Twists of supersymmetric gauge theories

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1 One-loop anomalies

1.1 Holomorphic Chern-Simons

Proposition 1.1. Let X be any flat Calabi-Yau manifold of odd complex dimension d. The one-loop anomaly to quantizing holomorphic Chern-Simons theory with values in \mathfrak{g} on X is proportional to the local cocycle

$$\int_X \operatorname{Tr}_{\operatorname{ad}}(\alpha \underbrace{\partial \alpha \cdots \partial \alpha}_{d}) \in C^{\bullet}_{\operatorname{loc}}(\mathscr{E}_{\operatorname{hCS}}).$$

where the trace is taken in the adjoint representation of g.

Remark 1.1. If we expand α in components as $\alpha = c + A^{0,1} + \sum_{i \geq 2} A^{0,i}$ where $c \in C^{\infty}(X) \otimes \mathfrak{g}$ and $A^{0,i} \in \Omega^{0,i}(X) \otimes \mathfrak{g}$, then up to some nonzero factor we can write the anomaly cocycle as

$$\int_{X} \operatorname{Tr}_{\operatorname{ad}}(c\partial A^{0,1} \cdots \partial A^{0,1}) = \int \operatorname{Tr}_{\operatorname{ad}}\left(c(F_{A^{0,1}})^{d}\right)$$