

Using **asremlPlus**, in conjunction with **asreml**, to do the analysis of a wheat experiment that includes choosing a local spatial variation model using AICs

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This vignette shows how to use **asremlPlus** (Brien, 2024), in conjunction with **asreml** (Butler et al., 2023), to select the terms to be included in a mixed model for an experiment that involves spatial variation by comparing of information criteria . It also illustrates diagnostic checking and prediction production and presentation for this experiment. Here, **asremlPlus** and **asreml** are packages for the R Statistical Computing environment (R Core Team, 2024).

It is divided into the following main sections:

1. Set up the initial model for this experiment
2. Compare a series of information criteria to select a linear mixed model for the data
3. Diagnostic checking using residual plots and variofaces
4. Prediction production and presentation

1. Set up the initial model for this experiment

```
library(asreml, quietly=TRUE)
```

```
## Offline License checked out Mon Jun 10 13:39:43 2024
```

```
## Loading ASReml-R version 4.2
```

```
library(asremlPlus)
library(parallel)
library(doParallel)
```

```
## Loading required package: foreach
```

```
## Loading required package: iterators
```

```
library(foreach)
suppressMessages(library(qqplotr, quietly=TRUE))
options(width = 100)
```

Get data available in asremlPlus

The data are from a 1976 spring wheat experiment and are taken from Gilmour et al. (1995). An analysis is presented in the `asreml` manual by Butler et al. (2023, Section 7.6), although they suggest that it is a barley experiment.

```
data(Wheat.dat)
```

Add row and column covariates for the spatial modelling

```
tmp.dat <- within(Wheat.dat,
  {
    cColumn <- dae::as.numfac(Column)
    cColumn <- cColumn - mean(unique(cColumn))
    cRow <- dae::as.numfac(Row)
    cRow <- cRow - mean(unique(cRow))
  })
```

Fit an initial model - Row and column random

In the following, an initial model is fitted that has the terms that would be included for a balanced lattice. In addition, a term `WithinColPairs` has been included to allow for extraneous variation arising between pairs of adjacent lanes.

```
current.asr <- do.call(asreml,
  list(yield ~ Rep + WithinColPairs + Variety,
    random = ~ Row + Column,
    residual = ~ Row:Column,
    data = tmp.dat))
```

```
## ASReml Version 4.2 10/06/2024 13:39:44
##      LogLik      Sigma2      DF      wall
##  1    -702.7015    26232.20    119    13:39:44
##  2    -700.4838    24394.12    119    13:39:44
##  3    -698.4908    22487.75    119    13:39:44
##  4    -697.5842    21242.11    119    13:39:44
##  5    -697.3673    20590.05    119    13:39:44
##  6    -697.3595    20477.28    119    13:39:44
```

Intialize a model sequence by loading the current fit into an asrtests object

In creating the `asrtests` object, `IClikelihood` is set to `full` so that the full Restricted Maximum Likelihood (full REML) of Verbyla, 2019 is incorporated into the `tests.summary` of the `asrtests` object.

```
current.asrt <- as.asrtests(current.asr, NULL, NULL, IClikelihood = "full",
  label = "Initial model")
```

```
## Warning in (function (fixed = ~1, random = ~NULL, sparse = ~NULL, residual = ~NULL, :
## Log-likelihood not converged
```

Check for and remove any boundary terms and print a summary of the fit in the `asrtests` object

```
current.asrt <- rmboundary(current.asrt)
print(current.asrt)
```

```
##
##
## ##### Summary of the fitted variance parameters
##
##           component std.error  z.ratio bound %ch
## Row           5943.898  3815.514  1.557824    P 0.0
## Column        12380.527  6323.542  1.957847    P 0.3
## Row:Column!R  20477.280  2896.642  7.069316    P 0.0
##
##
## ##### Pseudo-anova table for fixed terms
##
##
## Wald tests for fixed effects.
## Response: yield
##
##           Df denDF  F.inc    Pr
## (Intercept)  1  14.9 1390.00 0.0000
## Rep          5  25.3   6.04 0.0008
## WithinColPairs 1  10.4   0.49 0.4998
## Variety      24 104.8   4.71 0.0000
##
##
## ##### Sequence of model investigations
##
## (If a row has NA for p but not denDF, DF and denDF relate to fixed and variance parameter numbers)
##
##           terms DF denDF  p      AIC      BIC      action
## 1 Initial model 31      3 NA 1720.891 1823.253 Starting model
```

The `test.summary` output shows that no changes have been made to the model loaded using `as.asrtests`. The pseudo-anova table shows that Varieties are highly significant ($p < 0.001$)

2. Compare a series of information criteria to select a linear mixed model for the data

In this section, models are compared using Akaike Information Criterion (AICs) based on the full REML.

Check the need for the term for within Column pairs (a post hoc factor)

```
current.asrt <- changeModelOnIC(current.asrt, dropFixed = "WithinColPairs",
                                label = "Try dropping withinColPairs", IClkelihood = "full")
```

```
## ASReml Version 4.2 10/06/2024 13:39:45
##           LogLik           Sigma2           DF           wall
## 1         -697.3595         20472.09         119         13:39:45
```

```
## Warning in (function (fixed = ~1, random = ~NULL, sparse = ~NULL, residual = ~NULL, :
## Log-likelihood not converged
```

```
print(current.asrt)
```

```
##
##
## ##### Summary of the fitted variance parameters
##
##           component std.error  z.ratio bound %ch
## Row           5941.153  3813.586 1.557891    P    0
## Column        11165.335  5583.267 1.999785    P    0
## Row:Column!R  20472.402  2895.582 7.070219    P    0
##
##
## ##### Pseudo-anova table for fixed terms
##
##
## Wald tests for fixed effects.
## Response: yield
##
##           Df denDF    F.inc    Pr
## (Intercept) 1  15.3 1466.00 0e+00
## Rep          5  26.7   6.11 7e-04
## Variety      24 105.3   4.73 0e+00
##
##
## ##### Sequence of model investigations
##
## (If a row has NA for p but not denDF, DF and denDF relate to fixed and variance parameter numbers)
##
##           terms DF denDF  p      AIC      BIC      action
## 1           Initial model 31      3 NA 1720.89134 1823.252944 Starting model
## 2 Try dropping withinColPairs -1      0 NA  -2.28193  -5.292565      Swapped
```

It is clear in the call to `changeModelOnIC` that the model is being changed by dropping the `withinColPairs` term, which could also be achieved using `update.asreml`. However, an `asremlPlus` model-changing function operates on an `asrtests` object, that includes an `asreml` object, and, except for `changeTerms.asrtests`, results in an `asrtests` object that may contain the changed model or the supplied model depending on the results of hypothesis tests or comparisons of information criteria. In addition, the result of the test or comparison will be added to a `test.summary` data.frame stored in the new `asrtests` object and, if the model was changed, the `wald.tab` in the new `asrtests` object will have been updated for the new model.

In this case, as can be seen from the summary of `current.asrt` after the call, the model without `withinColPairs` had a smaller AIC and so now the model stored in `current.asrt` does not include `withinColPairs`. The `wald.tab` has been updated for the new model.

Choose a model for local spatial variation from several potential models

This example has been analyzed using a model for the local spatial variation based on a separable auto-correlation process of order one (Butler et al., 2023). The need for this model can be assessed using the function `addSpatialModelOnIC` from `asremlPlus` that uses a forward selection strategy for fitting a correlation model (see output below). For this function, the spatial model to be fitted, the centred covariates for the two dimensions of the grid, and the factors corresponding to the covariates must be specified. Also, `IClikelihood` is set to `full` so that the likelihood will be based on the full REML. Because the model that incorporates the spatial model has a smaller AIC, it is the model returned in `spatial.ar1.asrt`.

The `print` of `spatial.ar1.asrt` shows that an `ar1` model for `Row` was tried first and was found to reduce the AIC by 11.898 and so became the current model. Next a model that incorporates an `ar1` function for `Column` was similarly tried and became the current model. This model includes a nugget variance, unless either the variance for `Row:Column` term or the Residual is bound or fixed. The Residual is the nugget term and represents non-spatial variance, such as measurement error. An appraisal of the need for a nugget term was made by comparing the fits with the residual variance unfixed and fixed at one. The model with the unfixed residual variance was chosen and is the model to be returned. That is, this model includes a term for nugget variance.

```
spatial.ar1.asrt <- addSpatialModelOnIC(current.asrt, spatial.model = "corr",
                                       row.covar = "cRow", col.covar = "cColumn",
                                       row.factor = "Row", col.factor = "Column",
                                       IClikelihood = "full")
```

```
## Warning in (function (fixed = ~1, random = ~NULL, sparse = ~NULL, residual = ~NULL, : Some
## components changed by more than 1% on the last iteration
## Warning in (function (fixed = ~1, random = ~NULL, sparse = ~NULL, residual = ~NULL, : Some
## components changed by more than 1% on the last iteration
```

```
## Warning in infoCriteria.asreml(asreml.obj, IClikelihood = ic.lik, bound.exclusions = bound.exclusion
## Column
```

```
## Warning in infoCriteria.asreml(asreml.obj, IClikelihood = ic.lik): The following bound terms were di
## Column
```

```
## Warning in (function (fixed = ~1, random = ~NULL, sparse = ~NULL, residual = ~NULL, : Warning :
## Oscillating parameter(s) reset to average value (iteration 7)
## Warning in (function (fixed = ~1, random = ~NULL, sparse = ~NULL, residual = ~NULL, : Warning :
## Oscillating parameter(s) reset to average value (iteration 7)
```

```
## Warning in (function (fixed = ~1, random = ~NULL, sparse = ~NULL, residual = ~NULL, :
## Log-likelihood not converged
```

```
## Warning in (function (fixed = ~1, random = ~NULL, sparse = ~NULL, residual = ~NULL, : Some
## components changed by more than 1% on the last iteration
```

```
## Warning in (function (fixed = ~1, random = ~NULL, sparse = ~NULL, residual = ~NULL, : Warning :
## Oscillating parameter(s) reset to average value (iteration 7)
## Warning in (function (fixed = ~1, random = ~NULL, sparse = ~NULL, residual = ~NULL, : Warning :
## Oscillating parameter(s) reset to average value (iteration 7)
```

```
## Warning in (function (fixed = ~1, random = ~NULL, sparse = ~NULL, residual = ~NULL, :
## Log-likelihood not converged
```

```
## Warning in (function (fixed = ~1, random = ~NULL, sparse = ~NULL, residual = ~NULL, : Some
## components changed by more than 1% on the last iteration

## Warning in newfit.asreml(asreml.obj, random. = mod.ran, trace = trace, update = update, :

## Warning in changeTerms.asrtests(asrtests.obj, dropFixed = dropFixed, addFixed = addFixed, : In
## analysing yield, boundary terms removed

## Warning in (function (fixed = ~1, random = ~NULL, sparse = ~NULL, residual = ~NULL, : Some
## components changed by more than 1% on the last iteration

## Warning in (function (fixed = ~1, random = ~NULL, sparse = ~NULL, residual = ~NULL, : Some
## components changed by more than 1% on the last iteration

## Warning in infoCriteria.asreml(asreml.obj, ICLikelihood = ic.lik, bound.exclusions = bound.exclusions,
## Row:Column!R

## Warning in infoCriteria.asreml(new.asrtests.obj$asreml.obj, ICLikelihood = ic.lik, : The following b
## Row:Column!R
```

```
spatial.ar1.asrt <- rmboundary(spatial.ar1.asrt)
infoCriteria(list(nonspatial = current.asrt$asreml.obj,
                 ar1 = spatial.ar1.asrt$asreml.obj))
```

```
##           fixedDF varDF NBound      AIC      BIC    loglik
## nonspatial      0     3      0 1409.023 1417.386 -701.5117
## ar1              0     5      0 1353.762 1367.700 -671.8811
```

```
print(spatial.ar1.asrt)
```

```
##
##
## ##### Summary of the fitted variance parameters
##
##           component      std.error    z.ratio bound %ch
## Row           2.202326e+03 8.222412e+03 0.2678443    P 0.3
## Row:Column     5.181864e+04 3.381226e+04 1.5325399    P 0.0
## Row:Column!Row!cor 7.121302e-01 9.570910e-02 7.4405694    U 0.0
## Row:Column!Column!cor 8.599232e-01 1.103950e-01 7.7895102    U 0.0
## Row:Column!R      4.821622e+03 1.717158e+03 2.8079077    P 0.0
##
##
## ##### Pseudo-anova table for fixed terms
##
##
## Wald tests for fixed effects.
## Response: yield
##
##           Df denDF    F.inc      Pr
```

```
## (Intercept)  1    1.8 194.600 0.0076
## Rep          5    21.9   0.559 0.7303
## Variety      24    74.2  10.560 0.0000
##
##
## ##### Sequence of model investigations
##
## (If a row has NA for p but not denDF, DF and denDF relate to fixed and variance parameter numbers)
##
##               terms DF denDF  p          AIC          BIC          action
## 1               Initial model 31      3 NA 1720.891344 1823.252944 Starting model
## 2           Try dropping withinColPairs -1      0 NA   -2.281930   -5.292565      Swapped
## 3               Try ar1(Row)  0      2 NA  -11.897710   -5.876439      Swapped
## 4           Try ar1(Column)  0      0 NA  -55.371326  -55.371326      Swapped
## 5                   Column  1     NA NA 1651.310695 1756.682931      Boundary
## 6 Try fixed nugget (residual) variance  0     -1 NA   1.802928   -1.207707      Unswapped
```

However, the spatial models that are available in `asremlPlus` also include those based on two-dimensional tensor-product natural cubic smoothing splines (TPNCSS), as described by Verbyla et al. (2018), and on two-dimensional tensor-product P-splines (TPPS), as described by Rodriguez-Alvarez et al. (2018) and Piepho, Boer and Williams (2022). The P-splines have been implemented using functions from the R package `TPSbits` authored by Sue Welham (2022)

The `asremlPlus` function `chooseSpatialModelOnIC` allows one to select the best model from amongst these spatial correlation models using the AIC or the BIC, the AIC being the default. The four models from which it selects are (i) a separable autocorrelation model on both row and column dimensions (`corr`), (ii) a two-dimensional tensor-product natural cubic smoothing spline (TPNCSS), (ii) a two-dimensional tensor-product cubic P-spline with second-difference penalties (TPPSC2), and (iii) a tensor-product two-dimensional linear P-spline with first-difference penalties (TPPSL1). By default all four are fitted and compared, but the `trySpatial` argument can be used to specify a subset of them.

The call to `chooseSpatialModelOnIC`, in addition to the arguments specifying covariates and factors, has further arguments: (i) `dropFixed` and `dropRandom` that are relevant to fitting P-splines and are used here to remove the random, overall Row and Column terms that were included in the original model so that the fitted P-spline model will be compared to a model including them and because the code for fitting P-splines also automatically includes these terms so that they would be included twice in the model to be fitted resulting in model singularities, (ii) `rotateX` and `ngridangles` so that the angles in both row and column directions for rotating the eigenvectors of the penalty matrix for the linear component of the P-splines is optimized (setting `ngridangles` to `NULL` causes the R function `nloptr::bobyqa` to be invoked for finding the optimal angles), (iii) an `asreml.option` argument to specify that the `grp` method be used in fitting the P-spline terms, this being safer because the data.frame needed for any post-fitting computation is available in the `asreml` object, and (iv) `return.asrts` to specify which `asrtests` objects are to be returned. Here we specify `all` so that `asrtests` objects for the best fits between the model supplied in the `current.asrt` and each of the spatial models is returned for all four spatial models. In this case, neither the `checkboundaryonly` nor the `IClikelihood` arguments were set because their defaults for `chooseSpatialModelOnIC` are appropriate.

```
suppressWarnings(
  spatial.asrts <- chooseSpatialModelOnIC(current.asrt,
    row.covar = "cRow", col.covar = "cColumn",
    row.factor = "Row", col.factor = "Column",
    dropRandom = "Row + Column",
    rotateX = TRUE, ngridangles = NULL,
    asreml.option = "grp", return.asrts = "all"))
```

```
## Notice : Spline design points closer than .000900 have been merged
```

```
## Notice : Spline design points closer than .001400 have been merged
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## Notice : Spline design points closer than .000900 have been merged
## Notice : Spline design points closer than .001400 have been merged
## Notice : Spline design points closer than .000900 have been merged
## elapsed time for bobyqa: 14.64 seconds
##
## ##### Optimal thetas: 20.2026926907242, 64.9728898361927 with criterion 1659.433
##
## Notice : Spline design points closer than .000900 have been merged
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## Notice : Spline design points closer than .000900 have been merged
```

Output the results

```
print(spatial.asrts$spatial.IC)
```

	fixedDF	varDF	AIC	BIC	loglik
## nonspatial	30	3	1718.609	1817.960	-826.3047
## corr	30	5	1651.317	1756.689	-790.6583
## TPNCSS	33	6	1639.489	1756.904	-780.7445
## TPPSC2	33	7	1642.838	1763.263	-781.4190
## TPPSL1	30	3	1708.443	1807.794	-821.2214

```
print(R2adj(spatial.asrts$asrts$TPNCSS$asreml.obj, include.which.random = ~.))
```

```
## ASReml Version 4.2 10/06/2024 13:40:36
##          LogLik          Sigma2      DF      wall
## 1      -662.1988        7657.986    117    13:40:36
```



```
## 2      -662.1988      7657.984    117    13:40:36
## 3      -662.1988      7657.977    117    13:40:36
## 4      -662.1988      7657.975    117    13:40:36
## 5      -662.1988      7657.975    117    13:40:36
## 6      -662.1988      7657.974    117    13:40:36
## 7      -662.1988      7657.974    117    13:40:36
## Notice : Spline design points closer than .001400 have been merged
## Notice : Spline design points closer than .000900 have been merged
## [1] 93.3036
## attr("fixed")
## ~.
## <environment: 0x0000023ab8a88748>
## attr("random")
## ~.
```

```
print(spatial.asrts$best.spatial.mod)
```

```
## [1] "TPNCSS"
```

```
print(spatial.asrts$asrts$TPNCSS)
```

```
##
##
## ##### Summary of the fitted variance parameters
##
##               component std.error  z.ratio bound %ch
## spl(cRow):cColumn      523.2004  372.8016 1.403429    P    0
## dev(cRow)              7664.0211 4442.0646 1.725329    P    0
## spl(cColumn)          13338.7348 9236.0511 1.444203    P    0
## spl(cColumn):cRow       366.7707  322.7484 1.136399    P    0
## spl(cRow):spl(cColumn)  3630.2187 2186.1497 1.660554    P    0
## Row:Column!R           7658.0113 1312.2026 5.835998    P    0
##
##
## ##### Pseudo-anova table for fixed terms
##
##
## Wald tests for fixed effects.
## Response: yield
##
##           Df denDF   F.inc    Pr
## (Intercept)  1   6.8 2645.00 0.0000
## Rep          5  41.3  20.45 0.0000
## Variety      24  86.5  10.15 0.0000
## cRow         1   7.0   0.07 0.7954
## cColumn      1  30.1  20.30 0.0001
## cRow:cColumn  1  64.3  22.00 0.0000
##
##
## ##### Sequence of model investigations
##
## (If a row has NA for p but not denDF, DF and denDF relate to fixed and variance parameter numbers)
##
```

		terms	DF	denDF	p	AIC	BIC	action
## 1		Initial model	31	3	NA	1720.89134	1823.252944	Starting model
## 2	Try dropping withinColPairs	-1		0	NA	-2.28193	-5.292565	Swapped
## 3	Try tensor NCS splines	3		3	NA	-79.12043	-61.056614	Swapped
## 4	dev(cColumn)	1		NA	NA	1639.48896	1756.903736	Boundary
## 5	spl(cRow)	1		NA	NA	1639.48901	1756.903788	Boundary

```
printFormulae(spatial.asrts$asrts$TPNCSS$asreml.obj)
```

```
##
##
## ##### Formulae from asreml object
##
## fixed: yield ~ Rep + Variety + cRow + cColumn + cRow:cColumn
## random: ~ spl(cColumn) + dev(cRow) + spl(cRow):cColumn + spl(cColumn):cRow + spl(cRow):spl(cColumn)
## residual: ~ Row:Column
```

The output shows that the TPNCSS model has the lowest AIC and so is selected as the best model. The adjusted R^2 value shows that the fixed and random terms in the fitted model account for 93% of the total variation in the yield. The model fitted for the TPNCSS model has been printed using `printFormulae.asreml`. The fitted model includes the term `dev(cRow)` that is equivalent to a random Row term and measures the deviations of the Row trend from a linear trend, the `spl(cRow)` term having been dropped because it was estimated to be zero. The Wald F -statistic for `Variety` is now 10.15 with 86.5 denominator degrees of freedom, as compared to 10.56 and 74.3 for the correlation model and 4.71 and 105.3 for the initial nonspatial model.

3. Diagnosing checking using residual plots and variofaces

Get current fitted asreml object and update to include standardized residuals

```
current.asr <- spatial.asrts$asrts$TPNCSS$asreml.obj
current.asr <- update(current.asr, aom=TRUE)
```

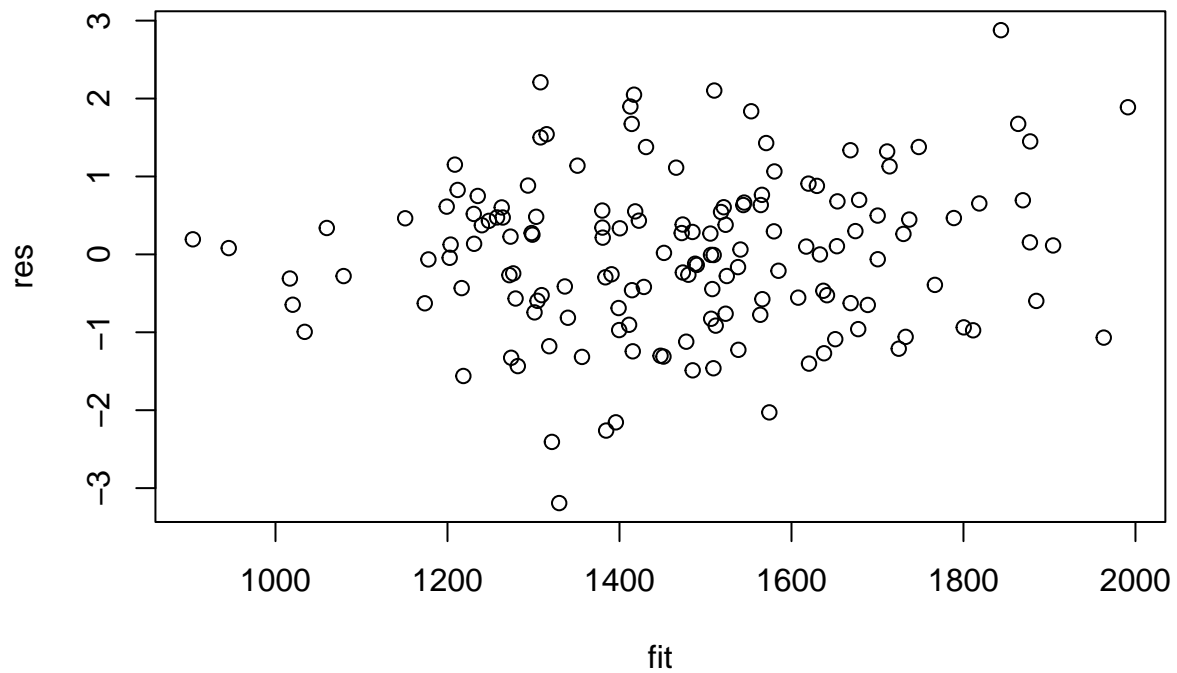
```
## ASReml Version 4.2 10/06/2024 13:40:37
##      LogLik      Sigma2      DF      wall
##  1    -662.1988      7657.986    117    13:40:37
##  2    -662.1988      7657.984    117    13:40:37
##  3    -662.1988      7657.977    117    13:40:37
##  4    -662.1988      7657.975    117    13:40:37
##  5    -662.1988      7657.975    117    13:40:37
##  6    -662.1988      7657.974    117    13:40:37
##  7    -662.1988      7657.974    117    13:40:37
## Notice : Spline design points closer than .001400 have been merged
## Notice : Spline design points closer than .000900 have been merged
```

```
Wheat.dat$res <- residuals(current.asr, type = "stdCond")
Wheat.dat$fit <- fitted(current.asr)
```

Do diagnostic checking

Do residuals-versus-fitted values plot

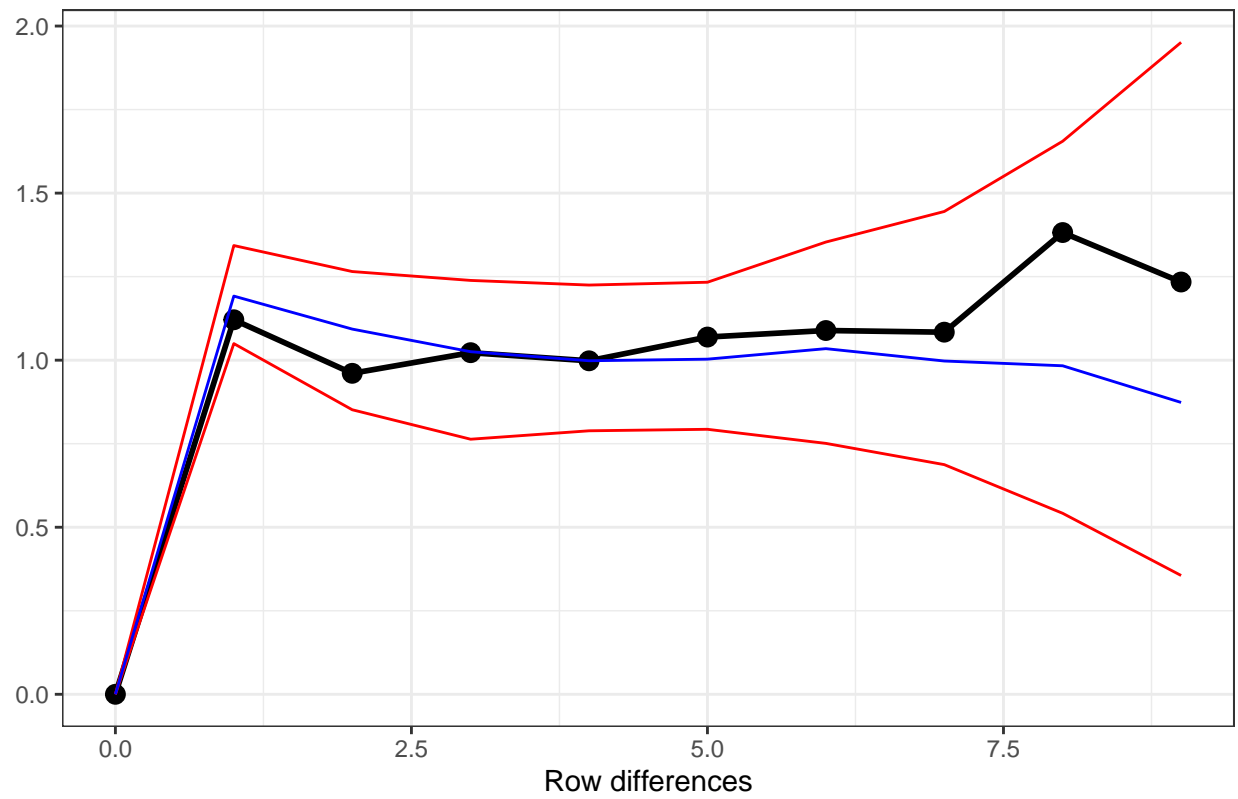
```
with(Wheat.dat, plot(fit, res))
```

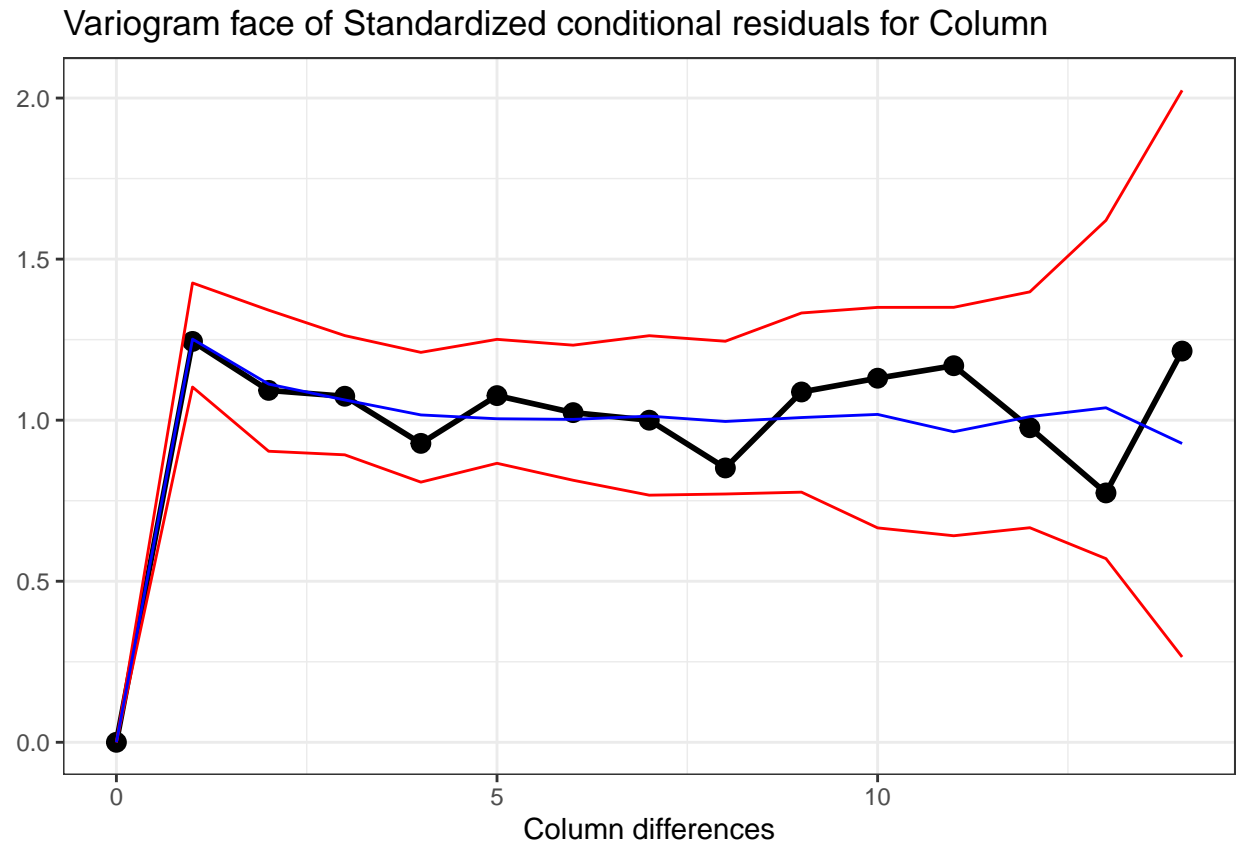


Plot varifaces

```
variofaces(current.asr, V=NULL, units="addtores",  
            maxiter=50, update = FALSE,  
            ncores = parallel::detectCores())
```

Variogram face of Standardized conditional residuals for Row



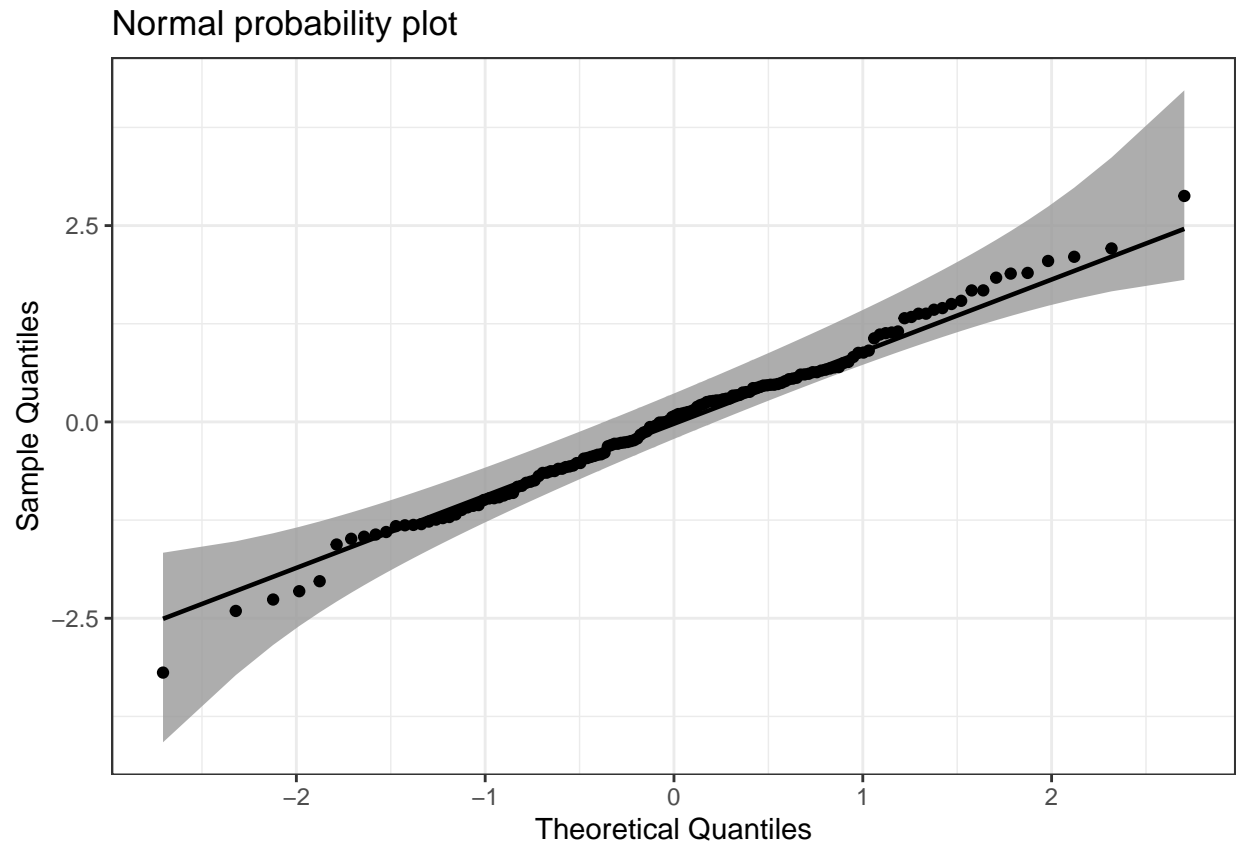


The variofaces are the lag 1 plots of the sample semivariogram with simulated confidence envelopes (Stefanova et al., 2009).

Plot normal quantile plot

The plot is obtained using the `ggplot` function with extensions available from the `qqplotr` package (Almeida et al., 2023).

```
ggplot(data = Wheat.dat, mapping = aes(sample = res)) +
  stat_qq_band(bandType = "ts") + stat_qq_line() + stat_qq_point() +
  labs(x = "Theoretical Quantiles", y = "Sample Quantiles",
       title = "Normal probability plot") +
  theme(plot.title = element_text(size = 12, face = "bold")) + theme_bw()
```



4. Prediction production and presentation

Get Variety predictions and all pairwise prediction differences and p-values

```
Var.diffs <- predictPlus(classify = "Variety",
  asreml.obj=current.asr,
  error.intervals="halfLeast",
  wald.tab=current.asrt$wald.tab,
  sortFactor = "Variety",
  tables = "predictions")
```

```
## Notice : Spline design points closer than .001400 have been merged
## Notice : Spline design points closer than .000900 have been merged
##
##
## #### Predictions for yield from Variety
##
##
## Notes:
## - The predictions are obtained by averaging across the hypertable
##   calculated from model terms constructed solely from factors in
##   the averaging and classify sets.
```

```

## - Use 'average' to move ignored factors into the averaging set.
## - spl(cRow) evaluated at average value of 0.00000
## - spl(cColumn) evaluated at average value of 0.00000
## - The simple averaging set: Rep
##
##      Variety predicted.value standard.error upper.halfLeastSignificant.limit
## 1         10         1196.214          64.11316          1255.028
## 2          9         1266.442          69.62176          1325.255
## 3         16         1268.206          74.01873          1327.020
## 4          1         1273.794          69.30592          1332.608
## 5         14         1310.806          70.75661          1369.620
## 6         23         1329.114          73.70238          1387.928
## 7         11         1340.668          75.07426          1399.481
## 8          4         1406.408          76.83336          1465.222
## 9          3         1408.543          71.85868          1467.356
## 10         7         1414.471          72.23082          1473.284
## 11        12         1423.303          71.18629          1482.117
## 12         8         1445.533          75.37805          1504.347
## 13         5         1480.687          70.31499          1539.500
## 14        15         1485.249          74.28025          1544.063
## 15        17         1495.212          71.09930          1554.026
## 16        21         1512.767          72.53109          1571.580
## 17         6         1520.841          72.50721          1579.654
## 18        24         1563.649          65.33308          1622.463
## 19        18         1568.887          71.06200          1627.700
## 20        25         1579.929          70.52509          1638.743
## 21         2         1585.398          74.96623          1644.211
## 22        22         1633.080          71.38265          1691.894
## 23        13         1637.119          66.24775          1695.932
## 24        19         1651.533          74.93686          1710.347
## 25        20         1657.612          68.65452          1716.426
##      lower.halfLeastSignificant.limit est.status
## 1              1137.400 Estimable
## 2              1207.628 Estimable
## 3              1209.393 Estimable
## 4              1214.980 Estimable
## 5              1251.993 Estimable
## 6              1270.300 Estimable
## 7              1281.854 Estimable
## 8              1347.594 Estimable
## 9              1349.729 Estimable
## 10             1355.657 Estimable
## 11             1364.490 Estimable
## 12             1386.720 Estimable
## 13             1421.873 Estimable
## 14             1426.435 Estimable
## 15             1436.399 Estimable
## 16             1453.953 Estimable
## 17             1462.027 Estimable
## 18             1504.836 Estimable
## 19             1510.073 Estimable
## 20             1521.115 Estimable
## 21             1526.584 Estimable
## 22             1574.266 Estimable

```

```
## 23                1578.305  Estimable
## 24                1592.719  Estimable
## 25                1598.798  Estimable
##
##
## LSD values
##
## minimum LSD =  109.6008
##
## mean LSD =  117.6273
##
## maximum LSD =  126.3422
##
## (sed range / mean sed =  0.142 )
```

We have set `error.intervals` to `halfLeast` so that the limits for each prediction $\pm(0.5 \text{ LSD})$ are calculated. When these are plotted overlapping error bars indicate predictions that are not significant, while those that do not overlap are significantly different (Snee, 1981).

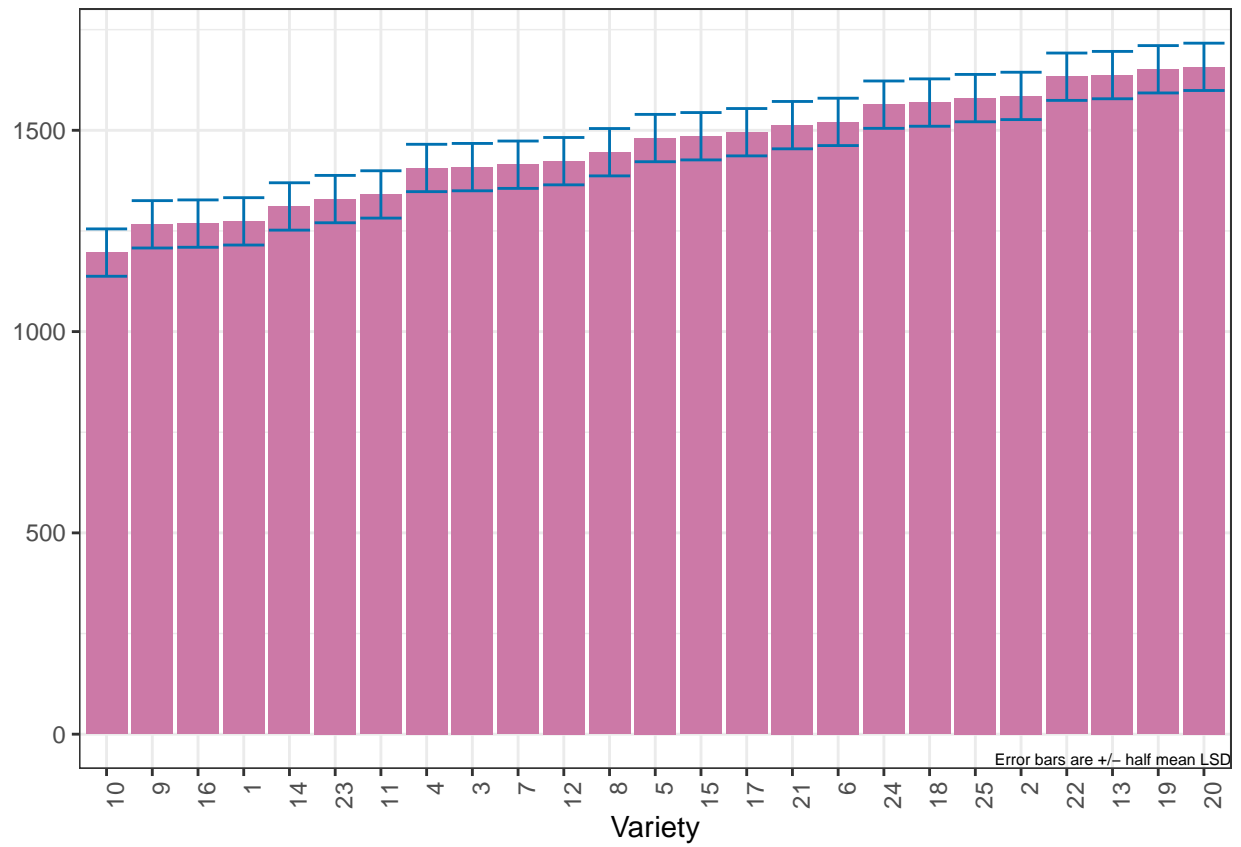
Also set was `sortFactor`, so that the results would be ordered for the values of the predictions for Variety.

The function `predictPlus` returns an `alldiffs` object, a list consisting of the following components:

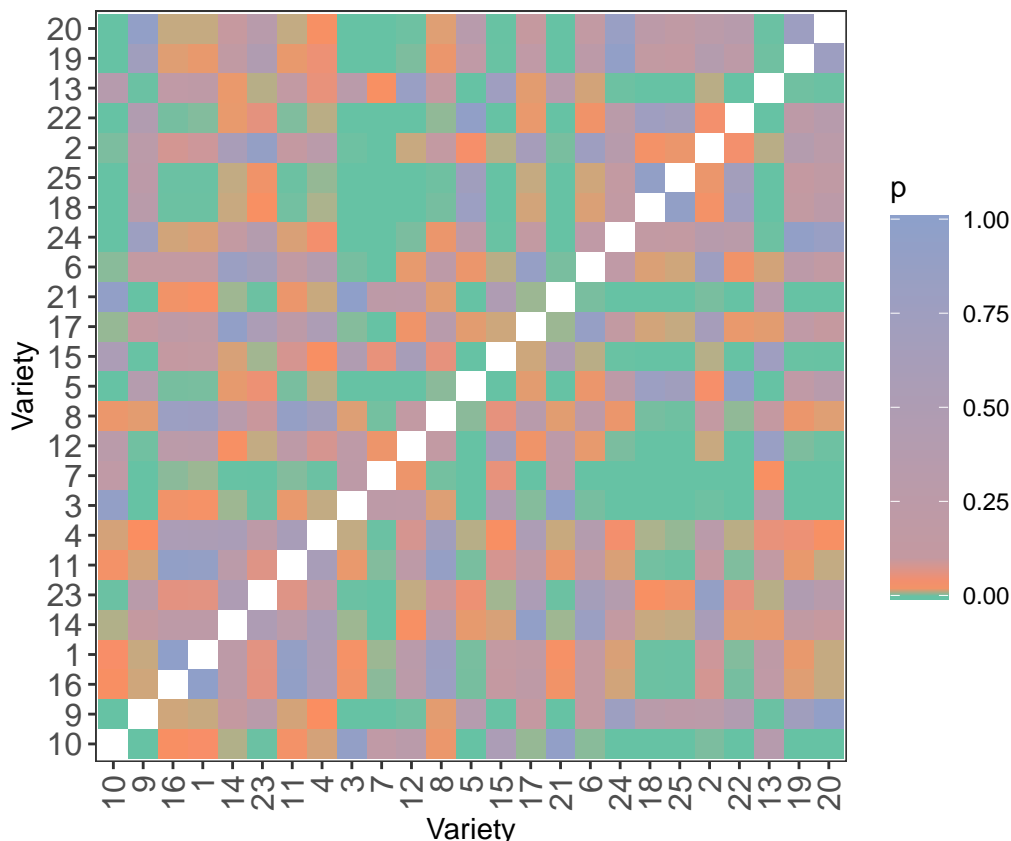
- `predictions`: the predictions, their standard errors and error intervals;
- `vcov`: the variance matrix of the predictions;
- `differences`: all pairwise differences between the predictions,
- `p.differences`: p-values for all pairwise differences between the predictions;
- `sed`: the standard errors of all pairwise differences between the predictions;
- `LSD`: the mean, minimum and maximum of the LSDs.

Plot the Variety predictions, with halfLSD intervals, and the p-values

```
plotPredictions(Var.diffs$predictions,
  classify = "Variety", y = "predicted.value",
  error.intervals = "half")
```

```
plotPvalues(Var.diffs)
```



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