Fields and supercharges live in the following places.

	6d	4d	3d
$A \in V_n \otimes \mathfrak{g}$	$V_6 \otimes \mathfrak{g}$	$V_4 \otimes \mathfrak{g}$	$V_3 \otimes \mathfrak{g}$
$\lambda \in \Sigma \otimes \mathfrak{g}$	$S_+ \otimes W_+ \otimes \mathfrak{g}$	$(S_+ \oplus S_-) \otimes \mathfrak{g}$	$S \otimes \mathfrak{g}$
$\psi \in \Psi$	$S_- \otimes U$	$(S_+ \otimes R) \oplus (S_- \otimes R^*)$	$S \otimes R$
$\phi \in \Phi$	$W_+ \otimes U$	$R \oplus R^*$	R
$Q \in \Sigma$	$S_+ \otimes W_+$	$S_+ \oplus S_-$	S
$Q^* \in S^*$	S_-	$S_+ \oplus S_-$	S

Here Φ is a representation of $\mathfrak{g}$ and Ψ is a representation of $\mathfrak{so}(n) \oplus \mathfrak{g}$. To define the action, including the background field Q, we need the following structures, all $\mathfrak{g}$-equivariant.

1. A first-order differential operator $\not{\partial}\colon \Psi \to \Psi^*$.
2. An operator $\rho_\Psi\colon V_n \otimes \Psi^* \to \Psi$.
3. A gamma pairing $\Gamma_\Psi\colon \operatorname{Sym}^2 \Psi \to V_n$.
4. A moment map pairing $\mu\colon \operatorname{Sym}^2 \Phi \to \mathfrak{g}$.
5. An additional $\mathfrak{g}$-equivariant map $Y\colon \Sigma \otimes \Psi \to \Phi$, or equivalently, using μ, either a map $Y_\mu\colon \Sigma \otimes \Psi \otimes \Phi \to \mathfrak{g}$ or $Y_\mu^*\colon \Sigma \otimes \Phi \to \mathfrak{g} \otimes \Psi^*$.

With this data in mind, we define the action, and the coupling to supercharges Q, as follows. I'll write η for the pairing on $V_n \otimes \mathfrak{g}$ induced from the metric and the Killing form, and κ for the Killing form alone.

$$
\begin{aligned}
S_{\text{gauge}} &= \int \left\langle -\frac{1}{4} F_A \wedge * F_A + \frac{1}{2}(\lambda, \not{D}_A \lambda) \right\rangle - (\mathrm{d}_A c, A^*) + ([\lambda, c], \lambda^*) + \frac{1}{2}([c,c], c^*) \\
S_{\text{matter}} &= \int -\frac{1}{2}(\mathrm{d}\phi \wedge * \mathrm{d}\phi) + \frac{1}{2}(\psi, \not{\partial}\psi) \\
I_{\text{couple}} &= \int g\big(\eta(A, \Gamma_\Psi(\psi,\psi)) - \frac{1}{2}\eta(A, \mathrm{d}_A \mu(\phi,\phi)) - \kappa \circ Y_\mu(\lambda, \psi, \phi) + (\psi^*, [c, \psi])_U + \\
&\qquad + (\phi^*, [c, \phi])\big) + g^2\left(\mu(\phi,\phi), \mu(\phi,\phi)\right) \\
I^{(1)}_{\text{gauge}}(Q) &= \int (\Gamma(Q, \lambda), A^*) + \frac{1}{2}(\rho(F_A), \lambda^*) \\
I^{(1)}_{\text{matter}}(Q) &= \int (\phi^*, Y(Q, \psi)) + (\psi^*, \rho(\mathrm{d}\phi) Q) \\
I^{(1)}_{\text{couple}}(Q) &= \int g(\psi^*, \kappa \circ \rho_\Psi \circ (1 \otimes Y_\mu^*)(A, Q, \phi)) + \frac{1}{2} g(\lambda^*, Q \otimes \mu(\phi,\phi)) \\
I^{(2)}_{\text{gauge}}(Q_1, Q_2) &= \int \frac{1}{4}(\Gamma(Q_1, Q_2), \Gamma(\lambda^*, \lambda^*)) - \frac{1}{2}(Q_1, \lambda^*)(Q_2, \lambda^*) - (\iota_{\Gamma(Q_1,Q_2)} A, c^*) \\
I^{(2)}_{\text{matter}}(Q_1, Q_2) &= \int \frac{1}{4}(\Gamma(Q_1, Q_2), \Gamma_{\Psi^*}(\psi^*, \psi^*)).
\end{aligned}
$$

With all of this, along with an appropriate version of the "3ψ-rule" – which should say something about the compatibility of these structures, along with the claim that the map from either $\wedge^2 S_+$, $S_+ \otimes S_-$ or $\operatorname{Sym}^2 S$ to V_n is an isomorphism – I speculate the supersymmetry calculation should work symmetrically. The 3ψ rule should just be the following.

Proposition 0.1. Suppose $Q_1, Q_2 \in \Sigma$ and $Q^* \in S$. Then

$$(Q_1, Q^*) Q_2 + (Q_2, Q^*) Q_1 = \rho(\Gamma(Q_1, Q_2)) Q^*$$