Exercise set 2 - Week 36

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This is an R Markdown Notebook. When you execute code within the notebook, the results appear beneath the code.

This notebook is shows my solution to the exercise sheet for week 35 in STA510 course at the University of Stavanger.

Import libraries

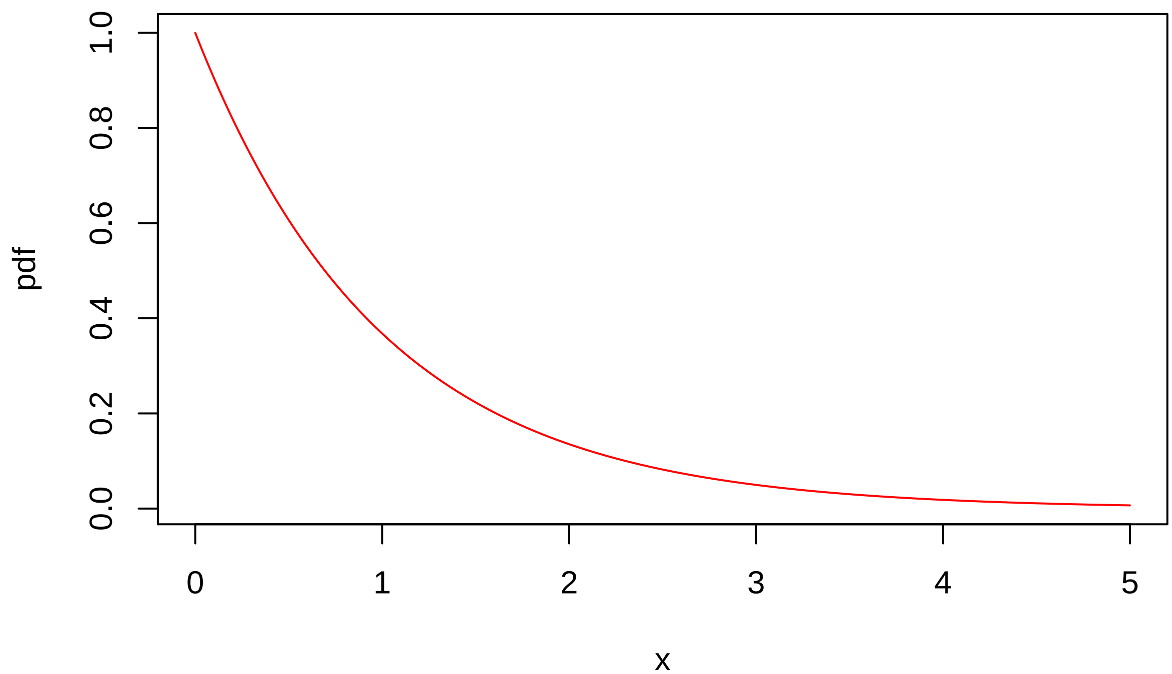
```
library(ggplot2)     # Library for plotting
```

Problem 1

Let X be exponentially distributed with density function $f(x) = \lambda e^{-\lambda x}$ for $x > 0$

a. Make a sketch of this density (e.g. for $\lambda = 1$) and compute the expectation and the variance of X as a function of λ.

```
fexp <- function (x, lambda = 1) {
  return(lambda * exp(1)^(-lambda*x))
}

x<-seq(0, 5, 0.01); y1=fexp(x, 1);

plot(x, y1, col="red", type="l", xlab="x", ylab="pdf")
```



Calculate E(X)

```
integrate(function(x) x*fexp(x, lambda = 1), 0, Inf)
```

```
## 1 with absolute error < 6.4e-06
```

```
integrate(function(x) x*fexp(x, lambda = 2), 0, Inf)
```

```
## 0.5 with absolute error < 8.6e-06
```

```
integrate(function(x) x*fexp(x, lambda = 3), 0, Inf)
```

```
## 0.3333333 with absolute error < 8.1e-08
```

```
integrate(function(x) x*fexp(x, lambda = 4), 0, Inf)
```

```
## 0.25 with absolute error < 3.2e-05
```

The lifetime of a type of light bulbs follows an exponential distribution with expectation 10 000 hours.

b. Let X be the random variable for lifetime of a light bulb. Calculate the probability:

$$P(X < 10000)$$

```
pexp(10000, rate = 1/10000, lower.tail = TRUE, log.p = FALSE)
```

```
## [1] 0.6321206
```

$P(X > 5000)$

```
pexp(5000, rate = 1/10000, lower.tail = FALSE, log.p = FALSE)
```

```
## [1] 0.6065307
```

$P(5000 < X < 10000)$

```
pexp(10000, rate = 1/10000, lower.tail = TRUE, log.p = FALSE) -
pexp(5000, rate = 1/10000, lower.tail = TRUE, log.p = FALSE)
```

```
## [1] 0.2386512
```

Problem 2

a. Calculate the probability that there will be exactly 2 accidents during one year.

Calculate the probability that there will be at least 3 accidents during one year.

$P(X = 2)$

```
dpois(2, lambda = 2, log = FALSE)
```

```
## [1] 0.2706706
```

$P(X < 3)$

```
ppois(3, lambda = 2, lower.tail = TRUE, log = FALSE)
```

```
## [1] 0.8571235
```

b. Calculate the probability that there will be exactly 2 accidents during half a year.

2 accidents during half a year gives us $\lambda = 1$

$P(X = 2)$

```
dpois(2, lambda = 1, log = FALSE)
```

```
## [1] 0.1839397
```

c. Calculate the probability that there will be at least 20 accidents during ten years (Hint for the manual calculation: By the Central Limit Theorem a Poisson distribution with expectation larger than 15 can be approximated by a normal distribution with the same expectation and variance as the Poisson distribution)

$P(X < 20)$ **when** $\lambda = 20$

```
ppois(20, lambda = 20, lower.tail = FALSE, log = FALSE)
```

```
## [1] 0.4409074
```

d. Calculate the probability that there will be more than 2 accidents on this road during a year when we know that there has been at least one accident.

This gives us:

$$P(X > 2|X > 1) = \frac{P(X > 2 \cap X > 1)}{P(X > 1)} = \frac{P(X > 2)}{P(X > 1)} \tag{1}$$

```
a <- ppois(2, lambda = 2, lower.tail = FALSE, log = FALSE)
b <- ppois(1, lambda = 2, lower.tail = FALSE, log = FALSE)
s <- a/b
s
```

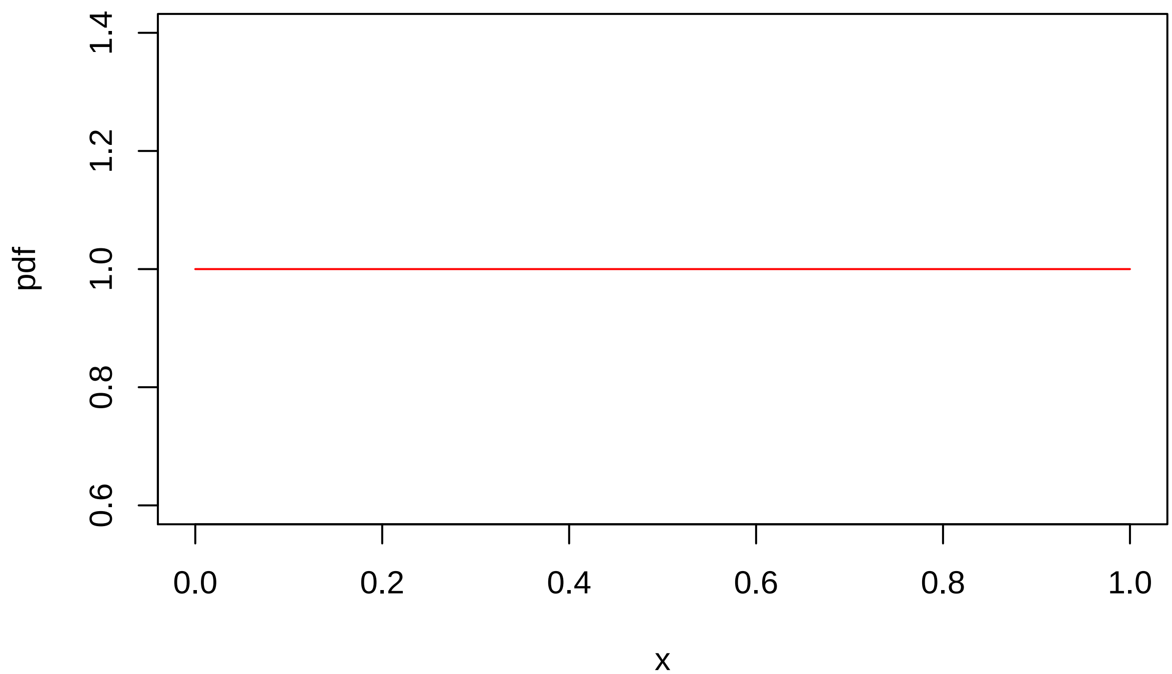
```
## [1] 0.5443212
```

Problem 3

```
betaf <- function(alfa) {
  helpf <- function(t) {
    t^(alfa - 1) * exp(1)^(-t)
  }

  return(integrate(helpf, 0, Inf)$value)
}

f <- function(x, alfa, beta) {
  return((betaf(alfa + beta) / betaf(alfa)*betaf(beta)) * x^(alfa - 1) * (1 - x)^(beta - 1))
}
```

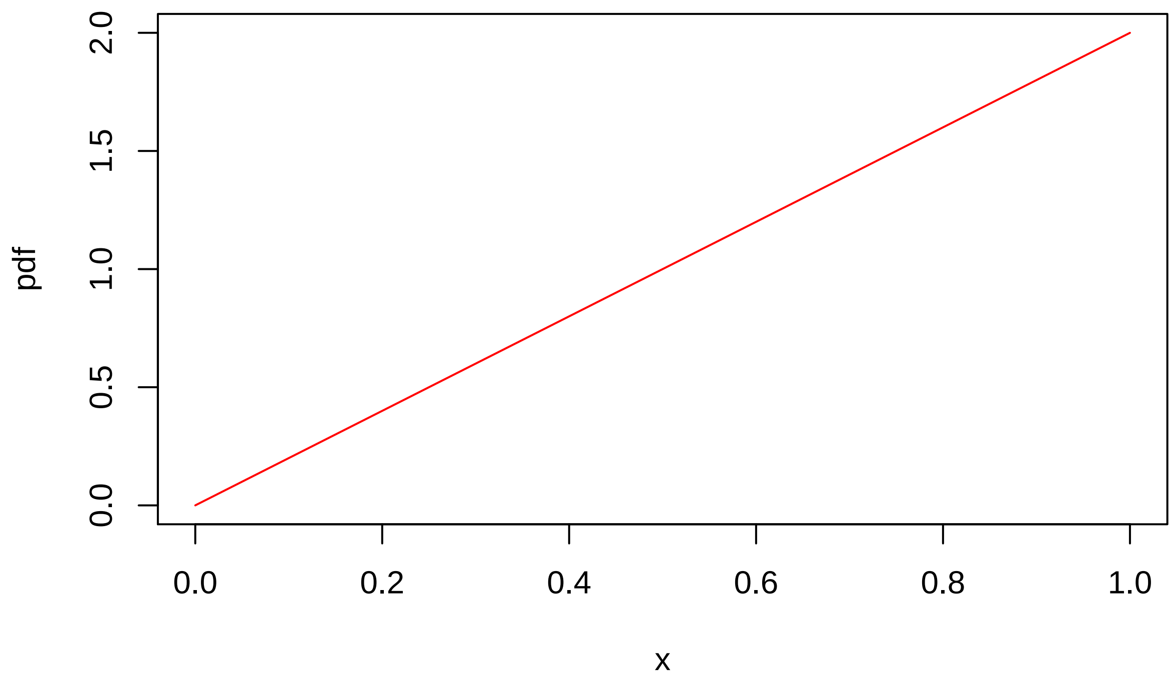
$\alpha = \beta = 1$

```
x<-seq(0, 1, 0.001); y1=f(x, 1, 1);

plot(x, y1, col="red", type="l", xlab="x", ylab="pdf")
```



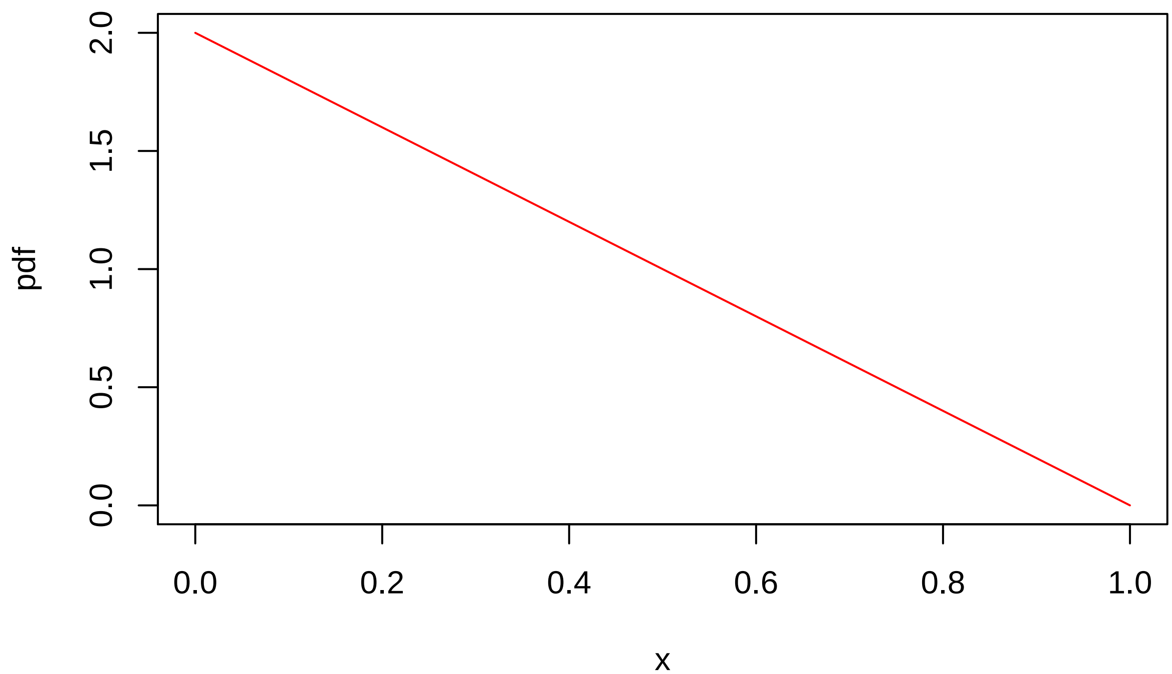
\# $\alpha = 2\beta = 1$

```
x<-seq(0, 1, 0.001); y1=f(x, 2, 1);

plot(x, y1, col="red", type="l", xlab="x", ylab="pdf")
```



$\alpha = 1 \beta = 2$

```r
x<-seq(0, 1, 0.001); y1=f(x, 1, 2);

plot(x, y1, col="red", type="l", xlab="x", ylab="pdf")
```



b. Draw samples and calculate sample mean and compare to expectation

```
samp10_1_1 = sum(rbeta(10, 1, 1, ncp = 0)) / 10
samp10_2_1 = sum(rbeta(10, 2, 1, ncp = 0)) / 10
samp10_1_2 = sum(rbeta(10, 1, 2, ncp = 0)) / 10
samp100_1_1 = sum(rbeta(100, 1, 1, ncp = 0)) / 100
samp100_2_1 = sum(rbeta(100, 2, 1, ncp = 0)) / 100
samp100_1_2 = sum(rbeta(100, 1, 2, ncp = 0)) / 100
samp1000_1_1 = sum(rbeta(1000, 1, 1, ncp = 0)) / 1000
samp1000_2_1 = sum(rbeta(1000, 2, 1, ncp = 0)) / 1000
samp1000_1_2 = sum(rbeta(1000, 1, 2, ncp = 0)) / 1000
samp10000_1_1 = sum(rbeta(10000, 1, 1, ncp = 0)) / 10000
samp10000_2_1 = sum(rbeta(10000, 2, 1, ncp = 0)) / 10000
samp10000_1_2 = sum(rbeta(10000, 1, 2, ncp = 0)) / 10000

samp10_1_1
```

```
## [1] 0.4712811
```

```
samp10_2_1
```

```
## [1] 0.6456831
```

```
samp10_1_2
```

```
## [1] 0.4024883
```

```
samp100_1_1
```

```
## [1] 0.4519164
```

```r
samp100_2_1
```

```
## [1] 0.6363363
```

```r
samp100_1_2
```

```
## [1] 0.2822957
```

```r
samp1000_1_1
```

```
## [1] 0.4994862
```

```r
samp1000_2_1
```

```
## [1] 0.6551235
```

```r
samp1000_1_2
```

```
## [1] 0.3327129
```

```r
samp10000_1_1
```

```
## [1] 0.4994846
```

```r
samp10000_2_1
```

```
## [1] 0.6649825
```

```r
samp10000_1_2
```

```
## [1] 0.3339194
```

Problem 4