a) 2 countries h, f. 2 goods x, y. $i = h, f$

$$x_i = z_{xi} K_{ix}^{\alpha} l_{ix}^{1-\alpha}$$
$$y_i = z_{yi} K_{yi}^{\beta} l_{iy}^{1-\beta}$$

with $\beta > 0$, $\alpha > 0$

i) firms maximization problem. Let prices be p_{xi}, p_{yi} and r_i, w_i prices of l and k respectively

firm x

$$\max_{l,K} \; p_{xi} z_{xi} K_{ix}^{\alpha} l_{ix}^{1-\alpha} - w_i l_{ix} - r_i K_{ix}$$

foc

$$[l_{ix}] \quad (1-\alpha) p_{xi} z_{xi} K_{ix}^{\alpha} l_{ix}^{-\alpha} = w_i$$
$$[K_{ix}] \quad \alpha p_{xi} z_{xi} K_{ix}^{\alpha-1} l_{ix}^{1-\alpha} = r_i$$

firm y

$$\max_{l,K} \; p_{yi} z_{yi} K_{iy}^{\beta} l_{iy}^{1-\beta} - w_i l_{iy} - r_i K_{iy}$$

foc

$$[l_{iy}] \quad (1-\beta) p_{yi} z_{yi} K_{iy}^{\beta} l_{iy}^{-\beta} = w_i$$
$$[K_{iy}] \quad \beta p_{yi} z_{yi} K_{iy}^{\beta-1} l_{iy}^{1-\beta} = r_i$$

ii)

$$\frac{\alpha}{1-\alpha} K_{ix}^{-1} l_{ix}^{1} = \frac{r_i}{w_i}$$
$$\Leftrightarrow \frac{\alpha}{1-\alpha} \frac{l_{ix}}{K_{ix}} = \frac{r_i}{w_i}$$
$$\Leftrightarrow \frac{\alpha}{1-\alpha} \frac{w_i}{r_i} = \frac{K_{ix}}{l_{ix}}$$

Similarly we get

$$\frac{\beta}{1-\beta} \frac{w_i}{r_i} = \frac{K_{iy}}{l_{iy}}$$

iii) we know that in equilibrium firms make 0 profits therefore

$$p_{xi} \, z_{xi} \, k_{ix}^{\alpha} \, l_{ix}^{1-\alpha} - w_i l_{ix} - r_i k_{ix} = 0$$

$$\rightarrow p_{xi} = \frac{w_i l_{ix} + r_i k_{ix}}{z_{xi} \, k_{ix}^{\alpha} \, l_{ix}^{1-\alpha}}$$

on the numerator we have

$$(w_i l_{ix} + r_i k_{ix}) \frac{l_i}{l_i}$$

$$w_i l_{ix} + r_i \frac{k_{ix}}{l_{ix}} l_i$$

$$w_i l_{ix} + r_i \left(\frac{\alpha}{1-\alpha} \frac{w_i}{r_i}\right) l_i$$

$$w_i l_{ix} + \frac{\alpha}{1-\alpha} w_i l_i$$

$$w_i l_{ix} \left(1 + \frac{\alpha}{1-\alpha}\right)$$

$$\frac{w_i l_{ix}}{1-\alpha}$$

also we know that

$$x_i = z_{xi} \, k_{ix}^{\alpha} \, l_{ix}^{1-\alpha} \cdot \frac{l_{ix}^{\alpha}}{l_{ix}^{\alpha}}$$

$$x_i = z_{xi} \left(\frac{k_i}{l_i}\right)^{\alpha} l_{ix}$$

$$x_i = z_{xi} \left(\frac{\alpha}{1-\alpha} \frac{w_i}{r_i}\right)^{\alpha} l_{ix}$$

$$p_{xi} = \frac{\dfrac{w_i l_{ix}}{1-\alpha}}{z_{xi} \left(\frac{\alpha}{1-\alpha} \frac{w_i}{r_i}\right)^{\alpha} l_{ix}}$$

$$p_{xi} = \frac{w_i l_{ix}}{(1-\alpha) z_{xi} \left(\frac{\alpha}{1-\alpha} \frac{w_i}{r_i}\right)^{\alpha} l_{ix}}$$

$$p_{xi} = \frac{w_i}{(1-\alpha) z_{xi} \left(\frac{\alpha}{1-\alpha} \frac{w_i}{r_i}\right)^{\alpha}}$$

Similarly

$$p_{yi} = \frac{w_i}{(1-\beta) z_{yi} \left(\frac{\beta}{1-\beta} \frac{w_i}{r_i}\right)^{\beta}}$$

Then

$$\frac{p_{xi}}{p_{yi}} = \frac{\dfrac{w_i}{(1-\alpha) Z_{xi} \left(\frac{\alpha}{1-\alpha} \frac{w_i}{r_i}\right)^{\alpha}}}{\dfrac{w_i}{(1-\beta) Z_{yi} \left(\frac{\beta}{1-\beta} \frac{w_i}{r_i}\right)^{\beta}}}$$

$$\frac{p_{xi}}{p_{yi}} = \frac{(1-\beta) Z_{yi} \left(\frac{\beta}{1-\beta} \frac{w_i}{r_i}\right)^{\beta}}{(1-\alpha) Z_{xi} \left(\frac{\alpha}{1-\alpha} \frac{w_i}{r_i}\right)^{\alpha}}$$

iv) What forces explain comparative advantages?

It depends on mainly 2 things, first which good uses one factor intensively this is (α, β) levels, and which factor is more abundant (K, L, endowments).

v) By FPE theorem by Samuelson, yes as long as they share the same Z and FIR doesn't occur.