Quantum Programming (4ETI) **Spring 2020**

Project #2

Instructor: Dr. Fedrici Due date: 3^{rd} May, 2020

Problem 1

In this project you will consider the Clauser-Horne-Shimony-Holt (CHSH) inequality to perform a simple Bell test on a 2-register quantum circuit with Qiskit.

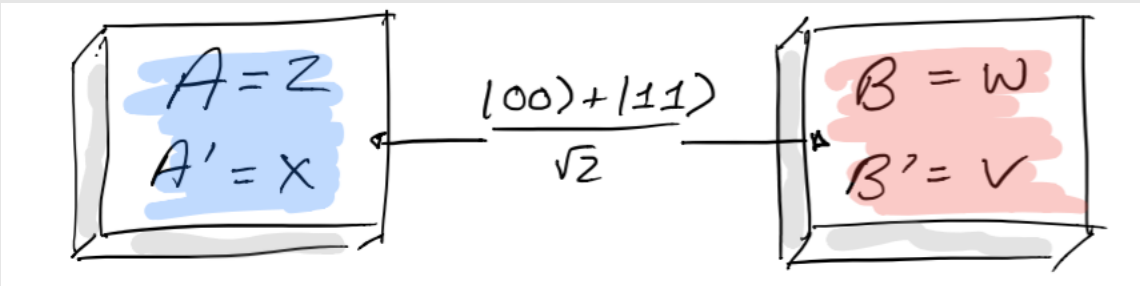
As a reminder, the usual form of the CHSH inequality is $|S| \leq 2$, where $S = E(A,B) + E(A,B') + E(A',B) - E(A',B')$ and:

- ***A*,*A'***: measurement settings associated with your first register - outcomes +1 and -1;
- ***B*,*B'***: measurement settings associated with your second register - outcomes +1 and -1;
- ***E*(*A*,*B*)**: expectation value of the product of the outcomes when measuring in the AB basis.

For a given configuration, the experimental estimate of E is then calculated in the following way - it's simply the statistical average:

$$E = \frac{N_{++} + N_{--} - N_{+-} - N_{-+}}{N_{++} + N_{--} + N_{+-} + N_{-+}}$$

where N_{++} is the number of recorded '11' measurements, N_{+-} the number of recorded '10' measurements, *etc.* Ideally the measurement settings have to be chosen uniformly at random for each run of the experiment.



A choice of measurement settings for which the quantum theory gives the greatest violation of the inequality is:

$$\begin{cases} A = Z \\ A' = X \\ B = W := (Z+X)/\sqrt{2} \\ B' = V := (Z-X)/\sqrt{2} \end{cases}$$

hence $S = \langle ZW \rangle + \langle ZV \rangle + \langle XW \rangle - \langle XV \rangle$. For a maximally entangled state of the form $|\psi\rangle = \big(|00\rangle + |11\rangle\big)/\sqrt{2}$, we expect $S = 2\sqrt{2} \approx 2.82$. This result can not be explained by any local hidden variable theory, and therefore there must be correlations that go beyond classical.

In the following sections your task is to to write a program that computes the S parameter for a given two-qubit input state. This assignment sheet is accompanied by **random_settings_a.txt** and **random_settings_b.txt** files that provide each a sequence of 2^{15} true random numbers for you to select your measurement settings in a random fashion. Those numbers have been generated by quantum means with the qRNG module - `https://pypi.org/project/qrng/`, an open-source quantum random number generator written in python.

In order to be graded correctly, your submitted code must include a **CHSH.py** file with the appropriate functions as described below.

a. Write a function in CHSH.py called **circuit_definition** whose arguments are a 2-register circuit object - your input state, and a string object of the form 'ZW', 'ZV', 'XW' or 'XV' - your measurement settings for a given shot of the experiment. This function returns a new circuit object allowing the measurement of your input state in the basis that have been specified.

b. Write a function in CHSH.py called **circuit_execution** that repeats 2^{15} times the following operation: select uniformly at random a settings configuration, call **circuit_definition** with those settings, and execute a single run of the circuit. **circuit_execution** arguments are a 2-register circuit object - your input state to be passed to **circuit_definition**, two file names as string objects (default **'random_settings_a.txt'** and **'random_settings_b.txt'**) - those files contain for each run your measurement settings to be passed to **circuit_definition**, and a string object for backend definition (default **'qasm_simulator'**). This function returns a list of 2^{15} tuples in the form ('settings', 'results') - for instance ('XW', '11') for a measurement in the XW basis and a 11 result.

Hint: Use the following mapping to associate random numbers with measurement settings: A(B): 0→ Z(W) and 1→ X(V)

c. Write a function in CHSH.py called **S_parameter** that compute the value of S from the results returned by **circuit_execution**. The arguments of **S_parameter** are the same as those of **circuit_execution**. This function returns the value of S and a summary of your different measures in the following form (this is an example, don't expect the same values):

	P(00)	P(11)	P(01)	P(10)	⟨AB⟩
ZW	0.434	0.380	0.070	0.116	0.629
ZV	0.409	0.415	0.100	0.076	0.648
XW	0.452	0.375	0.090	0.083	0.654
XV	0.110	0.077	0.451	0.36	−0.626

To check the validity of your results, remember that for an input state of the form $|\psi\rangle = \big(|00\rangle + |11\rangle\big)/\sqrt{2}$, we expect S as close as possible of $2\sqrt{2} \approx 2.82$ when running your program on the qasm_simulator. On a noisy QPU this result would be a bit lower, but still greater than 2 if the QPU is not too noisy.