Assignment #8

Due Wednesday, 16 November 2022, at the start of class

Please read sections 11.4, 11.5, 12.1, 12.2, and 12.3 from the textbook.[1] Though quasi-Newton methods are intimidating on the first look, please read 12.3 carefully so that you see how they are generalizations of the secant method (**P17** below).

DO THE FOLLOWING EXERCISES from section 11.4, pages 374–375:

- Exercise 4.4

DO THE FOLLOWING EXERCISES from section 11.5, pages 385–391:

- Exercise 5.2
- Exercise 5.5

DO THE FOLLOWING EXERCISES from section 12.2, pages 408–411:

- Exercise 2.3

DO THE FOLLOWING EXERCISES from section 12.3, pages 420–421:

- Exercise 3.7

Problem P16. Suppose $x_k \in \mathbb{R}^n$ is any iterate, and suppose that $\nabla f(x_k)$ is a nonzero vector. Compute all $p \in \mathbb{R}^n$ which solve the problem

$$\min_{p \neq 0} \frac{p^\top \nabla f(x_k)}{\|p\| \|\nabla f(x_k)\|}.$$

(*Hint.* The solution is brief. Remember $x^\top y$ is the dot product of vectors! Explain why you have found all minima.)

Problem P17. Consider the one-variable problem

$$\min_{x \in \mathbb{R}} f(x)$$

where $f : \mathbb{R} \to \mathbb{R}$ is twice continuously-differentiable. Recall that Newton method for the above minimization problem solves $f'(x) = 0$ by the formulas $p_k = -f'(x_k)/f''(x_k)$ and $x_{k+1} = x_k + p_k$.

[1] Griva, Nash, and Sofer, *Linear and Nonlinear Optimization*, 2nd ed., SIAM Press 2009.

The *secant method* for minimization only differs from the Newton method by replacing the second derivative with a difference quotient approximation based on the last two iterates:

$$f''(x_k) \approx \frac{f'(x_k) - f'(x_{k-1})}{x_k - x_{k-1}}.$$

Thus the secant method computes the step (search vector) by

$$p_k = -\frac{(x_k - x_{k-1})f'(x_k)}{f'(x_k) - f'(x_{k-1})},$$

with $x_{k+1} = x_k + p_k$ as before.

a) Implement the secant method. Include a stopping criterion $|f'(x_k)| <$`tol`.

b) Use your code to accurately solve $\min_{x\in\mathbb{R}} f(x)$, e.g. with `tol`$= 10^{-8}$, for the following functions and initial iterates:

i) $f(x) = x^3 - 2\sin x, \quad x_0 = 0, \quad x_1 = 1$

ii) $f(x) = 3x^4 - 4x^3 + 3x^2 - 6x, \quad x_0 = -1, \quad x_1 = 0$

c) In part ii) above the exact minimum is at $x_* = 1$. Compute the errors $e_k = x_k - x_*$. Give evidence that the convergence is superlinear. Using the notation of section 2.5, what is your estimate of the rate (exponent) r?