NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2020-2021

EE7403 – IMAGE ANALYSIS AND PATTERN RECOGNITION

April / May 2021 Time Allowed: 3 hours

INSTRUCTIONS

1. This paper contains 5 questions and comprises 4 pages.
2. Answer all 5 questions.
3. All questions carry equal marks.
4. This is a closed book examination.
5. Unless specifically stated, all symbols have their usual meanings.

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1. A digital greyscale image has the image histogram $P_{of}(f)$ given by

$$P_{of}(f) = 0.2\delta[f-0.7] + 0.1\delta[f] + 0.3\delta[f-0.8] + c\delta[f-1] + 0.2\delta[f-0.9],$$

where f is the grey level, and c is a positive real number.

(a) Determine the minimal and the maximal grey levels, f_{min}, f_{max}, of the image. How many different grey levels does the image have?

(5 Marks)

(b) Plot the histogram $P_{of}(f)$.

(5 Marks)

(c) A gamma correction $g = f^2$ is applied to the image. Compute and plot the histogram of the gamma-corrected image $P_{\gamma g}(g)$.

(5 Marks)

(d) Histogram equalization is applied to correct the image. Compute and plot the histogram of the histogram-equalized image $P_{heg}(g)$.

(5 Marks)

2. (a) Some expressions that relate to binary morphological image processing are shown in Table 1 and Table 2 below. Copy the Answer Table in Table 3 and complete it with letters a, b, c, d, e and f to associate each expression in Table 1 with that in Table 2.

Table 1

1	$A \circ B$
2	$A \oplus B$
3	$A \bullet B$
4	$A - B$
5	$A \ominus B$
6	$(B)_z$

Table 2

a	$\{w - z \mid w \in B\}$
b	$(A \oplus B) \ominus B$
c	$\bigcup \{(B)_z \mid (B)_z \subseteq A\}$
d	$\{z \mid (\hat{B})_z \cap A \neq \varnothing\}$
e	$A \cap B^c$
f	$\{z \mid (B)_z \subseteq A\}$

Table 3

Answer Table					
1	2	3	4	5	6

(10 Marks)

(b) A binary image shown in Figure 1 is represented by a set A where the shaded squares are the elements of set A. Let $C = A - (A \ominus B)$.

(i) Shade the elements of set C using a copy of Figure 4 based on the structuring element B shown in Figure 2.

(ii) Shade the elements of set C using another copy of Figure 4 based on the structuring element B shown in Figure 3. How are the two results in parts (b) (i) and (b) (ii) related to image A?

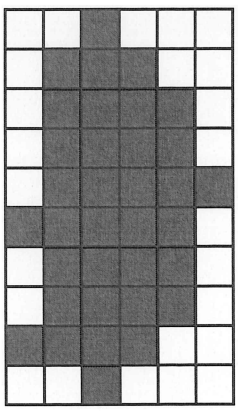

Figure 1

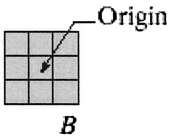

Figure 2

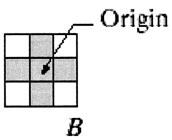

Figure 3

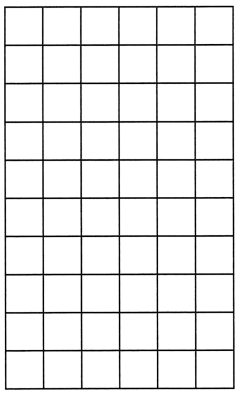

Figure 4

(10 Marks)

3. An image contains 40 fishes of type ω_1 and 60 fishes of type ω_2. You need to identify the type of each fish in the image. An image segmentation algorithm is used to separate each fish and compute its size. The two PDFs of the size x of the two types of fishes, $p(x|\omega_1)$ and $p(x|\omega_2)$, are shown in Figure 5, where 2 straight lines can be observed.

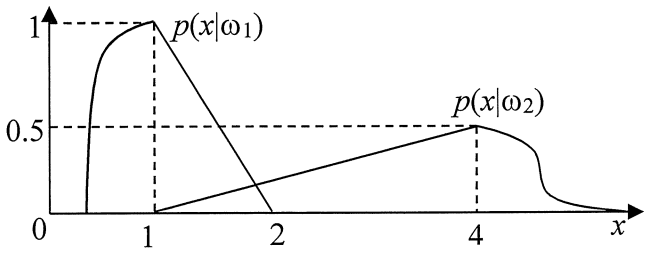

Figure 5

(a) Determine the type of a fish with x = 1.5 and the corresponding probability of making a wrong decision using Bayes' theorem.

(10 Marks)

(b) Generate a decision rule to determine the type of fish using Bayes' theorem and evaluate the probability of making a wrong decision using this decision rule.

(10 Marks)

4. A linear transform $\mathbf{W}$ is applied to transform feature vector $\mathbf{x}$ into feature vector $\mathbf{y}$ by $\mathbf{y}=\mathbf{W}^T\mathbf{x}$.

(a) Express the Euclidean distance between two linearly transformed feature vectors $d_u(\mathbf{y}_1,\mathbf{y}_2)$ in terms of the two corresponding feature vectors $\mathbf{x}_1,\mathbf{x}_2$ before the transform. Derive the conditions under which $d_u(\mathbf{y}_1,\mathbf{y}_2)=d_u(\mathbf{x}_1,\mathbf{x}_2)$ for any $\mathbf{x}_1,\mathbf{x}_2$.

(10 Marks)

(b) The Mahalanobis distance is defined as $d_m(\mathbf{x}_1,\mathbf{x}_2)=(\mathbf{x}_1-\mathbf{x}_2)^T\Sigma_x^{-1}(\mathbf{x}_1-\mathbf{x}_2)$ where Σ_x is the covariance matrix of the feature vector $\mathbf{x}$. Express the Mahalanobis distance $d_m(\mathbf{y}_1,\mathbf{y}_2)$ in terms of $\mathbf{x}_1$, $\mathbf{x}_2$ and derive the condition under which $d_m(\mathbf{y}_1,\mathbf{y}_2)=d_m(\mathbf{x}_1,\mathbf{x}_2)$ for any $\mathbf{x}_1,\mathbf{x}_2$. Compare the results with those in part (a).

5. The inputs and outputs of a typical fully-connected layer of neural network are denoted by $\mathbf{x} = [x_1, x_2, ..., x_{100}]^T$ and $\mathbf{y} = [y_1, y_2, ..., y_{98}]^T$. Linear activation function is applied in this layer and the network parameters to connect the inputs and outputs are given by $\mathbf{W} = [w_{ij}]$ and $\boldsymbol{\theta} = [\theta_1, \theta_1, ..., \theta_{98}]^T$. The matrix $\mathbf{W} = [w_{ij}]$ has 98 rows and 100 columns.

(a) (i) Express the outputs in term of inputs in vector-matrix form and scalar form.

(ii) Compute the number of trainable parameters and the number of multiplications and summations required in this layer to compute the outputs from the inputs.

(iii) What is the ratio of the number of outputs to the number of trainable parameters?

(10 Marks)

(b) Now replace this layer by a convolutional neural network layer that has 20 learnable filters of size 3 with trainable parameters, $\mathbf{w}^k = [w_{-1}^k, w_0^k, w_1^k]^T$ and θ^k, $1 \le k \le 20$. This generates 20 output feature maps, denoted by $\mathbf{y}^k = [y_2^k, y_3^k, ..., y_{99}^k]^T$, $1 \le k \le 20$. Linear activation function is applied in this layer.

(i) Express the outputs in term of inputs in scalar form.

(ii) Compute the number of trainable parameters and the number of multiplications and summations required in this layer to compute the outputs from the inputs.

(iii) What is the ratio of the number of outputs to the number of trainable parameters?

(10 Marks)

END OF PAPER

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Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.