Let's analyze the EOM for the field

$\mu^{1,1} \in \Omega^0(\mathbb{R}) \otimes PV^{1,1}(\mathbb{C}^5)$.

Kinetic term $\displaystyle\int_{\mathbb{R}\times\mathbb{C}^5} \gamma\left(d\mu + \tfrac{1}{2}\{\mu,\mu\} \vee \Omega\right)$

$\Rightarrow$ $\mu^{1,1}$ pairs against the component of γ in $\Omega^0(\mathbb{R}) \otimes \Omega^{1,3}(\mathbb{C}^5)$

What components of γ can this interact w/ ?

Expanding $\gamma\,\partial\gamma\,\partial\gamma$, find

$$\gamma^{1,3}\left(\partial\gamma^{1,1}\partial\gamma^{1,1} + \partial\gamma^{1,0}\partial\gamma^{1,2}\right)$$

w/ $\gamma^{p,q} \in \Omega^{\cdot}(\mathbb{R}) \otimes \Omega^{p,q}(\mathbb{C}^5)$

second of these involves fields of odd ghost #.

$\Rightarrow$ EOM from varying $\gamma^{1,3}$ is

$$\left(d\mu^{1,1} + \tfrac{1}{2}\{\mu^{1,1},\mu^{1,1}\}\right)\vee\Omega + \partial\gamma^{1,0}\partial\gamma^{1,2} = 0.$$

Kevin's suggestion

Consider the equation

$$\left(\bar\partial\mu + \tfrac{1}{2}\{\mu,\mu\}\right)\vee\Omega + B\wedge\eta = 0 \qquad \left(\begin{array}{l}\mu \in PV^{1,1}\\ B \in \Omega^{2,0}_{cl}\\ \eta \in \Omega^{2,2}\end{array}\right).$$

Then $\mathrm{Im}(B\wedge -)$ gives a foliation.

(maybe need to divide by Ω?) and eq^n describes def of hol str transverse to it.