P-unit model by Henriette Walz & Alexander Ott

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1 The model

The input into the P-unit model, $x(t)$, is

- the fish's own EOD
$$x(t) = x_{EOD}(t) = \cos(2\pi f_{EOD} t) \tag{1}$$
with EOD frequency f_{EOD} and amplitude normalized to one.

- the EOD multiplied with an amplitude modulation $AM(t)$:
$$x(t) = (1 + AM(t)) \cos(2\pi f_{EOD} t) \tag{2}$$
For a random amplitude modulaten ($AM(t) = RAM(t)$) random numbers are drawn for each frequency up to $f_{EOD}/2$ in Fourier space. After backtransformation the resulting signal is scaled to the desired standard deviation relative to the EOD carrier.

- a superposition of two EODs
$$x(t) = x_{EOD}(t) + x_{EOD1}(t) = \cos(2\pi f_{EOD} t) + \alpha_1 \cos(2\pi f_{EOD1} t) \tag{3}$$
with the EOD of the second fish having frequency f_{EOD1} and amplitude α_1 relative to the amplitude of the receiving fish.

- a superposition of many EODs
$$x(t) = x_{EOD}(t) + \sum_{i=1}^{n} x_{EODi}(t) = \cos(2\pi f_{EOD} t) + \sum_{i=1}^{n} \alpha_i \cos(2\pi f_{EODi} t) \ . \tag{4}$$
For $n = 2$ this is our cocktail-party problem.

The input $x(t)$ is a normalized EOD and thus is unitless.

First, the input $x(t)$, is thresholded potentially at the synapse between the receptor cells and the afferent, and then low-pass filtered with time constant τ_d by the afferent's dendrite:

$$\tau_d \frac{dV_d}{dt} = -V_d + \lfloor x(t) \rfloor_0^p \tag{5}$$

Because the input is unitless, the dendritic voltage is unitless, too. $\lfloor x(t) \rfloor_0$ denotes the threshold operation that sets negative values to zero:

$$\lfloor x(t) \rfloor_0 = \begin{cases} x(t) & ; \quad x(t) \geq 0 \\ 0 & ; \quad x(t) < 0 \end{cases} \tag{6}$$

Usually the exponent p is set to one (pure threshold). In our advanced models p is set to three in order to reproduce responses to beats with difference frequencies above half of the EOD frequency.

This thresholding and low-pass filtering extracts the amplitude modulation of the input $x(t)$. The dendritic voltage $V_d(t)$ is the input to a leaky integrate-and-fire (LIF) model

$$\tau_m \frac{dV_m}{dt} = -V_m + \mu + \alpha V_d - A + \sqrt{2D}\xi(t) \tag{7}$$

where τ_m the membrane time constant, μ a fixed bias current, α a scaling factor for V_d, and $\sqrt{2D}\xi(t)$ a white noise of strength D. All terms in the LIF are unitless.

The adaptation current A follows

$$\tau_A \frac{dA}{dt} = -A \tag{8}$$

with adaptation time constant τ_A.

Whenever the membrane voltage $V_m(t)$ crosses the threshold $\theta = 1$ a spike is generated, $V_m(t)$ is reset to 0, the adaptation current is incremented by ΔA, and integration of $V_m(t)$ is paused for the duration of a refractory period t_{ref}:

$$V_m(t) \geq \theta \ : \begin{cases} V_m & \mapsto & 0 \\ A & \mapsto & A + \Delta A/\tau_A \end{cases} \tag{9}$$

2 Parameter values

[JB: Sascha, list all parameters (table or itemize) plus time step of the model with typical values and the right (time) units]

3 Numerical implementation

[JB: Sascha: This is Alexander Ott's code from the git repository, right?]

The ODEs are integrated by the Euler forward method with time-step Δt.

For the intrinsic noise of the model $\xi(t)$ in each time step i a random number is drawn from a normal distribution $\mathcal{N}(0,\,1)$ with zero mean and standard deviation of one. This number is multiplied with $\sqrt{2D}$ and divided by $\sqrt{\Delta t}$:

$$V_{m_{i+1}} = V_{m_i} + \left(-V_{m_i} + \mu + \alpha V_{d_i} - A_i + \sqrt{\frac{2D}{\Delta t}} \mathcal{N}(0,\,1)_i \right) \frac{\Delta t}{\tau_m} \tag{10}$$

[JB: Benjamin: wir haben das Rauschen innerhalb der Klammer. Damit ist zwar das $\sqrt{\Delta t}$ richtig, aber wir teilen noch durch τ_m. Damit sind effektiv unsere D Werte andere als wenn wir den Rauschterm ausserhalb der Klammer haetten (so machst du das glaube ich immer).]

The noise strength values from the table in fact equal $\sqrt{2D}$ and not D.

[JB: we need to clean up that noise issue]