Problem of the Month – March

Let $[n] = \{1, 2, \ldots, n\}$. For two sets A and B, we write $A \subset B$ if A is a proper subset of B, namely $A \subseteq B$ and $A \neq B$. Determine the number of 3-*chains* $\emptyset \subset A \subset B \subset [n]$. In other words, determine the number of proper non-empty subsets A and B of $[n]$ such that A is a proper subset of B. For example, $\emptyset \subset \{1\} \subset \{1, 2\} \subset [n]$ is a 3-Chain.

You answer should be a simple function of n instead of a complicated summation. Hint: you may want to determine the number of 2-chains $\emptyset \subset A \subset [n]$ first and use a similar idea for 3-chains.

Deadline: March 31, 2009, 5:00pm.

- You may get a copy of this from the wall behind you.
- Solution and Problem of April will be posted by April 10, 2009.
- Submit your solution to Dr. Yi Zhao by *yzhao6@gsu.edu* or drop a hard copy in his mailbox before the deadline.

The problem of January:

Problem: Evaluate $\sum_{n=1}^{\infty} \frac{n}{2^{n-1}}$ *exactly.*

Solution: We know that geometric series converges:

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$$

for any x with $0 < |x| < 1$. We differentiate both sides obtaining

$$\sum_{n=1}^{\infty} n x^{n-1} = \frac{1}{(1-x)^2}.$$

Now with $x = \frac{1}{2}$, we have $\sum_{n=1}^{\infty} \frac{n}{2^{n-1}} = \frac{1}{(1-1/2)^2} = 4$.

Winner: Anthony Angelucci.