Consider infinitely many two states switches S_i and infinitely many bulbs B_i, where i ranges over the positive integers. The switches control the bulbs in an unknown manner; however for every integer n there exists a combination of the switches S_i which turns on the n first bulbs (and possibly others). Show that there exists a combination of the switches S_i which turns on simultaneously all the bulbs .

- Please **Submit** your solution to
  - *Dr. Tirtha Timsina,* ttimsina@gsu.edu or
  - *Dr. Christian Avart,* cavart@gsu.edu

  before the deadline: **Friday, January 28th, 7:00PM.**
- The WINNER will be awarded with a $15 gift certificate and will be announced in the NEXT issue.

Solution to the November Problem of the Month: Let the line $y = mx + b$ intersect the parabola $y = kx^2$ at two points A and B. Let C be the point on the parabolic arc AB that is furthest from the line $y = mx + b$.
Find the ratio of the area of the region bounded by the parabola and the line, and the area of the triangle ABC.
⇒Let the line and the parabola intersect at $x = x_1$ and $x = x_2$ respectively. Then the area of the bounded region between the line and the parabola is $A_1 = \int_{x_1}^{x_2}(mx + b - kx^2)dx$. After rigorous algebra and simplification the integral simply becomes
$A_1 = \int_{x_1}^{x_2}(mx + b - kx^2)dx = \frac{k}{6}(x_2 - x_1)^3$.
Now, let the vertices of the triangle ABC be $A(x_1, kx_1^2)$, $B(x_2, kx_2^2)$, $C(x, kx^2)$ (here, A and B are the intersection points and C is any point on the parabolic arc). Area of triangle ABC is then given by A=$\left|det\begin{pmatrix} x_1 & x_2 & x \\ kx_1^2 & kx_2^2 & kx^2 \\ 1 & 1 & 1 \end{pmatrix}\right| =$
$1/2|kx_1x_2^2 - kx_2x_1^2 - kx_1x^2 + kxx_1^2 + kx_2x^2 - kxx_2^2|$.

We want to maximize the area by setting $\frac{dA}{dx} = 0$ and solving for x. This gives us $x = \frac{x_1+x_2}{2}$.
Using this x value in the formula for area of the triangle ABC, we get $A_2 = (\frac{k}{8}(x_2 - x_1)^3)$.
Therefore, $\frac{A_1}{A_2} = \frac{8}{6} = \frac{4}{3}$

Winner: Joseph Moravitz