INTERACTION OF GOLDSTONE PARTICLES IN TWO DIMENSIONS. APPLICATIONS TO FERROMAGNETS AND MASSIVE YANG-MILLS FIELDS

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Interaction of Goldstone particles in two dimensions lead to the infrared catastrophe. In order to analyze it we apply to the problem the method of the renormalization group. It is shown that due to interaction the regime of the "asymptotic freedom" arises. The continuation to higher dimensions and the applications of the result are briefly discussed.

In this note we shall investigate the infrared catastrophe which occurs in two dimensions with the Goldstone particles. (G.P.). We fix our attention on the group SU(2) but all results obtained below will be valid for the arbitrary group.

Let us consider the chiral lagrangian:

$$\mathcal{L} = \frac{1}{2f}\sum_{a=1}^{N}(\partial_\mu n^a)^2. \tag{1}$$

(Here $\Sigma_a (n^a)^2 = 1$; f is a coupling constant). The Schwinger-Weinberg lagrangian describing SU(2) $\times$ SU(2) breakdown corresponds to $N = 4$, but it is convenient to work with unspecified N. We shall assume also that the Wick rotation can be done and work with the euclidean field theory.

After the usual parametrization of the unit vector the uselessness of the perturbation theory in two dimensions become evident since all terms of the perturbation expansion contain infrared divergences.

Because of this complication we shall use the method of the renormalization group.

The idea is as follows. In (1) fields should have f Fourier-component cut off at some momentum Λ. (In the statistical physics Λ is the inverse lattice spacing). Let us integrate the theory on the fields $n^a(x)$ with the wave lengths between Λ^{-1} and $\tilde{\Lambda}^{-1}$. It will give us the new effective lagrangian by the formula [1]:

$$\exp\left(-\int \tilde{\mathcal{L}}\,\mathrm{d}^2x\right) = \int \prod_{\tilde{\Lambda}<|k|<\Lambda} \mathrm{d}n^a(k)\exp\left(-\int \mathcal{L}\,\mathrm{d}^2x\right). \tag{2}$$

The new lagrangian should have the form which follows from O(N)-symmetry:

$$\tilde{\mathcal{L}} = \frac{1}{2\tilde{f}}(\partial_\mu n^a)^2 + c_1(\partial_\mu n)^4 + \dots . \tag{3}$$

Our aim will be to connect $\tilde{f}$ and f. Then repeating the process we obtain the renormalization group equation for f.

There are several methods to put this into practice. The most convenient one is to use the expansion for $\boldsymbol{n}$ first proposed by Berezinsky and Blank [2]. Let us introduce the slow varying background field $\boldsymbol{n}(x)$ with the wave lengths greater than $\tilde{\Lambda}^{-1}$. The arbitrary configuration may be represented as following:

$$\boldsymbol{n}(x) = \tilde{\boldsymbol{n}}(x)(1-\varphi^2)^{1/2} + \sum_{a=1}^{N-1}\varphi_a \boldsymbol{e}_a(x) \tag{4}$$

where $\{e_\alpha\} \equiv \{\tilde{n}(x), \boldsymbol{e}_a(x)\}$; is some orthonormal basis. The field φ_a possesses wave lengths from Λ^{-1} to $\tilde{\Lambda}^{-1}$ only. If we integrate over φ_a we get the effective lagrangian, depending on the background field.

Substituting (4) into (1) we get

$$\mathcal{L} = \frac{1}{2f}(\partial_\mu \nu^\alpha + A_\mu^{\alpha\beta}\nu^\beta)^2$$

where "gauge" field $A_\mu^{\alpha\beta}$ is given by:

$$A_\mu^{\alpha\beta} = (\boldsymbol{e}^\alpha \cdot \partial_\mu \boldsymbol{e}^\beta) = -A_\mu^{\beta\alpha} \tag{5}$$

$$\{\nu^\alpha\} = ((1-\varphi^2)^{1/2}, \varphi_a), \quad \varphi^2 = \sum_{a=1}^{N-1}(\varphi_a)^2. \tag{6}$$

After substituting of (4) into (5) we can perform the functional integration over φ-field since for small coupling the mean fluctuations of the φ-field is of the order:

$$\overline{\varphi^2} = (N-1)f\int\frac{\mathrm{d}^2k}{k^2} = \frac{(N-1)}{2\pi}f\log\frac{\Lambda}{\tilde{\Lambda}}. \tag{7}$$

Due to this condition the perturbation theory is applicable. The corresponding calculations are very simple if we assume that

$$f \ll f \log \Lambda/\widetilde{\Lambda} \ll 1 \tag{8}$$

and retain only the terms proportional to $f \log \Lambda/\widetilde{\Lambda}$. The result is of the form:

$$\mathcal{L} = \frac{1}{2\widetilde{f}}(A_\mu^{ao})^2 = \frac{1}{2\widetilde{f}}(\partial_\mu \widetilde{\boldsymbol{n}})^2 \tag{9}$$

where

$$1/\widetilde{f} = 1/f - \frac{N-2}{2\pi} \log \Lambda/\widetilde{\Lambda}. \tag{10}$$

After repeating the partial integration sufficient number of times we get the coupling constant for the momentum q:

$$f(q) = \frac{f}{1-(N-2)(1/2\pi)\log \Lambda/q}. \tag{11}$$

It is evident, that (11) is correct only for such q for which $f(q) \ll 1$.

Now let us calculate the correlation function:

$$G(R) = \langle \boldsymbol{n}(0)\, \boldsymbol{n}(R) \rangle. \tag{12}$$

From the dimensional argument

$$G(R, \Lambda) = G(\Lambda R). \tag{13}$$

Now, using (4) we get:

$$\begin{aligned} G(R,\Lambda) &\approx \langle 1 - \tfrac{1}{2}\varphi^2(0) - \tfrac{1}{2}\varphi^2(R) \rangle G(R, \widetilde{\Lambda}) \\ &= \left(1 - f\frac{(N-1)}{2\pi}\log \Lambda/\widetilde{\Lambda}\right) G(R,\widetilde{\Lambda}) . \end{aligned} \tag{14}$$

From (14) it follows the renormalization group equations:

$$\frac{\mathrm{d}}{\mathrm{d}l}\log G(l) = -\frac{N-1}{2\pi} f(l). \tag{15}$$

Here $l = \log(\Lambda R)$.

This gives the result for correlation function:

$$\langle \boldsymbol{n}(0)\, \boldsymbol{n}(R) \rangle = \left[1 - \frac{N-2}{2\pi} f \log\frac{R}{a}\right]^{(N-1)/(N-2)} \tag{16}$$

$a \equiv \Lambda^{-1}$.

As we noticed earlier, this formula is valid for:

$$1 - \frac{N-2}{2\pi} f \log R/a \gg f. \tag{17}$$

Due to the second limitation (17) we cannot answer the question whether the vacuum is degenerate and whether the phase transition take place when we are changing the coupling f. The answer depends on what happens with (16) for:

$$R \gg a \exp\left\{\frac{2\pi}{(N-2)f}\right\} \equiv R_c.$$

If there are no vacuum degeneracy (no G.P.) then

$$G(R) \propto \exp\{-R/R_c)\}. \tag{18}$$

In this case G.P. acquire the mass:

$$m = R_c^{-1} = \Lambda \exp\left\{-\frac{2\pi}{(N-2)f}\right\}.$$

Another possibility is the case of degenerate vacuum in which $G(R)$ damps more slowly then the exponent. In this case the phase transition for some coupling f exist.

It is worth to notice that the lagrangian (1) describes the isotropic planar Heizenberg ferromagnet and in this case the constant f is just the temperature. Hence the dilemma mentioned above could be resolved experimentally.

Another application of the chiral lagrangians is the massive Yang-Mills problem. It was shown by Veinstein [3] that the interaction of the most singular longitudinal polarisations of the massive Yang-Mills field is given precisely by chiral lagrangian.

For both mentioned applications it is interesting to continue the above theory in higher dimensions. If we set dimension of space equal to $2+\epsilon$, then in the leading order in

$$f(q) = f\Big/\left\{1 - \frac{N-2}{2\pi}\frac{f}{\epsilon}[\Lambda^\epsilon - q^\epsilon]\right\} \tag{19}$$

Now it is evident, that there exist the phase transition point:

$$f_c = \frac{2\pi\epsilon}{N-2}\Lambda^{-\epsilon}. \tag{20}$$

For $f < f_c$ G.P. interact weakly for all distances, and not only vacuum is degenerate but also the symmetry is violated. (The last, of course, was impossible in two dimensions). For $f > f_c$ formula (2.5) will be valid for sufficiently large q only and detailed situation is not clear. The method permits us to give the new estimates for critical exponents in 3-dimensional isotropic ferromagnet. This will be done elsewhere.

From the point of view of the massive Yang-Mills theory in 4-dimensions the result (19) (if one can trust it for $\epsilon = 2$) means the existence of the critical coupling f_c for which effective coupling $f(q) \propto q^{-2}$ for $q \to \infty$. Such a behaviour of the effective charge will provide the natural cut off for the ultraviolet divergencies, and will determine in principle all subtraction constants in the perturbation theory.

References

[1] J. Kogut and K. Wilson, Phys. Rep. 12 (1974) 75.
[2] V.L. Berezinsky and A. Blank, Zhetf 64 (1973) 725. Our results disagree with that of this authors since they have not taken into account the renormalization of temperature.
[3] A. Vainstein, Yadernaya Fizika, in print.