Online appendix for “Shocking Stuff: Technology, Hours, and Factor Substitution

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Contents

1 Selected Derivations

Derivation of (23) to (26). Re-write production function (14) as:

$$Y_t = \underbrace{\left(\frac{\eta}{\eta-1}\right)}_{\mu} \left[\alpha_0 \left(e^{z_t^K} \frac{r_0}{\alpha_0} K_{t-1}\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha_0)\left(e^{z_t^H} H_t\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} - \frac{1}{\eta-1} \tag{A.1}$$

$$\Rightarrow d\log Y_t = \left(\frac{\eta}{\eta-1}\right) d\log CES_t = \left(\frac{\eta}{\eta-1}\right)\left[\alpha_0 dz_t^K + (1-\alpha_0)\left(dz_t^H + d\log H_t\right)\right] \tag{A.2}$$

To proceed, note that the *rhs* of (11) is the normalized CES condition for F_H and, hence, the FOC for labor, we can write,

$$d\log w_t = \frac{\sigma-1}{\sigma} dz_t^H + \frac{1}{\sigma} d\log CES_t - \frac{1}{\sigma} d\log H_t + d\log mc_t \tag{A.3}$$

Now (A.2) and (A.3) imply the following labor demand relation corresponding to (23) in the main text:

$$d\log w_t = d\log mc_t + \frac{\alpha_0}{\sigma} dz_t^K + \frac{\sigma-\alpha_0}{\sigma} dz_t^H - \frac{\alpha_0}{\sigma} d\log H_t. \tag{A.4}$$

Equation (9) implies the labor supply relation (equation (24) in the main text):

$$d\log w_t = \gamma d\log H_t + \sigma_c d\log C_t. \tag{A.5}$$

The equilibrium change in hours, thus, can be found by combining (A.4) and (A.5). After some transformations, defining $\Upsilon = \frac{\gamma\sigma+\alpha_0}{\sigma} > 0$, this yields:

$$d\log H_t = \frac{1}{\Upsilon}\left[d\log mc_t + \frac{\alpha_0}{\sigma} dz_t^K + \frac{\sigma-\alpha_0}{\sigma} dz_t^H - \sigma_c d\log C_t\right], \tag{A.6}$$

from which we derive the derivatives associated with equations (25) and (26).

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Derivation of (28) and (29). Given (27), we can substitute $\frac{d\log Y_t}{dz_t^i}$ in this expression by making use of (A.2). The resulting expression can then be used in conjunction with (25) and (26). This yields the expressions for the impact of both shocks as functions of the parameters of the model, mpc_t, and the reaction of the marginal cost:

$$\frac{d\log H_t}{dz_t^H} = \frac{1}{\Upsilon + \sigma_c \frac{mpc_t}{apc}\mu(1-\alpha_0)}\left[\frac{d\log mc_t}{dz_t^H} + \frac{\sigma-\alpha_0}{\sigma} - \sigma_c \frac{mpc_t}{apc}\mu(1-\alpha_0)\right]. \tag{A.7}$$

$$\frac{d\log H_t}{dz_t^K} = \frac{1}{\Upsilon + \sigma_c \frac{mpc_t}{apc}\mu(1-\alpha_0)}\left[\frac{d\log mc_t}{dz_t^K} + \frac{\alpha_0}{\sigma} - \sigma_c \frac{mpc_t}{apc}\mu\alpha_0\right], \tag{A.8}$$

Accordingly,

$$\frac{d\log H_t}{dz_t^H} > 0 \quad \text{if} \quad \sigma - \frac{\alpha_0}{1+\frac{d\log mc_t}{dz_t^H} - \sigma_c \frac{mpc_t}{apc}\mu(1-\alpha_0)} > 0,$$
$$\frac{d\log H_t}{dz_t^K} > 0 \quad \text{if} \quad \sigma - \frac{\alpha_0}{\sigma_c \frac{mpc_t}{apc}\mu\alpha_0 - \frac{d\log mc_t}{dz_t^K}} < 0.$$

2 A Labor Demand & Supply Interpretation

Consider labor demand and supply under the flexible prices RBC case. From (23) and (24), we obtain:

$$d\log H_t^D = -\frac{\sigma}{\alpha_0} d\log w_t + \left(\frac{\sigma-\alpha_0}{\alpha_0}\right) dz_t^H + dz_t^K \tag{B.1}$$

$$d\log H_t^S = \frac{1}{\gamma + \sigma_c \frac{mpc_t}{apc}(1-\alpha_0)} \left\{ d\log w_t - \sigma_c \frac{mpc_t}{apc} \left[(1-\alpha_0)\, dz_t^H + \alpha_0 dz_t^K \right] \right\} \tag{B.2}$$

Straightforwardly, these two schedules can be graphed in the hours-wage space and the resulting shifts from changes in shocks and parameters analyzed accordingly. We can analyze each schedule in turn quite simply.

2.1 Technology Shocks and Hours Demand

Whilst positive capital-augmenting shocks unambiguously raise labor demand one-for-one, labor-augmenting shocks increase labor demand only if the substitution elasticity exceeds the capital income share. This property arises if there is complementarity between labor demand and labor-augmenting technical progress, $\frac{\partial^2 Y}{\partial \Gamma^H \partial H} > 0$.[1] Complementarity, though, is neither necessary nor sufficient to produce a positive hours response following $dz_t^H > 0$ since the net effect depends on labor *supply* reactions and – in the NK case – the reaction of real marginal costs on labor demand. For Cobb Douglas (respectively, Leontief) the complementarity between labor demand and labor-augmenting technical progress is always positive (negative) since $\sigma = 1 > \alpha_0$ (and $\sigma = 0 < \alpha_0$). More generally, outside these two restrictive cases, a labor-augmenting technology shock will decrease labor demand within the interval $\sigma \in [0, \alpha_0)$. In the Leontief case, labor demand is unresponsive to real wages (as expected under a fixed-proportions production) and falls one-for-one with positive labor augmenting technical improvements.

2.2 Technology Shocks and Hours Supply

Looking at equation (B.2), shifts in labor supply depend on the Frisch elasticity and risk aversion. They also depend on the endogenous responses of marginal consumption decisions. In the conventional $mpc_t > 0$ case, labor supply is positively related to the real wage but is decreasing in both technical progress terms (the latter reflects the familiar dominance of

[1] Recall again the proof in footnote 12.

the income effect over the substitution effect).[2] For the most part, it can be shown that the strength of the *mpc* responses depend on the values of investment adjustment costs and risk aversion and also on how persistent the technology shocks are. Investment adjustment costs inhibit capital accumulation and hence future consumption possibilities. Such costs raise the trade-off between current and future consumption in favor of the former: if $\psi \to \infty$, the investment/saving motive disappears, leaving all extra income to be consumed, $mpc \to 1$. However, for any finite adjustment costs, consumption smoothing works to decrease the initial fraction of income consumed. When technology shocks are temporary, mpc_t is positive and small with all feasible parameters.[3] Euler equation (8) reminds us that current consumption depends positively on future consumption and negatively on future capital productivity. Via (10) and (12), current output and the marginal product of capital increase inducing a positive capital-stock effect. If the shock is temporary, most of the positive output effect rapidly disappears while the increased capital stock remains at a persistently higher level. Hence, next-period marginal product of capital decreases marginally below the steady state. Because of the increasing next-period consumption and the decreasing next-period marginal product of capital, current consumption rises.

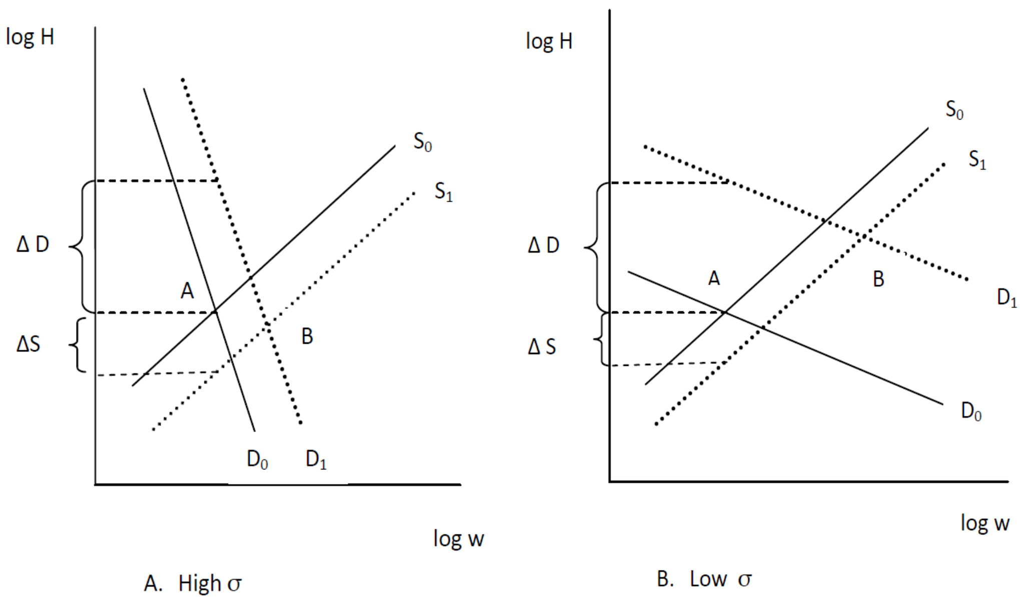


Figure 1: Capital augmenting shock.

[2]In the context of a permanent (or highly persistent) productivity shock the consumption effect is ambiguous: future marginal product of capital increases with a negative impact on current period consumption. The higher is the intertemporal substitution (i.e. the inverse of risk aversion σ_c) the stronger is that negative effect raising the possibility that $mpc_t < 0$ (e.g., agents deny themselves current consumption to accelerate the transition to a higher level of capital and income).

[3]The following is based on extensive simulation exercises. See our earlier working paper version for the full set of robustness analysis.

2.3 The Capital Augmenting Shock

As a graphical illustration, we examine capital-augmenting shocks. The semi-elasticity of the labor demand with respect to the capital augmenting shock is unity (see (B.1)). Hence, looking at equation (B.2), the hours impact is unambiguously positive only if the *mpc*-response is negative: labor demand and supply shift upwards. As extensive numerical simulations confirm, this requires that the shock is highly persistent and risk aversion not too high. In all other cases, the *mpc*-response will be positive meaning that labor supply shifts downward. The sign of the shift of the intersection point is ambiguous. Equation (B.1) makes clear that the downward slope of the demand curve in wage space is high (steep) with high σ and flat when $\sigma \approx 0$ (see **Figure 1A and 1B**). To repeat, in both cases, the upward shift of labor demand remains always the same. Hence, when the downward shift of the supply curve ($S_0 \to S_1$) is coupled with the upward shift of two alternative demand curves (steep and flat), the new equilibrium point B related to the steeper demand curve (panel A) is to the south-west of the new equilibrium point corresponding to the flat demand curve (panel B). Therefore with high σ as in panel A the hours impact can be negative. Thus, this implies that when substitution elasticity is low (i.e. labor demand is flat) the possibility that the hours-technology impact of the capital augmenting shock is positive increases compared to the case when the substitution elasticity is higher (i.e. demand is steep).

2.4 Labor Demand and Supply in the NK Model

The impact effects of technology shocks on labor demand and supply are defined by equations (23) and (24), the latter common to both models. Price stickiness, however, implies that real marginal costs deviate from their steady-state level and affect labor demand. Interestingly the marginal cost channel affects the shift of the demand curve but not its slope. The crucial questions are: (i) whether marginal cost reactions strengthen or compensate the shift effects that could otherwise occur without price stickiness, and; (ii) how big these effects are. There is no clear-cut answer to the first issue and, as our own (unreported) numerical analysis indicates, real marginal costs may either increase or decrease in response to technology shocks. The answer to the second is more clear-cut. The importance of the marginal cost channel is by definition positively related to Calvo-parameter θ. With small θ, the marginal cost channel, independently from the sign of the effect, has only a minor effect on the shift of the demand curve and our previous analysis remains qualitatively unchanged. The importance of the marginal cost channel strengthens as $\theta \to 1$. However, to validate even approximately the conventional wisdom that the hours impact of labor augmenting shock is negative, would require that only a small deviation of θ from zero suffices to render the marginal cost channel dominating and that with a very wide range of feasible parameter values of the model the effect of this channel on labor demand is negative. To provide further evidence regarding this point, **Figure 2** plots the impact effect of both technology shocks in the NK model using the baseline values of the parameters in **Table 2** but changing simultaneously the Calvo parameter (θ) and the investment adjustment cost parameter (ψ). Parameter ψ is important as it affects the marginal propensity to consume. One result worth mentioning is that, for capital-augmenting shocks with low values of σ, the response remains positive even for high values of θ and for all values of ψ considered. It is also noticeable that when investment adjustment costs are low, very high levels of price rigidity generate a positive rather than negative hours effect for high

σ values.

3 Threshold with Non-Separable Preferences

In the non-separable case, we define the utility function as,

$$U\left(C_t, H_t\right) = \frac{\left[C_t^{\gamma}\left(T - H_t\right)^{1-\gamma}\right]^{1-\sigma_c}}{1-\sigma_c} \tag{C.1}$$

where T represents the total number of hours available in each period and thus $\Re = \frac{T-H}{H}$ represents the ratio of non-market-to-market activities. Corresponding to the steady state $H_0 = 1$, which we used earlier, a natural choice of T would be around 3, i.e. $\Re \approx 2$. Note, we could just as easily have set $T = 1$ as commonly done, although for familiarity with our earlier case, we chose $H_0 = 1$. This yields the following labor supply equation (in log-difference form):

$$d\log w_t = -d\log\left(T - H_t\right) + d\log C_t. \tag{C.2}$$

The labor demand equation is obviously not affected by the change in the utility function. Around the steady state, $\Re = \frac{T-H}{H}$ and hence the growth of w_t can be expressed as:

$$d\log w_t = \frac{1}{\Re} d\log\left(T - H_t\right) + d\log C_t. \tag{C.3}$$

This implies that the expression for the labor supply schedule now becomes:

$$d\log H_t = \frac{1}{1/\Re + \frac{mpc_t}{apc}\phi(1-\alpha_0)}\left[d\log w_t - \frac{mpc_t}{apc}\phi\left(\alpha_0 dz_t^K + (1-\alpha_0)dz_t^H\right)\right]. \tag{C.4}$$

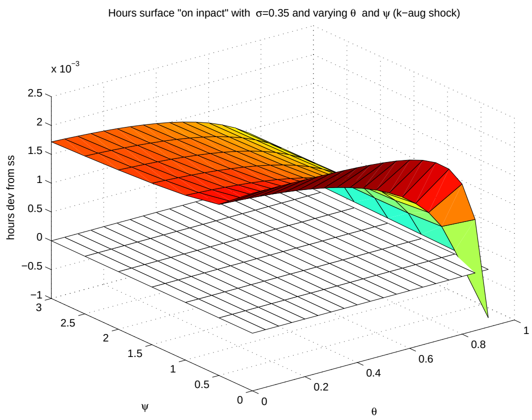
It is clear that the rest of the analysis in the main text then remains the same, as this case of utility function is simply a limiting case of the separable utility function with $\sigma_c = 1$ and $\gamma = 1/\Re$.[1] Hence, the new *general* threshold condition now becomes:

$$\frac{d\log H_t}{dz_t^H} > 0 \quad \text{if} \quad \sigma > \frac{\alpha_0}{1+\frac{d\log mc_t}{dz_t^H} - \frac{mpc_t}{apc}\phi(1-\alpha_0)}. \tag{C.5}$$

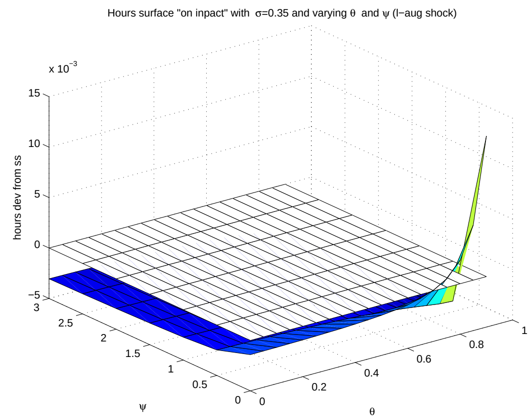
$$\frac{d\log H_t}{dz_t^K} > 0 \quad \text{if} \quad \sigma < \frac{\alpha_0}{\frac{mpc_t}{apc}\phi\alpha_0 - \frac{d\log mc_t}{dz_t^K}} \tag{C.6}$$

[1] This is logical as this form of utility function goes back to Prescott (1986) and is restricted to display both constant intertemporal and unit intra-temporal elasticities of substitution. This function implies an elasticity of substitution between consumption and leisure of 1 and is consistent with the observation that leisure per capita has shown virtually no secular trend. $1/\sigma_c > 0$ is the elasticity of substitution between different date composite commodities $C_t^{\gamma}\left(1 - H_t\right)^{1-\gamma}$ where γ is the consumption share parameter.

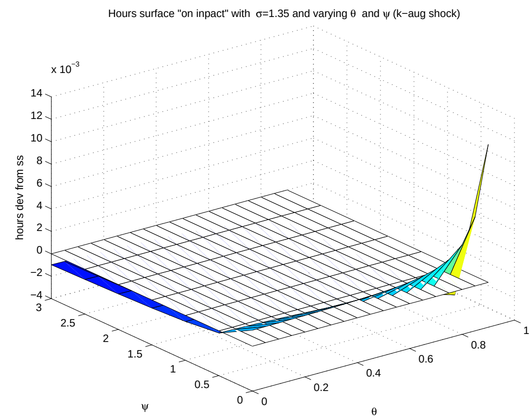
4 Sensitivity Graphs



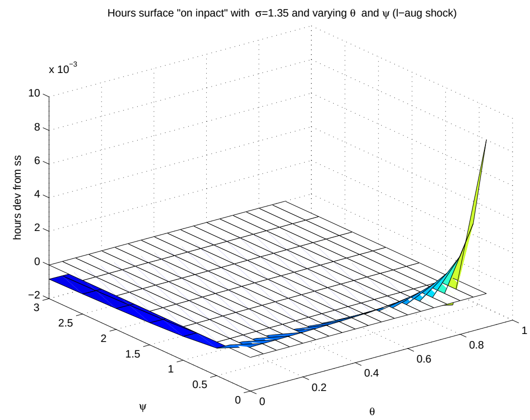
(a) K-aug $\sigma = 0.35$



(b) L-aug $\sigma = 0.35$

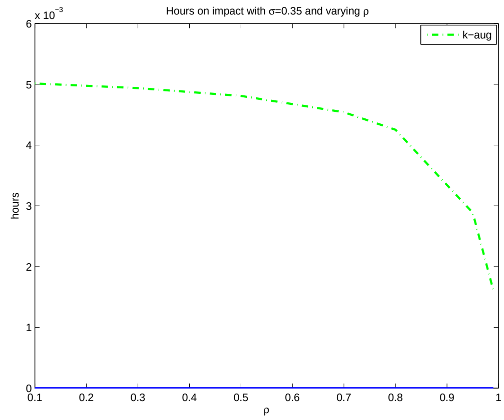


(c) K-aug $\sigma = 1.35$

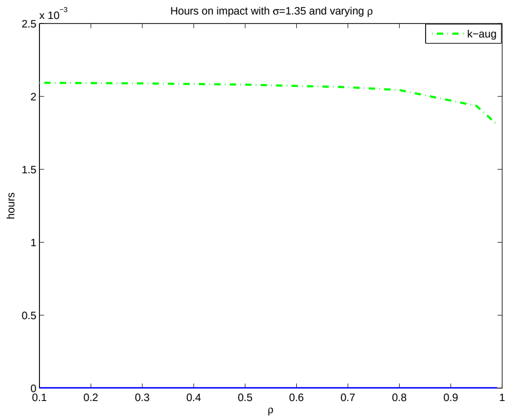


(d) L-aug $\sigma = 1.35$

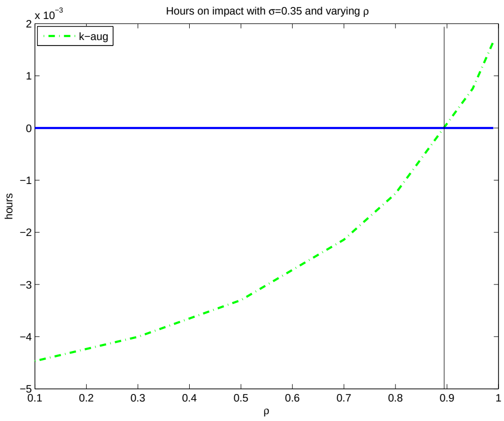
Figure 2: Sensitivity analysis in the NK model on the "impact"of Hours for Calvo parameter and Investment Adjustment Costs



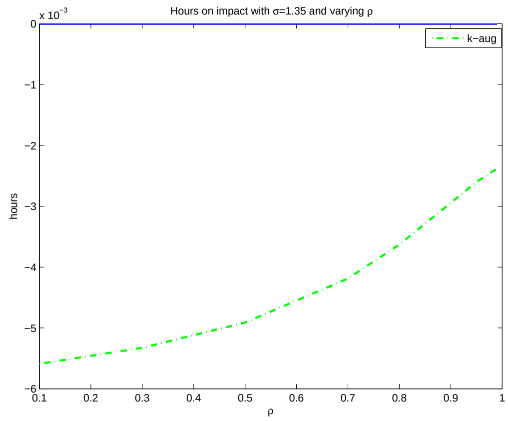
(a) Sensitivity for ρ with $\sigma = 0.35$, RBC Model



(b) Sensitivity for ρ with $\sigma = 1.35$, RBC Model

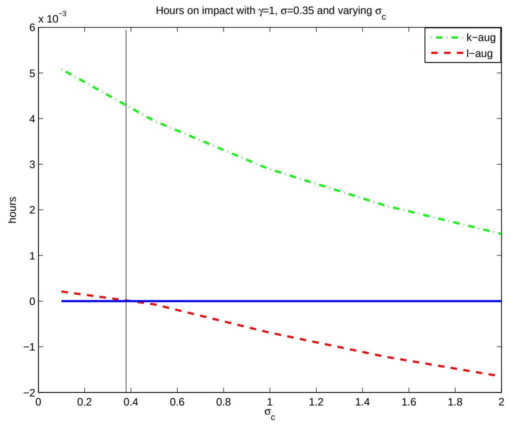


(c) Sensitivity for ρ with $\sigma = 0.35$, NK Model

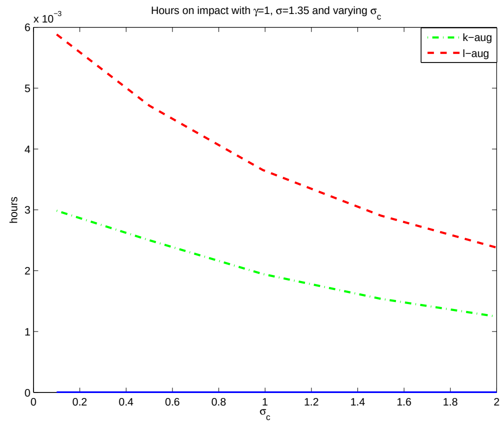


(d) Sensitivity for ρ with $\sigma = 1.35$, NK Model

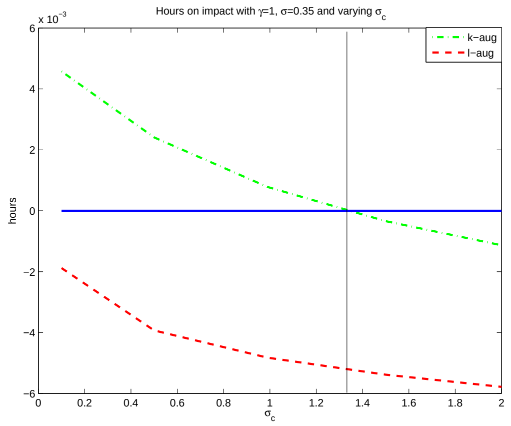
Figure 3: Sensitivity analysis for ρ, baseline value: $\rho = 0.95$



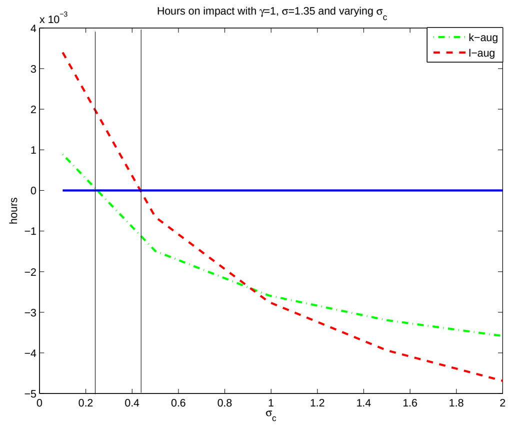
(a) Sensitivity for σ_c with $\sigma = 0.35$, RBC Model



(b) Sensitivity for σ_c with $\sigma = 1.35$, RBC Model

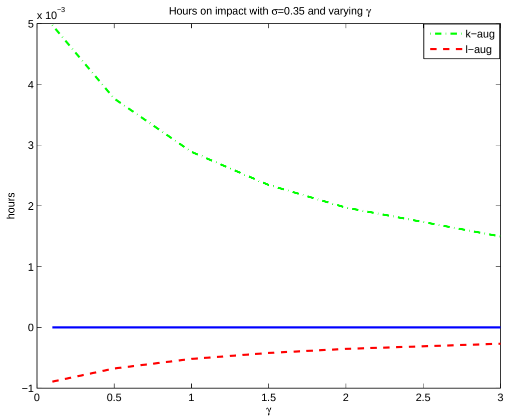


(c) Sensitivity for σ_c with $\sigma = 0.35$, NK Model

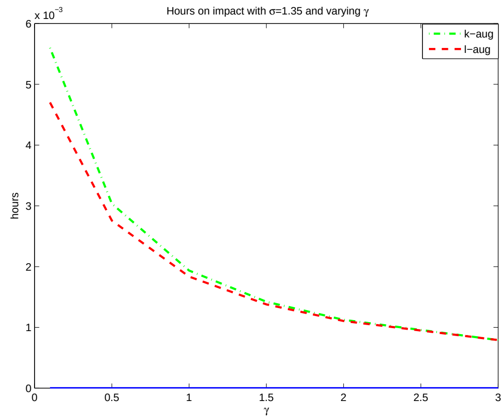


(d) Sensitivity for σ_c with $\sigma = 1.35$, NK Model

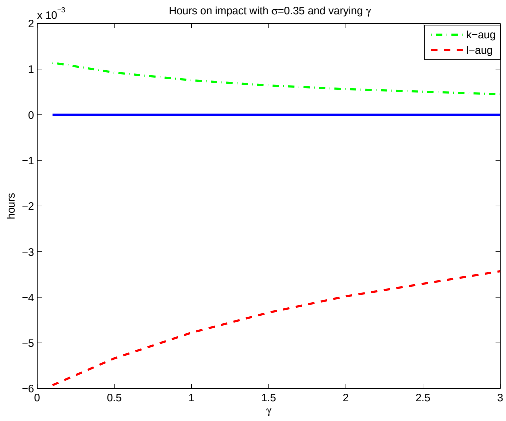
Figure 4: Sensitivity analysis for σ_c, baseline value: $\sigma_c = 1$ (assuming an AR(1) process for z_t^H).



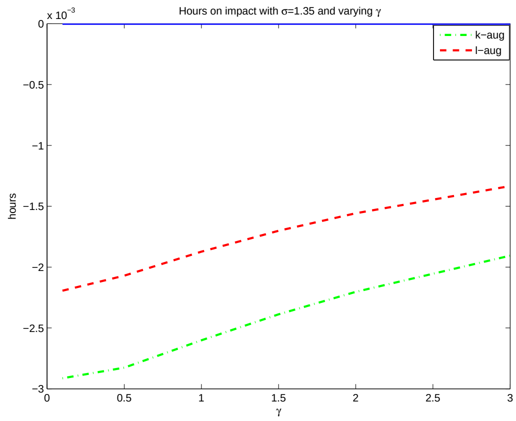
(a) Sensitivity for γ with $\sigma = 0.35$, RBC Model



(b) Sensitivity for γ with $\sigma = 1.35$, RBC Model

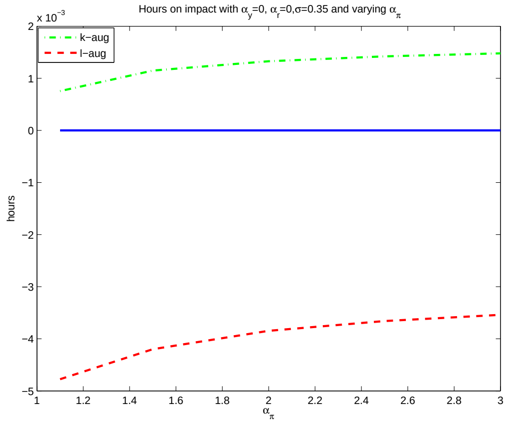


(c) Sensitivity for γ with $\sigma = 0.35$, NK Model

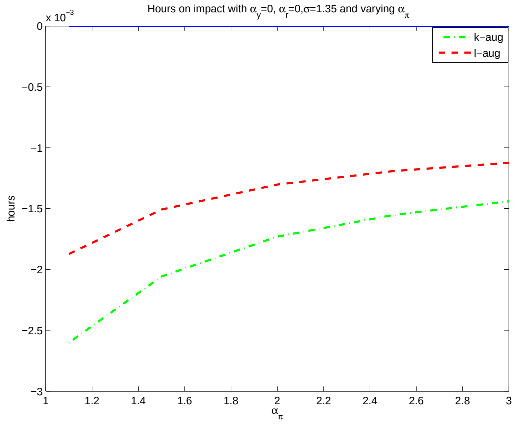


(d) Sensitivity for γ with $\sigma = 1.35$, NK Model

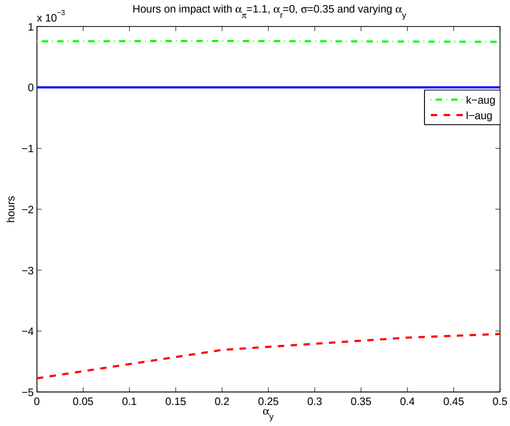
Figure 5: Sensitivity analysis for γ, baseline value: $\gamma = 1$



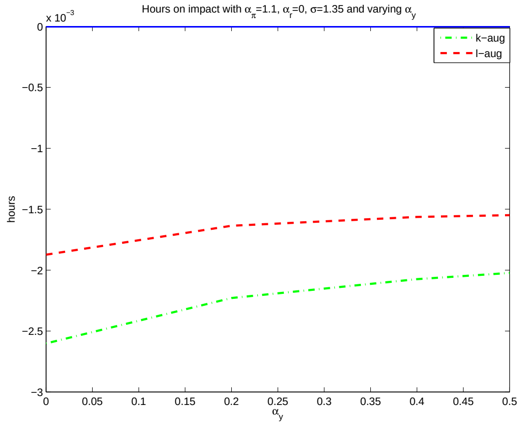
(a) Sensitivity for α_π when $\sigma = 0.35$



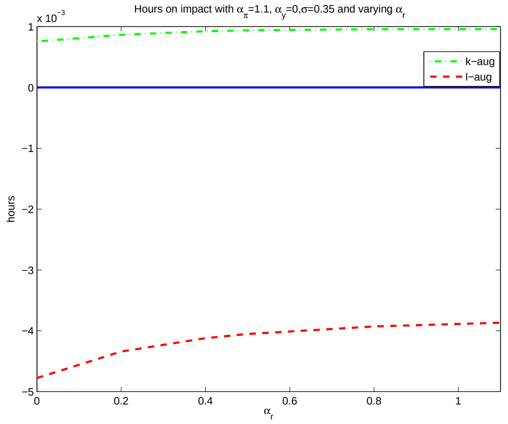
(b) Sensitivity for α_π when $\sigma = 1.35$



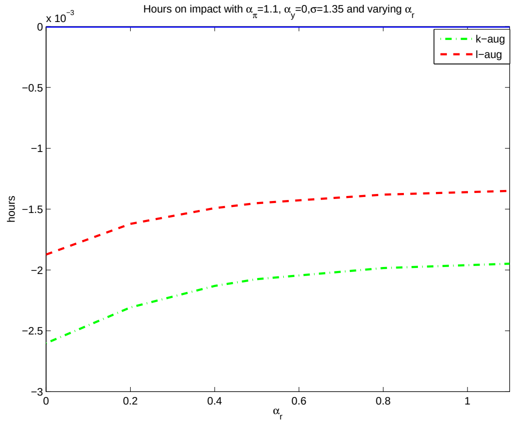
(c) Sensitivity for α_y when $\sigma = 0.35$



(d) Sensitivity for α_y when $\sigma = 1.35$

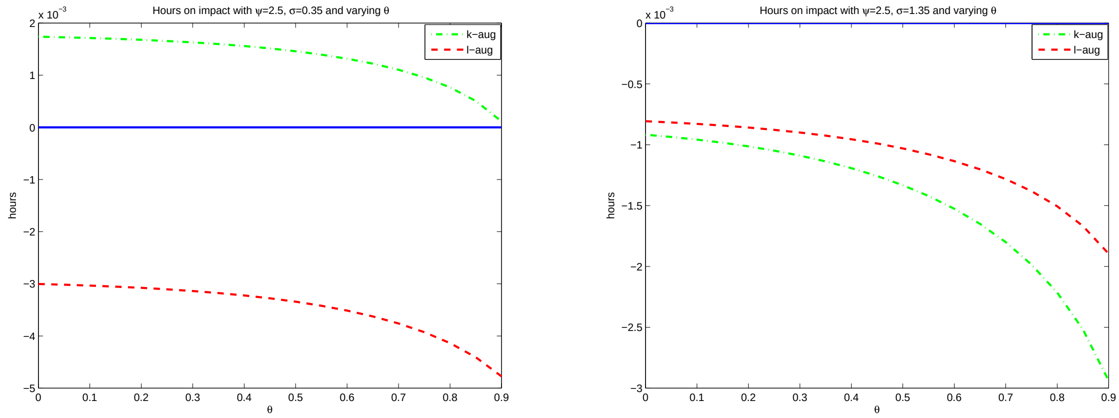


(e) Sensitivity for α_r when $\sigma = 0.35$



(f) Sensitivity for α_r when $\sigma = 1.35$

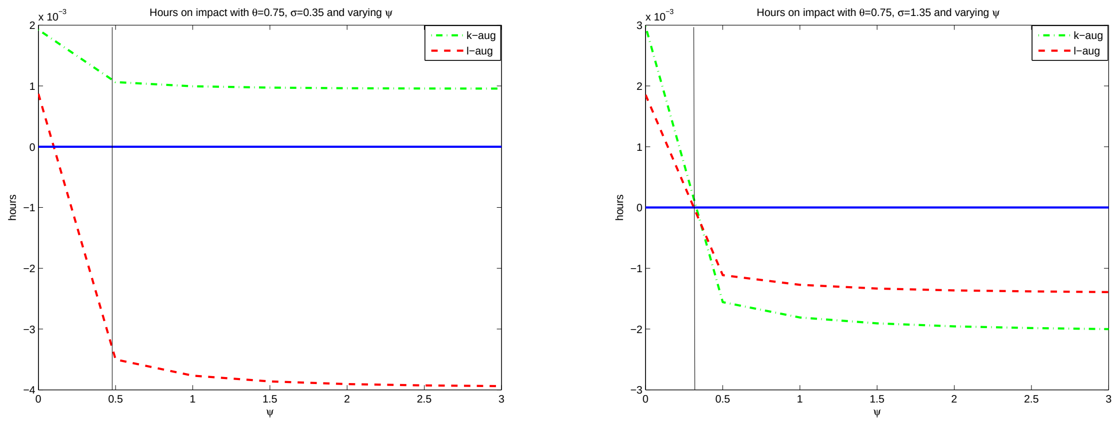
Figure 6: Sensitivity analysis in the NK model for monetary policy parameters, baseline values: $\alpha_\pi = 1.1$, $\alpha_y = \alpha_r = 0$



(a) Sensitivity for θ with $\sigma = 0.35$, NK Model

(b) Sensitivity for θ with $\sigma = 1.35$, NK Model

Figure 7: Sensitivity analysis for θ, baseline value: $\theta = 0.75$



(a) Sensitivity for ψ with $\sigma = 0.35$, NK Model

(b) Sensitivity for ψ with $\sigma = 1.35$, NK Model

Figure 8: Sensitivity analysis for ψ, baseline value: $\psi = 2.5$

5 Hicks-Neutral shocks

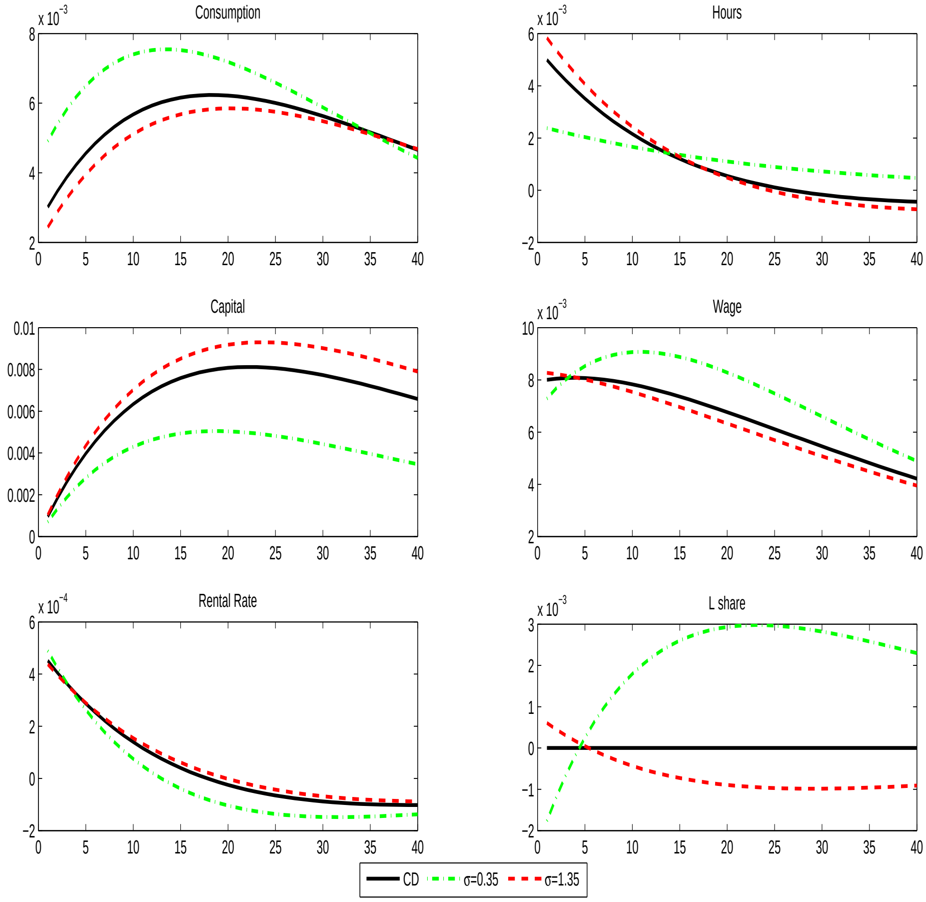


Figure 9: RBC model – Hicks Neutral Shock

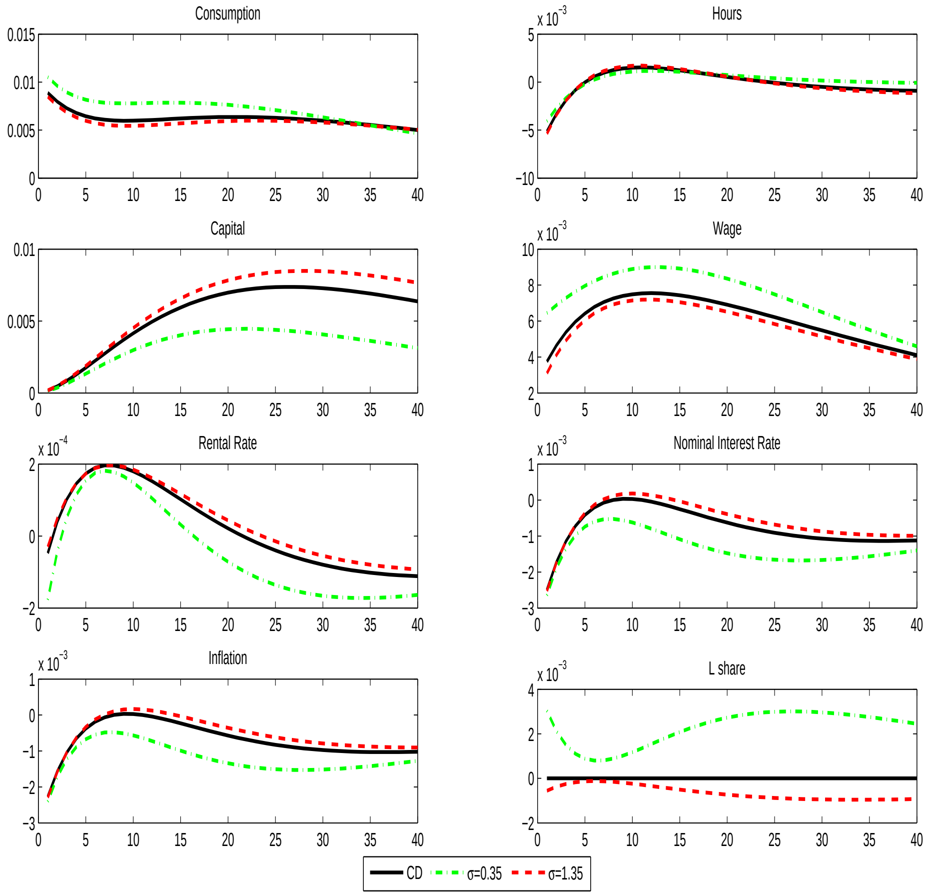


Figure 10: NK model – Hicks Neutral Shock

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Prescott, E. C. (1986). Theory Ahead of Business Cycle Measurement. *Quarterly Review, Federal Reserve Bank of Minneapolis*, Fall:9–22.