1. INTRODUCTION

This paper describes the CMC of the Time Department Laboratory ROA (LHROA) in time difference source measurements using a time interval counter in time interval (TI) measurement mode between pulses generated by independent start and stop outputs from the source under calibration.

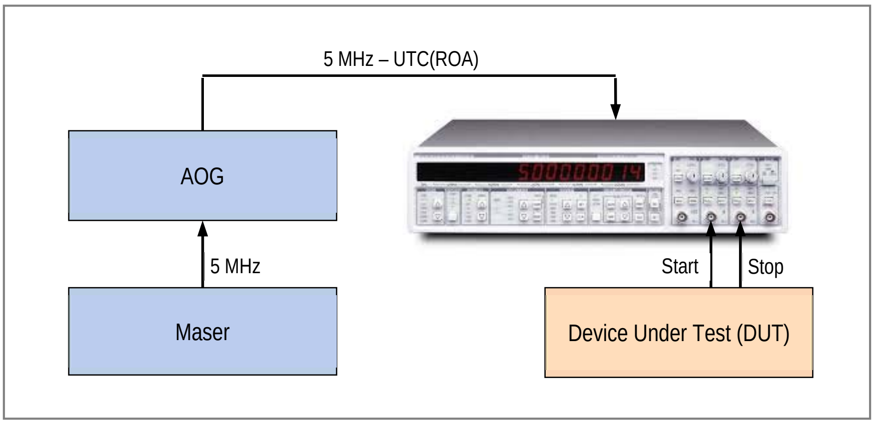

Figure 1. Chart of the measurement method.

This service according to the BIPM classification for the metrological area of Time and Frequency is identified with code 3.2.3 *Time difference source*.

2. MEASUREMENT CONDITIONS

El presente documento es aplicable a generadores de ancho de pulsos dentro de los siguientes rangos y parámetros de funcionamiento:

Time Range:	1×10^{-9} s to 1 s
Amplitude of DUT signal:	± 5 V
Slew rate of signal:	> 0.5 V/ns
Rise time of signal:	≥1 ns
Sample size:	≥10,000
Measuring time:	≥10,000 s

The calculations shall take into account the delay due to the asymmetry between the TI counter channels and the difference produced between the cables used in the calibration.

The uncertainty to be declared under the above conditions ($k=2$) shall be:

$$U = 0.5 \times 10^{-9}\ s$$

3. UNCERTAINTY BUDGET

The analysis is performed for a laboratory thermal variation of 1 °C (±0.5 °C). For the sensitivity study it is considered that, in the instruments, the worst-case thermal variation can occur with a duration of 10,000 s (10,000 samples in 1 s time interval).

<u>Uncertainty contributions</u>

– UTC(ROA) time scale accuracy.
– UTC(ROA) stability during the measurement period of time.

- MHM 2010 temperature sensitivity.
- AOG-110 temperature sensitivity.
- SR620 counter uncertainties.
- Asymmetry due to the counter and the cables.

Contribution of uncertainties to the frequency derived from the UTC(ROA) scale

- Temperature variation in the laboratory Δt: ± 1,0 ºC
- UTC(ROA) accuracy: 7.5×10^{-15} (τ = 86,400 s)

The fractional frequency deviation of the UTC(ROA) scale averaged over one day compared to UTC is treated as uncertainty, thus considering a zero offset of the frequency.

$$u_{UTC(ROA)} = \sqrt{\left(\frac{\Delta f}{f}\right)^2 + \left(\sigma_y\right)^2} = 7.5\times10^{-15}$$

- MHM 2010 short-term stability (10,000 s): $< 1.47\times10^{-15}$ (obtained by calibration)
- MHM 2010 temperature sensitivity: $< 1.0\times10^{-14}$ /ºC (in the range 20°C to 24°C)
- AOG-110 temperature sensitivity: < 10 ps/ºC

In case of 10000 measurements of a nominal TI of 1s, a variation of 1°C during the measurement period could cause an instability in the frequency of:

$$u_{temp\_AOG110} = \frac{10\times10^{-12}(\mathrm{s}/{}^{\circ}\mathrm{C})}{10000\ (s)} \times 1({}^{\circ}\mathrm{C}) = 1\times10^{-16}\left(\frac{Hz}{Hz}\right)$$

- AOG-110 Jitter: 1×10^{-15}

Contribution of uncertainties of the TI Counter (SR620)

Calculated for the worst-case scenario, with a signal amplitude of 1 V, trigger level of 0.5 V and a rise time of 1 ns (lower limit of the counter) and considering the following parameters:

- Measuring Time Interval: 1,000 s
- Start/stop trigger level error): 0.5 V
- Internal noise in the counter ($E_{internal}$): 350×10^{-6} V
- Slew rate os start/stop signals: $\Delta V/t_{rise\text{-}fall}$
- Input signal jitter (33622A generator): 3 ps

In the calculation, it has been regarded to the error formula for time interval measurement**:**

$$Error_{SR620} = \pm resolution \pm (timebase\ error \times time\ interval) \pm start\ trigger\ level\ error \pm stop\ trigger\ level\ error \pm 0{,}5\ ns$$

Being:

- ***Systematic error*** (0.5 given in the $Error_{SR620}$ *formula*) is cancelled by considering the asymmetry between counter channels and the cables used, both in the calculation of the final result of IT (systematic error) and by considering its corresponding uncertainty in the calculation of u_c (random error), being:

$$\overline{IT}_{IEC} = \overline{M}_{IEC} - \overline{R}_{BA} - \bar{C}_{BA}\ ^*$$

***** For additional information about the R_{BA} and C_{BA} estimations see section 3.1.

- ***Trigger level error*** *is calculated, according to the manufacturer, as*:

$$triggerlevelerror = \frac{15\ mV + 0.5\%\ of setting}{InputSlewRate}$$

- ***Resolution**:* Function of the time interval, calculated according to the manufacturer as:

$$Resolution = \pm\sqrt{\frac{(25ps)^2 + (time\ interval\ \times short\ term\ stability)^2 + (start\ trigger\ jitter)^2 + (stop\ trigger\ jitter)^2}{N}}$$

That a breakdown of the terms of the expression of the resolution shows that:

- ***25 ps:*** uncertainty due to the device's resolution entered into the measurement.
- ***Time interval× short term stability***: uncertainty caused by the standard reference frequency (external frequency).
- ***Start/stop trigger jitter***: uncertainties caused by the random noise of the input signals and the internal noise of the trigger circuits.
- ***N:*** number of measurements performed.
- ***Trigger jitter*** of input signals. For the measurement of time differences, **square signals** are considered, the signal noise (jitter) is expressed directly in time units. In this case, for a channel:

$$triggerjitter = \sqrt{\left(\frac{E_{internal}}{InputSlewRate}\right)^2 + jitter_{signal}^2}$$

Derived from the above calculations, the uncertainty balance in Figure 2 is obtained:

Magnitude X_i	Estimation x_i	Standart uncertainty $u(x_i)$	Probability distribution	Conversion factor	Uncertainty contribution $u_i(y)$
Exactitud UTC(ROA) @ 1 día	-	7,500E-15	Normal	1,0	7,500E-15
Estab. UTC(ROA) @ $\tau_{medición}$		1,470E-15	Normal	1,0	1,470E-15
Maser temperature sensivity		1,000E-14	Rectangular	0,3	2,887E-15
AOG Temperature sensivity	-	1,000E-16	Rectangular	0,3	2,887E-17
No linealidad diferencial SR620		5,000E-11	Normal	1,0	5,000E-11
Resol. display (LSD)		1,000E-12	Rectangular	0,3	2,887E-13
Ret. Dif. SR620	1,380E-10	8,217E-11	Normal	1,0	8,217E-11
Ret. Dif. Cables	-4,500E-11	8,217E-11	Normal	1,0	8,217E-11

Figure 2. Uncertainty budget (except SR620 error and resolution).

Considering the counter error, and calculated the uncertainties for every TI considered in the CMC, the final calculation of the combined uncertainty of the measurement is obtained as follows (type A uncertainty is not considered in this case):

$$u_c(\overline{M}_{IEC}) = u_c(\overline{IT}_{IEC}) = \sqrt{Error^2(\overline{M}_{IEC}) + u_A^2(\overline{M}_{IEC}) + \sum_{i=0}^{n} u_i^2}$$

Finally obtaining the expanded uncertainty value calculated for $k=2$ in Figure 3:

IT nominal (s)	Desv. Típ. (jitter)	incert. Tipo A	Resolution (s)	u_{SR620} (s)	IT	u_c(IT)	U (k=2)
1,0E-09	0,0E+00	0,00E+00	2,54E-13	3,53E-11	9,07E-10	1,31E-10	2,63E-10
1,0E-08	0,0E+00	0,00E+00	2,54E-13	3,53E-11	9,91E-09	1,31E-10	2,63E-10
1,0E-07	0,0E+00	0,00E+00	2,54E-13	3,53E-11	9,99E-08	1,31E-10	2,63E-10
1,0E-06	0,0E+00	0,00E+00	2,54E-13	3,53E-11	1,00E-06	1,31E-10	2,63E-10
1,0E-05	0,0E+00	0,00E+00	2,54E-13	3,53E-11	1,00E-05	1,31E-10	2,63E-10
1,0E-04	0,0E+00	0,00E+00	2,54E-13	3,53E-11	1,00E-04	1,31E-10	2,63E-10
1,0E-03	0,0E+00	0,00E+00	2,54E-13	3,53E-11	1,00E-03	1,31E-10	2,63E-10
1,0E-02	0,0E+00	0,00E+00	2,54E-13	3,53E-11	1,00E-02	1,31E-10	2,63E-10
1,0E-01	0,0E+00	0,00E+00	2,54E-13	3,55E-11	1,00E-01	1,31E-10	2,63E-10
1,0E+00	0,0E+00	0,00E+00	2,54E-13	3,73E-11	1,00E+00	1,32E-10	2,64E-10

Figure 3. CMC 3.2.3 calculations

In the end, it is finally justified:

$$\boxed{U_{calculated}(k=2) = 2.63 \times 10^{-10}\ \mathrm{s}}$$

Therefore, with the given conditions it will be fulfilled:

$$\boxed{\boldsymbol{U_{CMC3.2.3}(k=2) \leq 5 \times 10^{-10}\ \mathrm{s}}}$$

3.1. Measurement of delay due to the counter and the cables

To measure the delays due to the asymmetry of the SR620 counter and the cables used (equal length), the measurements shown in the diagrams in Figure 4 are carried out:

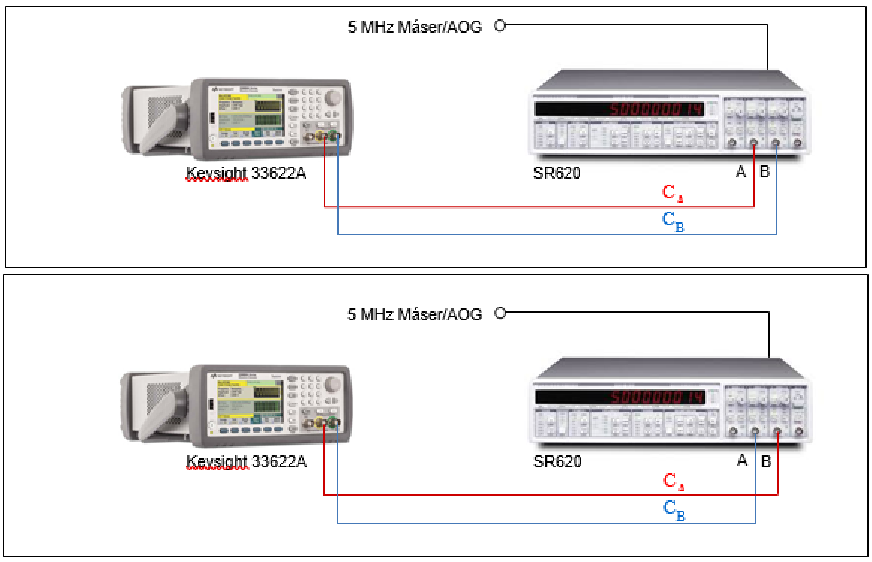

Figura 4. M_1 (on top) y M_2 (under) measurements to obtain the delay

In which identical signals (PPS) are injected from the function generator into the two counter channels, triggering on channel A of the counter, for a total of 100 measurements.

The sources of uncertainty of these measurements are equivalent to those derived from the UTC(ROA) scale, with the exception of considering the following:

- UTC(ROA) stability @ τ = 100 s: 4.00 ×10^{-14} (obtained by calibration)
- 33622A generator asymmetry: 100 ps
- SR620 differential non-linearity: 50 ps

– SR620 Display Resolution: 1.00×10^{-12}

Magnitude X_i	Estimation x_i	Standart uncertainty $u(x_i)$	Probability distribution	Conversion factor	Uncertainty contribution $u_i(y)$
Exactitud UTC(ROA) @ 1 día	-	7,500E-15	Normal	1,0	7,500E-15
Estab. UTC(ROA) @ $\tau_{medición}$		4,000E-14	Normal	1,0	4,000E-14
Maser temperature sensivity		1,000E-14	Rectangular	0,3	2,887E-15
AOG Temperature sensivity	-	1,000E-14	Rectangular	0,3	2,887E-15
33622A Channel-to-channel skew	-	1,00E-10	Normal	1,0	1,00E-10
No linealidad diferencial SR620		5,000E-11	Normal	1,0	5,000E-11
Resol. display (LSD) SR620		1,000E-12	Rectangular	0,3	2,887E-13

Figura 5. Uncertainty Budget for delay measurements

M_1 y M_2 obtained measurements are:

$$\bar{M}_1 = C_B - C_A + R_B - R_A$$

$$\bar{M}_2 = C_B - C_A + R_A - R_B$$

where:

$\bar{M}_1, \bar{M}_2$ average value of the N measurements.

$C_B - C_A = C_{BA}$, delay of the signals at the input to the counter, due to the asymmetry between the generator channels and the cables used.

$R_B - R_A = R_{BA}$, delay caused by asymmetries of the counter channels. Includes the differential trigger error of the counter channels.

Then:

$$\bar{M}_1 = C_{BA} + R_{BA}$$

$$\bar{M}_2 = C_{BA} - R_{BA}$$

From which we obtain:

$$R_{BA} = \frac{\bar{M}_1 - \bar{M}_2}{2} \qquad C_{BA} = \frac{\bar{M}_1 + \bar{M}_2}{2}$$

Regarding the uncertainties of these measurements, in accordance with the law of propagation of uncertainties and taking into account the independence of them:

$$u_c^2(R_{BA}) = \left(\frac{\partial R_{BA}}{\partial \bar{M}_1}\right)^2 u^2(\bar{M}_1) + \left(\frac{\partial R_{BA}}{\partial \bar{M}_2}\right)^2 u^2(\bar{M}_2)$$

Being:

$$\frac{\partial R_{BA}}{\partial \bar{M}_1} = \frac{1}{2}\frac{\partial R_{BA}}{\partial \bar{M}_2} = -\frac{1}{2} \qquad u(\bar{M}_1) = \frac{\sigma(M_1)}{\sqrt{n}} \quad u(\bar{M}_2) = \frac{\sigma(M_2)}{\sqrt{n}}$$

With n as the number of measurements performed, the following uncertainties are obtained for asymmetry (R_{BA}) and delay of cables (C_{BA}):

$$\boxed{u_c(R_{BA}) = \frac{1}{2}\sqrt{u^2(\bar{M}_1) + u^2(\bar{M}_2)}} \qquad \boxed{u_c(C_{BA}) = \frac{1}{2}\sqrt{u^2(\bar{M}_1) + u^2(\bar{M}_2)}}$$

As a result of the measurements carried out, the following results are obtained:

Cálculo de las asimetrías - SR620						
Magnitud	IT promedio (s)	Desv. Típ. (jitter)	Incert. Tipo A	Resolución SR620 (s)	u_{SR620} (s)	u_c(IT)
M_1	9,30E-11	5,00E-12	5,00E-13	2,54E-12	3,17E-11	1,16E-10
M_2	-1,83E-10	6,00E-12	6,00E-13	2,54E-12	3,17E-11	1,16E-10

Magnitud (s) X_i	Estimación (s) x_i	Incertidumbre $u(x_i)$	Distribución	Coeficiente c_i	Contribución a $u_i(y)$
M_1	9,30E-11	1,16E-10	Normal	0,5	5,81E-11
M_2	-1,83E-10	1,16E-10	Normal	-0,5	-5,81E-11
Ret. dif. SR620	1,38E-10	Incertidumbre combinada u_c:			8,22E-11

Magnitud (s) X_i	Estimación (s) x_i	Incertidumbre $u(x_i)$	Distribución	Coeficiente c_i	Contribución a $u_i(y)$
M_1	9,30E-11	1,16E-10	Normal	0,5	5,81E-11
M_2	-1,83E-10	1,16E-10	Normal	0,5	5,81E-11
Ret. dif. cables	-4,5E-11	Incertidumbre combinada u_c:			8,22E-11

Figura 6. Results of performed delays measurements.

4. REFERENCE DOCUMENTATION

EURAMET.TF-S1_ROA/001.20: Supplementary Comparison. ”Comparison of time interval measurements”, report of the Real Instituto y Observatorio de la Armada (ROA). (performed within EURAMET Project #1485)