```
In[1]:= Clear["Global`*"]

In[2]:= freq = 25; nSamples = 100;
(* This will make a "fake" wave signal (optical or audio signal). *)
(* Its frequency should be 25. *)
(*   Note, we are sampling the wave, so, actually, the frequency is *)
(*   25/100, as per y = A Cos(2*Pi*f*t) *)
(*   However, we are sampling our made-up wave - it is a discrete *)
(*   representation, so we have divided by nSamples to get an *)
(*   easy-to-understand frequency. *)
(*   Perhaps a clearer thing to say is that we are at a frequency *)
(*   of 25 cycles per 100 units of time. *)
(*   Table: Make it have a nice output *)
(*   N: Give the numberic value *)
(*   Cos: not too hard; the cosine function*)
(*   RandomReal: a random number *)
(*   to add "real-life" noise. *)
(*   Without it, the output *)
(*   would simply be: *)
(*   {0,-1,0,1,0,-1,0,1,...} *)
waveformWithXAxisTime =
  Table[N[Cos[2 * Pi * freq * t/nSamples] + RandomReal[] - 0.5], {t, nSamples}]
(* If this were audio output, the x-axis would be time, and the *)
(* y-axis would be (something related to) air pressure. *)
```

Out[3]= {-0.36486, -0.648403, -0.396836, 1.23881, 0.342068, -1.13419, 0.392898, 1.05064,
 -0.25348, -0.685096, 0.397843, 1.27155, -0.496049, -0.818067, -0.310042, 0.512795,
 0.216035, -1.046, -0.119668, 0.93829, 0.278568, -1.2363, 0.345026, 0.715119,
 0.26566, -1.3238, -0.0650755, 0.647469, -0.40178, -1.04048, 0.105636, 1.28159,
 -0.061007, -1.41506, -0.0191707, 0.896015, 0.342626, -0.801747, 0.371791,
 0.972851, 0.406637, -1.16023, -0.486096, 1.07916, 0.457143, -0.684228, 0.48586,
 0.644662, 0.231389, -0.805899, -0.0778726, 0.894136, -0.192847, -0.601711,
 0.160441, 1.38643, 0.40191, -1.01813, -0.433877, 1.40624, -0.44611, -1.32542,
 0.355899, 0.67695, -0.238991, -0.725832, 0.117066, 1.07671, -0.205068, -1.11087,
 0.290457, 0.960511, 0.423141, -0.529253, -0.0154628, 0.964296, 0.153138,
 -1.20842, 0.0731447, 1.43679, -0.422058, -1.46253, 0.122983, 0.729838, 0.401219,
 -0.70178, 0.095036, 0.717772, -0.0980891, -0.532807, -0.0976844, 0.89304,
 -0.0465112, -1.23261, 0.191923, 1.10104, 0.16775, -0.682639, -0.0548461, 1.17925}

In[4]:= `ListLinePlot[waveformWithXAxisTime, AxesOrigin → {1, 0}]`

Out[4]=
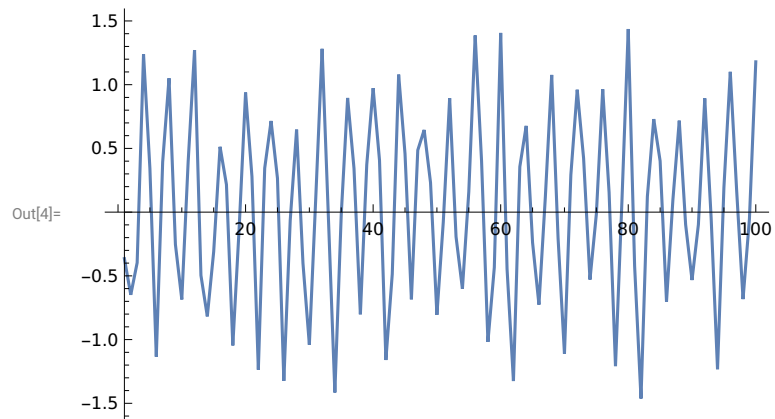
In[5]:= `(* Running a Discrete Fourier Transform. Note that we get complex values. *)`
`fourierTransformNowXAxisFreq = Fourier[waveformWithXAxisTime]`

Out[5]= $\{0.303025 + 0.\,i, -0.0788184 - 0.120385\,i, 0.273989 - 0.0698019\,i, 0.0267862 + 0.277852\,i,$

$0.0993218 - 0.0771232\,i, -0.0891472 - 0.0718325\,i, -0.0861473 + 0.348756\,i,$

$-0.025465 - 0.0674206\,i, -0.117247 - 0.28216\,i, 0.0979374 - 0.242297\,i,$

$-0.226255 - 0.110062\,i, 0.207117 + 0.234586\,i, -0.398628 - 0.0890934\,i,$

$0.160186 + 0.0438754\,i, -0.128123 + 0.204033\,i, -0.336882 - 0.301765\,i,$

$-0.173111 - 0.153429\,i, -0.131271 + 0.313048\,i, 0.310336 - 0.00870918\,i,$

$0.0818762 - 0.122505\,i, 0.353313 - 0.116102\,i, -0.173254 + 0.112998\,i,$

$-0.0764432 - 0.204795\,i, 0.0276179 + 0.269019\,i, -0.0937553 - 0.0471179\,i,$

$-0.056894 - 4.86034\,i, -0.0772726 + 0.123345\,i, 0.0950518 - 0.0732112\,i,$

$-0.124681 - 0.0982841\,i, 0.345192 - 0.210455\,i, 0.0937386 - 0.373958\,i,$

$-0.127874 + 0.411686\,i, 0.259327 + 0.25403\,i, -0.0255474 + 0.44143\,i,$

$0.0709624 - 0.0554208\,i, -0.189285 - 0.141046\,i, -0.147544 + 0.345096\,i,$

$0.396003 - 0.135013\,i, -0.164596 + 0.156145\,i, -0.0150639 + 0.00707736\,i,$

$-0.414499 - 0.0711398\,i, -0.359455 - 0.128501\,i, -0.173258 + 0.270357\,i,$

$-0.342158 - 0.0387509\,i, -0.0414707 - 0.168694\,i, 0.0891059 - 0.225836\,i,$

$0.00156622 + 0.243988\,i, -0.13661 - 0.0230807\,i, -0.355337 + 0.212988\,i,$

$-0.156618 - 0.20449\,i, 0.154936 - 4.44089 \times 10^{-17}\,i, -0.156618 + 0.20449\,i,$

$-0.355337 - 0.212988\,i, -0.13661 + 0.0230807\,i, 0.00156622 - 0.243988\,i,$

$0.0891059 + 0.225836\,i, -0.0414707 + 0.168694\,i, -0.342158 + 0.0387509\,i,$

$-0.173258 - 0.270357\,i, -0.359455 + 0.128501\,i, -0.414499 + 0.0711398\,i,$

$-0.0150639 - 0.00707736\,i, -0.164596 - 0.156145\,i, 0.396003 + 0.135013\,i,$

$-0.147544 - 0.345096\,i, -0.189285 + 0.141046\,i, 0.0709624 + 0.0554208\,i,$

$-0.0255474 - 0.44143\,i, 0.259327 - 0.25403\,i, -0.127874 - 0.411686\,i,$

$0.0937386 + 0.373958\,i, 0.345192 + 0.210455\,i, -0.124681 + 0.0982841\,i,$

$0.0950518 + 0.0732112\,i, -0.0772726 - 0.123345\,i, -0.056894 + 4.86034\,i,$

$-0.0937553 + 0.0471179\,i, 0.0276179 - 0.269019\,i, -0.0764432 + 0.204795\,i,$

$-0.173254 - 0.112998\,i, 0.353313 + 0.116102\,i, 0.0818762 + 0.122505\,i,$

$0.310336 + 0.00870918\,i, -0.131271 - 0.313048\,i, -0.173111 + 0.153429\,i,$

$-0.336882 + 0.301765\,i, -0.128123 - 0.204033\,i, 0.160186 - 0.0438754\,i,$

$-0.398628 + 0.0890934\,i, 0.207117 - 0.234586\,i, -0.226255 + 0.110062\,i,$

$0.0979374 + 0.242297\,i, -0.117247 + 0.28216\,i, -0.025465 + 0.0674206\,i,$

$-0.0861473 - 0.348756\,i, -0.0891472 + 0.0718325\,i, 0.0993218 + 0.0771232\,i,$

$0.0267862 - 0.277852\,i, 0.273989 + 0.0698019\,i, -0.0788184 + 0.120385\,i\}$

In[6]:= `ListLinePlot[Abs[fourierTransformNowXAxisFreq], PlotRange → All]`

Out[6]=
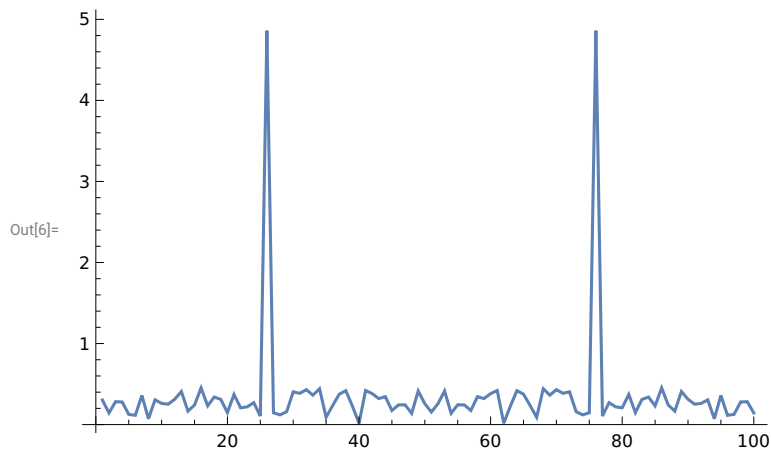


In[7]:= `(*We've now gone from a time profile to a frequency profile,`
`      where we can see that the most important frequency is 25. I'`
`    ll have to review my sampling theory or ask David, but I'm`
`  pretty sure that the peak at 75 is just a reflection of the`
`  frequency component and has to do with our sampling,`
`i.e. n_samples - freq = 75 *)`
`(* A conceptual explanation is that, with a "window" of 100 (100 samples),`
`it would be impossible to draw out frequencies greater than`
`  50/100. That (pure) frequency of 50 would mean that we would need`
`  to "see" the graph going from 1 to -1 with every tick of the`
`  axis: {1,-1,1,-1,1,…}. (Maybe you can see why I used cosine instead of sine. *)`
`(* Hopefully, it's obvious that we can't get a frequency higher than that the`
`  granularity of our sampling makes it impossible to go "all the way" to 100. *)`
`(* Also remember the concept that Cos[x - Pi/2] = Sin[x] *)`

In[8]:= `(* Let's make two main frequencies and decide what it does.*)`

`(* I did, and decided that we need to up nSamples *)`

```
freq1 = 25; amp1 = 10; freq2 = 75; amp2 = 5; nSamplesMix = 500;
waveformMixed = Table[N[amp1 * Cos[2 * Pi * freq1 * t / nSamplesMix] + (RandomReal[] - 0.5) +
        amp2 * Cos[2 * Pi * freq2 * t / nSamplesMix] + (RandomReal[] - 0.5)], {t, nSamplesMix}];
ListLinePlot[waveformMixed, AxesOrigin → {1, 0}]
fourierMixed = Fourier[waveformMixed];
ListLinePlot[Abs[fourierMixed], PlotRange → All]
```

Out[10]=
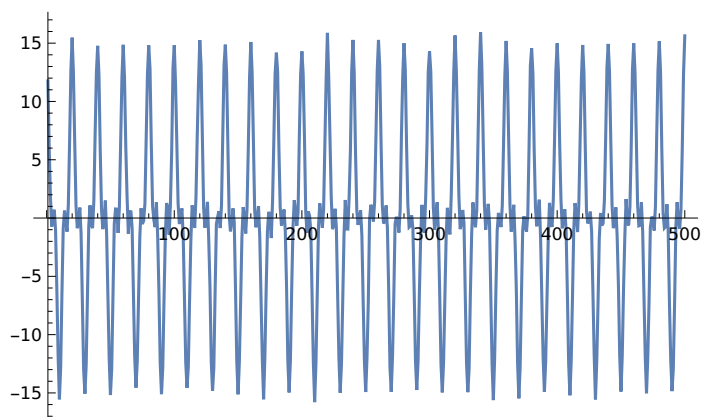


Out[12]=
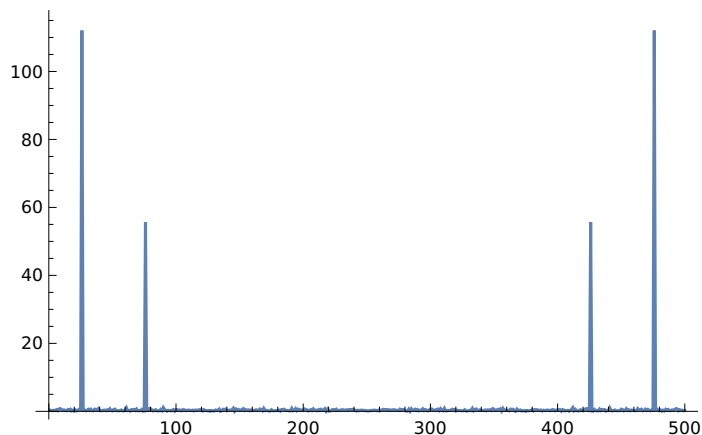


**Before executing these two cells below, you'll need to go down to the bottom of the notebook, where I've put the bunches of data that would be way too much in the way here. It's under the only other cell that has plain input and the white equals sign with the orange background. Evaluate those, then evaluate these.**

In[15]:= `pianoA440 = pianoA440PreData;`
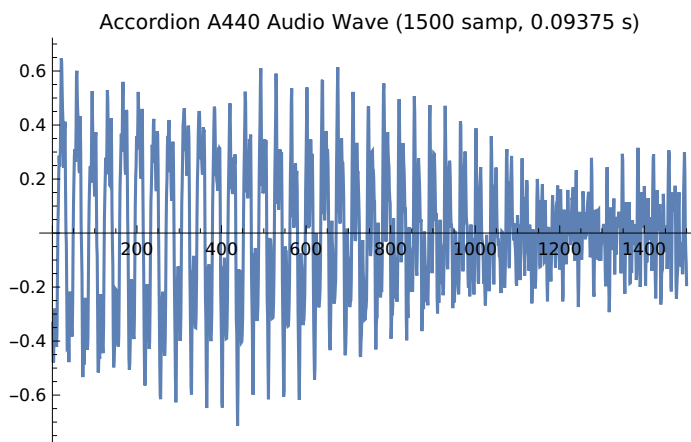
In[16]:= `accordionA440 = accordionA440PreData;`

In[17]:= `ListLinePlot[pianoA440, PlotRange → All,`

`    PlotLabel → "Piano A440 Audio Wave (1500 samp, 0.09375 s)"]`

Out[17]=

Piano A440 Audio Wave (1500 samp, 0.09375 s)
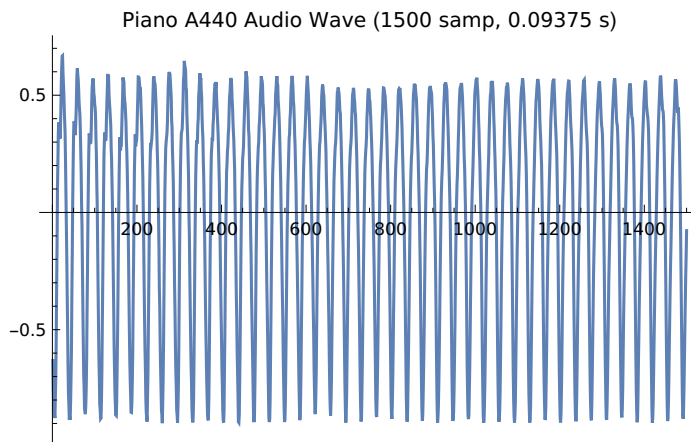


In[18]:= `ListLinePlot[accordionA440, PlotRange → All,`

`    PlotLabel → "Accordion A440 Audio Wave (1500 samp, 0.09375 s)"]`

Out[18]=

Accordion A440 Audio Wave (1500 samp, 0.09375 s)

In[19]:=
```
pianoFourier = Abs[Fourier[pianoA440]];
accordionFourier = Abs[Fourier[accordionA440]];
ListLinePlot[pianoFourier,
  PlotRange → {{0, 1500./2}, {0, 13}}, PlotLabel → "Piano A440 Fourier"]
lineStyle1 = {Thick, Red, Dashed};
lineStyle2 = {Automatic};
line = Line[{{42, 0}, {42, 13}}];
ListLinePlot[pianoFourier, PlotRange → {{30, 50}, {0, 13}},
  PlotLabel → "Piano Fourier Zoom, line: x=42",
  PlotStyle → {Directive[lineStyle2], Directive[lineStyle1]},
  Epilog → {Directive[lineStyle1], line}]
Print["Frequency is 42 cycles per 0.09375 seconds = ",
  42/0.09375, " Hz. I only need to tune my piano a little."]
Print["Actually, since 1 \"tick\" on the x-axis is 1/0.09375 = ",
  1/0.09375, " Hz, I'm right on, within sampling error."]
```

Out[21]=



Piano A440 Fourier

Piano Fourier Zoom, line: x=42
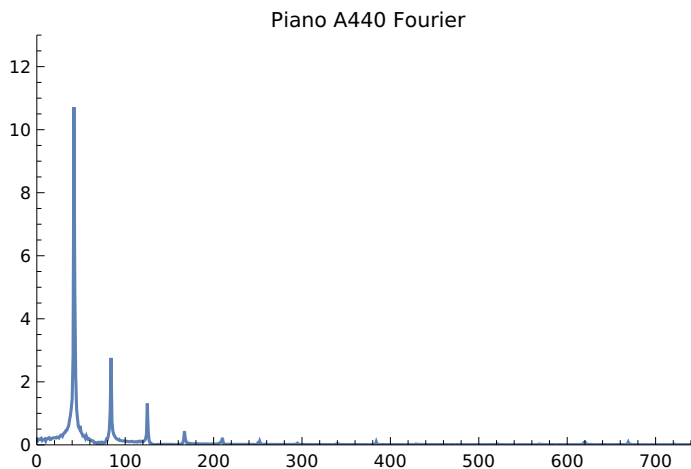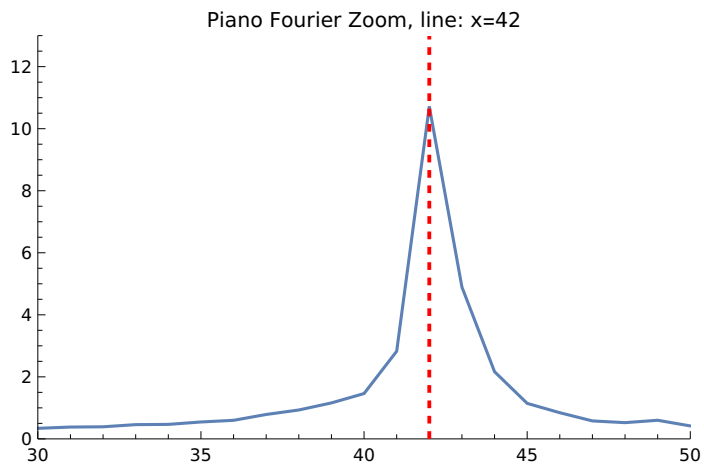
Frequency is 42 cycles per 0.09375 seconds =
448.  Hz. I only need to tune my piano a little.

Actually, since 1 "tick" on the x-axis is 1/0.09375 =
10.6667  Hz, I'm right on, within sampling error.

In[28]:= 
```
(*Accordion time*)
line2 = Line[{{42, 0}, {42, 5}}];
ListLinePlot[accordionFourier,
  PlotRange → {{0, 1500./2}, {0, 5}}, PlotLabel → "Accordion A440 Fourier"]
ListLinePlot[accordionFourier, PlotRange → {{30, 50}, {0, 5}},
  PlotLabel → "Accordion Fourier Zoom, line: x=42",
  PlotStyle → {Directive[lineStyle2], Directive[lineStyle1]},
  Epilog → {Directive[lineStyle1], line2}]
Print["It seems as if my wife and I
    can play together without too much cacaphony. Hooray!"]
```

Out[29]=

Accordion A440 Fourier

Out[30]=

Accordion Fourier Zoom, line: x=42
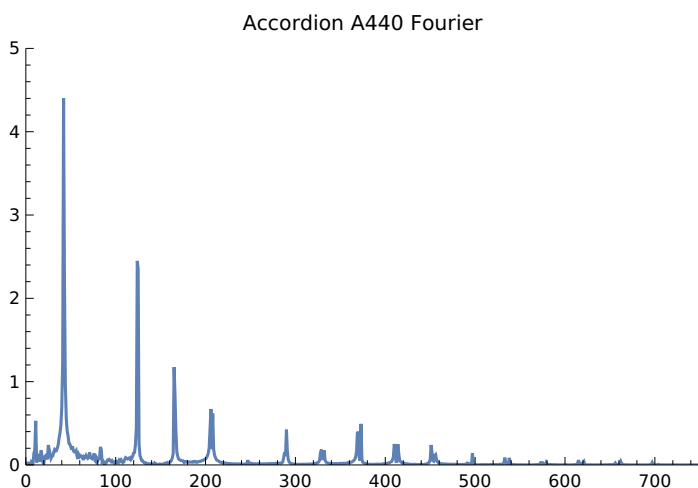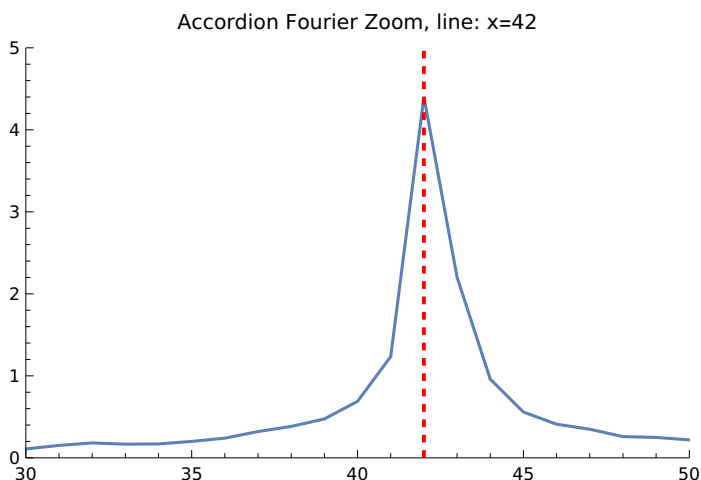
It seems as if my wife and I can play together without too much cacaphony. Hooray!

**Here below are the data for the piano and the accordion. You'll have to evaluate these cells before the one that loads the pianoA440 and accordionA440.**

In[13]:= 
```
pianoA440PreData = {-0.63177, -0.74435, -0.83264, -0.85211, -0.86359, -0.87689,
    -0.78519, -0.70773, -0.54391, -0.33185, -0.14603, 0.10275, 0.21585, 0.33972,
```

0.38388, 0.36969, 0.35489, 0.31470, 0.36145, 0.43430, 0.54309, 0.61490,
0.66663, 0.66837, 0.60025, 0.55310, 0.50909, 0.44269, 0.39230, 0.33795,
0.22232, 0.08707, −0.05246, −0.18832, −0.34576, −0.49463, −0.65637, −0.77451,
−0.82132, −0.88495, −0.87149, −0.88406, −0.81351, −0.71402, −0.59161, −0.36569,
−0.17416, 0.04361, 0.19220, 0.32034, 0.38849, 0.38412, 0.36917, 0.32968,
0.36914, 0.41937, 0.49280, 0.58588, 0.61496, 0.58783, 0.56805, 0.52792, 0.47461,
0.44208, 0.41992, 0.35898, 0.26733, 0.17804, 0.03726, −0.05521, −0.22406,
−0.39355, −0.54337, −0.68875, −0.74799, −0.83746, −0.84930, −0.86029, −0.82080,
−0.75211, −0.62833, −0.41782, −0.24915, −0.03699, 0.11368, 0.24915, 0.33652,
0.32446, 0.30695, 0.29266, 0.29959, 0.35797, 0.46655, 0.53055, 0.55707,
0.57114, 0.55899, 0.50757, 0.49429, 0.46091, 0.44772, 0.41644, 0.31528,
0.25235, 0.12918, 0.03073, −0.13666, −0.31235, −0.46591, −0.61597, −0.70551,
−0.81845, −0.83960, −0.87973, −0.87714, −0.80527, −0.68567, −0.49420, −0.30991,
−0.11499, 0.08121, 0.24200, 0.30804, 0.34116, 0.33090, 0.30505, 0.32162,
0.41345, 0.47513, 0.53296, 0.59085, 0.56659, 0.56873, 0.49582, 0.46719,
0.44019, 0.40433, 0.37213, 0.27670, 0.22522, 0.12738, 0.03680, −0.11151,
−0.26373, −0.40497, −0.56903, −0.67761, −0.76755, −0.82117, −0.86426, −0.86633,
−0.81342, −0.68378, −0.54153, −0.36792, −0.15591, 0.03445, 0.17194, 0.27737,
0.32138, 0.26498, 0.28610, 0.28708, 0.35831, 0.43268, 0.48630, 0.57538,
0.55917, 0.56219, 0.50308, 0.47513, 0.46310, 0.42392, 0.38474, 0.30270,
0.25751, 0.18369, 0.08215, −0.03098, −0.16925, −0.34641, −0.49280, −0.62369,
−0.71201, −0.77444, −0.85208, −0.85300, −0.80832, −0.71970, −0.60065, −0.38361,
−0.21362, −0.00943, 0.17450, 0.26672, 0.33261, 0.28271, 0.30704, 0.30368,
0.33969, 0.42532, 0.48831, 0.57791, 0.57751, 0.57388, 0.54858, 0.52887,
0.53296, 0.48740, 0.42621, 0.37878, 0.30307, 0.22290, 0.14108, 0.00485,
−0.13077, −0.31091, −0.45374, −0.57898, −0.66742, −0.79208, −0.85327, −0.86981,
−0.89212, −0.78723, −0.68726, −0.49133, −0.30746, −0.10587, 0.10718, 0.19962,
0.28021, 0.27808, 0.29614, 0.31323, 0.34280, 0.43726, 0.50400, 0.56992,
0.57993, 0.56424, 0.55222, 0.54498, 0.50558, 0.46698, 0.40024, 0.34103,
0.26282, 0.20505, 0.10590, −0.00851, −0.14493, −0.32831, −0.41376, −0.56906,
−0.67099, −0.76035, −0.84540, −0.86084, −0.89886, −0.80789, −0.70474, −0.54105,
−0.32755, −0.12656, 0.08514, 0.18494, 0.27231, 0.30096, 0.31821, 0.33823,
0.38086, 0.45969, 0.54001, 0.57965, 0.59909, 0.59213, 0.59048, 0.56134, 0.52527,
0.49280, 0.39499, 0.36142, 0.25919, 0.20200, 0.12296, −0.04004, −0.14661,
−0.31354, −0.42386, −0.56763, −0.68179, −0.74283, −0.84277, −0.86475, −0.89615,
−0.82993, −0.73700, −0.57983, −0.35181, −0.13705, 0.05585, 0.19095, 0.28055,
0.33624, 0.34613, 0.35699, 0.40643, 0.47607, 0.56195, 0.58490, 0.64636,
0.62790, 0.61731, 0.60693, 0.53281, 0.52451, 0.42450, 0.36636, 0.31525,
0.23383, 0.15387, −0.00470, −0.10187, −0.26462, −0.40308, −0.53326, −0.64810,
−0.72690, −0.83203, −0.87692, −0.89545, −0.87189, −0.78232, −0.61713, −0.40192,
−0.17432, −0.00217, 0.15570, 0.24191, 0.30664, 0.29343, 0.30417, 0.37296,
0.40018, 0.49609, 0.52246, 0.56238, 0.59348, 0.55728, 0.57297, 0.52393,
0.49225, 0.40439, 0.35248, 0.31320, 0.21689, 0.14398, 0.02231, −0.08832,

−0.23926, −0.38287, −0.50037, −0.61050, −0.71680, −0.79590, −0.86377,
−0.88126, −0.88675, −0.79544, −0.63721, −0.43594, −0.20279, −0.04434, 0.15274,
0.25137, 0.28098, 0.31824, 0.30469, 0.37375, 0.40027, 0.45529, 0.50665,
0.53143, 0.55106, 0.53040, 0.55658, 0.50864, 0.46088, 0.41858, 0.37714,
0.33347, 0.26712, 0.19315, 0.10559, −0.03558, −0.16660, −0.30963, −0.42542,
−0.53830, −0.66690, −0.73676, −0.84225, −0.88455, −0.89575, −0.84268,
−0.65952, −0.51279, −0.29349, −0.10455, 0.05383, 0.17517, 0.20627, 0.26077,
0.28241, 0.33215, 0.36200, 0.42969, 0.49359, 0.50912, 0.54678, 0.57300,
0.56750, 0.53949, 0.48425, 0.45279, 0.39554, 0.34573, 0.31070, 0.24643,
0.18460, 0.04327, −0.07727, −0.20087, −0.35028, −0.44992, −0.59705, −0.67947,
−0.78391, −0.89294, −0.89572, −0.86353, −0.72180, −0.56744, −0.36319, −0.16855,
−0.00830, 0.10349, 0.15897, 0.22971, 0.25180, 0.28110, 0.35425, 0.40872,
0.47131, 0.51202, 0.56192, 0.60184, 0.58441, 0.57883, 0.52521, 0.48303,
0.42746, 0.35107, 0.32486, 0.23572, 0.15262, 0.04056, −0.09351, −0.18439,
−0.34656, −0.45050, −0.56339, −0.67050, −0.78741, −0.88348, −0.89438, −0.86002,
−0.73260, −0.57831, −0.37354, −0.16409, −0.02774, 0.10440, 0.18793, 0.23404,
0.27798, 0.30215, 0.36633, 0.39377, 0.43689, 0.49728, 0.53244, 0.58130,
0.56790, 0.56134, 0.54160, 0.47025, 0.44891, 0.38065, 0.35052, 0.28394,
0.17828, 0.09216, −0.02600, −0.15042, −0.29422, −0.39832, −0.51974, −0.64285,
−0.76495, −0.87616, −0.89377, −0.88272, −0.79678, −0.61609, −0.43695, −0.23434,
−0.07532, 0.05777, 0.16599, 0.21484, 0.27716, 0.31796, 0.35599, 0.40198,
0.42447, 0.50372, 0.52835, 0.56345, 0.58118, 0.55280, 0.54236, 0.47842,
0.45218, 0.41440, 0.36313, 0.29346, 0.21243, 0.13440, 0.00497, −0.11014,
−0.24100, −0.35556, −0.45792, −0.61374, −0.72601, −0.82687, −0.89383,
−0.88361, −0.81094, −0.65045, −0.48199, −0.30658, −0.11743, −0.00391, 0.11682,
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