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SHORT RANGE RESONATING VALENCE BOND THEORIES AND SUPERCONDUCTIVITY

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We consider the nature of superconductivity near a spin-liquid state with a large spin-excitation-gap. We argue that the quantum-dimer-model with holes is a good approximation in this limit. The insulator is shown to be *exactly* equivalent to compact quantum electrodynamics, and has a massive spectrum. The doped system is a superconductor with a low density phase characterized by tightly bound pairs and a high density phase with two weakly coupled condensates.

The nature of superconductivity in highly correlated two-dimensional electron systems is a fundamental problem which has attracted particular attention in the context of high temperature superconductivity in the cuprate superconductors. Despite considerable theoretical effort, there are few solid results, even for extremely simplified models. In this paper, we consider the nature of superconductivity in the vicinity of a spin-liquid state. Much of our picture is based on the idea that high temperature superconductors are strongly coupled systems with a coherence length of a few lattice spacings. In a "realistic" system, many processes and excitations compete. We find it useful to think of the limit in which there is a large gap in the spin excitation spectrum, and the spin correlation length ξ_s is the shortest physical length-scale. While it is likely that real materials are seldom so strongly coupled, we argue that results obtained in this limit apply for any disordered spin-state at temperatures and energies lower than the spin-gap and distances large compared to ξ_s. In this situation, the physics is dominated by spin singlet excitations. This is the essential idea underlying the Short Range Resonating-Valence-Bond (RVB) picture.[1–3]

In the large spin-gap limit, charge and spin degrees of freedom are decoupled[2] and the remaining low-energy degrees of freedom are electrically neutral valence-bonds ("dimers") plus charged, spinless "holons", all of which can be taken at a

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bare level tobe hardcore bosons.[4] The simplest model which incorporates the physics discussed above is the Quantum Dimer Model (QDM) which includes resonance between valence bonds (VB) only around short loops on the lattice and in which hole hopping is strictly local. This is a drastic approximation in the sense that the non-orthogonality of the VB's and virtual spinon excitations will induce VB resonances over arbitrarily long loops, albeit with exponentially small amplitudes.[a] One essential feature of the dimer model on a bipartite lattice is that there are two topologically distinct types of holes: those that live on the "red" and "black" sublattices,[2,4] respectively. An advantage of the QDM is that it has known solvable points[4] at which the system is probably critical.

The central idea in our approach is the realization that dimer models are *exactly* equivalent to Polyakov's compact electrodynamics in $2+1$ dimensions.[5] A key difference here is that one is not interested in the vacuum sector, but rather in the case in which there are sources of "electric" field on the lattice. These sources are responsible for enforcing the condition that exactly one dimer can touch a lattice site. This is a confining theory with at most a finite ground-state degeneracy and a gap in the excitation spectrum. From this we learn that the QDM also has a singlet ground-state with a non-zero energy gap, and that isolated holons or spinons are not part of the physical spectrum. For infinite spin-gap, strictly static holons are confined by a linear potential. The "resonon" of the QDM[4] can be identified with the "photon" of the gauge theory. However, since the theory is confining, there is actually no photon state but only local fluctuations of flux which are massive. While we find[6,7] that the simplest (undoped) dimer model has spin-Peierls order, we have not yet fully resolved the issue of the existence of a spin-liquid phase in the presence of longer range resonance. By means of a duality transformation we show that the gauge theory is equivalent to a frustrated quantum discrete Gaussian model.[6]

We also consider the effects of holes. The two different types of holons have opposite gauge charge, $\pm q$ (although both have electric charge $+e$). We show that at any non-zero hole density the linear potential between static holes is screened, that all excitations are gauge singlets, and that there are finite energy excitations with the quantum numbers of a holon.[b] We consider two possible ground-states; one in which the holons on opposite sublattices are intimately paired, and another in which they form two *nearly* separate condensates. Residual gauge interactions in effect produce locking of the red and black condensates. *Both states exhibit a Meissner effect and are superconducting.* We derive a Landau-Ginzburg theory for the high density phase and find that many of the phenomenological implications are reminiscent of some of the observed properties of the cuprate superconductors.

[a]We will discuss the effect of longer loops in Ref. 7.

[b]This is a well known effect in Lattice Gauge Theories with matter fields. See, E. Fradkin and S. Shenker, *Phys. Rev.* **D19** (1979) 3682.

All of our results were obtained for the simplest dimer model. The addition of extra terms representing the effect of some longer loops and bonds will not change our results qualitatively.

Preliminary results of this work have been presented elsewhere.[8] Ideas similar to the ones presented in this work have also been discussed by several groups. Preliminary ideas concerning the relation of a U(1) gauge theory to spin liquids were discussed by Baskaran and Anderson.[3] Read and Sachdev[9] have reached a closely related picture (i.e., connections with $2+1$ QED and confinement in the hole-less system) within a $1/N$ expansion for the SU(N) antiferromagnet in a disordered phase. R. Shankar,[7] X. Wen,[10] and P. Lee,[11] elaborating ideas of Wiegmann,[12] have noted the existence of two distinct types of holons in a nearly antiferromagnetic (spin-liquid) background, which they describe in terms of a U(1) gauge theory. In our view, the striking similarities between the Wiegmann-Shankar-Lee theory and the present theory are a consequence of the fact that actually both are describing the same physics. However, our description is best when the spin-spin correlation length is of the order of a lattice constant while their description assumes that the correlation length for antiferromagnet fluctuations is large compared to a lattice constant. The dimer problem has also been considered by Ioffe and Larkin[13] who reach similar conclusions to the present ones using a more directly physical formalism.

To begin with, we consider the simplest version of the QDM described in Ref. 4. The states are specified by configurations of a hard-core dimer gas on a square lattice, and the simplest interactions represent resonances between different VB states

$$H_d = J \sum_{\text{plaquettes}} [\,|\,\|\rangle\langle = | + h.c.] \,. \tag{1a}$$

The dynamics of the holes are governed by the shortest range hole hopping term in the dimer Hilbert space

$$H_h = -t \sum [\,|\,.\,|\rangle\langle \dot{-} \,| + h.c.] \,, \tag{1b}$$

where the sum runs over all nearest-neighbor holon-dimer pairs. Note that holes hop only between sites on the same sublattice. This is a consequence of the large spin-gap and the nature of the dimer-gas on a bipartite lattice; there is a topological charge of $+$ or $-$ associated with holes on the two sublattices.[2] In fact, t should be viewed as an effective parameter which already includes the effect of high energy processes in which the holons hop between the different sublattices. The total Hamiltonian is $H = H_d + H_h$. Every site on the lattice is occupied either by a hole or touched by one dimer.

To map this problem onto compact QED, let us define a Hilbert space on links and sites as follows: On each link $(\mathbf{r},\mathbf{r}')$, let $L(\mathbf{r},\mathbf{r}')$ be an angular-momentum operator and $a(\mathbf{r},\mathbf{r}')$ be the canonically conjugate phase variable (i.e., $[a,L] = i$);

it is useful to relable $L(\mathbf{r},\mathbf{r}+\hat{e}_j)=L_j(\mathbf{r})$ (and a_j analogously) where $j=1,2$. Define $\Psi^\dagger(\mathbf{r})$ to be the creation operator for a hole on lattice site $\mathbf{r}$; Ψ satisfies Bose commutation relations. In terms of these variables, the following Hamiltonian $H = H_0 + H_1$ is a faithful representation of (1a) and (1b) in the limit that K and M go to zero

$$H = \frac{1}{2K}\sum_{\mathbf{r},j}[L_j(\mathbf{r})^2 - 2L_j(\mathbf{r})\alpha_j(\mathbf{r})] + 2J\sum_{\mathbf{r}}\cos[f_{1,2}(\mathbf{r})]$$

$$-2t\sum_{\langle \mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3\rangle}[\Psi^\dagger(\mathbf{r}_1)e^{iq(\mathbf{r}_1)\int_{r_1}^{r_3}\mathbf{a}(\mathbf{r}')\cdot d\mathbf{r}'}\Psi(\mathbf{r}_3) + h.c.]$$

$$+\frac{1}{2M}\sum_{\mathbf{r}} n(\mathbf{r})[n(\mathbf{r})-1]\,. \tag{2}$$

Here, $n(\mathbf{r})=\Psi^\dagger(\mathbf{r})\Psi(\mathbf{r})$ is the hole density on site $\mathbf{r}$, $\alpha_j(\mathbf{r}) = 1/2\, e^{i\mathbf{Q}\cdot\mathbf{r}}$ is a background electric field, with $\mathbf{Q}=(\pi,\pi)$, $f_{1,2}$ is the magnetic flux

$$f_{1,2}(\mathbf{r}) = [\Delta_1\, a_2(\mathbf{r}+\hat{e}_2) - \Delta_2\, a_1(\mathbf{r}+\hat{e}_1)]\,, \tag{3a}$$

where the lattice derivative defined as

$$\Delta_j f(\mathbf{r}) = f(\mathbf{r}) - f(\mathbf{r}-\hat{e}_j)\,. \tag{3b}$$

The sum over $\langle \mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3\rangle$ runs over neighboring triplets of sites, with

$$\int_{r_1}^{r_3}\mathbf{a}(\mathbf{r}')\cdot d\mathbf{r}' = [a(\mathbf{r}_1,\mathbf{r}_2) + a(\mathbf{r}_2,r_3)]\,. \tag{3c}$$

$q(\mathbf{r}) = e^{i\mathbf{Q}\cdot\mathbf{r}}$ is the gauge charge of a hole, which is $+1$ (-1) for holes on the red (black) sublattice. To see the correspondence between the Hamiltonians in Eqs. (1) and (2), identify $e^{i\mathbf{Q}\cdot\mathbf{r}}L_j(\mathbf{r})$ with the number of dimers on link $(\mathbf{r},\mathbf{r}+\hat{e}_j)$ and $n(\mathbf{r})$ with the number of holes on site $\mathbf{r}$; in the limit $K\to 0$ and $M\to 0$ these operators are each constrained to take on values 0 or 1. It is easy to see that the remaining terms in Eq. (2) *exactly* reproduce the Hamiltonian of the QDM with holes, Eq. (1). The Hamiltonian in Eq. (2) has to be supplemented with the constraint

$$\mathbf{\Delta}\cdot L(\mathbf{r}) = e^{i\mathbf{Q}\cdot\mathbf{r}}[1-n(\mathbf{r})]\,. \tag{4}$$

which simply states that a site either has a hole or it is touched by a dimer.

The first two terms in Eq. (2) can be recognized as the Hamiltonian for Polyakov's lattice QED in 2 + 1 dimensions for the gauge field a_j. The main differences here are: (1) the presence of the shifts α_j; we will see that this has little effect on the interesting physics; (2) the constraint (Gauss's law) represents a system with a positive (negative) sources on red (black) sites and (3) the coupling constant J has the wrong sign. This final problem can be remedied by shifting the a_j field by a half flux quantum per plaquette. The penalty we pay is that this induces frustration in the hole-system; this reflects the competition between the magnetic interactions and the hole kinetic energy. The last two terms in Eq. (2) represent the coupling to dynamical holes.

We can gain much insight about this problem by performing a duality transformation. This amounts to solving the constraint by going to the dual lattice.[14] For the sake of simplicity, in this paper we will consider the dual of the undoped system. In a forthcoming paper[6] we will also discuss duality with holes. Define a set of operators $\hat{s}(\mathbf{r})$, with integer spectrum, defined on the sites of the dual lattice with periodic boundary conditions. For the undoped system we can write

$$L_j = \varepsilon_{jk}(\Delta_k \hat{s} + B_k) \ , \tag{5}$$

where B_j is a classical integer-valued background field satisfying

$$\varepsilon_{jk}\,\Delta_j\,B_k(\mathbf{r}) = e^{\mathbf{Q}\cdot\mathbf{r}} \ . \tag{6}$$

It is easy to see that the number of possible solutions of (6) is in one-to-one correspondence with the number of classical dimer configurations. By substituting (5) into (2) we find, for the undoped case, that after shifting the a's

$$H = \frac{1}{2K}\sum_{\mathbf{r},i}\left[(\Delta_i \hat{s} + B_i - \Gamma_i)^2 - \Gamma_i^2\right] - 2J\sum_{\mathbf{r}}\cos\hat{R}(\mathbf{r}) \ , \tag{7}$$

where $\mathbf{r}$ runs over the sites of the dual lattice and $\hat{R}(\mathbf{r})$ is canonically conjugate to $\hat{s}(r)$ (i.e., $[\hat{R}(\mathbf{r}), \hat{s}(r)] = i$). We have also defined $\Gamma_j = \varepsilon_{jk}\,\alpha_k$. This is the Hamiltonian for a frustrated Quantum Discrete Gaussian Model (DGM). The frustration here is created by the B-fields whose curl equals $+1$ (-1) on plaquettes dual to red (black) sites. The classical part of this Hamiltonian has, for a given choice of B's, an infinite number of classical ground states, the classical dimer configurations. (These correspond to the $(1.79)^A$ classical dimer configurations, where A is the area of the system.) In the unfrustrated case, Polyakov[5] showed that the ground state is unique, that there is a non-zero gap and confinement. In the frustrated case there is numerical evidence[15,6] that the ground state is a VB crystal, which in axial gauge, $B_1 = 0$, corresponds to the state with constant $\hat{s}(\mathbf{R})$. Fluctuations about this state are small with a correlation length of order one lattice constant.[6] We expect, and have confirmed numerically,[6] that there is a melting transition at a critical temperature T_m of order J. Above T_m, the system is effectively equivalent to a frustrated classical DGM, which is always critical (i.e., has power-law correlations). This suggests the possible existence of a new finite temperature phase of a frustrated spin-1/2 Heisenberg model having short-range spin-spin correlations and power-law four-spin correlations.

It is also possible to write down a Euclidean path integral for (7) and, in turn, to study its behavior in terms of a gas of topological excitations, magnetic monopoles.[16] Physically magnetic monopoles are Euclidean histories in which the flux changes by 2π. In a discrete time path integral, the result is

$$Z = (\text{const.}) \sum_{\{m\}} e^{2\pi^2 \Sigma_{x,x'} m(x)\, G_0(x,x')m(x') - i2\pi\, \Sigma_x m(x)\varphi(x)} \,, \tag{8}$$

where $G_0(x,x')$ is the $2+1$ dimensional lattice Green function for our (anisotropic) problem. This "Coulomb Gas" (monopole gas) has complex amplitudes which are the Euclidean version of Berry phases. These phases follow directly from the formalism without need for an adiabatic approximation.[17] The phases $\varphi(x)$ are

$$\varphi(x) = \frac{1}{4}\,(-1)^{x_2}\left[\frac{(-1)^{x_1}}{2} - 1\right] \tag{9}$$

for all times. (Similar results were obtained in Ref. 9.)

We now consider the effect of holes. Holes couple to the gauge field a_j through their gauge current. (See Eqs. (2) and (4).) We can use a path integral description of the system in which the holes are represented by a set of closed world-lines in the background of the fluctuating gauge fields. This coupling produces a shift in the hole mass (self-energy) and a phase factor which is equal to the Aharonov-Bohm phase produced by the flux of the a-field through the world-lines. There are two sources of flux: a slowly varying piece due to "resonons", and a rapidly varying piece due to monopoles. At low densities, $(\langle n(\mathbf{R})\rangle \equiv n_0 \ll J/t)$ the path integral is dominated by histories in which red and black holes move in tightly bound pairs; the net contribution from histories with unbound holes is exponentially small due to the rapid fluctuations in the Aharonov-Bohm phases. These pairs have electric charge $+2e$, are gauge-neutral, and will Bose-condense.[18] We expect the zero temperature correlation length ξ_0 to be of order $n_0^{-1/2}$, and $T_c \propto n_0$.

At finite hole density, the a field acquires a Higgs mass (as shown below); this implies that the monopoles are confined in neutral pairs.[19] In this limit the fluctuations of the gauge field are mild, so a mean-field picture is reasonable. Let us consider the solutions of the classical equations of motion corresponding to Eqs. (2) and (4) in the presence of a chemical potential which fixes the average density of holes, n_0. Solutions with static, uniform flux Φ per plaquette have the form

$$\Psi(\mathbf{R}) = \sqrt{n_0}\, e^{-ia_0(\mathbf{R})t} \tag{10a}$$

$$a_1(\mathbf{R}) = -a_2(\mathbf{R}) = \frac{1}{4}\,\Phi\, e^{i\mathbf{Q}\cdot\mathbf{R}} \tag{10b}$$

and $a_0(\mathbf{R})$ is the solution to the constraint equation, and equals

$$a_0(\mathbf{R}) = \frac{1}{8}\,[3 - n_0]\, e^{i\mathbf{Q}\cdot\mathbf{R}}\ . \tag{10c}$$

In the high density phase, $n_0 > 2J/t$, the action has a minimum when the flux $\Phi = 0$. For $n_0 < 2J/t$ we find a minimum when the flux

$$\Phi = 2\cos^{-1}\left(\frac{n_0 t}{2J}\right)\ . \tag{11}$$

This regime is reminiscent of the flux phases that are so popular these days.[11,12,20–22]

The high density phase ($n_0 > J/2t$) is a superconductor with two weakly coupled (red and black) Bose condensates. In this regime, we find that the effective action for small fluctuations about the mean field has the form

$$S^{\text{eff}} = \int d\mathbf{r}dt \left\{ \frac{f_{\mu,\nu}^2}{4g} + \Psi^*\left[(i\partial_t - a_0\sigma_3 - A_0) - \frac{\tau}{2}\,(i\nabla - \mathbf{a}\sigma_3 - \mathbf{A})^2\right]\Psi \right.$$
$$\left. - \frac{\lambda}{4}\,[\Psi^*\Psi - n_0]^2 - \frac{\gamma}{4}\,[\Psi^*\sigma_3\Psi]^2 \right\}\ , \tag{12}$$

where we have kept only terms quadratic in the gauge fluctuations and have rescaled the time to bring the first term into a relativistic form. Here $\Psi(\mathbf{r})$ is the two component (red and black) Bose field, σ_3 is a Pauli matrix, and A_μ is the background electromagnetic field. The coupling constants in Eq. (12) are smooth functions of the microscopic parameters. However, they are heavily renormalized by short-distance fluctuations. Still we expect τ and $\lambda \sim tl^2$ where l is a lattice constant.[6] In the broken symmetry phase, the out-of-phase oscillation of the two condensates gets "eaten" by a and becomes massive. The in-phase oscillation is "eaten" by the electromagnetic field A which develops a mass $\sqrt{2n_0\tau}$.

There are several phenomenological consequences of these results. Most importantly, *we find a standard two-component charge 2e superconducting order parameter and a Meissner effect.* In addition, we find two branches of spin zero, electrically neutral, massive excitations (the longitudinal and transverse components of the gauge field a). T_c is found to be $\sim \tau n_0$, and ξ_0 is very small, $\xi_0 \sim n_0^{-1/2}$. To compute the London penetration depth λ we consider a three dimensional stack of planes which interact only electromagnetically. In this case, $\lambda = (4\pi n_0\tau e^2)/(l_c c^2)$ in units $\hbar = 1$, where l_c is the spacing between planes. This superconductor is strongly type II, with λ/ξ_0 of order 100–1000 for a holon effective mass of order an electron mass, and $l_c \sim 10$ Å. These results show that a model based on a valence bonds plus holes can produce a phenomenology which is reminiscent of some of the most striking features of high temperature superconductors. Other features of the experiments, such as the observed Pauli susceptibility above T_c, are not present in a model with a very large spin-gap.

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