Product

Find the following product.

$$\sqrt{72+\sqrt{72+\sqrt{72+\sqrt{72+\sqrt{\ldots}}}}}\cdot\sqrt{56-\sqrt{56-\sqrt{56-\sqrt{56-\sqrt{\ldots}}}}}=?$$

- Please **Submit** your solution to
  - *Dr. Erol Akbas,* eakbas@gsu.edu or
  - *Dr. Tirtha Timsina,* ttimsina@gsu.edu

  before the deadline: **Monday, October 31st , 7:00PM.**
- The WINNER will be awarded with a $25 gift certificate and will be announced in the NEXT issue.

Problem of the last month: Real Roots

Let C denotes the set of complex numbers and let $f:C\to C$ be a function defined as $f(x)=(x-i)^{10}+(x+i)^{10}$. Show that f has a real zero.

Solution: The fundamental theorem of algebra states that every non-constant single-variable polynomial with complex coefficients has at least one complex root.

Let $f(x)=0 \Leftrightarrow (x-i)^{10}=-(x+i)^{10} \Rightarrow \|(x-i)^{10}\|=\|-(x+i)^{10}\| \Leftrightarrow$

$\|(x-i)^{10}\|=\|(x+i)^{10}\| \Leftrightarrow \|x-i\|^{10}=\|x+i\|^{10} \Leftrightarrow \|x-i\|=\|x+i\|.$

Let $x=a+bi$ where a, b are real numbers.

$\Rightarrow \|(a+bi-i)\|=\|(a+bi+i)\| \Leftrightarrow \|(a+bi-i)\|^2=\|(a+bi+i)\|^2$

$\Leftrightarrow a^2+(b-1)^2=a^2+(b+1)^2 \Leftrightarrow a^2+b^2-2b+1=a^2+b^2+2b+1$

$\Leftrightarrow -2b=2b \Leftrightarrow b=0$

$\Rightarrow$ Any zero of the polynomial $f(x)=(x-i)^{10}+(x+i)^{10}$ has to be a real number.

Winner: Daniel Balena

Participants with correct solutions: Daniel Balena, John Hull, Wenyan Zhou