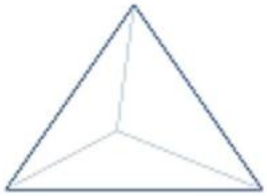
Problem of the Month

Car Crashing?

Februrary 2010



Imagine a planet in the shape of a regular tetrahedron (its surface consists of 4 equilateral triangles). Suppose that on each face there is a car traveling at a constant speed in clockwise direction along the edges bounding the face. Can they travel without crashing?

SUBMIT your solution to

- Dr. Erol Akbas @ matexa@langate.gsu.edu **or**
- Dr. Yuanhui Xiao @ matyxx@langate.gsu.edu

before the **deadline**: **Friday, Februrary 26, 2010, 5:00PM**.

You may get a copy of this problem from **the wall behind you**.

Problem of Last Month: An Equation

Let x be a real number and $frac(x) = x - [|x|]$ (fractional part of x) where $[|x|]$ is the greatest integer function. For example: $frac(5.87) = 0.87$ or $frac(\frac{7}{5}) = \frac{2}{5}$ or $frac(-\frac{1}{3}) = \frac{2}{3}$.

Find a positive real number x such that $frac(x) + frac(\frac{1}{x}) = 1$.

Winner: Reimbay Reiimbayev.

Participant(s) with Correct Solution: Robert Xu, Donald Davis.

Solution. We can safely exclude 0 and 1 as solutions. Suppose that $n = [|1/x|]$ and $y = frac(1/x)$. If we look for a solution x in $(0,\ 1)$, then $frac(x) = x$. So,

$$x + y = x + 1/x - n = 1.$$

Doing some algebra would result in the following quadratic equation

$$x^2 - (n+1)x + 1 = 0,$$

which has the solutions

$$x = \frac{n+1 \pm \sqrt{(n+3)(n-1)}}{2}, \quad n = 2, 3, \ldots.$$

A solution is

$$\frac{2+1-\sqrt{(2+3)(2-1)}}{2} = \frac{3-\sqrt{5}}{2} = 0.381966011\ldots$$