Cross-Correlated Responses of Topological Superconductors and Superfluids

Kentaro Nomura,[1] Shinsei Ryu,[2,3] Akira Furusaki,[3] and Naoto Nagaosa[1,4,5]

[1]*Correlated Electron Research Group (CERG), RIKEN-ASI, Wako 351-0198, Japan*
[2]*Department of Physics, University of Illinois, 1110 West Green Street, Urbana Illinois 61801, USA*
[3]*Condensed Matter Theory Laboratory, RIKEN, Wako, Saitama 351-0198, Japan*
[4]*Cross-Correlated Material Research Group (CMRG), RIKEN-ASI, Wako 351-0198, Japan*
[5]*Department of Applied Physics, The University of Tokyo, Hongo, Bunkyo-ku, Tokyo 113-8656, Japan*
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We study nontrivial responses of topological superconductors and superfluids to the temperature gradient and rotation of the system. In two-dimensional gapped systems, the Středa formula for the electric Hall conductivity is generalized to the thermal Hall conductivity. Applying this formula to the Majorana surface states of three-dimensional topological superconductors predicts cross-correlated responses between the orbital angular momentum and thermal polarization (entropy polarization). These results can be naturally related to the gravitoelectromagnetism description of three-dimensional topological superconductors and superfluids, analogous to the topological magnetoelectric effect in $\mathbb{Z}_2$ topological insulators.

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Introduction.—The quantum Hall effect (QHE) [1] is a prominent example of quantum phenomena characteristic of insulators with topologically nontrivial electronic wave functions, a class of materials called topological insulators (TIs) [2]. In the QHE, the Hall conductivity is quantized in units of e^2/h at integer values equal to the topological number of bulk wave functions [3]. The two-dimensional (2D) topological superconductors (TSCs) and topological superfluids (TSFs) with chiral (p-wave) Cooper pairing are superconductor analogues of the QHE and considered to be realized, e.g., in a thin film of ^{3}He A phase [4,5], Sr_2RuO_4 [6], and the $\nu = 5/2$ fractional QHE [7]. The topological nature of such TSCs and TSFs will manifest itself in thermal transport properties, such as quantization of the thermal Hall conductivity [8].

Recent studies have shown that topological states exist in time-reversal invariant and three-dimensional (3D) cases as well [2], and the systematic classification of them is established in terms of symmetries and dimensionality [9,10]. A key experimental signature of 3D TIs is the topological magnetoelectric (ME) effect. Namely, the electromagnetic response of 3D TIs is described by the axion electrodynamics [2,11,12],

$$S_\theta^{\rm EM} = \int dt d^3\boldsymbol{x} \frac{e^2}{4\pi^2\hbar c}\theta \boldsymbol{E}\cdot\boldsymbol{B}, \tag{1}$$

with $\theta = \pi$, the possible nonzero value in time-reversal invariant systems (mod 2π). The effective action (1) leads to the surface QHE that induces the topological ME effect as $\boldsymbol{M} = (e^2/2hc)\boldsymbol{E}$ and $\boldsymbol{P} = (e^2/2hc)\boldsymbol{B}$ where $\boldsymbol{M}$ and $\boldsymbol{P}$ are the magnetization and electric polarization, respectively.

An example of 3D TSFs [13] is the B phase of superfluid ^{3}He [9]. In addition, the newly found superconducting phase in Cu-doped Bi_2Se_3 [14] has been proposed to be a 3D TSC [15]. The topological nature of 3D TSCs manifests itself in their coupling with the gravitational field, which is described by the term similar to Eq. (1), the gravitational instanton term [16,17].

In this Letter, we shall develop the response theory of TSCs, which reveals the cross correlation between thermal and mechanical (rotational) responses, and is the direct equivalent of the topological ME effect in TIs. We first generalize the Středa formula [18] to the thermal Hall conductivity in 2D systems. By applying it to the surface states of a 3D TSC, the cross-correlated responses of 3D TSCs are identified with those between the orbital angular momentum and the thermal (entropy) polarization, generated by temperature gradient ∇T and by rotating the system with angular velocity $\boldsymbol{\Omega}$, respectively, as shown in Fig. 1. Our main findings are summarized in Table I. Their derivations will be given below.

Thermal Hall conductivity.—Relation between the Hall conductivity σ_H and the magnetization $\boldsymbol{M}$ in the quantum Hall regime is known as the Středa formula [18]:

$$\sigma_H = ec\frac{\partial M^z}{\partial \mu}, \tag{2}$$

where μ is the chemical potential, e (< 0) is the electric charge, and c is the speed of light. The magnetization is evaluated by $\boldsymbol{M} = (e/2c){\rm Tr}[\theta(\mu - \mathcal{H})\boldsymbol{x}\times\boldsymbol{v}]$, where $\boldsymbol{v} = (i/\hbar)[\mathcal{H}, \boldsymbol{x}]$, $\mathcal{H}$ is the single-particle Hamiltonian, and $\theta(x)$ is the step function. The relation (2) can be understood by identifying the Hall current $\boldsymbol{j} = \sigma_H \boldsymbol{E}\times\hat{\boldsymbol{z}}$ with $\boldsymbol{j} = c\nabla\times\boldsymbol{M} = -c\partial\boldsymbol{M}/\partial\mu\times\nabla\mu$.

For the thermal conductivity $\kappa_H = j_T^x/\partial_y T$, we will show below that a similar relation holds:

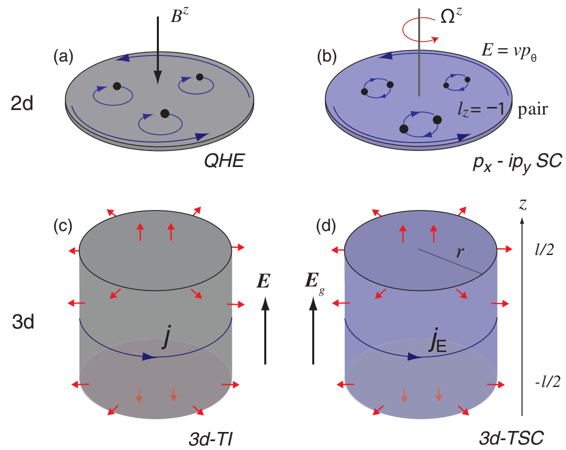

FIG. 1 (color online). Electronic responses in (a) two-dimensional (2D) and (c) three-dimensional (3D) topological insulators (TIs) (left) and thermal and mechanical (rotating) responses in (b) 2D and (d) 3D topological superconductors (TSCs) (right). In (b), the arrow along the boundary indicates the chiral Majorana edge channel with dispersion $E = vp_\theta$. In the 3D TSC (d), temperature gradient induces the surface thermal Hall current $\boldsymbol{j}_E$. A uniform mass gap is induced in the surface fermion spectrum by doping magnetic impurities near the surface of the 3D TI and TSC such that spins are all pointing out or in (arrows on the surface).

$$\kappa_H = c\frac{\partial M_T^z}{\partial T}. \tag{3}$$

The thermal current is $\boldsymbol{j}_T = \boldsymbol{j}_E - (\mu/e)\boldsymbol{j}$, where $\boldsymbol{j}$ is the electric charge current. The energy current $\boldsymbol{j}_E$ is defined as $j_E^a = cT^{a0}$, where T^{a0} is a spatiotemporal component of the energy-momentum tensor, $T^{\mu\nu} = (\delta\mathcal{L}/\delta\partial_\mu\Psi)\partial^\nu\Psi - g^{\mu\nu}\mathcal{L}$, satisfying the continuity equation, $\partial_\mu T^{\mu\nu} = 0$; $\mathcal{L}[\Psi, \partial_\mu\Psi]$ is the Lagrangian density of the system. The moment of the thermal current in Eq. (3) is defined as $\boldsymbol{M}_T = \boldsymbol{M}_E - (\mu/e)\boldsymbol{M}$, where $M_E^{\mu\nu} = \frac{1}{2}\langle x^\mu T^{\nu 0} - x^\nu T^{\mu 0}\rangle$,

TABLE I. Comparison between the cross correlation in topological insulators (TIs) and topological superconductors (TSCs) in two (2D) and three spatial dimensions (3D). In TSCs the orbital angular momentum $\boldsymbol{L}$ and entropy S (thermal polarization $\boldsymbol{P}_E$ in 3D) are generated by temperature gradient ($\boldsymbol{E}_g = -T^{-1}\nabla T$) and by rotating the system with angular velocity Ω^a. In analogy with the magnetoelectric polarizability χ_θ^{ab} in 3D TI gravitomagnetoelectric polarizability $\chi_{\theta,g}^{ab}$ can be introduced in the 3D TSC (Right-bottom). Note that the relations for TSCs hold also for the thermal response of TIs. These responses are characterized by the topological integers and quantized. See Eq. (6) for 2D TSCs and Eq. (10) for the surface of 3D TSCs.

	TI	TSC
2D	$\sigma_H = ec\frac{\partial M^z}{\partial\mu} = ec\frac{\partial N}{\partial B^z}$	$\kappa_H = \frac{v^2}{2}\frac{\partial L^z}{\partial T} = \frac{v^2}{2}\frac{\partial S}{\partial\Omega^z}$
3D	$\chi_\theta^{ab} = \frac{\partial M^a}{\partial E^b} = \frac{\partial P^a}{\partial B^b}$	$\chi_{\theta,g}^{ab} = \frac{\partial L^a}{\partial E_g^b} = \frac{\partial P_E^a}{\partial\Omega^b}$

and $M_E^z = M_E^{12}$. In contrast to the charge Hall conductivity (2), the average has to be taken at finite temperature: $\langle\cdots\rangle \equiv \sum_n f(\varepsilon_n)\langle n|\cdots|n\rangle$ where ε_n and $|n\rangle$ are the eigenvalue and eigenstate of the Hamiltonian $\mathcal{H}$, and $f(\varepsilon_n)$ is the Fermi distribution function. Below, we will consider the part of the thermal current carried by $\boldsymbol{j}_E$, specializing to the case of $\mu = 0$. For insulators, this means we count the total number N of particles such that when the chemical potential is within a gap, $N = 0$. In the Bogoliubov–de Genne theory of superconductors, due to particle-hole symmetry, this is always true, and hence $\boldsymbol{j}_T = \boldsymbol{j}_E$ and $\boldsymbol{M}_T = \boldsymbol{M}_E$.

Let us now introduce a *gravitomagnetic* field $\boldsymbol{B}_g$ [19] which is conjugate to $\boldsymbol{M}_E$ so that the variation of free energy is $dF = -SdT - \boldsymbol{M}_E \cdot d\boldsymbol{B}_g$. Equation (3) is written as $\kappa_H = c(\partial M_E^z/\partial T)_{B_g^z} = c(\partial S/\partial B_g^z)_T$, or

$$\kappa_H = \frac{c}{T}\left(\frac{\partial M_E^z}{\partial\phi}\right)_{B_g^z} = \frac{c}{T}\left(\frac{\partial Q}{\partial B_g^z}\right)_\phi, \tag{4}$$

by introducing $dQ = TdS$ and $d\phi = dT/T$, where ϕ is a fictitious gravitational potential that couples to thermal energy density Q [20]. Equation (4) is the thermal analogue of the Strěda formula for the charge Hall conductivity, $\sigma_H = ec\partial M^z/\partial\mu = ec\partial N/\partial B^z$, in that Q is the zeroth component of the energy current as eN is in the charge current.

To see the physical meaning of $\boldsymbol{B}_g$ and $\boldsymbol{M}_E$, note that the definition of $M_E^{\mu\nu}$ is similar to the orbital angular momentum: $L^{\mu\nu} = (1/c)\langle x^\mu T^{0\nu} - x^\nu T^{0\mu}\rangle$. Indeed, when there is a relativistic invariance, the energy-momentum tensor can be symmetrized so that $T^{\mu\nu} = T^{\nu\mu}$, and thus $M_E^{\mu\nu} = (c/2)L^{\mu\nu}$. While this is not the case in condensed-matter systems in general, in so-called pseudorelativistic systems where the Lorentz invariance is realized at low energies, electrons or quasiparticles obey the Dirac or Majorana equation. In these systems the Fermi velocity v plays a role of the speed of light c, and the $M_E^{\mu\nu}$ tensor is related to the orbital angular momentum as $\boldsymbol{M}_E = (v/2)\boldsymbol{L}$, and $\boldsymbol{B}_g$ can be understood as the angular velocity vector of rotating systems, $\boldsymbol{B}_g = (2/v)\boldsymbol{\Omega}$. For a system rotating with the frequency $\boldsymbol{\Omega} = \Omega^z\hat{\boldsymbol{z}}$, this can also be understood by making a coordinate transformation from the rest frame to the rotating frame in which the metric in the polar coordinates $(t,\ r,\ \varphi)$ takes the form $ds^2 \simeq v^2dt^2 - 2\Omega^z r^2 d\varphi dt - r^2d\varphi^2 - dr^2$. One can then read off, from the definition of the gravitoelectromagnetic field, the nonzero gravitogauge potential $A_g^\varphi = \Omega^z r/v$ [19]. In Cartesian coordinates, $\boldsymbol{A}_g = (1/v)\Omega^z\hat{\boldsymbol{z}}\times\boldsymbol{x}$, and $\boldsymbol{B}_g = \nabla\times\boldsymbol{A}_g = (2/v)\Omega^z\hat{\boldsymbol{z}}$. Consequently, κ_H can be written as

$$\kappa_H = \frac{v^2}{2}\left(\frac{\partial L^z}{\partial T}\right)_{\Omega^z} = \frac{v^2}{2}\left(\frac{\partial S}{\partial\Omega^z}\right)_T. \tag{5}$$

This is one of the main results of this Letter.

Although it is necessary to have (pseudo) Lorentz invariance to identify, at the operator level, $\boldsymbol{M}_E$ with the angular momentum $\boldsymbol{L}$, the relation (5) can in fact be derived for a wider range of systems, e.g., by assuming that the edge state in the disk geometry is described by chiral conformal field theory [21]. This indicates the validity of Eq. (5) in many 2D topological phases, a representative example of which is the 2D chiral p-wave superconductor with $l_z = -1$ pairing as shown in Fig. 1(b). Near the edge, there exist chiral Majorana modes [8] described by $H_{\mathrm{edge}} = (1/2)\int_0^L dx\,\psi(-i\hbar v \partial_x)\psi$, where ψ is a single-component real fermionic field and L is the circumference of the edge. Since $T^{00} = T^{10} = T^{01} = (-i\hbar v/2)\psi\partial_x\psi$, $\langle j_E\rangle = (v/L)\langle H_{\mathrm{edge}}\rangle = (v^2/2)\times \langle L^z_{\mathrm{edge}}\rangle = \pi^2 k_B^2 T^2/12h$, leading to

$$\kappa_H = \frac{\partial\langle j_E\rangle}{\partial T} = \frac{\pi^2 k_B^2 T}{6h}, \tag{6}$$

which is a half of the quantized value for conventional chiral fermions. Here, L^z_{edge} is the edge mode contribution to the total orbital angular momentum per unit area.

Majorana fermion on the surface of 3D TSC/TSF.—Let us now derive Eqs. (4) and (5) from a microscopic theory via the Kubo formula. We work with an example, the 2D massive Majorana fermion system realized on the surface of 3D TSCs [2,9,22], anticipating applying (4) and (5) in the derivation of the cross-correlated responses of the 3D TSCs. It is described by $H = (1/2)\int d^2\boldsymbol{x}\,\psi^T\mathcal{H}\psi$, where $\psi^T = (\psi_\uparrow, \psi_\downarrow)$ is the real spinor field satisfying $\{\psi_\alpha(\boldsymbol{x}), \psi_\beta(\boldsymbol{x}')\} = \delta_{\alpha\beta}\delta(\boldsymbol{x}-\boldsymbol{x}')$, and

$$\mathcal{H} = -i\hbar v(\sigma_z\partial_x + \sigma_x\partial_y) + m\sigma_y. \tag{7}$$

The mass m is due to the interaction with magnetic fields or magnetic moments perpendicular to the surface [Fig. 1(d)] and thus breaks time-reversal symmetry. To study thermal transport, we introduce coupling with the fictitious gravitational potential ϕ in the Lagrangian [20,23]: $\mathcal{L} = (1/2)\psi^T[i\hbar\partial_t - \mathcal{H} - (1/2)\{\phi, \mathcal{H}\}]\psi$. The energy-current operator is given by ($a, b = x, y$)

$$\begin{aligned} j_E^a = {}& \psi^T\frac{1}{4}\{v^a, \mathcal{H}\}\psi - \psi^T\frac{i\hbar}{8}[v^a, v^b]\psi\partial_b\phi \\ &+ \psi^T\frac{1}{8}[\mathcal{H}(v^a x^b + 3x^b v^a) + \mathrm{H.c.}]\psi\partial_b\phi. \end{aligned} \tag{8}$$

The first term is nonvanishing even in the absence of gravitational potential; the second and the third terms are proportional to $\nabla\phi$, which are analogous to the diamagnetic charge current in the presence of a magnetic field. To evaluate $\kappa_H = -\langle j_E^x\rangle/(T\partial_y\phi)$, we apply the Kubo linear response formula to the first term, while the second and third terms need to be averaged in thermal equilibrium [21]. When the mass gap is large enough, the density of states of Majorana fermions vanishes at $\varepsilon = 0$, and the thermal Hall conductivity is given by $\kappa_H = v\partial M_E^z/\partial T$, where $M_E^z = (1/4v)\sum_n f(\varepsilon_n)\varepsilon_n\langle n|(xv^y - yv^x)|n\rangle$, and $\langle\boldsymbol{x}|n\rangle = u_n(\boldsymbol{x})$ is the exact eigenstate of the Majorana Hamiltonian (7), $\mathcal{H}u_n(\boldsymbol{x}) = \varepsilon_n u_n(\boldsymbol{x})$.

Quite generally, one can derive, in the limit $T \to 0$, the relation

$$\kappa_H = \frac{\hbar\pi^2 k_B^2 T}{6L^2}\sum_{n,m}\theta(-\epsilon_n)\frac{2\mathrm{Im}[\langle n|v^x|m\rangle\langle m|v^y|n\rangle]}{(\epsilon_n - \epsilon_m)^2}, \tag{9}$$

where L^2 is the area of the surface. Apart from the factor $\pi^2 k_B^2 T/6$, the right hand side resembles the Kubo formula for the electrical Hall conductivity, which, however, is not a well-defined quantity for Majorana fermions; nevertheless Eq. (9) can be regarded as the generalized Wiedemann-Franz law to Majorana fermions [20,23]. Compared to the electron systems, there is an extra factor of $1/2$ due to Majorana nature. Since σ_H of the massive Dirac fermion is $\sigma_H = \mathrm{sgn}(m)e^2/(2h)$, Eq. (9) immediately gives

$$\kappa_H = \mathrm{sgn}(m)\frac{\pi^2}{6}\frac{k_B^2}{2h}T \tag{10}$$

for the massive Majorana fermion. There is a factor $1/2$ compared to the 2D result of Eq. (6).

Cross-correlated response of 3D TSC/TSF.—Let us illustrate the physical implications of Eq. (5) by studying the responses of 3D TSCs to the temperature gradient and the rotation. For simplicity, we consider a sample with cylindrical geometry with height ℓ and radius r as depicted in Fig. 1(d). We assume that magnetic impurities are doped near the surface and their spins are all pointing out or in so that uniform mass gap is formed on the surface. Let us first introduce the temperature gradient in the z direction, which generates the energy current $j_E = \kappa_H\partial_z T$ on the surface. Since j_E/v^2 corresponds to the momentum per unit area, total momentum due to the surface energy current is $P_\varphi = (2\pi r\ell)j_E/v^2$ and thus, the induced orbital angular momentum per volume is given by

$$L^z|_{\Omega^z} = \frac{rP_\varphi}{\pi r^2\ell} = \frac{2}{v^2}\kappa_H\partial_z T. \tag{11}$$

Similarly, upon rotating the cylinder with $\boldsymbol{\Omega} = \Omega^z\hat{\boldsymbol{z}}$ (without temperature gradient), applying Eqs. (4) and (5) to the top and bottom surfaces, we obtain the induced thermal energy density (the induced entropy change) localized on the top and bottom surfaces,

$$\Delta Q(z)|_T = \frac{2T\Omega^z}{v^2}[\kappa_H^t\delta(z - \ell/2) + \kappa_H^b\delta(z + \ell/2)], \tag{12}$$

where $\kappa_H^t = -\kappa_H^b$ as the spins on the top and bottom surfaces are pointing to the opposite directions (different signs of m) [24]; see Fig. 1(d) .

In terms of the gravitoelectric field $\boldsymbol{E}_g = -T^{-1}\nabla T$ and the momentum of the energy current $\boldsymbol{M}_E$, Eq. (11) can be written as $\boldsymbol{M}_E = (T\kappa_H/v)\boldsymbol{E}_g$. Further introducing the thermal polarization $\boldsymbol{P}_E$ by $\Delta Q = -\nabla \cdot \boldsymbol{P}_E$, Eq. (12) can be written similarly as $\boldsymbol{P}_E = (T\kappa_H/v)\boldsymbol{B}_g$. These highlight the correspondence between TIs and TSCs,

$$\mathrm{TI}: \frac{\partial M^a}{\partial E^b} = \frac{\partial P^a}{\partial B^b} \Leftrightarrow \mathrm{TSC}: \frac{\partial M_E^a}{\partial E_g^b} = \frac{\partial P_E^a}{\partial B_g^b}. \tag{13}$$

Since the orbital angular momentum is given from the internal energy functional by $L^a = -\delta U_\theta/\delta\Omega^a$, the coupling energy of the temperature gradient and rotation velocity is written as

$$U_\theta = -\int d^3\boldsymbol{x} \frac{2}{v^2}\kappa_H \nabla T \cdot \boldsymbol{\Omega} = \int d^3\boldsymbol{x} \frac{k_B^2 T^2}{24\hbar v}\theta \boldsymbol{E}_g \cdot \boldsymbol{B}_g. \tag{14}$$

This is analogous to Eq. (1) with $e^2/\hbar c \leftrightarrow (\pi k_B T)^2/6\hbar v$ and $\theta = \pm\pi$ playing the same role as in Eq. (1). In 2D TSC cases, the corresponding term is written as $U_{\mathrm{TSC}}^{2d} = \int d^2x (2/v^2) T\kappa_H \phi \Omega^z$. This is the thermodynamical analogue of the Chern-Simons term [2,5,25].

Angular momentum paradox in 2D TSC/TSF.—These terms can be related to the problem of total angular momentum of the ground state of chiral superfluids. For simplicity we consider the 2D case. The angular momentum per unit area of the ground state of the chiral p-wave state has been proposed to behave as $L^z(T=0) = -(\hbar n/2)(k_B T_c/E_F)^\gamma$ with different exponents $\gamma = 2$, 1, and 0 [4], where n is the number of particles. The controversy of γ has not yet resolved. By integrating Eq. (5) from 0 to T, and using $\Delta = \hbar v k_F$, one obtains $L^z(T) - L^z(0) = (\pi\hbar k_F^2/6)(k_B T/\Delta)^2$, at low temperature, which is consistent with a numerical simulation [26]. If we extrapolate this relation to the critical temperature T_c at which $L^z(T_c)$ vanishes, we obtain the angular momentum of the ground state, up to a numerical factor, $L^z(T=0) \sim -\hbar(\pi k_F^2/2)(k_B T_c/\Delta)^2 \sim -\hbar n/2$, indicative of $\gamma = 0$. (Here we should keep in mind that we used $\kappa_H \propto T$ which is, however, valid only at low temperature $T \ll T_c$.) Since the chiral Majorana edge modes of a 2D TSC are moving in the opposite direction to the rotation of Cooper pairs [Fig. 1(b)], at finite temperature, they contribute to reducing the total angular momentum from its ground state value at $T = 0$.

Possible experiments.—We conclude by discussing possible experiments to probe the cross-correlated response of TSCs. For the 2D case, let us assume a 2D TSC is rotating with a frequency of Ω^z. As Ω^z increases by $\Delta\Omega^z$, we predict the temperature changes as $\Delta T = \Delta Q/C = (2\kappa_H T/Cv^2)\Delta\Omega^z$, where C is the heat capacity of the 2D TSC sample. As the heat capacity of a fully gapped superconductor is small [6], this temperature change may not be so difficult to measure.

For 3D TSCs, the Einstein–de Hass effect will reveal a relationship between magnetism and angular momentum of the system, as it has been used for the study of ferromagnetic materials. We assume a cylindrical 3D TSC suspended by a thin string and apply a thermal gradient [Fig. 1(d)]. This induces a surface energy current with the angular momentum L^z, according to Eq. (11). By the conservation law of total angular momentum, it must be compensated by a mechanical angular momentum of the material, which can be directly measured in principle. Its inverse effect is the generation of thermal polarization (entropy polarization) by rotating the system. To observe the effect, the frequency Ω^z should be lower than the critical frequency Ω_{c1} above which vortices are introduced in the bulk of the sample that will generate the energy current in the z direction. The key point is that magnetization has to be induced properly on the surface, all pointing out or in [24].

In summary, we have found nontrivial correlated responses of TSCs to thermal gradient and mechanical rotation which are analogous to the topological ME effect. We have proposed possible experiments which probe these topological responses.

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Note added.—While we were finalizing the Letter, a preprint [27] appeared in which the energy magnetization ME is discussed while its relation to the angular momentum L and TSCs are not discussed.

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