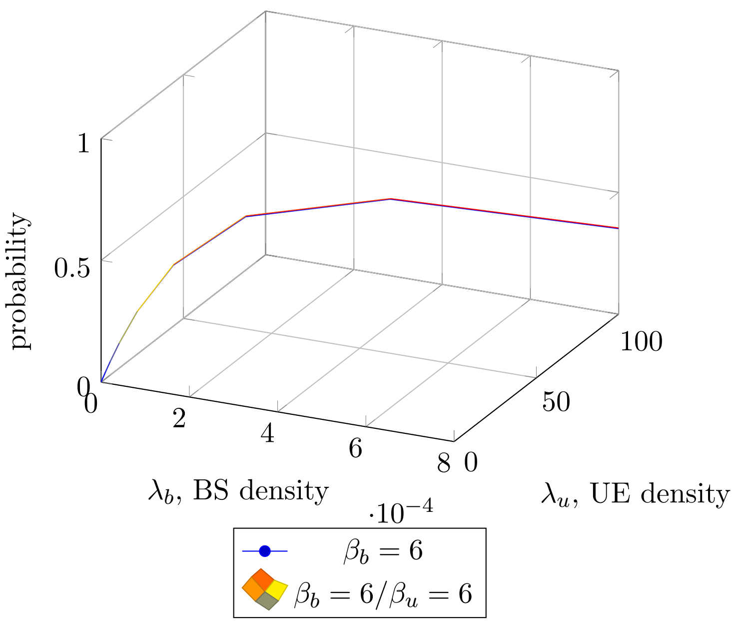
$\beta_b = 2.7$:

λ_b	$\mathbb{P}$
0.00000001	0.290
0.00000002	0.496
0.00000004	0.746
0.00000008	0.935
0.00000016	0.996

$\beta_b = 2.7/\beta_u = 2.86$:

λ_b	λ_u	$\mathbb{P}$
0.00000001	0.000001	0.290
0.00000002	0.000010	0.496
0.00000004	0.000100	0.746
0.00000008	0.001000	0.936
0.00000016	0.010000	0.996
0.00000032	0.100000	1.000

Analysis: This experiment takes a different scenario. While the performance remains identical for both the direct cellular and D2D relay connections, here an increased density of mobile users is considered. While the previous experiment handled the ratio of the number of base stations to the number of mobile users at 1/1000, this experiment drastically decreases the ratio down to 1/1000000. The result indicates a better connectivity for D2D relay link.



1
0.5
0
probability
0
2
4
6
8
$\cdot 10^{-4}$
λ_b, BS density
0
50
100
λ_u, UE density
$\beta_b = 6$
$\beta_b = 6/\beta_u = 6$

Figure 8:

$\beta_b = 6$:

λ_b	$\mathbb{P}$
0.00000032	0.001
0.00000064	0.003
0.00000128	0.006
0.00000256	0.012
0.00000512	0.023
0.00001024	0.046
0.00002048	0.090
0.00004096	0.172
0.00008192	0.315
0.00016384	0.530
0.00032768	0.779
0.00065536	0.951
0.00131072	0.998

$\beta_b = 6/\beta_u = 6$:

λ_b	λ_u	$\mathbb{P}$
0.00000001	0.000001	0.000
0.00000002	0.000002	0.000
0.00000004	0.000004	0.000
0.00000008	0.000008	0.000
0.00000016	0.000016	0.001
0.00000032	0.000032	0.001
0.00000064	0.000064	0.003
0.00000128	0.000128	0.006
0.00000256	0.000256	0.012
0.00000512	0.000512	0.023
0.00001024	0.001024	0.046
0.00002048	0.002048	0.090
0.00004096	0.004096	0.172
0.00008192	0.008192	0.315
0.00016384	0.016384	0.532
0.00032768	0.032768	0.782
0.00065536	0.065536	0.953
0.00131072	0.131072	1.000

Analysis: In this experiment for the seriously obstructed environment, the strategy was to keep the ratio of the number of base stations to the number of mobile users at the value 1/100. The previous experiment handled the same ratio at the value of 1/10000. Naturally, the result of this experiment indicates the demand of a more dense base station network in order to provide a reliable connectivity.

7 Conclusion

The consideration of SNR in cellular and D2D link cases resulted in a derivation of analytical formulas that allow to perform numerical evaluation of the performance of the respective case. It also allowed to carry out the numerical experiments. The strategy of choosing the parameters to vary is that modern types of network (3G/LTE) have already got the necessary values of transmission power of both the base stations and mobile users, so the trasmit power was decided to keep as a constant. The same goes for the service threshold. As for thermal noise, then its value is also calculated, so it stays constant as well. So the densities and propagation exponents are left to be varied. Based on the results of numerical experiments, a D2D enabled cellular network becomes useful when the signal propagation is seriously obstructed, say in the building of any kind. Also, based on the table results for figures 6 and 8, a D2D enabled cellular network yields better performance, when the ratio of the number of base stations to the number of mobile users is around 1/10,000.

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Future Work.

The future work may include the consideration of interference, a more sophisticated point processes (determinantal point process known to address the repulsion effect). Also, the stationarity of the point process might be changed to the consideration of location changes over time.

A new layer of relay transmission can be added considering the developing of the project OneWeb that aims to provide Internet access from small satellites orbiting the Earth.

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A Proofs.

Theorem 1.

$$p_{nf}^{cel} = \mathbb{P}\left[SNR_{nf} \geq \Theta_b\right] = \mathbb{P}\left[d \leq R_1\right] = 1 - \mathbb{P}\left[d > R_1\right] = 1 - \mathbb{P}\left[\Phi_b(A) = 0\right]$$

$$= 1 - e^{-\lambda_b \pi R_1^2}$$

where A is the area of search disk of radius R_1.

Theorem 2.

The scenario of getting connectivity by means of a D2D relay is represented by event $C = A \cap B$, where event A is $R_1 < d \leq R_1 + R_2$; and event B represents the existence of at least 1 user in the area of $b(O, R_2) \cap b(B_{nst}, R_1)$ - let us denote this area as D.

More precisely, the following probability is considered:

$$\mathbb{P}[C] = \mathbb{P}[A \cap B] =$$

$$\mathbb{P}\left[(R_1 < d \leq R_1 + R_2) \cap (\Phi_u(D(d))) \geq 1\right] \tag{9}$$

Here, d is a random variable. And consequently, the area of D depends on the value of d.

Consider $f_d(r)$ - probability density function of the random variable d:

$$f_d(r) = \frac{d}{dr} F_d(r) = \frac{d}{dr}\left(\mathbb{P}(d \leq r)\right)$$

$$= \frac{d}{dr}\left(1 - \mathbb{P}(d > r)\right) = \frac{d}{dr}\left(1 - \mathbb{P}(\Phi_b(b(O, r)) = 0)\right)$$

$$= \frac{d}{dr}\left(1 - e^{-\lambda_b \pi r^2}\right) = 2\lambda_b \pi r e^{-\lambda_b \pi r^2}$$

Here, $F_d(r)$ is a cumulative distribution function of the random variable d.

Interpreting (1) in terms of (4), denoting event A as $\Phi_u(D(d)) \geq 1$, and subset $B = (R_1; R_1 + R_2]$:

$$\mathbb{P}\Big[(R_1 < d \leq R_1 + R_2) \cap (\Phi_u(D(d)) \geq 1)\Big]$$

$$= \mathbb{P}(\Phi_u(D(d)) \geq 1, d \in (R_1; R_1 + R_2])$$

$$= \int_{(R_1;R_1+R_2]} f_d(r)\mathbb{P}\Big(\Phi_u(D(d) \geq 1)\Big| d = r\Big) dr$$

$$= \int_{(R_1;R_1+R_2]} f_d(r)\mathbb{P}(\Phi_u(D(r)) \geq 1) dr$$

$$= \int_{(R_1;R_1+R_2]} f_d(r)\big(1 - \mathbb{P}(\Phi_u(D(r)) = 0)\big) dr$$

$$= \int_{(R_1;R_1+R_2]} 2\lambda_b \pi r e^{-\lambda_b \pi r^2}\Big(1 - e^{-\lambda_u |D(r)|}\Big) dr \tag{10}$$

where

$$|D(r)| = R_2^2 \cos^{-1}\left(\frac{r^2 + R_2^2 - R_1^2}{2rR_2}\right) + R_1^2 \cos^{-1}\left(\frac{r^2 + R_1^2 - R_2^2}{2rR_1}\right)$$

$$-\frac{1}{2}\sqrt{(-r + R_2 + R_1)(r + R_2 - R_1)(r - R_2 + R_1)(r + R_1 + R_2)}$$

Theorem 4. Conditioning on the nearest base station being at a distance r from a typical user:

$$p_c = \mathbb{P}\big[SNR \geq \Theta_b \big| d = r\big]$$

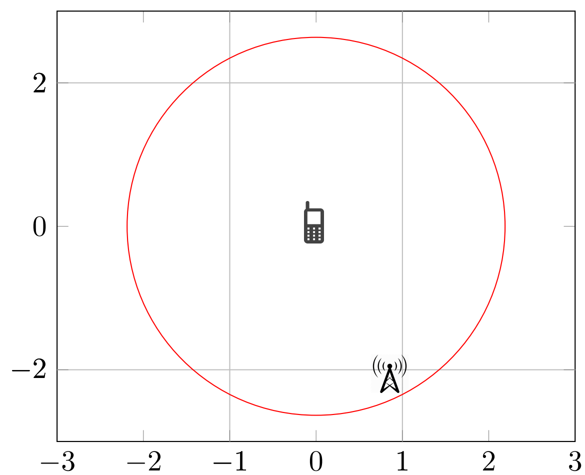
$$= \mathbb{P}\Big[\frac{P_b h d^{-\beta_b}}{N_b} \geq \Theta_b \Big| d = r\Big]$$

$$= \int_{r>0} \mathbb{P}\left[h \geq \frac{\Theta_b N_b r^{\beta_b}}{P_b}\right] f_d(r) dr$$

$$= 2\lambda_b \pi \int_{r>0} e^{-\frac{\mu \Theta_b N_b r^{\beta_b}}{P_b}} r e^{-\lambda_b \pi r^2} dr$$

B Plots.

Picture 1.



Picture 2.