Matter, uniformly

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Definition 0.1. Let \mathfrak{g} be a Lie algebra. An even (resp. odd) **supersymmetric matter pair** is a pair (S, U) where

(1) S is a $\mathbb{Z}/2$ -graded spinorial representation equipped with an even (resp. odd) nondegenerate V-valued pairing

$$\Gamma: \wedge^2(\mathbb{S})^0 \xrightarrow{\cong} V$$

(resp. $\Gamma : \wedge^2(\mathbb{S})^1 \xrightarrow{\cong} V$), and

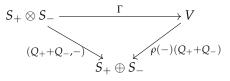
(2) *U* is an even (resp. odd) symplectic g-representation.

Lemma 0.1. For a fixed Lie algebra \mathfrak{g} , a complete classification of the supersymmetric matter pairs (Brian: Should we say minimal here? What about dimension 3 and 2?) (for dimensions $n \ge 2$) are:

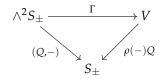
- (1) Dimension n=4. There is an odd supersymmetric matter pair $(S=S_+\oplus S_-, U=R\oplus R^*)$ where the odd pairing $\Gamma:S_+\otimes S_-\stackrel{\cong}{\to} V$ is the usual Γ pairing. Here, R is a \mathfrak{g} -representation, and in the odd symplectic representation $U=R\oplus R^*$ we view R as even and R^* as odd.
- (2) Dimension n=6. There are two even supersymmetric matter pairs: $(S=S_+\otimes W_+,U)$ and $(S=S_-\otimes W_-,U)$ where W_\pm is a 2-dimension symplectic vector space and U is a symplectic \mathfrak{g} -representation. The even nondegenerate pairings are the usual Γ pairings $\Gamma: \wedge^2(S_\pm) \xrightarrow{\cong} V$

Proposition 0.2. (Brian: how to state 3ψ -rule)

(1) Dimension n=4. Suppose $Q_++Q_-\in S_+\oplus S_-$. Then, the following diagram commutes



(2) Dimension n = 6. Suppose $Q \in S_{\pm}$. Then, the following diagram commutes



To a supersymmetric matter pair we will associate a free BV theory whose underlying BRST fields consist of:

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- a scalar $\phi \in C^{\infty}(\mathbb{R}^n; \mathbb{S}^*)$;
- a negative Weyl spinor valued in U; $\psi \in C^{\infty}(\mathbb{R}^4; S_- \otimes U)$.

Definition 0.3. The BRST theory associated to a supersymmetric matter pair (even or odd) (S, U) has underlying bundle of BRST fields: