From: **Chris Elliott** celliott@ihes.fr
Subject: Re: Twisted gauge theory paper
Date: February 22, 2019 at 5:11 AM
To: Pavel Safronov psafronov@gmail.com
Cc: Brian Williams brianwilliams.math@gmail.com

CE

Hi Pavel,

I'm with you almost entirely, especially your summary of points 1-4, except that Berkovits's theory doesn't look equivalent to 10d SYM to me -- isn't it literally the sum 10d SYM with a bunch of free scalars, and with a modified supersymmetry action? I'd suggest modifying your proposal to say that there's an equivalence between Baulieu's theory and the SYM theory, and a projection map from Berkovits's theory to the SYM theory (with a section), then apply homotopy transfer for the appropriate subalgebras of the supersymmetry algebras under those two maps and ask whether they coincide after restricting to their intersection.

Best,

Chris

On 21/02/2019 20:53, Pavel Safronov wrote:

> Thanks for your answer. Let me quickly reply to some of the points you mention, I'll need to do more computations to answer the rest. I won't be around tomorrow, so probably I'll answer next week.
>
>> On Feb 21, 2019, at 7:46 PM, Chris Elliott <celliott@ihes.fr> wrote:
>>
>> First of all, for meeting, how about 9am PST on Wednesday, which is 6pm in Europe?
>
> Yes, it works for me.
>
>> Thanks for your detailed response Pavel, maybe I can try to explain what I was thinking again, sorry for not being very clear. We would like to say, given a super Yang-Mills theory and a Lorentz x R-symmetry orbit Q in the space of square zero supercharges, there is a unique thing called the Q-twisted theory. The obstacle we're running into is that the supertranslation action typically isn't defined off-shell without introducing auxiliary fields, and for different Q's in the same orbit we might need to choose different choices for the auxiliary fields.
>>
>> So I guess what you're talking about in your e-mail is trying to quantify how bad this problem is, is that right?
>
> I would formulate it slightly differently, but the content is the same. There's the 10d SYM which admits an action of SO(10), but it's not supersymmetric (or at least not obviously supersymmetric). Then there's Baulieu's theory with 1 auxiliary field which admits an action of SU(5). We know that this theory is equivalent to 10d SYM (as theories with an SU(5) action). But Baulieu's theory in addition has an action of the scalar supercharge. Finally, there's Berkovits's theory with 7 auxiliary fields which admits an action of Spin(7) x SO(2). Again, it is equivalent to 10d SYM as a theory with a Spin(7) x SO(2)-action. But it also admits an action of 9 supercharges (1 scalar and 8 in the spin representation of Spin(7)). More precisely, there's a partial supertranslation algebra with a 9d odd space and a 1d even space which is Spin(7) x SO(2)-equivariant (the even space has, say, weight 1 with respect to SO(2)) and Berkovits's theory has an action of this supertranslation algebra.
>
>> So let me think about that. As you point out, in dimension 9 and below, in the maximally SUSY case, we can always use Berkovits's auxiliary fields, so we have a uniform choice for all such twists. You asked whether the SU(4) action was the same whether you come via Spin(7) or SU(5) -- but aren't the embeddings in SO(10) isomorphic? The embedding in SO(10) is a choice of 10d rep of SU(4), and I think in both cases we're defining the rep V (x)_R C + C^2 where V is the 4d complex fundamental rep of SU(4)?
>
> Yes, they are. I haven't done this computation previously, but I agree that it splits like you say. Here's the computation. The 10d defining representation of SO(10) splits as L + L^* as a representation of SU(5), where L is the 5d defining representation. Next, let's say M is the 4d defining rep of SU(4). Then L + L^* splits as (M + C) + (M^* + C) as an SU(4) rep.
>
> Next, the 10d defining representation of SO(10) splits as V_8 + C^2 as a representation of SO(8) or S_8 + C^2 as a representation of Spin(7), where S_8 is the spin representation. But then it further splits as (M + M^*) + C^2 as a rep of SU(4) since M and M^* are the semi-spin representations of Spin(6) = SU(4). It is also easy to see that the two representations have the same bilinear form, so the two embeddings of SU(4) in SO(10) are conjugate.
>
> You can also do the same computation for the 10d semi-spin representation. What you observe is that the SU(5) scalar supercharge lies in C + S_8, i.e. the 9d space of supercharges in Berkovits's theory. So, here's a precise thing one can prove:
>
> 1) Enhance the action of the single supercharge in Baulieu's theory to a Lie action (this is an L_infinity action since the action is only preserved up to gauge transformations)
>
> 2) Enhance the action of the partial supertranslation algebra in Berkovits's theory to a Lie action (again, it has the same problem that the action is only preserved up to gauge transformations)
>
> 3) Homotopy transfer both supertranslation actions to the plain 10d SYM (in the BV formulation; note that the two points above can be done in the BRST language)
>
> 4) Show that if you restrict to SU(4) < SU(5) and SU(4) < Spin(7) x SO(2), then the single Baulieu supercharge acts in the same way

> 4) Show that if you restrict to $SU(4) < SU(5)$ and $SU(4) < Spin(7) \times SO(2)$, then the single Baulieu supercharge acts in the same way as Berkovits tells us it should
>
> Then one can do the 10d twist in Baulieu’s language (which Baulieu has done and we have repeated the computation) and all lower-dimensional twists in Berkovits’s language. Everything will be consistent by point 4 above. You seem to point out there’s some problem in 8d with supercharges, I’ll think about it.
>
> Will try to reply to the rest soon.