Manual Calibration: Circle-band objective (contrast with background)

This note documents the objective used by `musclex/ui/ManualCalibrationDialog.py` in `ManualCalibrationDialog._circle_band_objective(center, radius, Q)` (the "circle band" optimizer).

1 What it computes

Given:

- circle center $c = (c_x, c_y)$
- radius r
- band scale $Q > 0$ (roughly a ring-width scale, in pixels)
- `objective_alpha` α
- `objective_bg_k` k
- `objective_nphi` N_ϕ (number of angular samples)

Let angles be uniformly sampled:

$$\phi_j = \frac{2\pi j}{N_\phi}, \quad j = 0, \ldots, N_\phi - 1$$

Let the bilinear-sampled image intensity at a point be $I(x, y)$.

1.1 Signal bands

The "signal" is sampled on 5 concentric circles near the candidate ring, using offsets

$$o \in \{-Q,\ -0.5Q,\ 0,\ 0.5Q,\ Q\}.$$

The signal mean is

$$\mu_{\text{sig}} = \frac{1}{5N_\phi} \sum_o \sum_j I\left(c_x + (r + o)\cos\phi_j,\ c_y + (r + o)\sin\phi_j\right).$$

1.2 Background bands

The "background" is sampled on 2 concentric circles farther away (symmetric inside/outside), using offsets

$$o \in \{-kQ,\, +kQ\}.$$

The background mean is

$$\mu_{\mathrm{bg}} = \frac{1}{2N_\phi} \sum_o \sum_j I\left(c_x + (r+o)\cos\phi_j,\ c_y + (r+o)\sin\phi_j\right).$$

1.3 Objective value (maximize)

The returned objective is:

$$J(c, r; Q) = \mu_{\mathrm{sig}} - \alpha\, \mu_{\mathrm{bg}}.$$

2 Why subtract a symmetric background

2.1 The problem: "maximize the integration" is biased by radial background

If you maximize a raw ring integral/mean, you are effectively optimizing

$$\int \left(S(r,\phi) + B(r,\phi)\right) d\phi,$$

where S is the ring/peak contribution you care about and B is baseline/background intensity.

In real diffraction images, B commonly varies strongly with radius (beam halo, small-angle scatter, detector response, broad diffuse scatter). That means the optimizer can increase the integral by moving the circle to a location where B is larger—even if the ring alignment is worse.

Concretely, small changes to center/radius can "ride" the background gradient and produce a larger sum/mean without actually matching the ring. This can pull the solution toward bright halos, broad low-q scatter, nearby rings with higher baseline, or detector shading artifacts.

2.2 What the background bands do

Subtracting $\alpha\, \mu_{\mathrm{bg}}$ makes the objective approximate *local contrast*: "how much brighter is the ring neighborhood compared to nearby off-ring samples".

Sampling background at both $r - kQ$ and $r + kQ$ is important:

- **Symmetry cancels first-order radial slope:** if $B(r)$ is roughly smooth, then averaging inside/outside approximates the local baseline at r and reduces bias from dB/dr.
- **Stays local but avoids the peak:** using $\pm kQ$ ties the background separation to the ring-width scale Q, so the off-ring samples remain "nearby" across different rings/images while being far enough not to sit on the peak itself.

2.3 Why not subtract a full radial profile?

A full radial background model can work, but it requires additional assumptions (masking peaks, robust fits, handling anisotropy). The two-band symmetric sampling is a lightweight, optimization-friendly proxy that is local (responds to local baseline), robust to global illumination changes, and cheap to compute.

2.4 Assumptions and practical caveats

The interpretation above relies on two practical assumptions:

- **Background smoothness:** the baseline varies reasonably smoothly with radius so that sampling at $r \pm kQ$ approximates the local baseline at r.
- **Background samples are off-peak:** the offsets $\pm kQ$ are far enough from the ring so they do not land on the ring itself or a nearby ring/feature.

If rings are thick (e.g., $\sim$20 pixels wide) or nearby features exist, increase Q and/or k so the background bands are truly outside the ring, or set $\alpha = 0$ to disable the background term.

3 Parameter intuition

- Q**:** sets the "thickness" scale of the signal sampling band; also scales how far away background is measured.
- k **(`objective_bg_k`):** how far from the ring to sample background (in units of Q). Larger k reduces contamination from the ring peak but can make the background less "local".

- α **(`objective_alpha`):** weight of the background penalty. Higher α emphasizes contrast; lower α behaves more like raw intensity maximization.
- N_ϕ **(`objective_nphi`):** angular sampling density; higher values reduce noise at higher compute cost.