ECS253 - Homework 1

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April 25, 2016

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Problem 1

```
In [43]: %matplotlib inline
         import numpy as np
         import networkx as nx
         import seaborn as sns
         sns.set_style('ticks')
         sns.set_context('poster')
```

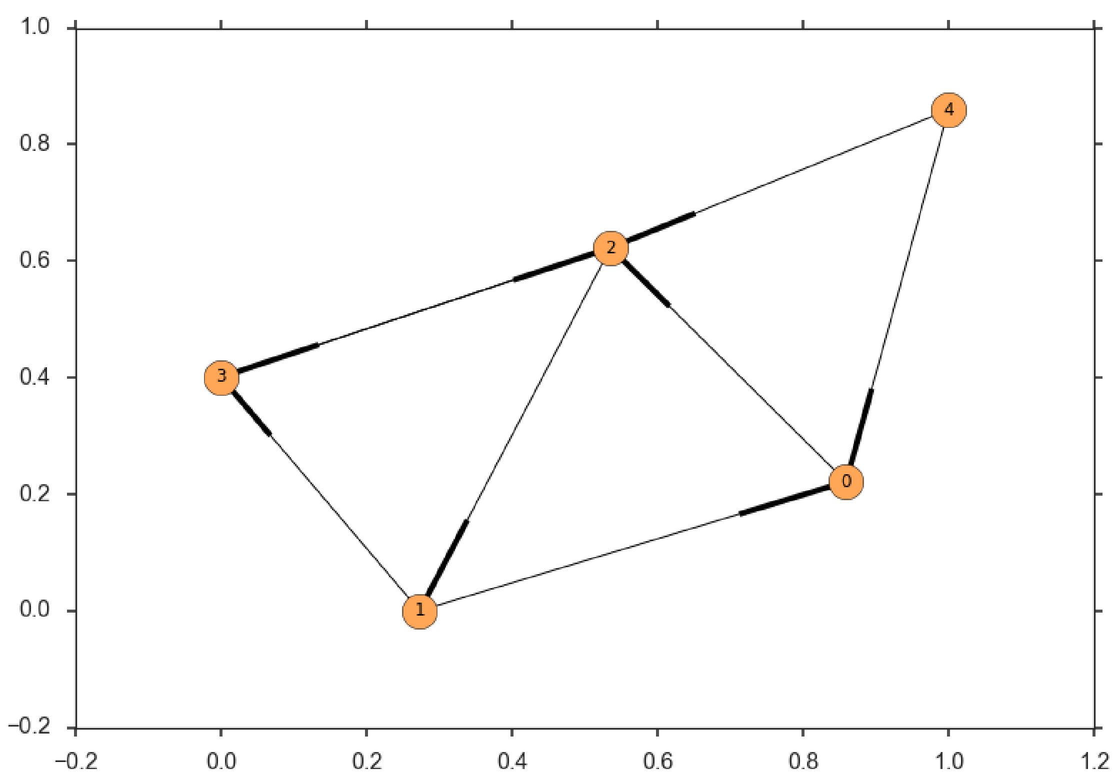
1.1 The Adjacency Matrix

```
In [21]: A_directed = np.array( [[0, 1, 0, 0, 1],
                                  [0, 0, 1, 0, 0],
                                  [1, 0, 0, 1, 1],
                                  [0, 1, 1, 0, 0],
                                  [0, 0, 0, 0, 0]] )
         print(A_directed)
```

```
[[0 1 0 0 1]
 [0 0 1 0 0]
 [1 0 0 1 1]
 [0 1 1 0 0]
 [0 0 0 0 0]]
```

A little visualization, just to double check.

```
In [36]: nx.draw_networkx(nx.DiGraph(data=A_directed.T),
                          node_size=600,
                          node_color=sns.xkcd_rgb["pale orange"])
```



1.2 Steady-State Probability of Random Walker

We can calculate the steady state probabilities quite simply by dividing out the column sums of the adjacency matrix A to convert it into a state transition matrix M and then computing the matrix power M^i. With i as a reasonably high number, the result will converge to the steady state probabilities.

```
In [5]: def calc_steady_state(A, i=100):
            M = A / A.sum(axis=0)
            M = np.linalg.matrix_power(M, i)
            return M
```

```
In [40]: def print_probs(probs):
             print('\n'.join('Node {0}: {1:.4f}'.format(node, p) for node, p in \
                         zip(range(1, len(probs)+1), probs)))
```

The resulting probabilities:

```
In [41]: probs = calc_steady_state(A_directed)[:,0]
         print_probs(probs)
```

```
Node 1: 0.1000
Node 2: 0.2000
Node 3: 0.4000
Node 4: 0.3000
Node 5: 0.0000
```

1.3 Steady-State Probabilities for Undirected Graph

For the undirected variant, we just mirror across the diagnol.

```
In [15]: A_undirected = np.array( [[0, 1, 1, 0, 1],
                                   [1, 0, 1, 1, 0],
                                   [1, 1, 0, 1, 1],
                                   [0, 1, 1, 0, 0],
                                   [1, 0, 1, 0, 0]] )
         print(A_undirected)
```

```
[[0 1 1 0 1]
 [1 0 1 1 0]
 [1 1 0 1 1]
 [0 1 1 0 0]
 [1 0 1 0 0]]
```

```
In [42]: probs = calc_steady_state(A_undirected)[:,0]
         print_probs(probs)
```

```
Node 1: 0.2143
Node 2: 0.2143
Node 3: 0.2857
Node 4: 0.1429
Node 5: 0.1429
```

Problem 2

2.1 Rate Equations

$k > m$: $n_{k,t+1} = n_{k,t} + \frac{1}{t}n_{k-1,t} - \frac{1}{t}n_{k,t}$
$k = m$: $n_{m,t+1} = n_{m,t} + 1 - \frac{1}{t}n_{m,t}$

2.2 Probabilistic Treatment

$k > m$: $(t+1)P_{k,t+1} = tP_{k,t} + \frac{1}{t}tP_{k-1,t} - \frac{1}{t}tP_{k,t}$
$\Rightarrow (t+1)P_{k,t+1} = (t-1)P_{k,t} + P_{k-1}t$
$k = m$: $(t+1)P_{m,t+1} = tP_{m,t} + 1 - \frac{1}{t}tP_{m,t}$
$\Rightarrow (t+1)P_{m,t+1} = (t-1)P_{m,t} + 1$

2.3 Solving for P_k

$k > m$: $tP_k + P_k = tP_k - P_k + P_k - 1$
$\Rightarrow P_k = \frac{P_{k-1}}{2}$
$k = m$: $tP_m + P_m = tP_m - P_m + 1$
$\Rightarrow P_m = \frac{1}{2}$

2.4 Final Expression

$P_k = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times ... \times P_m$
$\Rightarrow P_k = \frac{1}{2^k}$