Holomorphic M-theory and the SU(4)-invariant twist of type IIA

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BCOV with potentials refers to a modification of minimal BCOV theory where we impose certain constraints on the fields so as to make the Poisson BV structure of the theory invertible. These constraints amount to requiring that certain fields lie in the image of the divergence operator ∂, or better yet replacing ∂-closed fields in a summand $\mathrm{PV}^{d,\bullet}$ with all of $\mathrm{PV}^{d,\bullet}$ and using a fixed choice of splitting of $\partial : \mathrm{PV}^{d,\bullet} \to \mathrm{PV}^{d-1,\bullet}$ to rewrite $\mathrm{PV}^{d,\bullet} \cong \mathrm{im}\partial \oplus \ker \partial$.

Under the conjectures of Costello-Li that describe twisted type II supergravity in terms of BCOV theory, these primitives correspond to certain components of Ramond-Ramond fields, which are chosen as potentials for Ramond-Ramond field strengths.

1 BCOV theory with potentials on a CY4

Let X be a CY4. BCOV theory on X with potentials will be the $\mathbb{Z}/2$ graded BV theory defined as follows. Fix splittings $C_1 : (\mathrm{im}\partial \subset \mathrm{PV}^{3,\bullet}) \to \mathrm{PV}^{4,\bullet}$ of

$$0 \to \ker \partial \to \mathrm{PV}^{4,\bullet} \to (\mathrm{im}\partial \subset \mathrm{PV}^{3,\bullet} \to 0$$

and $C_2 : (\mathrm{im}\partial \subset \mathrm{PV}^{2,\bullet}) \to \mathrm{PV}^{3,\bullet}$ of

$$0 \to \ker \partial \to \mathrm{PV}^{3,\bullet} \to (\mathrm{im}\partial \subset \mathrm{PV}^{2,\bullet}(X)) \to 0.$$

And let $\phi_{C_1} : \mathrm{PV}^{4,\bullet} \cong (\ker \partial \subset \mathrm{PV}^{4,\bullet}) \oplus (\mathrm{im}\partial \subset \mathrm{PV}^{3,\bullet})$, $\phi_{C_2} : \mathrm{PV}^{3,\bullet} \cong (\ker \partial \subset \mathrm{PV}^{3,\bullet}) \oplus (\mathrm{im}\partial \subset \mathrm{PV}^{2,\bullet})$ be the resulting isomorphisms.

- The fields of the theory are

$$\begin{aligned}\mathcal{E}^{C_1,C_2}_{mBCOV} = \mathrm{PV}^{0,\bullet} \oplus (\ker \partial \subset \mathrm{PV}^{1,\bullet}) &\oplus (\mathrm{im}\partial \subset \mathrm{PV}^{2,\bullet} \oplus \ker \partial \subset \mathrm{PV}^{3,\bullet}) \\ &\oplus (\mathrm{im}\partial \subset \mathrm{PV}^{3,\bullet} \oplus \ker \partial \subset \mathrm{PV}^{4,\bullet})\end{aligned}$$

- The Poisson kernel is given by $(\partial \otimes 1)\delta_{Diag}$
- The L_∞ structure is defined as follows
  - $\ell_1 = \bar{\partial}$
  - ℓ_2 is a certain modification of the Schouten bracket, defined as follows. Let $[-,-]$ denote the usual Schouten bracket of polyvector fields. Then
    1. $\mu \in \mathrm{PV}^{0,\bullet}, \nu \in \ker\partial \subset \mathrm{PV}^{1,\bullet}$, $\ell_2(\mu,\nu) = [\mu,\nu]$.
    2. $\mu \in \mathrm{PV}^{0,\bullet}, \nu \in \mathrm{im}\partial \subset \mathrm{PV}^{2,\bullet}$, $\ell_2(\mu,\nu) = (-1)^{|\mu|-1}\partial[\mu, \phi^{1}_{C_1}\nu]$ and $\ell_2(\nu,\mu) = \partial[\mu, \phi^{-1}_{C_1}\nu]$
    3. $\mu \in \mathrm{PV}^{0,\bullet}, \nu \in (\mathrm{im}\partial \subset \mathrm{PV}^{3,\bullet}) \oplus (\ker\partial \subset \mathrm{PV}^{4,\bullet})$, $\ell_2(\mu,\nu) = (-1)^{|\mu|-1}\phi_{C_1}[\mu, \phi^{-1}_{C_2}\nu]$ and $\ell_2(\nu,\mu) = \phi_{C_1}[\mu, \phi^{-1}_{C_2}\nu]$.
    4. $\mu,\nu \in \ker\partial \subset \mathrm{PV}^{1,\bullet}$, $\ell_2(\mu,\nu) = [\mu,\nu]$.
    5. $\mu \in \ker\partial \subset \mathrm{PV}^{1,\bullet}, \nu \in (\mathrm{im}\partial \subset \mathrm{PV}^{2,\bullet}) \oplus (\ker\partial \subset \mathrm{PV}^{3,\bullet})$, $\ell_2(\mu,\nu) = (-1)^{|\mu|-1}\phi_{C_1}[\mu, \phi^{-1}_{C_1}\nu]$ and $\ell_2(\nu,\mu) = \phi_{C_1}[\phi^{-1}_{C_1}\nu, \mu]$
    6. $\mu \in \ker\partial \subset \mathrm{PV}^{1,\bullet}, \nu \in (\mathrm{im}\partial \subset \mathrm{PV}^{3,\bullet}) \oplus (\ker\partial \subset \mathrm{PV}^{5,\bullet})$, $\ell_2(\mu,\nu) = (-1)^{|\mu|-1}\phi_{C_2}[\mu, \phi^{-1}_{C_2}\nu]$ and $\ell_2(\nu,\mu) = \phi_{C_2}[\phi^{-1}_{C_2}\nu, \mu]$
    7. All other brackets vanish for degree reasons.

Proposition 1.1. The above in fact defines a (shifted) L_∞-structure.

Remark 1.2.
- Note that the complex underlying $\mathcal{E}^{C_1,C_2}_{mBCOV}$ does not arise as sections of a graded vector bundle due to the presence of the constraints.
- Recall that minimal BCOV theory has fields $\mathcal{E}_{mBCOV} = \oplus_{i\leq 3}(\ker\partial \subset \mathrm{PV}^{\text{ß},\bullet}$. As we have mentioned above, we may view the underlying complex of $\mathcal{E}^{C_1,C_2}_{mBCOV}$ as gotten by replacing $\ker\partial \subset \mathrm{PV}^{2/3,\bullet} \subset \mathcal{E}_{mBCOV}$ with $\mathrm{PV}^{3/4,\bullet}$ in the same degree, and using the isomorphisms ϕ_{C_1,C_2}. The L_∞ structure is gotten by simply transporting the ordinary Schouten-Nijenhuis bracket on

$$\mathrm{PV}^{0,\bullet} \oplus (\ker\partial \subset \mathrm{PV}^{1,\bullet} \oplus \mathrm{PV}^{3,\bullet}[] \oplus \mathrm{PV}^{4,\bullet}[]$$

  and applying the fact that since ∂ is a derivation of $[-,-]$, for $\mu \in \ker\partial$, $[\mu,\partial\gamma] = (-1)^{|\mu|-1}\partial[\mu,\gamma]$ and $[\partial\gamma,\mu] = \partial[\gamma,\mu]$. Hopefully this demystifies the above formulas.

- Suppose we were to naively try to define a BV pairing ω on $\mathcal{E}^{C_1,C_2}_{mBCOV}$ by $\omega(-,-) = \int(-) \wedge \partial^{-1}(-)$ and write an "action functional" using the above L_∞-structure. Then the resulting action functional would be equivalent to one coming from an honest BV pairing and an L_∞-structure involving a composition of ∂ and the ordinary Schouten-Nijenhuis bracket. Note that we could have accomplished the same thing by choosing splittings of ∂ from any $\mathrm{im}\partial \subset \mathrm{PV}^{d_1,\bullet}$, $\mathrm{im}\partial \subset \mathrm{PV}^{d_2,\bullet}$ such that $d_1 + d_2 \neq d-1$. However, it seems like these two choices of splittings are favored in a sense (articulated below).

2 Dimensional Reduction

Let's consider the holomorphic twist of M-theory on $\mathbb{R} \times \mathbb{C}^\times \times \mathbb{C}^4$. We may decompose the fields as

- $$\mu = \mu_{01} + \mu_{10} \in \Omega^\bullet(\mathbb{R}) \otimes \begin{pmatrix} (\mathrm{PV}^{0,\bullet}(\mathbb{C}^\times) \otimes \mathrm{PV}^{1,\bullet}(\mathbb{C}^4)) \\ \oplus(\mathrm{PV}^{1,\bullet}(\mathbb{C}^\times) \otimes \mathrm{PV}^{0,\bullet}(\mathbb{C}^4)) \end{pmatrix} \to \mathrm{PV}^{0,\bullet}(\mathbb{C}^\times \times \mathbb{C}^4) \Bigg).$$

- $\gamma = \gamma_{01} + \gamma_{10} \in \Omega^\bullet(\mathbb{R}) \otimes (\Omega^{0,\bullet}(\mathbb{C}^\times) \otimes \Omega^{1,\bullet}(\mathbb{C}^4) \oplus \Omega^{1,\bullet}(\mathbb{C}^\times) \otimes \Omega^{0,\bullet}(\mathbb{C}^4))$.

Proposition 2.1. There is a homomorphism of L_∞-algebras from the $\bar{\partial}_{\mathbb{C}^\times}$-cohomology of M theory on $\mathbb{R} \times \mathbb{C}^\times \times \mathbb{C}$ to $\Omega^\bullet(\mathbb{R}^2) \times \mathcal{E}^{C_1,C_2}_{mBCOV}$ given by

- $[\mu_{01}] \mapsto \mu^1 \in \ker \partial \subset \mathrm{PV}^{1,\bullet}(\mathbb{C}^4) \subset \mathcal{E}^{C_1,C_2}_{mBCOV}$
- $[\mu_{10}] \mapsto \mu^3 = \partial_{\mathbb{C}^4}(\mu_{10}\Omega^{-1}_{\mathbb{C}^4}) \subset \mathrm{im}\partial \subset \mathrm{PV}^{3,\bullet} \subset \mathcal{E}^{C_1,C_2}_{mBCOV}$ where $\Omega_{\mathbb{C}^4}$ denotes the holomorphic volume form on $\mathbb{C}^4$.
- $[\gamma_{01}] \mapsto \mu^2 = \partial_{C^4}(\gamma_{01} \vee \Omega^{-1}_{\mathbb{C}^4}) \subset \mathrm{im}\partial \subset \mathrm{PV}^{2,\bullet} \subset \mathcal{E}^{C_1,C_2}_{mBCOV}$.
- $[\gamma_{10}] \mapsto \mu^0 \in \mathrm{PV}^0 \subset \mathcal{E}^{C_1,C_2}_{mBCOV}$

preserving the relevant pairings.

That is, the reduction of the holomorphic M theory on a holomorphic circle should be the SU(4) invariant twist of IIA.