Using `asremlPlus`, in conjunction with `asreml`, to do a linear mixed model analysis of a wheat experiment using hypothesis tests

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This vignette shows how to use `asremlPlus` (Brien, 2025), in conjunction with `asreml` (Butler et al., 2020), to employ hypothesis tests to select the terms to be included in a mixed model for an experiment that involves spatial variation. It also illustrates diagnostic checking and prediction production and presentation for this experiment. Here, `asremlPlus` and `asreml` are packages for the `R` Statistical Computing environment (R Core Team, 2025).

It is divided into the following main sections:

1. Set up the maximal model for this experiment
2. Perform a series of hypothesis tests to select a linear mixed model for the data
3. Diagnostic checking using residual plots and variofaces
4. Prediction production and presentation

1. Set up the maximal model for this experiment

```
library(knitr)
opts_chunk$set("tidy" = FALSE, comment = NA)
suppressMessages(library(asreml, quietly=TRUE))
```

```
## Offline License checked out Sun Aug 31 13:16:32 2025
```

```
packageVersion("asreml")
```

```
## [1] '4.2.0.370'
```

```
suppressMessages(library(asremlPlus))
packageVersion("asremlPlus")
```

```
## [1] '4.4.53'
```

```
suppressMessages(library(qqplotr, quietly=TRUE))
options(width = 100)
```

Get data available in asremlPlus

The data are from a 1976 spring wheat experiment and are taken from Gilmour et al. (1995). An analysis is presented in the `asreml` manual by Butler et al. (2020, Section 7.6), although they suggest that it is a barley experiment.

```
data(Wheat.dat)
```

Fit the maximal model

In the following a model is fitted that has the terms that would be included for a balanced lattice. In addition, a term WithinColPairs has been included to allow for extraneous variation arising between pairs of adjacent lanes. Also, separable ar1 residual autocorrelation has been included. This model represents the maximal anticipated model,

```
current.asr <- asreml(yield ~ WithinColPairs + Variety,
                      random = ~ Rep/(Row + Column) + units,
                      residual = ~ ar1(Row):ar1(Column),
                      maxit = 30, data=Wheat.dat)
```

```
ASReml Version 4.2 31/08/2025 13:16:34
          LogLik        Sigma2    DF      wall
  1     -724.1213     23034.14   124   13:16:34
  2     -717.4149     9206.931   124   13:16:34  (  2 restrained)
  3     -694.8752     26492.99   124   13:16:34  (  2 restrained)
  4     -694.1600     33101.80   124   13:16:34  (  1 restrained)
  5     -692.0020     36912.26   124   13:16:34  (  1 restrained)
  6     -691.7892     46701.51   124   13:16:34  (  2 restrained)
  7     -691.8336     46208.51   124   13:16:34  (  1 restrained)
  8     -691.7749     47698.26   124   13:16:34
  9     -691.7711     47041.85   124   13:16:34
```

Warning in asreml(yield ~ WithinColPairs + Variety, random = ~Rep/(Row + : Some components changed by more than 1% on the last iteration

The warning from `asreml` is probably due to a bound term.

Initialize a testing sequence by loading the current fit into an asrtests object

A label and the information criteria based on the full likelihood (Verbyla, 2019) are included in the `test.summary` stored in the `asrtests` object.

```
current.asrt <- as.asrtests(current.asr, NULL, NULL,
                            label = "Maximal model", IClikelihood = "full")
```

Warning in infoCriteria.asreml(asreml.obj, IClikelihood = ic.lik, bound.exclusions = bound.exclusions):
 Rep

Warning in asreml(fixed = yield ~ WithinColPairs + Variety, random = ~Rep/(Row + : Log-likelihood not converged

Check for and remove any boundary terms

```
current.asrt <- rmboundary(current.asrt, IClikelihood = "full")
```

Warning in infoCriteria.asreml(asreml.obj, IClikelihood = ic.lik): The following bound terms were discou
 Rep

```
ASReml Version 4.2 31/08/2025 13:16:35
          LogLik        Sigma2    DF      wall
  1     -691.7710     47071.42   124   13:16:35
```

Warning in asreml(fixed = yield ~ WithinColPairs + Variety, random = ~Rep/(Row + : Log-likelihood not converged

```
summary(current.asrt$asreml.obj)$varcomp
```

```
                        component    std.error    z.ratio bound %ch
Rep:Row              4.293282e+03 3.199458e+03  1.3418779     P 0.0
Rep:Column           1.575689e+02 1.480357e+03  0.1064398     P 0.7
units                5.742689e+03 1.652457e+03  3.4752438     P 0.0
Row:Column!R         4.706787e+04 2.515832e+04  1.8708669     P 0.0
Row:Column!Row!cor   7.920301e-01 1.014691e-01  7.8056280     U 0.0
Row:Column!Column!cor 8.799559e-01 7.370402e-02 11.9390486     U 0.0
```

```
print(current.asrt, which = "testsummary")
```

```
####  Sequence of model investigations for yield

(If a row has NA for p but not denDF, DF and denDF relate to fixed and variance parameter numbers)

          terms DF denDF  p      AIC      BIC        action
1 Maximal model 26     6 NA 1646.129 1742.47 Starting model
2           Rep  1    NA NA 1646.129 1742.47       Boundary
```

Rep has been removed because it has been constrained to zero. Following the recommendation of Littel et al. (2006, p. 150), the bound on all variance components is set to unconstrained (U) using `setvariances.asreml` so as to avoid bias in the estimate of the residual variance. Alternatively, one could move Rep to the fixed model.

Unbind Rep, Row and Column components and reload into an asrtests object

```
current.asr <- setvarianceterms(current.asr$call,
                                terms = c("Rep", "Rep:Row", "Rep:Column"),
                                bounds = "U")
```

```
ASReml Version 4.2 31/08/2025 13:16:36
         LogLik       Sigma2     DF     wall
   1   -724.1213     23034.14    124   13:16:36
   2   -717.4149      9206.931   124   13:16:36  (  2 restrained)
   3   -694.8752     26492.99    124   13:16:36  (  2 restrained)
   4   -693.9744     33129.65    124   13:16:36  (  1 restrained)
   5   -692.8856     39662.12    124   13:16:36
   6   -691.4276     53103.83    124   13:16:36
   7   -691.2387     48092.17    124   13:16:36
   8   -691.1808     47278.94    124   13:16:36
   9   -691.1710     46850.98    124   13:16:36
  10   -691.1700     46690.46    124   13:16:36
```

```
Warning in asreml(fixed = yield ~ WithinColPairs + Variety, random = ~Rep/(Row + : Some components
changed by more than 1% on the last iteration

WARN [2025-08-31 13:16:36] Some components changed by more than 1% on the last iteration

Warning in asreml(fixed = yield ~ WithinColPairs + Variety, random = ~Rep/(Row + : Some components
changed by more than 1% on the last iteration

WARN [2025-08-31 13:16:36] Some components changed by more than 1% on the last iteration

Warning in asreml(fixed = yield ~ WithinColPairs + Variety, random = ~Rep/(Row + : Some components
changed by more than 1% on the last iteration
```

```
current.asrt <- as.asrtests(current.asr, wald.tab = NULL, test.summary = current.asrt$test.summary,
                            IClikelihood = "full", label = "Max model & Unbound components")
current.asrt <- rmboundary(current.asrt)
summary(current.asrt$asreml.obj)$varcomp
```

```
                         component     std.error   z.ratio bound %ch
Rep                   -2462.3785858 1.191435e+03 -2.066734     U 0.2
Rep:Row                5012.4021415 3.396848e+03  1.475604     U 0.1
Rep:Column              920.5936391 1.704008e+03  0.540252     U 1.1
units                  5964.9099377 1.608792e+03  3.707695     P 0.1
Row:Column!R          46690.4620387 2.731906e+04  1.709080     P 0.0
Row:Column!Row!cor        0.8152180 9.988929e-02  8.161216     U 0.1
Row:Column!Column!cor     0.8857252 7.487875e-02 11.828793     U 0.0
```

```
print(current.asrt, which = "testsummary")
```

```
####  Sequence of model investigations for yield

(If a row has NA for p but not denDF, DF and denDF relate to fixed and variance parameter numbers)

                            terms DF denDF  p      AIC      BIC         action
1                   Maximal model 26     6 NA 1646.129 1742.470 Starting model
2                             Rep  1    NA NA 1646.129 1742.470       Boundary
3 Max model & Unbound components 26     7 NA 1647.200 1746.551 Starting model
```

```
print(current.asrt, which = "pseudoanova")
```

```
####  Pseudo-anova table for fixed terms


Wald tests for fixed effects.
Response: yield

              Df denDF  F.inc     Pr
(Intercept)    1   1.7 153.400 0.0115
WithinColPairs  1  15.6   2.543 0.1308
Variety        24  76.1  10.110 0.0000
```

Now the Rep component estimate is negative.

The `test.summary` output has been extended, by supplying the previous `test.summary` to `as.asrtests`, to show that there is a new starting model. The pseudo-anova table shows that Varieties are highly significant ($p < 0.001$)

2. Perform a series of hypothesis tests to select a linear mixed model for the data

The hypothesis tests in this section are Wald tests for fixed terms, with denominator degrees of freedom calculated using the Kenward-Rogers adjustment (Kenward and Rogers (1997)), and Restricted Maximum Likelihood Ratio Tests (REMLRT) for random terms.

Check the term for within Column pairs (a post hoc factor)

The information criteria based on the full likelihood (Verbyla, 2019) is also included in the `test.summary` stored in the `asrtests` object.

```
current.asrt <- testranfix(current.asrt, term = "WithinColPairs",
                           drop.fix.ns=TRUE, IClikelihood = "full")
```

WARN [2025-08-31 13:16:38] Some components changed by more than 1% on the last iteration

Warning in asreml(fixed = yield ~ Variety, random = ~Rep/(Row + Column) + : Some components changed by more than 1% on the last iteration

WARN [2025-08-31 13:16:38] Some components changed by more than 1% on the last iteration

Warning in asreml(fixed = yield ~ Variety, random = ~Rep/(Row + Column) + : Some components changed by more than 1% on the last iteration

```
print(current.asrt)
```

Summary of the fitted variance parameters for yield

	component	std.error	z.ratio	bound	%ch
Rep	-2392.1616314	1.199592e+03	-1.9941460	U	0.4
Rep:Row	5033.2850607	3.408523e+03	1.4766764	U	0.2
Rep:Column	760.1498938	1.617038e+03	0.4700879	U	2.5
units	5929.0518909	1.609478e+03	3.6838361	P	0.0
Row:Column!R	45940.6913910	2.634982e+04	1.7434920	P	0.0
Row:Column!Row!cor	0.8101561	9.995026e-02	8.1055925	U	0.1
Row:Column!Column!cor	0.8846454	7.504265e-02	11.7885681	U	0.0

Pseudo-anova table for fixed terms

Wald tests for fixed effects.
Response: yield

	Df	denDF	F.inc	Pr
(Intercept)	1	1.7	159.00	0.0112
Variety	24	76.8	10.27	0.0000

Sequence of model investigations for yield

(If a row has NA for p but not denDF, DF and denDF relate to fixed and variance parameter numbers)

	terms	DF	denDF	p	AIC	BIC	action
1	Maximal model	26	6.0	NA	1646.129	1742.470	Starting model
2	Rep	1	NA	NA	1646.129	1742.470	Boundary
3	Max model & Unbound components	26	7.0	NA	1647.200	1746.551	Starting model
4	WithinColPairs	1	15.6	0.1308	1645.326	1741.666	Dropped

It is clear in the call to `testranfix` that the model is being changed by dropping the `withinColPairs` term, which could also be achieved using `update.asreml`. However, an `asremlPlus` model-changing function operates on an `asrtests` object, that includes an `asreml` object, and, except for `changeTerms.asrtests`,

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results in an `asrtests` object that may contain the changed model or the supplied model depending on the results of hypothesis tests or comparisons of information criteria. In addition, the result of the test or comparison will be added to a `test.summary` data.frame stored in the new `asrtests` object and, if the model was changed, the `wald.tab` in the new `asrtests` object will have been updated for the new model.

In this case, as can be seen from the summary of `current.asrt` after the call, the *p*-value for the `withinColPairs` was greater than 0.05 and so now the model stored in `current.asrt` does not include `withinColPAirs`. The `wald.tab` has been updated for the new model.

Test the nugget term

The nugget term represents non-spatial variance, such as random plot and measurement error. It is fitted using the `asreml` reserved word `units`.

```
current.asrt <- testranfix(current.asrt, "units", positive=TRUE, IClikelihood = "full")
```

WARN [2025-08-31 13:16:40] Some components changed by more than 1% on the last iteration

Warning in asreml(fixed = yield ~ Variety, random = ~Rep + Rep:Row + Rep:Column, : Some components changed by more than 1% on the last iteration

WARN [2025-08-31 13:16:40] Some components changed by more than 1% on the last iteration

Warning in asreml(fixed = yield ~ Variety, random = ~Rep + Rep:Row + Rep:Column, : Some components changed by more than 1% on the last iteration

Test Row autocorrelation

We begin testing the autocorrelation by dropping the Row autocorrelation. Because of messages about the instability of the fit, `iterate.asrtests` is used to execute extra iterations of the fitting process.

```
current.asrt <- testresidual(current.asrt, "~ Row:ar1(Column)",
                             label="Row autocorrelation",
                             simpler=TRUE, IClikelihood = "full")
```

WARN [2025-08-31 13:16:41] Some components changed by more than 1% on the last iteration

Warning in asreml(fixed = yield ~ Variety, random = ~Rep/(Row + Column) + : Some components changed by more than 1% on the last iteration

WARN [2025-08-31 13:16:42] Some components changed by more than 1% on the last iteration

Warning in asreml(fixed = yield ~ Variety, random = ~Rep/(Row + Column) + : Some components changed by more than 1% on the last iteration

```
current.asrt <- iterate(current.asrt)
```

WARN [2025-08-31 13:16:43] Some components changed by more than 1% on the last iteration

Warning in asreml(fixed = yield ~ Variety, random = ~Rep/(Row + Column) + : Some components changed by more than 1% on the last iteration

WARN [2025-08-31 13:16:43] Some components changed by more than 1% on the last iteration

Warning in asreml(fixed = yield ~ Variety, random = ~Rep/(Row + Column) + : Some components changed by more than 1% on the last iteration

Test Column autocorrelation (depends on whether Row autocorrelation retained)

The function `getTestPvalue` is used to get the p-value for the Row autocorrelation test. If it is significant then the Column autocorrelation is tested by by dropping the Column autocorrelation, while retaining

the Row autocorrelation. Otherwise the model with just Row autocorrelation, whose fit is returned via `current.asrt` after the test, is compared to one with no autocorrelation.

```
(p <- getTestPvalue(current.asrt, label = "Row autocorrelation"))
```

```
[1] 4.675632e-06
```

```
{ if (p <= 0.05)
  current.asrt <- testresidual(current.asrt, "~ ar1(Row):Column",
                               label="Col autocorrelation",
                               simpler=TRUE, IClikelihood = "full")
  else
    current.asrt <- testresidual(current.asrt, "~ Row:Column",
                                 label="Col autocorrelation",
                                 simpler=TRUE, IClikelihood = "full")
}
```

```
Warning in DFdiff(bound.h1, bound.h0, DF = DF, bound.exclusions = bound.exclusions): There were a total
   The following bound terms occur in only one of the models compared and so were discounted:
    Row:Column!Row!cor
```

Output the results

```
print(current.asrt)
```

```
####  Summary of the fitted variance parameters for yield

                           component     std.error      z.ratio bound %ch
Rep                    -2392.1616314 1.199592e+03  -1.9941460     U 0.4
Rep:Row                 5033.2850607 3.408523e+03   1.4766764     U 0.2
Rep:Column               760.1498938 1.617038e+03   0.4700879     U 2.5
units                   5929.0518909 1.609478e+03   3.6838361     P 0.0
Row:Column!R           45940.6913910 2.634982e+04   1.7434920     P 0.0
Row:Column!Row!cor         0.8101561 9.995026e-02   8.1055925     U 0.1
Row:Column!Column!cor      0.8846454 7.504265e-02  11.7885681     U 0.0


####  Pseudo-anova table for fixed terms


Wald tests for fixed effects.
Response: yield

            Df denDF  F.inc      Pr
(Intercept)  1   1.7 159.00 0.0112
Variety     24  76.8  10.27 0.0000


####  Sequence of model investigations for yield

(If a row has NA for p but not denDF, DF and denDF relate to fixed and variance parameter numbers)

                          terms DF denDF      p      AIC      BIC         action
```

```
1                    Maximal model 26   6.0     NA 1646.129 1742.470 Starting model
2                              Rep  1    NA     NA 1646.129 1742.470       Boundary
3 Max model & Unbound components 26   7.0     NA 1647.200 1746.551 Starting model
4                    WithinColPairs  1  15.6 0.1308 1645.326 1741.666        Dropped
5                            units  1    NA 0.0006 1645.326 1741.666       Retained
6              Row autocorrelation  1    NA 0.0000 1645.326 1741.666      Unswapped
7              Col autocorrelation  2    NA 0.0000 1645.326 1741.666      Unswapped
```

```
printFormulae(current.asrt$asreml.obj)
```

```
#### Formulae from asreml object

fixed: yield ~ Variety
random: ~ Rep/(Row + Column) + units
residual: ~ ar1(Row):ar1(Column)
```

```
print(R2adj(current.asrt$asreml.obj, include.which.random = ~ .))
```

```
ASReml Version 4.2 31/08/2025 13:16:45
        LogLik        Sigma2    DF    wall
 1    -694.6149      45873.83   125   13:16:45
 2    -694.6149      45868.25   125   13:16:45
[1] 44.62078
attr(,"fixed")
~.
<environment: 0x000001fce0e0d6c8>
attr(,"random")
~.
```

The `test.summary` shows is that both Row and Column autocorrelation were retained in the model, as was the units term. The `asreml.obj` in `current.asrt` contains the model selected by the selection process, which has been printed using `printFormulae.asrtests`. It is clear that no changes were made to the variance terms. The adjusted R^2 value shows that the fixed and random terms in the fitted model account for 45% of the total variation in the yield.

3. Diagnostic checking using residual plots and variofaces

Get current fitted asreml object and update to include standardized residuals

```
current.asr <- current.asrt$asreml.obj
current.asr <- update(current.asr, aom=TRUE)
```

```
ASReml Version 4.2 31/08/2025 13:16:45
        LogLik        Sigma2    DF    wall
 1    -694.6149      45873.83   125   13:16:45
 2    -694.6149      45868.25   125   13:16:45
 3    -694.6149      45854.78   125   13:16:45
```
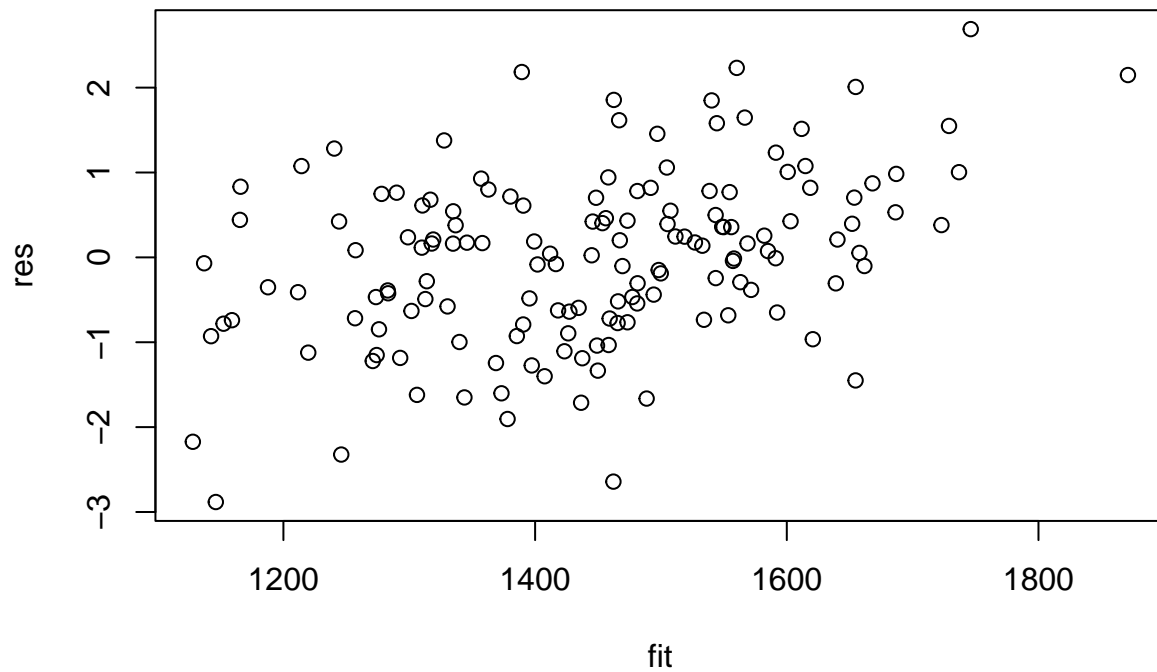
```
Wheat.dat$res <- residuals(current.asr, type = "stdCond")
Wheat.dat$fit <- fitted(current.asr)
```

Do diagnostic checking

Do residuals-versus-fitted values plot
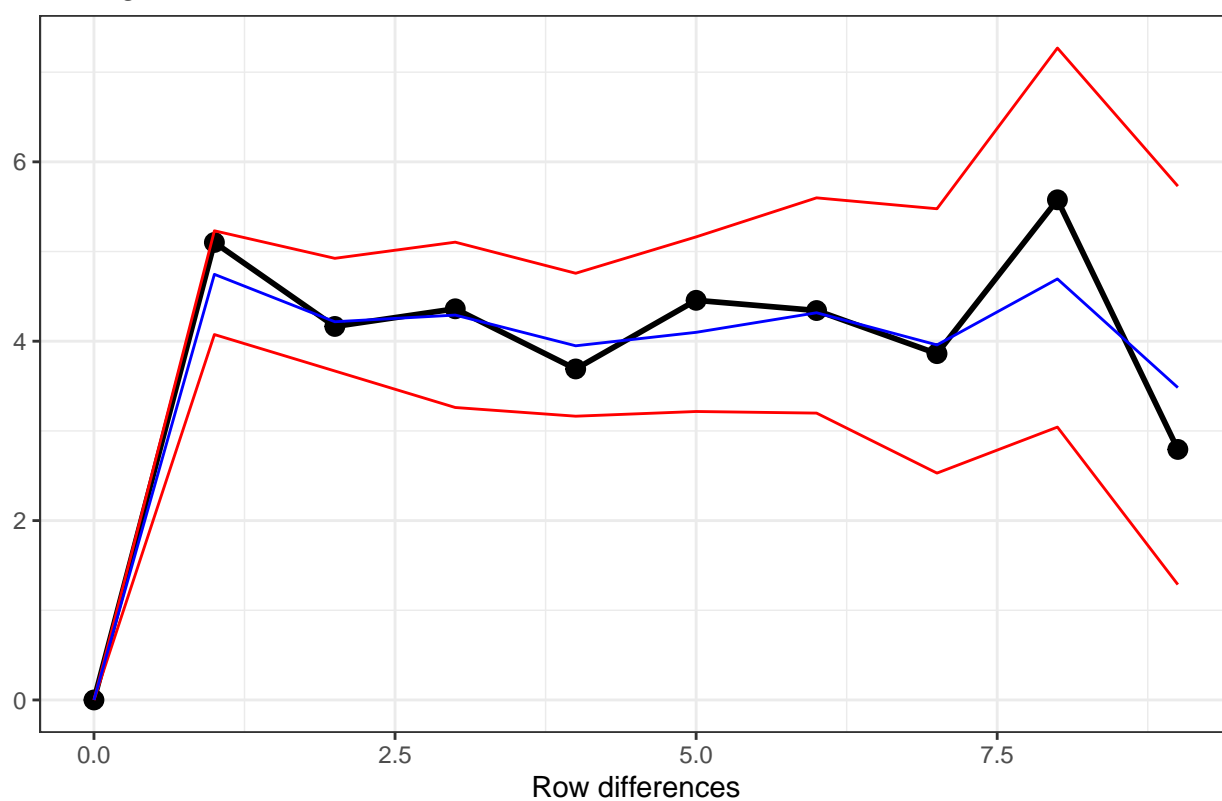
```r
with(Wheat.dat, plot(fit, res))
```
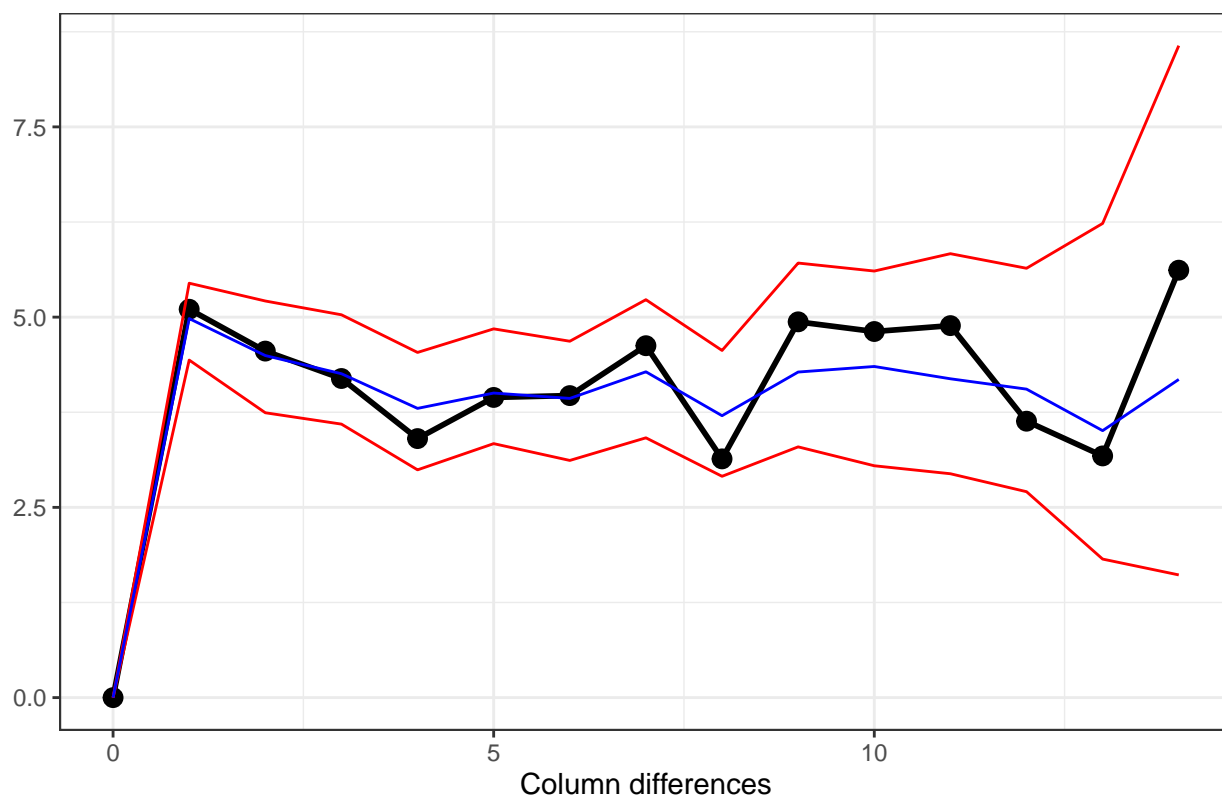

Plot variofaces

```r
variofaces(current.asr, V=NULL, units="addtores",
           maxiter=50, update = FALSE,
           ncores = parallel::detectCores())
```

Variogram face of Standardized conditional residuals for Row

Variogram face of Standardized conditional residuals for Column

The variofaces are the lag 1 plots of the sample semivariogram with simulated confidence envelopes (Stefanova et al., 2009).

Plot normal quantile plot

The plot is obtained using the `ggplot` function with extensions available from the `qqplotr` package (Almeida, A., Loy, A. and Hofmann, H., 2023).

```
suppressWarnings(
  ggplot(data = Wheat.dat, mapping = aes(sample = res)) +
    stat_qq_band(bandType = "ts") + stat_qq_line() + stat_qq_point() +
    labs(x = "Theoretical Quantiles", y = "Sample Quantiles",
         title = "Normal probability plot") +
    theme(plot.title = element_text(size = 12, face = "bold")) + theme_bw())
```
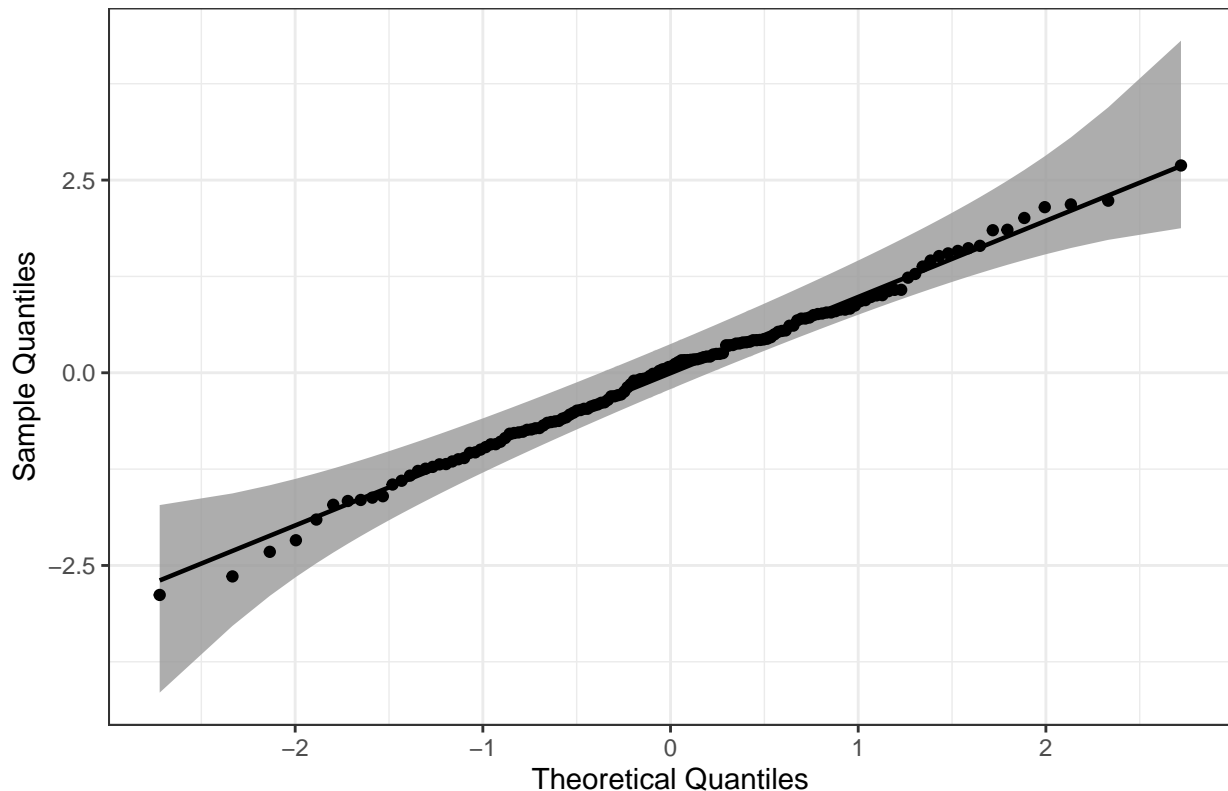

4. Prediction production and presentation

Get Variety predictions and all pairwise prediction differences and p-values

```
Var.diffs <- predictPlus(classify = "Variety",
                         asreml.obj=current.asr,
                         error.intervals="halfLeast",
                         wald.tab=current.asrt$wald.tab,
                         sortFactor = "Variety",
                         tables = "predictions")
```

Predictions for yield from Variety

Notes:
- The predictions are obtained by averaging across the hypertable
  calculated from model terms constructed solely from factors in
  the averaging and classify sets.
- Use 'average' to move ignored factors into the averaging set.
- The ignored set: Rep,Row,Column,units
- Variety is included in this prediction
- (Intercept) is included in this prediction
- units is ignored in this prediction

	Variety	predicted.value	standard.error	upper.halfLeastSignificant.limit
1	10	1168.989	120.4776	1228.315
2	1	1242.750	119.8112	1302.076
3	9	1257.137	119.9716	1316.463
4	16	1285.719	119.9407	1345.045
5	14	1293.527	119.9234	1352.853
6	23	1313.653	120.2937	1372.979
7	11	1322.159	120.1971	1381.485
8	7	1374.446	120.2415	1433.773
9	3	1394.070	120.4040	1453.396
10	4	1410.980	120.1063	1470.306
11	12	1444.557	120.6042	1503.883
12	8	1453.396	120.5948	1512.722
13	15	1458.383	120.4354	1517.709
14	5	1473.782	120.4462	1533.108
15	17	1487.828	120.2904	1547.154
16	6	1498.294	120.1196	1557.621
17	21	1517.121	120.2270	1576.447
18	2	1520.466	119.6330	1579.792
19	24	1533.768	120.3003	1593.095
20	18	1541.148	120.3671	1600.474
21	25	1575.795	120.5149	1635.121
22	22	1610.482	120.3289	1669.808
23	13	1610.761	120.4582	1670.088
24	20	1627.971	120.2336	1687.297
25	19	1652.992	120.3443	1712.318

	lower.halfLeastSignificant.limit	est.status
1	1109.662	Estimable
2	1183.424	Estimable
3	1197.811	Estimable
4	1226.392	Estimable
5	1234.200	Estimable
6	1254.327	Estimable
7	1262.832	Estimable
8	1315.120	Estimable
9	1334.743	Estimable
10	1351.654	Estimable
11	1385.231	Estimable
12	1394.070	Estimable
13	1399.057	Estimable

```
14                              1414.456  Estimable
15                              1428.501  Estimable
16                              1438.968  Estimable
17                              1457.795  Estimable
18                              1461.140  Estimable
19                              1474.442  Estimable
20                              1481.822  Estimable
21                              1516.468  Estimable
22                              1551.156  Estimable
23                              1551.435  Estimable
24                              1568.645  Estimable
25                              1593.666  Estimable


LSD values

minimum LSD =   114.0129

mean LSD =   118.6523

maximum LSD =   123.3577

(sed range / mean sed =   0.0788 )
```

We have set `error.intervals` to `halfLeast` so that the limits for so that the limits for each prediction $\pm$ (0.5 LSD) are calculated. When these are plotted overlapping error bars indicate predictions that are not significant, while those that do not overlap are significantly different (Snee, 1981).

Also set was `sortFactor`, so that the results would be ordered for the values of the predictions for Variety.

The function `predictPlus` returns an `alldiffs` object, a list consisting of the following components:

- `predictions`: the predictions, their standard errors and error intervals;

- `vcov`: the variance matrix of the predictions;

- `differences`: all pairwise differences between the predictions,

- `p.differences`: p-values for all pairwise differences between the predictions;

- `sed`: the standard errors of all pairwise differences between the predictions;

- `LSD`: the mean, minimum and maximum LSDs.

Plot the Variety predictions, with halfLSD intervals, and the p-values

```
plotPredictions(Var.diffs$predictions,
                classify = "Variety", y = "predicted.value",
                error.intervals = "half")
```

Error bars are +/- half mean LSD
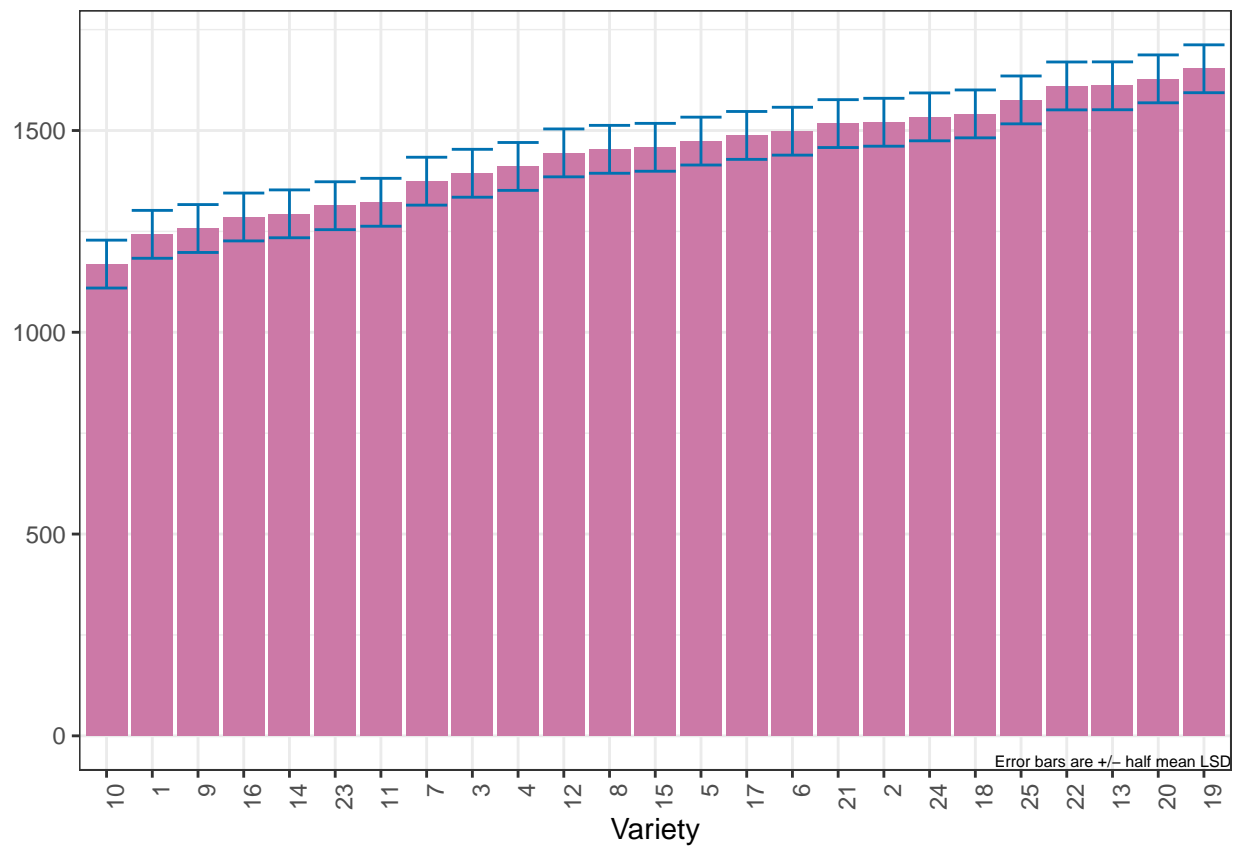
```
plotPvalues(Var.diffs)
```
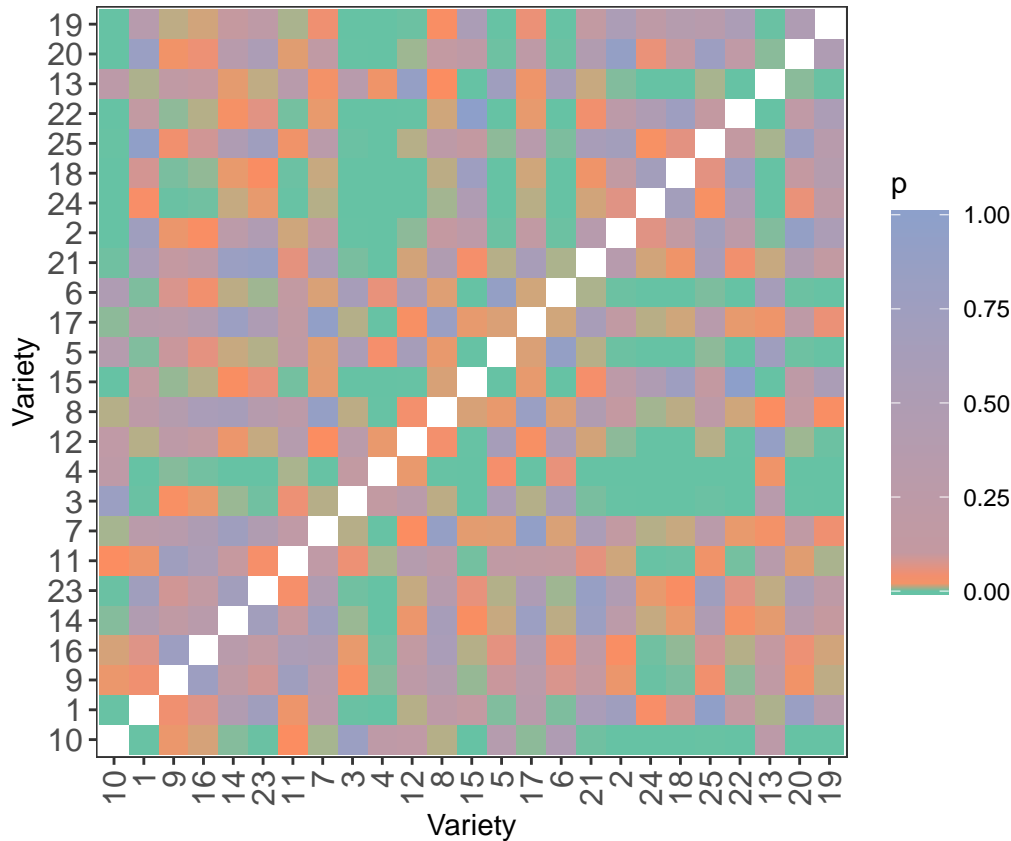
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