There are N light bulbs lined up in a row in a room. Each bulb has its own switch and they are all currently switched off. The room has separate entrance and exit doors. There are N students lined up outside the door. Each bulb and each student is numbered consecutively from 1 to N.
Student 1 enters the room, turns on all the bulbs and exits. Student 2 enters and flips the switch on every second bulb (turning off 2,4,6,8 and so on) and exits. Student 3 enters and flips the switch on every third bulb (changing the state on bulb 3,6,9,12 and so on). This continues until all students have passed through the room. If there were total of 100 students, which bulbs are still illuminated after the last person passed through the room?

Please submit your solution to:

- Dr. Tirtha Timsina, ttimsina@gsu.edu
- Dr. Christian Avart, cavart@gsu.edu

before the deadline: October 28th, 7:00PM. The WINNER will be awarded with a certificate and will be announced in the NEXT issue.

Solution to the September Problem of the Month:

Let $f : [0, 1] \to [0, 1]$ be any continuous function with the property that $f(0) = f(1)$. Prove that for every positive integer n there exists $x_n \in [0, 1]$ satisfying $f(x_n) = f(x_n + 1/n)$.

For $n = 1$, this follows from the assumption $f(0) = f(1)$. Fix an integer n greater than 1 and consider the function $g(x) = f(x) - f(x + \frac{1}{n})$. If on the interval $[0, \frac{n-1}{n}]$ the function g has a zero c, then $g(c) = 0$, i.e, $f(c) = f(c + \frac{1}{n})$ and we are done.

We now assume $g(x)$ has no zero on $[0, \frac{n-1}{n}]$ and proceed by contradiction. In particular $g(0), g(1), \cdots g(\frac{n-1}{n})$ are all non zero. But it easy to check that $g(0) + g(1) + \cdots + g(\frac{n-1}{n}) = 0$, and since all the terms are non zero, one (at least) must be negative and one (at least) must be positive. Since g is continuous, the intermediate value theorem implies that g must have a 0 somewhere in the interval $[0, \frac{n-1}{n}]$ contradicting our assumption.

Winner: none.
