Rational Point: Show that in the xy-plane, for odd integers A, B and C, the line $Ax + By + C = 0$ does not intersect the parabola $y = x^2$ in a rational point.

- Please **Submit** your solution to
  - *Dr. Erol Akbas,* eakbas@gsu.edu or
  - *Dr. Tirtha Timsina,* ttimsina@gsu.edu

  before the deadline: **Wednesday, November 30th, 7:00PM.**
- The WINNER will be awarded with a $25 gift certificate and will be announced in the NEXT issue.

Problem of the last month:

Product: Find the following product. $\sqrt{72+\sqrt{72+\sqrt{\ldots}}}\cdot\sqrt{56-\sqrt{56-\sqrt{\ldots}}} = ?$

Solution: Let $a_1 = \sqrt{72}$ and $a_{n+1} = \sqrt{72+a_n}$ for $n \geq 1$. Notice that $\lim_{n\to\infty} a_{n+1} = \sqrt{72+\sqrt{72+\sqrt{\ldots}}}$. It is clear that $8 < a_1 < a_2 < \ldots < a_n < a_{n+1} < \ldots$.

Claim: $a_n < 9$ for $n \geq 1$.

Proof of Claim: By induction, for $n = 1$, $a_1 = \sqrt{72} = \sqrt{9(9-1)} < 9$. Assume that for $n = k$, $a_k < 9$.

Then $a_{k+1} = \sqrt{72+a_k} = \sqrt{9(9-1)+a_k} = \sqrt{9^2 + a_k - 9} < 9$ since $a_k - 9 < 0$.

So by induction, $8 < a_1 < a_2 < \ldots < a_n < a_{n+1} < \ldots < 9$. Since $\{a_n\}$ is a monotone, increasing, bounded sequence, $\lim_{n\to\infty} a_n = L$ for some $L \leq 9 \quad\Rightarrow\quad L = \lim_{n\to\infty} a_n = \lim_{n\to\infty} a_{n+1} = \lim_{n\to\infty}\sqrt{72+a_n} = \sqrt{72+L} \quad\Rightarrow$

$L = \sqrt{72+L} \quad\Rightarrow\quad L^2 - L - 72 = 0 \quad\Rightarrow\quad L = 9$. By a similar argument, $\sqrt{56-\sqrt{56-\sqrt{\ldots}}} = 7$

$\Rightarrow \quad \sqrt{72+\sqrt{72+\sqrt{\ldots}}}\cdot\sqrt{56-\sqrt{56-\sqrt{\ldots}}} = 9\cdot 7 = 63$.

Winner: Thomas Polstra

Participants with correct solutions: Thomas Polstra, Max Suica, Wenyan Zhou, Shadi Renno, John Hull, David Lim, Joshua Tomy, Ajene Ennis, Daniel Balena.