The underlying supervector space for the $6d$ $\mathcal{N}=(1,0)$ supersymmetry algebra has the following description.

1. The bosonic piece of the 6d $\mathcal{N}=(1,0)$ supersymmetry algebra is
$$(\mathfrak{so}(6,\mathbb{C}) \times \mathfrak{sl}(2,\mathbb{C})_R) \ltimes V$$
where
   - $V=\mathbb{C}^6$ is the fundamental representation of $\mathfrak{so}(6,\mathbb{C})$.
   - $\mathfrak{sl}(2,\mathbb{C})_R$ is the R-symmetry algebra. It acts trivially on V.
2. The fermionic piece of the algebra is
$$S_+ \otimes_{\mathbb{C}} W$$
where $S_+=\mathbb{C}^4$ is the chiral irreducible spin representation of $\mathfrak{so}(6,\mathbb{C})$ and $W_R=\mathbb{C}^2$ is the two-dimensional vector space.

The Lie bracket for the superalgebra is determined by the rules:

1. There is a term in the Lie bracket for the fundamental action of $\mathfrak{so}(6,\mathbb{C})$ on V and the spinorial action of $\mathfrak{so}(6,\mathbb{C})$ on S_+.
2. There is a term in the Lie bracket given by the action of $\mathfrak{sl}(2,\mathbb{C})$ on $S_+ \otimes_{\mathbb{C}} W_R$ by rotating the $W_R=\mathbb{C}^2$ factor.
3. The final term in the Lie bracket is defined by the composition
$$\Gamma : (S_+ \otimes \mathbb{C}^2) \otimes (S_+ \otimes \mathbb{C}^2) = (S_+ \otimes S_+) \otimes (\mathbb{C}^2 \otimes \mathbb{C}^2) \xrightarrow{\wedge \otimes \omega_{std}} V$$
where we have used the isomorphism $\wedge^2 S_+ \cong V$ and the standard symplectic form on $\mathbb{C}^2$.

0.1 Pure gauge theory

The field content for $6d$ $\mathcal{N}=(1,0)$ pure gauge theory consists of a gauge field
$$A \in C^\infty(\mathbb{R}^4, V) \otimes \mathfrak{g} = \Omega^1(\mathbb{R}^6, \mathfrak{g})$$
and a spinor
$$\psi \in C^\infty(\mathbb{R}^4, S_+ \otimes W_R) \otimes \mathfrak{g}.$$
As a convention, we choose a basis for W_R given by symbols $\{+,-\}$ and we accordingly decompose the spinor as $\psi = \psi_+ + \psi_-$ where $\psi_\pm \in C^\infty(\mathbb{R}^4, S_+) \otimes \mathfrak{g}$. BW: yikes, bad notation?

0.2 The hypermultiplet

0.3 Holomorphic language

A complex structure on $\mathbb{R}^6 \cong \mathbb{C}^3$ determines an embedding $\mathfrak{gl}(3,\mathbb{C}) \hookrightarrow \mathfrak{so}(6,\mathbb{C})$. With respect to this inclusion, the $\mathfrak{so}(6,\mathbb{C})$ representations split as
$$V = V^{1,0} \oplus V^{0,1}$$
$$S_+ = \mathbb{C} \oplus \wedge^2(V^{0,1})$$

where $V^{1,0}$ is the fundamental representation of $\mathfrak{gl}(3,\mathbb{C})$ and $V^{0,1}$ its complex conjugate.

Accordingly, the fields of the pure gauge theory decompose as

$$A = A^{1,0} + A^{0,1} \in \Omega^{1,0}(\mathbb{C}^3, \mathfrak{g}) \oplus \Omega^{0,1}(\mathbb{C}^3, \mathfrak{g})$$

and

$$\psi = \phi + \chi^{0,2} \in C^{\infty}(\mathbb{C}^3, W_R) \otimes \mathfrak{g} \oplus \Omega^{0,2}(\mathbb{C}^3, W_R) \otimes \mathfrak{g}.$$

In our notation above, after we choose a basis $\{+,-\}$ for W_R we can decompose the spinor further into $\phi_{\pm} \in C^{\infty}(\mathbb{C}^3) \otimes \mathfrak{g}$ and $\chi_{\pm}^{0,2} \in \Omega^{0,2}(\mathbb{C}^3) \otimes \mathfrak{g}$.