
Let S be a finite set and $f : S \to S$ be any map. We write f^n for the map obtained by composing f with itself n times. Show that there exists an integer k such that $f^{2k} = f^k$.

Please submit your solution to:

- Dr. Tirtha Timsina, ttimsina@gsu.edu
- Dr. Christian Avart, cavart@gsu.edu

before the deadline: March 28th, 7:00PM. The WINNER will be awarded with a \$15 gift certificate and will be announced in the NEXT issue.

Solution to the February Problem of the Month:

Without using derivative find the exact minimum value of the function $\dfrac{(9x^2\sin^2 x) + 4}{x\sin x}$ for $0 < x < \pi/2$ and show that this minimum value is attainable in the given interval.

We can approach this problem couple of ways. Here, we will use the fact that $Arithmetic\ Mean(AM) \geq Geometric\ Mean(GM)$.

Let $y = x\sin x$. Then the problem becomes $\dfrac{9y^2+4}{y} = 9y + \dfrac{4}{y}$. Let two numbers be $9y$ and $\dfrac{4}{y}$. The geometric mean of these two numbers will be $\sqrt{(9y)\dfrac{4}{y}} = 6$ and arithmetic mean will be $\dfrac{9y^2+4}{2y}$. Since $AM \geq GM$, we have

$\dfrac{9y^2+4}{2y} \geq 6$, or $9y^2 - 12y + 4 \geq 0$ or $(3y-2)^2 \geq 0$. Equality holds when $y = \dfrac{2}{3}$. This leads to the minimum value of the given function to be 12.

Since $y = x\sin x$, we now need to show that there is $0 < x < \pi/2$ such that $x\sin x = \dfrac{2}{3}$. This can be easily shown using the intermediate value theorem for the function $f(x) = x\sin x - \dfrac{2}{3}$.

Winner: Joseph Moravitz.
