The Lagrangian density is, in a local coordinate patch, given by

$$\mathcal{L}(\omega, e) = \varepsilon_{abcd}\Omega^{ab} \wedge e^c \wedge e^d.$$

Compute its first variation under the gauge transformation $e^a \mapsto e^a + \partial f^a + \omega^{ab} f_b$. We find

$$\begin{aligned}
\delta\mathcal{L}(\omega, e) &= \varepsilon_{abcd}\Omega^{ab} \wedge (\partial^c f \wedge e^d + f\omega^c \wedge e^d + e^c \wedge \partial^d f + fe^c \wedge \omega^d) \\
&= (\varepsilon_{abcd} + \varepsilon_{abdc})\Omega^{ab} \wedge (\partial^c f \wedge e^d + f\omega^c \wedge e^d) \\
&= 0.
\end{aligned}$$

In the index notation, the order in which we write the forms isn't important, only the order of the indices.

Now, here's what I think this calculation really means. First, how should we interpret the Palatini action? I'll explain it in n dimensions. We can take e, which is an $\mathbb{R}^n$ valued 1-form, and produce a $\wedge^{n-2}\mathbb{R}^n$ valued $n-2$ form, which I'll write as

$$e^{a_1} \wedge e^{a_2} \wedge \cdots \wedge e^{a_{n-2}}.$$

This is totally antisymmetric in the a_i indices. What's more, the curvature Ω of the connection ω is an $\mathfrak{so}(n)$ valued 2-form, and $\mathfrak{so}(n)$ is canonically isomorphic to $\wedge^2\mathbb{R}^n$. Thus we produce a $\wedge^n\mathbb{R}^n$ valued n form which I'll write as

$$\Omega^{a_1 a_2} \wedge e^{a_3} \wedge e^{a_4} \wedge \cdots \wedge e^{a_n}.$$

Our choice of basis gives us a canonical isomorphism $\wedge^n\mathbb{R}^n \cong \mathbb{R}$, thus an $\mathbb{R}$ valued n form which I could reasonably write as

$$\frac{1}{n!}\varepsilon_{a_1\cdots a_n}\Omega^{a_1 a_2} \wedge e^{a_3} \wedge e^{a_4} \wedge \cdots \wedge e^{a_n}.$$

Note that while the wedge product of 1-forms remains antisymmetric, $e^a \wedge e^b$ is still not zero if $a \neq b$, because we're pairing different components of the vielbein. The symmetry I used to see that the variation vanishes is on the level of components: it was that

$$e^a \wedge (d_\omega f)^b = -e^b \wedge (d\omega f)^a.$$