Lunar Ascent: From Earth to Moon's Orbit

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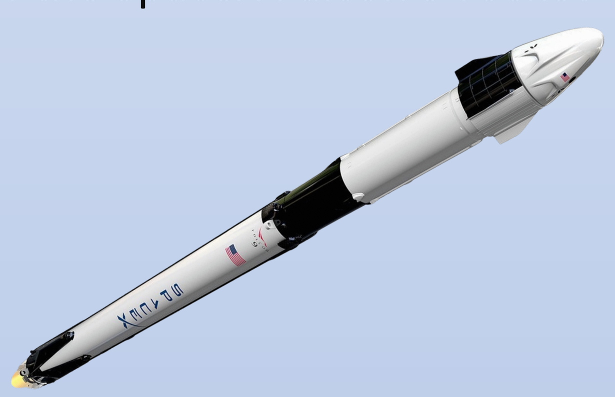
ABSTRACT

We present a simulation of a rocket's path from Earth's surface to the orbit of the Moon. We model the complex dynamics involved in rocket propulsion, gravitational interactions, and orbital mechanics by leveraging differential equations and numerical methods. Our simulation accurately captures the stages of ascent, including liftoff, trajectory optimization, and orbital insertion. Through detailed analysis and visualization, we explore the critical factors influencing orbital maneuvers required for this orbital trajectory. The results offer valuable insights into the challenges and considerations of space exploration, paving the way for future missions to celestial bodies beyond Earth's orbit.

MODEL ROCKET

Our specs used in this project are based off the SpaceX Falcon 9 rocket

Height: 70.0 m
Diameter: 3.7 m
Mass: 549,054 kg
Max Thrust (Stage 1): 7.560 millinewtons
Max Thrust (Stage 2): 981 kilonewtons



PONTRYAGIN'S MAXIMUM PRINCIPLE

Pontryagin's maximum principle tells us that given an optimal control problem where $\mathbf{x}' = \mathbf{f}(t;\mathbf{x},\mathbf{u})$ with $\mathbf{x}(t_0) = \mathbf{x}_0$ that is seeking to minimize the cost functional

$$J[\mathbf{u}] = \int_{t_0}^{t_f} L(t;\mathbf{x},\mathbf{u})dt,$$

via the control $\mathbf{u}(t) \in U$ where U is the admissible control region, we have that

(i)

$$\tilde{\mathbf{x}}' = \frac{DH}{d\mathbf{p}}, \qquad \tilde{\mathbf{x}}(t_0) = \mathbf{x}_0,$$

$$\tilde{\mathbf{p}}' = -\frac{DH}{D\mathbf{x}}, \qquad \tilde{\mathbf{p}}(t_f) = 0,$$

(ii) The Hamiltonian $H = \mathbf{p}\cdot\mathbf{f} - L$ has a global maximum for each $t \in [t_0, t_f]$ at the optimal control $\tilde{\mathbf{u}}(t)$, i.e. $H(\tilde{\mathbf{u}}) \geq H(\mathbf{u})$ for all $\mathbf{u}(t) \in U$.

REFERENCES

- SpaceX. (n.d.). Falcon 9. Retrieved from https://www.spacex.com/vehicles/falcon-9/
- Braeunig, R. A. (n.d.). Constants, Units & Physical Properties. Retrieved from http://www.braeunig.us/space/constant.htm

OUR SET UP

We defined our cost functional as

$$J[u] = \int_0^{t_f} \|\mathbf{u}(t)\|^2 dt$$

with $\mathbf{u}(t) = [u_x \;\; u_y]$. We define our initial conditions to be

$$\begin{aligned} &x(0) = R_e, \quad y(0) = 0, \quad x'(0) = 0, \quad y'(0) = \omega_e \\ &x(t_f) = L_m, \quad y(t_f) = R_m + h_f, \quad x'(t_f) = v_f, \quad y'(t_f) = 0 \end{aligned}$$

Since we only consider 2D motion in the first quadrant with the Earth at the origin and the moon starting on the x axis, we define our state equations as

$$\mathbf{x}' = \begin{bmatrix} x' \\ y' \\ x'' \\ y'' \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ -\frac{GM_e x}{(x^2+y^2)^{3/2}} + \frac{GM_m(L_m - x)}{((L_m-x)^2+y^2)^{3/2}} + \frac{u_x}{m_r} \\ -\frac{GM_e y}{(x^2+y^2)^{3/2}} - \frac{GM_m y}{((L_m-x)^2+y^2)^{3/2}} + \frac{u_y}{m_r} \end{bmatrix}$$

Leading to our Hamiltonian

$$\begin{aligned} H = p_1 x' + p_2 y' - p_3 &\left(\frac{GM_e x}{(x^2+y^2)^{3/2}} - \frac{GM_m(L_m - x)}{((L_m-x)^2+y^2)^{3/2}} - \frac{u_x}{m_r} \right) \\ - p_4 &\left(\frac{GM_e y}{(x^2+y^2)^{3/2}} + \frac{GM_m y}{((L_m-x)^2+y^2)^{3/2}} - \frac{u_y}{m_r} \right) - u_x^2 - u_y^2 \end{aligned}$$

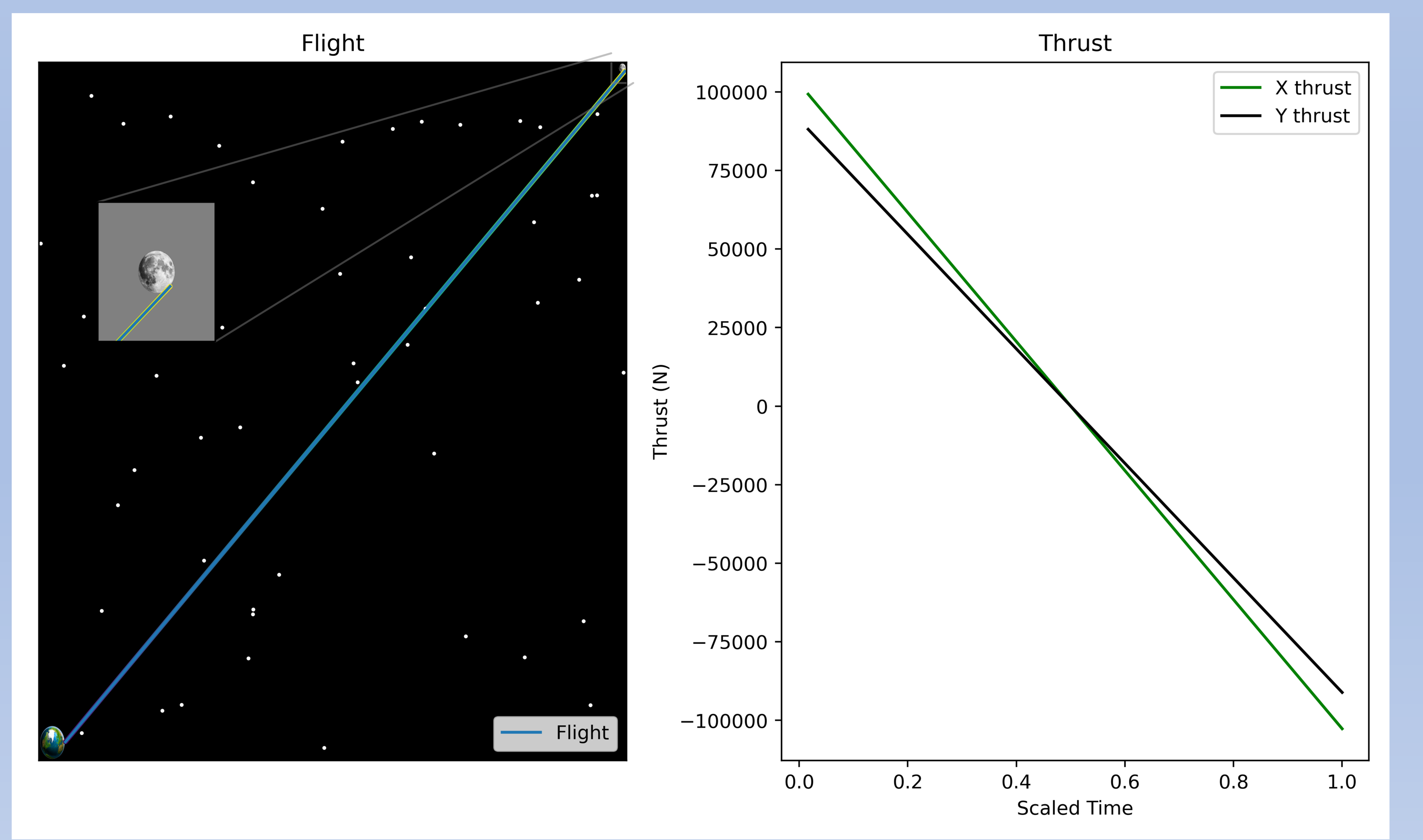
PARAMETERS

- R_e radius of Earth (kilometers)
- L_m distance between the center of the Earth and moon (kilometers)
- R_m radius of the moon (kilometers)
- h_f final distance between the rocket and the moon (kilometers)
- v_f final velocity of the rocket (kilometers per second)
- ω_e angular velocity of the Earth at the equator (radians per second)

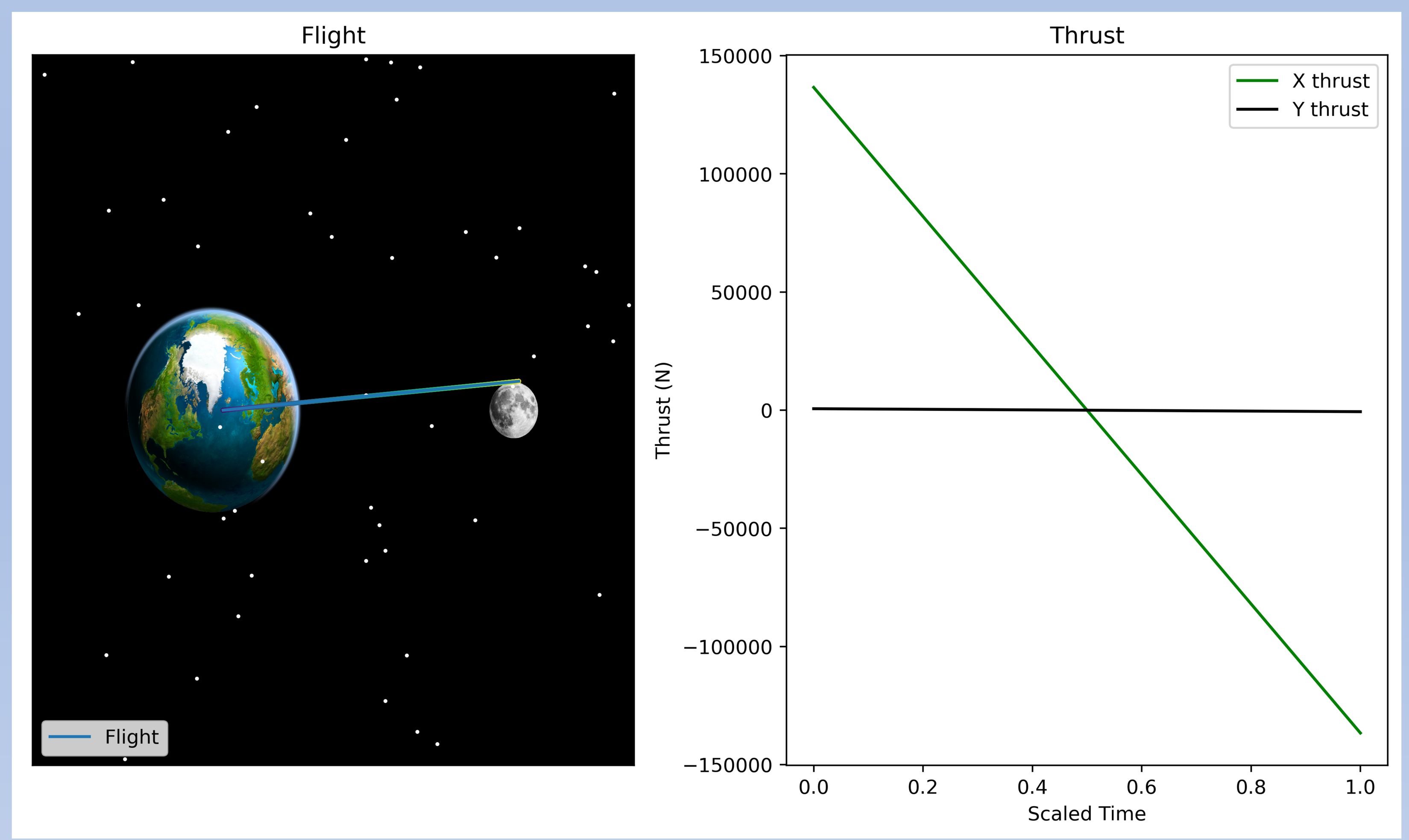
FUTURE WORK

- Set the moon's motion as a function of time rather than stationary
- Set the mass of rocket as a function of control variable
- Optimize over final time
- Incorporate a bound on the thrust
- Incorporate discontinuous control (i.e., thrust to get into space and then float for a while)
- Validate effect of gravity on model

RESULTS



Example of flight path from Earth to moon where the moon is at a 42-degree angle from the x-axis. We see that the x-thrust and y-thrust start off very positive, then change linearly in time so that at the midway point thrust is in opposite direction to slow rocket down in time to reach the moon. In this picture, the distance between the earth and the moon is scaled based on size of earth and moon.



Example of flight path from Earth to moon where final position of the moon is on x-axis and does not move. We see very little y-thrust so the change in y is small compared to the massive change in x. There is a very high x-thrust until the midway point when x-thrust reverses to slow down the rocket. Here the size of the earth and moon are not to scale for readability.