Exercise set 8

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Clear R environment

```
rm(list = ls())
```

Exercise 8.4

Note: MLE for $\hat{\lambda} = n/\sum_{i=1}^{n} X_i$, where X denote time between failure

Refer to the air-conditioning data set **aircondit** provided in the boot library. The 12 observations are the times in hours between failures of air-conditioning equipment

3 5 7 18 43 85 91 98 100 130 230 487

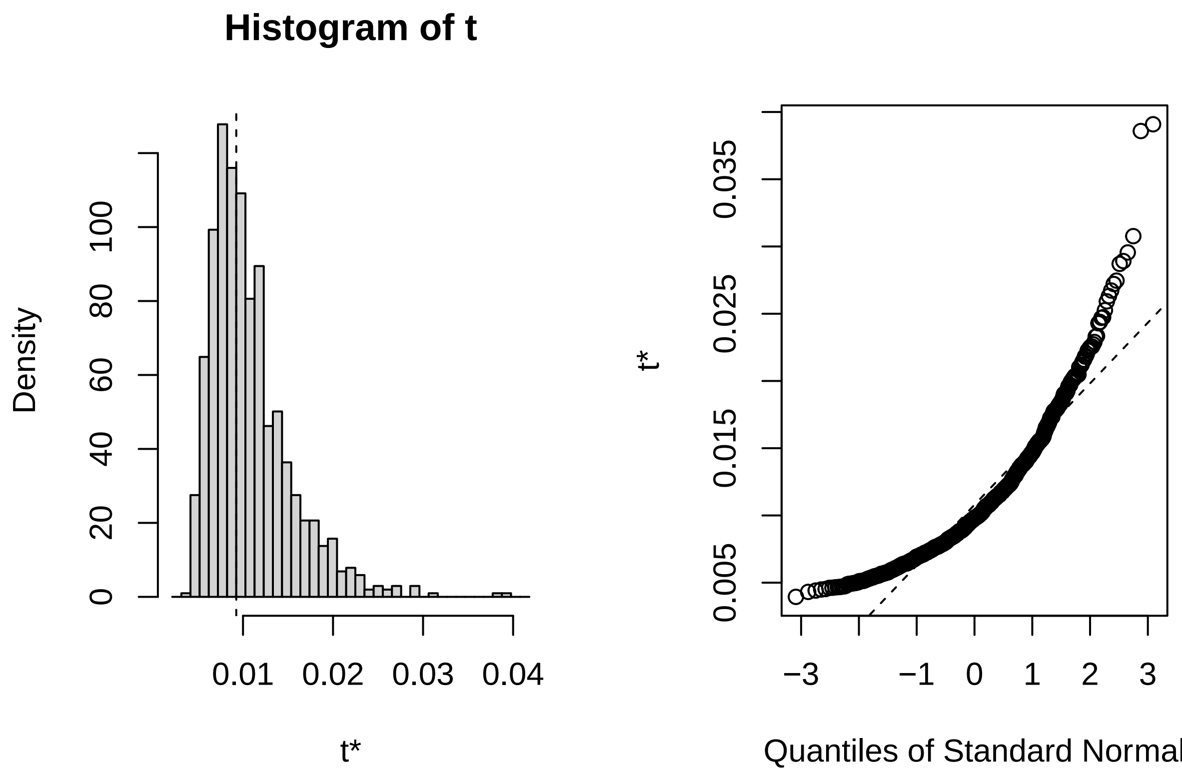
Assume that the times between failures follow an exponential model with rate λ. Obtain the MLE of the hazard rate λ and use bootstrap to estimate the bias and standard error of the estimate.

in R:

```
library(boot)

# MLE for hazard rate of exponential distributed data
mle <- function(data, i) {
  return(length(data[i])/sum(data[i]))
}

# bootstrapping with 1000 replications
results <- boot(data=aircondit$hours, statistic = mle, R=1000)

# view results
results
```

```
##
## ORDINARY NONPARAMETRIC BOOTSTRAP
##
##
## Call:
## boot(data = aircondit$hours, statistic = mle, R = 1000)
##
##
## Bootstrap Statistics :
##       original      bias    std. error
## t1* 0.00925212 0.001541358 0.004513273
```

```
plot(results)
```




```
# get 95% confidence interval
boot.ci(results, type=c("bca", "norm", "perc"))
```

```
## BOOTSTRAP CONFIDENCE INTERVAL CALCULATIONS
## Based on 1000 bootstrap replicates
##
## CALL :
## boot.ci(boot.out = results, type = c("bca", "norm", "perc"))
##
## Intervals :
## Level      Normal             Percentile            BCa
## 95%   (-0.0011,  0.0166 )   ( 0.0051,  0.0223 )   ( 0.0045,  0.0183 )
## Calculations and Intervals on Original Scale
## Some BCa intervals may be unstable
```

Exercise 8.5

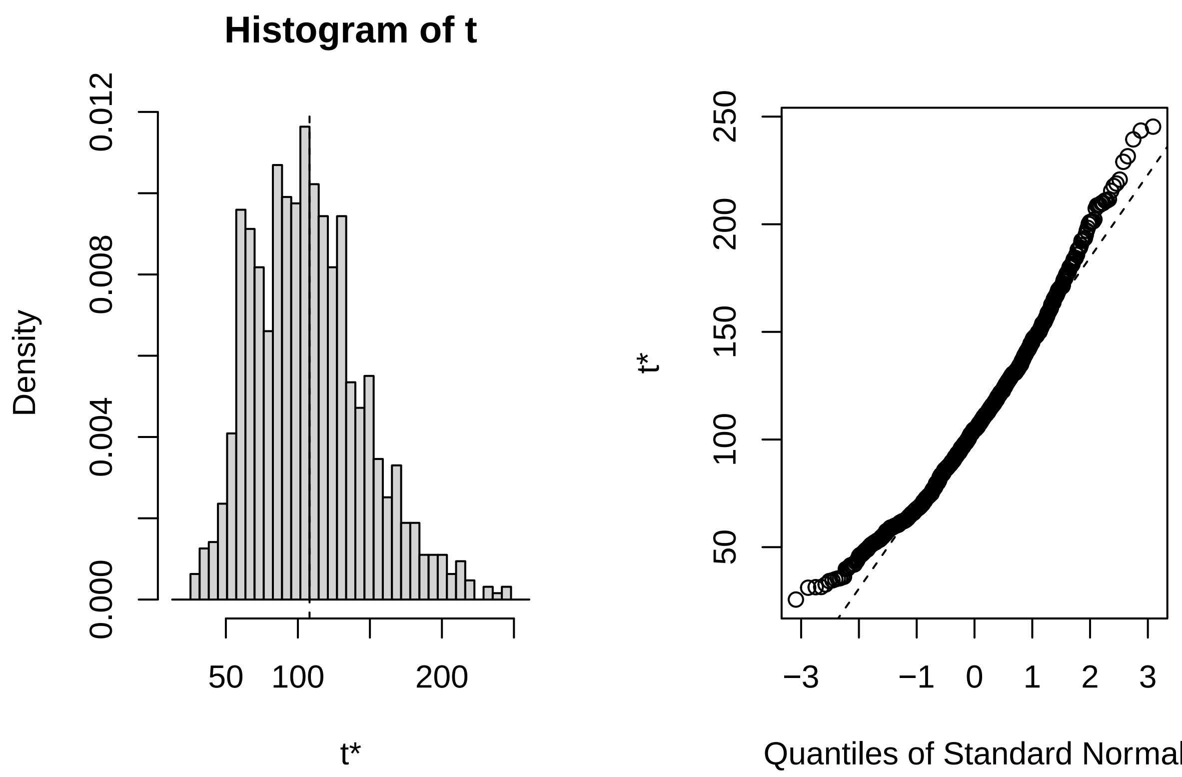
Refer to exercise 8.4. Compute 95% confidence interval for the mean time between failures by the standard normal, basic, percentile and BCa methods.

```
# MLE for hazard rate of exponential distributed data
meantimeest <- function(data, i) {
  rate <- length(data[i])/sum(data[i])
  return(1/rate)
}
```

```r
# bootstrapping with 1000 replications
results <- boot(data=aircondit$hours, statistic = meantimeest, R=1000)

# view results
results
```

```
##
## ORDINARY NONPARAMETRIC BOOTSTRAP
##
##
## Call:
## boot(data = aircondit$hours, statistic = meantimeest, R = 1000)
##
##
## Bootstrap Statistics :
##     original      bias    std. error
## t1* 108.0833 -0.4198333    38.45611
```

```r
plot(results)
```




```r
# get 95% confidence interval
boot.ci(results, type=c("norm", "basic", "perc", "bca"))
```

```
## BOOTSTRAP CONFIDENCE INTERVAL CALCULATIONS
## Based on 1000 bootstrap replicates
##
## CALL :
```

```
## boot.ci(boot.out = results, type = c("norm", "basic", "perc",
##     "bca"))
##
## Intervals :
## Level      Normal              Basic
## 95%   ( 33.1, 183.9 )   ( 17.9, 169.7 )
##
## Level     Percentile            BCa
## 95%   ( 46.4, 198.3 )   ( 58.1, 231.7 )
## Calculations and Intervals on Original Scale
## Some BCa intervals may be unstable
```

Exercise 11.3

Use metropolis-hastings sampler to generate random variables from a standard Cauchyy distribution. Discard the first 1000 of the chain, and compare the deciles of the generated observations with the deciles of the standard Cauchy distribution. Recall that a Cauchy(θ, η) has density

$$f(x) = \frac{1}{\theta\pi(1 + [(x - \eta)/\theta]^2)}, \quad -\infty < x < \infty, \quad \theta > 0. \tag{1}$$

Bibliography