

# Supplementary Material to “Hierarchical Gaussian Processes and Mixtures of Experts in Predicting COVID Patient Trajectories”

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## 1. Fitting Additional Covariates

We show the robustness of our hierarchical mixture of experts Gaussian process model by fitting and predicting patient trajectories on the following additional covariates: blood  $CO_2$ , fraction of oxygen inspired  $FIO_2$ , chloride, lactic acid, and creatinine. We re-present the results for albumin referenced in the main text as a point of reference. Again, we benchmark our mixture of experts (MOE) model against Gaussian process regression (GPR) and a simple hierarchical Gaussian process (HGP).

## 1.1. Albumin

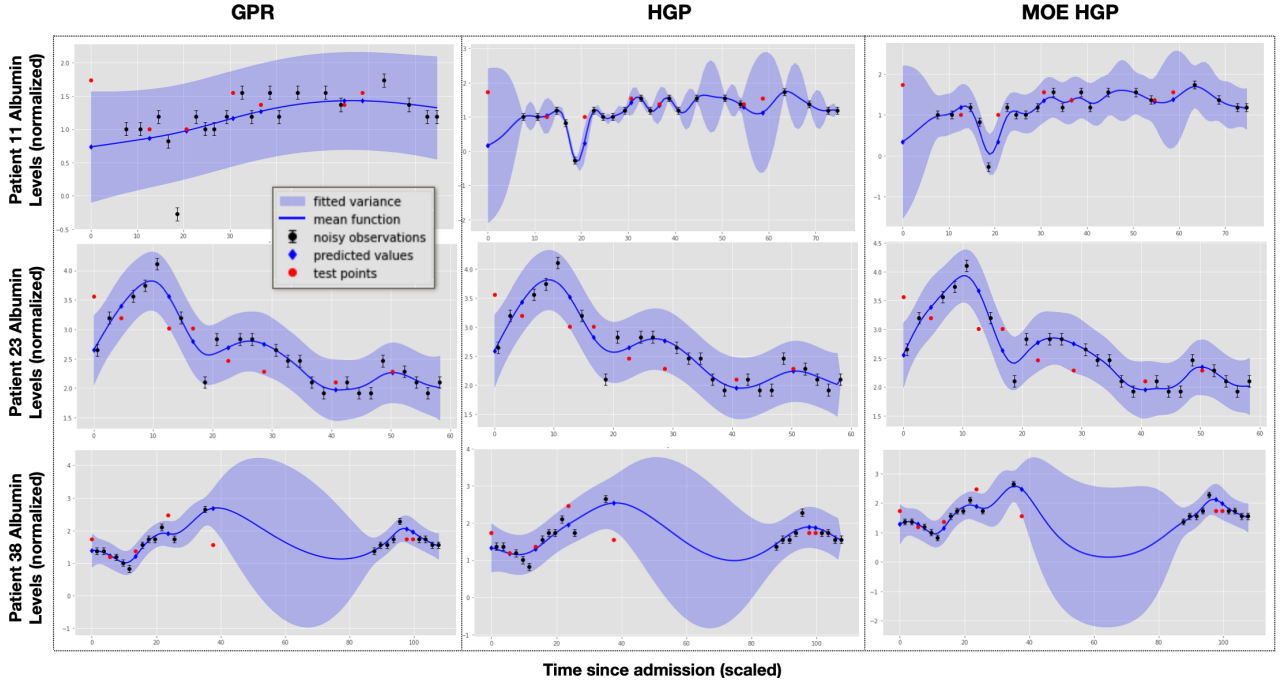


Fig. 1: Cluster representative fits for covariate **albumin**. Patient 11 is a representative of attribute 0 (male); Patient 23 is a representative of attributes 1 (female) and 3 (ethnically black). Patient 38 is a representative of attribute 2 (ethnically white). The train/test MSEs for Patient 11 are 0.026/0.188 (GPR), 0.498/0.198 (HGP) and 0.015/0.245 (MOE). The train/test MSEs for Patient 23 are 0.105/0.172 (GPR),  $2.83 \times 10^{-10}$ /0.204 (HGP) and 0.014/0.356 (MOE). The train/test MSEs for Patient 38 are 10.118/0.172 (GPR),  $3.46 \times 10^{-10}$ /0.204 (HGP) and 0.009/0.237 (MOE).

Model	Train MSE	Test MSE	% of Patient Train $R^2$ s for which Model > GPR	% of Patient Test $R^2$ s for which Model > GPR	% of Patient 95% CIs for which Model is better than GPR	% of Patient 95% CIs for which Model is same as GPR
GPR	0.04	0.21	—	—	—	—
HGP	0.03	0.23	73.17	58.54	21.95	60.98
MOE	0.02	0.21	60.98	53.66	21.95	68.29

Table 1: Model metrics for covariate **Albumin**

## 1.2. Blood $CO_2$

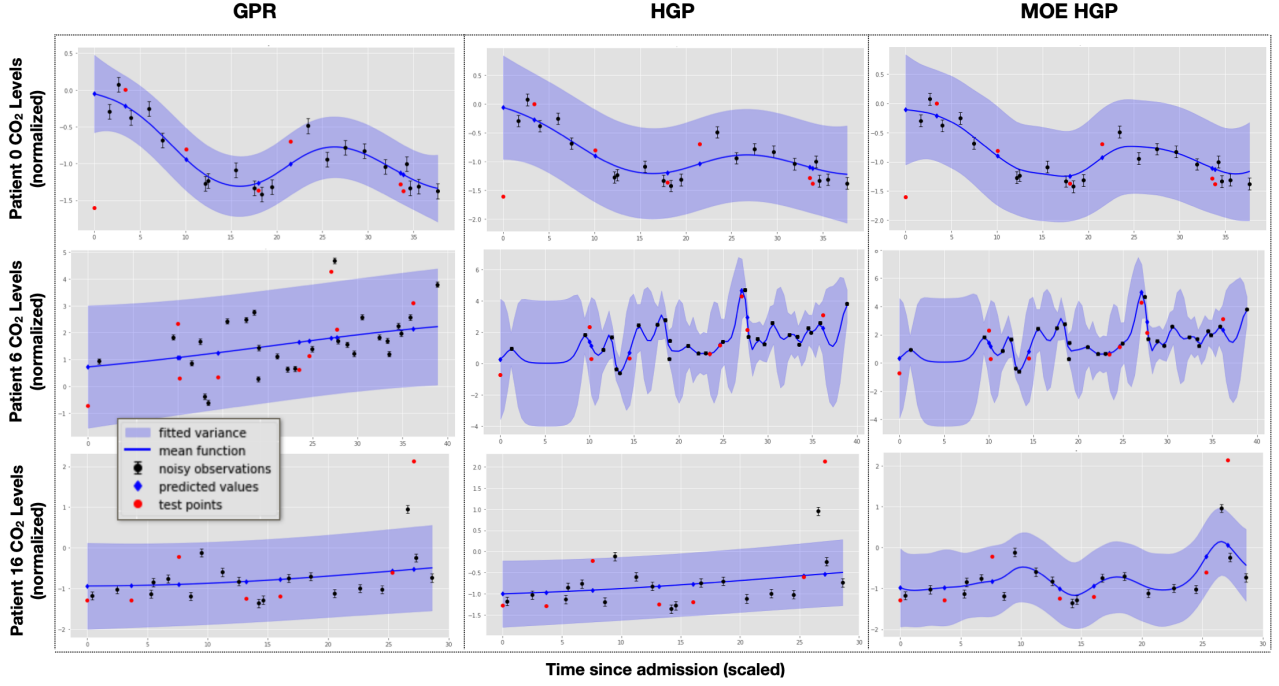


Fig. 2: Cluster representative fits for  $CO_2$ . Patient 0 is a representative of attributes 3 (ethnically black). Patient 6 representative of attributes 0 (male) and 2 (ethnically white). Patient 16 is a representative of attribute 1 (female). The train/test MSEs for Patient 0 are 0.025/0.380 (GPR), 0.688/0.389 (HGP) and 0.024/0.352 (MOE). The train/test MSEs for Patient 6 are 1.00/1.51 (GPR), 0.126/0.463 (HGP) and 0.043/0.514 (MOE). The train/test MSEs for Patient 16 are 0.226/1.16 (GPR), 0.228/1.15 (HGP) and 0.081/0.729 (MOE).

Model	Train MSE	Test MSE	% of Patient Train $R^2$ s for which Model > GPR	% of Patient Test $R^2$ s for which Model > GPR	% of Patient 95% CIs for which Model is better than GPR	% of Patient 95% CIs for which Model is same as GPR
GPR	0.20	0.70	—	—	—	—
HGP	0.09	0.61	47	47	24	65
MOE	0.058	0.57	76	47	29	65

Table 2: Model metrics for covariate  $CO_2$

### 1.3. Fraction of Oxygen Inspired ( $FIO_2$ )

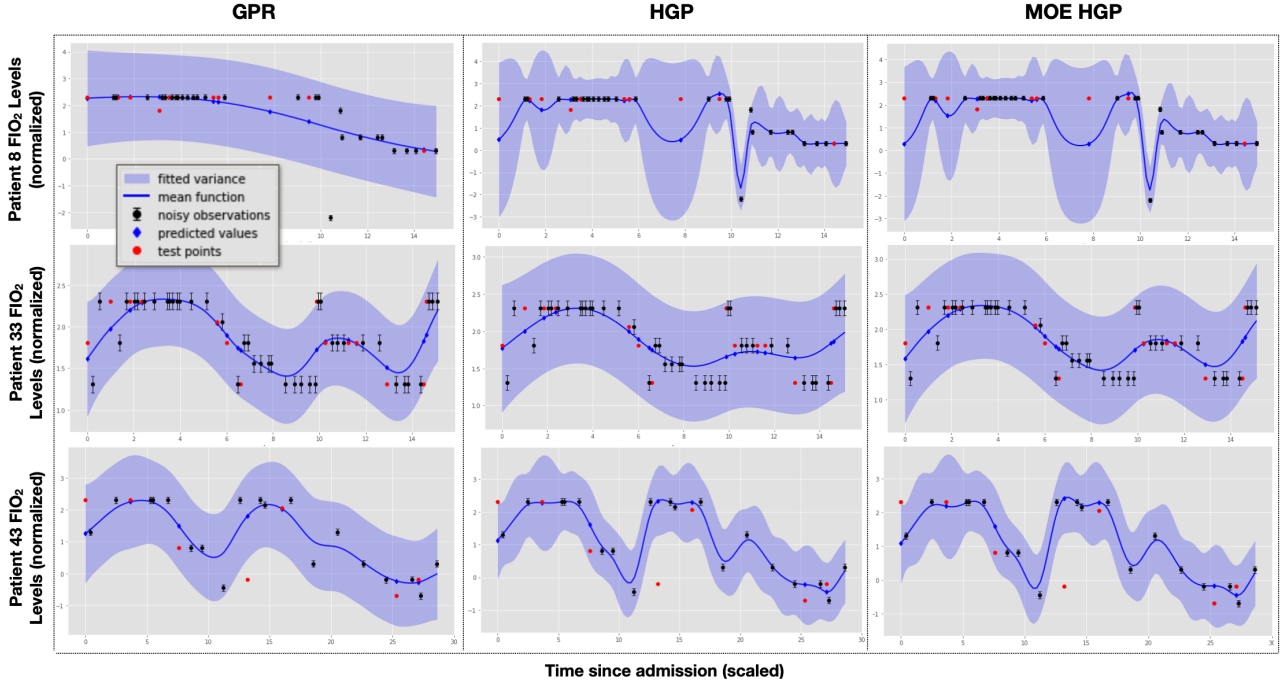


Fig. 3: Cluster representative fits for  $FIO_2$ . Patient 8 representative of attribute 0 (male). Patient 33 is a representative of attributes 1 and 3 (ethnically black). Patient 43 is a representative of attribute 2 (ethnically white). The train/test MSEs for Patient 8 are 0.547/0.144 (GPR), 0.106/0.731 (HGP) and 0.043/0.907 (MOE). The train/test MSEs for Patient 33 are 0.053/0.090 (GPR), 0.323/0.105 (HGP) and 0.058/0.004 (MOE). The train/test MSEs for Patient 43 are 0.181/0.830 (GPR), 0.593/0.125 (HGP) and 0.026/0.133 (MOE).

Model	Train MSE	Test MSE	% of Patient Train $R^2$ s for which Model > GPR	% of Patient Test $R^2$ s for which Model > GPR	% of Patient 95% CIs for which Model is better than GPR	% of Patient 95% CIs for which Model is same as GPR
GPR	0.14	0.48	—	—	—	—
HGP	0.07	0.51	47	53	34	40
MOE	0.06	0.49	49	55	36	43

Table 3: Model metrics for covariate  $FIO_2$

#### 1.4. Chloride

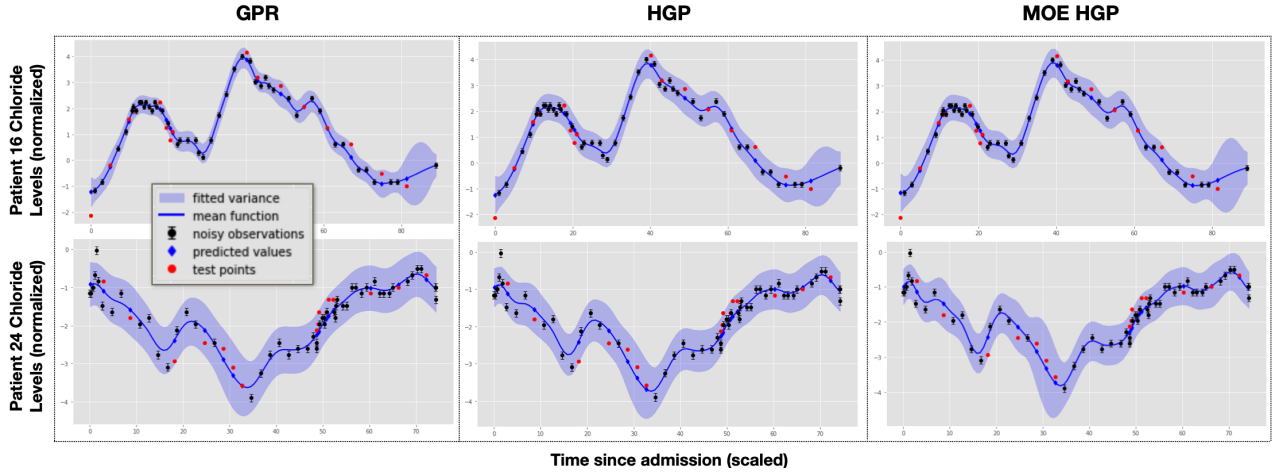


Fig. 4: Cluster representative fits for **Chloride**. Patient 16 representative of attributes 1 (female) and 3 (ethnically black). Patient 24 is a representative of attributes 0 (male) and 2 (ethnically white). The train/test MSEs for Patient 16 are 0.010/0.116 (GPR), 0.090/0.123 (HGP) and 0.016/0.128 (MOE). The train/test MSEs for Patient 24 are 0.049/0.074 (GPR), 0.024/0.077 (HGP) and 0.033/0.072 (MOE).

Model	Train MSE	Test MSE	% of Patient Train $R^2$ s for which Model > GPR	% of Patient Test $R^2$ s for which Model > GPR	% of Patient 95% CIs for which Model is better than GPR	% of Patient 95% CIs for which Model is same as GPR
GPR	0.05	0.22	—	—	—	—
HGP	0.04	0.23	33	50	31	52
MOE	0.03	0.23	71	57	29	55

Table 4: Model metrics for covariate **Chloride**

### 1.5. Lactic Acid

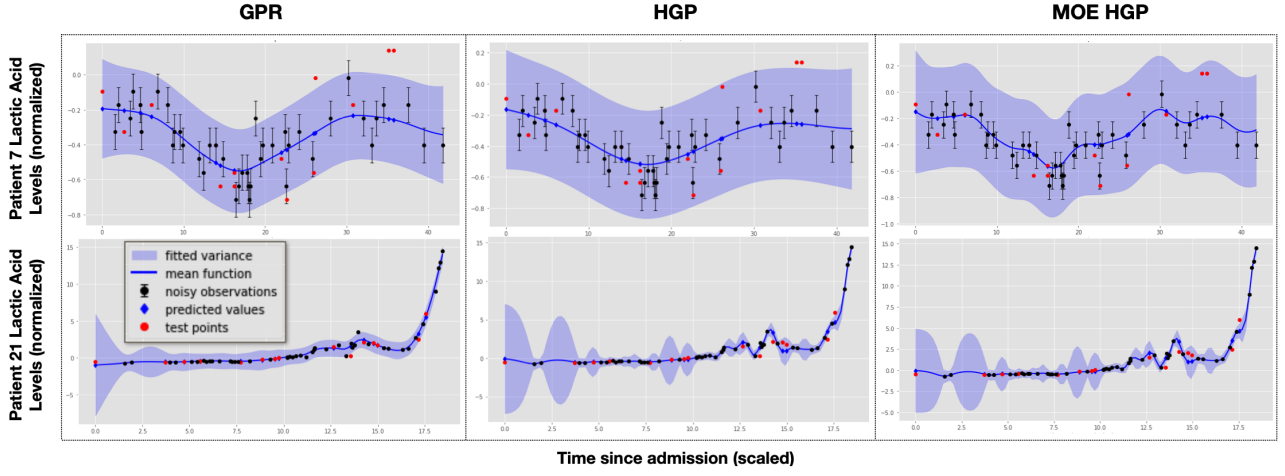


Fig. 5: Cluster representative fits for **Lactic Acid**. Patient 13 is a representative of attributes 0 (male) and 2 (ethnically white). Patient 21 is a representative of attributes 1 (female) and 3 (ethnically black). The train/test MSEs for Patient 13 are 0.012/0.040 (GPR), 0.055/0.042 (HGP) and 0.009/0.035 (MOE). The train/test MSEs for Patient 21 are 0.115/0.194 (GPR), 0.092/0.052 (HGP) and 0.180/0.052 (MOE).

Model	Train MSE	Test MSE	% of Patient Train $R^2$ s for which Model > GPR	% of Patient Test $R^2$ s for which Model > GPR	% of Patient 95% CIs for which Model is better than GPR	% of Patient 95% CIs for which Model is same as GPR
GPR	0.06	0.19	—	—	—	—
HGP	0.02	0.22	47	59	21	59
MOE	0.01	0.22	55	53	24	56

Table 5: Fit on 33 COVID patients for **Lactic Acid**

## 1.6. Creatinine

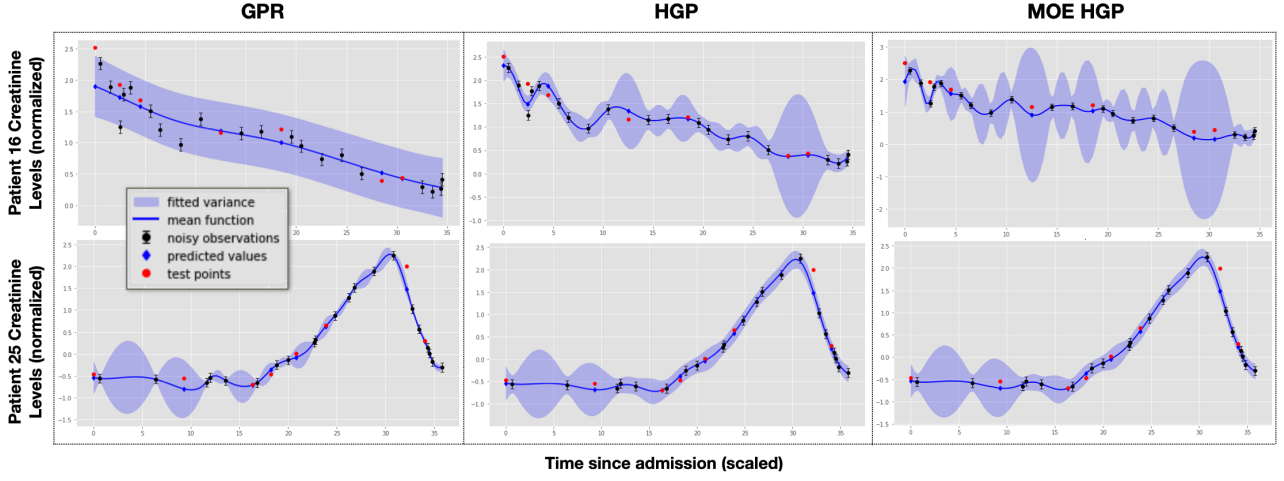


Fig. 6: Cluster representative fits for **Creatinine**. Patient 1 representative of attribute 0 (male). Patient 16 is a representative of attribute 1 (female). Patient 25 is a representative of attribute 2 (ethnically white). Patient 36 is a representative of attribute 3 (ethnically black). The train/test MSEs for Patient 16 are 0.037/0.070 (GPR), 0.104/0.043 (HGP) and  $5.20 \times 10^{-4}$ /0.141 (MOE). The train/test MSEs for Patient 25 are  $1.00 \times 10^{-5}$ /0.045 (GPR), 0.023/0.048 (HGP) and  $8.10 \times 10^{-4}$ /0.039 (MOE).

Model	Train MSE	Test MSE	% of Patient Train $R^2$ s for which Model > GPR	% of Patient Test $R^2$ s for which Model > GPR	% of Patient 95% CIs for which Model is better than GPR	% of Patient 95% CIs for which Model is same as GPR
GPR	0.05	0.07	—	—	—	—
HGP	0.003	0.13	51	47	27	53
MOE	0.001	0.14	89	44	29	44

Table 6: Model metrics for covariate **Creatinine**