Probability

Gate Notes

Standard : Division : Roll :

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Subject : probability.

22/06/2018 (Revise) (2 Day to revise)

PROBABILITY

INTRODUCTION

- **Experiment :-** An operation which can produce some well-defined outcomes call an experiment.

- **Random Experiment :-** An experiment in which all possible outcomes are known and the exact output cannot be predicted in advance, is called a random experiment.

  Examples -
  (i) Rolling a dice.
  (ii) Tossing a coin.
  (iii) picking a object.

- **Sample Space :-** set of all possible out comes of an experiment is called as sample space.
  (Set)

  Examples -
  (i) In tossing a coin, $S_1 = \{H, T\}$
  (ii) In tossing two coin, $S_2 = \{HH, HT, TH, TT\}$
  (iii) In rolling a dice, $S_3 = \{1, 2, 3, 4, 5, 6\}$
  (iv) rolling two dice, $S_4 = \{(1,1)\ (1,2)\ (1,3)\ (1,4), (1,5)\ (1,6)\ \ldots\ (6,6)\}$
  (v) Life time of a car, $S_5 = [0, \infty)$.

- **Event :-** Any subset of a sample space is an event.
  (Set)

  Examples -
  (i) Tossing a coin that can be event, $E_1 = \{H\}$, $E_6 = \{T\}$ $\boxed{E_1 \subseteq S_1}$
  (ii) $E_3 = \{2, 4, 6\}$, ~~even out~~ (set of even outcomes)
  (iii) $E_2 = \{HH, HT, TH\}$, (at least 1 head).
  (iv) $E_4 = \{(1,2)\ (2,1)\}$, (sum is ~~o~~ ~~odd~~ 3).
  (v) $E_5 = [3, 6]$, (car life time is atleast 3 and atmost 6 - year).

→ sample space and Events both are set, so we can apply all the rules of set. (union, intersection, complement)

Ex-

(1) $E_1 \cup E_6 = \{H, T\}$

(2) $E_1 \cap E_2 = \emptyset$ (Mutual Exclusion = no common - Events)

(3) $\overline{E_1} = S - E_1$ $(\overline{E_1} = \{T\})$

→ For each event 'E' in the sample Space 'S', we assign a number $P(E)$ such that.

(1) $0 \leq P(E) \leq 1$

(11) $P(S) = 1$

(111) For any sequence of events $E_1, E_2 ---$ that are mutually exclusive,

$$P\left(\bigcup_{n=1}^{\infty} E_n\right) = \sum_{n=1}^{\infty} P(E_n).$$

$\boxed{P(E) = \frac{n(E)}{n(S)}}$

Ex- tossing 1 coin-

$S = \{H, T\}$

$E_1 = \{H\}$, $E_2 = \{T\}$

$P(E_1) = \frac{1}{2}$, $P(E_2) = 1/2$

Ex- tossing 2 coin-

$S = \{HH, HT, TH, TT\}$
$\quad\; 1/4 \quad 1/4 \quad 1/4 \quad 1/4$

$E_1 = \{HH\}$, $P(E_1) = 1/4$

$E_2 = \{TT\}$, $P(E_2) = 1/4$

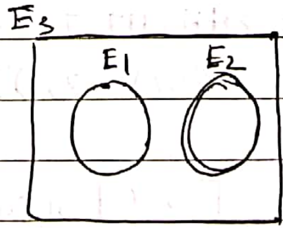
$E_3 = E_1 \cup E_2$

$P(E_3) = P(E_1 \cup E_2)$

* $\boxed{P(E_3) = P(E_1) + P(E_2)}$

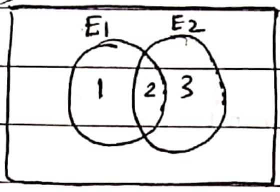
$= 1/4 + 1/4$

$= 1/2$



Individual probability (ME)

E_s



not individual probability (not-ME)

$P \; E_1 \cup E_2 = (1,2) \cup (2,3)$

** $\boxed{P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)}$

Ex- tossing two coin-

→ $S = \{HH, HT, TH, TT\}$

$E_1 = \{HH, HT\}$ – first coin going to head.

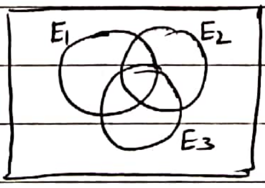
$E_2 = \{HH, TH\}$ – second coin going to be head.

$E_3 = \{HH, HT, TH\}$ at least one head.
$= E_1 \cup E_2$

$P(E_3) = \boxed{P(E_1 \cup E_2) \Rightarrow P(E_1) + P(E_2) - P(E_1 \cap E_2)}$

$\Rightarrow \frac{2}{4} + \frac{2}{4} - \frac{1}{4}$

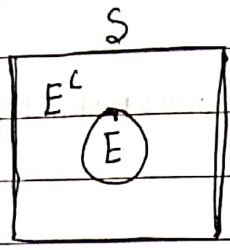
$\Rightarrow \frac{1}{2} + \frac{1}{2} - \frac{1}{4} \Rightarrow \frac{3}{4}$.



(not-Mutual exclusive)

** $\boxed{P(E_1 \cup E_2 \cup E_3) = P(E_1) + P(E_2) + P(E_3) - P(E_1 \cap E_2) - P(E_2 \cap E_3) - P(E_3 \cap E_1) + P(E_1 \cap E_2 \cap E_3)}$

→ When ever trying to take the union you have to see whether they are mutual exclusive or not.



(ME, so add them)

$S = E \cup E^c$

$P(S) = P(E \cup E^c)$

$P(S) = P(E) + P(E^c)$

$\Rightarrow \quad 1 = P(E) + P(E^c)$

$* \quad \boxed{P(E^c) = 1 - P(E)}$

CONDITIONAL - PROBABILITY

- Conditional probability –

$$\boxed{P(A/B) = \frac{P(A \cap B)}{P(B)}}$$

→ Consider two events A and B defined on a sample space S. The probability of occurence of event A given that event B has already occured is known as conditional probability of A relative to B.

Ex-1

- Rolling a dice :

→ Sample space(S) = $\{1, 2, 3, 4, 5, 6\}$
$\quad\frac{1}{6}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{6}\ \frac{1}{6}$

event(E) = $\{1\}$

Probability of E $\boxed{P(E) = \frac{n(E)}{n(S)}} \Rightarrow \frac{1}{6}$.

→ If you know something about the output,

→ for example – The dice are rolled and the output will be even number. In this condition the probability of getting output will change.

$S = \{1, 2, 3, 4, 5, 6\}$
$\downarrow\ \downarrow\ \downarrow\ \downarrow\ \downarrow\ \downarrow$
$0\ \ \frac{1}{3}\ \ 0\ \ \frac{1}{3}\ \ 0\ \ \frac{1}{3}$

New Sample space, $S_N = \{2, 4, 6\}$
$\frac{1}{3}\ \frac{1}{3}\ \frac{1}{3}$

→ The dice are rolled and the outcome will be prime number. so,

$S = \{1, 2, 3, 4, 5, 6\}$
$\downarrow\ \downarrow\ \downarrow\ \downarrow\ \downarrow\ \downarrow$
$0\ \ \frac{1}{3}\ \ \frac{1}{3}\ \ 0\ \ \frac{1}{3}\ \ 0$

new S going to change = $\{2, 3, 5\}$
$\frac{1}{3}\ \frac{1}{3}\ \frac{1}{3}$

** → We have a sample space and they have perform a experiment and said that something have occured then the new probability to the simple event should changed.

- $E = \{2, 4, 6\}$ (event to getting even number)

$$P(E) = \frac{n(E)}{n(S)} \Rightarrow \frac{3}{6} \Rightarrow \frac{1}{2}$$

- $F = \{2, 3, 5\}$ (event to getting prime no)

$$P(F) \Rightarrow \frac{n(F)}{n(S)} \Rightarrow \frac{3}{6} \Rightarrow \frac{1}{2}.$$

- $E \cap F = \{2\}$

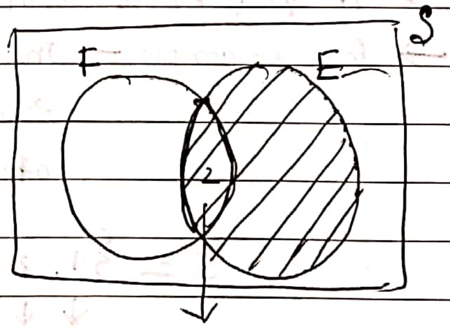
$$P(E \cap F) = \frac{n(E \cap F)}{n(S)} \Rightarrow \frac{1}{6}.$$

Q. What is the probability of getting a prime number given that an even number showed up.

$$\rightarrow \quad P(F/E) = \frac{n(F \cap E)}{n(E)} = \frac{1}{3}$$

$S = \{1, 2, 3, 4, 5, 6\}$

1	2	3	4	5	6
1/6	1/6	1/6	1/6	1/6	1/6
↓	↓	↓	↓	↓	↓
0	1/3	0	1/3	0	1/3



required probability

$$P(F/E) \Rightarrow \frac{\frac{n(F \cap E)}{n(S)}}{\frac{n(E)}{n(S)}}$$

**

$$\boxed{P(F/E) \Rightarrow \frac{P(F \cap E)}{P(E)}} \Rightarrow \frac{1/6}{1/2} \Rightarrow 1/3$$

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EX-2

Rolling two dice. It is already known that there is 4 on the first dice.

$\rightarrow S = \{6 \times 6\} = 36$

- $E = \{41, 42, 43, 44, 45, 46\}$ (4 on first dice)

$\quad \frac{1}{6} \quad \frac{1}{6} \quad \frac{1}{6} \quad \frac{1}{6} \quad \frac{1}{6} \quad \frac{1}{6}$

$$P(E) \Rightarrow \frac{n(E)}{n(S)} \Rightarrow \frac{6}{36} \Rightarrow \frac{1}{6}.$$

- $F = \{15, 51, 24, \cancel{42}, 42, 33\}$ [event the sum of out come 6

$$P(F) = \frac{5}{36}.$$

$$P(F/E) \Rightarrow \frac{P(E \cap F)}{P(F)} \Rightarrow \frac{\frac{n(E \cap F)}{n(S)}}{\frac{n(E)}{n(S)}} \Rightarrow \frac{1/36}{6/36} \Rightarrow 1/6.$$

Example-1

A family has two children. If one of them is known to be a boy, then what is the probability that both are boy.

$\rightarrow S = \{BB, BG, GB, GG\}$

$\quad \frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4}$

$\quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

$\quad \frac{1}{3} \quad \frac{1}{3} \quad \frac{1}{3} \quad 0$

$E = \{BB, BG, GB\}$ (event one of them are boy)

$F = \{BB\}$ (Both are boy) $\quad E \cap F = \{BB\}$

$$P(F/E) = \frac{P(F \cap E)}{P(E)} \Rightarrow \frac{\frac{n(F \cap E)}{n(S)}}{\frac{n(E)}{n(S)}}$$

$$= \frac{1/4}{3/4} \Rightarrow 1/3$$

Example-2

performing a experimen in which flipping 3-coins. known that at least ~~2 Head turn~~ two of them is turnout to be head, then ~~what is the prob~~ at least one of them turnout to be tail.

$\Rightarrow$ $S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

$\frac{1}{8} \quad \frac{1}{8} \quad \frac{1}{8} \quad \frac{1}{8} \quad \frac{1}{8} \quad \frac{1}{8} \quad \frac{1}{8} \quad \frac{1}{8}$

$E = \{HHT, HHT, HTH, THH\}$

$F = \{\cancel{HHT}, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

$$P(E/F) = \frac{P(E \cap F)}{P(F)} \Rightarrow \frac{3/8}{7/8} \Rightarrow 3/7.$$

$$F(F/E) = \frac{P(F \cap E)}{P(E)} \Rightarrow \frac{3/8}{4/8} \Rightarrow 3/4$$

Ex-3

given 10 cards which are no from 1-10 ~~and~~ then we ~~have~~ are drawn one of them, and found that already the number is at least 5. therefore what is the probability that it is actually 10.

$\rightarrow$ $S = \{1 \; 2 \; 3 \; 4 \; 5 \; 6 \; 7 \; 8 \; 9 \; 10\}$

$\frac{1}{10} \; \frac{1}{10} \; \ldots \; \frac{1}{10}$

$0 \; 0 \; 0 \; 0 \; \frac{1}{6} \; \frac{1}{6} \; \frac{1}{6} \; \frac{1}{6} \; \frac{1}{6} \; \frac{1}{6}$

$E = \{5, 6, 7, 8, 9, 10\}$ (atleast 5)

$F = \{10\}$

$(F \cap E) = \{10\}$

$$P(F/E) = \frac{P(E \cap F)}{P(E)} \Rightarrow \frac{1/10}{6/10} \Rightarrow 1/6$$

** Multiplication Theorem:

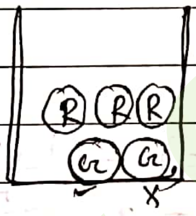
$$P(A/B) = \frac{P(A\cap B)}{P(B)}$$
$$P(B/A) = \frac{P(A\cap B)}{P(A)}$$
both are conditional probability.

Multiplication theorem (Dependent) –

$$\boxed{\begin{array}{l} P(A\cap B) = P(B)\ P(A/B) \\ P(A\cap B) = P(A)\ P(B/A) \end{array}}$$
→ for 2 event.

$\boxed{\text{Ex-1}}$ The Box containing 2 Green and 3 Red balls. 2 balls are randomly drawn from the box, without replacement. What is the probability that ~~the balls are~~ ~~of~~ the first ball are green and second ball are red.

→



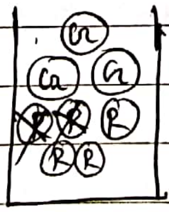
$$P(G_1 \cap R_2) = P(G_1)\ P(R_2/G_1)$$
$$= \frac{2}{5} \times \frac{3}{4}$$
$$= \frac{3}{10}$$

(*) $\boxed{P(E_1 \cap E_2 \cap E_3) = P(E_1)\ P(E_2/E_1)\ P(E_3/E_1\cap E_2)}$ → for 3 events

Chain rule

$\boxed{\text{Ex-2}}$ The Box containing total 8 balls. Where 5 Red balls and 3 Green balls. 3 balls are drawn one by one from the box without replacement. What is the propability that 3 balls are red.

→



$$P(R_1 \cap R_2 \cap R_3) = P(R_1)\ P(R_2/R_1)\ P(R_3/R_1\cap R_2)$$
$$= \left(\frac{5}{8} \times \frac{4}{7} \times \frac{3}{6}\right)$$
$$= \frac{5}{28}.$$

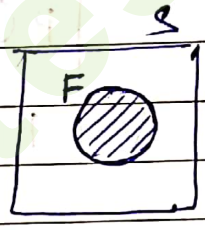
• PROPERTIES OF CONDITIONAL PROBABILITY:

(1) $P(S/F) = 1$

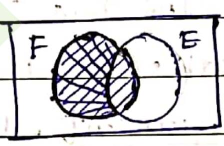
(2) $P(\bar{E}/F) = 1 - P(E/F)$.

(3) $P(A \cup B/F) = P(A/F) + P(B/F) - P\left(\frac{A \cap B}{F}\right)$.

(1) $P(S/F) \Rightarrow \frac{P(S \cap F)}{P(F)} \Rightarrow \frac{P(F)}{P(F)} \Rightarrow \underline{1}$



(2) $P(\bar{E}/F) = 1 - P(E/F)$

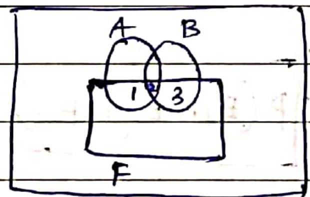


$P(F) = 1$

$\Rightarrow P(F) = P(\bar{E}/F) + P(E/F)$

$\Rightarrow 1 = P(\bar{E}/F) + P(E/F)$

(3) $P(A \cup B/F) = \underset{1,2}{\underline{P(A/F)}} + \underset{2,3}{\underline{P(B/F)}} - \underset{2}{\underline{\frac{P(A \cap B)}{F}}}$



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Ex-3

Ravi can either take biology or calculus. If he takes biology, then ~~to~~ he will get an 'A' grade with probability $1/2$. If he ~~to~~ takes calculus, then he will get an 'A' grade with probability $1/3$. He decides to base his decision on the flip of a fair coin. What is the probability that,

(a) He will get an 'A' in calculas.
(b) He will get an 'A' in biology.

→ Probability to take Biology(B) and calculas by flip of a coin,

$$P(B) = 1/2$$
$$P(C) = 1/2$$

given,

$P(A/B) = 1/2$ (Prob to get 'A' grade when he take Bio.)

$P(A/C) = 1/3$

(a) $P(C \cap A) = P(C)\ P(A/C)$

$= 1/2 \times 1/3 \Rightarrow 1/4$

(b) $P(B \cap A) = P(B)\ P(A/B)$

$= 1/2 \times 1/3 \Rightarrow 1/6$

Ex-4 suppose that each of 3 men at a party throws his hat into center of the room. The hats are first mixed up and each man randomly selects a hat. What is the probability that—

(a) All of the 3 men selects his own hat.

(b) Non of the 3 men selects his own hat.

→ E_1, E_2, E_3

(a) Three events that 3 man selects his own hat.

E_1, E_2, E_3

$P(E_1) = 1/3$ (Pro to get first person his own heat)

$P(E_2/E_1) = 1/2$ (" " secon " " ")

$P(E_3/E_1 \cap E_2) = 1$ (" " 3^{rd} " " ")

$$P(E_1 \cap E_2 \cap E_3) = P(E_1)\, P(E_2/E_1)\, P(E_3/E_1 \cap E_2)$$

$$= 1/3 \times 1/2 \times 1$$

$$= 1/6$$

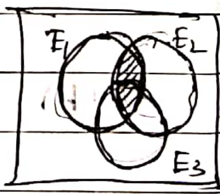
(b) Non of the 3 men selects his own hat –

$\bar{E}_1, \bar{E}_2, \bar{E}_3$

$$P(\bar{E}_1 \cap \bar{E}_2 \cap \bar{E}_3) = P(\overline{E_1 \cup E_2 \cup E_3})$$

** $\boxed{P(\bar{E}) = 1 - P(E)}$

$$= 1 - P(E_1 \cup E_2 \cup E_3)$$



$$= 1 - \frac{2}{3}$$

$$P(\bar{E}_1 \cap \bar{E}_2 \cap \bar{E}_3) = \frac{1}{3}$$

$P(E_1 \cup E_2 \cup E_3)$

$= P(E_1) + P(E_2) + P(E_3) - P(E_1 \cap E_2) - P(E_2 \cap E_3) - P(E_1 \cap E_3) + P(E_1 \cap E_2 \cap E_3)$

$= 1/3 + 1/3 + 1/3 - 1/6 - 1/6 - 1/6 + 1/6$

$= (1 - 1/3) = \frac{2}{3}$

$P(E_1 \cap E_2) = P(E_1)\ P(E_1/E_2)$
$= 1/3 \times 1/2 = 1/6$

$P(E_2 \cap E_3) = 1/3 \times 1/2 = 1/6$

$P(E_1 \cap E_3) = P(E_1)\ P(P_1/E_3)$
$= 1/3 \times 1/2 = 1/6$

- **Independent Events:**

→ Two events are said to be ~~…~~ Independent of each other, If the outcome of one event does not affect the outcome of other.

$\boxed{\text{Ex-}}$ A coin is tossed 2 time. What is the probability ~~that~~ of getting heads twice.

$P(H) = 1/2$
$P(T) = 1/2$

→ getting the head in 1'st flip $= H_1$
" " " " 2'nd flip $= H_2$

$P(H_1 \cap H_2) = P(H_1)\ P(H_2/H_1)$
$= P(H)\ P(H)$
$= \left(\frac{1}{2}\right)(1/2)$
$= \frac{1}{4}$

→ If A and B are two Independent event, then

$P(A \cap B) = P(A)\ P(B/A)$

$$\boxed{\begin{aligned} P(A/B) &= P(A) \\ P(B/A) &= P(B) \end{aligned}}$$

** $\boxed{P(A \cap B) = P(A)\ P(B)}$

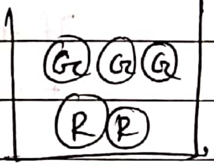
→ A, B and C are Independent events, then

** $\boxed{P(A \cap B \cap C) = P(A)\,P(B)\,P(C)}$

→ $E_1, E_2, E_3, \cdots\cdots E_n$ are Independent events, then

$$\boxed{P(E_1 \cap E_2 \cap E_3 \cap \cdots \cap E_n) = P(E_1)\,P(E_2)\,P(E_3) \cdots\cdots P(E_n)}$$

Example-1 In a bag there are 3 green balls and 2 red balls. ~~Dawn~~ Drawn balls randomly 2 times with replacement. Then ~~whath~~ what is the probability ~~In which~~ that first ball are green and second ball are red.



→ $P(G_1 \cap R_2) = P(G_1) \cdot P(R_2)$

$= \left(\frac{3}{5}\right) \times \left(\frac{2}{5}\right)$

$= \frac{6}{25}$.

→ Drawn balls randomly 3 times with replacement. Then what is the probability that first, second, third are ~~gren~~ green balls.

→ $P(G_1 \cap G_2 \cap G_3) = P(G_1)\,P(G_2)\,P(G_3)$

$= \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5}$

$= \frac{27}{125}$.

Ex-2 Three bags A, B and C. ~~A~~ In A contain 3 Red and 2 G ball, in B contain 3 red and 5 Green balls, in C bag contain 1 Red and 4 Green balls. Drawn ~~ba~~ 1 ball from each bag simultaneously, then what is the probability that Green ball from bag A, Red ball from bag B and Green ball from bag C.

$\rightarrow$

A	B	C
3R, 2G	3R, 5G	1R, 4G
G_A	R_B	G_C

$$P(G_A \cap R_B \cap G_C)$$

$$= P(G_A)\, P(R_B)\, P(G_C)$$

$$= \left(\frac{2}{5}\right) \times \left(\frac{3}{8}\right) \times \left(\frac{4}{5}\right)$$

$$= \frac{3}{25}$$

* If A, B are independent, then
(1) $A, \bar{B}$
(II) $\bar{A}, B$
(III) $\bar{A}, \bar{B}$ also independent.

[Ex-4] There is an interview and two people ~~are~~ A and B are ~~interviewed~~ getting interviewed and the probability that a getting job 1/2 and B geeting JOB 1/3. Both are independent events —

1) What is the prob that both of them selected.

$\rightarrow P(A \cap B) = P(A)\, P(B)$

$\Rightarrow \frac{1}{2} \times \frac{1}{2} \Rightarrow \frac{1}{4}$

II) what is the prob. that exactly One of them ~~at~~ will be selected.

$\rightarrow P(A \cap \bar{B}) + P(\bar{A} + B)$

$\rightarrow P(A)\, P(\bar{B}) + P(\bar{A})\, P(B)$

$\rightarrow \left(\frac{1}{2}\right)\left(\frac{2}{3}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{3}\right)$

$\Rightarrow \left(\frac{1}{3} + \frac{1}{6}\right)$

$P(\bar{B}) = 1 - P(3)$
$= 1 - \frac{1}{3}$
$= \frac{2}{3}$

$P(\bar{A}) = 1 - P(A)$
$= 1 - \frac{1}{2}$

(iii) What are the prov that both one are not go to be selected.

$\rightarrow P(\bar{A} \cap \bar{B})$

$\Rightarrow P(\bar{A})\ P(\bar{B})$

$\Rightarrow (1/2) \times (\frac{2}{3})$

$\Rightarrow 1/3$

(iv) What are the prov that at least one get selected.

$\rightarrow$ (i) + (ii).

(v) What are the prov that at most one get selected.

$\rightarrow$ (ii) + (iii).

$\boxed{Ex-5}$ We have lock and ~~we have Bunch~~ a bunch of 8 keys. Take a key and try to open it and even not abable to open it let the key stay in the bunch and then what is the propability that the lock will open in 7th try.

$$\rightarrow P(\bar{L}_1 \cap \bar{L}_2 \cap \bar{L}_3 \cap \bar{L}_4 \cap \bar{L}_5 \cap \bar{L}_6 \cap L_7)$$

$$= P(\bar{L}_1) \times P(\bar{L}_2) \times P(\bar{L}_3) \times P(\bar{L}_4) \times P(\bar{L}_5) \times P(\bar{L}_6) \times P(L_7)$$

$$= \left(\frac{7}{8}\right) \times \left(\frac{7}{8}\right) \times \left(\frac{7}{8}\right) \times \left(\frac{7}{8}\right) \times \left(\frac{7}{8}\right) \times \left(\frac{7}{8}\right) \times \left(\frac{1}{8}\right).$$

$$= \frac{(7)^6}{(8)^7}$$

(*) 90% of Question come from Independence like this –

→ 2 event are Independence then what is the prob that both this 2 event will happen simultaneously. ($P(A \cap B) = P(A)\,P(B)$)

Ex-6 Derive that the event are Independent or not.

• Rolling 2 dice.

$$S = \{(1\,1)(1\,2)(1\,3)(1\,4)(1\,5)(1\,6)$$
$$(2\,1)(2\,2)(2\,3)(2\,4)(2\,5)(2\,6)$$
$$\vdots$$
$$(6\,1)(6\,2)(6\,3)(6\,4)(6\,5)(6\,6)\}$$

• defind event E_1,

E_1 = Getting a '4' on 1st die.

$E_1 = \{(4,1)(4\,2)(4\,3)(4\,4)(4\,5)(4\,6)\}$

$$P(E_1) = \frac{6}{36} \Rightarrow \frac{1}{6}$$

• defind event E_2,

E_2 = Sum of two dice is '6'.

$E_2 = \{(1,5)(5,1)(2,4)(4,2)(3,3)\}$

$$P(E_2) = \frac{5}{36}$$

$$E_1 \cap E_2 = \{(4,2)\}$$

$$P(E_1 \cap E_2) = \frac{1}{36}.$$

$$\boxed{P(E_1 \cap E_2) \neq P(E_1)\,P(E_2)}$$

$\frac{1}{36}$ $\quad (1/6) \times (5/36)$

So, Event E_1 and E_2 dependent.

10 [Ex-7] Let a ball be drawn from an bag containing 4 balls numbered 1, 2, 3, 4. Let $E=\{1,2\}$, $F=\{1,3\}$, $G=\{1,4\}$. If all the outcomes are equally likely then are the events E, F and G independent.

→ To verify all three events E, F, G are Independent.

$$\begin{cases} P(E\cap F) = P(E)\,P(F) \\ P(F\cap G) = P(F)\,P(G) \\ P(E\cap G) = P(E)\,P(G) \\ P(E\cap F\cap G) = P(E)\,P(F)\,P(G) \end{cases}$$

→ It should be satisfy to prove E, F, G are Independent.

*

$$\boxed{\begin{array}{l} \text{For 4 event } E_1, E_2, E_3, E_4. \\ P(E_1\cap E_2) = P(E_1)\,P(E_2) \\ P(E_2\cap E_3) = P(E_2)\,P(E_3) \\ P(E_3\cap E_4) = P(E_3)\,P(E_4) \\ P(E_1\cap E_2\cap E_3\cap E_4) = P(E_1)\,P(E_2)\,P(E_3)\,P(E_4). \end{array}}$$

$P(1) = P(2) = P(3) = P(4) = 1/4$

$$\begin{cases} P(E) = \frac{1}{4} + \frac{1}{4} = 1/2 \\ P(F) = \frac{1}{2} \\ P(G) = \frac{1}{2} \end{cases}$$

(1/4) $P(E\cap F) = P(E)\,P(F) \Rightarrow 1/2 \times 1/2 \Rightarrow 1/4$.

(1/4) $P(F\cap G) = P(F)\,P(G) \Rightarrow 1/2 \times 1/2 \Rightarrow 1/4$.

(1/4) $P(E\cap G) = P(E)\,P(G) \Rightarrow 1/2 \times 1/2 \Rightarrow 1/4$.

$$\boxed{\underset{1/4}{P(E\cap F\cap G)} \neq \underset{(1/2)}{P(E)}\;\underset{(1/2)}{P(F)}\;\underset{(1/2)}{(P(G))}}$$

They are pair wise Independent but all together not Independent.

→ This formula are not satisfied.

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ex-7 A bag contain 50 good bulbs and 50 defective dulbs. 2 bulb are drawn at randomly with replacement. Then Event A is first bulb Good, Event B is second bulb Good and event C is both are good and both are defective. So verify that all the events A, B, C are Independent or not.

$\rightarrow$ Sample Space $(S) = \{ \overset{1/4}{GG}, \overset{1/4}{GB}, \overset{1/4}{BG}, \overset{1/4}{BB} \}$

$\left(\frac{50}{100}\right)\times\left(\frac{50}{100}\right)$

event $A = \{ \overset{1/4}{GG}, \overset{1/4}{GB} \}$ $\quad P(A) = \frac{2}{4} = 1/2$

$B = \{ GG, BG \}$ $\quad P(B) = \frac{2}{4} = 1/2$

$C = \{ \underset{1/4}{GG}, \underset{1/4}{BB} \}$ $\quad P(C) = 1/2$

To verify A, B, C are Independent ~~or~~ or not -

✓ $1/4$ $P(A\cap B) = P(A)\,P(B) = 1/2 \times 1/2 = 1/4$

✓ $1/4$ $P(B\cap C) = P(B)\,P(C) = 1/2 \times 1/2 = 1/4$

✓ $1/4$ $P(C\cap A) = P(C)\,P(A) = 1/2 \times 1/2 = 1/4$

} they are pair wise independent

$1/4$ $\boxed{P(A\cap B\cap C) = P(A)P(B)\,P(C)}$ $\rightarrow$ not satisfied, all three events together dependent.

$1/4 \neq (1/2 \times 1/2 \times 1/2)$ $1/8$

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$\boxed{EX-8}$ ~~F~~ Let flip a fair ~~go~~ coin 5 times. probability of getting head and probability of getting ~~tail~~ $1/2$. ~~Cerry~~ Then what is the probability ~~to get~~ that not getting two head consecutively.

$\rightarrow$ HTHTHT + THTHT

$\rightarrow (1/2)(1/2)(1/2)(1/2)(1/2) + (1/2)\times(1/2)\times(1/2)\times(1/2)\times(1/2)$

$\rightarrow \frac{1}{2^5} + \frac{1}{2^5}$

$\rightarrow 1/2^4 \rightarrow 1/16$.

$\boxed{EX-9}$ Let A, B, C are three Independent events.
(means they are pair wise Independent, also all together Independent)

1) A and $(B \cup C)$
2) B and $(A \cup C)$
3) C and $(A \cup B)$

$\left.\right\}$ — also Independent.

~~P(A∩~~

1) $P(A \cap (B \cup C)) = P(A)\ P(B \cup C)$

2) $P(B \cap (A \cup C)) = P(B)\ P(A \cup C)$

3) $P(C \cap (A \cup B)) = P(C)\ P(A \cup B)$.