TWISTS

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Compactification:

- $\mathrm{Map}((N \times S^1)_{\mathrm{dR}}, X)$ compactifies to $\mathrm{Map}(N_{dR}, \mathrm{T}[-1]X)$.
- If Σ is a complex curve and L a 1-manifold, $\mathrm{Map}(\Sigma, X)$ compactifies to $\mathrm{Map}(L_{\mathrm{dR}}, X)$.

If an anomaly is not listed, the theory is not anomalous.

1. $d = 2$

To define an $\mathcal{N} = (2, 0)$ σ-model need a complex manifold X with a holomorphic vector bundle $\mathcal{E}$. If $\mathcal{E} = \mathrm{T}_X$, we can enhance it to $\mathcal{N} = (2, 2)$ supersymmetry. Let $E \to X$ be the total space.

- The rank $(1, 0)$ twist on Σ gives $\mathrm{T}^*[-1]\mathrm{Map}(\Sigma, E[-1])$. It only has a one-loop anomaly which is given by $\chi(\Sigma)c_1(E[-1])$ and $\mathrm{ch}_2(E[-1])$.
- If $\mathcal{E} = \mathrm{T}_X$, this is
$$\mathrm{T}^*[-1]\mathrm{Map}(\Sigma, \mathrm{T}[-1]X) \cong \mathrm{Map}(\Sigma_{\mathrm{Dol}}, \mathrm{T}^*[1]X).$$
It only has a one-loop anomaly given by $\chi(\Sigma)c_1(X)$.
- The rank $(1, 1)$ A-twist gives $\mathrm{T}^*[-1]\mathrm{Map}(\Sigma, X)_{\mathrm{dR}}$.
- The rank $(1, 1)$ B-twist gives $\mathrm{Map}(\Sigma_{\mathrm{dR}}, \mathrm{T}^*[1]X)$. The anomaly is the same as for the holomorphic twist.

To define an $\mathcal{N} = (2, 2)$ Yang–Mills theory we need a compact group K and a complex K-representation V. Let G be the complexification of K.

- The rank $(1, 0)$ twist on Σ gives
$$\mathrm{T}^*[-1]\mathrm{Map}(\Sigma, \mathrm{T}[-1][V/G]) \cong \mathrm{Map}(\Sigma_{\mathrm{Dol}}, \mathrm{T}^*[1][V/G]).$$
It only has a one-loop anomaly given by $\chi(\Sigma)c_1([V/G])$.
- The rank $(1, 1)$ A-twist gives $\mathrm{T}^*[-1]\mathrm{Map}(\Sigma, [V/G])_{\mathrm{dR}}$.
- The rank $(1, 1)$ B-twist gives $\mathrm{Map}(\Sigma_{\mathrm{dR}}, \mathrm{T}^*[1][V/G])$. The anomaly is the same as for the holomorphic twist

2. $d = 3$

To define an $\mathcal{N} = 2$ Yang–Mills theory we need a compact group K and a complex K-representation V. If V is the adjoint representation, the supersymmetry is enhanced to $\mathcal{N} = 4$.

- The rank 1 twist on $\Sigma \times L$ where Σ is a CY curve and L is a 1-manifold gives $\mathrm{T}^*[-1]\mathrm{Map}(\Sigma \times L_{\mathrm{dR}}, [V/G])$.
- For $V = \mathfrak{g}$ get
$$\mathrm{T}^*[-1]\mathrm{Map}(\Sigma \times L_{\mathrm{dR}}, \mathrm{T}[-1]\mathrm{B}G) \cong \mathrm{Map}(\Sigma_{\mathrm{Dol}} \times L_{\mathrm{dR}}, \mathrm{T}^*[2]\mathrm{B}G).$$

- The rank 2 A-twist gives $\mathrm{T}^*[-1]\mathrm{Map}(\Sigma \times L_{\mathrm{dR}}, \mathrm{B}G)_{\mathrm{dR}}$.
- The rank 2 B-twist on a 3-manifold M gives $\mathrm{Map}(M_{\mathrm{dR}}, \mathrm{T}^*[2]\mathrm{B}G)$.

We can also add $\mathcal{N}=4$ matter. For this pick a complex K-representation W. If $W=\mathfrak{g}$ we obtain $\mathcal{N}=8$ supersymmetry. Then:

- The rank 2 A-twist gives $\mathrm{T}^*[-1]\mathrm{Map}(\Sigma \times L_{\mathrm{dR}}, [W/G])_{\mathrm{dR}}$.
- The rank 2 B-twist gives $\mathrm{Map}(M_{\mathrm{dR}}, \mathrm{T}^*[2][W/G])$.

3. $d=4$

To define an $\mathcal{N}=1$ Yang–Mills theory we need a compact group K and a complex K-representation V. The theta-term is zero. If V is the adjoint representation, the supersymmetry is enhanced to $\mathcal{N}=2$. We consider a complex surface $M^4 = \Sigma_1 \times \Sigma_2$.

- The rank $(1,0)$ twist gives $\mathrm{T}^*[-1]\mathrm{Map}(M^4, [V/G])$. It only has a one-loop anomaly which in flat space is given by $\mathrm{ch}_3([V/G])$.
- For $V=\mathfrak{g}$ get
$$\mathrm{T}^*[-1]\mathrm{Map}(M, \mathrm{T}[-1]\mathrm{B}G) \cong \mathrm{T}^*[-1]\mathrm{Map}((\Sigma_1)_{\mathrm{Dol}} \times \Sigma_2, \mathrm{B}G).$$
It only has a one-loop anomaly given by $\chi(\Sigma_1)\mathrm{ch}_2(\mathrm{B}G)$.
- The rank $(2,0)$ twist gives $\mathrm{T}^*[-1]\mathrm{Map}(M^4, \mathrm{B}G)_{\mathrm{dR}}$. It compactifies to the 3d A-twist.
- The rank $(1,1)$ twist gives $\mathrm{T}^*[-1]\mathrm{Map}((\Sigma_1)_{\mathrm{dR}} \times \Sigma_2, \mathrm{B}G)$. It compactifies to the 3d B-twist or the 3d holomorphic twist. The anomaly is the same as in the holomorphic twist.

We can again add $\mathcal{N}=2$ matter given a complex K-representation W:

- The rank $(2,0)$ twist gives $\mathrm{T}^*[-1]\mathrm{Map}(M^4, [W/G])_{\mathrm{dR}}$.
- The rank $(1,1)$ twist gives $\mathrm{T}^*[-1]\mathrm{Map}((\Sigma_1)_{\mathrm{dR}} \times \Sigma_2, [W/G])$.

In the case $W=\mathfrak{g}$ (i.e. we have $\mathcal{N}=4$)

- The rank $(1,0)$ twist gives
$$\mathrm{T}^*[-1]\mathrm{Map}((\Sigma_1)_{\mathrm{Dol}} \times (\Sigma_2)_{\mathrm{Dol}}, \mathrm{B}G).$$
- The rank $(1,1)$ twist gives
$$\mathrm{T}^*[-1]\mathrm{Map}((\Sigma_1)_{\mathrm{dR}} \times (\Sigma_2)_{\mathrm{Dol}}, \mathrm{B}G).$$
- The rank $(2,0)$ twist gives
$$\mathrm{T}^*[-1]\mathrm{Map}((\Sigma_1)_{\mathrm{Dol}} \times \Sigma_2, \mathrm{B}G)_{\mathrm{dR}}.$$
- The special rank $(2,2)$ twist gives
$$\mathrm{T}^*[-1]\mathrm{Map}((\Sigma_1)_{\mathrm{dR}} \times (\Sigma_2)_{\mathrm{dR}}, \mathrm{B}G).$$
- The generic rank $(2,2)$ twist gives
$$\mathrm{T}^*[-1]\mathrm{Map}((\Sigma_1)_{\mathrm{dR}} \times \Sigma_2, \mathrm{B}G)_{\mathrm{dR}}.$$

4. $d = 5$

We consider $M^5 = M^4 \times L$ where $M^4 = \Sigma_1 \times \Sigma_2$ is a complex surface and L is a 1-manifold.

- The rank 1 twist gives $\mathrm{T}^*[-1]\mathrm{Map}(M^4 \times L_{\mathrm{dR}}, [W/G])$.
- If $W = \mathfrak{g}$ get $\mathrm{T}^*[-1]\mathrm{Map}(M \times L_{\mathrm{dR}}, \mathrm{T}[-1]\mathrm{B}G)$.
- The rank 2 topological twist gives $\mathrm{T}^*[-1]\mathrm{Map}(M^4 \times L_{\mathrm{dR}}, \mathrm{B}G)_{\mathrm{dR}}$.
- The rank 2 partially topological twist gives $\mathrm{T}^*[-1]\mathrm{Map}((\Sigma_1)_{\mathrm{dR}} \times \Sigma_2 \times L_{\mathrm{dR}}, \mathrm{B}G)$.
- The rank 4 twist gives $\mathrm{Map}(M^5_{\mathrm{dR}}, \mathrm{B}G)$. This is $\mathbf{Z}/2$-graded.

5. $d = 6$

Let $M^6 = M^4 \times \Sigma$ be a complex 3-fold.
$\mathcal{N} = (1, 0)$ Yang–Mills:

- The rank $(1, 0)$ twist gives $\mathrm{T}^*[-1]\mathrm{Map}(M^6, [W/G])$. It only has a one-loop anomaly. In flat spacetime it is given by $\mathrm{ch}_4([W/G])$.

$\mathcal{N} = (1, 1)$ Yang–Mills:

- The rank $(1, 1)$ topological twist gives $\mathrm{T}^*[-1]\mathrm{Map}(M^6, \mathrm{B}G)_{\mathrm{dR}}$.
- The rank $(1, 1)$ partially topological twist gives $\mathrm{T}^*[-1]\mathrm{Map}(M^4 \times \Sigma_{\mathrm{dR}}, \mathrm{B}G)$. It only has a one-loop anomaly when $\chi(\Sigma) \neq 0$ and $c_1(M) \neq 0$ in which case it is $\mathrm{ch}_2(\mathrm{B}G)$.
- The rank $(2, 2)$ twist gives $\mathrm{Map}(M^2 \times (M^4)_{\mathrm{dR}}, \mathrm{B}G)$. This is $\mathbf{Z}/2$-graded. It has a one-loop anomaly given by $\chi(M^4)\mathrm{ch}_2(\mathrm{B}G)$.

6. $d = 7$

- The topological twist is $\mathrm{Map}(M^6 \times L_{\mathrm{dR}}, \mathrm{B}G)_{\mathrm{dR}}$.
- The holomorphic rank 1 twist is $\mathrm{T}^*[-1]\mathrm{Map}(M^6 \times L_{\mathrm{dR}}, \mathrm{B}G)$.
- The rank 2 twist is $\mathrm{Map}(M^4 \times (M^3)_{\mathrm{dR}}, \mathrm{B}G)$. This is $\mathbf{Z}/2$-graded.

7. $d = 8$

- The twist by a pure spinor rank $(1, 0)$ supercharge gives $\mathrm{T}^*[-1]\mathrm{Map}(M^8, \mathrm{B}G)$.
- The twist by an impure spinor rank $(1, 0)$ supercharge gives $\mathrm{Map}(M^8, \mathrm{B}G)_{\mathrm{dR}}$.
- The twist by a rank $(1, 1)$ supercharge gives $\mathrm{Map}((M^6) \times (M^2)_{\mathrm{dR}}, \mathrm{B}G)$. This is $\mathbf{Z}/2$-graded. Its one-loop anomaly vanishes for instance when M^2 is flat. If M^2 is not flat while M^6 is, the one-loop anomaly is given by $\mathrm{ch}_4(\mathrm{B}G)$.

8. $d = 9$

- The twist by a pure spinor supercharge gives $\mathrm{Map}(M^8 \times L_{\mathrm{dR}}, \mathrm{B}G)$. This is $\mathbf{Z}/2$-graded.

9. $d = 10$

- The rank 1 twist gives $\mathrm{Map}(M^{10}, \mathrm{B}G)$. This is $\mathbf{Z}/2$-graded. Its one-loop anomaly in flat spacetime is $\mathrm{ch}_6(\mathrm{B}G)$.