Assignment #9

Due Wednesday, 30 November 2022, at the start of class

Please read sections 12.3, 12.4, 13.5, 14.1, 14.2, and 14.3 from the textbook.[1]

DO THE FOLLOWING EXERCISES from section 12.3, pages 420–421:

- Exercise 3.1 *Please write your own code, or modify* `sr1bt.m` *to use exact line search.*
- Exercise 3.3
- Exercise 3.4 *Hint. What do you know about a rank one matrix? Thus you can write it as an outer product.*
- Exercise 3.8

DO THE FOLLOWING EXERCISES from section 13.5, pages 473–474:

- Exercise 5.5 *Hint. Assume matrix B_k has a known inverse $H_k = B_k^{-1}$. Let $v = y_k - B_k s_k$. Then set $u = -v/(v^\top s_k)$ in Sherman-Morrison.*

DO THE FOLLOWING EXERCISES from section 14.2, pages 489–491:

- Exercise 2.1

Problem P18. *This problem simplifies/clarifies Exercise 4.4 in section 12.4.*

Apply the forward difference formula $f'(x) \approx (f(x+h) - f(x))/h$ to estimate the gradient of the function

$$f(x) = \exp(10x_1 + 2x_2^2),$$

at the point $x = (-1, 1)$. (Obviously, $f : \mathbb{R}^2 \to \mathbb{R}$.) Assuming that $\epsilon_{\text{mach}} = 2.2204 \times 10^{-16}$ on the computer you used, how accurate is the approximated gradient when you actually compute using the "best" value of h and the "simpler" value of h, from page 425? (*Use a norm to quantify the error in the gradient.*)

Problem P19. Suppose a user wants to solve $\min_{x\in\mathbb{R}^n} f(x)$ for a smooth objective function $f : \mathbb{R}^n \to \mathbb{R}$, but they do not provide a gradient function $\nabla f(x)$. We can still use quasi-Newton if we apply finite differences to replace the missing gradient (section 12.4). Use this idea to write a new code

```
function [xk, xklist] = sr1btfd(x0,f,tol,maxiters)
```

which replaces the user-provided gradient in `sr1bt.m` with a finite-difference gradient. Test it on the Rosenbrock example, as is done in `rosencompare.m`, and give iterations. (*Don't worry about visualizing, or the contours.*) How would it help if the user provided values for the typical scales of the gradient and the Hessian?

[1] Griva, Nash, and Sofer, *Linear and Nonlinear Optimization*, 2nd ed., SIAM Press 2009.