

Connectivity analysis of device-to-device cellular network using stochastic geometry

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Abstract

- Device-to-device (D2D) enables direct communication between nearby mobiles;
- D2D improves spectrum utilization, energy efficiency, coverage and overall throughput;
- Thesis considers D2D cellular network in comparison with the traditional cellular network;
- Model consists of independent homogeneous Poisson point processes with contact distance probability and noise-limited environment;
- Analytical way of calculating performance is developed;
- Using analytical results, numerical experiments are carried out and results are displayed.

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Device-to-device communication

- Growing demands of network capacity has led to evolution of networks from 1G to 5G;
- Device-to-device (D2D) communication becomes an effective facilitator of the upcoming high data rate;

Evolution: Four Mobile Generation
Core Services & Network Speed



Mobile Generation	Core Services & Applications	Network Speed
1G 1980s	Voice only (analog)	NA
2G 1990s	SMS / data (digital)	0.01-0.1M
3G 2000s	Internet / Multimedia	0.4-42M
4G 2010s	Cloud Computing	100M-1G

Overview of the existing studies on device-to-device communication

Device-to-device communication is classified into 2 major groups:

- ① D2D sharing cellular spectrum, *a.k.a* **inband**;
 - spectrum utilization;
 - power efficiency;
 - cellular coverage
 - interference.
- ② D2D exploits unlicensed spectrum, *a.k.a* **outband**;
 - video transmission;
 - average file transfer delay.

System model

- 2 stationery independent homogeneous PPPs of base stations and mobile users (Φ_b, Φ_u) ;
- Given parameters $(\lambda_b, P_b, N_b, \beta_b, \Theta_b), (\lambda_u, P_u, N_u, \beta_u, \Theta_u)$;
- Rayleigh fading with mean 1;
- Noise-limited environment;
- Receiver at the origin of \mathbb{R}^2 , nearest BS at B_{nst} ;
- Distance to the nearest BS as d_b , to the nearest D2D node - d_u

Performance evaluation

The key performance characteristics that the paper considers is Signal-to-Noise ratio (SNR).

$$SNR_{nf} = \frac{S}{N} = \frac{Pd^{-\beta}}{N},$$

for channel without fading; and

$$SNR_f = \frac{Phd^{-\beta}}{N},$$

for channel with fading where h - random variable that follows an exponential distribution with mean $1/\mu$ which we denote as $h \sim \exp(\mu)$.

Performance evaluation

The purpose of the thesis is to consider the following probabilities:

$$p_{nf} = \mathbb{P}\left[SNR_{nf} \geq \Theta\right] = \mathbb{P}\left[d \leq \left(\frac{P}{N\Theta}\right)^{1/\beta}\right], \quad (1)$$

and:

$$p_f = \mathbb{P}\left[SNR_f \geq \Theta\right] = \mathbb{P}\left[h \geq \frac{Nd^\beta\Theta}{p}\right]. \quad (2)$$

Considering (1) for BS and D2D node, we get:

$$\mathbb{P}\left[d_b \leq R_1\right], \text{ where } R_1 = \left(\frac{P_b}{N_b\Theta_b}\right)^{1/\beta_b}$$

$$\mathbb{P}\left[d_u \leq R_2\right], \text{ where } R_2 = \left(\frac{P_u}{N_u\Theta_u}\right)^{1/\beta_u}$$

Scenarios under consideration:

- ① $d_b \leq R_1$ —direct cellular connection;
- ② $R_1 < d_b \leq R_1 + R_2$ —single D2D relay connection.

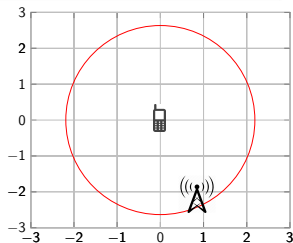
Direct cellular connection

Cellular connection with non-fading channel

$$p_{nf}^{cel} = 1 - \exp(-\lambda_b \pi R_1^2)$$

Idea: Using (1) and R_1 we get:

$$\mathbb{P}[d_b \leq R_1] = 1 - \mathbb{P}[\Phi_b(b(o, R_1)) = 0]$$



Single D2D relay link connection

Single D2D relay connection with non-fading channel

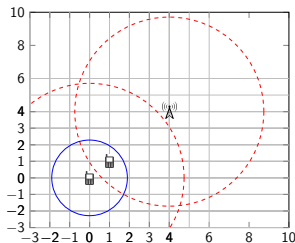
$$p_{nf}^{s-hop} = 2\lambda_b\pi \int_{R_1}^{R_1+R_2} r \exp(-\lambda_b\pi r^2) (1 - \exp(-\lambda_u |D(r)|)) dr,$$

where $|D(r)| = R_2^2 \cos^{-1} \left(\frac{r^2 + R_2^2 - R_1^2}{2rR_2} \right) + R_1^2 \cos^{-1} \left(\frac{r^2 + R_1^2 - R_2^2}{2rR_1} \right) - \frac{1}{2} \sqrt{(R_2 + R_1 - r)(r + R_2 - R_1)(r - R_2 + R_1)(r + R_1 + R_2)}.$

Single D2D relay link connection

Idea: We consider the event $C = [\Phi_u(b(o, R_2) \cap b(B_{nst}, R_1)) \geq 1, d_b \in (R_1; R_1 + R_2]]$. We then derive a PDF of (1) for case of a mobile user $f_{d_b}(r) = \frac{d}{dr}(\mathbb{P}(d_b \leq r)) = 2\lambda_b \pi r e^{-\lambda_b \pi r^2}$. We use the conditioning on the nearest BS to be at distance r , $d_b = r$ and compute the integral:

$$\int_{R_1}^{R_1+R_2} f_{d_u}(r) \mathbb{P}(\Phi_u(b(o, R_2) \cap b(B_{nst}, R_1)) \geq 1 \mid d_b = r)$$



Cellular connection with fading effect.

Cellular connection with fading in the channel

$$p_f^{cel} = 2\lambda_b\pi \int_{r>0} \exp\left(-\frac{\mu\Theta_b N_b r^{\beta_b}}{P_b}\right) r \exp(-\lambda_b\pi r^2) dr$$

Idea: We condition on the nearest BS to be at distance r , and using (2) we consider the following probability:

$$\mathbb{P}\left[h \geq \frac{N_b d^{\beta_b} \Theta_b}{P_b} \middle| d_b = r\right]$$

We then use the PDF of $d_b - f_{d_b}(r)$ and compute the integral:

$$\int_{r>0} f_{d_b}(r) \mathbb{P}\left[h \geq \frac{N_b r^{\beta_b} \Theta_b}{P_b}\right]$$

Testing environment

Simulations were conducted for cases:

- ① direct cellular connection (without fading);
- ② direct cellular connection (with fading);
- ③ direct cellular connection or single D2D relay link (without fading);

The value of the precision used during the numerical integration is 10^{-3} .

Initial testing parameters

The following data was used as a baseline sample for experiments:

Symbol	Simulation value
P_u	23 dBm
P_b	46 dBm+14 dBi
λ_u	5×10^{-5}
λ_b	10^{-6}
N_u	-105 dBm
N_b	-99 dBm
Θ_u	10 dB
Θ_b	5 dB
β_u	3.68
β_b	3.52

Testing scenarios.

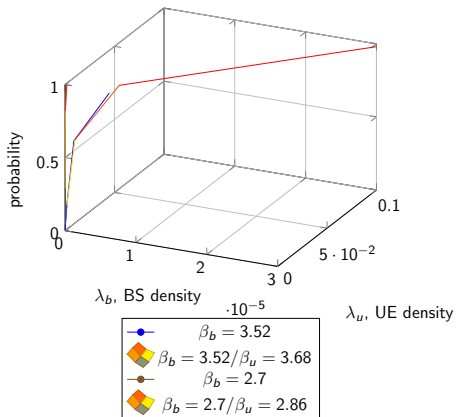
Parameters left to be constant:

- transmission power;
- thermal noise;
- service threshold.

Parameters being tested:

- density;
- propagation exponents.

Direct cellular connection or single D2D relay link without fading



Figure

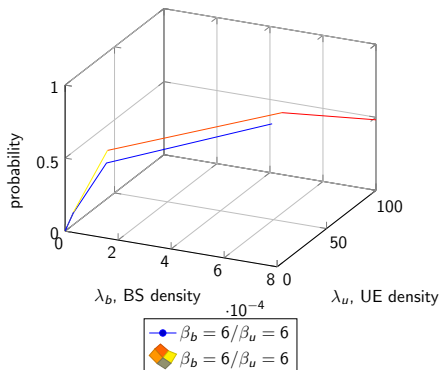
$$\beta_b = 3.52/\beta_u = 3.68:$$

λ_b	λ_u	\mathbb{P}
0.00000001	0.000001	0.008
0.00000005	0.000010	0.039
0.00000025	0.000100	0.178
0.00000125	0.001000	0.626
0.00000625	0.010000	0.993
0.00003125	0.100000	1.000

$$\beta_b = 2.7/\beta_u = 2.86:$$

λ_b	λ_u	\mathbb{P}
0.00000001	0.000001	0.290
0.00000005	0.000010	0.819
0.00000025	0.000100	1.000

Direct cellular connection or single D2D relay link without fading



Figure

$$\beta_b = 6/\beta_u = 6:$$

λ_b	λ_u	\mathbb{P}
0.00000025	0.000100	0.001
0.00000125	0.001000	0.006
0.00000625	0.010000	0.029
0.00003125	0.100000	0.139
0.00015625	1.000000	0.595
0.00078125	10.000000	0.997
0.00390625	100.000000	1.000

$$\beta_b = 6:$$

λ_b	\mathbb{P}
0.00000025	0.001
0.00000125	0.006
0.00000625	0.028
0.00003125	0.134
0.00015625	0.514
0.00078125	0.973

General results.

- Derivation of analytical formulas for performance evaluation;
- Numerical experiments are carried out;
- D2D enabled cellular network becomes useful when the signal propagation is seriously obstructed;
- D2D enabled cellular network yields good performance when the ratio of the number of base stations to the number of mobile users is around $1/10,000$.

Future work

- Consideration of n -D2D relay links in both non-fading channel and channel with fading;
- Interference modelling;
- Additional relaying layer can be added into the model and performance evaluation is also possible (like in project 'OneWeb').

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