# Using asreml and asremlPlus for the Ladybird example from Welham et al. (2014)

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# Introduction

This vignette shows how to use asremlPlus (Brien, 2024a), and dae (Brien, 2024b), for exploring and presenting predictions from a linear mixed model analysis, the predictions having been produced using asreml (Butler et al., 2023). Here, asremlPlus, dae and asreml are packages for the R Statistical Computing environment (R Core Team, 2024).

The context is a three-factor factorial experiment on ladybirds (Welham et al., 2014, Example 8.2) that aims to answer the question "Will ladybirds transfer fungus to aphids on plants?" The experiment consists of 2 runs of 36 containers, each with a plant and aphids. There are three factors that results in 12 treatments: Host plant (beans, trefoil), infected Cadavers (5, 10, 20), Ladybird (-, +). These are randomized to the containers within a run so that each is replicated 3 times within a run. The response to be analysed is the logit of the proportion of live aphids that were infected.

# Initialize

```
library(knitr)
opts_chunk$set("tidy" = FALSE, comment = NA)
suppressMessages(library(asreml))

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packageVersion("asreml")

## [1] '4.2.0.276'
suppressMessages(library(asremlPlus))
packageVersion("asremlPlus")

## [1] '4.4.30'
suppressMessages(library("dae"))
packageVersion("dae")

## [1] '3.2.23'
```

```
options(width = 95, show.signif.stars = FALSE)
```

# Get data available in asremlPlus

```
data("Ladybird.dat")
```

# Do an ANOVA of logits

```
Error: Run
         Df Sum Sq Mean Sq F value Pr(>F)
Residuals 1 0.06766 0.06766
Error: Run:Plant
                      Df Sum Sq Mean Sq F value
                                                  Pr(>F)
Host
                       1 13.599 13.599 59.172 1.82e-10
Cadavers
                                 8.514 37.044 3.78e-11
                       2 17.027
Ladybird
                       1 11.091 11.091 48.257 3.33e-09
Host:Cadavers
                       2 0.308
                                0.154
                                          0.670
                                                 0.5158
```

Host:Ladybird 1 0.228 0.228 0.992 0.3234 Cadavers:Ladybird 2 1.735 0.867 3.774 0.0287 Host:Cadavers:Ladybird 2 0.200 0.100 0.435 0.6493 Residuals 0.230 59 13.560

The anova table gives the F-tests for the three-factor effects and interactions. Note the Residuals Mean Sq value for Run:Plant of 0.230. Also, it is clear that the Run component is negative, given that the Residuals Mean Sq value for Run is less than that for Run:Plant; it is (0.06766-0.230)/36). From the table it is seen that the only significant interaction is Cadavers:Ladybird and that the Host main effect is significant.

# Use asreml to analyse the logits

# Mixed model analysis of logits

```
component std.error z.ratio bound %ch Run 2.298309e-08 NA NA B NA Run:Plant!R 2.271216e-01 0.04156985 5.463612 P 0
```

As expected the Run component is bound (B) at approximately zero. This results in a change in the estimate of the residual variance to 0.227. To allow for a negative estimate we will unconstrain the Run component. As Littell et al. (2006, p.150) say

if you do not set the negative variance component estimate to zero, but allow it to remain negative, you get better control over Type I error and, for cases of negative wholeplot error variance estimates, greater power. Therefore, this is the recommended procedure.

# Unconstrain Reps to make the analysis equivalent to ANOVA

```
m <- setvarianceterms(m$call, terms = "Run", bounds = "U")
summary(m)$varcomp</pre>
```

```
component std.error z.ratio bound %ch Run -0.004504789 0.002896281 -1.555370 U 0.1 Run:Plant!R 0.229834648 0.042316936 5.431269 P 0.0
```

Now the Run component is negative and the Run:Plant variance estimate is now equal to that for the Residuals Mean Sq for Run:Plant from the anova table.

# Set up an asrtests object

```
current.asrt <- as.asrtests(m)
print(current.asrt, which = "pseudoanova")</pre>
```

#### Pseudo-anova table for fixed terms

Wald tests for fixed effects. Response: logitP

	Df	denDF	F.inc	Pr
(Intercept)	1	1	1550.00	0.0162
Host	1	59	59.17	0.0000
Cadavers	2	59	37.04	0.0000
Ladybird	1	59	48.26	0.0000
Host:Cadavers	2	59	0.67	0.5158
Host:Ladybird	1	59	0.99	0.3234
Cadavers:Ladybird	2	59	3.77	0.0287
${\tt Host:Cadavers:Ladybird}$	2	59	0.44	0.6493

The asrtests object contains a wald.tab component which can be printed by specifying that the pseudoanova is printed. The F-values for the fixed terms in this table are the same as those in the anova table.

#### Obtain the marginality matrix for the fixed terms

The pstructure function from the dae package (Brien, 2024b) produce the marginality matrix for a formula as a side effect and we take advantage of that to obtain the matrix required here.

		~ .				
	Host	Cadavers	Host:Cadavers	Ladybird	Host:Ladybird	Cadavers:Ladybird
Host	1	0	1	0	1	0
Cadavers	0	1	1	0	0	1
Host:Cadavers	0	0	1	0	0	0
Ladybird	0	0	0	1	1	1
Host:Ladybird	0	0	0	0	1	0
Cadavers:Ladybird	0	0	0	0	0	1
Host:Cadavers:Ladybird	0	0	0	0	0	0
	Host	:Cadavers	:Ladybird			
Host			1			
Cadavers			1			
Host:Cadavers			1			
Ladybird			1			
Host:Ladybird			1			

This marginality matrix is interpreted by taking a row term and noting that it is marginal to any column term with a one in this row.

1

1

# Choose marginality-compliant model

Cadavers:Ladybird

Host:Cadavers:Ladybird

```
chosen <- chooseModel(current.asrt, terms.marginality = HCL.marg)
current.asrt <- chosen$asrtests.obj
print(current.asrt, which = "test", omit.columns = c("AIC", "BIC"))</pre>
```

#### #### Sequence of model investigations

```
terms DF denDF
                                    р
                                              action
1 Host:Cadavers:Ladybird 2
                             59 0.6493 Nonsignificant
      Cadavers:Ladybird 2
                                         Significant
                             59 0.0287
          Host:Ladybird 1
3
                             59 0.3234 Nonsignificant
          Host:Cadavers 2
                             59 0.5158 Nonsignificant
4
5
                  Host 1
                             59 0.0000
                                          Significant
```

```
(chosen\sig.terms)
```

[[1]]

```
[1] "Cadavers:Ladybird"
[[2]]
[1] "Host"
```

The chooseModel function produces a list with components sig.terms, a list with the terms in the marginality-compliant model, and asrtests.obj, the asrtests object resulting from the model selection. In particular, the asrtests object contains a test.summary that details the tests performed in choosing the model; the AIC and BIC columns are omitted from test.summary because their inclusion has not been requested. Note that chooseModel does not test the main effects for Cadavers or Ladybird, because these are marginal to the significant two-factor interaction Cadavers:Ladybird.

#### Form formula for selected model

```
chosen.mod <- paste(unlist(chosen$sig.terms), collapse = " + ")
(chosen.mod <- as.formula(paste("~", chosen.mod)))</pre>
```

~Cadavers:Ladybird + Host

Obtain predictions under the chosen model and form an alldiffs object

```
Joining with 'by = join_by(fac.comb)'
Joining with 'by = join_by(Host)'
```

#### Predictions for logitP transform(s) from Host:Ladybird:Cadavers

#### Notes:

The original predictions, obtained as described below, have been linearly transformed to form estimated marginal means.

- The predictions are obtained by averaging across the hypertable calculated from model terms constructed solely from factors in the averaging and classify sets.
- Use 'average' to move ignored factors into the averaging set.
- The ignored set: Run

```
Host Ladybird Cadavers predicted.value standard.error upper.halfLeastSignificant.limit

bean - 5 -1.6038338 0.1417454 -1.4080222

bean - 10 -1.1454308 0.1417454 -0.9496192
```

```
3
      bean
                           20
                                   -0.7448097
                                                   0.1417454
                                                                                     -0.5489981
4
                           5
                                   -1.0195475
                                                   0.1417454
                                                                                     -0.8237359
      bean
5
      bean
                           10
                                   -0.5983440
                                                   0.1417454
                                                                                     -0.4025323
6
                           20
                                    0.4786704
                                                   0.1417454
                                                                                     0.6744820
      bean
7
  trefoil
                           5
                                   -2.4730339
                                                   0.1417454
                                                                                     -2.2772223
  trefoil
                           10
                                                   0.1417454
8
                                   -2.0146309
                                                                                    -1.8188193
  trefoil
                                                   0.1417454
                           20
                                   -1.6140098
                                                                                     -1.4181982
10 trefoil
                           5
                                   -1.8887476
                                                   0.1417454
                                                                                     -1.6929360
11 trefoil
                           10
                                   -1.4675441
                                                   0.1417454
                                                                                     -1.2717325
                           20
                                   -0.3905297
12 trefoil
                                                   0.1417454
                                                                                     -0.1947181
   lower.halfLeastSignificant.limit est.status
                          -1.7996454
1
                                     Estimable
2
                          -1.3412425
                                      Estimable
3
                         -0.9406214 Estimable
4
                          -1.2153592 Estimable
5
                          -0.7941556 Estimable
6
                          0.2828588 Estimable
7
                          -2.6688455 Estimable
                          -2.2104426 Estimable
8
9
                          -1.8098215 Estimable
10
                          -2.0845593 Estimable
11
                          -1.6633557 Estimable
12
                          -0.5863414 Estimable
LSD values
minimum LSD = 0.3916233 0.3916233
mean LSD = 0.3916233 0.3916233
maximum LSD = 0.3916233 0.3916233
(sed range / mean sed = 9.92e-16 9.92e-16)
```

Setting the terms argument to Host:Ladybird:Cadavers requests predictions for all combinations of the three factors and the linear.transformation argument is used to obtain estimated marginal means (emm) that conform to the chosen model. The wald.tab is supplied so that it can be used to get the degrees of freedom for the t-value to be used in calculating the LSD; the degrees of freedom to the source for the terms argument will be used. The error.intervals argument has been set to "halfLeast", the LSDtype argument to "factor.combination" and the LSDby argument to "Host" so that the average LSD will be calculated for each Host. This necessary because, under the chosen model, the LSDs differ between Hosts. It results in lower.halfLeastSignificant.limit and upper.halfLeastSignificant.limit being added to the predictions component of the alldiffs object.

Or, caclulate predictions to check first and then transform to conform to chosen model

```
Joining with 'by = join_by(fac.comb)'
Joining with 'by = join_by(Host)'
```

#### Predictions for logitP transform(s) from Host:Ladybird:Cadavers

#### Notes:

The original predictions, obtained as described below, have been linearly transformed to form estimated marginal means.

- The predictions are obtained by averaging across the hypertable calculated from model terms constructed solely from factors in the averaging and classify sets.
- Use 'average' to move ignored factors into the averaging set.
- The ignored set: Run

```
Host Ladybird Cadavers predicted.value standard.error upper.halfLeastSignificant.limit
      bean
                                  -1.6038338
1
                           5
                                                  0.1417454
                                                                                   -1.4080222
2
      bean
                          10
                                  -1.1454308
                                                  0.1417454
                                                                                   -0.9496192
3
                          20
     bean
                                  -0.7448097
                                                  0.1417454
                                                                                   -0.5489981
4
                  +
                          5
     bean
                                  -1.0195475
                                                  0.1417454
                                                                                   -0.8237359
5
     bean
                          10
                                  -0.5983440
                                                  0.1417454
                                                                                   -0.4025323
6
     bean
                          20
                                  0.4786704
                                                  0.1417454
                                                                                   0.6744820
7 trefoil
                          5
                                  -2.4730339
                                                  0.1417454
                                                                                   -2.2772223
                                                  0.1417454
8 trefoil
                        10
                                  -2.0146309
                                                                                   -1.8188193
9 trefoil
                          20
                                  -1.6140098
                                                  0.1417454
                                                                                   -1.4181982
10 trefoil
                  +
                          5
                                  -1.8887476
                                                  0.1417454
                                                                                   -1.6929360
11 trefoil
                          10
                                  -1.4675441
                                                  0.1417454
                                                                                   -1.2717325
12 trefoil
                          20
                                  -0.3905297
                                                  0.1417454
                                                                                   -0.1947181
```

lower.halfLeastSignificant.limit est.status

```
-1.7996454 Estimable
1
2
                         -1.3412425 Estimable
3
                        -0.9406214 Estimable
4
                        -1.2153592 Estimable
                        -0.7941556 Estimable
5
6
                         0.2828588 Estimable
7
                        -2.6688455 Estimable
8
                        -2.2104426 Estimable
9
                         -1.8098215 Estimable
10
                        -2.0845593 Estimable
11
                        -1.6633557 Estimable
12
                        -0.5863414 Estimable
```

LSD values

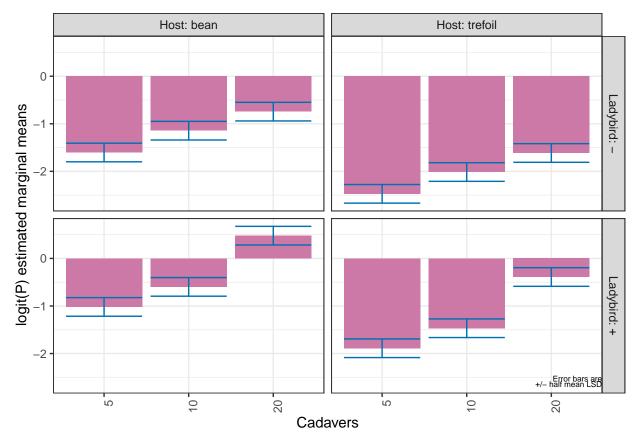
```
minimum LSD = 0.3916233 0.3916233

mean LSD = 0.3916233 0.3916233

maximum LSD = 0.3916233 0.3916233

(sed range / mean sed = 9.92e-16 9.92e-16)
```

# Plot the predictions

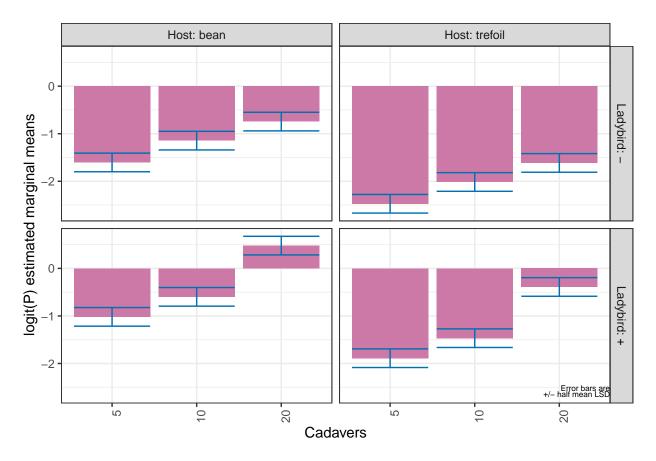


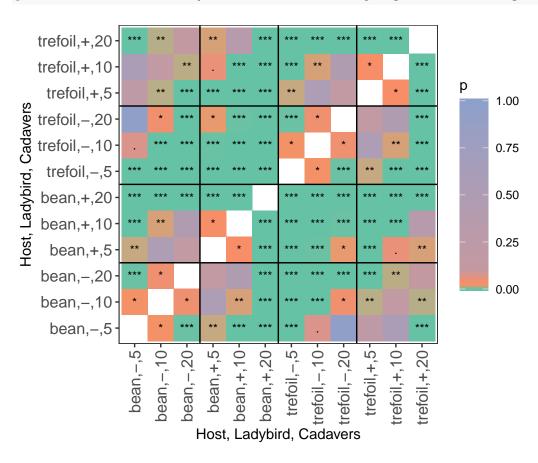
The function plotPredictions uses ggplot to produce the plot and the ggplotFuncs argument allows the addition of ggplot functions to modify the plot. In this case, the facet.grid function is respecified to include prepender functions that modify the labels of the facets to include the factor names. Note the the error bars in the plots are of  $\pm 0.5 LSD$  so that pairs of prediction with nonoverlapping bars are significantly different (Snee, 1981).

#### Get and plot the predictions with a single function call

The predictPresent function combines the functionality of predictPlus and plotPredictions, as demonstrated now. Also, the use of plotPvalues to plot the pairwise p-values is displayed. The predictPresent function has the capability of producing alldiffs objects for multiple terms and these are stored in a list each of which is named for the term whose alldiffs object it stores. Thus, the term has to be specified in referencing components of diffs.

Joining with 'by = join\_by(fac.comb)'
Joining with 'by = join\_by(Host)'





# options(width = 90) diffs\$Host.Ladybird.Cadavers\$differences

```
bean, -, 5 bean, -, 10 bean, -, 20
                                                bean,+,5 bean,+,10 bean,+,20
bean, -,5
              0.00000000 - 0.4584030 - 0.8590241 - 0.5842863 - 1.0054898 - 2.0825042
bean, -, 10
              0.45840297
                         0.0000000 - 0.4006211 - 0.1258833 - 0.5470869 - 1.6241012
              0.85902408
                         0.4006211
                                    0.0000000
                                               0.2747378 -0.1464657 -1.2234801
bean, -,20
                         bean,+,5
              0.58428627
bean, +, 10
              1.00548982
                         0.5470869
                                    0.1464657
                                               0.4212036
                                                          0.0000000 - 1.0770144
bean, +, 20
              2.08250420
                         1.6241012
                                    1.2234801
                                               1.4982179
                                                          1.0770144
                                                                     0.0000000
trefoil, -, 5 -0.86920012 -1.3276031 -1.7282242 -1.4534864 -1.8746899 -2.9517043
trefoil,-,10 -0.41079715 -0.8692001 -1.2698212 -0.9950834 -1.4162870 -2.4933014
trefoil,-,20 -0.01017604 -0.4685790 -0.8692001 -0.5944623 -1.0156659 -2.0926802
trefoil,+,5
            -0.28491385 -0.7433168 -1.1439379 -0.8692001 -1.2904037 -2.3674180
trefoil,+,10
             0.13628970 - 0.3221133 - 0.7227344 - 0.4479966 - 0.8692001 - 1.9462145
trefoil,+,20
             1.21330408
                         0.7549011 0.3542800 0.6290178 0.2078143 -0.8692001
            trefoil,-,5 trefoil,-,10 trefoil,-,20 trefoil,+,5 trefoil,+,10 trefoil,+,20
bean, -,5
               0.8692001
                            0.4107972
                                        0.01017604
                                                     0.2849139
                                                                 -0.1362897
                                                                              -1.2133041
               1.3276031
                            0.8692001
                                        0.46857901
                                                     0.7433168
                                                                              -0.7549011
bean, -, 10
                                                                  0.3221133
bean, -, 20
               1.7282242
                            1.2698212
                                        0.86920012
                                                     1.1439379
                                                                  0.7227344
                                                                              -0.3542800
bean,+,5
               1.4534864
                            0.9950834
                                        0.59446231
                                                     0.8692001
                                                                  0.4479966
                                                                              -0.6290178
bean, +, 10
               1.8746899
                            1.4162870
                                        1.01566586
                                                     1.2904037
                                                                  0.8692001
                                                                              -0.2078143
                                        2.09268024
bean,+,20
              2.9517043
                            2.4933014
                                                     2.3674180
                                                                  1.9462145
                                                                               0.8692001
```

```
trefoil,-,5
               0.0000000
                           -0.4584030
                                       -0.85902408
                                                     -0.5842863
                                                                  -1.0054898
                                                                                -2.0825042
                            0.0000000
trefoil,-,10
               0.4584030
                                       -0.40062111
                                                     -0.1258833
                                                                  -0.5470869
                                                                                -1.6241012
trefoil,-,20
               0.8590241
                            0.4006211
                                        0.00000000
                                                      0.2747378
                                                                  -0.1464657
                                                                                -1.2234801
trefoil,+,5
                            0.1258833
                                        -0.27473781
                                                      0.0000000
                                                                  -0.4212036
                                                                                -1.4982179
               0.5842863
trefoil,+,10
               1.0054898
                            0.5470869
                                         0.14646574
                                                      0.4212036
                                                                   0.0000000
                                                                                -1.0770144
trefoil,+,20
                            1.6241012
                                         1.22348012
                                                      1.4982179
                                                                   1.0770144
                                                                                0.0000000
               2.0825042
options(width = 90)
print(diffs$Host.Ladybird.Cadavers$sed)
              bean,-,5 bean,-,10 bean,-,20 bean,+,5 bean,+,10 bean,+,20 trefoil,-,5
bean, -,5
                    NA 0.1957142 0.1957142 0.1957142 0.1957142 0.1957142
                                                                            0.1129957
bean, -, 10
                              NA 0.1957142 0.1957142 0.1957142 0.1957142
                                                                            0.2259913
             0.1957142
bean,-,20
             0.1957142 0.1957142
                                        NA 0.1957142 0.1957142 0.1957142
                                                                            0.2259913
             0.1957142 0.1957142 0.1957142
                                                   NA 0.1957142 0.1957142
                                                                            0.2259913
bean,+,5
bean,+,10
             0.1957142 0.1957142 0.1957142 0.1957142
                                                             NA 0.1957142
                                                                            0.2259913
             0.1957142 \ 0.1957142 \ 0.1957142 \ 0.1957142 \ 0.1957142
                                                                            0.2259913
bean,+,20
                                                                       NA
trefoil, -, 5 0.1129957 0.2259913 0.2259913 0.2259913 0.2259913 0.2259913
                                                                                   NA
trefoil,-,10 0.2259913 0.1129957 0.2259913 0.2259913 0.2259913 0.2259913
                                                                            0.1957142
trefoil,-,20 0.2259913 0.2259913 0.1129957 0.2259913 0.2259913 0.2259913
                                                                            0.1957142
trefoil,+,5 0.2259913 0.2259913 0.1129957 0.2259913 0.2259913
                                                                            0.1957142
trefoil,+,10 0.2259913 0.2259913 0.2259913 0.1129957 0.2259913
                                                                            0.1957142
trefoil,+,20 0.2259913 0.2259913 0.2259913 0.2259913 0.2259913 0.1129957
                                                                            0.1957142
             trefoil,-,10 trefoil,-,20 trefoil,+,5 trefoil,+,10 trefoil,+,20
                                         0.2259913
bean, -, 5
                0.2259913
                             0.2259913
                                                       0.2259913
                                                                    0.2259913
bean,-,10
                0.1129957
                             0.2259913
                                         0.2259913
                                                       0.2259913
                                                                    0.2259913
bean, -, 20
                0.2259913
                             0.1129957
                                         0.2259913
                                                       0.2259913
                                                                    0.2259913
                                                                    0.2259913
bean,+,5
                0.2259913
                             0.2259913
                                         0.1129957
                                                       0.2259913
bean, +, 10
                0.2259913
                             0.2259913
                                         0.2259913
                                                       0.1129957
                                                                    0.2259913
bean, +, 20
                0.2259913
                             0.2259913
                                         0.2259913
                                                       0.2259913
                                                                    0.1129957
trefoil,-,5
                0.1957142
                             0.1957142
                                         0.1957142
                                                       0.1957142
                                                                    0.1957142
trefoil,-,10
                             0.1957142
                                         0.1957142
                                                       0.1957142
                                                                    0.1957142
                       NA
trefoil,-,20
                0.1957142
                                          0.1957142
                                                       0.1957142
                                                                    0.1957142
                                    NA
trefoil,+,5
                0.1957142
                             0.1957142
                                                       0.1957142
                                                                    0.1957142
                                                 NA
trefoil,+,10
                0.1957142
                             0.1957142
                                                                    0.1957142
                                         0.1957142
                                                              NA
trefoil,+,20
                                         0.1957142
                0.1957142
                             0.1957142
                                                       0.1957142
                                                                           NA
```

#### Perform the analysis with just selected model fitted

-0.004327122 0.002802876 -1.543815

Run:Plant!R 0.223431497 0.039503516 5.655990

Run

The model with nonsignificant fixed terms dropped is obtained in order to compare it with the fit when they are retained and the estimated marginal means for the chosen model are obtained.

```
ns.terms <- current.asrt$test.summary$terms[current.asrt$test.summary$action == "Nonsignificant"]
red.asrt <- changeTerms(current.asrt, dropFixed = paste(ns.terms, collapse = "+"))
summary(red.asrt$asreml.obj)$varcomp

component std.error z.ratio bound %ch</pre>
```

U 0.8

P 0.0

# print(red.asrt, which = "pseudoanova")

#### #### Pseudo-anova table for fixed terms

Wald tests for fixed effects.

Response: logitP

```
Df denDF F.inc Pr (Intercept) 1 1 1550.00 0.0162 Host 1 64 60.88 0.0000 Cadavers 2 64 38.12 0.0000 Ladybird 1 64 49.65 0.0000 Cadavers:Ladybird 2 64 3.88 0.0256
```

```
Joining with 'by = join_by(fac.comb)'
Joining with 'by = join_by(Host)'
```

#### Predictions for logitP from Host:Ladybird:Cadavers

#### Notes:

- The predictions are obtained by averaging across the hypertable calculated from model terms constructed solely from factors in the averaging and classify sets.
- Use 'average' to move ignored factors into the averaging set.
- The ignored set: Run

	Host	Ladybird	${\tt Cadavers}$	predicted.value	standard.error
1	bean	-	5	-1.6038338	0.1398332
2	bean	_	10	-1.1454308	0.1398332
3	bean	-	20	-0.7448097	0.1398332
4	bean	+	5	-1.0195475	0.1398332
5	bean	+	10	-0.5983440	0.1398332
6	bean	+	20	0.4786704	0.1398332
7	trefoil	_	5	-2.4730339	0.1398332
8	trefoil	-	10	-2.0146309	0.1398332
9	trefoil	-	20	-1.6140098	0.1398332
10	trefoil	+	5	-1.8887476	0.1398332
11	trefoil	+	10	-1.4675441	0.1398332
12	trefoil	+	20	-0.3905297	0.1398332

```
upper.halfLeastSignificant.limit lower.halfLeastSignificant.limit est.status
1
                         -1.4107942
                                                          -1.7968734 Estimable
2
                         -0.9523913
                                                          -1.3384704 Estimable
3
                         -0.5517702
                                                          -0.9378493 Estimable
4
                         -0.8265080
                                                          -1.2125871
                                                                      Estimable
5
                                                          -0.7913835 Estimable
                         -0.4053044
6
                          0.6717100
                                                           0.2856308 Estimable
7
                         -2.2799944
                                                          -2.6660735 Estimable
8
                         -1.8215914
                                                          -2.2076705
                                                                      Estimable
9
                         -1.4209703
                                                          -1.8070494
                                                                      Estimable
10
                         -1.6957081
                                                          -2.0817872
                                                                      Estimable
                                                                      Estimable
11
                         -1.2745045
                                                          -1.6605837
12
                         -0.1974902
                                                          -0.5835693 Estimable
LSD values
minimum LSD = 0.3860791 0.3860791
mean LSD = 0.3860791 0.3860791
maximum LSD = 0.3860791 0.3860791
(sed range / mean sed = 1.44e-16 1.44e-16)
options(width = 90)
print(diffs.red$sed)
              bean,-,5 bean,-,10 bean,-,20 bean,+,5 bean,+,10 bean,+,20 trefoil,-,5
bean, -, 5
                    NA 0.1929435 0.1929435 0.1929435 0.1929435 0.1929435
                                                                            0.1113960
                              NA 0.1929435 0.1929435 0.1929435 0.1929435
bean, -, 10
                                                                            0.2227920
             0.1929435
             0.1929435 0.1929435
                                        NA 0.1929435 0.1929435 0.1929435
bean, -, 20
                                                                            0.2227920
bean,+,5
             0.1929435 0.1929435 0.1929435
                                                  NA 0.1929435 0.1929435
                                                                            0.2227920
             0.1929435 0.1929435 0.1929435 0.1929435
                                                            NA 0.1929435
                                                                            0.2227920
bean,+,10
             0.1929435 \ 0.1929435 \ 0.1929435 \ 0.1929435
                                                                            0.2227920
bean,+,20
                                                                      NA
trefoil,-,5 0.1113960 0.2227920 0.2227920 0.2227920 0.2227920 0.2227920
trefoil,-,10 0.2227920 0.1113960 0.2227920 0.2227920 0.2227920 0.2227920
                                                                            0.1929435
trefoil,-,20 0.2227920 0.2227920 0.1113960 0.2227920 0.2227920 0.2227920
                                                                            0.1929435
trefoil,+,5 0.2227920 0.2227920 0.1113960 0.2227920 0.2227920
                                                                            0.1929435
trefoil,+,10 0.2227920 0.2227920 0.2227920 0.2227920 0.1113960 0.2227920
                                                                            0.1929435
trefoil,+,20 0.2227920 0.2227920 0.2227920 0.2227920 0.2227920 0.1113960
                                                                            0.1929435
             trefoil,-,10 trefoil,-,20 trefoil,+,5 trefoil,+,10 trefoil,+,20
bean, -, 5
                0.2227920
                             0.2227920
                                         0.2227920
                                                      0.2227920
                                                                    0.2227920
bean,-,10
                0.1113960
                             0.2227920
                                         0.2227920
                                                      0.2227920
                                                                    0.2227920
bean, -, 20
                0.2227920
                             0.1113960
                                         0.2227920
                                                      0.2227920
                                                                    0.2227920
                0.2227920
                             0.2227920
                                         0.1113960
                                                      0.2227920
                                                                    0.2227920
bean,+,5
bean, +, 10
                0.2227920
                             0.2227920
                                         0.2227920
                                                      0.1113960
                                                                    0.2227920
bean, +, 20
                0.2227920
                             0.2227920
                                         0.2227920
                                                      0.2227920
                                                                    0.1113960
trefoil,-,5
                0.1929435
                             0.1929435
                                         0.1929435
                                                      0.1929435
                                                                    0.1929435
                             0.1929435
trefoil,-,10
                       NA
                                        0.1929435
                                                      0.1929435
                                                                    0.1929435
trefoil,-,20
                0.1929435
                                         0.1929435
                                                      0.1929435
                                                                    0.1929435
trefoil,+,5
                0.1929435
                             0.1929435
                                                      0.1929435
                                                                    0.1929435
                                                NA
```

0.1929435

0.1929435

0.1929435

0.1929435

NΑ

trefoil,+,10

trefoil,+,20

0.1929435

0.1929435

0.1929435

0.1929435

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