Averaging Digits March 2010

How many four-digit numbers are composed of four distinct digits (no leading 0s), such that **one digit is the average of the other three**? Examples of such numbers: 3621 (3 is the average of 6, 2 and 1), 5210 (2 is the average of 5, 1 and 0).

SUBMIT your solution to

- Dr. Erol Akbas @ matexa@langate.gsu.edu **or**
- Dr. Yuanhui Xiao @ matyxx@langate.gsu.edu

before the **deadline**: **Friday, March 26, 2010, 5:00PM**.

You may get a copy of this problem from **the wall behind you**.

Problem of Last Month: Car Crashing?

Imagine a planet in the shape of a regular tetrahedron (its surface consists of 4 equilateral triangles). Suppose that on each face there is a car traveling at a constant speed in clockwise direction along the edges bounding the face. Can they travel without crashing?
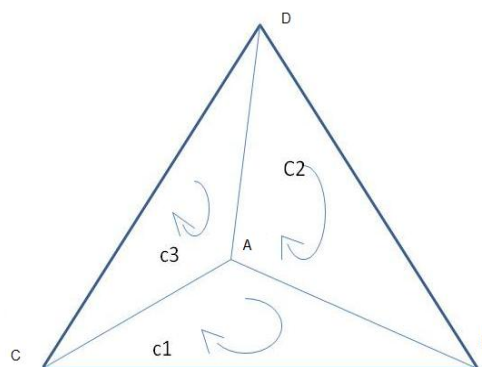
Winner: __Robert Xu__.

Solution.

Denote the vertices of the tetrahedron by A, B, C and D as in the above picture. Let c_1, c_2, c_3, c_4 be four cars traveling at constant speeds on the paths ABC, ADB, ACD and BDC respectively. Without loss of generality, assume that c_1 has the smallest speed. Examine the moment when c_1 is passing through the point A traveling toward the point B. The car c_2 can not be on AB because of unavoidable collision. It can not be on DB either because its speed is greater than or equal to speed of c_1. So the car c_2 has to be traveling on AD. Similarly c_3 can not be on AC and CD (running the cars in reverse direction proves this). In order to avoid collision with c_1, both c_2 and c_3 have to be on AD. Then collision between c_2, c_3 will be unavoidable. So this implies that even if there are three cars traveling with constant speed, collision is inevitable.