Friday 7 October

$\min c^\top x$ subject to $Ax = b$, $x \geq 0$ where

$$A = \begin{bmatrix} 2 & 1 & 1 & 0 \\ -1 & 1 & 0 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 3 \\ 2 \end{bmatrix}, \quad c = \begin{bmatrix} -2 \\ -4 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathcal{B} = \{3, 4\}, \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad c_B = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \underline{Bx_B = b} \implies x_B = \hat{b} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$\mathcal{N} = \{1, 2\}, \quad N = \begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix}, \quad c_N = \begin{bmatrix} -2 \\ -4 \end{bmatrix}$$

$$\underline{B^\top y = c_B} \implies y = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \implies \underline{\hat{c}_N = c_N - N^\top y} = \begin{bmatrix} -2 \\ -4 \end{bmatrix}$$

$\boxed{\hat{c}_N \geq 0\text{ ?: stop with optimum}}$ $\quad \hat{c}_N \xrightarrow[\text{min}]{\text{index of}} t = \boxed{2} \to \underline{B\hat{A}_t = A_t} \implies \hat{A}_t = \begin{bmatrix} \hat{a}_{1,t} \\ \vdots \\ \hat{a}_{m,t} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\boxed{\hat{A}_t \leq 0\text{ ?: stop, unbounded}}$ $\quad \left\{\frac{\hat{b}_i}{\hat{a}_{i,t}}\right\} = \left\{ \frac{3}{1}, \frac{2}{1} \right\} \xrightarrow[\text{min over } \hat{a}_{i,t} > 0]{\text{index of}} s = \boxed{4}$

$$\mathcal{B} = \{3, 2\}, \quad B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \quad c_B = \begin{bmatrix} 0 \\ -4 \end{bmatrix}, \quad \underline{Bx_B = b} \implies x_B = \hat{b} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\mathcal{N} = \{1, 4\}, \quad N = \begin{bmatrix} 2 & 0 \\ -1 & 1 \end{bmatrix}, \quad c_N = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$$

$$\underline{B^\top y = c_B} \implies y = \begin{bmatrix} 0 \\ -4 \end{bmatrix} \implies \underline{\hat{c}_N = c_N - N^\top y} = \begin{bmatrix} -6 \\ 4 \end{bmatrix}$$

$\boxed{\hat{c}_N \geq 0\text{ ?: stop with optimum}}$ $\quad \hat{c}_N \xrightarrow[\text{min}]{\text{index of}} t = \boxed{1} \to \underline{B\hat{A}_t = A_t} \implies \hat{A}_t = \begin{bmatrix} \hat{a}_{1,t} \\ \vdots \\ \hat{a}_{m,t} \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$

$\boxed{\hat{A}_t \leq 0\text{ ?: stop, unbounded}}$ $\quad \left\{\frac{\hat{b}_i}{\hat{a}_{i,t}}\right\} = \left\{ \frac{1}{3}, \times \right\} \xrightarrow[\text{min over } \hat{a}_{i,t} > 0]{\text{index of}} s = \boxed{3}$

$\mathcal{B} = \{ 1, 2 \}$, $B = \begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix}$, $c_B = \begin{bmatrix} -2 \\ -4 \end{bmatrix}$, $\underline{B x_B = b} \implies x_B = \hat{b} = \begin{bmatrix} \frac{1}{3} \\ 7/3 \end{bmatrix}$

$\mathcal{N} = \{ 3, 4 \}$, $N = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $c_N = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$X = \begin{bmatrix} 1/3 \\ 7/3 \\ 0 \\ 0 \end{bmatrix}$

$\underline{B^\top y = c_B} \implies y = \begin{bmatrix} -2 \\ -2 \end{bmatrix} \implies \underline{\hat{c}_N = c_N - N^\top y} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

$\boxed{\hat{c}_N \geq 0 \text{ ?: stop with optimum}}$ $\quad \hat{c}_N \xrightarrow[\text{min}]{\text{index of}} t = \boxed{} \to \underline{B\hat{A}_t = A_t} \implies \hat{A}_t = \begin{bmatrix} \hat{a}_{1,t} \\ \vdots \\ \hat{a}_{m,t} \end{bmatrix} = \begin{bmatrix} \end{bmatrix}$

$\boxed{\hat{A}_t \leq 0 \text{ ?: stop, unbounded}}$ $\quad \left\{ \frac{\hat{b}_i}{\hat{a}_{i,t}} \right\} = \{ \qquad \qquad \} \xrightarrow[\text{min over } \hat{a}_{i,t} > 0]{\text{index of}} s = \boxed{}$

$\mathcal{B} = \{ \qquad \}$, $B = \begin{bmatrix} \end{bmatrix}$, $c_B = \begin{bmatrix} \end{bmatrix}$, $\underline{B x_B = b} \implies x_B = \hat{b} = \begin{bmatrix} \end{bmatrix}$

$\mathcal{N} = \{ \qquad \}$, $N = \begin{bmatrix} \end{bmatrix}$, $c_N = \begin{bmatrix} \end{bmatrix}$

$\underline{B^\top y = c_B} \implies y = \begin{bmatrix} \end{bmatrix} \implies \underline{\hat{c}_N = c_N - N^\top y} = \begin{bmatrix} \end{bmatrix}$

$\boxed{\hat{c}_N \geq 0 \text{ ?: stop with optimum}}$ $\quad \hat{c}_N \xrightarrow[\text{min}]{\text{index of}} t = \boxed{} \to \underline{B\hat{A}_t = A_t} \implies \hat{A}_t = \begin{bmatrix} \hat{a}_{1,t} \\ \vdots \\ \hat{a}_{m,t} \end{bmatrix} = \begin{bmatrix} \end{bmatrix}$

$\boxed{\hat{A}_t \leq 0 \text{ ?: stop, unbounded}}$ $\quad \left\{ \frac{\hat{b}_i}{\hat{a}_{i,t}} \right\} = \{ \qquad \qquad \} \xrightarrow[\text{min over } \hat{a}_{i,t} > 0]{\text{index of}} s = \boxed{}$