Using asremlPlus, in conjunction with asreml, to do the analysis of a wheat experiment that includes choosing a local spatial variation model using AICs

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This vignette shows how to use asremlPlus (Brien, 2024), in conjunction with asreml (Butler et al., 2023), to select the terms to be included in a mixed model for an experiment that involves spatial variation by comparing of information criteria . It also illustrates diagnostic checking and prediction production and presentation for this experiment. Here, asremlPlus and asreml are packages for the R Statistical Computing environment (R Core Team, 2024).

It is divided into the following main sections:

- 1. Set up the initial model for this experiment
- 2. Compare a series of information criteria to select a linear mixed model for the data
- 3. Diagnostic checking using residual plots and variofaces
- 4. Prediction production and presentation

1. Set up the initial model for this experiment

```
library(asreml, quietly=TRUE)

## Offline License checked out Thu Mar 14 14:20:56 2024

## Loading ASReml-R version 4.2

library(asremlPlus)
library(parallel)
library(doParallel)

## Loading required package: foreach

## Loading required package: iterators

library(foreach)
suppressMessages(library(qqplotr, quietly=TRUE))
options(width = 100)
```

Get data available in asremlPlus

The data are from a 1976 spring wheat experiment and are taken from Gilmour et al. (1995). An analysis is presented in the asreml manual by Butler et al. (2023, Section 7.6), although they suggest that it is a barley experiment.

```
data(Wheat.dat)
```

Add row and column covariates for the spatial modelling

Fit an initial model - Row and column random

In the following, an initial model is fitted that has the terms that would be included for a balanced lattice. In addition, a term WithinColPairs has been included to allow for extraneous variation arising between pairs of adjacent lanes.

```
## ASReml Version 4.2 14/03/2024 14:20:57
##
             LogLik
                            Sigma2
                                        DF
                                               wall
##
   1
          -702.7015
                          26232.20
                                       119
                                             14:20:57
##
   2
          -700.4838
                          24394.12
                                       119
                                             14:20:57
                          22487.75
##
    3
          -698.4908
                                       119
                                             14:20:57
##
   4
          -697.5842
                          21242.11
                                       119
                                             14:20:57
##
   5
          -697.3673
                          20590.05
                                       119
                                             14:20:57
          -697.3595
                          20477.28
##
    6
                                       119
                                             14:20:57
```

Intialize a model sequence by loading the current fit into an asrtests object

In creating the asrtests object, IClikelihood is set to full so that the full Restricted Maximum Likelihood (full REML) of Verbyla, 2019 is incorporated into the tests.summary of the asrtests object.

```
## Warning in (function (fixed = ~1, random = ~NULL, sparse = ~NULL, residual = ~NULL, :
## Log-likelihood not converged
```

Check for and remove any boundary terms and print a summary of the fit in the asrtests object

```
current.asrt <- rmboundary(current.asrt)</pre>
print(current.asrt)
##
##
## #### Summary of the fitted variance parameters
##
##
                component std.error z.ratio bound %ch
                 5943.898 3815.514 1.557824
                                                 P 0.0
## Row
                12380.527 6323.542 1.957847
                                                 P 0.3
## Column
## Row:Column!R 20477.280 2896.642 7.069316
                                                 P 0.0
##
##
## #### Pseudo-anova table for fixed terms
##
##
## Wald tests for fixed effects.
## Response: yield
##
##
                  Df denDF
                             F.inc
                   1 14.9 1390.00 0.0000
## (Intercept)
                   5 25.3
                              6.04 0.0008
                              0.49 0.4998
## WithinColPairs 1 10.4
## Variety
                  24 104.8
                              4.71 0.0000
##
## #### Sequence of model investigations
## (If a row has NA for p but not denDF, DF and denDF relate to fixed and variance parameter numbers)
             terms DF denDF p
##
                                             BIC
                                    AIC
                                                          action
## 1 Initial model 31
                          3 NA 1720.891 1823.253 Starting model
```

The test.summary output shows that no changes have been made to the model loaded using as.asrtests. The pseudo-anova table shows that Varieties are highly significant (p < 0.001)

2. Compare a series of information criteria to select a linear mixed model for the data

In this section, models are compared using Akaike Information Criterion (AICs) based on the full REML.

Check the need for the term for within Column pairs (a post hoc factor)

```
## ASReml Version 4.2 14/03/2024 14:20:58
##
             LogLik
                            Sigma2
                                       DF
                                               พลไไ
##
    1
          -697.3595
                          20472.09
                                      119
                                             14:20:58
## Warning in (function (fixed = ~1, random = ~NULL, sparse = ~NULL, residual = ~NULL, :
## Log-likelihood not converged
print(current.asrt)
##
##
##
   ####
         Summary of the fitted variance parameters
##
##
                component std.error z.ratio bound %ch
##
  Row
                 5941.153
                            3813.586 1.557891
                                                   Ρ
                                                       0
                11165.335
                            5583.267 1.999785
                                                   Ρ
                                                       0
##
  Column
   Row:Column!R 20472.402
                                                   Р
                                                       0
                            2895.582 7.070219
##
##
   #### Pseudo-anova table for fixed terms
##
##
##
## Wald tests for fixed effects.
##
  Response: yield
##
##
               Df denDF
                           F.inc
                                    Pr
##
  (Intercept)
                1
                    15.3 1466.00 0e+00
## Rep
                5
                   26.7
                            6.11 7e-04
## Variety
               24 105.3
                            4.73 0e+00
##
##
        Sequence of model investigations
##
##
  (If a row has NA for p but not denDF, DF and denDF relate to fixed and variance parameter numbers)
##
##
                            terms DF denDF
                                                      AIC
                                                                   BIC
                                                                               action
                                             р
## 1
                    Initial model 31
                                          3 NA 1720.89134 1823.252944 Starting model
## 2 Try dropping withinColPairs -1
                                          O NA
                                                 -2.28193
                                                             -5.292565
                                                                              Swapped
```

It is clear in the call to changeModelOnIC that the model is being changed by dropping the withinColPairs term, which could also be achieved using update.asreml. However, an asremlPlus model-changing function operates on an asrtests object, that includes an asreml object, and, except for changeTerms.asrtests, results in an asrtests object that may contain the changed model or the supplied model depending on the results of hypothesis tests or comparisons of information criteria. In addition, the result of the test or comparison will be added to a test.summary data.frame stored in the new asrtests object and, if the model was changed, the wald.tab in the new asrtests object will have been updated for the new model.

In this case, as can be seen from the summary of current.asrt after the call, the model without withinColPairs had a smaller AIC and so now the model stored in current.asrt does not include withinColPAirs. The wald.tab has been updated for the new model.

Choose a model for local spatial variation from several potential models

This example has been analyzed using a model for the local spatial variation based on a separable auto-correlation process of order one (Butler et al., 2023). The need for this model can be assessed using the function addSpatialModelOnIC from asremlPlus that uses a forward selection strategy for fitting a correlation model (see output below). For this function, the spatial model to be fitted, the centred covariates for the two dimensions of the grid, and the factors corresponding to the covariates must be specified. Also, IClikelihood is set to full so that the likelihood will be based on the full REML. Because the model that incorporates the spatial model has a smaller AIC, it is the model returned in spatial.ar1.asrt.

The print of spatial.ar1.asrt shows that an ar1 model for Row was tried first and was found to reduce the AIC by 11.898 and so became the current model. Next a model that incorporates an ar1 function for Column was similarly tried and became the current model. This model includes a nugget variance, unless either the variance for Row:Column term or the Residual is bound or fixed. The Residual is the nugget term and represents non-spatial variance, such as measurement error. An appraisal of the need for a nugget term was made by comparing the fits with the residual variance unfixed and fixed at one. The model with the unfixed residual variance was chosen and is the model to be returned. That is, this model includes a term for nugget variance.

```
spatial.ar1.asrt <- addSpatialModelOnIC(current.asrt, spatial.model = "corr",</pre>
                                        row.covar = "cRow", col.covar = "cColumn",
                                        row.factor = "Row", col.factor = "Column",
                                        IClikelihood = "full")
## Warning in (function (fixed = ~1, random = ~NULL, sparse = ~NULL, residual = ~NULL, : Some
## components changed by more than 1% on the last iteration
## Warning in (function (fixed = ~1, random = ~NULL, sparse = ~NULL, residual = ~NULL, : Some
## components changed by more than 1% on the last iteration
## Warning in infoCriteria.asreml(asreml.obj, IClikelihood = ic.lik, bound.exclusions = bound.exclusion
## Column
## Warning in infoCriteria.asreml(asreml.obj, IClikelihood = ic.lik): The following bound terms were di
## Column
## Warning in (function (fixed = ~1, random = ~NULL, sparse = ~NULL, residual = ~NULL, : Warning :
## Oscillating parameter(s) reset to average value (iteration 7)
## Warning in (function (fixed = ~1, random = ~NULL, sparse = ~NULL, residual = ~NULL, : Warning :
## Oscillating parameter(s) reset to average value (iteration 7)
## Warning in (function (fixed = ~1, random = ~NULL, sparse = ~NULL, residual = ~NULL, :
## Log-likelihood not converged
## Warning in (function (fixed = ~1, random = ~NULL, sparse = ~NULL, residual = ~NULL, : Some
## components changed by more than 1% on the last iteration
## Warning in (function (fixed = ~1, random = ~NULL, sparse = ~NULL, residual = ~NULL, : Warning :
## Oscillating parameter(s) reset to average value (iteration 7)
## Warning in (function (fixed = ~1, random = ~NULL, sparse = ~NULL, residual = ~NULL, : Warning :
```

Oscillating parameter(s) reset to average value (iteration 7)

```
## Warning in (function (fixed = ~1, random = ~NULL, sparse = ~NULL, residual = ~NULL, :
## Log-likelihood not converged
## Warning in (function (fixed = ~1, random = ~NULL, sparse = ~NULL, residual = ~NULL, : Some
## components changed by more than 1% on the last iteration
## Warning in newfit.asreml(asreml.obj, random. = mod.ran, trace = trace, update = update, :
## Warning in changeTerms.asrtests(asrtests.obj, dropFixed = dropFixed, addFixed = addFixed, : In
## analysing yield, boundary terms removed
## Warning in (function (fixed = ~1, random = ~NULL, sparse = ~NULL, residual = ~NULL, : Some
## components changed by more than 1% on the last iteration
## Warning in (function (fixed = ~1, random = ~NULL, sparse = ~NULL, residual = ~NULL, : Some
## components changed by more than 1% on the last iteration
## Warning in (function (fixed = ~1, random = ~NULL, sparse = ~NULL, residual = ~NULL, : Some
## components changed by more than 1% on the last iteration
## Warning in infoCriteria.asreml(asreml.obj, IClikelihood = ic.lik, bound.exclusions = bound.exclusion
## Row:Column!R
## Warning in infoCriteria.asreml(new.asrtests.obj$asreml.obj, IClikelihood = ic.lik, : The following b
## Row:Column!R
spatial.ar1.asrt <- rmboundary(spatial.ar1.asrt)</pre>
infoCriteria(list(nonspatial = current.asrt$asreml.obj,
                  ar1 = spatial.ar1.asrt$asreml.obj))
              fixedDF varDF NBound
                                        AIC
                                                 BIC
                                                        loglik
## nonspatial
                    0
                          3
                                 0 1409.023 1417.386 -701.5117
## ar1
                                 0 1353.762 1367.700 -671.8811
print(spatial.ar1.asrt)
##
##
## #### Summary of the fitted variance parameters
##
                                                     z.ratio bound %ch
##
                            component
                                         std.error
## Row
                         2.202326e+03 8.222412e+03 0.2678443
                                                                 P 0.3
## Row:Column
                         5.181864e+04 3.381226e+04 1.5325399
                                                                 P 0.0
## Row:Column!Row!cor
                         7.121302e-01 9.570910e-02 7.4405694
                                                                 U 0.0
## Row:Column!Column!cor 8.599232e-01 1.103950e-01 7.7895102
                                                                 U 0.0
## Row:Column!R
                         4.821622e+03 1.717158e+03 2.8079077
                                                                 P 0.0
##
##
## #### Pseudo-anova table for fixed terms
##
##
```

```
## Wald tests for fixed effects.
##
  Response: yield
##
##
                           F.inc
                                      Pr
               Df denDF
##
   (Intercept)
                     1.8 194.600 0.0076
                 5
                    21.9
  Rep
                           0.559 0.7303
##
##
   Variety
                    74.2
                         10.560 0.0000
##
##
##
   ####
         Sequence of model investigations
##
   (If a row has NA for p but not denDF, DF and denDF relate to fixed and variance parameter numbers)
##
##
                                      terms DF denDF
##
                                                      р
                                                                 AIC
                                                                              BIC
                                                                                           action
## 1
                                                    3 NA 1720.891344 1823.252944 Starting model
                             Initial model 31
## 2
              Try dropping withinColPairs -1
                                                    O NA
                                                           -2.281930
                                                                        -5.292565
                                                                                          Swapped
## 3
                              Try ar1(Row)
                                             0
                                                    2 NA
                                                          -11.897710
                                                                        -5.876439
                                                                                          Swapped
## 4
                           Try ar1(Column)
                                                    O NA
                                                          -55.371326
                                                                       -55.371326
                                                                                          Swapped
## 5
                                     Column
                                             1
                                                  NA NA
                                                         1651.310695 1756.682931
                                                                                         Boundary
## 6 Try fixed nugget (residual) variance
                                                  -1 NA
                                                            1.802928
                                                                        -1.207707
                                                                                        Unswapped
```

However, the spatial models that are available in asremlPlus also include those based on two-dimensional tensor-product natural cubic smoothing splines (TPNCSS), as described by Verbyla et al. (2018), and on two-dimensional tensor-product P-splines (TPPS), as described by Rodriguez-Alvarez et al. (2018) and Piepho, Boer and Williams (2022). The P-splines have been implemented using functions from the R package TPSbits authored by Sue Welham (2022)

The asremlPlus function chooseSpatialModelOnIC allows one to select the best model from amongst these spatial correlation models using the AIC or the BIC, the AIC being the default. The four models from which it selects are (i) a separable autocorrelation model on both row and column dimensions (corr), (ii) a two-dimensional tensor-product natural cubic smoothing spline (TPNCSS), (ii) a two-dimensional tensor-product cubic P-spline with second-difference penalties (TPPSC2), and (iii) a tensor-product two-dimensional linear P-spline with first-difference penalties (TPPSL1). By default all four are fitted and compared, but the trySpatial argument can be used to specify a subset of them.

The call to chooseSpatialModelOnIC, in addition to the arguments specifying covariates and factors, has further arguments: (i) dropFixed and dropRandom that are relevant to fitting P-splines and are used here to remove the random, overall Row and Column terms that were included in the original model so that the fitted P-spline model will be compared to a model including them and because the code for fitting P-splines also automatically includes these terms so that they would be included twice in the model to be fitted resulting in model singularities, (ii) rotateX and ngridangles so that the angles in both row and column directions for rotating the eigenvectors of the penalty matrix for the linear component of the P-splines is optimized (setting ngridangles to NULL causes the R function nloptr::bobyqa to be invoked for finding the optimal angles), (iii) an asreml.option argument to specify that the grp method be used in fitting the P-spline terms, this being safer because the data.frame needed for any post-fitting computation is available in the asreml object, and (iv) return.asrts to specify which asrtests objects are to be returned. Here we specify all so that asrtests objects for the best fits between the model supplied in the current.asrt and each of the spatial models is returned for all four spatial models. In this case, neither the checkboundaryonly nor the IClikelihood arguments were set because their defaults for chooseSpatialModelOnIC are appropriate.

```
## Notice : Spline design points closer than .000900 have been merged
## Notice : Spline design points closer than .001400 have been merged
## Notice : Spline design points closer than .000900 have been merged
## Notice : Spline design points closer than .001400 have been merged
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## Notice : Spline design points closer than .000900 have been merged
## Notice : Spline design points closer than .001400 have been merged
## Notice : Spline design points closer than .000900 have been merged
## elapsed time for bobyqa: 15.64 seconds
## #### Optimal thetas: 20.2026926907242, 64.9728898361927 with criterion 1659.433
## Notice : Spline design points closer than .000900 have been merged
## Notice : Spline design points closer than .001400 have been merged
## Notice : Spline design points closer than .000900 have been merged
## Notice : Spline design points closer than .001400 have been merged
## Notice : Spline design points closer than .000900 have been merged
## Notice : Spline design points closer than .000900 have been merged
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## Notice : Spline design points closer than .000900 have been merged
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## Notice : Spline design points closer than .001400 have been merged
## Notice : Spline design points closer than .000900 have been merged
## Notice : Spline design points closer than .001400 have been merged
## Notice : Spline design points closer than .000900 have been merged
```

rotateX = TRUE, ngridangles = NULL,

asreml.option = "grp", return.asrts = "all"))

Output the results

```
print(spatial.asrts$spatial.IC)
```

```
fixedDF varDF
                                  AIC
                                           BIC
                                                  loglik
                          3 1718.609 1817.960 -826.3047
## nonspatial
                   30
## corr
                   30
                          5 1651.317 1756.689 -790.6583
## TPNCSS
                   33
                          6 1639.489 1756.904 -780.7445
## TPPSC2
                   33
                          7 1642.838 1763.263 -781.4190
## TPPSL1
                   30
                          3 1708.443 1807.794 -821.2214
```

```
print(R2adj(spatial.asrts$asrts$TPNCSS$asreml.obj, include.which.random = ~ .))
## ASReml Version 4.2 14/03/2024 14:21:50
##
             LogLik
                           Sigma2
                                     DF
                                             wall
##
         -662.1988
                         7657.986
                                     117
                                           14:21:51
         -662.1988
                        7657.984
## 2
                                     117
                                           14:21:51
## 3
         -662.1988
                        7657.977
                                     117
                                           14:21:51
## 4
         -662.1988
                         7657.975
                                     117
                                           14:21:51
## 5
         -662.1988
                         7657.975
                                     117
                                           14:21:51
         -662.1988
## 6
                         7657.974
                                           14:21:51
                                     117
         -662.1988
                         7657.974
                                     117
                                           14:21:51
## 7
## Notice : Spline design points closer than .001400 have been merged
## Notice : Spline design points closer than .000900 have been merged
## [1] 93.3036
## attr(,"fixed")
## ~.
## <environment: 0x000002378bd46118>
## attr(,"random")
## ~.
print(spatial.asrts$best.spatial.mod)
## [1] "TPNCSS"
print(spatial.asrts$asrts$TPNCSS)
##
##
## #### Summary of the fitted variance parameters
##
##
                           component std.error z.ratio bound %ch
## spl(cRow):cColumn
                            523.2004 372.8016 1.403429
                                                                0
## dev(cRow)
                           7664.0211 4442.0646 1.725329
                                                            Ρ
                                                                0
## spl(cColumn)
                          13338.7348 9236.0511 1.444203
                                                                0
## spl(cColumn):cRow
                            366.7707 322.7484 1.136399
                                                                0
## spl(cRow):spl(cColumn) 3630.2187 2186.1497 1.660554
                                                            Ρ
                                                                0
## Row:Column!R
                           7658.0113 1312.2026 5.835998
                                                                0
##
##
## #### Pseudo-anova table for fixed terms
##
##
## Wald tests for fixed effects.
## Response: yield
##
                Df denDF
                           F.inc
## (Intercept)
                1
                     6.8 2645.00 0.0000
                5 41.3
                           20.45 0.0000
## Rep
## Variety
                24 86.5
                           10.15 0.0000
                           0.07 0.7954
                     7.0
## cRow
                1
## cColumn
                1 30.1
                           20.30 0.0001
```

```
## cRow:cColumn 1 64.3
                           22.00 0.0000
##
##
        Sequence of model investigations
##
  ####
##
  (If a row has NA for p but not denDF, DF and denDF relate to fixed and variance parameter numbers)
##
##
##
                           terms DF denDF p
                                                     AIC
                                                                  BIC
                                                                              action
## 1
                   Initial model 31
                                         3 NA 1720.89134 1823.252944 Starting model
## 2 Try dropping withinColPairs -1
                                         O NA
                                                -2.28193
                                                            -5.292565
                                                                             Swapped
          Try tensor NCS splines
                                         3 NA
                                               -79.12043
                                                          -61.056614
                                                                             Swapped
                    dev(cColumn)
## 4
                                        NA NA 1639.48896 1756.903736
                                                                            Boundary
## 5
                       spl(cRow)
                                        NA NA 1639.48901 1756.903788
                                                                            Boundary
printFormulae(spatial.asrts$asrts$TPNCSS$asreml.obj)
##
```

```
##
## #### Formulae from asreml object
##
## fixed: yield ~ Rep + Variety + cRow + cColumn + cRow:cColumn
## random: ~ spl(cColumn) + dev(cRow) + spl(cRow):cColumn + spl(cColumn):cRow + spl(cRow):spl(cColumn)
## residual: ~ Row:Column
```

The output shows that the TPNCSS model has the lowest AIC and so is selected as the best model. The adjusted R^2 value shows that the fixed and random terms in the fitted model account for 93% of the total variation in the yield. The model fitted for the TPNCSS model has been printed using printFormulae.asreml. The fitted model includes the term dev(cRow) that is equivalent to a random Row term and measures the deviations of the Row trend from a linear trend, the spl(cRow) term having been dropped because it was estimated to be zero. The Wald F-statistic for Variety is now 10.15 with 86.5 denominator degrees of freedom, as compared to 10.56 and 74.3 for the correlation model and 4.71 and 105.3 for the initial nonspatial model.

3. Diagnosting checking using residual plots and variofaces

Get current fitted asreml object and update to include standardized residuals

```
current.asr <- spatial.asrts$asrts$TPNCSS$asreml.obj</pre>
current.asr <- update(current.asr, aom=TRUE)</pre>
## ASReml Version 4.2 14/03/2024 14:21:52
##
              LogLik
                             Sigma2
                                         DF
                                                wall
          -662.1988
                           7657.986
##
    1
                                        117
                                              14:21:52
##
    2
          -662.1988
                           7657.984
                                        117
                                              14:21:52
##
    3
          -662.1988
                           7657.977
                                        117
                                              14:21:52
    4
          -662.1988
                           7657.975
##
                                        117
                                              14:21:52
##
    5
          -662.1988
                           7657.975
                                        117
                                              14:21:52
    6
          -662.1988
                           7657.974
                                        117
                                              14:21:52
##
##
   7
          -662.1988
                           7657.974
                                        117
                                              14:21:52
## Notice : Spline design points closer than .001400 have been merged
```

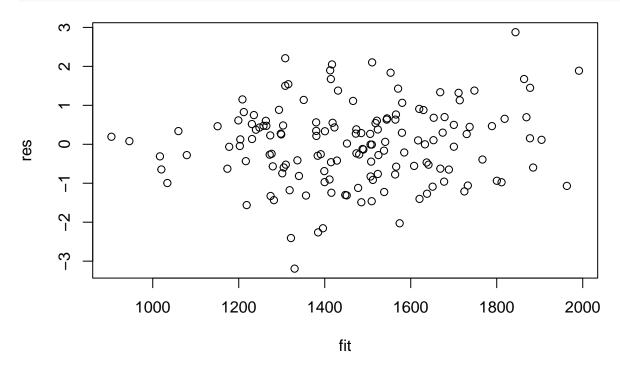
Notice : Spline design points closer than .000900 have been merged

```
Wheat.dat$res <- residuals(current.asr, type = "stdCond")
Wheat.dat$fit <- fitted(current.asr)</pre>
```

Do diagnostic checking

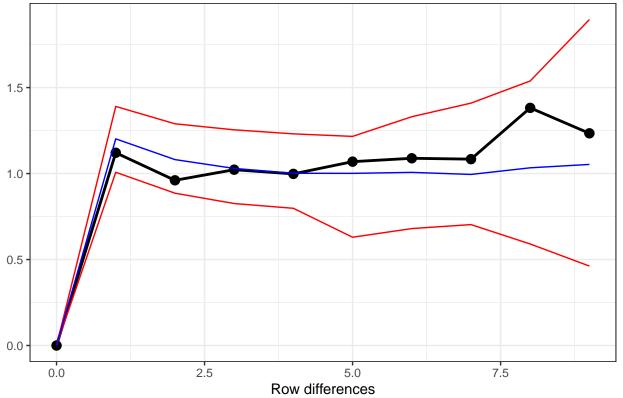
Do residuals-versus-fitted values plot

```
with(Wheat.dat, plot(fit, res))
```

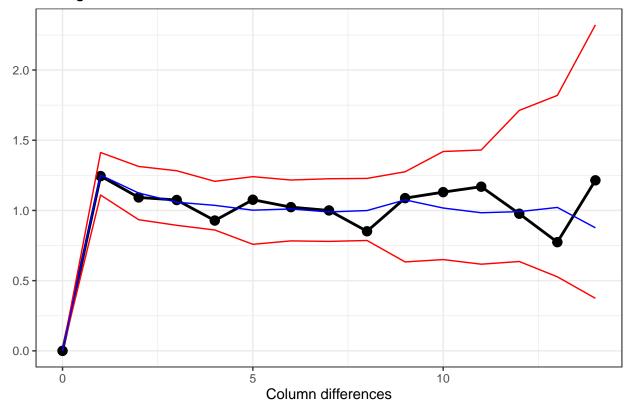


Plot variofaces

Variogram face of Standardized conditional residuals for Row



Variogram face of Standardized conditional residuals for Column



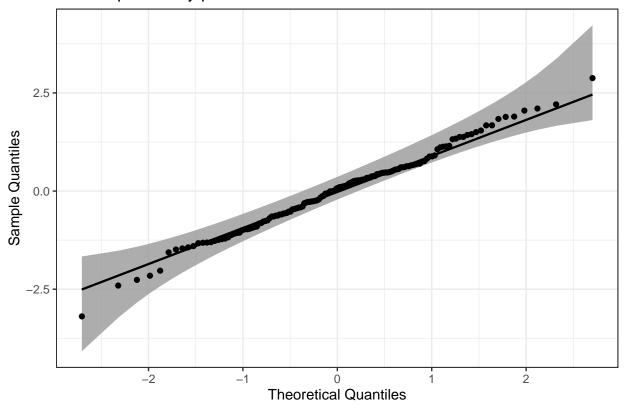
The variofaces are the lag 1 plots of the sample semivariogram with simulated confidence envelopes (Stefanova

et al., 2009).

Plot normal quantile plot

The plot is obtained using the ggplot function with extensions available from the qqplotr package (Almeida et al., 2023).

Normal probability plot



4. Prediction production and presentation

Get Variety predictions and all pairwise prediction differences and p-values

```
## Notice : Spline design points closer than .001400 have been merged
## Notice : Spline design points closer than .000900 have been merged
##
##
## #### Predictions for yield from Variety
##
##
## Notes:
## - The predictions are obtained by averaging across the hypertable
     calculated from model terms constructed solely from factors in
     the averaging and classify sets.
## - Use 'average' to move ignored factors into the averaging set.
## - spl(cRow) evaluated at average value of 0.00000
## - spl(cColumn) evaluated at average value of 0.00000
## - The simple averaging set: Rep
##
##
      Variety predicted.value standard.error upper.halfLeastSignificant.limit
## 1
                      1196.214
                                     64.11316
                                                                        1255.028
## 2
            9
                      1266.442
                                     69.62176
                                                                        1325.255
## 3
           16
                      1268.206
                                     74.01873
                                                                        1327.020
## 4
            1
                      1273.794
                                     69.30592
                                                                        1332.608
## 5
           14
                      1310.806
                                     70.75661
                                                                        1369.620
## 6
                                                                        1387.928
           23
                      1329.114
                                     73.70238
## 7
           11
                      1340.668
                                     75.07426
                                                                        1399.481
                                     76.83336
## 8
            4
                      1406.408
                                                                        1465.222
## 9
            3
                      1408.543
                                     71.85868
                                                                        1467.356
## 10
            7
                      1414.471
                                     72.23082
                                                                        1473.284
## 11
           12
                      1423.303
                                     71.18629
                                                                        1482.117
## 12
            8
                                     75.37805
                      1445.533
                                                                        1504.347
## 13
            5
                      1480.687
                                     70.31499
                                                                        1539.500
## 14
           15
                      1485.249
                                     74.28025
                                                                        1544.063
## 15
           17
                      1495.212
                                     71.09930
                                                                        1554.026
## 16
           21
                      1512.767
                                     72.53109
                                                                        1571.580
## 17
            6
                      1520.841
                                     72.50721
                                                                        1579.654
## 18
           24
                      1563.649
                                     65.33308
                                                                        1622.463
## 19
           18
                                     71.06200
                                                                        1627.700
                      1568.887
## 20
           25
                      1579.929
                                     70.52509
                                                                        1638.743
## 21
            2
                                     74.96623
                                                                        1644.211
                      1585.398
## 22
           22
                      1633.080
                                     71.38265
                                                                        1691.894
## 23
           13
                      1637.119
                                     66.24775
                                                                        1695.932
           19
## 24
                      1651.533
                                     74.93686
                                                                        1710.347
## 25
           20
                      1657.612
                                                                        1716.426
                                     68.65452
##
      lower.halfLeastSignificant.limit est.status
## 1
                               1137.400
                                         Estimable
## 2
                               1207.628
                                         Estimable
## 3
                               1209.393
                                         Estimable
## 4
                               1214.980
                                         Estimable
## 5
                               1251.993 Estimable
## 6
                               1270.300 Estimable
## 7
                               1281.854
                                         Estimable
## 8
                                         Estimable
                               1347.594
## 9
                               1349.729 Estimable
## 10
                               1355.657 Estimable
## 11
                               1364.490 Estimable
```

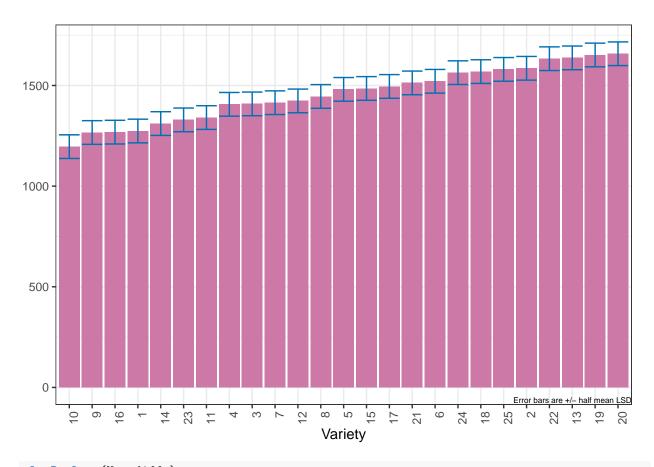
```
## 12
                               1386.720
                                         Estimable
## 13
                               1421.873
                                         Estimable
                               1426.435
## 14
                                         Estimable
                               1436.399
                                         Estimable
## 15
## 16
                               1453.953
                                         Estimable
                                         Estimable
## 17
                               1462.027
## 18
                               1504.836
                                         Estimable
## 19
                               1510.073
                                         Estimable
## 20
                               1521.115
                                         Estimable
## 21
                               1526.584
                                         Estimable
## 22
                               1574.266
                                         Estimable
## 23
                                         Estimable
                               1578.305
## 24
                               1592.719
                                         Estimable
## 25
                               1598.798 Estimable
##
##
## LSD values
##
## minimum LSD = 109.6008
## mean LSD = 117.6273
## maximum LSD = 126.3422
## (sed range / mean sed = 0.142)
```

We have set error.intervals to halfLeast so that the limits for each prediction $\pm (0.5 \text{ LSD})$ are calculated. When these are plotted overlapping error bars indicate predictions that are not significant, while those that do not overlap are significantly different (Snee, 1981).

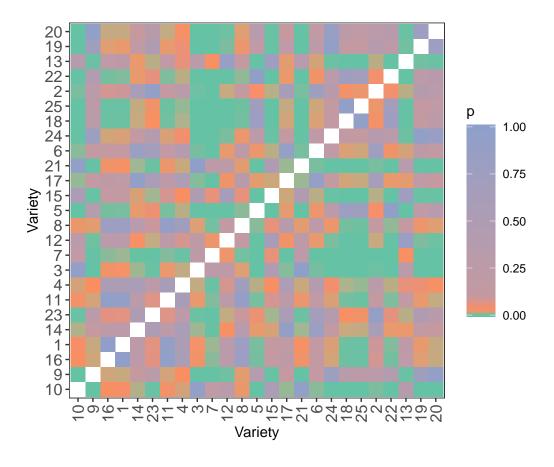
Also set was sortFactor, so that the results would be ordered for the values of the predictions for Variety. The function predictPlus returns an alldiffs object, a list consisting of the following components:

- predictions: the predictions, their standard errors and error intervals;
- vcov: the variance matrix of the predictions;
- differences: all pairwise differences between the predictions,
- p.differences: p-values for all pairwise differences between the predictions;
- sed: the standard errors of all pairwise differences between the predictions;
- LSD: the mean, minimum and maximum of the LSDs.

Plot the Variety predictions, with halfLSD intervals, and the p-values



plotPvalues(Var.diffs)



References

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