Written exam at Linear Algebra and Geometry

CEN 1.1, CEN 1.2, CEN 1.3

January 29, 2018, 08 PM, hall ACB

Without partial exam

1. The orthogonal complement of a subspace of an Euclidian space.

2. Let be quadratic form $f : \mathbf{R}^3 \to \mathbf{R}$,

$$f(\bar{x}) = (x^1)^2 - 2x^1x^2 + (x^2)^2 + 2x^2x^3 + 4x^3x^1 + 2(x^3)^2, \forall \bar{x} = (x^1, x^2, x^3) \in \mathbf{R}^3.$$

a) Find the matrix of f relative to the canonical basis of $\mathbf{R}^3$;
b) Find a canonical form for the quadratic form f;
c) Find the corresponding basis for the canonical form of f found above.

With partial exam

1. Distances in space (formulae, one proof).

2. Let us consider the point $A(1, -1, 1)$, the straight line $d : \begin{cases} x + y - z = 0 \\ x - y + z = 0 \end{cases}$ and the sphere $S : x^2 + y^2 + z^2 - 1 = 0$.

a) Write the equations of a line g which is parallel with line d and passes by the point A;

b) Find the coordinates of the orthogonal projection of the point A on the line d;

c) Find the equation of a sphere S' which is the symmetric sphere of the sphere S with respect to the point A.

Common subjects

3. Let be the following points $A(1, -1, 1)$, $B(1, 0, 1)$, $C(-1, 2, 1)$, $D(0, 1, 2)$ and the plane $\pi : x + y + z - 1 = 0$

a) Check if A, B, C, D are coplanar points or not;
b) Compute the distance form the point A to the plane (BCD);
c) Study the relative position of the line AB with respect to the plane π and find $\sin\varphi$, where $\varphi = m(\widehat{AB, \pi})$.

4. Let be the curve γ given by the scalar parametric equations:

$$\begin{cases} x = \cos t \\ y = \sin t \\ z = \cos t \end{cases}, \ t \in \mathbf{R}.$$

a) Find the unit vectors of the Frenét frame associated to the curve γ in the point $M(t = 0)$.

b) Write the equations of the coordinates lines and the coordinates planes of the Frenét frame associated to the curve γ in the point $M(1, 0, 1)$;

c) Compute the curvature and the torsion of the curve γ in the point $M(t = 0)$.