Algoritem potisni-povišaj za iskanje maksimalnih pretokov

Zagovor dela diplomskega seminarja

Marcel Čampa

Fakulteta za matematiko in fiziko Univerze v Ljubljani

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Pregled vsebine

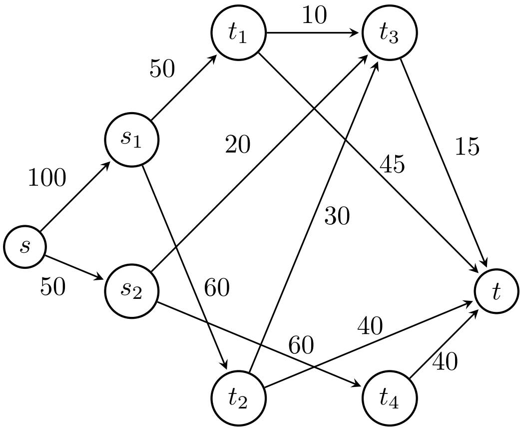
Graf in omrežje

Definicija

Graf *G je par množic $G = (V, E)$, kjer je V množica vozlišč grafa, $E \subseteq V \times V$ pa je množica usmerjenih povezav grafa G.*

Definicija

Naj bo $G = (V, E)$ graf. ***Omrežje*** *na grafu G je par (G, c), kjer je $c\colon V \times V \to \mathbb{R}_0^+ \cup \{\infty\}$* ***funkcija prepustnosti****, ki vsakemu paru vozlišč (u, v) priredi prepustnost $c(u, v)$ povezave od u do v. Prepustnost $c(u, v) = \infty$ natanko tedaj, ko prepustnost povezave ni omejena. Velja še, da $c(u, v) = 0$ natanko tedaj, ko povezava ne obstaja v G.*
Pretočno omrežje *na omrežju (G, c) je četverica (G, c, s, t), kjer je $s \in V$ začetno vozlišče pretočnega omrežja, rečemo mu* ***izvir****, $t \in V$ pa končno vozlišče pretočnega omrežja, ki mu pravimo* ***ponor****.*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(Maksimalni) pretok

Definicija

Pretok *f je taka funkcija $f\colon V \times V \to \mathbb{R}$, da velja naslednje.*

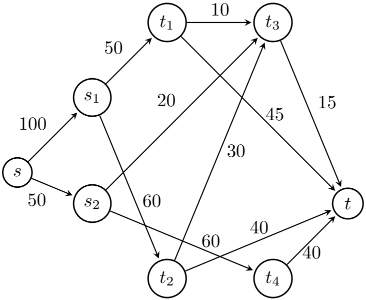
1. *Za vsaki vozlišči $u, v \in V$ velja $f(u, v) = -f(v, u)$.*
2. *Za vsaki vozlišči $u, v \in V$ velja $f(u, v) \leq c(u, v)$, kjer je c funkcija prepustnosti.*
3. *Za vsak $v \in V \setminus \{s, t\}$ velja, da je neto tok, ki priteče v vozlišče v, enak nič, torej da velja $e_f(v) = 0$.*

Definicija

Maksimalni pretok *je pretok f, za katerega velja*

$$|f| = \max_{f_i} |f_i|,$$

kjer f_i teče po vseh možnih pretokih skozi omrežje.



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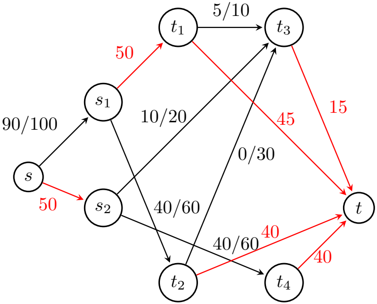

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s
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t_2
t_4

Pregled algoritmov

algoritem	*časovna zahtevnost*	*leto*
Ford-Fulkerson	$\mathcal{O}(E\|f\|)$	1956
Edmonds-Karp	$\mathcal{O}(VE^2)$	1972
Dinic	$\mathcal{O}(VE \log V)$	1970
potisni-povišaj (gen.)	$\mathcal{O}(V^2E)$	1986
potisni-povišaj	$\mathcal{O}(V^3)$	1988
KRT	$\mathcal{O}(VE \log_{\frac{E}{V \log V}} V)$	1994
Orlin + KRT	$\mathcal{O}(VE)$	2013

Osnovni algoritem

```
POTISNI-POVIŠAJ(G,s)
    INICIALIZIRAJ_PREDPRETOK(G,s)
    DOKLER obstaja mogoča operacija POTISNI ali POVIŠAJ
        izvedi mogočo operacijo
```

Operacija POTISNI

```
POTISNI (u, v)
    // Potisnemo lahko, če je e(u) > 0,
    // c_f(u,v) > 0 in h(u) = h(v) + 1.
    delta = min{ e(u), c(u,v) - f(u,v) }
    f(u,v) += delta
    f(v,u) -= delta
    e(u) -= delta
    e(v) += delta
```

Operacija POVIŠAJ

```
POVIŠAJ (u)
    // Vozlišče u povišamo, če je e(u) > 0 in
    // za vsak v iz V, (u,v) v E_f, velja h(u) <= h(v).
    h(u) = min{ h(v) : (u,v) v E_f } + 1
```

Operacija INICIALIZIRAJ_PREDPRETOK

```
INICIALIZIRAJ_PREDPRETOK(G,s)
   // V grafu G z izbranim izvirom s
   // inicializiramo predpretok.
   ZA vsak v v V(G)
       h(v) = 0
       e(v) = 0
   ZA vsak (u,v) v E(G)
       f(u,v) = 0
   h(s) = |V|
   ZA vsak v, za katerega obstaja (s,v) v E(G)
       f(s,v) = c(s,v)
       f(v,s) = -f(s,v)
       e(v) = f(s,v)
```

Časovna zahtevnost

Število operacij

- `POVIŠAJ` je kvečjemu $2|V|^2$;
- `POTISNI` je kvečjemu $2|V||E| + 4|V|^2(|V| + |E|)$.

Časovna zahtevnost

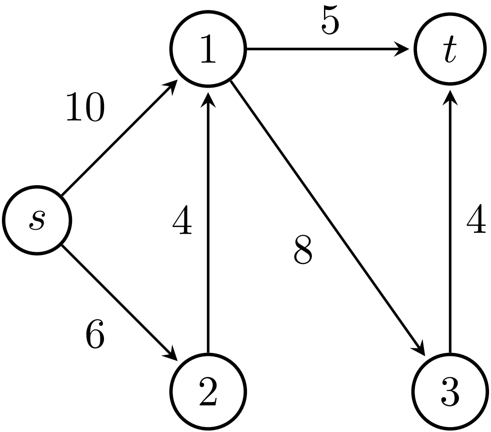
Število operacij

- `POVIŠAJ` je kvečjemu $2|V|^2$;
- `POTISNI` je kvečjemu $2|V||E| + 4|V|^2(|V| + |E|)$.

Izrek

Časovna zahtevnost algoritma POTISNI–POVIŠAJ je $\mathcal{O}(V^2E)$.

Začetno omrežje



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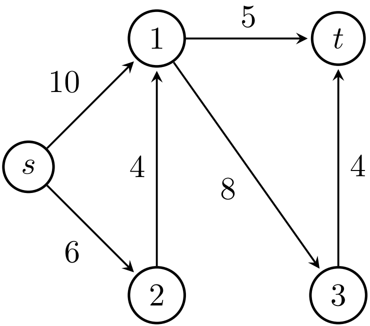
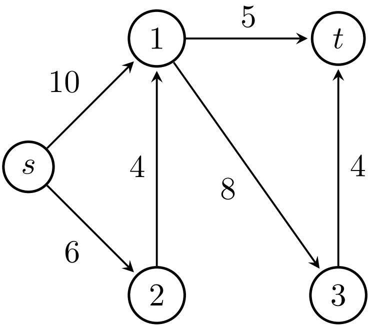
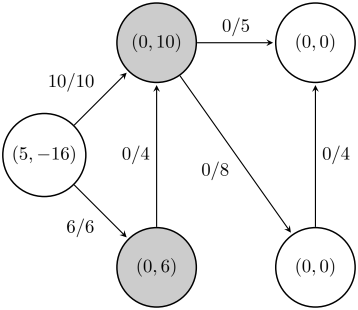


Inicializacija predpretoka





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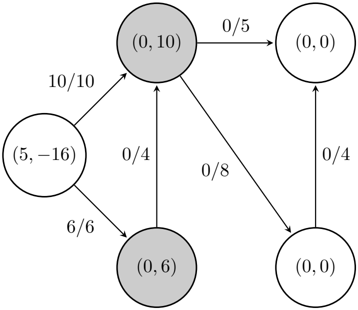
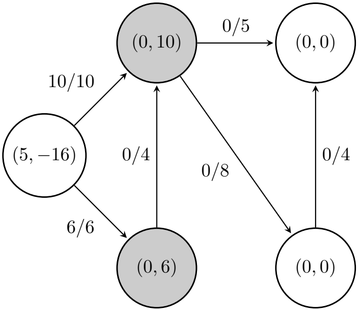


Korak 1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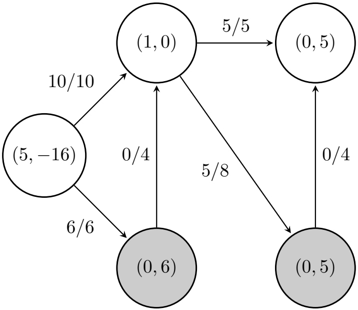
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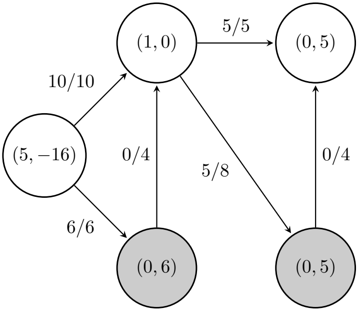

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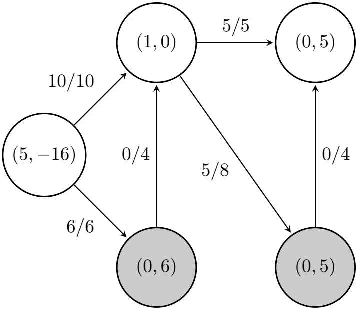
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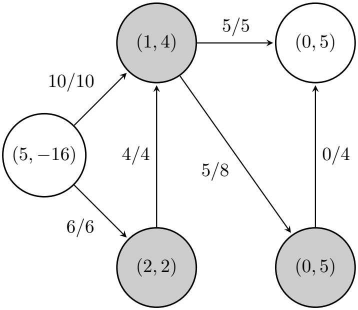
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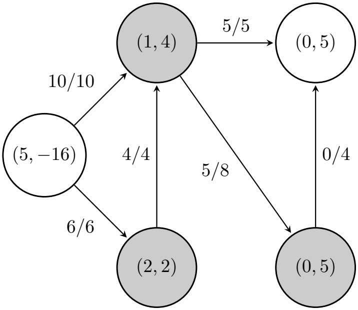

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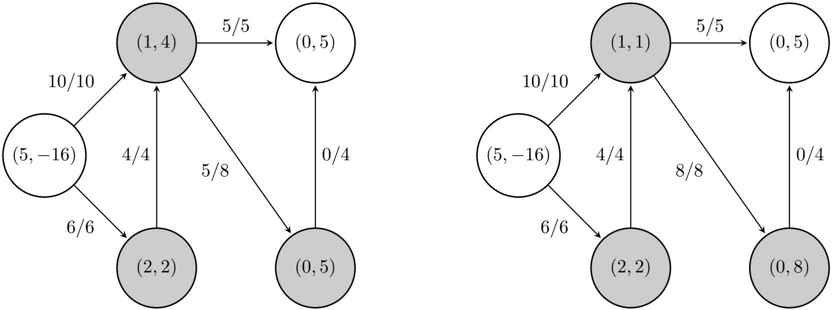
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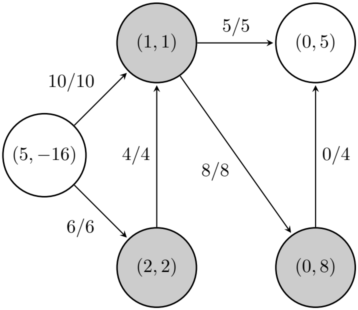

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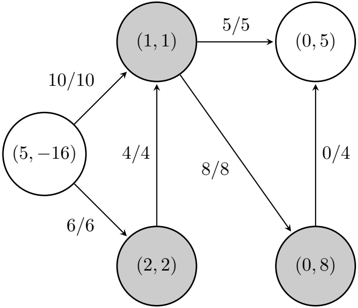
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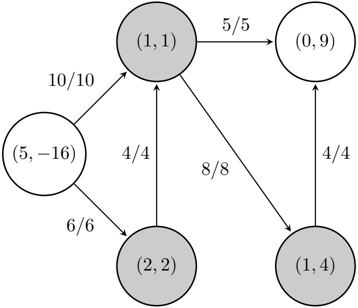
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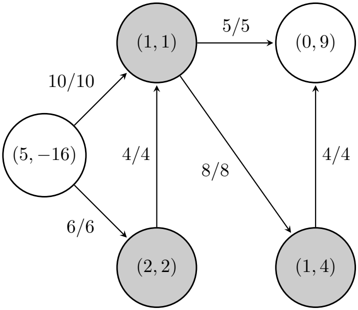

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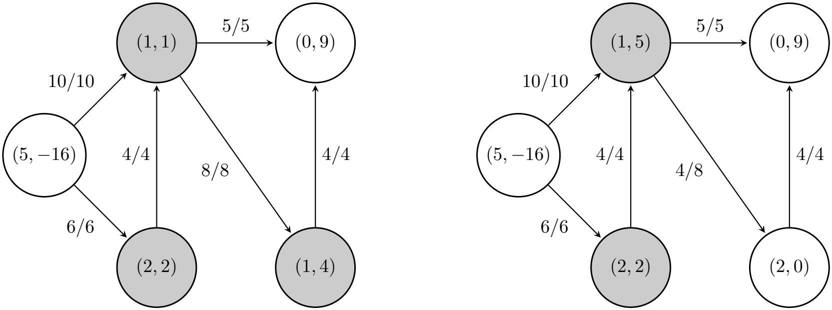

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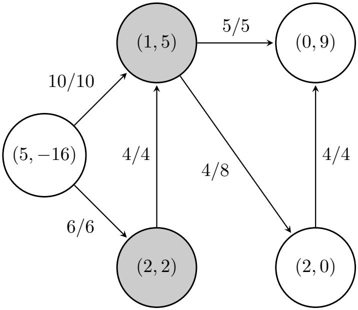

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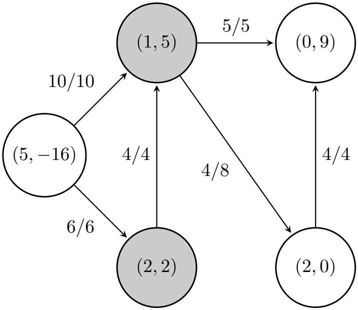
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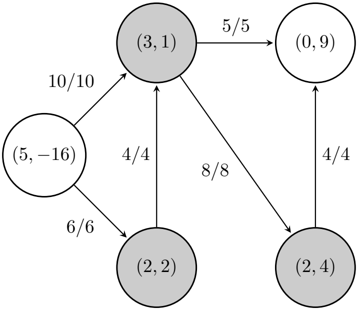
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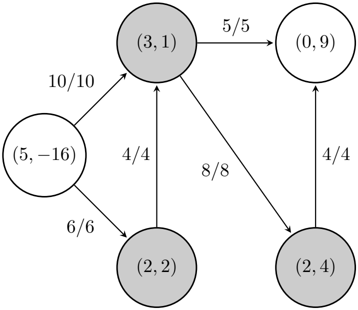
Korak 7



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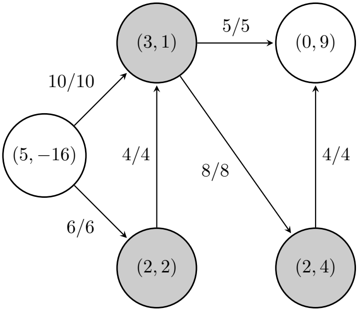
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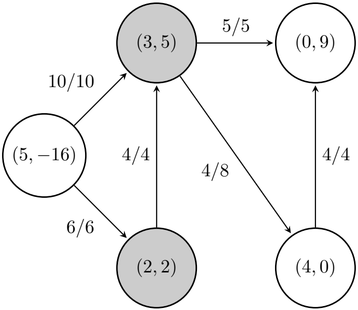
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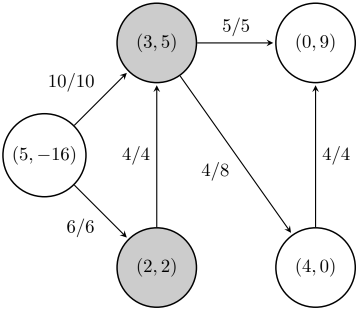

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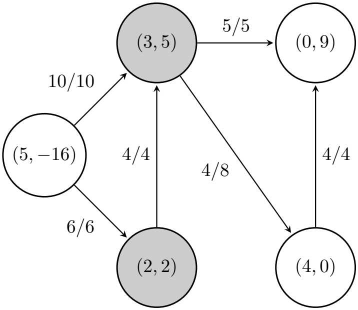
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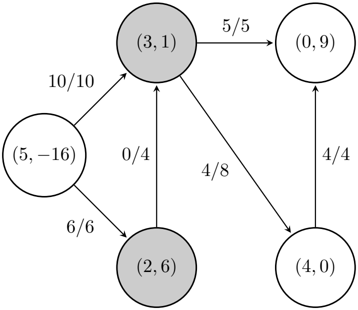
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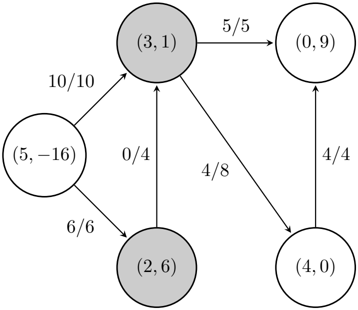

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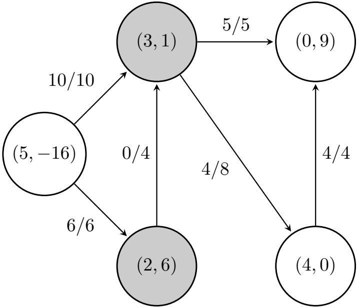
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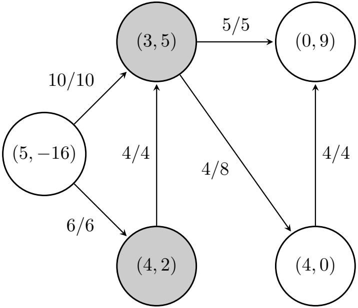
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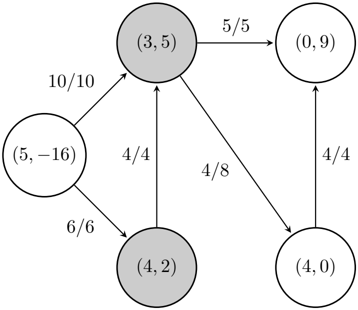

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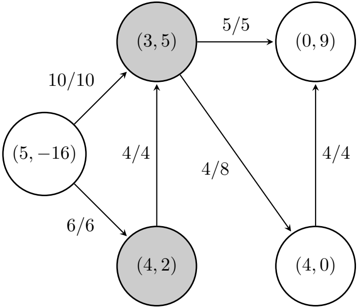
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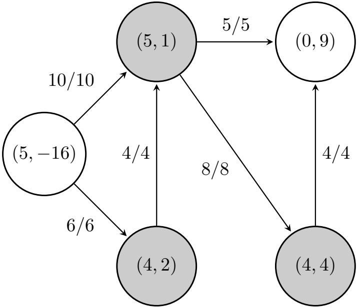
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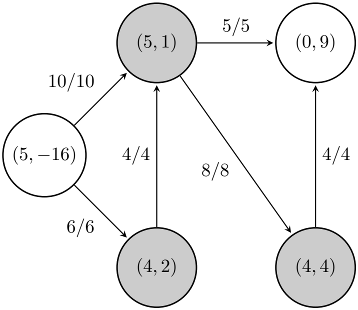
Korak 11



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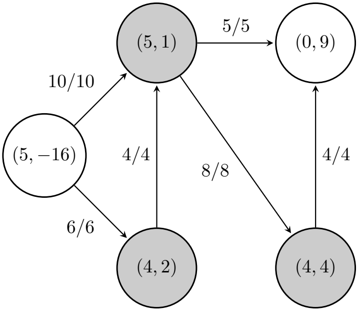
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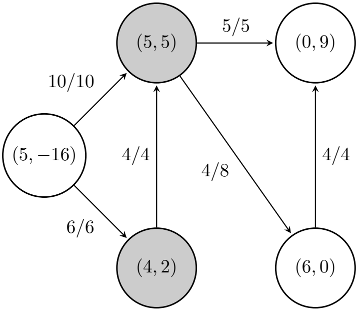
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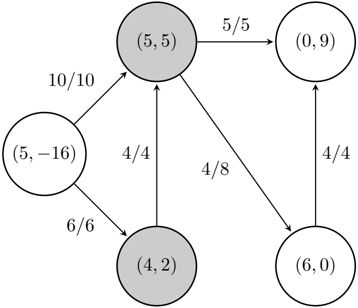

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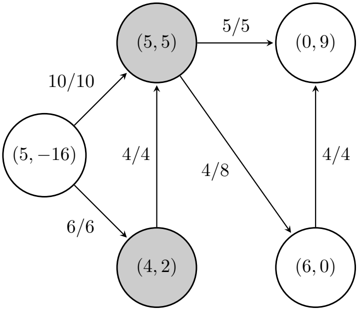
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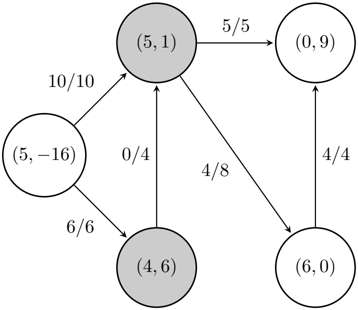
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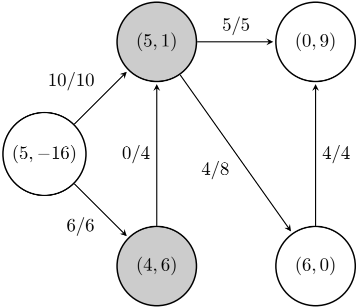

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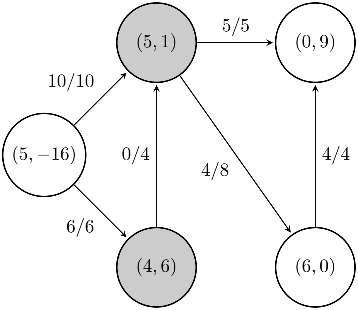
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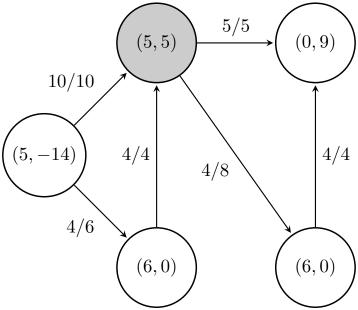
(6, 0)


(5, 5)
5/5
(0, 9)
10/10
(5, −14)
4/4
4/8
4/4
4/6
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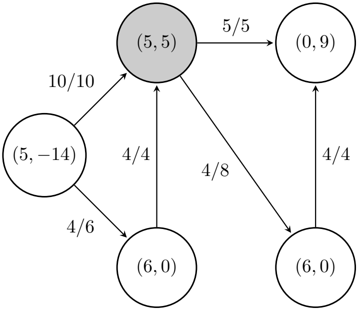
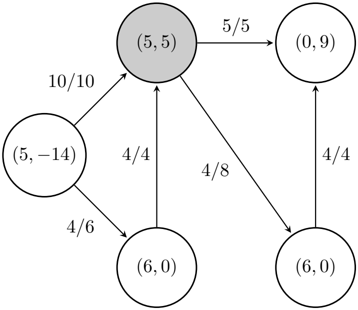
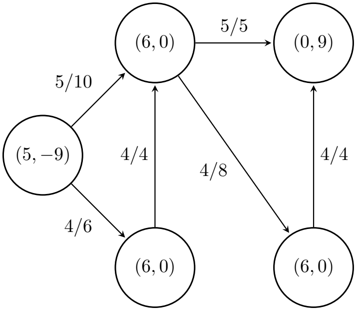
Korak 14



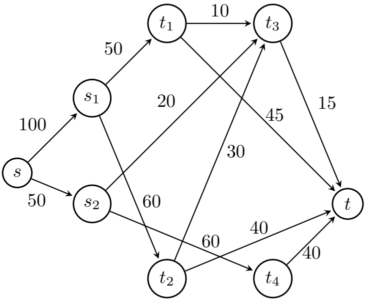
Korak 14



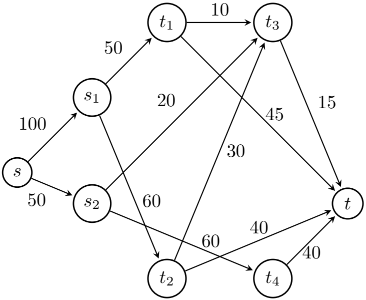


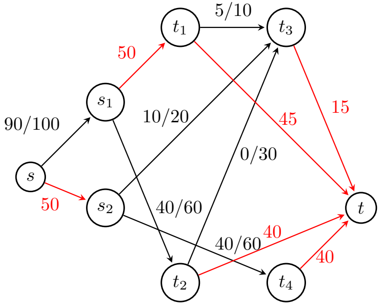
Problem ponudbe in povpraševanja

Problem ponudbe in povpraševanja



Problem ponudbe in povpraševanja





Baseball elimination

Baseball elimination