Basics On Optimisation - COMP24112

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1. General Form of Optimisation

The problem of ***optimisation*** finds the input variables that can given the smallest (minimum) or largest (maximum) output value of a real-valued function. It has the following general form:

$$\begin{aligned} \min \quad & O(x_1, x_2, \ldots, x_n), \\ \text{subject to} \quad & f_1(x_1, x_2, \ldots, x_n) \le 0, \\ & f_2(x_1, x_2, \ldots, x_n) \le 0, \\ & \quad \vdots \\ & f_m(x_1, x_2, \ldots, x_n) \le 0. \end{aligned} \tag{1}$$

The real-valued function $O(x_1, x_2, \ldots, x_n)$ that takes n real-valued variables as the input is called the ***optimisation objective function***. The different real-valued functions $f_i(x_1, x_2, \ldots, x_n) \le 0$ ($i = 1, 2, \ldots m$) are called the ***constraints***. They restrict the sets from which the input variables are allowed to choose their values. If all the input variables of the objective function $O(x_1, x_2, \ldots, x_n)$ are allowed to be chosen from the set of all real numbers such that $x_i \in R$ for $i = 1, 2, \ldots, n$, an ***unconstrained optimisation*** problem is to be solved, simply written as

$$\min O(x_1, x_2, \ldots, x_n). \tag{2}$$

2. Notations

Storing the n input variables in a vector such as $\boldsymbol{x} = [x_1, x_2, \ldots, x_n]$, the optimisation problem in Eq. (1) can be expressed as

$$\min \quad O(\boldsymbol{x}) \tag{3}$$

$$\text{subject to} \quad f_i(\boldsymbol{x}) \le 0, i = 1, 2, \ldots m. \tag{4}$$

This notation can also be simplified as

$$\min_{f_i(\boldsymbol{x}) \le 0, i=1,2,\ldots m} O(\boldsymbol{x}).$$

The unconstrained optimisation in Eq. (2) is simplified as $\min O(\boldsymbol{x})$.

3. Example

We look at the example of finding the maximum of the function $(x+1)^2 \sin(y)$, where the input x is allowed to be chosen from the set of real numbers between 0 and 5, expressed as $x \in [0,5]$ or $0 \le x \le 5$. The input y is allowed to be chosen from the set of real numbers between 0 and 3, expressed as $y \in [0,3]$ or $0 \le y \le 3$.

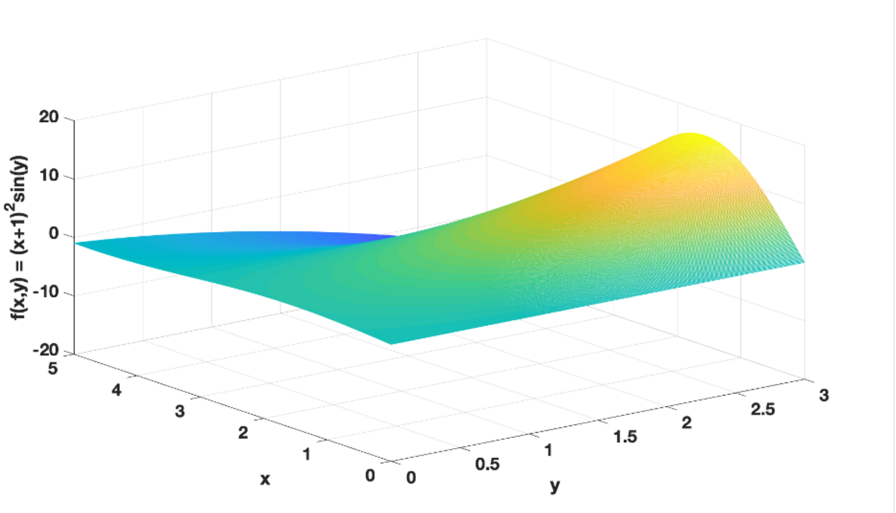

Figure 1: Example function plot for $f(x,y) = (x+1)^2 \sin(y)$ where $0 \le x \le 5$ and $0 \le y \le 3$.

To formulate it as an optimisation problem, the objective function is set as

$$O(x,y) = (x+1)^2 \sin(y).$$

The input x and y are restricted to the two sets $[0,5]$ and $[0,3]$, which can be converted to four constraint functions:

$$\begin{aligned} -x &\le 0, \\ x-5 &\le 0, \\ -y &\le 0, \\ y-3 &\le 0. \end{aligned}$$

Together, it gives the following general-form expression:

$$\max_{\substack{-x\le 0\\ x-5\le 0\\ -y\le 0\\ y-3\le 0}} (x+1)^2 \sin(y),$$

or

$$\min_{\substack{-x\le 0\\ x-5\le 0\\ -y\le 0\\ y-3\le 0}} -(x+1)^2 \sin(y).$$

Usually, you can just simply write it as

$$\max_{\substack{x\in[0,5]\\ y\in[0,3]}} (x+1)^2 \sin(y),$$

and

$$\max_{\substack{0\le x\le 5\\ 0\le y\le 3}} (x+1)^2 \sin(y),$$