System Anaylsis Exams

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Personal answers.

May not always be right.

2018-2019

1

(a)

(b)

initial solution:

	1	2	3
1		60	
2	40		30
3		5	45

optimal solution:

$$\begin{cases} u_1 + v_2 = 24 \\ u_2 + v_1 = 18 \\ u_2 + v_3 = 20 \\ u_3 + v_2 = 25 \\ u_3 + v_3 = 26 \end{cases} \quad \begin{cases} u_1 = 1 \\ u_2 = -4 \\ u_3 = 2 \end{cases} \quad \begin{cases} v_1 = 22 \\ v_2 = 23 \\ v_3 = 24 \end{cases}$$

$$\begin{cases} x_{11} = -3 \\ x_{13} = -1 \\ x_{22} = 4 \\ x_{31} = -4 \end{cases}$$

	1	2	3
1	40	15	5
2			70
3		50	

	1	2	3
1		55	5
2			70
3	40	10	

(c)

$\Delta \geq -1$

2

(a)

Min $8x_1^2 + 28x_1x_2$

Subject to:

$\frac{x_1^2 x_2}{2.5} \geq 98$

$x_1 \geq 0,\ x_1 \geq 0$

(b)

$L = 8x_1^2 + 28x_1x_2 + \lambda(\frac{2x_1^2x_2}{5} - 98)$

We have

$\frac{\partial L}{\partial x_1} = 16x_1 + 28x_2 + \lambda(\frac{4x_2x_1}{5}) = 0$

$\frac{\partial L}{\partial x_2} = 28x_1 + \lambda(\frac{2x_1^2}{5}) = 0$

$\frac{\partial L}{\partial \lambda} = \frac{2x_1^2x_2}{5} - 98 = 0$

$$\begin{cases} x_1 = 7.541 \\ x_2 = 4.309 \\ \lambda = -0.928 \end{cases}$$

$$\text{Hessian matrix } H = \begin{bmatrix} \dfrac{\partial^2 L}{\partial x_1^2} & \dfrac{\partial^2 L}{\partial x_1 \partial x_2} \\ \dfrac{\partial^2 L}{\partial x_2 \partial x_1} & \dfrac{\partial^2 L}{\partial x_2^2} \end{bmatrix} = \begin{bmatrix} -16 & -28 \\ -28 & 0 \end{bmatrix}$$

$$|\lambda E - H| = |\begin{bmatrix} \lambda + 16 & 28 \\ 28 & \lambda \end{bmatrix}| = 0$$

$\lambda_1 = -37.12,\ \lambda_2 = 21.12$

3

(a)

Yes, As the shop carries only 1 car in the shop, B_k only depends on the current week.

4

(a)

State space $S = \{0, 1, 2\}$

$$\begin{aligned} 0 &: \ \text{baking} \\ 1 &: \ \text{preparation} \\ 2 &: \ \text{repairting} \end{aligned}$$

(b)

$\pi_j(\sum_{k\neq j} q_{jk}) = \sum_{k\neq j} q_{kj}\pi_k$

$$Q = \begin{bmatrix} -b-f & b & f \\ p & -p & 0 \\ 0 & r & -r \end{bmatrix}$$

$$\pi Q = 0$$

$$\begin{cases} (-b-f)\pi_0 + p\pi_1 = 0 \\ b\pi_0 - p\pi_1 + r\pi_2 = 0 \\ f\pi_0 - r\pi_2 = 0 \end{cases}$$

$$\pi_0 + \pi_1 + \pi_2 = 1$$

(c)

Average production rate $R = b\pi_0$

(d)

R increase, as π_0 increase

5

(a)

	MP	TR
MP	1	0.2
TR	5	1
SUM	6	1.2

	MP	TR
MP	1/6	1/6
TR	5/6	5/6
SUM	6	1.2

	MP	TR	Priority
MP	1/6	1/6	1/6
TR	5/6	5/6	5/6

Level 3 priority matrix:

$$\begin{bmatrix} 0.129 & 0.545 \\ 0.277 & 0.273 \\ 0.595 & 0.182 \end{bmatrix}$$

composite priority:

$$\begin{bmatrix} 0.129 & 0.545 \\ 0.277 & 0.273 \\ 0.595 & 0.182 \end{bmatrix} \times \begin{bmatrix} 1/6 \\ 5/6 \end{bmatrix} = \begin{bmatrix} 0.476 \\ 0.274 \\ 0.251 \end{bmatrix}$$

I recommend company choose company U1.

(b)

$$\begin{bmatrix} 1 & 0.2 \\ 5 & 1 \end{bmatrix} \begin{bmatrix} 1/6 \\ 5/6 \end{bmatrix} = \begin{bmatrix} 1/3 \\ 5/3 \end{bmatrix}$$

$\lambda_{max} = (\frac{1}{3}/\frac{1}{6} + \frac{5}{3}/\frac{5}{6})/2 = 2$
$C.I. = \frac{\lambda_{max} - n}{n-1} = 0$
$CR = \frac{CI}{RI} = 0$

Matrix $A_{\mathbf{MP}}$ is perfectly consistent.

(c)

	MP	TR
MP	1	2
TR	0.5	1
SUM	1.5	3

2020-2021

1

(a)

Supposing that the factory sells x_1, x_2, x_3 packages of PA, PB, PC.

Max $2.5x_1 + 2.2x_2 + 2.1x_3$

Subject to:

$$\begin{cases} 0.2x_1 + 0.18x_2 + 0.16x_3 \leq 10 \\ 1.5x_1 + 1.7x_2 + 1.8x_3 \leq 100 \\ 0.5x_1 + 0.35x_2 + 0.6x_3 \leq 30 \\ x_1,\ x_2,\ x_3 \geq 0 \end{cases}$$

(b)

Max $2.5x_1 + 2.2x_2 + 2.1x_3$

Subject to:

$$\begin{cases} 0.2x_1 + 0.18x_2 + 0.16x_3 + x_4 = 10 \\ 1.5x_1 + 1.7x_2 + 1.8x_3 + x_5 = 100 \\ 0.5x_1 + 0.35x_2 + 0.6x_3 + x_6 = 30 \\ x_1,\ x_2,\ x_3,\ x_4,\ x_5,\ x_6 \geq 0 \end{cases}$$

		2.5	2.2	2.1	0	0	0		
C_B	X_B	x_1	x_2	x_3	x_4	x_5	x_6	b	θ
0	x_4	0.2	0.18	0.16	1	0	0	10	50
0	x_5	1.5	1.7	1.8	0	1	0	100	66.67
0	x_6	0.5	0.35	0.6	0	0	1	30	60
	σ	2.5	2.2	2.1	0	0	0		

		2.5	2.2	2.1	0	0	0		
C_B	X_B	x_1	x_2	x_3	x_4	x_5	x_6	b	θ
2.5	x_1	1	0.9	0.8	5	0	0	50	62.5
0	x_5	0	0.35	0.6	-7.5	1	0	25	41.67
0	x_6	0	-0.1	0.2	-2.5	0	1	5	25
	σ	0	-0.05	0.1	-12.5	0	0		

		2.5	2.2	2.1	0	0	0		
C_B	X_B	x_1	x_2	x_3	x_4	x_5	x_6	b	θ
2.5	x_1	1	1.3	0	15	0	-4	30	
0	x_5	0	0.65	0	0	1	-3	10	
2.1	x_3	0	-0.5	1	-12.5	0	5	25	
	σ	0	0	0	-11.25	0	-0.5		

$x_1 = 30,\ x_2 = 0,\ x_3 = 25$

(c)

$$b' = B^{-1}b = \begin{bmatrix} 15 & 0 & -4 \\ 0 & 1 & -3 \\ -12.5 & 0 & 5 \end{bmatrix} \begin{bmatrix} 10 \\ 100 \\ 30-\Delta \end{bmatrix} = \begin{bmatrix} 30+4\Delta \\ 10+3\Delta \\ 25-5\Delta \end{bmatrix}$$

$x_1 = 30 + 4\Delta,\ x_2 = 0,\ x_3 = 25 - 5\Delta$

Previous solution is neither optimal nor feasible.

2

(a)

Min $z = 2\pi C x_1^2 + 2\pi C x_1 x_2$

Subject to:

$\pi x_1^2 x_2 = V$

$x_1 \geq 0,\ x_2 \geq 0$

(b)

$L = 2\pi C x_1^2 + 2\pi C x_1 x_2 + \lambda(\pi x_1^2 x_2 - V)$

$\frac{\partial L}{\partial x_1} = 4\pi C x_1 + 2\pi C x_2 + \lambda(2\pi x_1 x_2) = 0$

$\frac{\partial L}{\partial x_2} = 2\pi C x_1 + \lambda(\pi x_1^2) = 0$

$\frac{\partial L}{\partial \lambda} = \pi x_1^2 x_2 - V = 0$

$$\begin{cases} x_1 = \dfrac{V(-\frac{16\pi C^3}{V})^{\frac{2}{3}}}{8C^2\pi} \\ \\ x_2 = \dfrac{V(-\frac{16\pi C^3}{V})^{\frac{2}{3}}}{4C^2\pi} \\ \\ \lambda = (-\frac{16\pi C^3}{V})^{\frac{1}{3}} \end{cases}$$

$$\text{Hessian matrix } H = \begin{bmatrix} \frac{\partial^2 L}{\partial x_1^2} & \frac{\partial^2 L}{\partial x_1 \partial x_2} \\ \\ \frac{\partial^2 L}{\partial x_2 \partial x_1} & \frac{\partial^2 L}{\partial x_2^2} \end{bmatrix} = \begin{bmatrix} -4\pi C & -2\pi C \\ -2\pi C & 0 \end{bmatrix}$$

$$|\lambda E - H| = | \begin{bmatrix} \lambda + 4\pi C & 2\pi C \\ 2\pi C & \lambda \end{bmatrix} | = 0$$

$$\lambda = (-2 \pm 2\sqrt{2})\pi C$$

(c)

$$z = 2\pi C x_1^2 + 2\pi C x_1 x_2 = \frac{3V^2(-\frac{16\pi C^3}{V})^{\frac{4}{3}}}{32C^3\pi}$$

If $V' = 0.95V$, $z' = 0.966z$

3

(a)

$$P_A = \begin{bmatrix} \frac{1}{5} & \frac{4}{5} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}, \; P_B = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{10} & \frac{9}{10} \end{bmatrix}$$

(b)

Strategy (A):

$E(T_0) = \frac{1}{1-p_{00}} = \frac{5}{4}$

$E(T_1) = \frac{1}{1-p_{11}} = 3$

Strategy (B):

$E(T_0) = \frac{1}{1-p_{00}} = 2$

$E(T_1) = \frac{1}{1-p_{11}} = 10$

(c)

Strategy (A):

$|\lambda E - P| = 0$

$\lambda_1 = -\frac{2}{15},\ \lambda_2 = 1$

$$Q = \begin{bmatrix} -12 & 1 \\ 5 & 1 \end{bmatrix},\ Q^{-1} = \begin{bmatrix} -\frac{1}{17} & \frac{1}{17} \\ \frac{5}{17} & \frac{12}{17} \end{bmatrix},\ D = \begin{bmatrix} -\frac{5}{12} & 0 \\ 0 & 1 \end{bmatrix}$$

when n goes to infinity:

$$P^n = QD^nQ^{-1} = \begin{bmatrix} \frac{5}{17} & \frac{12}{17} \\ \frac{5}{17} & \frac{12}{17} \end{bmatrix}$$

$\pi(n) = \pi(0)P^n = \begin{bmatrix} \frac{5}{17} & \frac{12}{17} \end{bmatrix}$

Strategy (B):

(d)

Strategy (A):

Expected profit $= \frac{5}{17} \times (500,000 - 200,000) + \frac{12}{17} \times 2,000,000 = 1,500,000$

Strategy (B):

2021-2022

1

(a)

(b)

2

(a)

(b)

3

(a)

(i) $P(a,b) = \sum_{i=a}^{b} P(X=i) = e^{-\lambda} \sum_{i=a}^{b} \frac{\lambda^i}{i!}$

(ii) $P(a,b) = \sum_{i=0}^{b} P(X=i) = e^{-\lambda} \sum_{i=0}^{b} \frac{\lambda^i}{i!}$

(iii) $P(a,b) = 1 - \sum_{i=0}^{a-1} P(X=i) = 1 - e^{-\lambda} \sum_{i=0}^{a-1} \frac{\lambda^i}{i!}$

(b)

4

(a)

(b)

(c)

5

(a)

(b)

(c)