1 Ideas

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2 Problem Formulation: OMUS as QMaxSAT

- Original Formula $\phi = \{c_1, ..., c_m\}$
- QBF Formulation of OMUS:
  - $S = \phi_R$
  - $\phi_R = \{\neg s_1 \vee c_1, ..., \neg s_m \vee c_m\}$
  - $\phi_{unsat} = \exists S \forall X . \neg \phi_R$
  - $f(s_1, ..., s_m) = \sum_{i=1}^{m} s_i \cdot a_i$
  - **Goal:** $\mathcal{A}_S \in \mathcal{M}(\phi_{unsat})$ s.t. $\forall \mathcal{B}_S \in \mathcal{M}(\phi_{unsat}) : f(\mathcal{A}_S) \leq f(\mathcal{B}_S)$
- Result QBF to decide iteratively
  - $\phi_{unsat} = \exists S \forall X . \neg \phi_R \wedge (f(s_1, ..., s_m) \leq k)$
  - $\phi_{unsat} = \exists S \forall X . \neg \phi_R \wedge \phi_S$, where $\phi_S = \{\neg s_1, ..., \neg s_m\}$

2.1 Digger - SOA SMUS

Algorithm 1: QMSU1 algorithm

$R_{all} \leftarrow \emptyset$
while *true* **do**
  $\phi'_R = \exists E \exists R_{all} \overrightarrow{Q} \cdot \phi$
  `// CEGAR 2 (or n) QBF oracle`
  (sat, ϕ_c, $\mathcal{A}_E$) $\leftarrow$ `QBF(`ϕ'_R`)`
  if *sat* **then**
    return $\mathcal{A}_E$
  end
  $R \leftarrow \emptyset$
  foreach $c \in \phi_c$ **do**
    let r be a new relaxation variable
    $R \leftarrow R \cup r$
    $\phi_S \leftarrow \phi_S \setminus \{c\} \cup \{c \wedge r\}$
  end
  $\phi \leftarrow \phi \wedge$ `CNF(`$\sum_{r \in R} \leq 1$`)`
  $R_{all} \leftarrow R_{all} \cup R$
end

Algorithm 2: CEGAR

$\omega \leftarrow \emptyset$ **while** *true* **do**
  $\phi_S \leftarrow$ `CNF(`$\bigwedge_{\nu \in \omega} \phi_H|_\nu$`)` $\cup$ `CNF(`$\bigwedge_{\nu \in \omega} \phi_S|_\nu$`)`
  (sat_1, μ, ϕ_C) $\leftarrow$ `SAT(`ϕ`)`
  if *not* sat_1 **then**
    $\phi'_S \leftarrow \{c \in \phi_S | c' \in \phi_C,\ \nu \in \omega c' = c|_\nu\}$
    return *(false,* $\phi_H \wedge \phi_S$*)*
  end
  (sat_2, μ, ϕ_C) $\leftarrow$ `SAT(`$\neg(\phi_H \wedge \phi_S))|_\nu$
  if *not* sat_2 **then**
    $\phi'_S \leftarrow \{c \in \phi_S | c' \in \phi_C,\ \nu \in \omega c' = c|_\mu\}$
    return *(true,* μ*)*
  end
  $\omega \leftarrow \omega \cup \{\nu\}$
end