Assignment #5

Due Monday, 17 October 2022, at the start of class

From the textbook[1] please read Chapter 4 and sections 5.1–5.2, but note that you can skip subsections 5.2.3 and 5.2.4 on tableaus.

DO THE FOLLOWING EXERCISES from section 4.2, pages 105–106:

- Exercise 2.1
- Exercise 2.2
- Exercise 2.4

DO THE FOLLOWING EXERCISES from section 4.3, pages 114–117:

- Exercise 3.1
- Exercise 3.12
- Exercise 3.16

DO THE FOLLOWING EXERCISES from section 5.2, pages 141–144. For *each* of these problems print out a simplex method template[2] and fill it in by hand:

- Exercise 2.2 (i)
- Exercise 2.2 (iii)
- Exercise 2.2 (vi)

Problem P9. On pages 90–91 the book describes how to use the QR decomposition to build a null-space matrix for A in a numerically-stable way:

> ...let A be an $m \times n$ matrix with full row rank. We perform an orthogonal factorization of $A^\top$:
> $$A^\top = QR.$$
> [Then let] $Q = (Q_1, Q_2)$, where Q_1 consists of the first m columns of Q and Q_2 consists of the last $n - m$ columns. [Then]
> $$Z = Q_2$$

[1]Griva, Nash, and Sofer, *Linear and Nonlinear Optimization*, 2nd ed., SIAM Press 2009.

[2]See the Worksheets tab at `bueler.github.io/opt`.

Note that an $m \times n$ matrix with full *row* rank has $m \leq n$, so in the description above $n-m$ is either zero or positive. As the book says, the columns of Z are not just a basis for the null space $\mathcal{N}(A)$, but a well-behaved *orthogonal* basis for $\mathcal{N}(A)$.

Write a MATLAB function[3]

```
function Z = mynull(A)
```

which implements the above strategy. In MATLAB the "orthogonal factorization" step can use the function `qr()`; you do not have to worry how `qr()` works. Your code should be quite short. Note that `size(A)` will tell you the values of m and n. Your code should stop with an error if $m > n$.

Test your `mynull()` on the matrices

$$A_1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 10 \\ 4 & 1 & 0 & 1 & 4 \end{pmatrix}$$

$$A_2 = \begin{pmatrix} 0 & 1 & 2 & 0 \end{pmatrix}$$

$$A_3 = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}.$$

Are the columns of Z actually in the null space of the matrix A_j in each case? (*Show command-line* MATLAB *verifications.*)

How does the result of `mynull()` differ from the result of the built-in command `null()` on the above matrices? (*Use* `norm` *to answer this.*) Is `null()` implemented the same way as `mynull()`?

[3] In Python, functions `qr()` and `null_space()` from `scipy.linalg` replace MATLAB commands `qr()` and `null()` in this problem. In Julia use `qr()` and `nullspace()`.