

Notes on 10d Super Yang-Mills

Chris Elliott, Pavel Safronov and Brian Williams

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1 Minimal Super Yang-Mills

Our starting point will be the theory complexifying the usual 10d super Yang-Mills theory. Fix a complex reductive gauge group G with Lie algebra \mathfrak{g} . The ordinary fields of super Yang-Mills theory on \mathbb{R}^{10} consist of a boson: a connection A on the trivial G -bundle, and a fermion: a \mathfrak{g} -valued section λ of the Weyl spinor bundle associated to the spinor representation S_+^{-1} . These fields are acted upon by the group of gauge transformations – G -valued functions on \mathbb{R}^{10} .

We can model the stack of fields modulo gauge transformations infinitesimally near the point 0 by the corresponding BRST complex. This is the local super Lie algebra

$$L_{\text{BRST}} = \Omega^0(\mathbb{R}^{10}; \mathfrak{g}) \rightarrow \Omega^1(\mathbb{R}^{10}; \mathfrak{g}) \oplus \Omega^0(\mathbb{R}^{10}; \Pi S_+ \otimes \mathfrak{g})$$

with the de Rham differential, placed in cohomological degrees 0 and 1, with bracket induced from the Lie bracket on \mathfrak{g} .

The action functional in 10d super Yang-Mills is given by

$$S(A, \lambda) = \int_{\mathbb{R}^{10}} \langle F_A \wedge *F_A + (\lambda, \not{D}_A \lambda) \rangle,$$

where $\langle - \rangle$ denotes an invariant pairing on \mathfrak{g} , and $(,)$ denotes a scalar-valued pairing $S_+ \otimes S_+ \rightarrow \mathbb{C}$ (there will be a unique such pairing, up to rescaling, characterized by the condition that $(\rho(v)\lambda_1, \rho(v)\lambda_2) = (\lambda_1, \lambda_2)$ for each $v \in \mathbb{C}^{10}$, where ρ denotes Clifford multiplication).

We can re-encode this data in terms of the classical BV complex. This is the L_∞ -algebra whose underlying cochain complex takes the form

$$\Omega^0(\mathbb{R}^{10}; \mathfrak{g}) \longrightarrow \Omega^1(\mathbb{R}^{10}; \mathfrak{g}) \xrightarrow{d \circ d} \Omega^2(\mathbb{R}^{10}; \mathfrak{g}) \longrightarrow \Omega^3(\mathbb{R}^{10}; \mathfrak{g})$$

$$\Omega^0(\mathbb{R}^{10}; \Pi S_+ \otimes \mathfrak{g}) \xrightarrow{*d} \Omega^1(\mathbb{R}^{10}; \Pi S_+ \otimes \mathfrak{g}),$$

with degree 2 and 3 brackets given by

¹If we didn't complexify we would instead consider $G_{\mathbb{R}}$ a compact connected Lie group, and a section of the Majorana-Weyl spinor bundle, which necessitates working in Lorentzian signature.

1.1 On-Shell Supersymmetry Action

2 Baulieu's 10 Model

2.1 Off-Shell Action of a Scalar Supersymmetry

3 Homotopy Data

3.1 Homotopy Transfer of the Scalar Supersymmetry