Assignment #2

Due Friday, 13 September 2024, at the start of class

Make sure you have a copy of the textbook:

> Griva, Nash, and Sofer, *Linear and Nonlinear Optimization*, 2nd ed., SIAM Press 2009.

Please read sections 2.1 through 2.4 and Appendices B.4 through B.8.

DO THE FOLLOWING EXERCISE from section 2.2, pages 47–48:

- Exercise 2.4
- Exercise 2.5
- Exercise 2.7

DO THE FOLLOWING EXERCISES from section 2.3, pages 52–54:

- Exercise 3.1
- Exercise 3.3
- Exercise 3.7
- Exercise 3.13
- Exercise 3.18

Problem P5. For each of the following functions, determine if it is convex, concave, both, or neither, on the real line $\mathbb{R}$. (Explain your answer.) If the function is convex or concave, indicated whether it is strictly so.

(a) $f(x) = 9x - 14$

(b) $f(x) = 1/(3 + x^4)$

(c) $f(x) = \sqrt{1 + x^2}$

(d) $f(x) = |x|$

Problem P6. (*This problem is related to Appendices B.4, B.6, B.7.*) Consider the scalar-valued function

$$f(x_1, x_2) = \exp(-x_1^2 - 2x_1 + 3x_2 - x_2^2)$$

Compute the gradient and Hessian of f. Find the location where f is maximum, and explain what properties of the gradient and Hessian show that it is a maximum.

Problem P7. (*This problem is related to Appendix B.4. Carefully read about the relationship between the gradient of a vector-valued function and its Jacobian.*)

(a) Consider the vector-valued function

$$f(x_1, x_2, x_3) = \begin{pmatrix} x_1^2 + x_2^2 + x_3^2 - 1 \\ x_2 - \arctan(x_1) \\ x_3^3 - x_3 - x_2 \end{pmatrix}$$

Compute the Jacobian of f.

(b) The Jacobian of f in part **(a)** is a 3×3 matrix, just like the Hessian of a scalar function. Is the Jacobian of f in part **(a)** actually the Hessian of some scalar function? This question can be answered by considering the symmetry of a matrix, for example at the point $x = (1, 1, 1)$ for concreteness.