Watt Balance Design

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1 Introduction

Since the late 1800's, the kilogram has been defined in terms of a physical platinum-iridium cylinder known as the International Prototype Kilogram (IPK). Over time, of course, the mass of this object has changed slightly, motivating the redefinition of the kilogram in terms of a physical constant. This year, a team at NIST will be redefining the kilogram in terms of Planck's Constant using a Watt Balance, which uses electricity and magnetism to determine the weight of an object by balancing it with a force created between a magnet and current carrying coil. We were inspired to build our own watt balance, and in particular, were inspired by the NIST team's construction of a LEGO Watt Balance. Our goal is to construct a tabletop-scale watt balance that has higher precision than the LEGO watt balance, while also being cheaper to construct.

The Watt Balance operates in two modes: velocity mode and force mode. In velocity mode, the coil is moved through the magnetic field by a motor and the voltage in the coil is measured. The purpose of this is to provide an indirect measurement of both the length of the coil and the strength of the magnetic field, as both quantities are difficult to measure directly with high accuracy. In force mode, we control a current through the loop with the coil centered between the magnets. This creates an upward force that we will use to counter the downward force of gravity on our test mass. In this way, we can determine the mass of the object in question.

2 Magnet and Coil Design

The design challenge for the magnet is that we would like the coil to be in a primarily radial magnetic field, and to have the field be as uniform as possible through the coil's range of motion in velocity mode. To achieve this magnetic field, we will be using the same design used by the LEGO watt balance team, which is two ring magnets with like poles facing each other such that the magnetic field in the space between the magnets is strongly radial. The magnets we will be using are neodymium N42 magnets which have 2" outer diameter, $\frac{1}{4}$" inner diameter, and $\frac{1}{4}$" thickness, and we plan to have $\frac{1}{16}$" clearance between the outer edge of the magnets and the 3D printed coil housing, which itself is $\frac{1}{16}$" thick. This makes the inner radius of the coil $\frac{1}{8}$" larger than the outer radius of the magnets. We are using 36 AWG wire, and are basing our calculations on having 3000 turns. This is where the coil design challenge comes in, as we will have to take into account the fact that the coil will have a non-negligible cross sectional area, and the magnetic field strength will vary throughout the coil. Using the Mathematica addon `Radia` created by researchers at the European Synchrotron Radiation Facility, we can model the magnetic field created by the magnets in order to observe the change in magnetic field strength over the coil, and approximate the magnitude of the voltage we can expect to measure.

In velocity mode, the EMF in the coil is caused by the force equation

$$\vec{F} = q(\vec{v} \times \vec{B})$$

For a single loop, we can integrate the force over the loop to find the total EMF

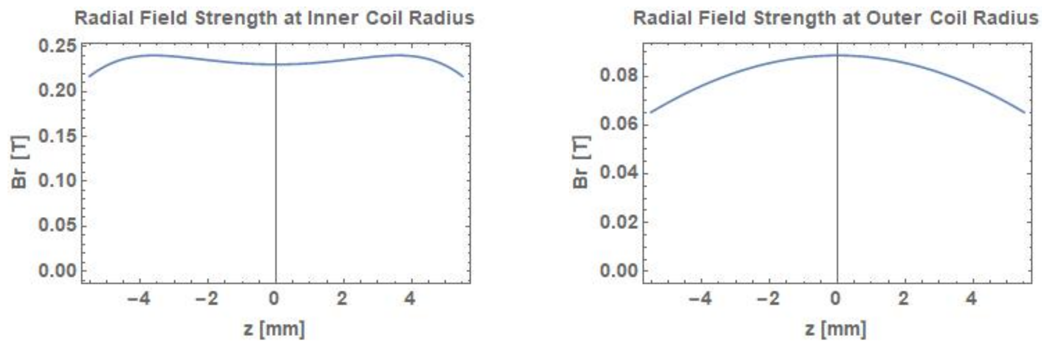
$$\mathcal{E} = \oint \vec{F} \cdot d\vec{l} = \oint (\vec{v} \times \vec{B}) \cdot d\vec{l} = BvL$$

We can make this last step because of the cylindrical symmetry of the magnetic field and coil. The goal now is to use this equation to determine the voltage we should expect from our coil and magnet, which is heavily dependent on a few geometric design decisions and considerations.

We will make a few approximations in order to simplify this calculation. First, we will assume that the coil is wound such that it is square-packed. This approximation makes the calculation of the EMF much simpler, as we will be able to assume that the radial component of the magnetic field does not change over a particular loop of the coil. We will also assume that the coil is wound such that the cross section is a square, with 55×55 turns. With each loop having a 0.005in diameter, we can determine exactly where each loop is located in space. To calculate the EMF, then, we can perform the sum

$$\mathcal{E} \approx \sum_{ij=1}^{55} B_r(\vec{r_{i,j}}) v L_{ij} = \sum_{ij=1}^{55} B_r(\vec{r_{i,j}}) v (2\pi r_{ij})$$

Where i and j index the loops of the coil and $\vec{r_{i,j}}$ is a position on a particular loop. Performing this sum when the coil is centered between the magnets, with a coil velocity of $10^{-2} ms^{-1}$, we get $\mathcal{E} \approx 0.877V$. We can perform the same sum with the coil positioned at many other locations through its range of motion to see how the voltage will vary as the coil moves.



(a) (b)

Figure 1: Profile of the radial component of the magnetic field at the inner radius of the coil (a) and outer radius of the coil (b). Notice on the vertical axis that the strength of the magnetic field decreases by more than a factor of two over the coil, making it necessary to consider the coil macroscopically.

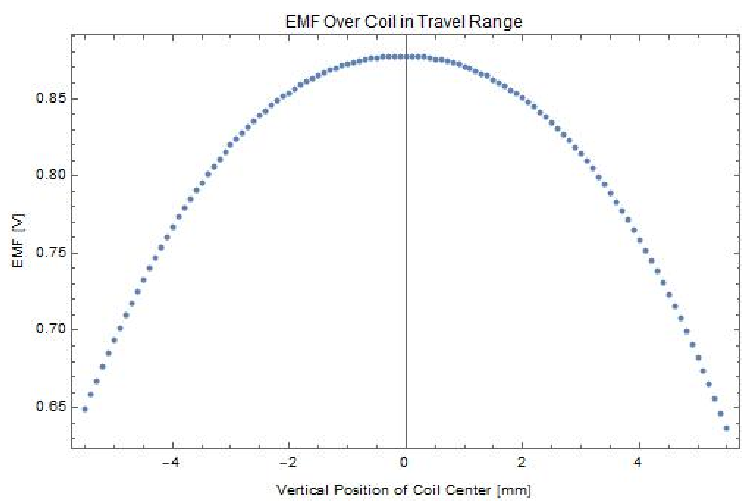


Figure 2: Plotted is the expected EMF in the coil as it moves through the 11mm between the magnets at a constant velocity of $10^{-2}ms^{-1}$

The maximum voltage occurs directly between the magnets at $0.877V$, as we determined in our first calculation. We will want to amplify this signal to $5V$ in order to increase the resolution we provide to our data acquisition device (DAQ). This can be done with a linear op-amp, and since our voltage will not be oscillating, we will not have to worry about the frequency dependence of the op-amp gain.

In force mode, we will be controlling the current through the coil as to exactly counter the force of gravity on our system. The force we will create is determined by

$$F = \oint q(\vec{v} \times \vec{B}) \cdot d\vec{l} = BLI$$

We would like to have an approximation for the maximum amount of current we may need to supply. To do this, we will assume the balance will need to counter the gravitational force from 50g of mass. We will ignore effects like self inductance for the purposes of this calculation. We can calculate the approximate length of the coil by again considering our 55×55 grid and summing over the circumference of each loop. This results in a length of 2000ft, and with 1.36Ω/ft, we find that the resistance in the coil will be 2.73kΩ. With our DAQ we will be able to apply a 10V voltage, which corresponds to a current of 3.7mA. The force will be

$$F \approx \sum_{ij=1}^{55} B_r(\vec{r_{i,j}}) L_{ij} I$$

We find that the force here is 0.345N, which would be capable of supporting 30g in the presence of Earth's gravity. This mass is achievable, and we can always make modifications such as hollowing out the nylon rods, which will be a significant portion of our mass, in the event that we are over weight.

3 Structure Design

Our design was originally based on the LEGO watt balance built by the NIST watt balance team. [1] Their design was symmetric to allow for simplicity in building. To improve our ability to reduce error, and to focus our spending on those improvements, we changed to an asymmetric design. Only one side of the watt balance needs to be used for measurement. The other side need only be a simple motor to provide the motion for the velocity measurement stage.

Due to the cost and design limitations of LEGO, we decided to build our watt balance using more traditional materials. For the main body, we will be using Baltic Birch plywood for its consistency, low cost, and dimensional stability. Where required, we will be using non-magnetic fasteners. Primarily, these will consist of glass-filled nylon bolts, with the bolt holding the magnet in place being made from titanium. To mount the coils to the wheel, we will be using titanium wire. If all goes as planned, our watt balance will be slightly above the cost of the LEGO watt balance, while being far more precise and durable.

The LEGO watt balance also used a shadow sensor and laser as an optical arm to measure velocity. [1] Brandon Grinkmeyer found that it was within our budget to build an interferometer. This change allows for much more precise velocity measurement and removes the need for a large beam-style balance. Because of this change, we were able to convert to a wheel-style balance. The advantage of the wheel is that it removes any horizontal motion, allowing for the tolerance between the magnets and the coil to be reduced, which increases the magnetic field strength at the coil. To maintain the position of the coil relative to the magnets, we added nylon guide rods to the coil housing. The rods slide in oil-impregnated nylon sleeve bearings mounted to the rigid body of the balance with a housing shown in Figure 3.

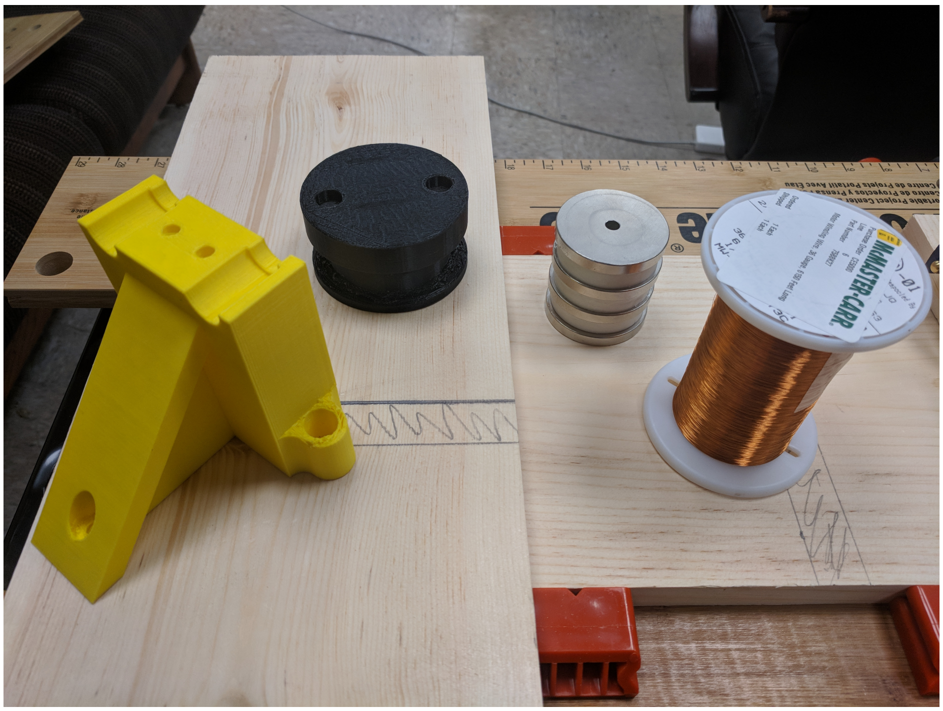


Figure 3: From left to right: 3D printed nylon sleeve bearing housing, 3D printed coil housing, set of 4 N42 magnets, 36AWG magnet wire.

We also changed the location of the mass pan from the LEGO design to below the coil. This change in location means that if the mass is off-center, it will result in less off-axis force because the ratio of the distance of the shifted center of mass to the length of the pendulum will be smaller.

The pivot for the beam in the LEGO balance was a knife edge resting in a v-block. [1] We researched the possibility of changing to a two-point pivot design using jewel bearing pivots, but found that the design has not been well characterized and may introduce hysteresis that we do not know how to model. Instead, we decided on a knife edge using a razor blade resting on a slab of tungsten carbide. This knife edge will be inexpensive and slow to wear.

In order to make the pivot adjustable, we will build a housing with set screws that will fit in our wheel such that the tip of the razor will be at the center of the wheel arc.

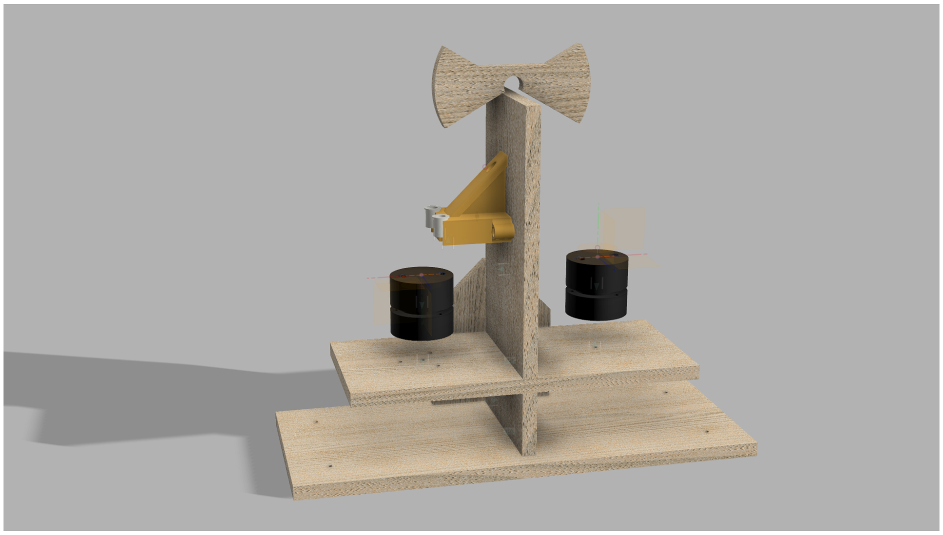


Figure 4: Preliminary CAD model. Includes all wooden components, the sleeve bearings, bearing mount, and coil housings.

4 Challenges

3D printing parts turned out to be far more time-consuming than originally anticipated. The process included a lot of trial and error due to the tolerances of the machines being an unknown that had to be tested. Parts also had to be designed such that they avoided the need for support material when printing. Overhangs had to be omitted and some parts needed to be made as separate components and glued back together.

While designing the main structure, our goal was to keep as many parts as adjustable as possible to allow for the tolerance levels of the different manufacturing processes. Many of the mechanisms for adjustment ended up getting in the way of other components and required redesigning multiple components.

5 Future Improvements

Currently, the wheel is designed such that the measurement side and the motor side have the same radius. After completion of our test model, we may find that their ratio should be something other than 1:1. We could likely confirm this with more simulation as well. At this point, whichever can get done more quickly will determine how we decide on how to change it.

We may also need to increase the size of the wheel to reduce the change in angle that the knife edge experiences. A high change in angle could cause the knife edge to "walk" on its bearing surface, and we need to avoid that to keep the movement of the coils smooth.

In terms of the magnets, we chose N42 grade based on its availability, but we could replace these with a higher grade to create a stronger magnetic field. This would result in a higher EMF in velocity mode, and also more force from the same current in force mode. This is important in force mode to give us greater control over the mass to hold it in place most effectively.

The electronics and velocity measurement system are out of the scope of this report, but both will need to interface with our design, and the design will likely need to be tweaked to accommodate these other components.

6 Task Breakdown

Brady:

- Magnet and Coil design
- Theory and Simulation

John:

- 3D CAD
- 3D printing
- Mechanical design
- Materials selection

7 Conclusion

Our simulations have shown that our current design strategy will produce a device that should be able to function properly. Given the shape of the magnetic field and the coil design, the voltage produced in the coil in velocity mode will change quadratically with the position of the coil, and the limits will be the same order of magnitude. Using a linear opamp we should be able to get the full benefit of the resolution of our data acquisition device. The simulations also showed that in force mode, the current we can apply has the capacity to support our coil housing and guide system.

The design decisions we chose to make kept us within our budget, and close to the cost of the LEGO watt balance due to the fact that LEGO bricks are very expensive, and that wood and 3D printing are very inexpensive. The design decisions have also brought us closer to the design of the part-per-billion precision watt balance at NIST. Building our watt balance has the added benefit of teaching us about the methods for decision and reasoning behind the design choices made by the NIST team.

Based on our initial research and the work that we have since done to bring this project to fruition, we should be able to build a desktop watt balance that will be able to measure a 10g mass to 10^{-4}g precision. This limitation is primarily due to the resolution of our data acquisition device from National Instruments. Our next goal after building the watt balance in its current form is to try to come up with a better data acquisition system to push past this limit.

Our total cost for the watt balance is less than $1000 and it can be built with limited tools. We plan to publish our designs and results to allow others to build and improve on our design. This design will ultimately help bridge the gap between the inexpensive, but imprecise LEGO watt balance, and the part-per-million precision desktop watt balance currently in development at NIST.

References

[1] L. S. Chao, S. Schlamminger, D. B. Newell, J. R. Pratt, F. Seifert, X. Zhang, G. Sineriz, M. Liu, and D. Haddad. A LEGO Watt balance: An apparatus to determine a mass based on the new SI. *American Journal of Physics*, 83(11):913–922, October 2015.