CMA package vignette

Martin Slawski * Anne-Laure Boulesteix †

Sylvia Lawry Centre, Hohenlindenerstr. 1, D-81677 Munich, Germany

1 Statistical background

For the last few years, microarray-based class prediction has been a major topic in statistics and machine learning. Traditional methods often yield unsatisfactory results or are even inapplicable in the $p\gg n$ setting. Hence, microarray studies have stimulated the development of new approaches and motivated the adaptation of known traditional methods to high-dimensional data.

Moreover, model selection and evaluation of prediction rules proves to be highly difficult in this situation for several reasons. Firstly, the hazard of overfitting, which is common to all prediction problems, is increased by high dimensionality. Secondly, the usual evaluation scheme based on the splitting into learning and test data sets often applies only partially in the case of small samples. Lastly, modern classification techniques rely on the proper choice of hyperparameters whose optimization is highly computer-intensive, especially in the case of high-dimensional data.

2 Class prediction based high-dimensional data with small samples

2.1 Settings

The classification problem can be briefly outlined as follows:

- we have a predictor space \mathcal{X} , here $\mathcal{X} \subseteq \mathbb{R}^p$
- we have a finite set of class labels $\mathcal{Y} = \{0, \dots, K-1\}$, with K denoting the total number of classes
- P(x,y) denotes the joint probability distribution on $\mathcal{X} \times \mathcal{Y}$
- we are given a finite sample $S = \{(x_1, y_1), \dots, (x_n, y_n)\}$ of n predictor-class pairs.

The task is to find a decision function

$$\widehat{f}:\; \mathcal{X}
ightarrow \mathcal{Y} \ oldsymbol{x} \mapsto \widehat{f}(oldsymbol{x})$$

^{*}Martin.Slawski@campus.lmu.de

[†]http://www.slcmsr.net/boulesteix

(the $\hat{\cdot}$ indicates estimation from the given sample S) such that the generalization error

$$R[f] = \mathbf{E}_{P(\boldsymbol{x},y)}[L(f(\boldsymbol{x}),y)] = \int_{\mathcal{X}\times\mathcal{Y}} L(y,f(\boldsymbol{x})) \ dP(\boldsymbol{x},y)$$
(1)

is minimized. $L(\cdot, \cdot)$ is a suitable loss function, usually taken to be the indicator loss $(1, \text{ if } f(x) \neq y, 0, \text{ otherwise}).$

2.2 Estimation of prediction accuracy

As we are only equipped with a finite sample S and the underlying distribution is unknown, approximations to (1) have to be found. Its empirical counterpart

$$R[f]_{\text{emp}} = n^{-1} \sum_{i=1}^{n} L(y_i, f(\mathbf{x}_i))$$
(2)

has a (usually large) negative bias for (1), thus model selection based on (2) leads to overfitting the sample S.

A first improvement involves a split of into parts \mathcal{L} (learning sample), \mathcal{T} (test sample) with the intention to separate model selection and -evaluation, by doing model selection only with \mathcal{L} and evaluating the resulting decision function $f(\cdot)$ only on \mathcal{T} .

For microarray data, the sample sizes n is usually very small, leading to serious problems when estimating prediction accuracy and when constructing a prediction rule based on the available data, a problem which is also related to model choice (see Section 2.3).

When splitting the original data into two approximately equally sized data sets (learning set and test set), the performance of $f(\cdot)$ is strongly diminuished due to a further reduction of the sample size and the evaluation is unreliable and highly variant. Hence, alternative designs are needed.

In the package CMA, we pursue the following route for mitigating at least the second consequence, by resampling and aggregating:

- Generate B splits of $S = (\mathcal{L}_b, \mathcal{T}_b), b = 1, \dots, B$ into learning- and test sample
- Obtain \widehat{f}_b from \mathcal{L}_b , $b = 1, \dots, B$
- Setting $\mathcal{I}_b = \{i : y_i \notin \mathcal{L}_b\}$, we use

$$\widehat{\varepsilon} = \frac{1}{B} \sum_{b=1}^{B} \frac{1}{|\mathcal{I}_b|} \sum_{i \in \mathcal{I}_b} I(y_i \neq \widehat{f}_b(\boldsymbol{x}_i))$$
(3)

as estimator for (1).

The strategy is to reduce the variance of the error estimator by averaging. If n is large, this procedure will not improve much on a simple splitting. As splitting rules, the function GenerateLearningsets implements:

• Leaving-one-out cross-validation :

 \mathcal{T}_b consists only of one observation, this is repeated for each observation in S, so that B = n.

- k-fold cross-validation (method = "CV", fold = , niter =):
 S is split into k parts of equal size. For each iteration b, the b-th part is used as
 T_b and the union of the remaining parts as L_b. Setting fold = n is equivalent to (method = "LOOCV"). As the splitting is not uniquely determined for fold < n, the whole procedure can be repeated niter times.
- Monte-Carlo-cross-validation (method = "MCCV", fold=, ntrain=, niter=): B=niter random learning samples of cardinality ntrain are generated.
- Bootstrap (method = "bootstrap", ntrain = , niter =):

 B=niter bootstrap samples of cardinality ntrain are used as learning samples.

Furhermore, stratified sampling is possible by setting the argument strat = TRUE. This implies, that for each \mathcal{L}_b , the proportion of the classes $\{0, \ldots, K-1\}$ is the same as for the full S. This option is very useful (and sometimes even necessary) in order to guarantee that each class is sufficiently often represented in each \mathcal{L}_b , in particular if there are classes that are small in size.

Benefits and drawbacks of above splitting rules are discussed in Braga-Neto and Dougherty [2003] and ?.

2.3 Constructing a prediction rule

The second main issue is the construction of a appropriate prediction rule. In microarray data analysis, we have to deal with the $n \ll p$ situation, i.e. the number of predictors exceeds by far the number of observations. Some class prediction methods only work for the case $n \ll p$, e.g. linear or qudratic discriminant analysis which are based on the inversion of a matrix of size $p \times p$ and rank n-1. In the $n \ll p$ setting, the hazard of overfitting is especially acute: perfect separation of the classes for a given sample based on a high number of predictors is always possible. However, the resulting classification rule may generalize poorly on independent test data.

There are basically three approaches to cope with the $n \ll p$ setting:

- 1. variable selection using, e.g., univariate statistical tests
- 2. regularization or shrinkage methods, such as the Support Vector Machine (Boser et al. [1992]), ℓ_2 or ℓ_1 penalized logistic regression (Zhu [2004]; Young-Park and Hastie [2007]) or Boosting (Friedman [2001]; Bühlmann and Yu [2003]) from which some also perform variable selection
- 3. dimension reduction or feature extraction. Most prominent is the Partial Least Squares method (?).

Most classification methods depend on a vector of hyperparameters λ that have to be correctly chosen. Together with variable selection, this is part of the model selection and has thus to be performed separately for each learning sample \mathcal{L}_b to avoid bias (?). An optimal vector of hyperparameters λ^{opt} is determined by defining a discrete set of values whose performance is then measured by cross-validation. This involves a further splitting step, which is sometimes called 'inner loop' or 'nested' cross-validation (Statnikov et al. [2005a]). More precisely, each learning set \mathcal{L}_b is divided into $l = 1, \ldots, k$ parts such that the l-th part forms the test sample for the l-th (inner) iteration. Analogously to (3), this procedure can be used to derive an error estimator for each value on the grid of candiate hyperparameter values. The optimal vector λ^{opt} is chosen to minimize this error criterion. Note that there are often several such minimizers. Furthermore, the minimizer found by this procedure

can be relatively far away from the true minimizer, depending on how fine the discrete grid has been chosen.

The choice of the inner cross-validation scheme is difficult. With a high k, computation times soon become prohibitively high. With a low k, the size of \mathcal{L}_{b_l} , $l=1,\ldots,k$ is strongly reduced compared to the complete sample S, which may have an impact on the derived optimal parameter values. Nevertheless, nested cross-validation is commonly used in this context (Statnikov et al. [2005b]).

Considering the computational effort required for hyperparameter optimization and the small sample sizes, one may prefer class prediction methods that do not depend on many hyperparameters and/or behave robustly against changes of the hyperparameter values.

3 Overview of CMA features

In a nutshell, the package has the following features.

- It offers a uniform, user-friendly interface to a total of 21 classification methods (3), comprising classical approaches (such as discriminant analysis) as well as more sophisticated methods, e.g. Support Vector Machines (SVM) or boosting techniques. User-friendliness means that the input formats are uniform among different methods, that the user may choose between three different input formats and that the output is highly self-explicable and informative.
- Probability estimations for predicted observations are provided by most of the classifiers, with only a few exceptions. This is more informative than only returning class labels and enables a more precise comparison of different classifiers.
- It automatically generates learning samples as explained in section 2, including the generation of stratified samples.
- Preliminary variable selection (if any) is performed for each iteration separately based on one of the following ranking procedures, using the method GeneSelection:

```
- ordinary two-sample t.test (method = "t.test")
```

- Welch modification of the t.test (method = "welch.test")
- Wilcoxon rank sum test (method = "wilcox.test")
- F test (method = "f.test") when K > 2
- Kruskal-Wallis test (method = "kruskal.test") when K > 2
- 'moderated' t and F test, respectively, using the package limma(Smyth [2005]) (method = "limma")
- One-step Recursive Feature Elimination (Guyon et al. [2002]) (method = "rfe")
- random forest variable importance measure (method = "rf")
- the Lasso (method = "lasso")
- the elastic net (method = "elasticnet")
- componentwise boosting (method = "boosting")
- the ad-hoc criterion used in Golub et al. [1999]

For most methods, the implementation is very fast. The package CMA uses own functions instead of the pre-defined R functions. Additionally, the multi-class case is fully supported, even if the chosen method is not defined for it. The workaround is realized by using either a pairwise or a one-vs-all scheme.

- Hyperparameter tuning is carried out using the scheme outlined in section section 2, for a *fixed* (sub)set of variables. It can be performed in a fully automatically manner using pre-defined grids. Alternatively, it can be completely customized by the user.
- The method classification enables the user to combine gene selection, hyperparameter tuning and class prediction into one single step. But each step can also be performed separately.
- Performance can be assessed using several performance measures which are commonly used in practice:
 - the misclassification rate
 - the sensitivity and specifity, for K=2
 - the empirical area under the curve (AUC), for K = 2, if the class prediction method returns a probability,
 - the Brier score
 - the average probability of correct classification

Each performance measure can be 1) averaged for all predictions globally, 2) averaged within each iteration first and then over all iterations or 3) averaged within each observation first and then over all observations. Based on the results, the function obsinfo can be used to identify observations that are frequenly misclassified (and are thus candidates for outliers).

- Comparison of the performance of several classifiers can be performed using one or several of the above performance measures. This comparison can be tabulated and visualized using the method comparison.
- Most results can quickly be summarized and visualized using pre-defined convenience methods, for example:
 - plot, cloutput-method produces a probability plot, also known as 'voting plot'
 - plot, genesel-method visualizes variable importance via a barplot
 - roc, cloutput-method draws the empirical ROC curve
 - toplist, genesel-method shows the most important variables
 - summary, evaloutput-method Makes a summary out of iteration- or observationwise performance measures
- The implementation is fully organized in S4 classes, thus making the extension of CMA very easy. In particular, own classification methods can easily be integrated if they return a proper object of class cloutput.
- The class prediction methods implemented in CMA are summarized in Table 3.

method name	CMA function name	Package	Reference
Componentwise Boosting	compBoostCMA	CMA	Bühlmann and Yu [2003]
Diagonal Discriminant Analysis	dldaCMA	CMA	McLachlan [1992]
Elastic Net	ElasticNetCMA	glmpath	Zhou and Hastie [2004]
Fisher's Discriminant Analysis	fdaCMA	CMA	?
Flexible Discriminant Analysis	flexdaCMA	mgcv	?
Tree-based Boosting	gbmCMA	gbm	Friedman [2001]
k-nearest neighbours	knnCMA	class	?
Linear Discriminant Analysis *	ldaCMA	MASS	McLachlan [1992]
Lasso	LassoCMA	glmpath	Young-Park and Hastie [2007]
Feed-Forward Neural Networks	${\tt nnetCMA}$	nnet	?
Probalistic nearest neighbours	pknnCMA	CMA	_
Penalized Logistic Regression	plrCMA	CMA	Zhu [2004]
Partial Least Squares $\star + *$	pls_ldaCMA	plsgenomics	?
\star + logistic regression	pls_lrCMA	plsgenomics	?
\star + Random Forest	pls_rfCMA	plsgenomics	?
Probabilistic Neural Networks	pnnCMA	CMA	Specht [1990]
Quadratic Discriminant Analysis *	qdaCMA	MASS	McLachlan [1992]
Random Forest	rfCMA	${\tt randomForest}$	Breiman [2001]
PAM	scdaCMA	CMA	?
Shrinkage Discriminant Analysis	${\tt shrinkldaCMA}$	CMA	_
Support Vector Machine	svmCMA	e1071	Schölkopf and Smola [2002]

4 Comparison with existing packages

The idea of an interace for the integration of classification methods for microarray data is not new and we here argue why CMA can be a significant improvement with respect to the following aspects: standardized and reproducible analysis, neutral comparisons of existing methods, comfortable use.

The CMA package shows similarities to the Bioconductor package MLInterfaces standing for 'An interace to various machine learning methods' (Mar et al. [2007]), see also the Bioconductor textbook for a presentation of an older version.

Contrary to CMA, MLInterfaces also offers access to popular 'unsupervised learning' methods such as clustering, independent component analysis etc. that can be beneficial for exploratory analyses as well as for revealing classes in a preparatory step preceding supervised classification. The package architecture is very similar the CMA structure in the sense that wrapper functions are used to call classification methods from other packages.

Up to now, CMA includes more predefined features than MLInterfaces as far as variable selection, hyperparameter tuning, classifier evaluation and comparison are concerned. While the method xval is flexible for experienced users, it provides only LOOCV or 'leave-one-group out' as predefined options. As this package addresses also unexperienced users, we decided to include the most common validation schemes in a standardized manner. As consequence, we additionally hope to increase reproducibility of results which is a major concern in statistics in general and for microarray data analysis in particular.

In the current version, variable selection can also be carried separately for each different learning set, but this seems not to be a standard procedure. In the examples presented in the book mentionned above, variable selection is only performed once using the complete sample S although this procedure is widely known to yield optimistically biased results (Ambroise and McLachlan [2002]).

Moreover, hyperparameter tuning is completely missing in MLInterfaces. In our opinion, this makes the objective comparison of different class prediction methods difficult. If tuning is ignored, simpler methods without (or with few) tuning parameters tend to perform seemingly better than more complex algorithms.

We 'borrowed' from MLInterface some ideas regarding the selection of classification methods (section 3) and some functionalities such as the variable importance plot, or the Planarplot.

We would also like to mention the package e1071 (Dimitriadou et al. [2006]) whose tune function served as the basic idea for the CMA tuning functionalities.

The package MCRestimate (Ruschhaupt et al. [2007]) emphasizes very similar aspects as CMA, focusing on the estimation of misclassification rates and cross-validation for model selection and evaluation. It is (to our knowledge) the only Biconductor package that supports hyperparameter tuning, but obviously referring to the function e1071:::tune. Compared to CMA, it is a bit less comprehensive, in particular with respect to variable selection. Moreover, the package structure is, to our opinion, less stringent than that of CMA, mainly because it lacks a class structure (neither S3 nor S4 is used).

5 Example 1: Focusing on one method

The following two sections demonstrate the usual workflow when using CMA and its essential features, methods and objects. While this section focusses on optimizing and evaluating one method, the following section is devoted to classifier comparison.

We use the famous leukaemia dataset of Golub et al. [1999] as a data example. The sample consists of 38 observations in total, from which 27 belong to class 0 (acute lymphoblastic leukemia) and 11 to class 1 (acute myeloid leukemia). We start by loading and the dataset and extracting gene expression and class labels, respectively.

```
> data(golub)
> golubY <- golub[, 1]
> golubX <- as.matrix(golub[, -1])</pre>
```

Following the approach described in section 2, we generate several learning samples by different splitting rules, taking into account that each of them has advantages and disadvantages. Learning samples are generated by the function GenerateLearningsets, which returns an object of class learningsets:

```
> loodat <- GenerateLearningsets(y = golubY, method = "LOOCV")
> class(loodat)
> getSlots(class(loodat))
> show(loodat)
```

For five-fold cross-validation, we use the following commands:

```
> set.seed(321)
> fiveCVdat <- GenerateLearningsets(y = golubY, method = "CV",
+ fold = 5, strat = TRUE)</pre>
```

Note that stratified learning sets are generated by setting the argument strat = TRUE. The random seed should be set for reproducibility. We can proceed analogously for Monte-Carlo cross-validation and bootstrap:

```
> set.seed(456)
> MCCVdat <- GenerateLearningsets(y = golubY, method = "MCCV",
+    niter = 3, ntrain = floor(2/3 * length(golubY)), strat = TRUE)
> set.seed(651)
> bootdat <- GenerateLearningsets(y = golubY, method = "bootstrap",
+    niter = 3, strat = TRUE)</pre>
```

In this example, we choose the Support Vector Machine with linear kernel as classification method. Variable selection is not strictly necessary, but is has empirically been shown that the performance of the SVM method can significantly be improved when noise features are removed (Hastie et al. [2001]). For simplicity, we choose the distribution free Wilcoxon-Test to rank the variables, separately for each learning sample

```
> varsel_fiveCV <- GeneSelection(X = golubX, y = golubY, learningsets = fiveCVdat,
+ method = "wilcox.test")
> varsel_MCCV <- GeneSelection(X = golubX, y = golubY, learningsets = MCCVdat,
+ method = "wilcox.test")
> varsel_boot <- GeneSelection(X = golubX, y = golubY, learningsets = bootdat,
+ method = "wilcox.test")</pre>
```

Now let us have a closer look at varsel_fiveCV. The toplist methods provides easy access to the top-ranked variables:

We see that no variable is among the top 10 in every learning sample; the highest frequency is 4 out of 5.

The next step is hyperparameter tuning. It is well-known that the SVM needs much tuning (Statnikov et al. [2005a]) and that its performance can decrease drastically without tuning. For simplicity, we use a linear kernel, thus avoiding the tuning of kernel parameters. The choice of the parameter C in the primal objective of the SVM

$$P(\mathbf{w}) = \|\mathbf{w}\|^2 + C \sum_{i=1}^{n} \xi_i, \ C > 0$$

has still to be done. \boldsymbol{w} denotes the weight vector of the maximum margin hyperplane, while the $\{\xi_i\}_{i=1}^n$ quantify the amount of violation of this hyperplane by the learning sample. Increasing C penalizes violations more severely, forcing the hyperplane to separate the learning sample (and thus probably producing overfitting).

In order to find an appropriate value for C (which corresponds to the argument cost in e1071:::svm), we take the best 100 genes from the previous variable ranking step. As hyperparameter tuning is very time-intensive, we use the five-fold CV procedure only in this demonstrating example. The resulting best value for cost will then also be used for the other other CV procedures. In this example, we consider five candidate values: 0.1,1,10,100,200. Due to the nested cross-validation procedure, the SVM is trained $3 \times 5 \times 5 = 75$ times (!) in the following example. Three is to the number of inner cross-validation folds, the first five is the number of candidate values and the second five is the number of outer cross-validation folds.

It is evident that the results are rather unstable. In order to decide for a suitable for the hyperparameter, we additionally visualize the results :

```
> par(mfrow = c(2, 2))
> for (i in 1:4) plot(tuningstep, iter = i, main = paste("iteration",
+ i))
```

Although it remains unclear which value is actually best, we take cost = 100 because this never produced bad results.

We can now turn the attention to class prediction:

The results of classification are lists where each element is an object of class cloutput, generated for each learning sample. For visualization purpose, we use the join function to combine the single list elements into 'big' objects. We first put the results from the four splitting schemes into one list and then use lapply():

```
> resultlist <- list(class_fiveCV, class_MCCV, class_boot)
> result <- lapply(resultlist, join)</pre>
```

The probability (or voting) plot is one of the most popular visualizatio method in microarray-based classification:

```
> schemes <- c("five-fold CV", "MCCV", "bootstrap")
> par(mfrow = c(3, 1))
> for (i in seq(along = result)) plot(result[[i]], main = schemes[i])
```

ftable applied to objects of class cloutput yields confusion matrices:

```
> invisible(lapply(result, ftable))
```

roc draws simple ROC cuves:

```
> par(mfrow = c(2, 2))
> for (i in seq(along = result)) roc(result[[i]])
```

We can now join again to aggregate over the different splitting rules:

```
> totalresult <- join(result)
> ftable(totalresult)
```

Confusion matrices implicity quantify performance via misclasification. For more advanced performance evaluation, one can use evaluation. Note that the input has to be a list (and not an object of class cloutput). For the Monte-Carlo cross-validation scheme, we have:

```
> av_MCCV <- evaluation(class_MCCV, measure = "average probability")
> show(av_MCCV)
> boxplot(av_MCCV)
> summary(av_MCCV)
```

measure = "average probability" stands for the average predicted probability for
the correct class, or formally:

$$\sum_{b=1}^{B} \sum_{i \in \mathcal{L}_b} \sum_{k=0}^{K-1} I(y_i = k) \widehat{p}(y_i = k | \mathbf{x}_i),$$

with $\widehat{p}(k|x)$ denoting the condtional predicted probability for class k, given x.

By default, the evalution scheme is iterationwise, but it can also be done observationwise:

```
> av_obs_MCCV <- evaluation(class_MCCV, measure = "average probability",
+ scheme = "obs")
> show(av_obs_MCCV)
```

One might also wonder which observations are misclassified very often. To find it out, one can use:

```
> obsinfo(av_obs_MCCV, threshold = 0.6)
```

6 Example 2: classifier comparison

CMA implements a complete bundle of methods based on the principle of discriminant analysis. Here, we compare six of them: diagonal-, linear- and quadratic discriminant analysis, discriminant analysis by Fisher, shrunken centroids discriminant analysis (also known as PAM) and Partial Least Squares followed by linear discriminant analysis, , applied to the small blue round cell tumour dataset of Khan et al. [2001] which comprises 65 samples from four tumour classes.

From a theoretical point of view, linear-, quadratic- and Fisher's discriminant analysis are apriori inferior due to the fact that they do not work in the $p\gg n$ without variable selection. Shruken centroids discriminant analysis is assumed to work better than the simple diagonal discriminant analysis because it can 'shrink-out' noise variables. Partial Least Squares is also expected to work well.

But let as see how this looks in pratice. As data basis, we will use (stratified) five-fold cross-validation, repeated ten times in order to achive more stable results.

Contrary to the step-by-step procedure in the previous example, we will here always use the flexible method classification. We start with diagonal discriminant analysis which neither needs variable selection nor tuning.

```
> class_dlda <- classification(X = khanX, y = khanY, learningsets = fiveCV5iter,
+ classifier = dldaCMA)</pre>
```

We now rank the variables (genes) according to the t statistic as basis for variable selection necessary for linear-, quadratic- and Fisher's linear discriminant analysis:

```
> genesel_da <- GeneSelection(X = khanX, y = khanY, learningsets = fiveCV5iter,
+ method = "t.test", scheme = "one-vs-all")</pre>
```

For the class prediction that follows, we pass the generated learning sets, the gene rankings (genesel_da) to classification. The number of genes that are retained is specified by the argument nbgene.

```
> class_lda <- classification(X = khanX, y = khanY, learningsets = fiveCV5iter,
+    classifier = ldaCMA, genesel = genesel_da, nbgene = 10)
> class_fda <- classification(X = khanX, y = khanY, learningsets = fiveCV5iter,
+    classifier = fdaCMA, genesel = genesel_da, nbgene = 10, comp = 2)
> class_qda <- classification(X = khanX, y = khanY, learningsets = fiveCV5iter,
+    classifier = qdaCMA, genesel = genesel_da, nbgene = 1)</pre>
```

Shrunken centroids discriminant analysis does not do variable selection, but hyperparameter tuning for the shinkage intensity. We here use the pre-specified grid by setting tuninglist to an empty list.

```
> set.seed(876)
> class_scda <- classification(X = khanX, y = khanY, learningsets = fiveCV5iter,
+ classifier = scdaCMA, tuninglist = list(grids = list()))</pre>
```

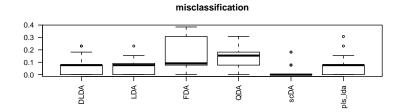
At last, we use partial least squares (with two components, which is the default).

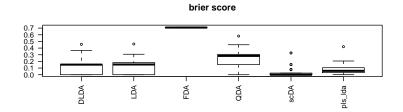
```
> class_plsda <- classification(X = khanX, y = khanY, learningsets = fiveCV5iter,
+ classifier = pls_ldaCMA)</pre>
```

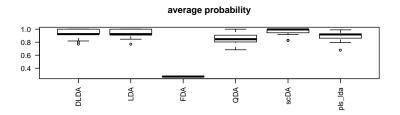
A comparison can be performed very quickly, needing only a very few lines. The method comparison does the whole job:

```
> dalike <- list(class_dlda, class_lda, class_fda, class_qda, class_scda,
+    class_plsda)
> par(mfrow = c(3, 1))
> comparison <- compare(dalike, plot = TRUE, measure = c("misclassification",
+    "brier score", "average probability"))
> print(comparison)
```

	misclassification	brier.score	average.probability
DLDA	0.06433566	0.12436384	0.9363004
LDA	0.07244755	0.14352016	0.9278874
FDA	0.15664336	0.70803592	0.2717210
QDA	0.14713287	0.26636674	0.8505199
scDA	0.02685315	0.04635105	0.9693266
pls_lda	0.06321678	0.08706237	0.8917140







References

- C. Ambroise and G.-J. McLachlan. Selection bias in gene extraction in tumour classification on basis of microarray gene expression data. *Proceedings of the National Academy of Science*, 99:6562–6566, 2002.
- B. Boser, I. Guyon, and V. Vapnik. A training algorithm for optimal margin classifiers. In *COLT '92: Proceedings of the fifth annual workshop on Computational learning theory*, New York, NY, USA, 1992. ACM Press.
- U.M. Braga-Neto and E.R. Dougherty. Is cross-validation valid for small-sample microarray classification? *Bioinformatics*, 20(3):374–380, 2003.
- L. Breiman. Random forests. Machine Learning, 45(1), 5-32, 2001.
- P. Bühlmann and B. Yu. Boosting with the l2 loss: Regression and classification. Journal of the American Statistical Association, 98:324–339, 2003.
- E. Dimitriadou, K. Hornik, F. Leisch, D. Meyer, and A. Weingessel. e1071: Misc Functions of the Department of Statistics (e1071), TU Wien, 2006. R package version 1.5-16.

- J. Friedman. Greedy function approximation: A gradient boosting machine. Annals of Statistics, 29(5):1189–1232, 2001.
- TR Golub, DK Slonim, P Tamayo, C Huard, M Gaasenbeek, JP Mesirov, H Coller, ML Loh, JR Downing, MA Caligiuri, CD Bloomfield, and ES Lander. Molecular classification of cancer: class discovery and class prediction by gene expression monitoring. *Science*, 15;286(5439):531-7., 1999.
- I. Guyon, J. Weston, S. Barnhill, and V. Vapnik. Gene selection for cancer classification using support vector machines. *Journal of Machine Learning Research*, 46: 389–422, 2002.
- T. Hastie, R. Tibshirani, and J. H. Friedman. *The elements of statistical learning*. Springer-Verlag, New York, 2001.
- J. Khan, J. Wei, M. Ringner, L.H. Saal, M. Ladanyi, F. Westermann, F. Berthold, M. Schwab, C.R. Antonescu, C. Peterson, and P.S. Meltzer. Classification and diagnostic prediction of cancers using gene expression profiling and artificial neural networks. *Nature Medicine*, 7;673-679., 2001.
- J. Mar, R. Gentleman, and V. Carey. MLInterfaces: Uniform interfaces to R machine learning procedures for data in Bioconductor containers, 2007. R package version 1.10.2.
- G.J. McLachlan. Discriminant Analysis and Statistical Pattern Recognition. Wiley, New York, 1992.
- M. Ruschhaupt, U. Mansmann, P. Warnat, W. Huber, and A. Benner. MCRestimate: Misclassification error estimation with cross-validation, 2007. R package version 1 10 2
- B. Schölkopf and A. Smola. Learning with Kernels. MIT Press, 2002.
- G.K. Smyth. Limma: linear models for microarray data, pages 397–420. Springer, New York, 2005.
- D.F. Specht. Probabilistic neural networks. Neural Networks, 3:109–118, 1990.
- A. Statnikov, C.-F. Aliferis, I. Tsamardinos, D. Hardin, and S. Levy. A comprehensive evaluation of multicategory classification methods for microarray gene expression cancer diagnosis. *Bioinformatics*, 21:631–643, 2005a.
- A. Statnikov, C.F. Aliferis, I.Tsamardinos, and S.Levy. A comprehensive evaluation of multicategory classification methods for microarray gene expression cancer diagnosis. *Bioinformatics*, 21:631–643, 2005b.
- M. Young-Park and T. Hastie. L1-regularization path algorithm for generalized linear models. *Journal of the Royal Statistical Society B*, 69(4):659–677, 2007.
- H. Zhou and T. Hastie. Regularization and variable selection via the elastic net. Journal of the Royal Statistical Society B, 67(2):301–320, 2004.
- J. Zhu. Classification of gene expression microarrays by penalized logistic regression. Biostatistics, 5:427–443, 2004.