Supplementary Material to "Hierarchical Gaussian Processes and Mixtures of Experts in Predicting COVID Patient Trajectories"

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1. Fitting Additional Covariates

We show the robustness of our mixture of experts hierarchical Gaussian process model by fitting and predicting patient trajectories on the following additional covariates: blood CO_2, fraction of oxygen inspired FIO_2, chloride, lactic acid, and creatinine. We present the results for albumin referenced in the main text as a point of reference. Again, we benchmark our mixture of experts (MOE) HGP model against Gaussian process regression (GPR) and a hierarchical Gaussian process (HGP).
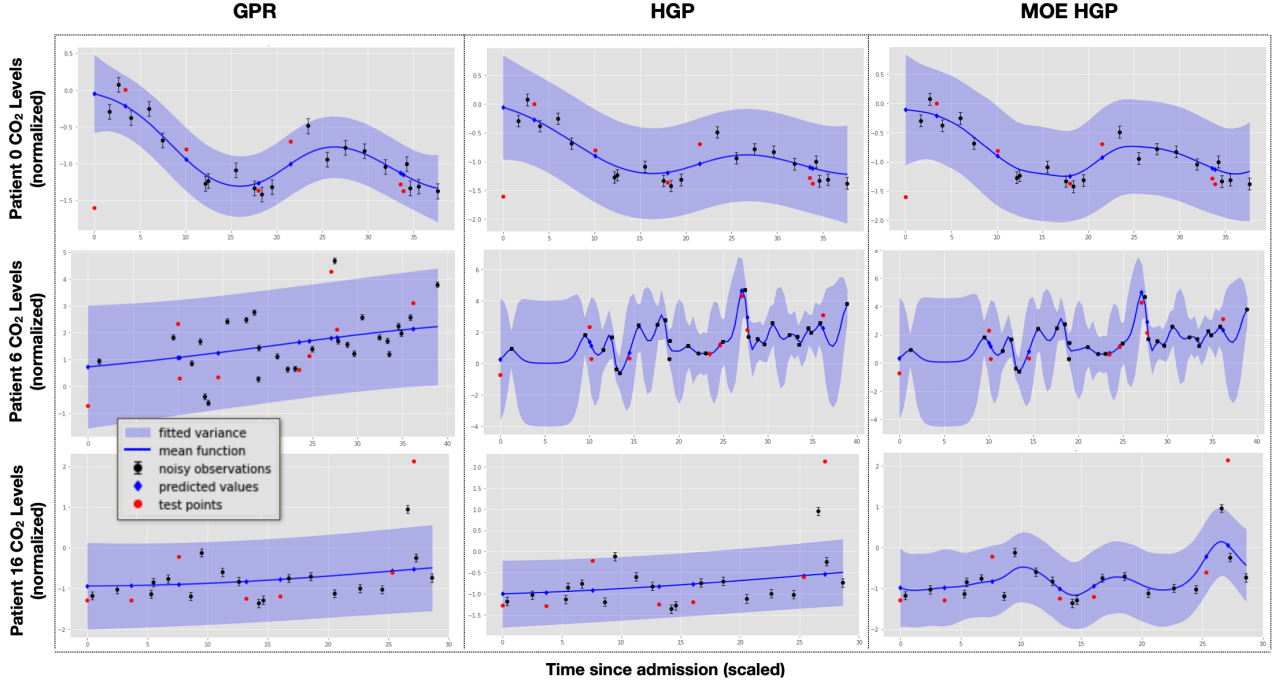
1.1. *Blood CO₂*

Fig. 1: Cluster representative fits for CO_2. Patient 0 is a representative of attributes 3 (ethnically Black). Patient 6 representative of attributes 0 (male) and 2 (ethnically white). Patient 16 is a representative of attribute 1 (female). The train/test MSEs for Patient 0 are 0.025/0.380 (GPR), 0.688/0.389 (HGP) and 0.024/0.352 (MOE). The train/test MSEs for Patient 6 are 1.00/1.51 (GPR), 0.126/0.463 (HGP) and 0.043/0.514 (MOE). The train/test MSEs for Patient 16 are 0.226/1.16 (GPR), 0.228/1.15 (HGP) and 0.081/0.729 (MOE).

Model	Train MSE	Test MSE	% of Patient Train R^2s for which Model > GPR	% of Patient Test R^2s for which Model > GPR	% of Patient 95% CIs for which Model is better than GPR	% of Patient 95% CIs for which Model is same as GPR
GPR	0.20	0.70	—	—	—	—
HGP	0.09	0.61	47	47	24	65
MOE	0.058	0.57	76	47	29	65

Table 1: Model metrics for covariate CO_2

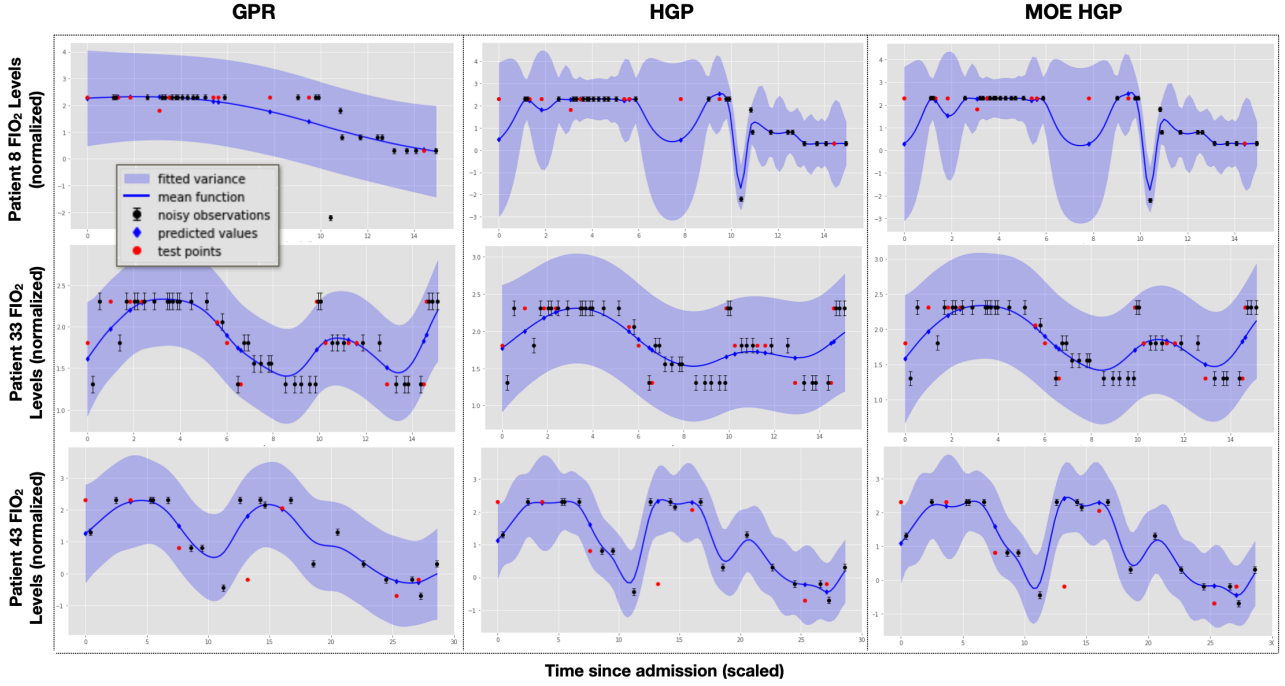## 1.2. *Fraction of Oxygen Inspired (FIO₂)*

Fig. 2: Cluster representative fits for FIO_2. Patient 8 is a representative of attribute 0 (male). Patient 33 is a representative of attributes 1 and 3 (ethnically Black). Patient 43 is a representative of attribute 2 (ethnically white). The train/test MSEs for Patient 8 are 0.547/0.144 (GPR), 0.106/0.731 (HGP) and 0.043/0.907 (MOE). The train/test MSEs for Patient 33 are 0.053/0.090 (GPR), 0.323/0.105 (HGP) and 0.058/0.004 (MOE). The train/test MSEs for Patient 43 are 0.181/0.830 (GPR), 0.593/0.125 (HGP) and 0.026/0.133 (MOE).

Model	Train MSE	Test MSE	% of Patient Train R^2s for which Model > GPR	% of Patient Test R^2s for which Model > GPR	% of Patient 95% CIs for which Model is better than GPR	% of Patient 95% CIs for which Model is same as GPR
GPR	0.14	0.48	—	—	—	—
HGP	0.07	0.51	47	53	34	40
MOE	0.06	0.49	49	55	36	43

Table 2: Model metrics for covariate FIO_2
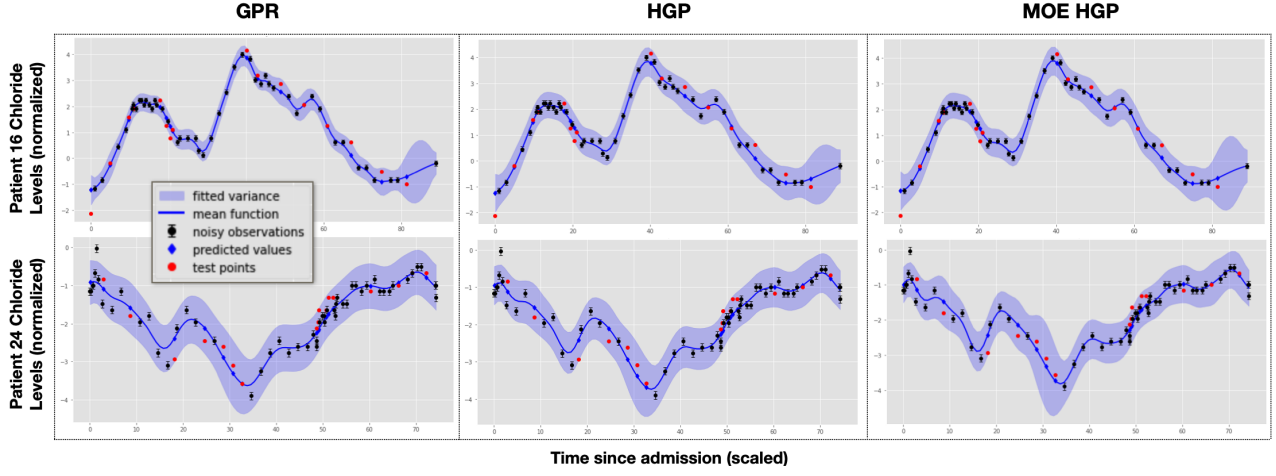
1.3. *Chloride*

Fig. 3: Cluster representative fits for **Chloride**. Patient 16 is a representative of attributes 1 (female) and 3 (ethnically Black). Patient 24 is a representative of attributes 0 (male) and 2 (ethnically white). The train/test MSEs for Patient 16 are 0.010/0.116 (GPR), 0.090/0.123 (HGP) and 0.016/0.128 (MOE). The train/test MSEs for Patient 24 are 0.049/0.074 (GPR), 0.024/0.077 (HGP) and 0.033/0.072 (MOE).

Model	Train MSE	Test MSE	% of Patient Train R^2s for which Model > GPR	% of Patient Test R^2s for which Model > GPR	% of Patient 95% CIs for which Model is better than GPR	% of Patient 95% CIs for which Model is same as GPR
GPR	0.05	0.22	—	—	—	—
HGP	0.04	0.23	33	50	31	52
MOE	0.03	0.23	71	57	29	55

Table 3: Model metrics for covariate **Chloride**
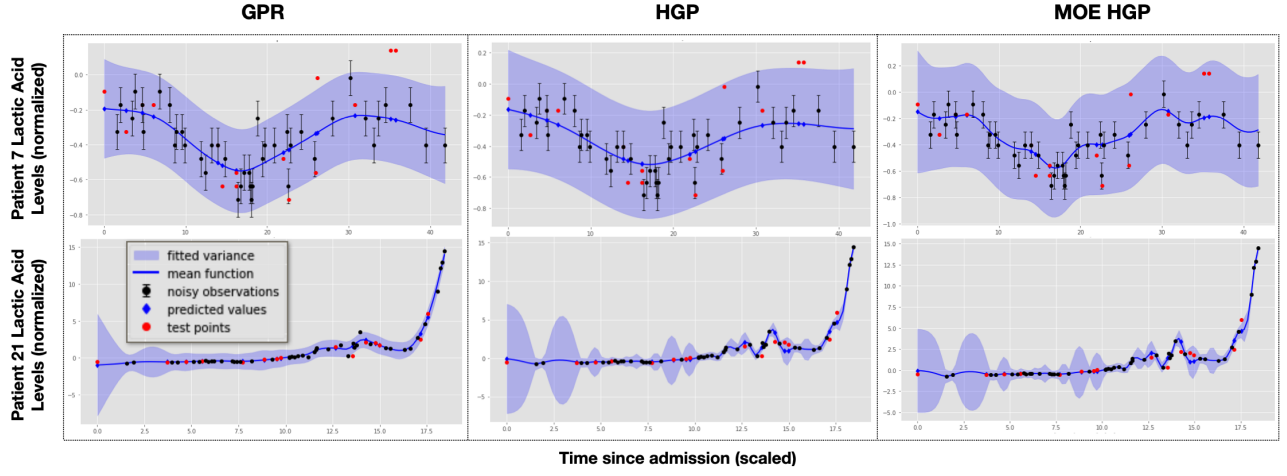
1.4. *Lactic Acid*

Fig. 4: Cluster representative fits for **Lactic Acid**. Patient 13 is a representative of attributes 0 (male) and 2 (ethnically white). Patient 21 is a representative of attributes 1 (female) and 3 (ethnically Black). The train/test MSEs for Patient 13 are 0.012/0.040 (GPR), 0.055/0.042 (HGP) and 0.009/0.035 (MOE). The train/test MSEs for Patient 21 are 0.115/0.194 (GPR), 0.092/0.052 (HGP) and 0.180/0.052 (MOE).

Model	Train MSE	Test MSE	% of Patient Train R^2s for which Model > GPR	% of Patient Test R^2s for which Model > GPR	% of Patient 95% CIs for which Model is better than GPR	% of Patient 95% CIs for which Model is same as GPR
GPR	0.06	0.19	—	—	—	—
HGP	0.02	0.22	47	59	21	59
MOE	0.01	0.22	55	53	24	56

Table 4: Fit on 33 COVID patients for **Lactic Acid**
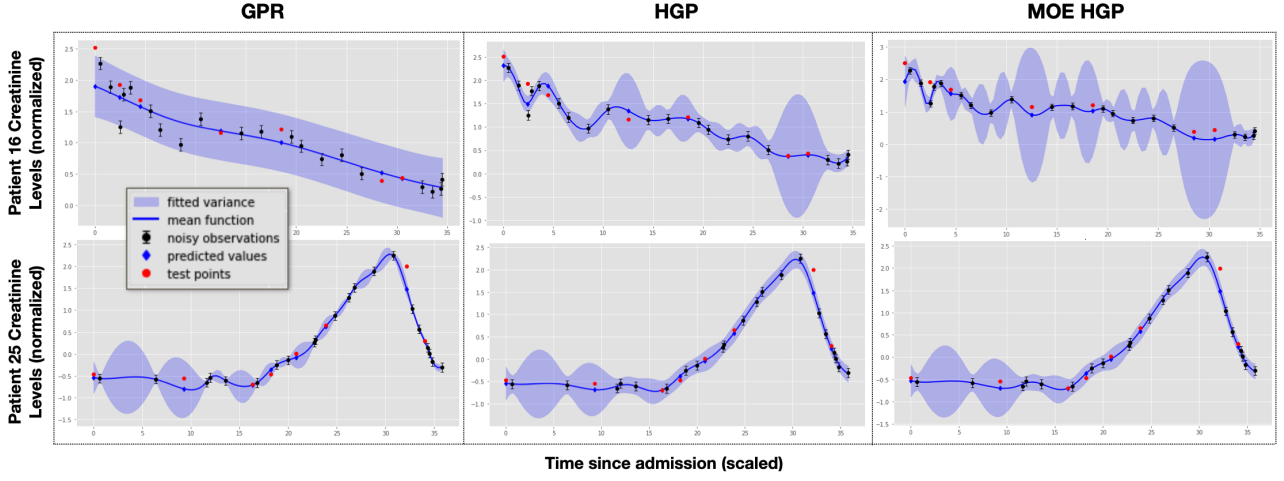
1.5. *Creatinine*

Fig. 5: Cluster representative fits for **Creatinine**. Patient 1 is a representative of attribute 0 (male). Patient 16 is a representative of attribute 1 (female). Patient 25 is a representative of attribute 2 (ethnically white). Patient 36 is a representative of attribute 3 (ethnically Black). The train/test MSEs for Patient 16 are 0.037/0.070 (GPR), 0.104/0.043 (HGP) and $5.20 \times 10^{-4}/0.141$ (MOE). The train/test MSEs for Patient 25 are $1.00 \times 10^{-5}/0.045$ (GPR), 0.023/0.048 (HGP) and $8.10 \times 10^{-4}/0.039$ (MOE).

Model	Train MSE	Test MSE	% of Patient Train R^2s for which Model > GPR	% of Patient Test R^2s for which Model > GPR	% of Patient 95% CIs for which Model is better than GPR	% of Patient 95% CIs for which Model is same as GPR
GPR	0.05	0.07	—	—	—	—
HGP	0.003	0.13	51	47	27	53
MOE	0.001	0.14	89	44	29	44

Table 5: Model metrics for covariate **Creatinine**