Written Exam at Linear Algebra and Geometry

Group **CEN 1.2**, 2017, February 03, 14 pm, Hall ACB

Without Partial Exam

1. Orthogonal complement of a subspace of a Euclidean space (definitions, two properties, one proof) [1p + 2p def. + 3p prop. + 4p proof]

2. Let $f : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear map with the matrix $A = \begin{pmatrix} 4 & -2 & 1 \\ -2 & 4 & 1 \\ 1 & 1 & 1 \end{pmatrix}$, relative to the canonical basis. Determine:

a) Eigenvalues of f;

b) Eigenvectors of f;

c) Compute A^n, $n \in \mathbb{N}^*$.

With Partial Exam

1. Scalar product, vector product, mixed product (definitions, formulas, properties) [1p + 3p + 3p +3p]

2. Let us consider the following points $A(1,0,0)$, $B(0,1,0)$, $C(0,0,1)$, $D(1,1,1)$.

a) Prove that these four points are not coplanar;

b) Compute the volume of the tetrahedron $OABC$;

c) Compute $d\,(D,(ABC))$.

Common Tasks

3. Let us consider a point $A(1,0,1)$ and a straight line d, given by the scalar parametric equations $\begin{cases} x &= & 1+t \\ y &= & -1+2t \\ z &= & t \end{cases}$, $t \in \mathbb{R}$.

a) Compute the distance from the point A to the line d;

b) Write the equation of the plan determined by the point A and the line d;

c) Find the coordinates of the orthogonal projection of the point A on the line d.

4. Let be Γ a quadratic surface, given by the cartesian equation:

$$2x^2 + 2y^2 + 2z^2 + 2xy + 2yz + 2zx + 4x + 4y + 4z + 2 = 0.$$

a) Compute δ şi Δ. Who is this quadratic surface?

b) Write the equation of the tangent plan at Γ in the point $A(0,0,-1)$;

c) Write the equations of the tangent plans at Γ who are parallel with the plan xOy.

Note: All tasks are compulsory. Each task is noted from 1 to 10. The note of the work is the arithmetic average of the scores of the four tasks. Working time is two hours.