

A Syntactic Proof Guide for Epistemic Logic

The strategy that Guanaco employs is building proofs bottom-up. This means that users start with the formula that they wish to derive. If this formula can only be derived by a rule, then Guanaco exploits the properties of this formula to extract new formulas the relevant rule is applied to. If the formula is an instantiation of an axiom, no new formulas are extracted. The same process is then applied until each formula has a justification and no new ones are produced. At that point, the proof is complete. Currently, axiom A1 for propositional tautologies is not yet validated automatically.

1	$\vdash \top$	A1	
2	$\vdash \neg p \rightarrow \neg(\neg p \rightarrow p)$	A1	
3	$\vdash [\neg p] p \leftrightarrow (\neg p \rightarrow p)$	A11	
4	$\vdash ([\neg p] p \leftrightarrow (\neg p \rightarrow p)) \rightarrow ((\neg p \rightarrow p) \leftrightarrow [\neg p] p)$	A1	
5	$\vdash (\neg p \rightarrow p) \leftrightarrow [\neg p] p$	R1	3, 4
6	$\vdash \neg(\neg p \rightarrow p) \leftrightarrow \neg[\neg p] p$	SUB	5
7	$\vdash \neg(\neg p \rightarrow p) \rightarrow \neg[\neg p] p$	EE	6
8	$\vdash \neg p \rightarrow \neg[\neg p] p$	HS	2, 7
9	$\vdash [\neg p] \neg p \leftrightarrow (\neg p \rightarrow \neg[\neg p] p)$	A12	
10	$\vdash ([\neg p] \neg p \leftrightarrow (\neg p \rightarrow \neg[\neg p] p)) \rightarrow ((\neg p \rightarrow \neg[\neg p] p) \leftrightarrow [\neg p] \neg p)$	A1	
11	$\vdash (\neg p \rightarrow \neg[\neg p] p) \leftrightarrow [\neg p] \neg p$	R1	9, 10
12	$\vdash (\neg p \rightarrow \neg[\neg p] p) \rightarrow [\neg p] \neg p$	EE	11
13	$\vdash [\neg p] \neg p$	R1	8, 12
14	$\vdash [\neg p] \neg p \rightarrow (\top \rightarrow [\neg p] \neg p)$	A1	
15	$\vdash \top \rightarrow [\neg p] \neg p$	R1	13, 14
16	$\vdash \top$	A1	

17 $\vdash K2 \top$

R2

16

18 $\vdash \top$

A1

19 $\vdash K1 \top$

R2

18

20 $\vdash K1 \top \rightarrow (K2 \top \rightarrow (K1 \top \wedge K2 \top))$

A1

21 $\vdash K2 \top \rightarrow (K1 \top \wedge K2 \top)$

R1

19, 20

22 $\vdash K1 \top \wedge K2 \top$

R1

17, 21

23 $\vdash (K1 \top \wedge K2 \top) \rightarrow ((\top \wedge \neg p) \rightarrow (K1 \top \wedge K2 \top))$

A1

24 $\vdash (\top \wedge \neg p) \rightarrow (K1 \top \wedge K2 \top)$

R1

22, 23

25 $\vdash E\top \leftrightarrow (K1 \top \wedge K2 \top)$

A6

26 $\vdash (E\top \leftrightarrow (K1 \top \wedge K2 \top)) \rightarrow ((K1 \top \wedge K2 \top) \leftrightarrow E\top)$

A1

27 $\vdash (K1 \top \wedge K2 \top) \leftrightarrow E\top$

R1

25, 26

28 $\vdash (K1 \top \wedge K2 \top) \rightarrow E\top$

EE

27

29 $\vdash (\top \wedge \neg p) \rightarrow E\top$

HS

24, 28

30 $\vdash \top \rightarrow [\neg p] C\neg p$

R5

15, 29

31 $\vdash [\neg p] C\neg p$

R1

1, 30

System

PAC

Agents

2

Goal

$[\neg p]C\neg p$