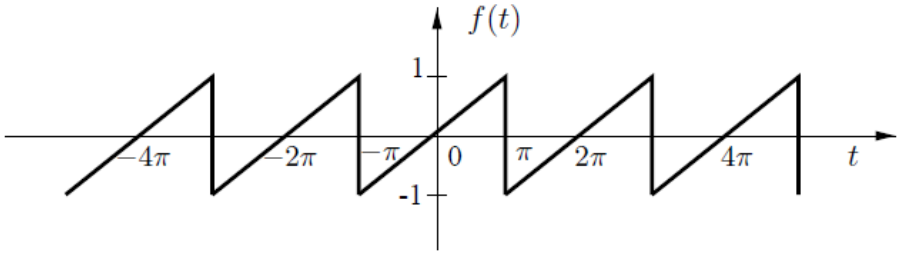
Homework Assignment 6

Instructions:

- Please ensure that your name and ID are clearly written on your assignment.
- Please submit your assignment before 16:00 (4:00 pm) on the due date. Submission is via the course's eClass webpage only.

Problems:

1. (30%) For the following signal, find the exponential Fourier series and sketch the corresponding spectra.
   Use $\int te^{at}dt = \frac{e^{at}}{a^2}(at-1)$.



2. (40%) The trigonometric Fourier series of a certain periodic signal is given by

$$f(t) = 3 + \sqrt{3}\cos(2t) + \sin(2t) + \sin(3t) - \frac{1}{2}\cos\left(5t + \frac{\pi}{3}\right)$$

   (a) Sketch the trigonometric Fourier spectra. (To express the Fourier series in compact form, combine the sine and cosine terms of the same frequency. All terms must appear in the cosine form with positive amplitudes).

   (b) By inspection of the spectra in (a), sketch the exponential Fourier series spectra.

   (c) By inspection of the spectra in (b), write the exponential Fourier series for $f(t)$.

3. (30%) Let $f(t)$ be a periodic signal with period 2. Inside the interval $[-1,1)$, $f(t) = At$. The compact trigonometric Fourier series for $f(t)$ is

$$f(t) = \frac{2A}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\cos\left[n\pi t - (-1)^n\frac{\pi}{2}\right].$$

   (a) Verify Parseval's theorem for this series.
   Use $\sum_{n=1}^{\infty}\frac{1}{n^2} = \frac{\pi^2}{6}$.

   (b) If $f(t)$ is approximated by the first N terms in this series, find N so that the power of the error signal is less than 10% of the power of $f(t)$, P_f.

Additional practice problems (not to be marked)

Lathi 3.5-1: For signals (b), (d) and (e) in Fig. P3.4-3, find the exponential Fourier series and sketch the corresponding spectra.

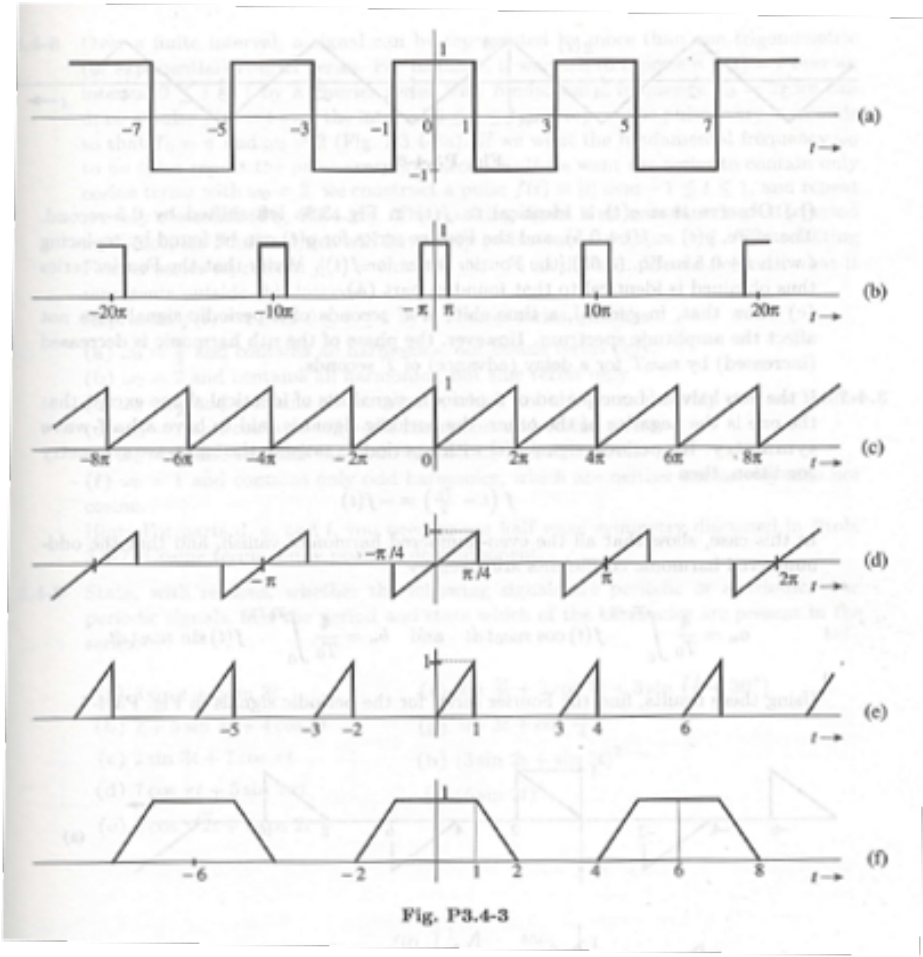


Fig. P3.4-3

Lathi 3.5-3: The exponential Fourier series of a certain periodic function is given as

$$f(t) = (2+2j)\,e^{-j3t} + 2je^{-jt} + 3 - 2je^{jt} + (2-2j)\,e^{j3t}$$

(a) Sketch the exponential Fourier spectra.

(b) By inspection of the spectra in part a, sketch the trigonometric Fourier spectra for $f(t)$.

(c) Find the compact trigonometric Fourier series from these spectra.