Let $f : [0,1] \to [0,1]$ be any continuous function with the property that $f(0) = f(1)$. Prove that for every positive integer n there exists $x_n \in [0,1]$ satisfying $f(x_n) = f(x_n + 1/n)$.

Please submit your solution to:

- Dr. Tirtha Timsina, ttimsina@gsu.edu
- Dr. Christian Avart, cavart@gsu.edu

before the deadline: September 28th, 7:00PM. The WINNER will be awarded with a \$15 gift certificate and will be announced in the NEXT issue.

Solution to the April Problem of the Month:

Show that $X^4Y^2Z^2 + X^2Y^4Z^2 + X^2Y^2Z^4 - 4X^2Y^2Z^2 + 1 \geq 0$ **for all** $X, Y, Z \in \mathbb{R}^3$**. The simplest proof applies the fact that arithmetic mean is always greater than or equal to geometric mean ie** $AM \geq GM$ **for any non-negative numbers. Clearly** $X^4Y^2Z^2, X^2Y^4Z^2, X^2Y^2Z^4, 1$ **are non-negative. Their arithmetic mean is**

$\dfrac{X^4Y^2Z^2 + X^2Y^4Z^2 + X^2Y^2Z^4 + 1}{4}$ **and their geometric mean is**

$\sqrt[4]{X^4Y^2Z^2 \times X^2Y^4Z^2 \times X^2Y^2Z^4 \times 1} = X^2Y^2Z^2$**.**

Since $AM \geq GM$**, we will have**

$$\frac{X^4Y^2Z^2 + X^2Y^4Z^2 + X^2Y^2Z^4 + 1}{4} \geq X^2Y^2Z^2$$

or $X^4Y^2Z^2 + X^2Y^4Z^2 + X^2Y^2Z^4 + 1 \geq 4X^2Y^2Z^2$

or $X^4Y^2Z^2 + X^2Y^4Z^2 + X^2Y^2Z^4 - 4X^2Y^2Z^2 + 1 \geq 0$ **for all** $X, Y, Z \in \mathbb{R}^3$**.**

Winner: Joseph Moravitz.