1. Wen finds 17 consecutive positive integers that sum to 2023. Compute the smallest of these integers.

   Answer: 111

   Solution: Notice that the average of these integers must be $\frac{2023}{17} = 119$. Since they are consecutive, we know that the middle, 9th, number out of our 17 integers must be 119. This will be 8 more than the smallest integer, so our answer is $119 - 8 = \boxed{111}$.

2. The polynomial $P(x) = 3x^3 - 2x^2 + ax + b$ has roots $\sin^2\theta$, $\cos^2\theta$, and $\sin\theta\cos\theta$ for some angle θ. Compute $P(1)$.

   Answer: $\mathbf{\frac{4}{9}}$

   Solution: Using Vieta's Formulas, we have

$$\begin{aligned}
-(1+\sin\theta\cos\theta) &= -\frac{2}{3}\\
\sin\theta\cos\theta(\sin\theta\cos\theta+1) = \sin\theta\cos\theta(\sin^2\theta+\cos^2\theta)+\sin^2\theta\cos^2\theta &= \frac{a}{3}\\
-(\sin\theta\cos\theta)^3 = -\sin^2\theta\cos^2\theta\,(\sin\theta\cos\theta) &= \frac{b}{3}.
\end{aligned}$$

   Solving the first equation gives $\sin\theta\cos\theta = -\frac{1}{3}$, which means that $a = -\frac{2}{3}$ and $b = \frac{1}{9}$. Then, $P(1) = 3 - 2 - \frac{2}{3} + \frac{1}{9} = \boxed{\frac{4}{9}}$.

3. Compute the real solution for x to the equation $(4^x + 8)^4 - (8^x - 4)^4 = (4 + 8^x + 4^x)^4$.

   Answer: $\mathbf{\frac{2}{3}}$

   Solution: Let $a = 4^x + 8$ and $b = 8^x - 4$. Then the equation becomes $a^4 - b^4 = (a+b)^4$. Then $(a^2 - b^2)(a^2 + b^2) = (a+b)^4$, and so $(a-b)(a+b)(a^2+b^2) = (a+b)^4$. Note that $a + b = (4^x + 8) + (8^x - 4) = 4^x + 8^x + 4 > 0$. Thus, we can divide by $a+b$ on both sides to get that $(a-b)(a^2+b^2) = (a+b)^3$. Expanding the LHS and RHS gives $a^3 - b^3 - a^2b + ab^2 = a^3 + 3a^2b + 3ab^2 + b^3$, which after simplification yields $2b^3 + 4a^2b + 2ab^2 = 0$. We can factor out a b and we have $b(2b^2 + 4a^2 + 2ab) = 0$. This gives a solution of $b = 0$, which implies that the only real solution here is $a = 0, b = 0$. The expression $2b^2 + 4a^2 + 2ab$ has no other real solutions. We can show this by factoring it as $2(2a^2 + ab + b^2)$: applying the quadratic formula by treating the b terms as constants would imply the discriminant, $-7b^2$, is negative. Since $b = 0$ is our only possibility, we have $8^x - 4 = 0$, so $8^x = 4$, which means $x = \boxed{\frac{2}{3}}$ is the solution for x.