

# **A Fiscal Stimulus and Jobless Recovery**

## *Appendix*

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## A Business Cycle Statistics

Table A.1: Business cycle properties of selected macroeconomic series.

	Standard deviations relative to real output	
	1995 - 2007	1995 - 2011
GDP	–	–
Private investment	4.16	4.90
Private capital	1.04	1.14
Hours per employee	0.55	0.53
Overtime hours per employee	4.20	5.84
Job openings	3.72	3.19
Unemployment rate	10.06	17.38

	Autocorrelations	
	1995 - 2007	1995 - 2011
GDP	0.93	0.91
Private investment	0.89	0.87
Private capital	0.97	0.96
Hours per employee	0.95	0.92
Overtime hours per employee	0.87	0.87
Job openings	0.94	0.96
Unemployment rate	0.96	0.98

	Correlations with real output	
	1995 - 2007	1995 - 2011
GDP	–	–
Private investment	0.88	0.86
Private capital	0.59	0.41
Hours per employee	0.88	0.91
Overtime hours per employee	0.34	0.35
Job openings	0.88	0.75
Unemployment rate	-0.74	-0.60

Note: The series of job openings starts from 2001.

Source: ALFRED, Federal Reserve Bank of St. Louis and authors' calculations. Quarterly data. Percentage deviations from HP-trend for GDP, private investment, private capital, hours per employee, and overtime hours per employee; percentage the sample mean for vacancies and the unemployment rate.

## B Sensitivity exercises

### B.1 Bargaining power

In Figure B.1 we show how the combination of different elasticities of substitutions ( $\sigma$ ) and different levels of firms' bargaining power ( $\varepsilon$ ) affect the response of output, unemployment and the real wage to a government spending shock. In the left column, the technology is almost Leontief ( $\sigma = 0.10$ ), in the right column it approximates a Cobb-Douglas ( $\sigma \rightarrow 1$ ), while the central column features an intermediate elasticity of substitution in the range of empirical estimates ( $\sigma = 0.40$ ). Impulse responses are drawn with  $\varepsilon = \{0.10, 0.50, 0.90\}$ .

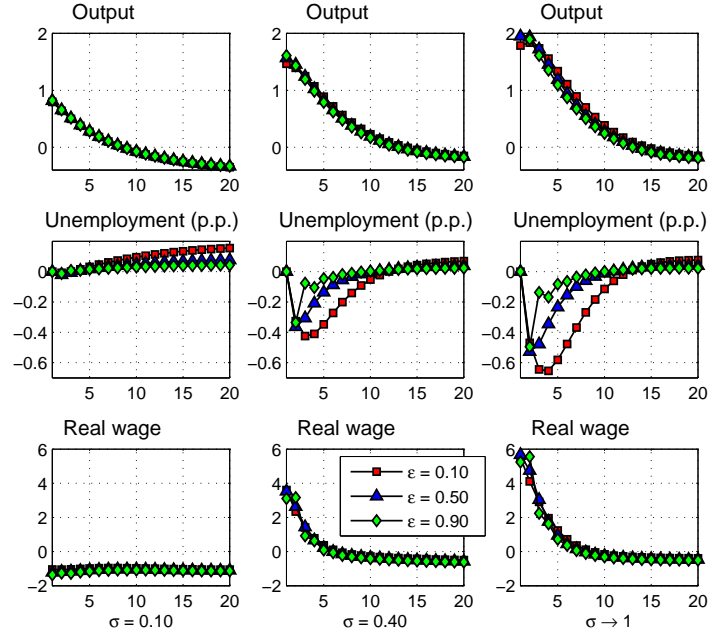
If  $\sigma = 0.10$ , despite the use of deep habits in consumption, by which the mark-up responds negatively to a government spending shock, the real wage declines as the strong complementarity between capital and labour induces firms to post relatively less vacancies and to a lower reservation wage for them. In other words labour demand (through vacancy posting) “shifts” less than labour supply. This scenario predicts an output multiplier less than unity and a small or even positive response of unemployment.

If the technology is sufficiently away from Leontief, the greater firms' share in the wage bargaining, the smaller the increase in the real wage and the reduction in unemployment, given the smaller incentive for households to sign labour contracts, keeping how they value non-work activities relative to work activities (replacement ratio) constant. While output is not greatly affected by the calibration of the factor elasticity of substitution and the bargaining parameter, the unemployment response is considerably affected by both choices. In addition, as the technology tends to Leontief, the calibration of the bargaining parameter becomes increasingly less important for the equilibrium outcome.

### B.2 Hagedorn and Manovskii effect

A common result in the MPMF literature is that unemployment volatility importantly depends on the calibration of the replacement ratio,  $\bar{\Theta}$ , i.e. the value of non-work to work activities. The higher the

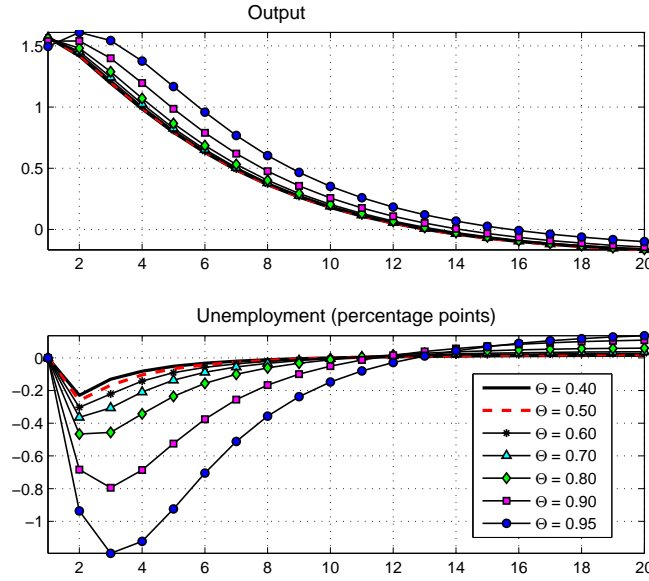
Figure B.1: Sensitivity of output and unemployment multipliers to changes in the firms' bargaining power.



Note: Fiscal policy: government spending expansion (1% of output, lump-sum taxes, balanced budget). Model: RBC with Mortensen-Pissarides Matching Friction (MPMF) and deep habits in consumption ( $\theta^c = 0.86$  and  $\rho^c = 0.85$ ). Responses of output and the real wage are in percentage deviations from steady state. For the unemployment rate, absolute changes in percentage points are reported.  $\varepsilon$  = firms' bargaining power;  $\sigma$  = elasticity of substitution between labour and capital.

steady-state value of non-work to work activities, the higher the volatility of unemployment. In the literature  $\bar{\Theta}$  ranges between Shimer (2005)'s 0.40 and Hagedorn and Manovskii (2008)'s 0.95. In Figure B.2 we show the sensitivity of the output and unemployment multipliers to the replacement ratio in the model with deep habits in consumption and the CES production function. Increasing  $\bar{\Theta}$  increases the magnitudes of both multipliers, however the output multiplier changes only marginally. Even when using the CES production function (with  $\sigma = 0.4$ ) – and hence incorporating a mechanism that moderates the unemployment multiplier *per se* – if the replacement ratio is calibrated in the high range of plausible values the flexible-price model augmented with MPMF and deep habits in consumption is able to reproduce a considerably higher unemployment multiplier. As a result, the Hagedorn and Manovskii effect proves to be a powerful tool if the goal is just to obtain higher

Figure B.2: Sensitivity of output and unemployment multipliers to changes in the magnitude of the replacement ratio.



Note: Fiscal policy: government spending expansion (1% of output, lump-sum taxes, balanced budget). Model: RBC with Mortensen-Pissarides Matching Friction (MPMF), deep habits in consumption ( $\theta^c = 0.86$  and  $\rho^c = 0.85$ ), and CES production function ( $\sigma = 0.40$ ). Responses of output are in percentage deviations from steady state. For the unemployment rate, absolute changes in percentage points are reported.  $\Theta$  = replacement ratio.

unemployment multipliers. Additional model features, such as deep habits, are needed to obtain higher output multipliers and impulse responses of the real wage, consumption and the mark-up featuring signs consistent with much of the empirical literature.

### B.3 Quantitative implications of the choice of the replacement ratio and the bargaining power

In Table B.1 we report the impact output multipliers and the unemployment peak multipliers obtained with different parametrisations: (i) our baseline value of the replacement ratio ( $\Theta = 0.7$ ), which is close to the estimate of 0.72 of Sala et al. (2008) versus the value used in the baseline calibration of Monacelli et al. (2010) ( $\Theta = 0.9$ ), which is in the high range of empirical estimates; (ii) our baseline value for the firms' bargaining power ( $\varepsilon = 0.5$ ) versus two extreme cases in which either

Table B.1: The impact of the fiscal stimulus in different scenarios

			(A)	(B)	
			$\sigma \rightarrow 1$	$\sigma = 0.4$	(B)/(A)
$\Theta = 0.7$	$\varepsilon = 0.1$	$\frac{\Delta Y}{\Delta G}$	1.78	1.46	0.82
		$\frac{\Delta u}{\Delta G}$	-0.65	-0.42	0.65
	$\varepsilon = 0.5$	$\frac{\Delta Y}{\Delta G}$	1.95	1.57	0.81
		$\frac{\Delta u}{\Delta G}$	-0.53	-0.35	0.66
	$\varepsilon = 0.9$	$\frac{\Delta Y}{\Delta G}$	2.02	1.61	0.80
		$\frac{\Delta u}{\Delta G}$	-0.49	-0.33	0.67
$\Theta = 0.9$	$\varepsilon = 0.1$	$\frac{\Delta Y}{\Delta G}$	1.63	1.38	0.85
		$\frac{\Delta u}{\Delta G}$	-1.27	-0.75	0.59
	$\varepsilon = 0.5$	$\frac{\Delta Y}{\Delta G}$	1.91	1.54	0.81
		$\frac{\Delta u}{\Delta G}$	-1.29	-0.79	0.61
	$\varepsilon = 0.9$	$\frac{\Delta Y}{\Delta G}$	2.05	1.62	0.79
		$\frac{\Delta u}{\Delta G}$	-1.01	-0.65	0.64

Note: Government spending expansion (1% of output, lump-sum taxes, balanced budget) in a model augmented with Mortensen-Pissarides Matching Frictions and deep habit formation. Impact output multipliers and peak unemployment multipliers are reported).

the workers or the firms get almost the whole surplus ( $\varepsilon = 0.1$  or  $\varepsilon = 0.9$ , respectively); (iii) the CD production function ( $\sigma \rightarrow 1$ ) versus a CES with  $\sigma = 0.4$  (our baseline value).

As noted above, while the unemployment multiplier is very sensitive to the choice of the replacement ratio, the output multiplier barely changes. Keeping  $\sigma$  constant, as firms gain a bigger share of the surplus from employment, while the output multiplier increases, the unemployment multiplier drops.

In relative terms (last column), almost irrespective of how the surplus is split between workers and firms ( $\varepsilon$ ) and how workers value non-work activities with respect to work activities ( $\Theta$ ), when  $\sigma$  drops from 1 (CD case) to 0.4, while the output multiplier is around 4/5 of the value obtained in the CD case, the unemployment multiplier is around or even below 2/3 of the value delivered by the CD case. In sum, the increasingly *jobless* stimulus obtainable as  $\sigma$  drops is robust to the calibration of the replacement ratio and the bargaining power parameter.

## B.4 Sensitivity to investment adjustment costs and the elasticity of substitution between leisure and consumption

In Table B.2 we report the impact output multiplier and the peak unemployment multiplier obtained using the CD production function ( $\sigma \rightarrow 1$ ) and the CES production function ( $\sigma = 0.4$ ) under the baseline parametrisation vis-a-vis two alternative scenarios: (i) with different calibrations of the elasticity of substitution between leisure and consumption, which are set to target a lower (greater)-than-baseline steady-state proportion of working time, i.e.  $\bar{h} = 0.25$  ( $\bar{h} = 0.40$ ); and (ii) in the absence of investment adjustment costs (obtained by setting  $\gamma = 0$ ).

A progressively lower elasticity of substitution between leisure and consumption,  $\rho$ , which implies a progressively greater steady-state proportion of working time, causes a *ceteris paribus* fall in the output multiplier, while the unemployment multiplier is almost insensitive to such a choice. In relative terms, when  $\sigma$  drops from 1 to 0.4, both the output and the unemployment multipliers fall by approximately the same proportion as in the baseline parametrisation. In other words, the increasingly *jobless* stimulus obtainable as  $\sigma$  drops is robust to the choice of  $\rho$  or, equivalently, to the calibration of hours worked.

Compared to baseline results, the absence of investment adjustment costs generates a *ceteris paribus* increase in the output multiplier and a fall in the unemployment multiplier. This is due to the fact that investment is allowed to respond more quickly to the fiscal stimulus and firms find it optimal to post relatively less vacancies. Compared to the baseline specification, when  $\sigma$  drops from 1 to 0.4, especially the output multiplier experiences a bigger reduction. The unemployment multiplier, however, is still reduced to a greater extent. From a quantitative perspective, investment adjustment costs prove important in delivering a *jobless* stimulus.

Another additional possible venue in modelling capital is the introduction of variable capital utilization. Such a feature may be used as a device to soften the *jobless* outcome of a fiscal stimulus. In fact if firms have an adequate capital buffer, they may use it in response to a fiscal stimulus. This, together with some complementarity between capital and labour may lead to higher vacancy posting and hence higher unemployment multipliers.

Table B.2: The impact of the fiscal stimulus in different scenarios

		(A)	(B)	
		$\sigma \rightarrow 1$	$\sigma = 0.4$	(B)/(A)
Baseline	$\frac{\Delta Y}{\Delta G}$	1.95	1.57	0.81
	$\frac{\Delta u}{\Delta G}$	-0.53	-0.35	0.66
$\rho$ s.t. $\bar{h} = 0.25$	$\frac{\Delta Y}{\Delta G}$	2.18	1.74	0.80
	$\frac{\Delta u}{\Delta G}$	-0.52	-0.35	0.67
$\rho$ s.t. $\bar{h} = 0.40$	$\frac{\Delta Y}{\Delta G}$	1.74	1.43	0.82
	$\frac{\Delta u}{\Delta G}$	-0.51	-0.34	0.67
No invest. adj. costs	$\frac{\Delta Y}{\Delta G}$	2.50	1.81	0.72
	$\frac{\Delta u}{\Delta G}$	-0.40	-0.25	0.63

Note: Government spending expansion (1% of output, lump-sum taxes, balanced budget) in a model augmented with Mortensen-Pissarides Matching Frictions and deep habit formation. Impact output multipliers and peak unemployment multipliers are reported).

## B.5 Debt-financed fiscal policy and distortionary taxation

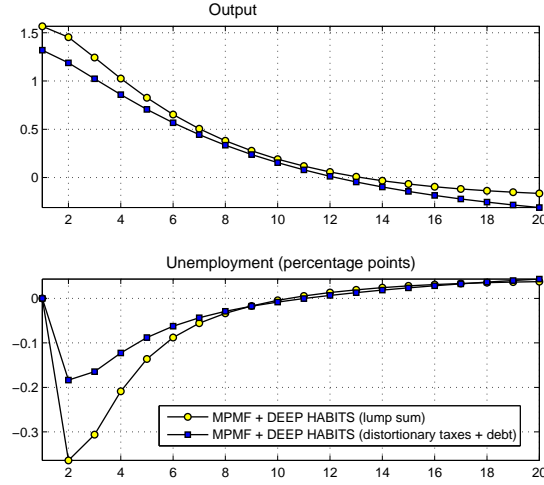
In order to introduce government debt and distortionary taxes we set the steady-state tax rates on consumption, the labour income and the return on capital to the values reported by Christiano et al. (2010), i.e.  $\bar{\tau}^c = 0.05$ ,  $\bar{\tau}^w = 0.24$ , and  $\bar{\tau}^k = 0.32$ . In addition, we let the government accumulate public debt, while tax rates react to government debt according to a feedback rule:

$$\log \left( \frac{X_t}{\bar{X}} \right) = \rho_X \log \left( \frac{X_{t-1}}{\bar{X}} \right) + \rho_{XB} \frac{B_{t-1}}{y_{t-1}}, \quad X_t = (\tau, \tau^c, \tau^w, \tau^k) \quad (\text{B.1})$$

We set the response coefficient of the tax rates to government debt  $\rho_{XB} = 0.02$ , the value used by Monacelli et al. (2010). Figure B.3 shows that the introduction of distortionary taxes alters the magnitude of the responses of output and unemployment, but unemployment is affected more. Such reductions (in absolute value) in the multipliers are due (i) to the distortion on equilibrium employment triggered by the increase in the tax rates following the fiscal expansion and (ii) to the dynamics of the fiscal instruments implied by the feedback rule. In fact, as consumption and the sources of income are taxed more, the tax-adjusted value of non-work activity increases. This reduces the total surplus of employment. In addition, as the imposed feedback rule implies a



Figure B.3: Sensitivity of output and unemployment multipliers to the introduction of distortionary taxation and government debt.



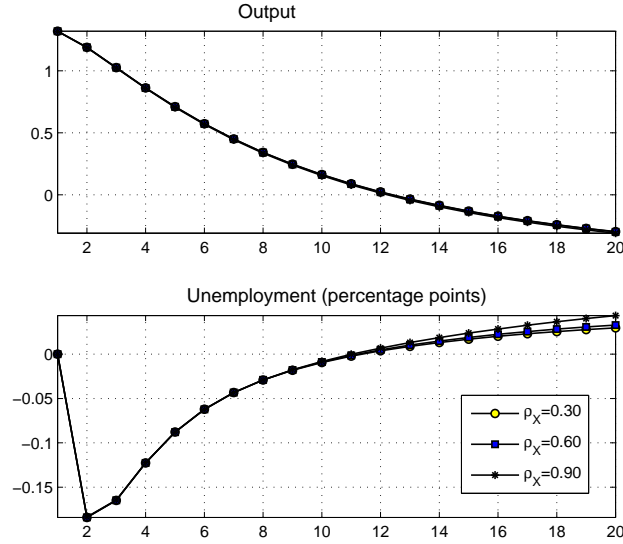
Note: Fiscal policy: government spending expansion (1% of output, distortionary taxes, partially debt-financed fiscal policy). Model: RBC with Mortensen-Pissarides Matching Friction (MPMF), deep habits in consumption ( $\theta^c = 0.86$  and  $\rho^c = 0.85$ ), and CES production function ( $\sigma = 0.40$ ). Responses of output are in percentage deviations from steady state. For the unemployment rate, absolute changes in percentage points are reported.

gradual return of the tax instruments to their steady-state value, this implies also postponement of work activities.

## B.6 Sensitivity to tax persistence and the tax responsiveness to government debt.

In the previous subsection we report output and unemployment multipliers in the case of distortionary taxes and in the presence of government debt. Results may be potentially affected by the assumed persistence of tax instruments or by the responsiveness of tax instruments to government debt. Hence, in this subsection, we show the sensitivity of the results to the choice of the two parameter values. Figure B.4 shows that both the output and the unemployment multipliers are very robust to the choice of the persistence of the tax instruments. When the persistence changes from low ( $\rho_X = 0.30$ ) to high ( $\rho_X = 0.90$ ), our baseline value, close to some empirical estimates, the plots of the output and the unemployment multipliers are almost indistinguishable. Figure B.5 shows that

Figure B.4: Sensitivity of output and unemployment multipliers to changes in the persistence of tax instruments.



Fiscal policy: government spending expansion (1% of output, distortionary taxes, government debt). Model: RBC with Mortensen-Pissarides Matching Friction (MPMF), deep habits in consumption ( $\theta^c = 0.86$  and  $\rho^c = 0.85$ ), and CES production function ( $\sigma = 0.40$ ). Responses of output are in percentage deviations from steady state. For the unemployment rate, absolute changes in percentage points are reported.

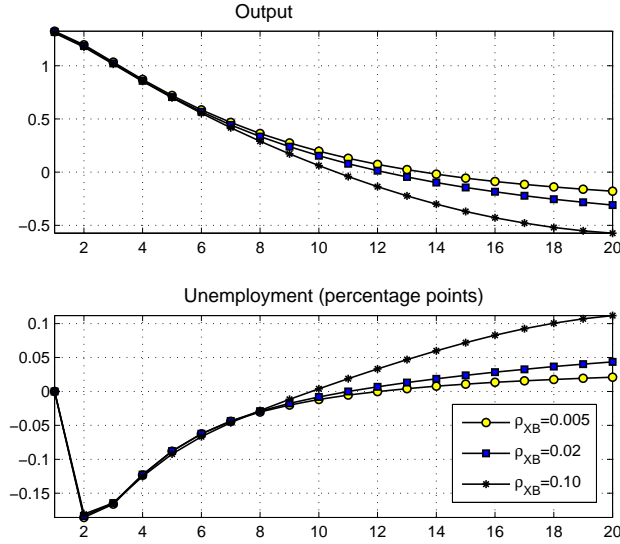
the output and the unemployment multipliers are quite robust also to the choice of the responsiveness of the tax instruments to government debt, especially up to a horizon of around two years. Starting from two years after the fiscal stimulus has occurred, a more aggressive responsiveness of the tax instruments of government debt (from  $\rho_{XB} = 0.005$  to  $\rho_{XB} = 0.10$ ) results into a progressively faster fall of output and a more pronounced rise in unemployment.

## C The Fiscal Stimulus in a NK Extension of the Model

This section offers a new-Keynesian (NK) extension of the model that includes sticky prices and monetary policy. Price stickiness is introduced as in Rotemberg (1982), i.e. by assuming that changing prices costs resources.<sup>1</sup>

<sup>1</sup>The use of price-adjustment costs as in Rotemberg (1982) is shared by virtually all papers featuring deep habits in consumption as it is a rather straight-forward addition from a technical point of view. By contrast using Calvo-type

Figure B.5: Sensitivity of output and unemployment multipliers to changes in the tax responsiveness to government debt.



Note: Fiscal policy: government spending expansion (1% of output, distortionary taxes, government debt). Model: RBC with Mortensen-Pissarides Matching Friction (MPMF), deep habits in consumption ( $\theta^c = 0.86$  and  $\rho^c = 0.85$ ), and CES production function ( $\sigma = 0.40$ ). Responses of output are in percentage deviations from steady state. For the unemployment rate, absolute changes in percentage points are reported.

## C.1 Introducing sticky prices

The introduction of sticky prices changes the problem of firms  $i \in (0, 1)$  in that they now choose the price level,  $P_{it}$ , instead of the relative price,  $p_{it}$ , and they face quadratic price adjustment costs  $\frac{\xi}{2} \left( \frac{P_{it}}{P_{it-1}} - 1 \right)^2$ , where parameter  $\xi$  measures the degree of price stickiness. Thus, the profit function now reads as follows:

$$J_t(n_{it}) = E_t \left\{ \sum_{s=0}^{\infty} D_{t,t+s} \left[ \begin{aligned} & \frac{P_{it+s}}{P_{t+s}} (C_{it+s} + G_{it+s} + I_{it+s}) - H C_{it+s} \\ & - w_{it+s} n_{it+s} h_{kt+s} - R_{t+s}^K K_{it+s} - \frac{\xi}{2} \left( \frac{P_{it+s}}{P_{it+s-1}} - 1 \right)^2 \end{aligned} \right] \right\}, \quad (\text{C.1})$$

The first-order conditions with respect to  $K_{it+s}$ ,  $n_{it+s}$ ,  $v_{kt+s}$ ,  $C_{it+s}$ ,  $S_{it+s}^c$ ,  $G_{it+s}$ ,  $S_{it+s}^g$  remain contracts introduces firm-specific habit effects which are more difficult to handle.

unaltered relative to the flexible-price case, while taking the first-order condition with respect to the price level  $P_{it+s}$  leads to the following:

$$\left\{ \begin{aligned} & \frac{P_{it}}{P_t} (C_{it} + G_{it}) - \xi \left( \frac{P_{it}}{P_{it+s}} - 1 \right) \frac{P_{it}}{P_{it-1}} + (1 - \eta) \left( \frac{P_{it}}{P_t} \right)^{1-\eta} I_t \\ & + \eta MC_t \left( \frac{P_{it}}{P_t} \right)^{-\eta} I_t - \eta v_t^c \left( \frac{P_{it}}{P_t} \right)^{-\eta} X_t^c - \eta v_t^g \left( \frac{P_{it}}{P_t} \right)^{-\eta} X_t^g \\ & + \xi \Lambda_{t,t+1} \left[ \left( \frac{P_{it+1}}{P_{it}} - 1 \right) \frac{P_{it+1}}{P_{it}} \right] \end{aligned} \right\} = 0. \quad (C.2)$$

Similar algebraic manipulations to those described in Section 2.4 lead to the following optimal pricing decision in the symmetric equilibrium:<sup>2</sup>

$$\left\{ \begin{aligned} & (X_t^c + X_t^g + I_t) \left[ 1 - \frac{\eta}{\eta-1} MC_t \right] \\ & + \frac{\eta}{\eta-1} (1 - \rho^c) [\lambda_t^c X_t^c + \lambda_t^g X_t^g] - \frac{\theta^c}{\eta-1} (S_{t-1}^c + S_{t-1}^g) \\ & + \xi E_t \Lambda_{t,t+1} [\Pi_{t+1} (\Pi_{t+1} - 1)] - \xi \Pi_t (\Pi_t - 1) \end{aligned} \right\} = 0, \quad (C.3)$$

where  $\Pi_t \equiv \frac{P_t}{P_{t-1}}$  is the gross inflation rate. Note that the pricing equation (C.3) collapses to the analogous flexible-price equation (32) in the paper when  $\xi = 0$ . Furthermore, when  $\xi > 0$ , real cost  $\frac{\xi}{2} \left( \frac{P_t}{P_{t-1}} - 1 \right)^2$  enters the economy's resource constraint.

The NK model is closed by the monetary policy, which is set by imposing a Taylor rule

$$\log \left( \frac{R_t^n}{\bar{R}^n} \right) = \rho_r \log \left( \frac{R_{t-1}^n}{\bar{R}^n} \right) + (1 - \rho_r) \left[ \rho_\pi \log \left( \frac{\Pi_t}{\bar{\Pi}} \right) + \rho_y \log \left( \frac{Y_t}{\bar{Y}} \right) \right], \quad (C.4)$$

where  $R_t^n$  is the nominal interest rate,  $\Pi_t$  is the gross inflation rate, and  $\rho_r$ ,  $\rho_\pi$  and  $\rho_y$  are parameters.

A Fisher equation links the ex-post real interest rate  $R_{t+1}$  to the nominal interest rate:

$$R_{t+1} = E_t \left[ \frac{R_t^n}{\Pi_{t+1}} \right]. \quad (C.5)$$

## C.2 Results

Woodford (2011) shows that adding sticky prices into an otherwise standard DSGE model enhances

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<sup>2</sup>Equation (C.3) is obtained by combining equations (28), (30) in the paper and (C.2), substituting for the demands for  $C_{it}$  and  $G_{it}$ , (8) and (17) in the paper, and rearranging.

the effects of a government spending expansion. Jacob (2011) argues that if price stickiness is added into a model with deep habit formation the countercyclical movement that the government spending shock induces in the mark-up is milder, that private consumption may still be crowded out as in traditional RBC and NK models and, consequently, the output multiplier becomes small. We show that, with deep habit formation, the addition of price stickiness may indeed soften the effects of a government spending expansion. However, we also find that (i) for an empirically plausible degree of deep habit formation and price stickiness the effects of a fiscal stimulus in terms of consumption and investment crowding-ins, the decline in the mark-up, the increase in the real wage, and the sizes of the output and unemployment multipliers are quite robust to the introduction of price stickiness; and (ii) Jacob's result (as evident also in the robustness exercises of his paper) is dependent on the assumption that the Taylor rule has a strong monetary response to the output gap that makes the nominal interest rate counteract the output expansion to an extent that the effects of the fiscal expansion are offset.

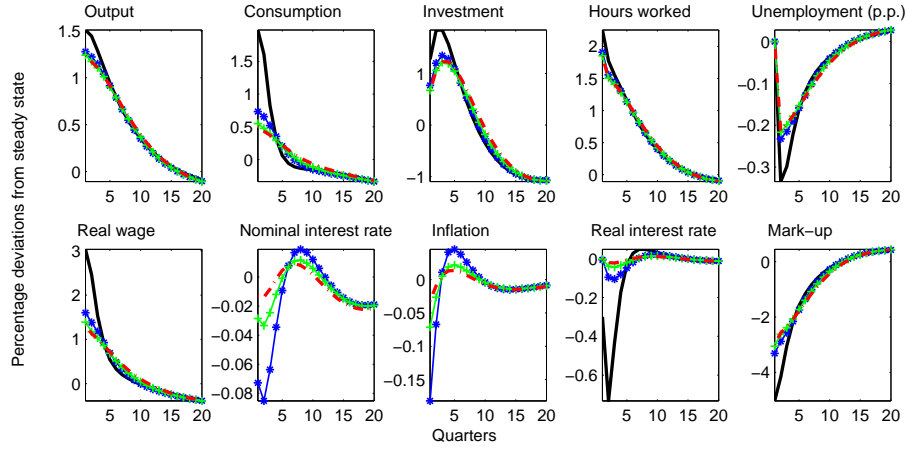
As a result, it is not price stickiness *per se* that subverts the effects of a government spending expansion, but an aggressive monetary response that goes exactly in the opposite direction of output growth, which is the primary goal of the fiscal stimulus itself. We explore these issues also in the context of a model with optimal monetary policy but without unemployment in Cantore et al. (2012).

Figure C.1 shows the effects of an expansion of government expenditures at different degrees of price stickiness in two alternative scenarios. First, in the top panel of Figure C.1, we assume that the nominal interest rate exhibits persistence in line with the data ( $\rho_r = 0.8$ ) and that the monetary authority reacts only to inflation ( $\rho_\pi = 2$ )<sup>3</sup> and not to the output gap ( $\rho_y = 0$ ). We explore increasing degrees of price stickiness,  $\xi$ . A  $\xi = 29.41$  corresponds to a Calvo contract average duration of around 3 quarters for our calibration.<sup>4</sup>

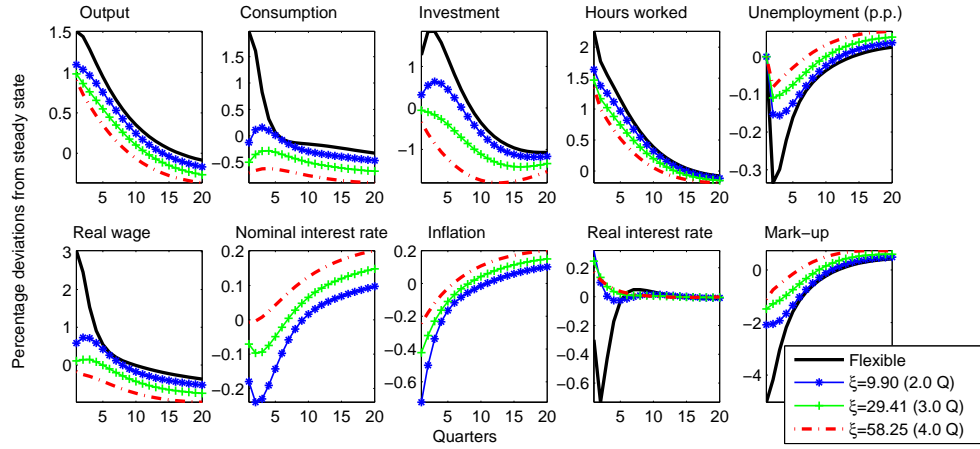
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<sup>3</sup>Both parameter values are the posterior estimates found by Smets and Wouters (2007).

<sup>4</sup>Jacob (2011) shows that for a given value of Rotemberg adjustment costs, the introduction of deep habits reduces the response of prices to the marginal cost and hence it is impossible to compare the deep habits New-Keynesian Phillips Curve (NKPC) slope to the Calvo analogue. Hence, following Jacob (2011), we interpret the slope of the standard forward-looking NKPC in quarterly terms. Namely, the log-linearised NKPC assumes the following form:  $\hat{\Pi}_t = \beta E_t \hat{\Pi}_{t+1} + \kappa \hat{M}C_t$ , where  $\kappa = \frac{\eta-1}{\xi}$  under Rotemberg pricing and  $\kappa = \frac{(1-\beta\xi^c)(1-\xi^c)}{\xi^c}$  under Calvo contracts, where  $\xi^c$  is the Calvo parameter that determines the average quarterly duration of contracts  $\frac{1}{1-\xi^c}$ . Given a certain  $\xi$ , it is straightforward to induce the implied analogous contract duration in the Calvo world.



(a) No monetary response to the output gap ( $\rho_y = 0$ ).



(b) Monetary response to the output gap ( $\rho_y = 0.5$ )

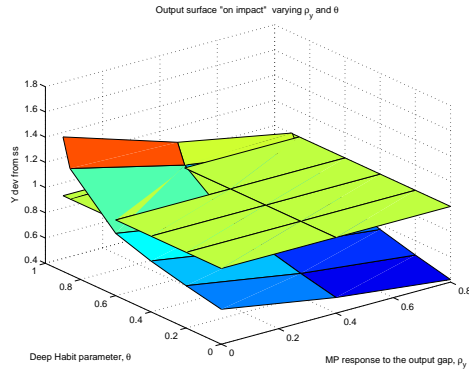
Figure C.1: A government spending expansion (1% of output, lump-sum taxes, balanced budget) in a model augmented with Mortensen-Pissarides Matching, deep habits and a CES production function: flexible vs. sticky prices.

When we introduce price stickiness, the effects of the fiscal expansion become softened by the decrease in the rate of inflation. This occurs if the shift in the aggregate supply, due to the presence of a high level of deep habits, is relatively strong given the shift in the aggregate demand due to the government spending expansion. However, the effects of a government spending expansion are similar to those obtained in the flexible-price case. In the lower panel of Figure C.1, we set a Taylor rule featuring a strong response to the output gap ( $\rho_y = 0.5$ ). In this case if prices are sticky, the nominal interest rate reacts positively to the rise in output despite the fall in inflation, the real interest rate reaction becomes positive and offsets the effects of the fiscal expansion.

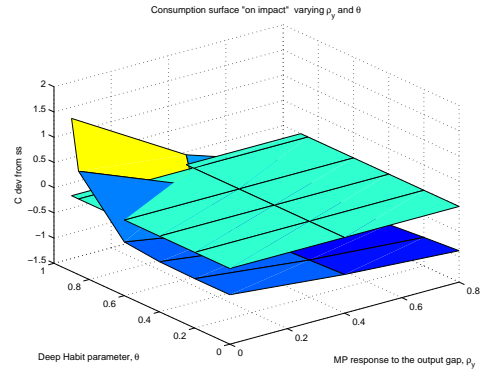
Figure C.2 shows the impact responses (peak responses for unemployment) (i) at different levels of monetary policy response to the output gap and (ii) at different degrees of deep habit formation. Surfaces show that, even at high degrees of deep habit formation, a substantial monetary policy response to the output gap may offset the expansionary effects of a government spending expansion. In particular, unemployment may also rise and consumption may be crowded out if  $\rho_y$  is above 0.4, while the output multiplier falls below one with a  $\rho_y$  above 0.6. Is the observed response parameter  $\rho_y$  so high and should it be so from an optimal policy perspective? In the empirical DSGE model literature, estimates of the value of  $\rho_y$  are typically low. For example in Smets and Wouters (2007) in a standard NK model with superficial habit, no unemployment and Cobb-Douglas production, estimated using US data by Bayesian methods over 1984:1-2004:1, a posterior mean corresponding to  $\rho_y = 0.08$  is obtained. These findings are typical of this literature. In the optimal policy literature optimised interest rate rules using a welfare criterion also find a weak long-run response of the interest rate to the output gap; for example, Schmitt-Grohe and Uribe (2007) find  $\rho_y = 0.1$  and Levin et al. (2006), Levine et al. (2008) and Levine et al. (2012) all find its welfare-improving contribution to be so small as to be ignored in their optimised rules.<sup>5</sup>

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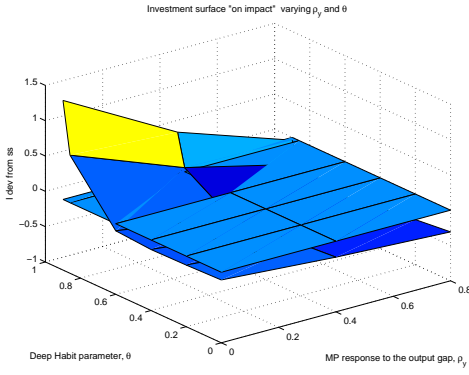
<sup>5</sup>These results abstract from active fiscal stabilisation policy. Our findings suggest that when fiscal rules are added, their efficacy would require an even weaker response of interest rates to the output gap. This is confirmed in Schmitt-Grohe and Uribe (2007). Indeed they devote a whole subsection to “the importance of (monetary policy) not responding to output”. For a study of optimal monetary and fiscal policy in a new Keynesian model with deep habit see Leith et al. (2009). In both these models, there is no unemployment and Cobb-Douglas production is assumed, so important features of our set-up are missing. Nonetheless their optimised interest rate rules in conjunction with fiscal stabilisation of debt also feature a weak long-run response to the output gap.



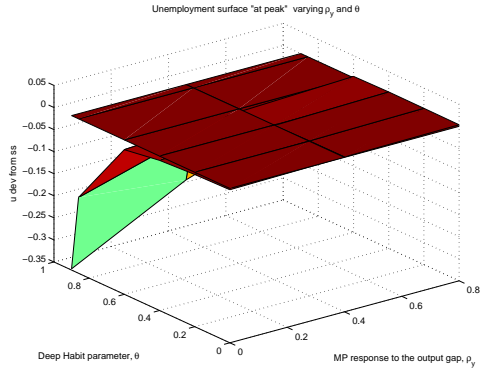
(a) Output



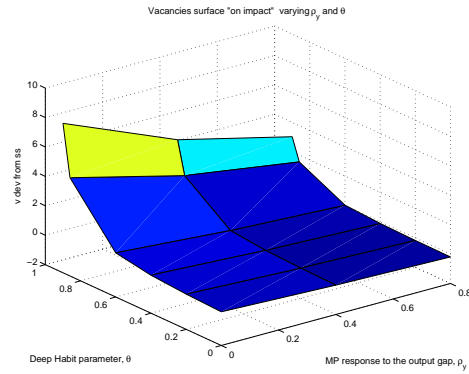
(b) Consumption



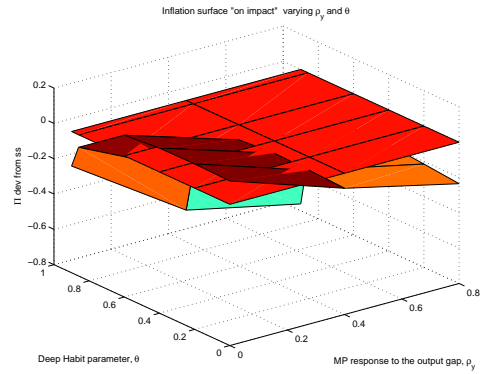
(c) Investment



(d) Unemployment



(e) Vacancies



(f) Inflation

Figure C.2: Sensitivity of impact responses to the deep habit parameter  $\theta^c$  and the monetary response to the output gap  $\rho_y$ .



## D Symmetric equilibrium

**Production function and marginal products:**

$$F((ZK)_t K_t, (ZN)_t n_t h_t) = \left[ \alpha_K ((ZK)_t K_t)^{\frac{\sigma-1}{\sigma}} + \alpha_N ((ZN)_t n_t h_t)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \quad (D.1)$$

$$F_{K,t} = \alpha_K (ZK)_t^{\frac{\sigma-1}{\sigma}} \left( \frac{Y_t}{K_t} \right)^{\frac{1}{\sigma}} \quad (D.2)$$

$$F_{N,t} = \alpha_N (ZN)_t^{\frac{\sigma-1}{\sigma}} \left( \frac{Y_t}{n_t h_t} \right)^{\frac{1}{\sigma}} \quad (D.3)$$

**Utility function, marginal utilities and deep habits in consumption:**

$$U(X_t^c, n_t, 1 - h_t) = n_t \frac{\left[ (X_t^c)^{(1-\rho)} (1 - h_t)^\rho \right]^{1-\sigma_c} - 1}{1 - \sigma_c} + (1 - n_t) \frac{(X_t^c)^{(1-\rho)(1-\sigma_c)} - 1}{1 - \sigma_c} \quad (D.4)$$

$$U_{x,t} = (1 - \rho) (X_t^c)^{(1-\rho)(1-\sigma)-1} \left[ 1 + n_t \left( (1 - h_t)^\rho (1-\sigma) - 1 \right) \right] \quad (D.5)$$

$$U_{n,t} = \frac{(X_t^c)^{(1-\rho)(1-\sigma)} \left[ (1 - h_t)^\rho (1-\sigma) - 1 \right]}{1 - \sigma} \quad (D.6)$$

$$U_{hn,t} = -\rho (X_t^c)^{(1-\rho)(1-\sigma)} (1 - h_t)^{\rho(1-\sigma)-1} \quad (D.7)$$

$$S_t^c = \rho^c S_{t-1}^c + (1 - \rho^c) C_t \quad (D.8)$$

$$C_t = X_t^c + \theta^c S_{t-1}^c \quad (D.9)$$

**Intertemporal investment/consumption decisions:**

$$K_{t+1} = (1 - \delta) K_t + I_t \left[ 1 - S \left( \frac{I_t}{I_{t-1}} \right) \right] \quad (D.10)$$

$$S \left( \frac{I_t}{I_{t-1}} \right) = \frac{\gamma}{2} \left( \frac{I_t}{I_{t-1}} - 1 \right)^2 \quad (D.11)$$

$$Q_t = E_t \left\{ D_{t,t+1} \left[ \left( 1 - \tau_{t+1}^k \right) R_{t+1}^K + (1 - \delta) Q_{t+1} \right] \right\} \quad (D.12)$$

$$1 = Q_t \left[ 1 - S \left( \frac{I_t}{I_{t-1}} \right) - S' \left( \frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \right] + E_t \left\{ D_{t,t+1} Q_{t+1} S' \left( \frac{I_{t+1}}{I_t} \right) \left( \frac{I_{t+1}}{I_t} \right)^2 \right\} \quad (\text{D.13})$$

$$D_{t,t+1} = \beta \frac{U_{x,t+1}}{U_{x,t}} \frac{1 + \tau_t^c}{1 + \tau_{t+1}^c} \quad (\text{D.14})$$

$$1 = E_t \left[ D_{t,t+1}^j R_{t+1} \right] \quad (\text{D.15})$$

$$MC_t F_{K,t} = R_t^K \quad (\text{D.16})$$

**Hiring decisions and wage bargaining:**

$$g_t = \frac{\chi}{1 + \psi} z_t^{1+\psi} \quad (\text{D.17})$$

$$g_{z,t} = \chi z_t^\psi \quad (\text{D.18})$$

$$n_{it+1} = (1 - \lambda) n_{it} + q(\theta_t) v_{it} \quad (\text{D.19})$$

$$\frac{g'(z_t)}{q(\theta_t)} = E_t \left\{ D_{t,t+1} \left[ \begin{array}{c} (MC_t F_{N,t+1} - w_{t+1}) h_{t+1} + g'(z_{t+1}) z_{t+1} \\ -g(z_{t+1}) + (1 - \lambda) \frac{g'(z_{t+1})}{q(\theta_{t+1})} \end{array} \right] \right\} \quad (\text{D.20})$$

$$w_t h_t = (1 - \varepsilon) \left[ MC_t F_{N,t} h_t - g(z_t) + g'(z_t) z_t + \theta_t g'(z_t) \right] + \varepsilon \left[ \frac{w_u - \frac{U_{n,t}}{U_{x,t}}}{1 - \tau_t^w} \right] \quad (\text{D.21})$$

$$F_{N,t} = - \frac{U_{nh,t}}{U_{C,t}} \quad (\text{D.22})$$

$$z_t = \frac{v_t}{n_t} \quad (\text{D.23})$$

$$\theta_t = \frac{v_t}{u_t} \quad (\text{D.24})$$

$$u_t = 1 - n_t \quad (\text{D.25})$$

$$q_t = k \theta_t^{-\omega} \quad (\text{D.26})$$

$$p_t = \theta_t q_t \quad (\text{D.27})$$

**Further firms' decisions:**

$$1 - MC_t + (1 - \rho^c)\lambda_t^c = v_t^c \quad (\text{D.28})$$

$$E_t D_{t,t+1}(\theta^c v_{t+1}^c + \rho^c \lambda_{t+1}^c) = \lambda_t^c \quad (\text{D.29})$$

$$1 - MC_t + (1 - \rho^c)\lambda_t^g = v_t^g \quad (\text{D.30})$$

$$E_t D_{t,t+1}(\theta^c v_{t+1}^g + \rho^c \lambda_{t+1}^g) = \lambda_t^g \quad (\text{D.31})$$

$$\left\{ \begin{array}{l} (X_t^c + X_t^g + I_t) \left[ 1 - \frac{\eta}{\eta-1} MC_t \right] \\ + \frac{\eta}{\eta-1} (1 - \rho^c) [\lambda_t^c X_t^c + \lambda_t^g X_t^g] - \frac{\theta^c}{\eta-1} (S_{t-1}^c + S_{t-1}^g) \\ + \xi E_t \Lambda_{t,t+1} [\Pi_{t+1} (\Pi_{t+1} - 1) - \Pi_t (\Pi_t - 1)] \end{array} \right\} = 0 \quad (\text{D.32})$$

**Government budget constraint and fiscal rules:**

$$B_t = R_t B_{t-1} + G_t + (1 - n_t)w_u - \tau_t - \tau_t^C C_t - \tau_t^W w_t n_t h_t - \tau_t^K R_t^K K_t \quad (\text{D.33})$$

$$S_t^g = \rho^c S_{t-1}^g + (1 - \rho^c) G_t \quad (\text{D.34})$$

$$G_t = X_t^g + \theta^c S_{t-1}^g \quad (\text{D.35})$$

$$\log \left( \frac{G_t}{\bar{G}} \right) = \rho_G \log \left( \frac{G_{t-1}}{\bar{G}} \right) + \varepsilon_t^g \quad (\text{D.36})$$

$$\log \left( \frac{X_t}{\bar{X}} \right) = \rho_X \log \left( \frac{X_{t-1}}{\bar{X}} \right) + \rho_{XB} \frac{B_{t-1}}{y_{t-1}} + \varepsilon_t^X, \quad X_t = (\tau, \tau^c, \tau^w, \tau^k) \quad (\text{D.37})$$

**Resource constraint:**

$$Y_t = C_t + I_t + G_t + g_t n_t + \frac{\xi}{2} \left( \frac{P_t}{P_{t-1}} - 1 \right)^2 \quad (\text{D.38})$$

**Taylor rule and Fisher equation (sticky-price model):**

$$\log \left( \frac{R_t^n}{\bar{R}^n} \right) = \rho_\pi \log \left( \frac{\Pi_t}{\bar{\Pi}} \right) + \rho_y \log \left( \frac{Y_t}{\bar{Y}} \right) \quad (\text{D.39})$$

$$R_{t+1} = E_t \left[ \frac{R_t^n}{\Pi_{t,t+1}} \right] \quad (\text{D.40})$$

## E Steady state

Steady-state values of the employment rate,  $n$ , hours worked,  $h$ , and the marginal cost,  $MC$ , solve simultaneously the wage equation, (D.21), the economy's resource constraint, (D.38), and the pricing equation, (D.32), while the value of the remaining unknowns in the system of equations reported in Appendix A can be found recursively by using the following relationships:

$$\overline{ZN} = (ZN)_0 \quad (\text{E.1})$$

$$\overline{ZK} = (ZK)_0 \quad (\text{E.2})$$

$$\overline{Y} = Y_0 \quad (\text{E.3})$$

$$\overline{D} = \beta \quad (\text{E.4})$$

$$\overline{Q} = 1 \quad (\text{E.5})$$

$$\overline{\Pi} = 1 \quad (\text{E.6})$$

$$\overline{R^K} = \frac{\overline{R} + \delta}{1 - \overline{\tau^K}} \quad (\text{E.7})$$

$$\overline{\left(\frac{K}{Y}\right)} = \overline{MC} \frac{\overline{S^K}}{\overline{R^K}} \quad (\text{E.8})$$

$$\overline{K} = \overline{\left(\frac{K}{Y}\right)} \overline{Y} \quad (\text{E.9})$$

$$\overline{I} = \delta \overline{K} \quad (\text{E.10})$$

$$\overline{G} = \overline{\left(\frac{G}{Y}\right)} \overline{Y} \quad (\text{E.11})$$

$$\overline{S^g} = \overline{G} \quad (\text{E.12})$$

$$\overline{X^g} = (1 - \theta^c) \overline{G} \quad (\text{E.13})$$

$$\overline{u} = 1 - \overline{n} \quad (\text{E.14})$$

$$\bar{p} = \frac{\lambda \bar{n}}{1 - \bar{n}} \quad (\text{E.15})$$

$$\bar{\theta} = \left( \frac{\bar{p}}{\kappa} \right)^{\frac{1}{1-\omega}} \quad (\text{E.16})$$

$$\bar{q} = \kappa \bar{\theta}^{-\omega} \quad (\text{E.17})$$

$$\bar{v} = \bar{u} \bar{\theta} \quad (\text{E.18})$$

$$\bar{z} = \frac{\bar{v}}{\bar{n}} \quad (\text{E.19})$$

$$\bar{g} = \frac{\chi}{1 + \psi} \bar{z}^{1+\psi} \quad (\text{E.20})$$

$$\bar{g}_z = \chi \bar{z}^\psi \quad (\text{E.21})$$

$$\overline{FN} = \alpha_N \overline{MC} (\overline{ZN})^{\frac{\sigma-1}{\sigma}} \left( \frac{\bar{Y}}{\overline{nh}} \right)^{\frac{1}{\sigma}} \quad (\text{E.22})$$

$$\overline{F}_n = \overline{F}_N \bar{h} \quad (\text{E.23})$$

$$\overline{X}^c = \frac{1 - \rho}{\rho} \overline{F}_N \frac{1 + \bar{n} \left( (1 - \bar{h})^{\rho(1-\sigma_c)} - 1 \right)}{(1 - \bar{h})^{\rho(1-\sigma_c)-1}} \quad (\text{E.24})$$

$$\overline{C} = \frac{\overline{X}^c}{1 - \theta^c} \quad (\text{E.25})$$

$$\overline{S}^c = \overline{C} \quad (\text{E.26})$$

$$\overline{U}_n = \frac{(\overline{X}^c)^{(1-\rho)(1-\sigma_c)} \left( (1 - \bar{h})^{\rho(1-\sigma_c)} - 1 \right)}{1 - \sigma_c} \quad (\text{E.27})$$

$$\overline{U}_x = (1 - \rho) (\overline{X}^c)^{(1-\rho)(1-\sigma_c)-1} \left( 1 + \bar{n} \left( (1 - \bar{h})^{\rho(1-\sigma_c)} - 1 \right) \right) \quad (\text{E.28})$$

$$\bar{w} = \frac{1}{\overline{Dh}} \left[ -\frac{\bar{g}_z}{\bar{q}} + \overline{D} \left( \overline{F}_n - \bar{g} + \bar{z} \bar{g}_z + (1 - \lambda) \frac{\bar{g}_z}{\bar{q}} \right) \right] \quad (\text{E.29})$$

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