Exercise set 6

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Clear R environment

```r
rm(list = ls())
```

Exercise 6.6 in Rizzo

In Exaple 6.7 the control variate approach was illustrated for monte carlo integration of

$$\theta = \int_0^1 e^x dx.$$

Now consider the antithetic variate approach. Compute $Cov(e^U, e^{1-U})$ and $Var(e^U + e^{1-U})$, where $U \sim \text{Uniform}(0,1)$. What is the percent reduction in variance of $\hat{\theta}$ that can be achieved using antithetic variates (compared with simple MC)?

```r
Nsim <- 10^4
u <- runif(Nsim, 0, 1)
x1 <- exp(1)^u
x2 <- exp(1)^(1-u)
cov <- cov(x1, x2)
var <- var(x1 + x2)
cov
```

```
## [1] -0.2314804
```

```r
# finne reduksjon i varians mellom simple MC og antithetic variate MC?
```

Exercise 6.7 in Rizzo

Refer to exercise 6.6. Use a Monte Carlo simulation to estimate θ by the antithetic variate approach and by the simple monte carlo method. Compute an empirical estimate of the percent reduction in variance using the antithetic variate. Compare the result with the theoritical value from Exercise 6.6.

```r
nrep <- 1000
intcMCvec <- numeric(nrep)
intaMCvec <- numeric(nrep)
for(i in 1:nrep){
  x <- runif(Nsim)
  intcMCvec[i] <- mean(exp(x)) # crude MC result
  x1 <- runif(Nsim/2)
  x2 <- 1-x1
  x <- c(x1,x2)
  intaMCvec[i] <- mean(exp(x)) # antitetic MC result
}
```

```r
mean(intcMCvec)
```

```
## [1] 1.718151
```

```r
mean(intaMCvec)
```

```
## [1] 1.718285
```

```r
1 - (var(intaMCvec) / var(intcMCvec))
```

```
## [1] 0.9680018
```

Exercise 6.10 in Rizzo

Use MC integration with antithetic variables to estimate

$$\int_0^1 \frac{e^{-x}}{1+x^2}dx,$$

and find the approximate reduction in variance as a percentage of the variance without variance reduction.

```r
intcMCvec <- numeric(nrep)
intaMCvec <- numeric(nrep)
for(i in 1:nrep){
  x <- runif(Nsim)
  intcMCvec[i] <- mean(exp(-x)/(1 + x^2)) # crude MC result
  x1 <- runif(Nsim/2)
  x2 <- 1-x1
  x <- c(x1,x2)
  intaMCvec[i] <- mean(exp(-x)/(1 + x^2)) # antitetic MC result
}

mean(intcMCvec)
```

```
## [1] 0.524818
```

```r
mean(intaMCvec)
```

```
## [1] 0.5248006
```

```r
1 - (var(intaMCvec) / var(intcMCvec))
```

```
## [1] 0.9686317
```

Exercise 6.12 in Rizzo

Let $\theta_f^{\hat{I}S}$ be an importans ampling estimator of $\theta = \int g(x)dx$, where the importance function f is a density. Prove that if $g(x)/f(x)$ is bounded, then the variance of the importance sampling estimator $\theta_f^{\hat{I}S}$ is finite.