# Real-Time DSP Design & Applications Exams

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## 2019 - 2020

### 1.

**(a)**

(i)

8

23

(ii)

dynamic range $= 20\ \log_{10}\frac{(-1)^0\times2^{254-127}\times(2-2^{-23})}{(-1)^0\times2^{1-127}\times1} = 1529.232$

(iii)

$(-1)^0 \times 2^{133-127} \times 1.5625 = 100$

(iv)

$(-1)^0 \times 2^{133-127} \times 1.5625 + (-1)^0 \times 2^{131-127} \times 1.25$

$= (-1)^0 \times 2^6 \times 1.5625 + (-1)^0 \times 2^4 \times 1.25$

$= (-1)^0 \times 2^6 \times 1.875$

$0 \mid (133)_{10} \mid (0.875)_{10}$

**(b)**

(i)

in Part 1 P90

$\eta_{\text{total}} = H(e^{j0})\eta_x + \eta_0 + \eta_1 + ... + \eta_{99} + \eta_y = -1.5625Q$

rounding => $\eta_x = 0,\ \eta_y = 0$

$\eta_i = -\frac{\frac{Q}{2^5}}{2}$

(ii)

$\sigma^2_{\text{total}} = ||H(e^{j\omega})||^2_2\sigma^2_x + \sigma^2_0 + \sigma^2_1 + ... + \sigma^2_{99} + \sigma^2_y = 0.133Q^2$

$\sigma^2_x = \sigma^2_y = \frac{Q^2}{12}$

$||H(e^{j\omega})||^2_2 = \frac{1}{2}$

$\sigma^2_i = \frac{(\frac{Q}{2^5})^2}{12}$

(iii)

$Q = 2^{-4}$

$\eta_{\text{total}} = -0.0977,\ \sigma^2_{\text{total}} = 0.00052$

(iv)

remains same

**2.**

**(a)**

(i)

end of Part 2

number of utilized stages $= 2$

maximum throughput $\leq \frac{1}{2}$

(ii)

forbidden set $= (0, 1, 3)$

scheduling strategy $= 0, 2, 4, 6...$

(iii)

latency $= 4$

Average initiation $= 2$ clocks/sample

Average throughput $= \frac{1}{2}$

**3.**

**(a)**

(i)

loop bound $= \frac{3+0+10}{1} = 13$

(ii)

critical path $= 14$

(iii)

critical path $= 13$

**5.**

**(a)**

in Part 5 P5

$L = 32,\ B = 128$

Processing time $T_p = [5 + 128 \times (5 + 32)]$ clock cycles $\times$ 10 ns = 47410 ns

$T_p \leq BT_s$

$T_s \geq 370.4$ ns

$f_s \leq 2.699$ MHz

## 2020 - 2021

### 1.

#### (a)

(i)

$-5.25$

(ii)

$(3)_{10} = (0011.000)_{2'}$

sum $= 11101.110$

No overflow

(iii)

$-5.25 - 3 = -8.25 < -8$

There is overflow

(iv)

$a = 1111010.11000$

#### (b)

(i)

$$\text{SQNR} = 10\ \log(\frac{3\ \sigma_x^2\ 2^{2n}}{V_{max}^2}) \geq 40$$

$n = 8$

(ii)

$\text{SQNR} = 43.73$ dB

$Q = \frac{V_{\max} - V_{\min}}{2^n} = 0.039$ volts

(iii)

$V_{\max} = 3$

$\frac{3}{Q} = 76.8$

For mid-rise quantizer, $nQ \sim (n+1)Q => (n+0.5)Q$

$76.5Q = 2.988$ volts

**2.**

**(a)**

(i)

$f_{\max} = 700$ Hz

$f_s \geq 2f_{\max} = 1400$ Hz

(ii)

in Part 2 P23

$f_c = 600$ Hz, $B = 200$ Hz

$\frac{2f_c + B}{m+1} = \frac{1400}{m+1},\ \frac{2f_c - B}{m} = \frac{1000}{m}$

No aliasing when $m = 2$

(iii)

$f_{s2} = \frac{1200}{n}$

**(b)**

(ii)

$4 + 6z^{-1} + 10z^{-2} + 15z^{-3} = (2 + 3z^{-1})(2 + 5z^{-2})$

Critical path $2T_M + 2T_A$, which is more than the earlier critical path $T_M + 3T_A$ since typically $T_M > T_A$

**3.**

**(a)**

(i)

$a_n = b_n = 1$

(ii)

$a_n = 1,\ b_n = 0 \quad \text{or} \quad a_n = 0,\ b_n = 1$

(iii)

in Part 3

**(b)**

in Part 5 P8

Minimum MIPS rating $= \frac{3300 \times 4200 \times 3 \times 9}{10^6} = 374.22$ MIPS

$\frac{500 \times 10^6}{3300 \times 4200 \times 3} = 12.025$ instructions/outputpixel/color

9 instructions/output pixel/color are required for filtering

overhead $= 12.025 - 9 = 3.025$ instructions/outputpixel/color

**4.**

**(a)**

40 bits

LO(ACx) = low bits, bits 15-0

HI(ACx) = high bits, bits 31-16

**5.**

**(b)**

(ii)

$T_s = 20$ ns

critical path $= 31$ ns

Not adequate

(iii)

$2 \times \frac{1}{31} > 50$

minimum block size $= 2$

$f_c = \frac{f_s}{2} = 25$ MHz

## 2021 - 2022

### 1.

**(a)**

(i)

0011.110

0111.111

1000.000

(ii)

$3.75 = 1.875 \times 2 = (-1)^0 \times 2^{128-127} \times 1.875$

S = 0

Exp = 128 = 1000 0000

D = 0.875 = 111 0000 0000 0000 0000 0000

Largest normalized number $= (-1)^0 \times 2^{254-127} \times (2 - 2^{-23}) = 3.403 \times 10^{38}$

**(b)**

(i)

in Part 1 P84, 74

$\eta_v = H(e^{j0})\eta_e = -\frac{Q}{2}$

$\sigma_v^2 = (\int_{-\pi}^{\pi} |H(e^{j\omega})|^2 \frac{d\omega}{2\pi})\sigma_e^2 = \frac{3}{4}\frac{Q^2}{12} = \frac{Q^2}{16}$

(ii)

$\eta_w = G(e^{j0})\eta_v = 0$

$\sigma_w^2 = (\int_{-\pi}^{\pi} |H(e^{j\omega})G(e^{j\omega})|^2 \frac{d\omega}{2\pi})\sigma_e^2 = \frac{1}{4}\frac{Q^2}{12} = \frac{Q^2}{48}$

(iii)

in Part 1 P84, 74

$$S_v(e^{j\omega}) = |H(e^{j\omega})|^2 S_e(e^{j\omega}) = \begin{cases} \frac{Q^2}{12} & 0 \leq \omega \leq \frac{3\pi}{4} \\ 0 & \frac{3\pi}{4} \leq \omega \leq \pi \end{cases}$$

$$S_w(e^{j\omega}) = |G(e^{j\omega})|^2 S_v(e^{j\omega}) = \begin{cases} \frac{Q^2}{12} & \frac{\pi}{2} \leq \omega \leq \frac{3\pi}{4} \\ 0 & \text{otherwise} \end{cases}$$

**2.**

**(a)**

(i)

$$\text{SQNR} = 10\ \log(\frac{3\ \sigma_x^2\ 2^{2n}}{V_{max}^2}) = 6.02n + 4.77 + 20\ \log\tfrac{\sigma}{V_{\max}}$$

$= 6.02n - 3.01 \geq 23$

$n = 5$

(ii)

$\sigma^2_{e,new} = \frac{1}{2}\sigma^2_{e,old}$

$$\text{SQNR} = 10\ \log(\frac{\sigma_x^2}{\sigma_e^2}) = 6.02n + 4.77 + 20\ \log\tfrac{\sigma}{V_{\max}}$$

$= 6.02n - 3.01 + 3.01 \geq 23$

$n = 4$

**3.**

**(a)**

(ii)

$s_0 : 3$ ns, $\quad s_1 : 5$ ns, $\quad s_2 : 7$ ns

$c_0 : 4$ ns, $\quad c_1 : 6$ ns, $\quad c_2 : 8$ ns

(iii)

No

**(b)**

(i)

critical path = 15

(iii)

in Part 4

critical path = 8

latch = 1

**4.**

**(a)**

(i)

$T_s = \frac{1}{f_c} = 100$ ns

$LT_s \geq T_c$

$L = 2$

(ii)

$f_s = 13.33$ MHz

**5.**

**(a)**

(i)

in Part5 P35

MOV AC0, AC1

or MOV #11, AC0

or MOV *AR0, AC0

(ii)

MOV XAR0, XAR1

or MOV AC0, XAR1

or MOV XAR0, AC0

or MOV dbl(*XAR0), XAR1

(iii)

MOV AR0, AR1

and many other cases

(iv)

Source bits are sign extended, e.g. MOV AR0, AC0. However, if source has 23 bits, then zero filled, e.g. MOV XAR0, AC0.

**(b)**

$$\begin{aligned}&\mathrm{XAR0} = 00\_0100\mathrm{h},\;\; \mathrm{XAR1} = 00\_0300\mathrm{h},\;\; \mathrm{XAR2} = 00\_0300\mathrm{h}\\ &\quad \mathrm{XAR0} = 00\_0100\mathrm{h},\;\; \mathrm{XAR1} = 00\_0301\mathrm{h},\;\; \mathrm{XAR2} = 00\_0200\mathrm{h}\\ &\quad \mathrm{XAR0} = 00\_0102\mathrm{h},\;\; \mathrm{XAR1} = 00\_0301\mathrm{h},\;\; \mathrm{XAR2} = 00\_0201\mathrm{h}\end{aligned}$$