



# Measuring the multi-faceted dimension of liquidity in financial markets: A literature review<sup>☆, ☆, ☆</sup>

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## ABSTRACT

This paper provides a thorough review of the liquidity measures that are used in the empirical literature to measure liquidity. A wide range of papers have emphasized its role and the need to manage and understand this topic, which had hitherto not been deeply explored. Literature on liquidity proposes a wide set of liquidity measures and proxies intended to measure the different characteristics and dimensions that liquidity presents. Early papers analyzing the liquidity issue were based on quotation data or on end-of-month prices, given that databases with widely complete transaction information were not available. The recent availability of high frequency databases has allowed researchers not only to develop new measures but also to adapt to other markets a comprehensive set of existing measures. In this paper, we classify and describe the variety of the existing liquidity measures and proxies depending on the aspect of liquidity that one wants to address.

## 1. Introduction

Liquidity is a key factor in the pricing and return of fixed income and equity assets. The most highly liquid assets are traded with a liquidity premium on prices compared with similar securities that differ in the degree of liquidity (e.g., Sarig and Warga, 1989; Krishnamurthy, 2002; Longstaff et al., 2005). Contrary, those assets that exhibit lower liquidity are traded with lower prices and higher liquidity premiums on yields to compensate their low liquidity levels (e.g., Silber, 1991; Amihud and Mendelson, 2006; Amihud et al., 2006; Vayanos and Weill, 2008; Díaz and Escribano, 2017). Its effects on prices and market behavior are relevant for a variety of market participants such as investors, broker-dealers, traders, and regulators.

During the last three decades, liquidity has been an increasingly relevant concept when valuing financial assets as well as a subject increasingly present in the literature. Despite the large number of articles on the subject, especially in the aftermath of the global financial crisis of 2008, there is a lack of a coherent understanding of what different aspects the concept of liquidity encompasses, what type of measures and proxies are most appropriate to measure each facet, what data and calculation methods they require, what

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types of results they produce, and under what circumstances. Understanding whether the liquidity measure used in the valuation of an asset is effective is a relevant practical issue for practitioners. The minimum liquidity standards proposed by regulators and international institutions and included in private investment policy statements are also other hot topics that require a good knowledge of the liquidity literature. This paper contributes to the understanding of liquidity by reviewing the existing body of theoretical and empirical research on the topic.

Indeed, because liquidity is present in many different financial and economic contexts, it depends on several factors and assets' features, such as the amount outstanding, age, term to maturity, issue status, economic activity cycle, interest rates volatility, or investor risk aversion, among others. This fact makes liquidity to be an elusive concept, subjective in nature and not easily measurable, which involves a wide range of different topics (see, e.g., Lippman and McCall, 1986; Houweling et al., 2005; Helwege et al., 2014). Accordingly, there is a wide range of definitions that try to deal with this multi-faceted issue, which is difficult to encapsulate in one single definition or measure (Amihud et al., 2005). Furthermore, asset's liquidity is not static. It varies over time both for individual securities and for the market as a whole.<sup>1</sup>

Despite the lack of consensus on liquidity literature about a one single and universal definition of liquidity, we find some approximations in prior research that try to give a description of term liquidity. In the pioneer paper of Fisher (1959), liquidity is defined as the ability of an asset to be negotiated quickly and without a significant loss on its value. Biais et al. (1997) state that "*an asset is liquid if it can be bought or sold quickly at low transaction costs and a reasonable price*". In other words, liquidity is related to the ability of an asset to be turned into money and in a brief period of time. In this sense, a liquid asset is a good substitute for money.

When dealing with market liquidity, we say that a market is liquid if it is possible to make and/or unwind positions in a short period of time and without causing significant losses in value. Demsetz (1968) defines market liquidity as the price concessions that a market participant has to incur to execute a trade. In this sense, in a liquid market, participants do not suffer large transactions costs and price concessions. This will depend on how liquid each of the assets traded are and on the degree of substitutability among them (see, e.g., Sarr and Lybek, 2002). Shen and Starr (2002) defines liquidity in a financial market as "*the ability to absorb smoothly the flow of buying and selling orders*". Fleming (2003) includes a definition of liquidity from O'hara (1995) and Engle and Lange (1997): "*a liquid market is defined as one in which trades can be executed with no cost*". On the other hand, Chordia et al. (2004) state that liquidity "*can be defined as the ability to buy or sell large quantities of an asset quickly and at low cost*". In other words, liquidity is related to the capacity of a market to absorb a large number of transactions without causing high movements in prices.

All these prior definitions of liquidity, some of them conditioned by the framework in which they have been made, present three main common elements, namely traded volume, transaction price and execution time. These elements are determinant and can be instrumentalized through different dimensions of liquidity as discussed in Oesterhelweg and Schiereck (1993). According to Sarr and Lybek (2002), liquidity has five dimensions, namely breadth (related with the number of orders and their volumes around equilibrium prices), depth (concerns the number of orders around equilibrium prices), immediacy (linked to the speed at which orders are executed), resilience (concerns the capacity of the market to recover from unexpected events) and tightness (related to the amount of transaction costs). Measuring these costs, price impacts, or transaction speed is not simple since they depend on numerous factors such as the size of the negotiation, time, the place of negotiation, or partners.

The literature on liquidity proposes different liquidity measures and proxies that attempt to capture these dimensions. The lack of complete high-frequency databases that enable to compute liquidity measures, makes that many of the academic surveys have been conducted using low-frequency data, which allow researchers to compute almost liquidity proxies.<sup>2</sup> The use and development of one or another measure is sometimes strongly driven by the availability of appropriate data. Most liquidity proxies need market-microstructure data, such as bid-ask spreads, daily or intra daily quotes, and/or transaction volumes to be accurately computed. These data are not always available for all the assets and markets, but even when they are accessible, filtering and analyzing such trade-by-trade information is a daunting and time-consuming task. This issue is the major driving force for using alternative and easily computable liquidity proxies that are based on more common data that are usually available in the vast number of markets.<sup>3</sup> The use and development of one or another measure is also driven by the specific facet of liquidity that one wants to address. Many liquidity measures and proxies are one-dimensional measures, meaning that capture only one dimension of liquidity. But there are also multi-dimensional measures (two- and three-dimensional measures) that are able to gather more than one dimension. This fact makes that some measures cannot be plainly assigned to one single dimension, since the same measure can capture more than one of the different facets of liquidity. Thus, there is not an unequivocal classification of measures regarding to which dimension is assessed by each one.

In this context and derived from the existence of the extensive number of liquidity measures, some papers study how properly several of them work well in measuring liquidity. The vast literature is focused on the US market, particularly on the Treasury and corporate bond markets, and on stock markets. For example, in the field of the fixed income markets, Fleming (2003) compares some direct and indirect liquidity measures and proxies for the US Treasury bond market, concluding that the simple bid-ask spread is a

<sup>1</sup> Some papers find evidence about the co-movements of individual liquidity measures with each other's (see, e.g., Huberman and Halka, 1999; Chordia et al., 2000; Hasbrouck and Seppi, 2001; Brockman et al., 2009; Karolyi et al., 2012; Foran et al., 2015).

<sup>2</sup> Liquidity measures are supposedly to be more accurate than liquidity proxies, which have seen to be less accurate than the firsts. However, recent empirical papers find evidence of high-quality liquidity proxies based on low-frequency data that are capable of measuring liquidity as accurately as high-frequency liquidity measures (see, e.g. Schestag et al., 2016; Fong et al., 2017).

<sup>3</sup> Early papers analyzing the bond liquidity issue were based on quotation data, given that databases with widely complete transaction information were not available until recent years. In fact, the recent availability of intraday transaction prices, such as those provided by the Trade Reporting and Compliance Engine (TRACE) dataset in the secondary US corporate bond market, has made it possible not only to compute traditional bond liquidity measures in an accurate way but also to incorporate and adapt from stock exchange markets other liquidity measures.

useful measure for assessing the degree of liquidity in this market. Díaz and Escribano (2017) provide evidence about the ability of different microstructure-based liquidity proxies to measure liquidity in the US Treasury bond market. They find that the market share liquidity proxy captures the liquidity level of bonds for different terms to maturity better than other proxies do. In the case of the US corporate bond market, Houweling et al. (2005) highlight that the yield volatility and the number of contributors perform better than other proxies of liquidity, although the differences among all tested proxies are small. Mahanti et al. (2008) develop a new measure and compare it with other existing liquidity measures. Among the most recent papers, and also for the US corporate bond market, Schestag et al. (2016) find that all the high-frequency liquidity measures analyzed are able to measure liquidity properly, meanwhile only some of the considered low-frequency liquidity proxies lead among the rest of them. More recently, Díaz and Escribano (2019) analyze the performance of sixteen of the most common liquidity measures in the US corporate bond market and find that those measures that account for the resilience and tightness dimensions of liquidity are able to reflect the liquidity differences between credit ratings. Further, Langedijk et al. (2018) investigate which liquidity proxy best measures liquidity in the context of EU sovereign bonds.

On the other hand, in the case of stock markets, Liu (2006) and Holden (2009) propose new liquidity measures and compare them with existing liquidity measures in the US market. Goyenko et al. (2009) analyze a wide range of both traditional and new microstructure liquidity measures and find that their new effective/realized spread measures win the majority of the horse races. Fong et al. (2017) and Fong et al. (2018) analyze several liquidity proxies based on low-frequency data to figure out which of them are the best liquidity proxies for both US and global research. Moreover, although the vast literature focuses on the US market, there are other recent empirical papers analyzing liquidity in other different markets. One example is the study of Florackis et al. (2011), who propose a new price impact measure and compare it with the Amihud measure for the UK market. In the same market, Foran et al. (2014) conduct an analysis of liquidity shocks using a range of tick data-based liquidity measures. They construct different market liquidity factors and find that the liquidity measures considered exhibit strong commonality suggesting that they may be capturing the same underlying property.<sup>4</sup>

The aim of this review is twofold. Firstly, we describe and highlight the different liquidity dimensions that this multi-faceted concept of liquidity presents. Secondly, we conduct a wide review of the liquidity measures that prior empirical literature has used to assess liquidity in both fixed income and stock markets, highlighting the multidimensional facet of liquidity. We present and classify the liquidity measures and proxies used in the literature according to the aspect of liquidity that each one wants to address. Our state-of-the-art review provides new insights to help researchers, practitioners and policymakers in the difficult task of measuring liquidity.

The remainder of this paper is organized as follows: Section 2 presents the different facets of liquidity. Section 3 includes the classification of liquidity measures and proxies. In Section 4, we present other liquidity measures used in the previous literature on liquidity. Section 5 provides some guidance for researchers, practitioners and policymakers. Finally, Section 6 concludes.

## 2. Dimensions of liquidity

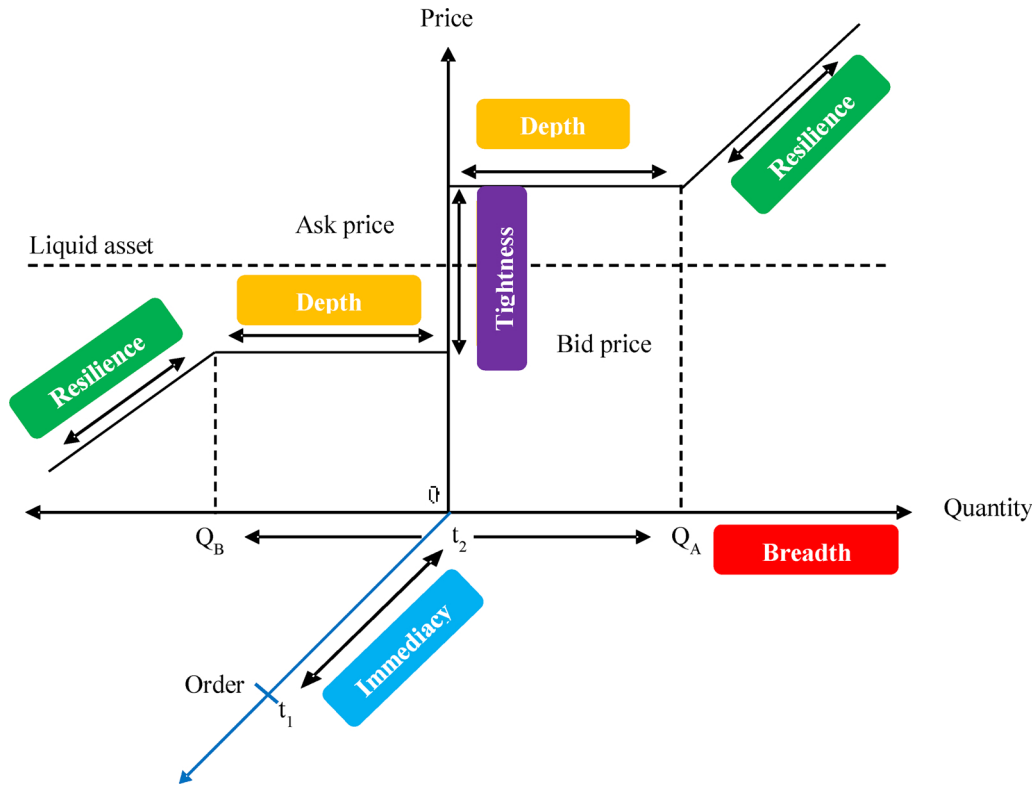
The vast body of literature on liquidity agree on the existence of different aspects of liquidity that determine its different dimensions, although there is no an unequivocal dimension list. According to Sarr and Lybek (2002) and Bervas (2006), a liquid market presents five different dimensions: *breadth*, *depth*, *immediacy*, *resilience* and *tightness*. Some prior literature identifies four different facets namely *tightness*, *depth* and *resilience* in addition to the *immediacy* dimension (Black, 1971; Grossman and Miller, 1988; Harris, 1990).<sup>5</sup> Contrary, other authors consider only three main dimensions of liquidity that correspond to *tightness*, *depth* and *resilience* (Garbade, 1982; Kyle, 1985; Holden, 1990), whereas Bernstein (1987) distinguish between those of *breadth*, *depth* and *resilience*. This discrepancy is mainly due to the fact that these last authors implicitly assume that immediacy is not a dimension itself but given in automated markets (Schwartz, 1988).

All these dimensions are different in nature, but some of them turn out to be interrelated.<sup>6</sup> First, *market breadth* concerns the trading volume of the existing orders at different prices. It refers to the possibility that market participants can suffer large price concessions when they want to sell discarded assets. A market is said to be broad when there are numerous buying and seller orders that at the same time present large volumes. Second, *market depth* addresses the number of orders around equilibrium prices and is also related to demand pressure and inventory risk. A market is deep when there are numerous buying and seller orders around equilibrium prices. Third, *market immediacy* is the speed at which the orders are executed in the market. It depends on its demand and supply, i.e. on the willingness of sellers and buyers to trade. Sellers claim and demand immediacy on their willingness to sell, whereas market makers are in charge of placing the immediacy supply. A market is more immediate if transactions between buyers and dealers, or between sellers and dealers can be executed in a brief period of time. Fourth, *market resilience* deals with a market's ability to absorb and recover from unexpected asset shocks. A market is resilient if there are many orders to respond to changes in prices and to correct order imbalances derived from asset shocks. Finally, *market tightness* is related with the amount of trading costs of turning

<sup>4</sup> In addition, there are recent empirical papers that study the ability of liquidity proxies in measuring liquidity in other markets, such as in Exchange Traded Funds (e.g., Marshall et al., 2018), in currency markets (e.g., Mancini et al., 2013), in commodity markets (e.g., Marshall et al., 2012; Karnaukh et al., 2015), and in emerging markets (e.g., Lesmond, 2005; Bekaert et al., 2007; Chai et al., 2010; Lee, 2011; Lang et al., 2012; Kang and Zhang, 2014; Ahn et al., 2018; Hilal Anwar and Hogholm, 2018; Bhattacharya et al., 2019; Będowska-Sójka and Echaust, 2019).

<sup>5</sup> Tightness is also referred in other papers as *width*, as in Harris (1990) and in Anderson and Lavoie (2004).

<sup>6</sup> Using a large battery of liquidity measures and proxies, Goyenko et al. (2009) show that there is close relationship among many of the measures used that account for different dimensions.



**Fig. 1. Dimensions of liquidity.** This figure shows four of the five dimensions of liquidity: depth, tightness, breadth, immediacy, and resilience.  $Q_1$  and  $Q_2$  represent different amounts that could be traded at standing bid and ask prices. These amounts represent the market depth. The difference between the bid and ask prices measures the tightness. The market breadth is the volume of orders at different prices. The capacity of the market to recover from unexpected events is measured by resilience. Finally,  $t_1$  and  $t_2$  represent the time at which an order is introduced and the time at which the order is executed, respectively. Thus, immediacy, i.e., the speed at which orders are executed, is measured by the difference between  $t_1$  and  $t_2$ . Adapted from [Bervas \(2006\)](#) and [Hibbert et al. \(2009\)](#).

around a position. Tighter markets are those in which market participants face large transaction costs when buy or sell an asset.

Among these dimensions, breadth and depth are interlinked since both facets rely on the number of orders at equilibrium prices. At the same time, both breadth and depth are related with immediacy as long as in the presence of numerous orders from buyers and sellers it would be possible to trade a given volume at the desired price immediately or in a later moment. Furthermore, immediacy is also related with resilience since a more resilient market will be able to quickly revert prices to their normal levels after shocks in assets, meaning that orders from sellers and buyers would be executed in a short period. The strong interrelation between some of the dimensions makes it desirable and necessary to have liquidity measures addressing each one of these dimensions. In fact, it serves as an indicator of the need that liquidity should be measured across the different dimensions simultaneously in order to have a global view of liquidity.

Fig. 1, adapted from Bervas (2006) and Hibbert et al. (2009), graphically represents the five aforementioned dimensions. The difference between the bid and ask prices represents market tightness. A lower bid-ask spread means lower transaction costs and hence a low level of tightness. As this spread decreases, it approaches the dotted line, which represents the perfect level of liquidity. Once an order is introduced in the system, the speed at which this order is executed assesses the immediacy of the market.  $t_1$  and  $t_2$  represent the time at which an order is introduced and executed, respectively. Thus, immediacy is measured by the difference between them. Additionally, the number of existing orders around equilibrium prices represents the market depth. The amounts  $Q_1$  and  $Q_2$  indicate the quantities that could be traded at the standing bid and ask prices with minimal impact on prices, and measure market breadth. Finally, the capacity of the market to recover from unexpected events is measured by resilience. It is represented from the threshold of trading volumes  $Q_1$  and  $Q_2$ . From these volumes, the marginal impact of an additional unit of trading volume increases (decreases) in the case of purchases (sells).

Due to this multidimensional nature of liquidity, the existing literature has developed and implemented different measures and proxies based on these attributes with the aim of addressing the different facets.<sup>7</sup> *Breadth* has been analyzed principally through trading volume-based measures that relate the volume of trades with the price variability. On the other hand, *depth* can be assessed through volume-based measures and through trading frequency measures. Volume-based measures are based on the trading volume

<sup>7</sup> However, there is no an unique measure that fully capture all the aspects of liquidity (Bao et al., 2011).

of the assets traded and are generally less sophisticated than other liquidity measures. Large volumes are associated with liquid and depth markets. However, large trading volumes could also be an indicator of other situations, for example, the arrival of new information in the market; therefore, other issues, such as the number of transactions, should be considered to control for other potential sources of the increasing volume traded. In addition, other volume-based measures include also transaction prices, which allow the price impact of different trading volume levels to be estimated. Conversely, trading frequency measures encompasses measures that account for the trading frequency of a market or an asset and basically take into account the number of transactions that occurred during a specific period of time. A large number of trades could deal with a large number of market participants, who trade different assets within a period. In addition, a higher number of trades could also come from a lower number of market participants that, nonetheless, are able to trade at a high frequency. In addition, *immediacy* has been documented through the opportunity cost of the continuous presence of market makers in providing prices as well as the demand preferences of customers. Moreover, *resilience* can be addressed through price-based measures that are able to capture the tendency of prices to revert to their fundamental or equilibrium values after shocks. The aim of these measures is to capture potential orderly movements directed to reach equilibrium prices. They try to separate liquidity from other factors that could move the price up or down, as for example, the economic cycle through which it is passing or the arrival of news that could affect transaction prices of securities. Finally, *tightness* has traditionally been assessed through bid-ask spread-based measures, i.e. transaction costs measures which may capture the implicit cost of trading. Higher transaction costs will be related to lower liquidity, as long as market participants will reduce their demand for trading, concentrating their operations on a few transactions to avoid larger costs.

All these different typologies of measures can be computed either directly, i.e. direct liquidity measures, or indirectly, i.e. indirect liquidity proxies. The difference between them basically concerns the frequency and the type of the data. Liquidity measures require high-frequency, trade and/or quote data to be accurately computed. High-frequency data are not always available, and their use requires computationally handling and cleaning procedures. These are the main reasons why other authors use daily or monthly data to compute proxies for liquidity. Additionally, in the case of transaction bond data, which usually come from over-the-counter markets, high-frequency measures are difficult to implement. In this case, most authors tend to employ liquidity proxies to proxy for liquidity. Liquidity proxies require low-frequency data to be computed and are usually less sophisticated and complex to compute than high-frequency liquidity measures. The availability and accessibility of low-frequency data is higher, and also the computational resources that are required are more straightforward, therefore it is easier to estimate them.

### 3. Classification of liquidity measures and proxies

The classification of the different liquidity dimensions serves as a guide to classify those liquidity measures that address one or more of the facets of liquidity. Besides, the classification of the different type of data required to compute liquidity measures serves also as another rule to properly classify them. However, there is no an unequivocal classification of measures regarding to which dimension is assessed by each one.<sup>8</sup> This discrepancy could be due because some measures as, for instance, complex volume- and/or price-based measures, cannot be plainly assigned to one dimension, since the same measure can capture more than one of the different facets of liquidity. There are one-dimensional measures which capture only one dimension of liquidity as for example the bid-ask spread that measures tightness, or the trading volume that accounts for depth. Besides, there are two-dimensional measures that are able to capture two of the dimensions of liquidity, as for example the auto-covariance in price changes from Bao et al. (2011), which captures both resilience and tightness. Moreover, there exist three-dimensional measures that may measure at least the three main dimensions of liquidity namely depth, immediacy and tightness, as for instance the Amihud (2002) measure or the Amivest liquidity ratio from Cooper et al. (1985).

In this section, we attempt to classify measures considering both the different dimensions that each one measures and the alternatives about their assignment to a particular dimension according to the previous literature on liquidity measures.

#### 3.1. High-frequency liquidity measures

Table 1 summarizes the most common high-frequency liquidity measures implemented by the existing literature based on this criterion. For each measure, we highlight some characteristics, such as the first author to propose the measure, other authors that use it, data requirements to compute it, the addressed facet/s of liquidity, and the market in which it has been implemented. Panel A displays the measures that measure transaction costs or bid-ask spreads and that address tightness. Among these measures, the *Quoted spread*, the *Effective spread* and the *Realized spread* are standard measures of liquidity from the microstructure literature (Fong et al., 2017).<sup>9</sup> The *Quoted spread* is the simplest measure of trading costs and is equal to the spread over the average between ask and bid

<sup>8</sup> This is the case of the Hui and Heubel's (1984) ratio. It can either be considered as a measure for the *breadth* dimension or as a measure for *resilience*. Other popular example is the Amihud's (2002) measure. Broto and Lamas (2016) consider it as a *breadth* dimension, meanwhile Black et al. (2016) include it within the measures to assess the *depth* dimension. Besides, Downing et al. (2005) employ the Range measure to account for *breadth*, whereas Broto and Lamas (2016) consider it as a measure for *immediacy*.

<sup>9</sup> Some authors refer to the *Effective spread* as the total component of the spread, meanwhile the *Realized spread* and *Price impact* represent the temporary and the permanent components of the spread respectively (see, e.g., Kraus and Stoll, 1972; Madhavan and Cheng, 1997; Boehmer et al., 2007; Bessembinder and Venkataraman, 2010). In addition, some other authors use the dollar and percent versions of these measures, i.e., the dollar quoted spread, the dollar effective spread, and the dollar realized spread; and the percent quoted spread, the percent effective spread, and the percent realized spread (see, e.g., Holden and Jacobsen, 2014; Fong et al., 2017).



Table 1

**Liquidity measures.** This table shows the expressions and models of the liquidity measures grouped by type. *Panel A* shows the transaction costs measures, *Panel B* the price-based measures, *Panel C* the volume-based measures and *Panel D* the trading frequency measures. The legend is the following: *In*, *Du*, *Li* and *Il* refers to intraday, daily, liquidity and illiquidity, respectively. *Ask<sub>t</sub>* (*Bid<sub>t</sub>*) is the best ask (bid) quote over *k* trades on day *t*; *D<sub>k</sub>* is a buy-sell indicator that takes value +1 (-1) if the *k*-th trade is a buy (sell); *p<sub>k</sub>* is the price of the *k*-th trade; *m<sub>k</sub>* is the midpoint of the best bid and offer at the moment in time of the *k*-th trade; *p<sub>k+5</sub>* and *m<sub>k+5</sub>* use a 5 min interval;  $\hat{p}_t^{buy}$  ( $\hat{p}_t^{sell}$ ) is the average customer buy (sell) price on day *t*;  $\Delta_k$  is the difference between the price of the *k*th trade and the estimated bid price; *D<sub>k</sub><sup>buy</sup>* is a dummy variable that takes the value of one if the *k*-th trade is a buy and zero otherwise; *D<sub>ts</sub>* is the number of days between trades *t* and *s*; *DR* is the bond coupon rate subtracted from 5%; *AI<sub>R<sub>ts</sub></sub>* is the index return for the average bond between trades *t* and *s*; *DD<sub>ts</sub>* and *CD<sub>ts</sub>* are the corresponding differences between index returns for long- and short-term bonds and high and low credit risk bonds;  $r_{ts}^P$  is the continuously compounded observed bond price return between trades *t* and *s*; *Q<sub>t</sub>* (*Q<sub>s</sub>*) is a buy-sell indicator of the trade *t* (*s*); *S<sub>t</sub>* (*S<sub>s</sub>*) is the size of the trade *t* (*s*) in dollar volume;  $p_t^{75th}$  ( $p_t^{25th}$ ) is the value of the 75th (25th) percentile on day *t*;  $\hat{p}_t$  is the average price on day *t*; *Pmax<sub>t</sub>* (*Pmin<sub>t</sub>*) is the maximum (minimum) trading price over *k* trades on day *t*; *Big Orders<sub>t</sub>* (*Small Orders<sub>t</sub>*) is the set of all orders in the range of 2000–9999 (100–499) shares that are executed within time interval *t*; *V<sub>t,k</sub>* is the signed dollar volume of the *k*-th trade within the period of time *t*; *Y<sub>t,k</sub>* is the yield quoted by the *k*-th trade on day *t*;  $\bar{y}_t$  is the average yield on day *t*; *K<sub>t</sub>* is the number of trades on day *t*; *TV<sub>t</sub>* (*TV<sub>t,k</sub>*) is the dollar volume traded over day *t* (on the *k*-th trade on day *t*). Dimensions in parenthesis refer to alternative classifications that have been considered in the literature.

Liquidity measure	Original author	Expression/model	Data frequency	Data requirements	Type	Facet	Other references	Market
<i>Panel A. Transaction costs measures</i>								
Quoted spread	Stoll (1989)	$QS_t = \frac{Ask_t - Bid_t}{Ask_t + Bid_t} \cdot \frac{1}{2}$	In	Quote prices	Il	Tightness	Fong et al. (2017); Schestag et al. (2016)	Stock Corporate
Effective spread	Lee (1993)	$ES_k = 2 \cdot D_k \cdot (ln(p_k) - ln(m_k))$	In	Transaction and quote prices and trade's sign	Il	Tightness	Beber et al. (2008); Hasbrouck (2009); Fong et al. (2017)	Treasury Stock
Realized spread	Huang and Stoll (1996)	$RS_k = 2 \cdot D_k \cdot (ln(p_k) - ln(m_{k+5}))$	In	Transaction and quote prices and trade's sign	Il	Tightness	Chordia et al. (2000); Goyenko et al. (2009)	Stock
Average bid-ask	Hong and Warga (2000)	$ABidAsk_t = \frac{\hat{p}_t^{buy} - \hat{p}_t^{sell}}{1 / (2(\hat{p}_t^{buy} + \hat{p}_t^{sell}))}$	In	Transaction and quote prices and trade's sign	Il	Tightness	Chakravarty and Sarkar (2003); Schestag et al. (2016)	Stock Corporate
Schultz's roundup transaction costs	Schultz (2001)	$\Delta_k = \alpha_0 + \alpha_1 D_k^{buy} + \varepsilon_k$	In	Transaction prices	Il	Tightness	Schestag et al. (2016)	Corporate
Edwards, Harris and Piwowar	Edwards et al. (2007)	$r_{ts}^V = D_s(DR) + \beta_1 AI_{R_{ts}} + \beta_2 DD_{ts} + \beta_3 CD_{ts} + \varepsilon_{ts}$ ; $c(S_t) = c_0 + c_1 \frac{1}{S_t} + c_2 \log S_t + c_3 S_t + c_3 S_t^2 + \kappa_1$ ; $r_{ts}^P - D_s(DR) = c_0(Q_t - Q_s) + c_1 \left( Q_t \frac{1}{S_t} - Q_s \frac{1}{S_s} \right)$ + $c_2(Q_t \log S_t - Q_s \log S_s) + c_3(Q_t S_t - Q_s S_s)$ + $c_4(Q_t S_t^2 - Q_s S_s^2) + \beta_1 AI_{R_{ts}} + \beta_2 DD_{ts} + \beta_3 CD_{ts} + \eta_{ts}$	In	Transaction prices	Il	Tightness	Schestag et al. (2016)	Corporate
Imputed roundup cost	Dick-Nielsen et al. (2012)	$IRC_t = \frac{P_{max} - P_{min}}{P_{max}}$	In	Transaction prices	Il	Tightness (Immediacy)	Goldstein et al. (2007); Bao and Pan (2013); Schestag et al. (2016)	Corporate
Static price impact	Goyenko et al. (2009)		In	Transaction and quote	Il	Tightness	Goyenko et al. (2009)	Stock

(continued on next page)

Table 1 (continued)

Liquidity measure	Original author	Expression/model	Data frequency	Data requirements	Type	Facet	Other references	Market
Huang and Stoll's price impact	Huang and Stoll (1996)	$SPI_t = \frac{\$ES_{Big Orders,t} - \$ES_{Small Orders,t}}{P_t}$ $H_k = 2 \cdot D_k \cdot (\ln(m_{k+5}) - \ln(m_k))$	In	prices and trade sizes	II	Tightness (Immediacy)	Goyenko et al. (2009); Fong et al. (2017)	Stock
<i>Panel B. Price-based measures</i>								
Interquartile range	Han and Zhou (2008)	$IQR_t = \frac{P_t^{75th} - P_t^{25th}}{P_t} \times 100$	In	Transaction prices	II	Resilience	Pu (2009); Helwege et al. (2014)	CDS Corporate
Gamma	Bao et al. (2011)	$\gamma_t = -cov(\Delta p_k, \Delta p_{k+1})$	In/Da	Transaction prices	II	Resilience and tightness (Immediacy)	Helwege et al. (2014); Díaz and Escribano (2017)	Stock Corporate Treasury
Lambda	Hasbrouck (2009)	$\eta = \lambda \cdot \sum_k \text{sign}(V_{t,k}) \sqrt{ V_{t,k} } + \varepsilon_t$	In	Transactions prices, volume and trade's sign	II	Resilience (Breadth)	Goyenko et al. (2009); Schestag et al. (2016); Fong et al. (2017)	Stock Corporate
Yield dispersion	Houweling et al. (2005)	$YD_t = \sqrt{\frac{1}{K_t} \sum_{k=1}^{K_t} \left( \frac{y_{t,k} - \bar{y}_t}{\bar{y}_t} \right)^2}$	In	Quote yields and transactions	II	Resilience	—	Corporate
Price dispersion	Jankowitsch et al. (2011)	$PD_t = \frac{1}{\sqrt{\sum_{k=1}^{K_t} TV_{t,k}}} \sum_{k=1}^{K_t} (\rho_k - m_k)^2 TV_{t,k}$	In	Transaction and quote prices and volume	II	Resilience and depth	Friewald et al. (2012); Jankowitsch et al. (2014); Schestag et al. (2016)	Corporate
<i>Panel C. Volume-based measures</i>								
Range measure	Downing et al. (2005)	$RG_{t,t} = \frac{P_{max,t} - P_{min,t}}{P_t} \times 100$	In	Transactions prices and volume	II	Breadth	Pu (2009); Helwege et al. (2014)	Corporate CDS
<i>Panel D. Trading frequency measures</i>								
Quote size	Fleming (2003)	$VI_{t,k}$	In	Quote size	Li	Depth	Goldreich et al. (2005); Li et al. (2009)	Treasury
Trade size	Fleming (2003)	$TV_{t,k}$	In/Da	Trade size	Li	Depth	Goldreich et al. (2005); Li et al. (2009)	Treasury
Number of trades	Chordia et al. (2001)	$K_t = \sum_{k=1}^{K_t} K_{t,k}$	In/Da	Number of trades	Li	Depth	Fleming (2003); Goldreich et al. (2005); Jankowitsch et al. (2014)	Treasury Corporate

prices.<sup>10</sup> The *Effective spread* is a useful measure to estimate transaction costs when trades occur either within or outside the quotes (Bessembinder and Venkataraman, 2010). Goyenko et al. (2009) implement a version of this measure, the *\$Effective spread*, using data from the Rule 605. The *Realized spread* captures the reversals of prices in the existence of informed traders. Goyenko et al. (2009) also compute an alternative version of the realized spread, taking  $p_{k+5}$ , that is, the price 5 min after the  $k$ th trade, instead of  $m_{k+5}$ . Other examples of this type of measures are the *Imputed Roundtrip Cost* and the Huang and Stoll (1996)'s *price impact* measure. The *Imputed Roundtrip Cost* (IRC) measure is a direct measure to estimate transaction costs. It was implemented by Dick-Nielsen et al. (2012), and requires previously computing the *Imputed Roundtrip Trades* (IRTs) to identify whether a trade is a buy or a sell.<sup>11</sup> On the other hand, the Huang and Stoll (1996)'s *price impact* measure represents the permanent component of the spread, and captures the price adjustments that market makers do in the presence of informed traders and their subsequent liquidity providers.

Panel B of Table 1 includes those price-based measures that measure mainly resilience. For example, the *Gamma* ( $\gamma$ ) from Bao et al. (2011) is proposed to be used either with trade-by-trade or daily data. The difference lies in taking changes in prices of consecutive trades or changes in consecutive daily prices. It is a variant of the Roll (1984) measure, although  $\gamma$  is able to capture a broader impact of illiquidity above and beyond the bid-ask spread effects. As a refinement, the *Lambda* ( $\lambda$ ) measure from the model of Hasbrouck (2009) introduces a time component.<sup>12</sup> Another measure in this class is the *Price Dispersion* (Jankowitsch et al., 2011), which captures significant price dispersion effects that cannot be captured by bid-ask spreads. This measure can be interpreted as the volatility of the price dispersion.

In Panel C of Table 1 we show those volume-based measures that assess market breadth.<sup>13</sup> The *Range* measure from Downing et al. (2005) derives from the daily range measure, which is the absolute difference between high and low prices of an asset on a particular day. Downing et al. (2005) compute the average daily range over the volume traded to compare the price variability with the volume of transactions.

On Panel D of Table 1, we list those trading frequency measures that account for market depth. The *Quote size* is the quantity of securities that could be susceptible to be traded at each bid and ask price. This measure is used to assess the market depth, but it is usually underestimated, given that market makers do not always want to reveal the quantities that they are willing to trade at given prices. The *Trade size* is the quantity of securities that in fact are traded at each bid and ask price. This measure could also underestimate the market depth, given that the quantity that is in fact traded could be lower than the quantity that could have been traded at such prices. The *Number of trades* or quotes can be computed across an interval of time to measure liquidity. In the presence of bonds with a low number of trades, it is difficult to sell or buy those bonds without incurring high transaction costs.<sup>14</sup>

### 3.2. Low-frequency liquidity proxies

Table 2 provides details of the liquidity proxies included under this criterion, for which we also list related subsequent papers, data requirements and facet/s of liquidity that they address. Panel A displays the first group of liquidity proxies, the transaction costs proxies. For instance, the Roll (1984) measure that was firstly designed to estimate transaction costs from daily price changes. Roll shows that the effective bid-ask spread equals to the square root of the negative covariance between price changes in adjacent trades. However, it can also be adapted to high-frequency data if the changes in prices are computed between consecutive adjacent trades instead of consecutive daily prices.<sup>15</sup> Another measure included in this class is the *LOT* measure (Lesmond et al., 1999), which follows an underlying idea similar to that of the Roll (1984) measure to assess transaction costs, but it is based on the occurrence of zero returns. These authors argue that in the presence of large trading costs, investors will tend not to trade, driving a large probability of the occurrence of zero return days. They estimate what they call “true returns” that are based on the observed returns and on the

<sup>10</sup> This measure, as well as other spread measures, can be computed over a period of time  $s$  instead than over a day, by aggregating all the  $k$ -th spreads on period  $s$  for asset  $i$ . It has also been used among others by Hong and Warga (2000); Chakravarty and Sarkar (2003); Chai et al. (2010) and Schestag et al. (2016).

<sup>11</sup> The IRT proposed by Feldhütter (2012) is based on finding two or three trades with a similar size close in time that are likely to be a buy and a sell from the same dealer since the information about the side of the trade is not known. When this occurs, these trades are considered as a part of an IRT. The highest price in an IRT would be the buyer price for the dealer, and the lowest would be the seller price from the dealer.

<sup>12</sup> Schestag et al. (2016) use a version of this estimator but including an intercept to control for the large costs that small trades bear in bond markets.

<sup>13</sup> Grossman and Miller (1988) develop a model to estimate market liquidity from the demand and supply of immediacy. As the demand for immediacy is large and the cost of maintaining a continuous presence of market makers is low, the market becomes more liquid as the number of transactions between final customers conducted through market makers is larger. See Grossman and Miller (1988) for details.

<sup>14</sup> Several papers use this measure, for example, Chordia et al. (2001); Strebulaev (2002); Fleming (2003); Goldreich et al. (2005); Pu (2009); Nashikkar et al. (2011) or Jankowitsch et al. (2014).

<sup>15</sup> The computation of the measure, as it deals with a covariance between price changes, requires a “sufficient number of observations”, that Roll chooses arbitrary of at least 21 observations in the rolling window, independently of the data frequency and market analyzed. Besides, when the covariance is positive, the Roll measure is undefined and thus set to zero. The Roll measure was originally applied for stock exchange markets, although the availability of daily and intraday databases in fixed income markets has made it possible to compute it in an accurate way in fixed income markets (see, e.g., Schestag et al., 2016; Han and Zhou, 2016). Furthermore, there are other papers that implement extended versions of the Roll measure, that take into account both idiosyncratic adjusted price changes and average prices within the considered window of the covariance (see, e.g., Holden, 2009; Fong et al., 2017).



proportional costs of trading, and that drive to three different scenarios of true returns. The difference between these costs is their measure of the proportional transaction costs for investors.<sup>16</sup> Both the Zeros measure (Lesmond et al., 1999) and Zeros2 (Goyenko et al., 2009) share the same underlying idea: an asset with large transaction costs (low liquidity) will tend to trade to a lesser extent than other assets with low transaction costs (high liquidity). If the trading frequency were low, it would be more likely that zero returns between trading days would occur, as long as the prices do not move.<sup>17</sup> The Holden measure (Holden, 2009) estimates the effective spread employing both serial price correlation and price clustering. Holden specifies the probability for the different spreads, which supposes being linked to price cluster regimes. The *Effective tick* proposed by Goyenko et al. (2009) and Holden (2009), is based on price clustering, and the intuition behind is that when prices tend to cluster, this means that there exist larger transaction costs.<sup>18</sup> Besides, the *FHT* measure (Fong et al., 2017) is another spread measure that follows the Lesmond et al. (1999) approach.

On Panel B of Table 2, we include those price-based proxies that assess liquidity in terms of resilience and breadth. One example is the Marsh and Rock (1986)'s ratio that is the absolute value of the return over a period divided by the number of transactions within that period. These authors consider that price changes are independent from trade size and rely on the price changes and the number of transactions in a given period of time, rather than in the trading volume, to measure liquidity. The *Market-Efficiency Coefficient* (MEC) is another example, which was proposed by Hasbrouck and Schwartz (1988) to capture the degree of resilience in a market. The essence of this coefficient is that for a given permanent price change, the transitory changes to that price should be minimal in a resilient market. One of the most recently developed proxies for liquidity is the *Latent liquidity* (Mahanti et al., 2008). It is defined as the "weighted average turnover of investors who hold a particular bond, in which the weights are the fractional investor holdings". This measure has predictive power for both transaction costs and price impact, and it can be used even when there are no transaction data. The *Closing Percent Quoted Spread* (CPQS) proxy was developed by Chung and Zhang (2014). This measure is based on closing bid and ask prices over a period. The authors test this measure jointly with others in US data and find that generally, but not always, it performs better in than any other proxy tested. One of the most recently developed measures based on prices is that of Abdi and Rinaldo (2017). The authors provide a new method to estimate bid-ask spreads using daily close, high and low prices, which improves the Roll (1984) measure since it uses wider prices and it is independent of the direction of the trade.

Panel C of Table 2 summarizes the volume-based proxies that measures the breadth and depth dimensions of liquidity. One of the most relevant proxies that also serves as inspiration for other developed later is the *Kyle's lambda* (Kyle, 1985). Kyle proposes a measure of liquidity,  $\lambda$ , which is typically estimated from the regression of the price changes on the net volume of trades during fixed intervals of time. Kyle considers that prices drop when the trade is seller-initiated and increase when it addresses buyer-initiated trades. The slope of this regression would indicate the relationship between prices and volumes. Other relevant proxies are the *Amivest Liquidity* ratio and the *Amihud* measure from Amihud (2002). The Amivest liquidity ratio was first employed by Cooper et al. (1985) following the Amivest Corporation's monthly Liquidity Report published since 1972. This ratio captures how well an asset can absorb trading volumes without a significant move in its daily price. In particular, the ratio indicates the dollar volume required to move prices 1% up or down. Large portions of illiquid assets are sometimes sold with a forced price drop. At the same time, a high number of liquid assets could be bought with a considerable upward price trend. Therefore, given a volume traded, higher prices would indicate that they deal with liquid assets.<sup>19</sup> The Amihud measure is one of the most widely used proxies both in the stock and in the fixed income markets. Inspired by Kyle (1985)'s concept of illiquidity, it relates to the average association between the price change and a unit of trading volume, i.e., represents the price change required to move a unit of dollar trading volume.<sup>20</sup> The *Turnover* is the ratio of volume traded over the amount outstanding. Since the ratio relates the trading volume of one asset in a period over the amount outstanding of such asset, it is often used to reflect the average holding period of assets. The higher the turnover, the lower average holding period. Thus, the turnover is a liquidity measure since lower holding periods would be associated with lower spreads (Amihud and Mendelson, 1986). On the other hand, the *Turnover price impact ratio* implemented in Florackis et al. (2011) is inspired by the shortcomings of the Amihud (2002)'s ratio. This ratio overcomes the ability of measuring liquidity since it is unbiased by size and is unequivocal to construct and interpret. The *Turnover-adjusted zero daily volumes* developed by Liu (2006) captures multiple dimensions of liquidity such as immediacy, breadth and tightness, and is able to capture market liquidity conditions. Finally,

<sup>16</sup> Goyenko et al. (2009) implement the LOT measure and also develop an alternative version, the *LOT Y-split* measure that breaks out the three scenarios of the true returns.

<sup>17</sup> Hilal Anwar and Hogholm (2018) propose a version of the Zeros measure for small open stock markets, the *dollar zero-return* that is a better proxy for liquidity in small markets where investors hold a lot of investments in foreign currency. In these cases, investors could benefit not only from a change in the local price but also from a change in the exchange rate.

<sup>18</sup> Goyenko et al. (2009) and Holden (2009) also implement a version of *Effective tick*, *Effective tick2*. This second version differs to the first in that it includes not only positive-volume days but also zero-volume trading days. Holden (2009) also develop other version of effective tick, as *Effective tick3* and *Effective tick4* which integrates other attributes of the daily data.

<sup>19</sup> The Amivest liquidity ratio was firstly implemented in stock markets and it has been also used, among others, by Kluger and Stephan (1997); Fraser et al. (1997); Amihud et al. (1997); Berkman and Eleswarapu (1998), and Goyenko et al. (2009).

<sup>20</sup> Several papers have used this measure, such as those of Sadka (2004); Acharya and Pedersen (2005); Downing et al. (2005); Sadka (2006); Mahanti et al. (2008); Korajczyk and Sadka (2008); Pu (2009); Lin et al. (2011); Jankowitsch et al. (2011); Dick-Nielsen et al. (2012); Friewald et al. (2012); Bao and Pan (2013); Jankowitsch et al. (2014); Flood et al. (2015), or Han and Zhou (2016). Additionally, several papers implement and develop extended versions of the Amihud measure, that are basically computed as the average of a daily transaction cost proxy over the average daily dollar volume (see, e.g., Goyenko et al., 2009; Schestag et al., 2016; Han and Zhou, 2016; Fong et al., 2017).

Table 2

**Liquidity proxies.** This table shows the expressions and models of the liquidity proxies grouped by type. *Panel A* shows the transaction costs measures, *Panel B* the price-based measures, *Panel C* the volume-based measures and *Panel D* the trading frequency measures. The legend is the following: *In*, *Da*, *Li* and *Il* refers to intraday, daily, liquidity and illiquidity, respectively.  $\Delta p_t$  refers to the change in prices from day  $t-1$  to day  $t$ ;  $R^*_{i,t}$  are the true returns for asset  $i$  on day  $t$ ;  $R_{m,t}$  is the market return on day  $t$ ;  $ZR$  means zero return;  $T$  is the number of observations;  $\hat{p}_j$  are the  $j$  probabilities;  $s_j$  are the  $j$  effective spreads;  $D_i$  is a sell side indicator;  $\beta = \sum_{j=0}^1 [ln(H^0_{t+j}/L^0_{t+j})]^2$  and  $\gamma = [ln(H^0_{t+j}/L^0_{t+j})]^2$  where  $H^0_{t+j}$  ( $L^0_{t+j}$ ) are the observed high (low) prices from  $t$  to  $t+1$ ;  $B_i$  ( $A_i$ ) is the bid (ask) quote of dealer  $i$ ;  $N$  is the number of available pricing sources;  $c$  is the daily close log-price;  $\eta$  is the average of daily high and low log-prices;  $M$  is the total number of trades of an asset within a specific period of time;  $Pmin_{i,t}$  ( $Pmax_{i,t}$ ) is the maximum (minimum) price over  $k$  trades on day  $t$ ;  $TV_i$  ( $TV_{i,k}$ ) is the dollar volume traded over day  $t$  (by the  $k$ -th trade on day  $t$ );  $n$  is the number of securities outstanding;  $\hat{p}_i$  is the average price on day  $t$ ;  $\hat{p}_j$  is the average price within interval  $j$ ;  $u_i$  ( $x_i$ ) denotes the orders on day  $t$  from noise (informed) traders;  $K_i$  is the bond return on day  $t$  ( $k$ -th trade return on day  $t$ );  $R^L_i$  ( $r^L_i$ ) means long-(short)-period returns;  $T_p$  is the number of short periods in a long period;  $r^L_{i,t}$  is the excess return of bond  $i$  over the market return on day  $t$ ;  $v_{i,t}$  is the dollar volume of bond  $i$  on day  $t$ ;  $\pi^L_{i,t}$  is the part of bond  $i$  owned by fund  $j$  at the end of  $t$ ;  $T_{i,t}$  is the average portfolio turnover of fund  $j$  from  $t$  to  $t-12$ ;  $AmOut$  is the dollar volume amount outstanding;  $NZVx$  is the number of zero daily volumes in prior  $x$  months;  $x$ -month turnover is the turnover over the prior  $x$  months;  $NoTD$  is the total number of trading days in the market over the prior  $x$  months;  $TTV_i$  is the total dollar volume traded by the market on day  $t$ ;  $d_s$  is the number of days traded over period  $s$ ;  $db_i$  ( $db_j$ ) is the number of days over  $s$  in which bond  $i$  (bonds of firm  $j$ ) is (are) not traded;  $D$  is the number of days that contains  $s$ . Dimensions in parenthesis refer to alternative classifications that have been considered in the literature.

Liquidity measure	Original author	Expression/mode	Data frequency	Data requirements	Type	Facet	Other references	Market
<i>Panel A. Transaction costs proxies</i>								
Roll	Roll (1984)	$Roll_t = 2\sqrt{-cov(\Delta p_t, \Delta p_{t-1})}$	In/Da	Transaction prices	Il	Tightness	Lesmond et al. (1999); Goyenko et al. (2009); Díaz and Escribano (2017)	Stock Treasury Corporate
LOT	Lesmond et al. (1999)	$R^*_{i,t} = \beta_i R_{m,t} + \varepsilon_{i,t}$	In/Da	Transaction prices	Il	Tightness	Chen et al. (2007); Frielwald et al. (2012)	Corporate Stock
Zeros	Lesmond et al. (1999)	$Zeros_t = \frac{\# \text{ of days with } ZR}{T}$	Da	Transaction prices	Il	Tightness and depth	Bekaert et al. (2007); Frielwald et al. (2012); El Kalak et al. (2017)	Corporate Stock
Zeros2	Lesmond et al. (1999)	$Zeros2_t = \frac{\# \text{ of positive volume days with } ZR}{T}$	Da	Transaction prices and volume	Il	Tightness and depth	Goyenko et al. (2009); Fong et al. (2017)	Stock
Holden	Holden (2009)	$S_H = \sum_{j=1}^J \hat{p}_j s_j$	Da	Transaction prices and number of trades	Il	Tightness	Goyenko et al. (2009)	Stock
Effective tick	Holden (2009) and Goyenko et al. (2009)	Effective Tick = $> \sum_{j=1}^J \hat{p}_j s_j / \hat{p}_j$	Da	Transaction prices and quote prices	Il	Tightness	Fong et al. (2017); Abdi and Rinaldo (2017)	Corporate Stock
Gibbs	Hasbrouck (2009)	$\eta_t = c \Delta D_t + \beta_m R_{m,t} + \varepsilon_t$	In/Da	Transaction prices, market index and trade's sign	Il	Tightness	Goyenko et al. (2009); Schestag et al. (2016)	Corporate Stock
High-Low ratio	Corwin and Schultz (2012)	$S = \frac{2(e^\alpha - 1)}{1 + e^\alpha}$ ; $\alpha = \frac{\sqrt{3} - \sqrt{2}}{3 - 2\sqrt{2}} \sqrt{\frac{\gamma}{3 - 2\sqrt{2}}}$	Da	Transaction prices	Il	Tightness	Schestag et al. (2016)	Corporate Stock
FHT	Fong et al. (2017)	$S = 2\alpha\Phi^{-1} \frac{1 + \frac{\# \text{ of days with } ZR}{T}}{2}$	Da	Transaction prices	Il	Tightness and depth	Schestag et al. (2016)	Corporate tock
<i>Panel B. Price-based proxies</i>								
Quote dispersion	Garbade and Silber (1976)	$QD_t = 2 \sqrt{\frac{1}{2N} \sum_{i=1}^N \left( \left( \frac{B_i - m}{m} \right)^2 + \left( \frac{A_i - m}{m} \right)^2 \right)}$	Da	Quote prices	Il	Resilience and breadth	Schestag et al. (2016)	Treasury Corporate
Marsh and Rock ratio	Marsh and Rock (1986)	$LR = \frac{1}{M} \sum_{m=1}^M \left  \frac{p_m - p_{m-1}}{p_{m-1}} \right  \cdot 100$	Da/In	Transaction prices and number of transactions	Li	Resilience and breadth	Pop et al. (2014)	Stock
MEC	Hasbrouck and Schwartz (1988)	$MEC_t = \frac{Var(R^L_t)}{T_p Var(r^L_t)}$	Da	Transaction prices	Il	Resilience	Porter and Weaver (1997); Broto and Lamas (2016); Vortelinos et al. (2018)	Corporate Stock Treasury
Pastor and Stambaugh	Pastor and Stambaugh (2003)	$r^L_{i,t+1} = \theta + \varphi r_{i,t} + \chi_{i,t} sign(r^L_{i,t}) v_{i,t} + \varepsilon_{i,t}$	In	Transaction prices and index prices	Il	Breadth	Goyenko et al. (2009); Lin et al. (2011); Hu et al. (2013)	Corporate Stock
Latent liquidity	Mahanti et al. (2008)	$L^L_t = \sum_{j=1}^J \pi^L_{j,t} T_{j,t}$	Da	Transaction prices and bondholder information	Il	Breadth and tightness	Nashikkar et al. (2011)	Corporate

(continued on next page)

Table 2 (continued)

Liquidity measure	Original author	Expression/mode	Data frequency	Data requirements	Type	Facet	Other references	Market
Closing Percent Quoted Spread	Chung and Zhang (2014)	$CPOQS_t = \frac{1}{N} \left( \frac{ClosingAsk_t - ClosingBid_t}{(ClosingAsk_t + ClosingBid_t)/2} \right)$	Da	Closing Bid and Ask prices	II	Tightness	Schstag et al. (2016); Fong et al. (2017); Abdi and Rinaldo (2017)	Stock Corporate
Spread	Abdi and Rinaldo (2017)	$Spread_t = 2 \sqrt{E[(\epsilon_t - \eta_t)(\epsilon_t - \eta_{t+1})]}$	Da	Closing, high and low prices	II	Tightness	Fong et al. (2018)	Stock
Panel C. Volume-based proxies								
Martin index	Martin (1975)	$M_t = \sum_{i=1}^N \frac{(p_{i,t} - p_{i,t-1})^2}{TV_{i,t}}$	Da	Transaction prices and volumes	Li	Breadth and depth	Pop et al. (2014); Vortelinos et al. (2018)	Stock
Hui-Heubel ratio	Hui and Heubel (1984)	$LR_t = \left( \frac{P_{max} - P_{min}}{P_{min}} \right) \left( \frac{TV_t}{r_t P_t} \right)$	Da	Transaction prices and volumes	II	Breadth and depth	Broto and Lamas (2016); Vortelinos et al. (2018)	Corporate Stock
Kyle's lambda	Kyle (1985)	$p_t = p_{t-1} + \lambda_t (u_t + x_t) + u_t$	In	Transactions prices, volume and sign of the trade	II	Breadth and depth	Amihud et al. (2005); Flood et al. (2015)	Corporat Stock
Trading volume	Kamara (1994)	$TV_t = \sum_{k=1}^n TV_{t,k}$	In/Da	Volume	Li	Depth	Elton and Green (1998); Alexander et al. (2000); Chordia et al. (2001)	Corporate Stock Treasury
Amihud	Amihud (2002)	$AH_t = \frac{1}{K} \sum_{k=1}^{D_t} \frac{r_{t,k}  I }{p_{t,k} \cdot TV_{t,k}}$	In/Da	Transaction prices and volumes	II	Breadth and depth	Mahanti et al. (2008); Dick-Nielsen et al. (2012); Han and Zhou (2016)	Corporate Stock Treasury
Amivest	Cooper et al. (1985)	$AV_t = \frac{1}{K} \sum_{k=1}^{D_t} \frac{p_{t,k} \cdot TV_{t,k}}{r_{t,k}  I }$	In/Da	Transaction prices and volumes	Li	Breadth and depth	Amihud et al. (1997); Custódio et al. (2011); Díaz and Escribano (2017)	Corporate Stock Treasury
Turnover	Gravelle (1999)	$TN_t = TV_t / AmOut$	In/Da	Volume and amount	Li	Depth	Streblaeve (2002); Downing et al. (2005); Sadka (2006)	Corporate Stock Treasury
Turnover price impact ratio	Florackis et al. (2011)	$RtoTR_t = \frac{1}{K} \sum_{k=1}^{D_t} \frac{r_{t,k}  I }{TN_{t,k}}$	Da	Transaction prices, volume and amount outstanding	II	Breadth and depth	Fernández-Amador et al. (2013); El Kalak et al. (2017); Ellington (2018)	Stock
Turnover-adjusted zero daily volumes	Liu (2006)	$LM_k = \left[ \frac{1}{NZVX_t \cdot \frac{x - \text{month turnover}}{\text{Deflator}}} \right] \frac{21x}{NoTD}$	Da	Trading volume	Li	Depth, immediacy and tightness	Chai et al. (2010); Vortelinos et al. (2018)	Stock
Market share	Díaz et al. (2006)	$MS_t = TV_t / TTV_t$	In/Da	Trade and market volumes	Li	Depth	Díaz and Escribano (2017)	Corporate Treasury
Panel D. Trading frequency proxies								
Days traded	Nashikkar et al. (2011)	$Days_s = \sum_{s=1}^n d_s$	Da	Trading days	Li	Depth	Hou and Moskowitz (2005); Pu (2009)	Corporate Stock
Bond zero-trading days	Dick-Nielsen et al. (2012)	$\text{BondZero}_s = (db/D) \times 100$	Da	Trading days	II	Depth	Bao and Pan (2013); Helwege et al. (2014)	Corporate
Firm zero-trading days	Dick-Nielsen et al. (2012)	$\text{FirmZero}_s = (df/D) \times 100$	Da	Trading days	II	Depth	Dionne and Chun (2013)	Corporate

the *Trading volume* is the most used trading frequency proxy to assess liquidity.<sup>21</sup> Although it does not include the cost of trading, some papers find evidence of higher trading volumes linked to lower bid-ask spreads (e.g., Foster and Viswanathan, 1990; Admati and Pfleiderer, 1988).

Finally, on Panel D of Table 2 we include those proxies that measures the trading frequency. *Days traded* counts for the number of days that an asset is traded over a period. Opposite, *Bond Zero-Trading Days* and *Firm Zero-Trading Days* calculate the percentage of days during a period of time in which a bond, or the bonds of a firm, present zero-trading days.

#### 4. Other liquidity proxies

In this section, we summarize liquidity proxies that are independent of the trading activity and that have been widely used in the traditional literature.

On the one hand, Fisher (1959) propose *Issue size*, *Issued amount*, and *Amount outstanding* as proxies of liquidity. They are based on the idea that large issues are usually the most frequently traded and, hence, the most liquid.<sup>22</sup> The *Age* of an asset is another widely used proxy for liquidity. It can be computed either in absolute term, i.e., expressed as years since issuance, or in relative term, expressed as a percentage of the original term maturity.<sup>23</sup> The *Term to maturity* of a bond is also another liquidity proxy. The preference of investors for bonds with shorter maturities make them more demanded than those with a longer term to maturity. Thus, the longer the term to maturity is, the lower the liquidity.<sup>24</sup>

Additionally, there are special features that can be used as proxies of liquidity, as for example, the *Issue status*, the *Specialness*, the *Number of contributors* and the *Order imbalance*. The *Issue status* is a feature of fixed income assets. The on-the-run property addresses the most recently issued bonds within an original term to maturity. The previous literature has provided wide evidence of the high liquidity level of bonds embedded in the on-the-run category.<sup>25</sup> The *Specialness* feature refers to the cost of shorting individual bonds. It is also a feature of fixed income assets, reflected by the existing spread between on-the-run and off-the-run bonds (see Duffie, 1996). The term *Specialness* also refers to the fact that on-the-run bonds sometimes present price advantages in the special repo market (see, e.g., Jordan and Jordan, 1997; Krishnamurthy, 2002). In fact, a bond is *on special* when it is possible to find its repo rate quote under the repo rate of other similar bonds.<sup>26</sup> The *Number of contributors* indicate the number of potential counterparties that would allow investors to unwind positions. A large number of contributors would hence indicate that investors can unwind positions easily and without incurring in large transaction costs.<sup>27</sup> The *Order imbalance* can be a signal of private information that could affect market prices and, hence, reduce market liquidity. The excess of buying or selling drives price movements, leading to drops in liquidity (see, e.g., Chordia et al., 2002).<sup>28</sup>

#### 5. Discussion

The liquidity of an asset or a financial market is a concept that encompasses different facets or aspects and is difficult to measure with a single measure. The literature emphasizes that there are five different aspects that characterize liquidity, namely, breadth, depth, immediacy, resilience, and tightness. Some authors point out the pitfalls of using one single measure or proxy when it comes to measuring liquidity (Bernstein, 1987). There are multiple measures proposed from high-frequency data, called liquidity measures, and others that only require low-frequency data, called liquidity proxies. There is no consensus on which measure is best suited to measure the liquidity of an asset or market, or even to decide which is best when it comes to showing one of the facets of liquidity. In fact, new liquidity measures are appearing in recent papers.

In any case, liquidity is a crucial issue for researchers, practitioners and policymakers. For researchers, liquidity measures are a topic of study or simply a necessary tool for financial asset valuation proposals. The choice between different liquidity measures is often made arbitrarily. It is difficult to propose a measure among the existing ones as the most appropriate for any proposal globally. Certainly, the selection of the liquidity measures depends on the research question. Existing papers obtain different results depending on the asset analyzed, the database available and even the frequency of observations. For example, recent empirical evidence shows that there are many liquidity proxies that are able to measure liquidity as accurately as liquidity measures in the US corporate bond

<sup>21</sup> Authors that use this proxy are, for instance, Chakravarty and Sarkar (1999); Strebulaev (2002) or Goldreich et al. (2005) in the Treasury market; those of Alexander et al. (2000); Fleming (2003); Chordia et al. (2004); Sadka (2004); Goldstein et al. (2007); Nashikkar et al. (2011) or Jankowitsch et al. (2014), in the corporate bond market; or the papers of Chordia et al. (2001) or Sadka (2006) in the stock market.

<sup>22</sup> A number of papers have used these measures, such as Wargha (1992); Crabbe and Turner (1995); Kempf and Uhrig-Homburg (2000); Houweling et al. (2005); Longstaff et al. (2005), and Bao and Pan (2013). Meanwhile, Jankowitsch et al. (2006) and Nashikkar et al. (2011) apply the log of the issue size in their respective papers.

<sup>23</sup> For instance, Wargha (1992); Chakravarty and Sarkar (1999); Houweling et al. (2005); Longstaff et al. (2005); Ericsson and Renault (2006), and Bao and Pan (2013) use the age to proxy for liquidity.

<sup>24</sup> Sarig and Wargha (1989); Díaz and Navarro (2002) and Longstaff et al. (2005) also use this measure.

<sup>25</sup> See, e.g., Amihud and Mendelson (1986); Kamara (1994); Krishnamurthy (2002); Strebulaev (2002); Fleming (2003); Amihud and Mendelson (2006); Jankowitsch et al. (2006); Goldreich et al. (2005), and Pasquariello and Vega (2009).

<sup>26</sup> Other papers using this measure are Longstaff et al. (2005) and Jankowitsch et al. (2006), among others.

<sup>27</sup> Houweling et al. (2005) and Jankowitsch et al. (2006) also use this measure in their respective analysis.

<sup>28</sup> Chordia and Subrahmanyam (2004), and Chordia et al. (2004) also use this proxy of liquidity. Besides, Kaul et al. (2008) and Holden and Jacobsen (2014) use the absolute order imbalance in their corresponding analysis.

market (Schestag et al., 2016) and in the US stock market (Fong et al., 2017). Therefore, the lack of high-frequency data is not a drawback in order to assess the degree of liquidity. These results are of interest to researchers, in the sense that they can avoid intensive data processing and cleansing procedures that many large databases require for being processing.

As for practitioners, they need measures that are simple and fast to compute with the information available. We outline a complete classification of liquidity measures according to the specific liquidity characteristic to be measured and the frequency of the data. In this sense, Schestag et al. (2016) provide some guidance in order to choose the appropriate liquidity measure. They stress that the correct measurement of transaction costs is the most relevant aspect for trading strategies, portfolio allocation, intermediaries and investors. Daily data are enough for a correct measurement of these transaction costs. Impact measures “should be used with great caution”. Finally, in markets without trade reports, they recommend using supply and demand spreads derived from executable quotes.

Regarding policymakers, it should be noted that the Financial Stability Board (FSB), the Basel Committee on Banking Supervision (BCBS), the International Organization of Securities Commissions (IOSCO), 2013 and other international standard-setters have adopted measures to require minimum liquidity requirements and establish rules for their proper valuation.<sup>29</sup> In this regard, the investment policy statements (IPS) used in delegated private management usually include minimum liquidity standards. Both the measures taken by international regulators and those appearing in IPS generally classify financial assets held in the portfolio into two or three categories. For example, Basel III establishes different discounts when calculating the liquidity coverage ratio (LCR) depending on the asset category.<sup>30</sup> Among other implications, all investment-grade corporate bonds are included in the same category. Many IPS set a minimum credit rating threshold for assets held in the portfolio to ensure adequate liquidity and credit risk. These regulations probably impose excessive granularity. As Díaz and Escribano (2019) show, there are differences in all dimensions of liquidity between credit rating categories, although there is no simple progressive behavior of liquidity by credit rating. Therefore, the policymaker could allow for greater granularity in setting liquidity standard requirements based on some easily computable liquidity proxy. This would move from a naive classification between two or three large asset blocks to a continuous and smooth application of requirements according to the values of an effective liquidity measure.

## 6. Conclusions

This paper provides a detailed overview of the different dimensions of liquidity, namely, breadth, depth, immediacy, resilience and tightness, as well as the existing liquidity measures and proxies that are able to empirically capture each of them, jointly with the statistical procedures and requirements of each measure. We also include a compilation of papers that have used each one and the context or market in which they have been used. Some of these papers aim to answer the question of what is the best measure for measuring liquidity. Although the literature agrees on the existence of different dimensions of liquidity, it seems that there is no consensus with regard to the suitability of using each measure to capture each dimension. The measure depends on the specific dimension that one wants to address and on the characteristics of the market studied, i.e., depends on the research question.

More generally, our study highlights that given that liquidity is a multi-faceted phenomenon and that financial markets are quite heterogeneous, there is not a universal measure able to compute liquidity. In fact, this multi-faceted view serves as an indicator of the need that liquidity should be measured across the different dimensions simultaneously in order to have a global view of liquidity. This rigorous literature review provides guidance for researchers, practitioners and policymakers interested in measuring liquidity.

## References

- Abdi, F., Rinaldo, A., 2017. A simple estimation of bid-ask spreads from daily close, high, and low prices. *Rev. Financ. Stud.* 30 (12), 4437–4480.
- Acharya, V.V., Pedersen, L.H., 2005. Asset pricing with liquidity risk. *J. financ. econ.* 77 (2), 375–410.
- Admati, A.R., Pfleiderer, P., 1988. A theory of intraday patterns: volume and price variability. *Rev. Financ. Stud.* 1, 3–40.
- Ahn, H.J., Cai, J., Yang, C.W., 2018. Which liquidity proxy measures liquidity best in emerging markets? *Economies* 6 (4), 67.
- Alexander, G.J., Edwards, A.K., Ferri, M.G., 2000. The determinants of trading volume of high-yield corporate bonds. *J. Financ. Mark.* 3 (2), 177–204.
- Amihud, Y., 2002. Illiquidity and stock returns: cross-section and time-series effects. *J. Financ. Mark.* 5 (1), 31–56.
- Amihud, Y., Mendelson, H., 1986. Asset pricing and the bid-ask spread. *J. financ. econ.* 17 (2), 223–249.
- Amihud, Y., Mendelson, H., 2006. Stock and bond liquidity and its effect on prices and financial policies. *Financ. Mark. Portf. Manag.* 20 (1), 19–32.
- Amihud, Y., Mendelson, H., Lauterbach, B., 1997. Market microstructure and securities values: evidence from the Tel Aviv stock exchange. *J. financ. econ.* 45, 365–390.
- Amihud, Y., Mendelson, H., Pedersen, L.H., 2005. Illiquidity and stock returns: liquidity and asset pricing. *Foundation and Trends in Finance* 1, 269–364.
- Amihud, Y., Mendelson, H., Pedersen, L.H., 2006. Liquidity and asset prices. *Found. Trends® Financ.* 1 (4), 269–364.
- Anderson, S., Lavoie, S., 2004. The evolution of liquidity in the market for government of Canada bonds. *Bank of Canada Review* 19–31.
- Bao, J., Pan, J., 2013. Bond illiquidity and excess volatility. *Rev. Financ. Stud.* 26 (12), 3068–3103.

<sup>29</sup> Notable examples are the FSB policy recommendations to address structural vulnerabilities arising from asset management activities, relating in particular to the transformation of liquidity by investment funds (Financial Stability Board – FSB, 2017), the BCBS reforms to the international framework for measuring liquidity risk (Basel Committee on Banking Supervision, 2013) and to the minimum capital requirement for market risk reform (Basel Committee on Banking Supervision, 2019), which incorporate both market risk and liquidity risk aspects, and the recommendations of the International Organization of Securities Commissions (IOSCO, 2013, International Organization of Securities Commissions (IOSCO), 2018) include the obligation to hold a minimum amount of liquid assets and rules on valuation techniques.

<sup>30</sup> Basel III classifies assets at level 1 (e.g., government-guaranteed securities), at level 2A (e.g., securities issued or guaranteed by specific multilateral development banks or sovereign entities), at level 2B (e.g., publicly-traded common stock and investment-grade corporate debt securities).



- Bao, J., Pan, J., Wang, J., 2011. The illiquidity of corporate bonds. *J. Finance* 66 (3), 911–946.
- Basel Committee on Banking Supervision (BCBS), 2013. Basel III: the Liquidity Coverage Ratio and Liquidity Risk Monitoring Tools. January. Bank for International Settlements.
- Basel Committee on Banking Supervision (BCBS), 2019. Minimum Capital Requirements for Market Risk. January 2019 (rev. February 2019). Bank for International Settlements.
- Beber, A., Brandt, M.W., Kavajecz, K.A., 2008. Flight-to-quality or flight-to-liquidity? Evidence from the euro-area bond market. *Rev. Financ. Stud.* 22 (3), 925–957.
- Będowska-Sójka, B., Echaust, K., 2019. Commonality in liquidity indices: the emerging european stock markets. *Systems* 7 (2), 24.
- Bekaert, G., Harvey, C.R., Lundblad, C., 2007. Liquidity and expected returns: lessons from emerging markets. *Rev. Financ. Stud.* 20 (6), 1783–1831.
- Berkman, H., Eleswarapu, V.R., 1998. Short-term traders and liquidity: a test using Bombay stock exchange data. *J. financ. econ.* 47, 339–355.
- Bernstein, P.L., 1987. Liquidity, stock markets, and market makers. *Financ. Manage.* 54–62.
- Bervas, A., 2006. Market liquidity and its incorporation into risk management. *Financial Stability Review* 8 (May), 63–79.
- Bessembinder, H., Venkataraman, K., 2010. Bid-ask spreads: measuring trade execution costs in financial markets. *Encyclopedia of Quantitative Finance*. pp. 184–190.
- Bhattacharya, S.N., Bhattacharya, M., Basu, S., 2019. Stock market and its liquidity: evidence from ARDL bound testing approach in the Indian context. *Cogent Econ. Financ.* 7 (19), 1–12.
- Biais, B., Foucault, T., Hillion, P., 1997. Microstructure Des Marchés Financiers: Institutions, Modèles Et Tests Empiriques. Presses universitaires de France, Paris.
- Black, F., 1971. Towards a fully automated exchange, part I. *Financ. Anal. J.* 27, 29–34.
- Black, J.R., Stock, D., Yadav, P.K., 2016. The pricing of different dimensions of liquidity: evidence from government guaranteed bonds. *J. Bank. Financ.* 71, 119–132.
- Boehmer, E., Jennings, R., Wei, L., 2007. Public disclosure and private decisions: equity market execution quality and order routing. *Rev. Financ. Stud.* 20 (2), 315–358.
- Brockman, P., Chung, D.Y., Perignon, C., 2009. Commonality in liquidity: a global perspective. *J. Financ. Quant. Anal.* 44 (4), 851–882.
- Broto, C., Lamas, M., 2016. Measuring market liquidity in US fixed income markets: a new synthetic indicator. *Spanish Rev. Financ. Econ.* 14 (1), 15–22.
- Chai, D., Faff, R., Gharghori, P., 2010. New evidence on the relation between stock liquidity and measures of trading activity. *Int. Rev. Financ. Anal.* 19 (3), 181–192.
- Chakravarty, S., Sarkar, A., 1999. Liquidity in US Fixed Income Markets: a Comparison of the Bid-ask Spread in Corporate, Government and Municipal Bond Markets. Working Paper. Purdue University and Federal Reserve Bank of New York.
- Chakravarty, S., Sarkar, A., 2003. Trading costs in three US bond markets. *J. Fixed Income* 13 (1), 39–48.
- Chen, L., Lesmond, D., Wei, J., 2007. Corporate yield spreads and bond liquidity. *J. Finance* 62, 119–149.
- Chordia, T., Subrahmanyam, A., 2004. Order imbalance and individual stock returns: theory and evidence. *J. financ. econ.* 72 (3), 485–518.
- Chordia, T., Roll, R., Subrahmanyam, A., 2000. Commonality in liquidity. *J. financ. econ.* 56, 3–28.
- Chordia, T., Roll, R., Subrahmanyam, A., 2001. Market liquidity and trading activity. *J. Finance* 56 (2), 501–530.
- Chordia, T., Roll, R., Subrahmanyam, A., 2002. Order imbalance, liquidity, and market returns. *J. financ. econ.* 65 (1), 111–130.
- Chordia, T., Sarkar, A., Subrahmanyam, A., 2004. An empirical analysis of stock and bond market liquidity. *Rev. Financ. Stud.* 18 (1), 85–129.
- Chung, K.H., Zhang, H., 2014. A simple approximation of intraday spreads using daily data. *J. Financ. Mark.* 17, 94–120.
- Cooper, S., Groth, K., Avera, W., 1985. Liquidity, exchange listing and common stock performance. *J. Econ. Bus.* 37, 19–33.
- Corwin, S.A., Schultz, P., 2012. A simple way to estimate bid-ask spreads from daily high and low prices. *J. Finance* 67 (2), 719–760.
- Crabbe, L.E., Turner, C.M., 1995. Does the liquidity of a debt issue increase with its size? Evidence from the corporate bond and medium-term note markets. *J. Finance* 50 (5), 1719–1734.
- Custódio, C., Ferreira, M.A., Laureano, L., 2011. The Declining Corporate Debt Maturity: the Impact of Asymmetric Information and New Listings. Working Paper. Demsetz, H., 1968. The cost of transacting. *Q. J. Econ.* 82, 33–53.
- Díaz, A., Escribano, A., 2019. Credit rating and liquidity in the US corporate bond market. *J. Fixed Income* 28, 46–59.
- Díaz, A., Escribano, A., 2017. Liquidity measures throughout the lifetime of the US Treasury bond. *J. Financ. Mark.* 33, 42–74.
- Díaz, A., Navarro, E., 2002. La prima de liquidez en la Deuda del Estado. *Revista de Economía Aplicada* 10 (29).
- Díaz, A., Merrick, J.J., Navarro, E., 2006. Spanish Treasury bond market liquidity and volatility pre- and post-European Monetary Union. *J. Bank. Financ.* 30, 1309–1332.
- Dick-Nielsen, J., Feldhütter, P., Lando, D., 2012. Corporate bond liquidity before and after the onset of the subprime crisis. *J. financ. econ.* 103 (3), 471–492.
- Dionne, G., Chun, O.M., 2013. Default and liquidity regimes in the bond market during the 2002–2012 period. *Canadian Journal of Economics/Revue canadienne d'économique* 46 (4), 1160–1195.
- Downing, C., Underwood, S., Xing, Y., 2005. Is Liquidity Risk Priced in the Corporate Bond Market?, Working Paper. Rice University.
- Duffie, D., 1996. Special repo rates. *J. Finance* 51 (2), 493–526.
- Edwards, A.K., Harris, L.E., Piwowar, M.S., 2007. Corporate bond market transaction costs and transparency. *J. Finance* 62 (3), 1421–1451.
- El Kalak, I., Azevedo, A., Hudson, R., Karim, M.A., 2017. Stock liquidity and SMEs' likelihood of bankruptcy: evidence from the US market. *Res. Int. Bus. Financ.* 42, 1383–1393.
- Ellington, M., 2018. Financial market illiquidity shocks and macroeconomic dynamics: evidence from the UK. *J. Bank. Financ.* 89, 225–236.
- Elton, E.J., Green, T.C., 1998. Tax and liquidity effects in pricing government bonds. *J. Finance* 53 (5), 1533–1562.
- Engle, R.F., Lange, J., 1997. Measuring, Forecasting and Explaining Time Varying Liquidity in the Stock Market (No. w6129). National Bureau of Economic Research.
- Ericsson, J., Renault, O., 2006. Liquidity and credit risk. *J. Finance* 61 (5), 2219–2250.
- Feldhütter, P., 2012. The same bond at different prices: identifying search frictions and selling pressures. *Rev. Financ. Stud.* 25, 1155–1206.
- Fernández-Amador, O., Gächter, M., Larch, M., Peter, G., 2013. Does monetary policy determine stock market liquidity? New evidence from the euro zone. *J. Empir. Finance* 21, 54–68.
- Financial Stability Board – (FSB), 2017. Policy Recommendations to Address Structural Vulnerabilities From Asset Management Activities. January. .
- Fisher, L., 1959. Determinants of risk premiums on corporate bonds. *J. Polit. Econ.* 67 (3), 217–237.
- Fleming, M., 2003. Measuring treasury market liquidity. *Economic Policy Review* 9, 83–108.
- Flood, M.D., Liechty, J.C., Piontek, T., 2015. Systemwide Commonalities in Market Liquidity. Working Paper. <http://ssrn.com/abstract=2612348>.
- Florackis, C., Gregoriou, A., Kostakis, A., 2011. Trading frequency and asset pricing on the London Stock Exchange: evidence from a new price impact ratio. *J. Bank. Financ.* 35 (12), 3335–3350.
- Fong, K.Y., Holden, C.W., Tobek, O., 2018. Are Volatility Over Volume Liquidityproxies Useful for Global or US Research? Working Paper. Available at SSRN: <https://ssrn.com/abstract=2989367>.
- Fong, K.Y., Holden, C.W., Trzcinka, C.A., 2017. What are the best liquidity proxies for global research? *Rev. Financ.* 21 (4), 1355–1401.
- Foran, J., Hutchinson, M.C., O'Sullivan, N., 2014. The asset pricing effects of UK market liquidity shocks: evidence from tick data. *Int. Rev. Financ. Anal.* 32, 85–94.
- Foran, J., Hutchinson, M.C., O'Sullivan, N., 2015. Liquidity commonality and pricing in U.K. Equities. *Res. Int. Bus. Financ.* 34, 281–293.
- Foster, F.D., Viswanathan, S., 1990. A theory of the intraday variations in volume, variance, and trading costs in securities markets. *Rev. Financ. Stud.* 3, 593–624.
- Fraser, D.R., Groth, J.C., Byers, S.S., 1997. Listing and the liquidity of bank stocks: revisited. *Appl. Financ. Econ.* 7 (2), 165–172.
- Friedwald, N., Jankowitsch, R., Subrahmanyam, M., 2012. Illiquidity or credit deterioration: a study of liquidity in the US corporate bond market during financial crises. *J. Financ. Econ.* 105 (1), 18–36.
- Garbade, K., 1982. Securities Markets. McGraw-Hill, New York.
- Garbade, K.D., Silber, W.L., 1976. Price dispersion in the government securities market. *J. Polit. Econ.* 84 (4, Part 1), 721–740.
- Goldreich, D., Hanke, B., Nath, P., 2005. The price of future liquidity: time-varying liquidity in the US Treasury market. *Rev. Financ.* 9, 1–32.
- Goldstein, M.A., Hotchkiss, E.S., Sirri, E.R., 2007. Transparency and liquidity: a controlled experiment on corporate bonds. *Rev. Financ. Stud.* 20 (2), 235–273.
- Goyenko, R., Holden, C., Trzcinka, C., 2009. Do liquidity measures measure liquidity? *J. financ. econ.* 92, 153–181.
- Gravelle, T., 1999. Liquidity of the government of Canada securities market: stylized facts and some market microstructure comparisons to the United States treasury

- market. Bank of Canada. Working Paper. 11–99.
- Grossman, S.J., Miller, M.H., 1988. Liquidity and market structure. *J. Finance* 43 (3), 617–633.
- Han, S., Zhou, H., 2008. Effects of Liquidity on the Non-default Component of Corporate Yield Spreads: Evidence From Intraday Transactions Data. Working Paper.
- Han, S., Zhou, H., 2016. Effects of bond liquidity on the non-default component of corporate yield spreads: evidence from intraday transactions data. *Q. J. Finance* 6 (03), 1650012.
- Harris, L.E., 1990. Liquidity, trading rules and electronic trading systems. NYU Salomon Center Monograph Series in Finance and Economics, 4, (No. 91-8), 1990-1994.
- Hasbrouck, J., 2009. Trading costs and returns for US equities: estimating effective costs from daily data. *J. Finance* 64, 1445–1477.
- Hasbrouck, J., Schwartz, R.A., 1988. Liquidity and execution costs in equity markets. *J. Portf. Manage.* 14 (3), 10–16.
- Hasbrouck, J., Seppi, D.J., 2001. Common factors in prices, order flows and liquidity. *J. financ. econ.* 59, 383–411.
- Helwege, J., Huang, J.Z., Wang, Y., 2014. Liquidity effects in corporate bond spreads. *J. Bank. Financ.* 45, 105–116.
- Hibbert, J., Kirchner, A., Kretschmar, G., Li, R., McNeil, A., 2009. Liquidity premium: literature review of theoretical and empirical evidence. Barrie & Hibbert Research Report.
- Hilal Anwar, B., Hogholm, K., 2018. The impact of illiquidity risk for the Nordic markets. *Spanish Journal of Finance and Accounting/Revista Española de Financiación y Contabilidad* 1–20.
- Holden, C.W., 1990. Intertemporal Arbitrage Trading: Theory and Empirical Tests, Discussion Paper. Indiana University.
- Holden, C.W., 2009. Low low-frequency spread measures. *J. Finance* 64, 778–813.
- Holden, C.W., Jacobsen, S., 2014. Liquidity measurement problems in fast, competitive markets: expensive and cheap solutions. *J. Finance* 69 (4), 1747–1785.
- Hong, G., Warga, A., 2000. An empirical study of bond market transactions. *Financ. Anal. J.* 56 (2), 32–46.
- Hou, K., Moskowitz, T.J., 2005. Market frictions, price delay, and the cross-section of expected returns. *Rev. Financ. Stud.* 18 (3), 981–1020.
- Houweling, P., Mentink, A., Vorst, T., 2005. Comparing possible proxies of corporate bond liquidity. *J. Bank. Financ.* 29, 1331–1358.
- Hu, G.X., Pan, J., Wang, J., 2013. Noise as information for illiquidity. *J. Finance* 68 (6), 2341–2382.
- Huang, R.D., Stoll, H.R., 1996. Dealer versus auction markets: a paired comparison of execution costs on NASDAQ and the NYSE. *J. financ. econ.* 41 (3), 313–357.
- Huberman, G., Halka, D., 1999. Systematic Liquidity. Columbia Business School.
- Hui, B., Heubel, B., 1984. Comparative Liquidity Advantages Among Major US Stock Markets. Data Resources, Inc., Financial Information Group Study Series, 84081, Lexington, MA.
- International Organization of Securities Commissions (IOSCO), 2013. Principles of Liquidity Risk Management for Collective Investment Schemes - Final Report, No. FR03/13, 2013.
- International Organization of Securities Commissions (IOSCO), 2018. Recommendations for Liquidity Risk Management for Collective Investment Schemes – Final Report, No. FR01/2018.
- Jankowitsch, R., Mösenbacher, H., Pichler, S., 2006. Measuring the liquidity impact on EMU government bond prices. *Eur. J. Finance* 12 (2), 153–169.
- Jankowitsch, R., Nagler, F., Subrahmanyam, M.G., 2014. The determinants of recovery rates in the US corporate bond market. *J. financ. econ.* 114 (1), 155–177.
- Jankowitsch, R., Nashikkar, A., Subrahmanyam, M.G., 2011. Price dispersion in OTC markets: a new measure of liquidity. *J. Bank. Financ.* 35 (2), 343–357.
- Jordan, B.D., Jordan, S.D., 1997. Special repo rates: an empirical analysis. *J. Finance* 52 (5), 2051–2072.
- Kamara, A., 1994. Liquidity, taxes, and short-term treasury yields. *J. Financ. Quant. Anal.* 29 (3), 403–417.
- Kang, W., Zhang, H., 2014. Measuring liquidity in emerging markets. *Pacific-basin Financ. J.* 27, 49–71.
- Karnaukh, N., Ranaldo, A., Söderlind, P., 2015. Understanding FX liquidity. *Rev. Financ. Stud.* 28 (11), 3073–3108.
- Karolyi, G.A., Lee, K.-H., van Dijk, M.A., 2012. Understanding commonality in liquidity around the world. *J. financ. econ.* 105 (1), 82–112.
- Kaul, G., Lei, Q., Stoffman, N., 2008. Aiming at Pin: Order Flow, Information, and Liquidity. Information, and Liquidity. Working Paper. Available at: <https://doi.org/10.2139/ssrn.872945>.
- Kempf, A., Uhrig-Homburg, M., 2000. Liquidity and its impact on bond prices. *Schmalenbach Business Review: ZFBF* 52 (1), 26.
- Kluger, B.D., Stephan, J., 1997. Alternative liquidity measures and stock returns. *Rev. Quant. Finance* 8, 19–36.
- Korajczyk, R.A., Sadka, R., 2008. Pricing the commonality across alternative measures of liquidity. *J. financ. econ.* 87 (1), 45–72.
- Krishnamurthy, A., 2002. The bond/old-bond spread. *J. financ. econ.* 66 (2), 463–506.
- Kyle, A.S., 1985. Continuous auctions and insider trading. *Econometrica: Journal of the Econometric Society* 1315–1335.
- Lang, M., Lins, K.V., Maffett, M., 2012. Transparency, liquidity, and valuation: international evidence on when transparency matters most. *J. Account. Res.* 50 (3), 729–774.
- Langedijk, S., Monokroussos, G., Papanagiotou, E., 2018. Benchmarking liquidity proxies: the case of EU sovereign bonds. *Int. Rev. Econ. Financ.* 56, 321–329.
- Lee, Charles M.C., 1993. Market integration and price execution for NYSE-listed securities. *J. Finance* 48, 1009–1038.
- Lee, K.H., 2011. The world price of liquidity risk. *J. financ. econ.* 99 (1), 136–161.
- Lesmond, D.A., 2005. Liquidity of emerging markets. *J. financ. econ.* 77 (2), 411–452.
- Lesmond, D.A., Ogden, J.P., Trzinka, C.A., 1999. A new estimate of transaction costs. *Rev. Financ. Stud.* 12 (5), 1113–1141.
- Li, H., Wang, J., Wu, C., He, Y., 2009. Are liquidity and information risks priced in the treasury bond market? *J. Finance* 64 (1), 467–503.
- Lin, H., Wang, J., Wu, C., 2011. Liquidity risk and expected corporate bond returns. *J. financ. econ.* 99, 628–650.
- Lippman, S.A., McCall, J.J., 1986. An operational measure of liquidity. *Am Econ Rev* 76 (1), 43–55.
- Liu, W., 2004. A liquidity-augmented capital asset pricing model. *J. financ. econ.* 82 (3), 631–671.
- Longstaff, F.A., Mithal, S., Neis, E., 2005. Corporate yield spreads: default risk or liquidity? New evidence from the credit default swap market. *J. Finance* 60 (5), 2213–2253.
- Mahanti, S., Nashikkar, A., Subrahmanyam, M., Chacko, G., Mallick, G., 2008. Latent liquidity: a new measure of liquidity, with an application to corporate bonds. *J. financ. econ.* 88, 272–298.
- Martin, P., 1975. Analysis of the Impact of Competitive Rates on the Liquidity of NYSE Stocks. *Economic Staff Paper* 75 (3).
- Mancini, L., Ranaldo, A., Wrampelmeyer, J., 2013. Liquidity in the foreign exchange market: measurement, commonality, and risk premiums. *J. Finance* 68 (5), 1805–1841.
- Marsh, T.A., Rock, K., 1986. Exchange Listing and Liquidity: a Comparison of the American Stock Exchange With the NASDAQ National Market System. American Stock Exchange.
- Marshall, B.R., Nguyen, N.H., Visaltanachoti, N., 2012. Commodity liquidity measurement and transaction costs. *Rev. Financ. Stud.* 25 (2), 599–638.
- Marshall, B.R., Nguyen, N.H., Visaltanachoti, N., 2018. Do liquidity proxies measure liquidity accurately in ETFs? *J. Int. Financ. Mark. Inst. Money* 55, 94–111.
- Nashikkar, A., Subrahmanyam, M.G., Mahanti, S., 2011. Liquidity and arbitrage in the market for credit risk. *J. Financ. Quant. Anal.* 46 (3), 627–656.
- Oesterhelweg, O., Schiereck, D., 1993. Measuring concept for the liquidity of financial markets. *Die Bank* 33, 390–397.
- O'hara, M., 1995. Market Microstructure Theory, vol. 108 Blackwell, Cambridge, MA.
- Pasquariello, P., Vega, C., 2009. The on-the-run liquidity phenomenon. *J. financ. econ.* 92 (1), 1–24.
- Pastor, L., Stambaugh, R., 2003. Liquidity risk and expected returns. *J. Polit. Econ.* 111, 642–685.
- Pop, C., Georgescu, M.A., Pop, I., 2014. Investigating Liquidity in a Frontier Government Bond Market: the Case of Bucharest Stock Exchange. Available at SSRN: <https://ssrn.com/abstract=2535224>.
- Porter, D.C., Weaver, D.G., 1997. Tick size and market quality. *Financ. Manage.* 5–26.
- Pu, X., 2009. Liquidity commonality across the bond and CDS markets. *J. Fixed Income* 19 (1), 26.
- Roll, R., 1984. A simple implicit measure of the effective bid-ask spread in an efficient market. *J. Finance* 39 (4), 1127–1139.
- Sadka, R., 2004. Liquidity Risk and Asset Pricing. Working Paper. Northwestern University.
- Sadka, R., 2006. Momentum and post-earnings-announcement drift anomalies: the role of liquidity risk. *J. financ. econ.* 80 (2), 309–349.
- Sarig, O., Warga, A., 1989. Bond price data and bond market liquidity. *J. Financ. Quant. Anal.* 24 (3), 367–378.

- Sarr, A., Lybek, T., 2002. Measuring Liquidity in Financial Markets. Working Paper. [https://papers.ssrn.com/sol3/papers.cfm?abstract\\_id=880932](https://papers.ssrn.com/sol3/papers.cfm?abstract_id=880932).
- Schestag, R., Schuster, P., Uhrig-Homburg, M., 2016. Measuring liquidity in bond markets. *Rev. Financ. Stud.* 29 (5), 1170–1219.
- Schultz, P., 2001. Corporate bond trading costs: a peek behind the curtain. *J. Finance* 56 (2), 677–698.
- Schwartz, R.A., 1988. A proposal to stabilize stock prices. *J. Portf. Manage.* 15 (1), 4–11.
- Shen, P., Starr, R.M., 2002. Market-makers' supply and pricing of financial market liquidity. *Econ. Lett.* 76 (1), 53–58.
- Silber, W., 1991. Discounts on restricted stock: the impact of illiquidity on stock prices. *Financ. Anal. J.* 47, 60–64.
- Stoll, H.R., 1989. Inferring the components of the bid-ask spread: theory and empirical tests. *J. Finance* 44 (1), 115–134.
- Strebulaev, I.A., 2002. Many Faces of Liquidity and Asset Pricing: Evidence From the US Treasury Securities Market. Working Paper. London Business School.
- Vayanos, D., Weill, P.O., 2008. A search-based theory of the on-the-run phenomenon. *J. Finance* 63 (3), 1361–1398.
- Vortelinos, D.I., Gkillas, K., Konstantatos, C., Peppas, G., 2018. The efficiency in liquidity measures during the US monetary announcements. *Theor. Econ. Lett.* 8 (01), 98.
- Warga, A., 1992. Bond returns, liquidity, and missing data. *J. Financ. Quant. Anal.* 27 (4), 605–617.