Problem of the Month

# Distributing Books

11/2010

A teacher has three (3) different books to distribute among her twelve (12) students in an English literature class. In how many ways can she give the books to her students? Assume that a student may have more than one book.

SUBMIT your solution to

- Dr. Erol Akbas @ matexa@langate.gsu.edu **or**
- Dr. Yuanhui Xiao @ matyxx@langate.gsu.edu

before the **deadline**: **Friday, November 26, 2010, 5:00PM**.

You may get a copy of this problem from **the wall behind you**.

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## Problem of Last Month: Round Robin Tournament

*In a round-robin tournament, each team plays with all of the other teams exactly once.*

*Consider the following round-robin tournament setup:*

- *$N$ teams play a round-robin and exactly one team is eliminated from further play.*
- *The remaining $N-1$ teams play another round-robin tournament. And then a second team is eliminated.*
- *Round-robin tournaments continue, with one team eliminated at the conclusion of each round-robin, until one team (the champion) remains.*

*What percentage of total games played in the multiple round-robin tournaments does the champion play?*

## Winner: Jeremy Brown.

## Participant(s) with Correct Solution: Reimbay Reiimbayev.

**Solution**. Let $T_i$ be the $i^{th}$ triangular number $\frac{i(i+1)}{2}$. When there are $i$ teams in a round, the total number of games is $T_{i-1}$, and the champion has to play $i-1$ games. Thus, finally, there are

$$\sum_{i=2}^{N} T_{i-1} = \frac{1}{2}\left[\sum_{i=1}^{N-1} i^2 + \sum_{i=1}^{N-1} i\right] = \frac{1}{2}\left[\frac{(N-1)N(2N-1)}{6} + \frac{N(N-1)}{2}\right] = \frac{N(N-1)(N+1)}{6}$$

games and the champion has to play

$$1 + 2 + \ldots + (N-1) = T_{N-1} = \frac{N(N-1)}{2}$$

games. Consequently, the percentage of total games played in the multiple round-robin tournaments is

$$100 \times \frac{T_{N-1}}{\sum_{i=2}^{N} T_i}\%. = 100 \times \frac{N(N-1)/2}{N(N-1)(N+1)/6}\% = 100 \times \frac{3}{N+1}\%$$