1.-

i) Household

$$C = \left[\sum_{i=1}^{N} C_i^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

Production

$$\frac{P_i}{w} = \left(\frac{1}{\varphi} + \frac{f}{q_i}\right) \qquad \frac{P_i}{w} = \frac{\sigma}{\sigma-1}\frac{1}{\varphi}$$

Mkt clearing

$$q = (L + L^{*})C$$

$$q = 2LC$$

$$\frac{\sigma}{\sigma-1}\frac{1}{\varphi} = \frac{1}{\varphi} + \frac{f}{q_i}$$

$$\frac{\sigma}{\sigma-1}\frac{1}{\varphi} = \frac{1}{\varphi} + \frac{f}{2LC}$$

$$C_i^{T} = \frac{f\varphi(\sigma-1)}{2L}$$

$$q^{T} = f\varphi(\sigma-1)$$

$$q_1 = LC_1 + L^{*}C_1$$

$$\frac{q_1}{L^{*}C_1} = \frac{LC_1}{L^{*}C_1} + 1 \quad \Rightarrow \quad \frac{q_1}{L^{*}C_1} = 2$$

$$1 = \frac{LC_1 + L^*C_1}{q_1} \rightarrow 1 - \frac{LC_1}{q_1} = \frac{L^*C_1}{q_1}$$

$$1 - \frac{L\left(\frac{f\varphi(\sigma-1)}{2\ell}\right)}{f\varphi(\sigma-1)} = \frac{L^*C_1}{q_1} = 1/2$$

ii)

$$\ell_1 = \frac{q_1}{\varphi} + f$$

$$L = \sum_{i=1}^{N} \ell_i$$

$$L = N\ell_i \Rightarrow L = N\left(\frac{q_i}{\varphi} + f\right) \quad ; \quad L = N\left(\frac{f\varphi(\sigma-1)}{\varphi} + f\right)$$

$$L = N(f((\sigma-1)+1))$$

$$N = \frac{L}{f\sigma}$$ if $\uparrow\sigma \rightarrow$ N increases

$\uparrow\sigma$ means that the household can substitute better between varieties
This reduces firm mark up, making some leave the market before $\Pi = 0$

$$\frac{P_i}{W} = \frac{\sigma}{\sigma-1}\frac{W}{\varphi}$$ $\uparrow\sigma \rightarrow$ Less markup $\downarrow p_i$

$$P = \left[\sum_{i=1}^{n} P_i^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \qquad P_i = P \ \forall i \rightarrow P\left[\sum_{i=1}^{n} P^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$$

$$(NP^{1-\sigma})^{1/1-\sigma}$$

$$N^{1/1-\sigma}P$$

$\uparrow \sigma \Rightarrow \downarrow p$

$\uparrow \sigma$ makes mark up smaller, this makes the price of the firms smaller. This means an increase in house hold welfare because they can consume more

iii)

$q_i = (L + L^*) c_i$

$\Rightarrow q_i = f\varphi(\sigma - 1) = (L + L^*) c_i \Rightarrow c_i = \dfrac{f\varphi(\sigma-1)}{L + L^*}$

if

$q_i =$ output and $L^* c_i =$ exports

Then $= \dfrac{\dfrac{L^*(f\varphi)(\sigma-1)}{L+L^*}}{f\varphi(\sigma-1)} = \dfrac{L^*}{L + L^*} > \dfrac{1}{2}$