Matter, uniformly

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Definition 0.1. Let $\mathfrak{g}$ be a Lie algebra. An even (resp. odd) **supersymmetric matter pair** is a pair $(\mathbb{S}, U)$ where

(1) $\mathbb{S}$ is a $\mathbb{Z}/2$-graded spinorial representation equipped with an even (resp. odd) nondegenerate V-valued pairing

$$\Gamma : \wedge^2(\mathbb{S})^0 \xrightarrow{\cong} V$$

(resp. $\Gamma : \wedge^2(\mathbb{S})^1 \xrightarrow{\cong} V$), and

(2) U is an even (resp. odd) symplectic $\mathfrak{g}$-representation.

Lemma 0.1. For a fixed Lie algebra $\mathfrak{g}$, a complete classification of the supersymmetric matter pairs (Brian: Should we say minimal here? What about dimension 3 and 2?) (for dimensions $n \geq 2$) are:

(1) Dimension $n = 4$. There is an odd supersymmetric matter pair $(\mathbb{S} = S_+ \oplus S_-, U = R \oplus R^*)$ where the odd pairing $\Gamma : S_+ \otimes S_- \xrightarrow{\cong} V$ is the usual Γ pairing. Here, R is a $\mathfrak{g}$-representation, and in the odd symplectic representation $U = R \oplus R^*$ we view R as even and R^* as odd.

(2) Dimension $n = 6$. There are two even supersymmetric matter pairs: $(\mathbb{S} = S_+ \otimes W_+, U)$ and $(\mathbb{S} = S_- \otimes W_-, U)$ where $W_\pm$ is a 2-dimension symplectic vector space and U is a symplectic $\mathfrak{g}$-representation. The even nondegenerate pairings are the usual Γ pairings $\Gamma : \wedge^2(S_\pm) \xrightarrow{\cong} V$

Proposition 0.2. (Brian: how to state 3ψ-rule)

(1) Dimension $n = 4$. Suppose $Q_+ + Q_- \in S_+ \oplus S_-$. Then, the following diagram commutes

$$\begin{tikzcd} S_+ \otimes S_- \arrow[rr, "\Gamma"] \arrow[dr, "(Q_+ + Q_-, -)"'] & & V \arrow[dl, "\rho(-)(Q_+ + Q_-)"] \\ & S_+ \oplus S_- & \end{tikzcd}$$

(2) Dimension $n = 6$. Suppose $Q \in S_\mp$. Then, the following diagram commutes

$$\begin{tikzcd} \wedge^2 S_\pm \arrow[rr, "\Gamma"] \arrow[dr, "(Q, -)"'] & & V \arrow[dl, "\rho(-)Q"] \\ & S_\pm & \end{tikzcd}$$

To a supersymmetric matter pair we will associate a free BV theory whose underlying BRST fields consist of:

- a scalar $\phi \in C^\infty(\mathbb{R}^n; \mathbb{S}^*)$;
- a negative Weyl spinor valued in U; $\psi \in C^\infty(\mathbb{R}^4; S_- \otimes U)$.

Definition 0.3. The BRST theory associated to a supersymmetric matter pair (even or odd) $(\mathbb{S}, U)$ has underlying bundle of BRST fields: