

Consistent hydrodynamic theory of chiral electrons in Weyl semimetals

E. V. Gorbar,^{1,2} V. A. Miransky,³ I. A. Shovkovy,^{4,5} and P. O. Sukhachov³¹*Department of Physics, Taras Shevchenko National Kiev University, Kiev 03680, Ukraine*²*Bogolyubov Institute for Theoretical Physics, Kiev 03680, Ukraine*³*Department of Applied Mathematics, Western University, London, Ontario, Canada N6A 5B7*⁴*College of Integrative Sciences and Arts, Arizona State University, Mesa, Arizona 85212, USA*⁵*Department of Physics, Arizona State University, Tempe, Arizona 85287, USA*

(Received 12 December 2017; published 12 March 2018)

The complete set of Maxwell's and hydrodynamic equations for the chiral electrons in Weyl semimetals is presented. The formulation of the Euler equation takes into account the explicit breaking of the Galilean invariance by the ion lattice. It is shown that the Chern-Simons (or Bardeen-Zumino) contributions should be added to the electric current and charge densities in Maxwell's equations that provide the information on the separation of Weyl nodes in energy and momentum. On the other hand, these topological contributions do not directly affect the Euler equation and the energy conservation relation for the electron fluid. By making use of the proposed *consistent hydrodynamic* framework, we show that the Chern-Simons contributions strongly modify the dispersion relations of collective modes in Weyl semimetals. This is reflected, in particular, in the existence of distinctive *anomalous Hall waves*, which are sustained by the local anomalous Hall currents.

DOI: [10.1103/PhysRevB.97.121105](https://doi.org/10.1103/PhysRevB.97.121105)

Introduction. The formulation of relativistic hydrodynamics was proposed more than half a century ago (see, for example, Ref. [1]). It is widely used in various fields of physics, ranging from nuclear physics to astrophysics and cosmology. In interacting systems close to equilibrium, the hydrodynamic equations describe the space-time evolution of conserved quantities (e.g., energy, momentum, electric charge, etc.) in the limit of long wavelengths and long time scales. Recently, relativistic hydrodynamics was also generalized to plasmas made of chiral fermions [2–5] where the chiral charge is included as an additional degree of freedom, whose conservation is violated only by the chiral anomaly.

The potential relevance of hydrodynamics for the electron transport in solids is an old idea too [6]. However, it can be realized only when the electron-electron scattering rate is much larger than the rate of electron scattering on phonons and impurities. Experimentally, the hydrodynamic transport of relativisticlike electrons was recently observed in graphene [7,8], which is a two-dimensional Dirac material.

It is reasonable to expect that the hydrodynamics is also relevant for the electron fluid in Dirac and Weyl semimetals. As in graphene, their low-energy quasiparticles are relativisticlike fermions, although propagating in the three rather than two dimensions. Moreover, the time-reversal (TR) symmetry and/or the parity inversion (PI) are broken in Weyl semimetals. As a result, the Weyl nodes come in pairs of opposite chirality that are separated in momentum and/or energy. The corresponding distances are quantified by the chiral shift \mathbf{b} and parameter b_0 , respectively. (For recent reviews of Weyl semimetals, see Refs. [9–11].)

The experimental confirmation of the hydrodynamic regime in the Weyl semimetal tungsten diphosphide was recently reported in Ref. [12], where the dependence of the electrical

resistivity on the channel width provides compelling evidence for the hydrodynamic transport. This interpretation is further supported by the observation of the Wiedemann-Franz law violation with the lowest value of the Lorenz number ever reported.

Previously, the equations of relativistic hydrodynamics were used to describe the negative magnetoresistance [13,14] and the thermoelectric transport [14] in Weyl semimetals. However, the corresponding approach lacks important information regarding the separation of the Weyl nodes in energy and momentum. The situation is similar to the conventional chiral kinetic theory [15,16] in which this information is also missing. In addition, such a kinetic theory suffers from an acute problem of the local nonconservation of the electric charge when both electromagnetic and strain-induced pseudoelectromagnetic fields are applied to the system [17]. We resolved this problem in Ref. [18] by amending the electric charge and current densities with the Chern-Simons contributions [19,20]. The latter are also known as the Bardeen-Zumino terms connected with the consistent anomaly in high-energy physics [21]. In this Rapid Communication, by using the consistent chiral kinetic theory [18] in the relaxation time approximation, we derive the hydrodynamic equations and study the implications of the Chern-Simons contributions on the properties of the collective excitations of the electron fluid in Weyl semimetals.

Origin of the Chern-Simons contributions. In order to demonstrate the need for the Chern-Simons contributions in the hydrodynamic theory of the electrons in Weyl materials, let us consider a two-band model of a Weyl semimetal defined by the following Hamiltonian [22,23]:

$$\mathcal{H}_{\text{latt}} = t_0 \sin(ak_z) + \sum_{i=x,y,z} \sigma_i d_i(\mathbf{k}), \quad (1)$$

where the first term $t_0 \sin(ak_z)$ is responsible for breaking the PI symmetry, $\sigma = (\sigma_x, \sigma_y, \sigma_z)$ are the Pauli matrices, and $\mathbf{d}(\mathbf{k})$ is a periodic function of the quasimomentum \mathbf{k} . A simple Weyl semimetal with a broken TR symmetry is defined by $d_x = \Lambda \sin(ak_x)$, $d_y = \Lambda \sin(ak_y)$, and $d_z = t_0 + t_1 + \sum_{i=x,y,z} \cos(ak_i)$, where a denotes the lattice spacing, whereas Λ , t_0 , and t_1 are material-dependent parameters. In this model, the opposite-chirality Weyl nodes are separated by $2eb_z/(c\hbar)$ in momentum and by $2eb_0$ in energy, where $b_0 = t_0 \sin(ab_z)/e$, $b_z = c\hbar \arccos[-(t_0 + 2t_1)/t_1]/(ea)$, and e is the absolute value of the electron charge.

The topological features of Weyl semimetals are captured by a nontrivial Berry curvature [24], i.e.,

$$\Omega_i = \sum_{l,m=x,y,z} \frac{\epsilon_{ilm}}{4\hbar|\mathbf{d}|^3} (\mathbf{d} \cdot [\partial_{k_l} \mathbf{d} \times \partial_{k_m} \mathbf{d}]). \quad (2)$$

This result is valid in the whole Brillouin zone and, unlike its simplified monopole analogs in linearized low-energy models, it captures the nontrivial dependence on b_0 and \mathbf{b} . By making use of the chiral kinetic theory approach [25], in the limit of zero temperature ($T = 0$) and at the neutrality point (with the vanishing electric and chiral chemical potentials $\mu = \mu_5 = 0$), we find the following topological charge and current densities determined by the Berry curvature:

$$\rho_{CS} = -\frac{e^2}{c} \int \frac{d^3k}{(2\pi)^3} (\mathbf{B} \cdot \boldsymbol{\Omega}) = -\frac{e^3}{2\pi^2 \hbar^2 c^2} (\mathbf{b} \cdot \mathbf{B}), \quad (3)$$

$$\begin{aligned} \mathbf{j}_{CS} &= -e^2 \int \frac{d^3k}{(2\pi)^3} \left\{ \frac{(\mathbf{v} \cdot \boldsymbol{\Omega}) \mathbf{B}}{c} + [\mathbf{E} \times \boldsymbol{\Omega}] \right\} \\ &= -\frac{e^3}{2\pi^2 \hbar^2 c} b_0 \mathbf{B} + \frac{e^3}{2\pi^2 \hbar^2 c} [\mathbf{b} \times \mathbf{E}], \end{aligned} \quad (4)$$

where \mathbf{E} and \mathbf{B} are the electric and magnetic fields, respectively, and $\mathbf{v} = \hbar^{-1} [at_0 \cos(ak_z) \hat{\mathbf{z}} + \partial_{\mathbf{k}} |\mathbf{d}|]$. (Here, $\hat{\mathbf{z}}$ denotes the unit vector in the $+z$ direction.) The topological nature of these terms as well as the absence of the “matter” contributions is evident from the lack of the distribution functions in Eqs. (3) and (4). The Chern-Simons contributions ρ_{CS} and \mathbf{j}_{CS} play an essential role in the consistent chiral kinetic theory [18]. As is easy to check, they originate primarily from the filled electron states deep below the Fermi surface. This explains why such contributions to the charge and current densities are usually missing in linearized semiclassical low-energy models, such as the chiral kinetic theory.

In this connection, we should also emphasize that the expressions for ρ_{CS} and \mathbf{j}_{CS} are proportional to a winding number of the mapping of a two-dimensional section of the Brillouin zone onto the unit sphere [26]. This fact accounts for the topological nature of ρ_{CS} and \mathbf{j}_{CS} and explains their robustness.

Hydrodynamic equations. In order to derive the hydrodynamic equations for the chiral electron fluid in Weyl semimetals, we utilize the consistent chiral kinetic theory [18] in the relaxation-time approximation. By following the standard approach [27,28], the Euler equation and the energy conservation relation are obtained by multiplying the corresponding kinetic equation with the quasiparticle momentum and energy, respectively, and integrating over the momentum (for details, see the Supplemental Material [29]). As expected, many terms

of the resulting hydrodynamic equations agree with those in Refs. [2–4,13,14]. In order to highlight the distinctive features of our framework, here we present the abbreviated version of the equations in which the terms with spatial derivatives are omitted, i.e.,

$$\begin{aligned} \frac{1}{v_F} \partial_t \left(\frac{\epsilon + P}{v_F} \mathbf{u} + \sigma^{(\epsilon,B)} \mathbf{B} \right) \\ = -en \left(\mathbf{E} + \frac{1}{c} [\mathbf{u} \times \mathbf{B}] \right) \\ + \frac{\sigma^{(B)} (\mathbf{E} \cdot \mathbf{B})}{3v_F^2} \mathbf{u} - \frac{\epsilon + P}{\tau v_F^2} \mathbf{u} + O(\nabla_{\mathbf{r}}), \end{aligned} \quad (5)$$

and

$$\partial_t \epsilon = -\mathbf{E} \cdot (en\mathbf{u} - \sigma^{(B)} \mathbf{B}) + O(\nabla_{\mathbf{r}}). \quad (6)$$

(The full expressions including also the vorticity effects, such as the chiral vortical effect [30] and the terms with magnetovorticity coupling [31], are given in the Supplemental Material [29].) In these equations, ϵ and P are the electron energy density and pressure, respectively, n is the electron number density, \mathbf{u} is the electron fluid velocity, τ is the relaxation time, and v_F is the Fermi velocity. The effects of the chiral anomaly including the chiral magnetic effect current [32–34] as well as the energy-momentum flow are captured in the hydrodynamic equations by the terms $\propto \sigma^{(B)} \mathbf{B}$ and $\propto \sigma^{(\epsilon,B)} \mathbf{B}$, respectively, with $\sigma^{(B)} = e^2 \mu_5 / (2\pi^2 \hbar^2 c)$ and $\sigma^{(\epsilon,B)} = -e \mu \mu_5 / (2\pi^2 \hbar^2 v_F c)$. The coefficients $\sigma^{(B)}$ and $\sigma^{(\epsilon,B)}$ agree with those obtained in Refs. [25,35,36] in the “no-drag” frame [36–38].

We would like to point out that the Chern-Simons terms ρ_{CS} and \mathbf{j}_{CS} do not directly contribute to the hydrodynamic parts of the charge and current densities in the Euler equation (5) and the energy conservation relation (6). Conceptually, this is the consequence of the specific topological origin of ρ_{CS} and \mathbf{j}_{CS} associated with the filled electron states deep below the Fermi surface. In this connection, we note that Eqs. (5) and (6) originate only from the states near the Fermi surface (see also the Supplemental Material [29]). Indeed, this follows from the fact that the corresponding chiral kinetic equations contain the derivatives from the Fermi-Dirac distribution functions that are insensitive to the details of the energy spectrum at the bottom of the valence band. Therefore, it is sufficient to use the linearized theory for the matter parts of the consistent hydrodynamics.

One of the key features of the Euler equation (5) is the dissipative term proportional to \mathbf{u}/τ on its right-hand side, which was also introduced phenomenologically in Ref. [13]. In the hydrodynamic regime, when the electron-electron scattering is primarily responsible for the formation of the electron fluid, such a term captures the dominant dissipative effects due to electron scattering on phonons and impurities [6] and explicitly breaks the Galilean invariance. From a physics viewpoint, this is connected with the existence of the preferred frame in which the ion lattice of a solid is at rest. In the absence of electromagnetic fields, this term ensures that $\mathbf{u} = \mathbf{0}$ in the global equilibrium state of the electron fluid.

It should be emphasized that the hydrodynamic equations (5) and (6) lead to the conventional Ohm’s law and Joule’s

heating. Indeed, in the presence of an external electric field, the steady state of the electron fluid is reached when the right-hand side of the Euler equation (5) vanishes, i.e., at $\mathbf{u}_{\text{ave}} = -en\tau v_F^2 \mathbf{E}/(\epsilon + P)$. The latter is analogous to the average velocity in the Drude model and reproduces Ohm's law when the definition for the electric current $\mathbf{J} = -en\mathbf{u}_{\text{ave}}$ is taken into account. By substituting \mathbf{u}_{ave} into Eq. (6), one also reproduces the correct local form of Joule's heating.

Maxwell's equations. Since the electron fluid carries a nonzero charge, the hydrodynamic set of equations is incomplete without Maxwell's equations, i.e.,

$$\epsilon_e \nabla \cdot \mathbf{E} = 4\pi(\rho + \rho_b), \quad (7)$$

$$\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}, \quad (8)$$

$$\frac{1}{\mu_m} \nabla \times \mathbf{B} = \frac{4\pi}{c} \mathbf{J} + \epsilon_e \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t}, \quad (9)$$

together with $\nabla \cdot \mathbf{B} = 0$. Here ϵ_e and μ_m denote the electric permittivity and magnetic permeability, respectively, which originate from the nonitinerant electrons. We note that the complete electric charge density in Gauss's law (7) must include the contribution of the electrons in the inner shells and the ions of the lattice ρ_b . The latter ensures that the Weyl material is electrically neutral in equilibrium, i.e., $\rho + \rho_b = 0$.

Maxwell's equations should be supplemented by the expressions for the *total* electron charge and current densities that include both the hydrodynamic contributions and the Chern-Simons terms, i.e.,

$$\rho = -en + \frac{(\mathbf{B} \cdot \mathbf{u})\sigma^{(B)}}{3v_F^2} + \rho_{\text{CS}}, \quad (10)$$

$$\mathbf{J} = -en\mathbf{u} + \sigma^{(B)}\mathbf{B} + \mathbf{j}_{\text{CS}}. \quad (11)$$

In order to be consistent with Eqs. (7) and (9), these electric charge and current densities should satisfy the usual continuity relation $\partial_t \rho + \nabla \cdot \mathbf{J} = 0$. (Note that the Chern-Simons contributions by themselves satisfy the continuity relation.)

Because of the chiral nature of the electron fluid in Weyl materials, the complete set of hydrodynamic equations should also include the anomalous continuity relation for the chiral charge,

$$\partial_t \rho_5 + \nabla \cdot \mathbf{J}_5 = -\frac{e^3}{2\pi^2 \hbar^2 c} (\mathbf{E} \cdot \mathbf{B}), \quad (12)$$

where the chiral charge and current densities are

$$\rho_5 = -en_5 + \frac{(\mathbf{B} \cdot \mathbf{u})\sigma_5^{(B)}}{3v_F^2}, \quad (13)$$

$$\mathbf{J}_5 = -en_5\mathbf{u} + \sigma_5^{(B)}\mathbf{B}. \quad (14)$$

Here $\sigma_5^{(B)} = e^2\mu/(2\pi^2\hbar^2c)$ is the anomalous transport coefficient responsible for the chiral separation effect [39–41].

The complete set of the hydrodynamic and Maxwell's equations presented above is one of the key results of this Rapid Communication. Unlike the previous formulations of such equations in the literature, it incorporates several distinctive features of the chiral electron fluid in Weyl semimetals: (i) the Chern-Simons contributions affecting Maxwell's equations

and (ii) the broken Galilean invariance due to the ion lattice and the electron scattering on phonons/impurities.

It is interesting to explore specific observable predictions of the proposed framework. One of them is the unusual spectrum of collective modes in Weyl semimetals. A comprehensive study of such modes will be presented elsewhere. Here, in order to support the general claim and to illuminate the vital role of the Chern-Simons contributions, we will consider only a few specific modes that propagate transverse to the direction of a static background magnetic-field $\mathbf{B}_0 \parallel \hat{\mathbf{z}}$.

Transverse collective excitations. In the state of local equilibrium, the local chemical potentials μ and μ_5 deviate from μ_0 and $\mu_{5,0}$ in global equilibrium. The latter state is characterized by the vanishing electric current density [20,23,42]

$$\mathbf{J}_0 = \left(\sigma^{(B)} - \frac{e^3}{2\pi^2 \hbar^2 c} b_0 \right) \mathbf{B}_0 = \mathbf{0}, \quad (15)$$

and by the condition of electric neutrality,

$$-en_0 + \rho_{\text{CS}} + \rho_b = 0. \quad (16)$$

Equation (15) is satisfied by setting $\mu_{5,0} = eb_0$. Enforcing Eq. (16) fixes the value of μ_0 , which becomes a function of temperature, the external magnetic field \mathbf{B}_0 , the chiral shift \mathbf{b} , and the energy separation b_0 .

In the study of collective modes, the deviations of the local thermodynamic parameters from their equilibrium values are small. Then, the use of linearized hydrodynamic equations is sufficient. They are obtained by looking for a solution in the form of plane waves, i.e., $\delta\mu(\mathbf{r}) = \delta\mu e^{-i\omega t + i\mathbf{k}\mathbf{r}}$ together with similar expressions for other oscillating variables (see Sec. II in the Supplemental Material [29]). In such an approach, we analyze the spectrum of transverse (i.e., $\mathbf{k} \perp \mathbf{B}_0$) collective excitations in Weyl semimetals.

By solving the characteristic equation at $n_0 = n_{5,0} = 0$, we find the dissipative magnetoacoustic waves with the dispersion relations given by

$$\omega_{s,\pm} = -\frac{i}{2\tau} \pm \frac{i}{2\tau} \sqrt{1 - \frac{4}{3}\tau^2 v_F^2 k_\perp^2 + \frac{8\tau^2 v_F^2 k_\perp^2 \sigma^{(\epsilon,u)} B_0^2}{3(\epsilon + P)}}. \quad (17)$$

Most importantly, there are also collective waves that are strongly affected by the chiral shift \mathbf{b} . One of such modes is realized when the wave vector \mathbf{k} is parallel to the chiral shift (i.e., $\mathbf{k} \parallel \mathbf{b} \perp \mathbf{B}_0$ and $b = b_\perp$). The corresponding collective excitation has the following dispersion relation linear in k_\perp :

$$\omega_{\text{tAHW}} = \frac{c|k_\perp| \sqrt{3v_F^3 \hbar^3 B_0}}{\sqrt{\mu_m} \sqrt{4\pi e^2 T^2 b_\perp^2 + 3\epsilon_e v_F^3 \hbar^3 B_0^2}} + O(k_\perp^2). \quad (18)$$

As we show below, the propagation of such a mode in Weyl semimetals is sustained by local currents associated with the anomalous Hall effect (AHE). Therefore, we call it the *transverse anomalous Hall wave* (tAHW). (We note that the longitudinal anomalous Hall waves also exist and their study will be reported elsewhere.)

To clarify the physical origin of the tAHW, it is instructive to present the relevant hydrodynamic and Maxwell's equations. By setting $\mu_0 = \mu_{5,0} = 0$ and $\mathbf{b} \perp \mathbf{B}_0$, we arrive at

$$\frac{4\epsilon\omega}{T}\delta T - k_{\perp}(\epsilon + P - 2B_0^2\sigma^{(\epsilon,u)})\delta u_{\perp} = 0, \quad (19)$$

$$\frac{k_{\perp}}{T}\delta T - \frac{i + \omega\tau}{v_F^2\tau}\delta u_{\perp} + \frac{5ck_{\perp}^2 B_0\sigma^{(\epsilon,u)}}{\omega(\epsilon + P)}\delta \tilde{E}_{\perp} = 0, \quad (20)$$

$$\frac{T^2\omega}{3v_F^3\hbar}\delta\mu_5 - i\frac{e^2 B_0}{2\pi^2 c}\delta E_{\parallel} = 0, \quad (21)$$

$$\left(\omega^2 - \frac{c^2 k_{\perp}^2}{\epsilon_e \mu_m}\right)\delta \tilde{E}_{\perp} - \frac{2ie^3 b_{\perp}}{\pi c \epsilon_e \hbar^2}\delta E_{\parallel} = 0, \quad (22)$$

$$\left(\omega^2 - \frac{c^2 k_{\perp}^2}{\epsilon_e \mu_m}\right)\delta E_{\parallel} + \frac{2ie^2 \omega}{\pi c \epsilon_e \hbar^2}(B_0\delta\mu_5 + eb_{\perp}\delta \tilde{E}_{\perp}) = 0, \quad (23)$$

where subscripts \parallel and \perp denote the vector components parallel and perpendicular to \mathbf{B}_0 . In addition, $\delta \tilde{E}_{\perp}$ denotes the component of the oscillating electric field perpendicular to both \mathbf{B}_0 and \mathbf{b} . The first two equations originate from the energy conservation relation and the Euler equation, respectively. The third one is the anomalous continuity relation for the chiral charge, and the last two equations are Maxwell's equations. In the derivation, we also used Faraday's law $\delta \mathbf{B} = (c/\omega)[\mathbf{k} \times \delta \mathbf{E}]$ and took into account that the only oscillating variables in the tAHW are $\delta\mu_5$, δT , δu_{\perp} , δE_{\parallel} , and $\delta \tilde{E}_{\perp}$.

The tAHW is a rather unusual mode that relies on the dynamical electromagnetism and the Chern-Simons currents in the chiral electron fluid. Its unique nature is clear from the modified Maxwell's equations (22) and (23) in which the AHE mixes the transverse and longitudinal components of the oscillating electric field. (Note that such a mixing occurs even in the absence of an external magnetic field.) Because of the presence of the magnetic field \mathbf{B}_0 , the oscillations of the electric fields, in turn, drive the local oscillations of the chiral charge, temperature, and the fluid velocity. In essence, therefore, the tAHW is a collective excitation that is strongly affected by the topological AHE currents in Weyl semimetals.

The representative dispersion relation of the tAHW for several values of the chemical potential μ_0 is shown in Fig. 1. Because of a finite relaxation time τ , the frequencies generically get nonzero imaginary parts. Although this is always the case for the magnetoacoustic waves, the tAHW at the neutrality point $\mu_0 = \mu_{5,0} = 0$ is not affected by the dissipation effects encoded in τ . By taking into account that the propagation of the tAHW is accompanied by oscillations of the electron fluid, this fact is quite amazing.

From Fig. 1, we see that the linear dispersion relation of the tAHW is transformed into a quadratic one at nonzero values of μ_0 . In addition, the frequency of the wave obtains a small but nonzero imaginary part, i.e., the tAHW becomes dissipative. Let us note that, at small enough values of the wave vector, the real part of the tAHW vanishes and the mode becomes completely diffusive at $\mu_0 \neq 0$.

Weyl semimetals with multiple pairs of Weyl nodes. So far, we discussed only the simplest model of a Weyl semimetal with a broken TR symmetry and a single pair of Weyl nodes. Most Weyl materials, however, have multiple pairs of Weyl nodes. Moreover, some of them have a broken PI, but intact

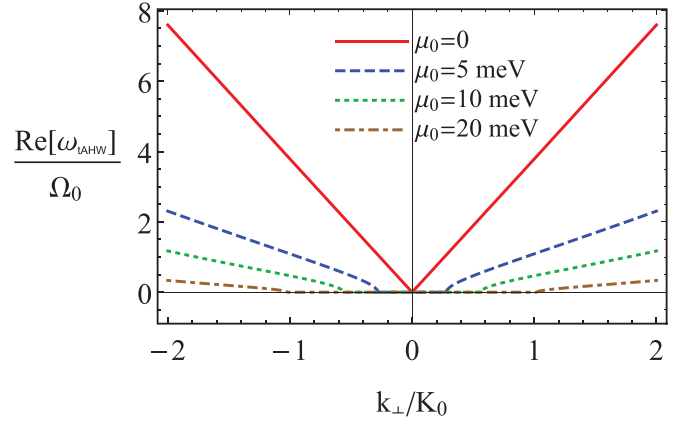


FIG. 1. The real part of the tAHW frequency for several values of μ_0 . The values of the parameters are $T_0 = 10$ K, $B_0 = 10^{-2}$ T, $b = b_{\perp} = 0.3 \times \pi \hbar c/(ea)$, $\mu_{5,0} = 0$, $a = 25.5 \times 10^{-8}$ cm $^{-1}$, and $\tau = 10^{-12}$ s [43]. The frequency and the wave vector are given in units of $\Omega_0 = 10^{-6} v_F \pi/a \approx 1.84$ GHz and $K_0 = 10^{-6} \pi/a \approx 12.3$ cm $^{-1}$, respectively.

TR symmetry (see, e.g., Ref. [11]). When the TR symmetry is present, the total number of Weyl nodes is a multiple of 4 implying that the sum of all chiral shifts vanishes $\sum_n \mathbf{b}^{(n)} = \mathbf{0}$ (here the sum runs over all pairs of Weyl nodes). If this is the case, the net sum of all Chern-Simons contributions, which are linear in the chiral shifts $\mathbf{b}^{(n)}$, should vanish too. Then, all distinctive features stemming from a nonzero \mathbf{b} will also disappear. Of course, this does not apply to Weyl semimetals in which both the PI and TR symmetries are broken. The hydrodynamic properties of such materials should be similar to those with only a broken TR symmetry, albeit with the chiral shift replaced by $\mathbf{b}_{\text{eff}} \equiv \sum_n \mathbf{b}^{(n)} \neq \mathbf{0}$.

Experimental setup. We would like to note that the existence of the tAHW and other collective modes can be tested experimentally in Weyl materials with a broken TR symmetry. As in usual metals [44], an experimental setup requires measuring the transmission amplitude of an electromagnetic wave through a Weyl crystal as a function of an applied magnetic field or as a function of the frequency at a fixed field. Because of the interference of standing waves inside the sample, the resulting signal should oscillate with the magnetic field. The effects of the chiral shift can be studied by changing the orientation of the crystal and/or magnetic field.

The promising materials for studying the effects of the Chern-Simons terms in the hydrodynamic regime might be magnetic Heusler compounds with a broken TR symmetry [45,46]. Moreover, such Weyl semimetals have only one pair of well-separated Weyl nodes near the Fermi level. One might also use antiferromagnetic half-Heusler compounds, which are predicted to be Weyl semimetals in an applied magnetic field [47–49].

Conclusion. In this Rapid Communication, we formulated a consistent hydrodynamic framework for the chiral electron fluid in Weyl semimetals. It systematically incorporates the Chern-Simons contributions, the vorticity effects, and takes into account the breaking of the Galilean invariance by the

ion lattice. As argued, the topological Chern-Simons terms affect the electron motion in the hydrodynamic regime only indirectly via the Maxwell's equations. Nevertheless, they lead to observable effects that are sensitive to the energy and momentum separations between the Weyl nodes.

By making use of the proposed hydrodynamic theory, we studied the transverse (with respect to the background magnetic field \mathbf{B}_0) collective excitations in Weyl materials. In addition to the magnetoacoustic waves (which are not modified by the chiral shift \mathbf{b} at the neutrality point), a class of modes that are significantly altered by the Chern-Simons terms was found. As a striking example, we mentioned a rather unusual transverse anomalous Hall wave obtained at $\mathbf{k} \parallel \mathbf{b}$. At the neutrality point, the latter is a gapless mode with a linear dispersion relation that remains dissipationless even at finite values of the relaxation time. Most interestingly, the anomalous Hall effect currents play the key role in the physics of the tAHW. At nonzero values of the electric charge density, the

frequency of the tAHW becomes quadratic in the wave vector and acquires a small imaginary part.

Our study in this Rapid Communication was limited to the case of transverse collective excitations. This was sufficient to illustrate the potential importance of the topological Chern-Simons terms. Nevertheless, it would be interesting to investigate the spectrum of collective modes in the most general case by using the consistent hydrodynamic theory proposed here. Such a study is underway and will be reported elsewhere.

Acknowledgments. The work of E.V.G. was supported partially by the Program of Fundamental Research of the Physics and Astronomy Division of the National Academy of Sciences of Ukraine. The work of V.A.M. and P.O.S. was supported by the Natural Sciences and Engineering Research Council of Canada. The work of I.A.S. was supported by the U.S. National Science Foundation under Grants No. PHY-1404232 and No. PHY-1713950.

-
- [1] L. D. Landau and E. M. Lifshitz, *Fluid Mechanics* (Pergamon, New York, 1959).
 - [2] D. T. Son and P. Surowka, *Phys. Rev. Lett.* **103**, 191601 (2009).
 - [3] A. V. Sadofyev and M. V. Isachenkov, *Phys. Lett. B* **697**, 404 (2011).
 - [4] Y. Neiman and Y. Oz, *J. High Energy Phys.* **03** (2011) 023.
 - [5] Y. Hidaka, S. Pu, and D. L. Yang, *Phys. Rev. D* **97**, 016004 (2018).
 - [6] R. N. Gurzhi, *J. Exp. Theor. Phys.* **17**, 521 (1963).
 - [7] J. Crossno, J. K. Shi, K. Wang, X. Liu, A. Harzheim, A. Lucas, S. Sachdev, P. Kim, T. Taniguchi, K. Watanabe, T. A. Ohki, and K. C. Fong, *Science* **351**, 1058 (2016).
 - [8] F. Ghahari, H.-Y. Xie, T. Taniguchi, K. Watanabe, M. S. Foster, and P. Kim, *Phys. Rev. Lett.* **116**, 136802 (2016).
 - [9] B. Yan and C. Felser, *Annu. Rev. Condens. Matter Phys.* **8**, 337 (2017).
 - [10] M. Z. Hasan, S.-Y. Xu, I. Belopolski, and C.-M. Huang, *Annu. Rev. Condens. Matter Phys.* **8**, 289 (2017).
 - [11] N. P. Armitage, E. J. Mele, and A. Vishwanath, *Rev. Mod. Phys.* **90**, 15001 (2018).
 - [12] J. Gooth, F. Menges, C. Shekhar, V. Süß, N. Kumar, Y. Sun, U. Drechsler, R. Zierold, C. Felser, and B. Gotsmann, *arXiv:1706.05925*.
 - [13] K. Landsteiner, Y. Liu, and Y. W. Sun, *J. High Energy Phys.* **03** (2015) 127.
 - [14] A. Lucas, R. A. Davison, and S. Sachdev, *Proc. Natl. Acad. Sci. USA* **113**, 9463 (2016).
 - [15] D. T. Son and N. Yamamoto, *Phys. Rev. D* **87**, 085016 (2013).
 - [16] M. A. Stephanov and Y. Yin, *Phys. Rev. Lett.* **109**, 162001 (2012).
 - [17] D. I. Pikulin, A. Chen, and M. Franz, *Phys. Rev. X* **6**, 041021 (2016).
 - [18] E. V. Gorbar, V. A. Miransky, I. A. Shovkovy, and P. O. Sukhachov, *Phys. Rev. Lett.* **118**, 127601 (2017).
 - [19] K. Landsteiner, *Phys. Rev. B* **89**, 075124 (2014).
 - [20] K. Landsteiner, *Acta Phys. Pol., B* **47**, 2617 (2016).
 - [21] W. A. Bardeen, *Phys. Rev.* **184**, 1848 (1969); W. A. Bardeen and B. Zumino, *Nucl. Phys. B* **244**, 421 (1984).
 - [22] K.-Y. Yang, Y.-M. Lu, and Y. Ran, *Phys. Rev. B* **84**, 075129 (2011).
 - [23] M. M. Vazifeh and M. Franz, *Phys. Rev. Lett.* **111**, 027201 (2013).
 - [24] M. V. Berry, *Proc. R. Soc. London, Ser. A* **392**, 45 (1984).
 - [25] D. T. Son and N. Yamamoto, *Phys. Rev. Lett.* **109**, 181602 (2012).
 - [26] E. V. Gorbar, V. A. Miransky, I. A. Shovkovy, and P. O. Sukhachov, *Phys. Rev. B* **96**, 085130 (2017).
 - [27] E. M. Lifshitz and L. P. Pitaevskii, *Physical Kinetics* (Pergamon, New York, 1981).
 - [28] K. Huang, *Statistical Mechanics* (Wiley, New York, 1987).
 - [29] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/PhysRevB.97.121105> for the details of the derivation of the consistent hydrodynamics equations as well as their linearized versions.
 - [30] J. Y. Chen, D. T. Son, M. A. Stephanov, H. U. Yee, and Y. Yin, *Phys. Rev. Lett.* **113**, 182302 (2014).
 - [31] K. Hattori and Y. Yin, *Phys. Rev. Lett.* **117**, 152002 (2016).
 - [32] D. Kharzeev and A. Zhitnitsky, *Nucl. Phys. A* **797**, 67 (2007).
 - [33] D. E. Kharzeev, L. D. McLerran, and H. J. Warringa, *Nucl. Phys. A* **803**, 227 (2008).
 - [34] K. Fukushima, D. E. Kharzeev, and H. J. Warringa, *Phys. Rev. D* **78**, 074033 (2008).
 - [35] K. Landsteiner, E. Megías, and F. Peña-Benitez, in *Strongly Interacting Matter in Magnetic Fields*, edited by D. Kharzeev, K. Landsteiner, A. Schmitt, and H. U. Yee, Lecture Notes in Physics Vol. 871 (Springer, Berlin, Heidelberg, 2013), pp. 433–468.
 - [36] M. A. Stephanov and H. U. Yee, *Phys. Rev. Lett.* **116**, 122302 (2016).
 - [37] K. Rajagopal and A. V. Sadofyev, *J. High Energy Phys.* **10** (2015) 018.
 - [38] A. V. Sadofyev and Y. Yin, *Phys. Rev. D* **93**, 125026 (2016).
 - [39] A. Vilenkin, *Phys. Rev. D* **22**, 3080 (1980).
 - [40] M. A. Metlitski and A. R. Zhitnitsky, *Phys. Rev. D* **72**, 045011 (2005).
 - [41] G. M. Newman and D. T. Son, *Phys. Rev. D* **73**, 045006 (2006).
 - [42] G. Basar, D. E. Kharzeev, and H. U. Yee, *Phys. Rev. B* **89**, 035142 (2014).

- [43] M. Neupane, S.-Y. Xu, R. Sankar, N. Alidoust, G. Bian, C. Liu, I. Belopolski, T.-R. Chang, H.-T. Jeng, H. Lin, A. Bansil, F. Chou, and M. Z. Hasan, *Nat. Commun.* **5**, 3786 (2014).
- [44] B. W. Maxfield, *Am. J. Phys.* **37**, 241 (1969).
- [45] G. Chang, S.-Y. Xu, H. Zheng, B. Singh, C.-H. Hsu, G. Bian, N. Alidoust, I. Belopolski, D. S. Sanchez, S. Zhang, H. Lin, and M. Z. Hasan, *Sci. Rep.* **6**, 38839 (2016).
- [46] Z. Wang, M. G. Vergniory, S. Kushwaha, M. Hirschberger, E. V. Chulkov, A. Ernst, N. P. Ong, R. J. Cava, and B. A. Bernevig, *Phys. Rev. Lett.* **117**, 236401 (2016).
- [47] M. Hirschberger, S. Kushwaha, Z. Wang, Q. Gibson, S. Liang, C. A. Belvin, B. A. Bernevig, R. J. Cava, and N. P. Ong, *Nature Mater.* **15**, 1161 (2016).
- [48] C. Shekhar, A. K. Nayak, S. Singh, N. Kumar, S.-C. Wu, Y. Zhang, A. C. Komarek, E. Kampert, Y. Skourski, J. Wosnitza, W. Schnelle, A. McCollam, U. Zeitler, J. Kubler, S. S. P. Parkin, B. Yan, and C. Felser, *arXiv:1604.01641*.
- [49] T. Suzuki, R. Chisnell, A. Devarakonda, Y.-T. Liu, W. Feng, D. Xiao, J. W. Lynn, and J. G. Checkelsky, *Nat. Phys.* **12**, 1119 (2016).