NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2021-2022

EE6204 – SYSTEMS ANALYSIS

April / May 2022 Time Allowed: 3 hours

INSTRUCTIONS

1. This paper contains 5 questions and comprises 7 pages.
2. Answer all 5 questions.
3. All questions carry equal marks.
4. This is a closed book examination.
5. Unless specifically stated, all symbols have their usual meanings.

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1. This question consists of two parts.

(a) Company X produces both interior and exterior paints from two raw materials, M1 and M2. The following Table 1 provides the basic data of the problem:

Table 1

	Tons of raw materials per ton of		Maximum daily availability (tons)
	Exterior paint	Interior paint	
Raw material, M1	6	4	24
Raw material, M2	1	2	6
Profit per ton ($1000)	5	4	

A market survey indicates that the daily demand for interior paint cannot exceed that for exterior paint by more than 1 ton. Also, the maximum daily demand for interior paint is 2 tons. Company X wants to determine the optimum (best) product mix of interior and exterior paints that maximizes the total daily profit. Formulate the Linear Programming problem for this optimization problem without solving it.

(10 Marks)

Note: Question No. 1 continues on page 2.

(b) Use the SIMPLEX method to solve the following Linear Programming problem: (show your working)

$$\begin{aligned} &\text{Min} \quad z = -4y_1 - 5y_2 \\ &\text{Subject to} \\ &\quad 4y_1 + 6y_2 \leq 24 \\ &\quad 2y_1 + y_2 \leq 6 \\ &\quad y_1 - y_2 \leq 1 \\ &\quad y_1 \leq 2 \\ &\quad y_1 \geq 0,\ y_2 \geq 0 \end{aligned}$$

(10 Marks)

2. This question consists of two parts.

(a) Joe Klyne's four children, John, Karen, Terri and Jean, want to earn some money for personal expenses. Mr. Klyne has chosen four chores for his children: mowing the lawn, painting the garage door, washing the family cars, and cleaning the kitchen. To avoid anticipated sibling competition, he asks them to submit individual (secret) bids for what they feel is a fair pay for each of the four chores. Table 2 summarizes the bids received. The children will abide their father's decision regarding the assignment of chores. Help Mr. Klyne determine the assignments that minimize the total pay.

Table 2

		Chores			
		Mow	Paint	Wash	Clean
Children	John	\$1	\$4	\$6	\$3
	Karen	\$9	\$7	\$10	\$9
	Terri	\$4	\$5	\$11	\$7
	Jean	\$8	\$7	\$8	\$5

(10 Marks)

(b) Let $x = [x_1, x_2]^T$. A quadratic optimization problem is given as follows,

$$\text{Min } z = \frac{1}{2}x^T \begin{bmatrix} 4 & 0 \\ 0 & 8 \end{bmatrix} x + \begin{bmatrix} 1 \\ 2 \end{bmatrix}^T x + 2$$

Subject to: $x_i \leq 2, i = 1,2$

Formulate the above problem using the method of Lagrange multipliers and find an optimal solution.

(10 Marks)

3 (a) Let X be a Poisson random variable with parameter λ. Let a and b be positive integers and denote the probability $P(a \le X \le b)$ by $P(a,b)$. Express the following probabilities in terms of λ.

(i) $P(a,b)$

(ii) $P(0,b)$

(iii) $P(a,\infty)$

(2 Marks)

(b) A car dealer decides to give some incentive to his sales team in the form of lucky draw prizes at the end of each month. The following terms and conditions of the lucky draw prizes are imposed:

- The car dealer starts with 3 lucky draw prizes, and he will not replenish until all the 3 lucky draw prizes have been given out. If all the lucky draw prizes have been given out in a particular month, the car dealer will order another 3 lucky draw prizes for the following month.
- Criterion of availability of lucky draw prizes for the sales team: There will be 1 lucky draw prize available for every 10 sales, subject to the availability of prizes. No lucky draw prize is available if the total number of sales achieved at the end of the month is below 10.
- The number of lucky draw prizes available for each month is at most 3.
- If there are some lucky draw prizes left at the end of the month (after giving out to the sales team), the remaining lucky draw prizes will be channeled to the following month, subject to the criterion mentioned above.

Let X_k denote the total number of sales achieved in the k-th month, and W_k denote the number of lucky draw prizes left at the end of the k-th month, after giving out the lucky draw prizes to the sales team.

Suppose that X_k's are independent and identical Poisson random variables with parameter λ.

(i) Give a state space for the discrete time Markov chain (DTMC) $\{W_k\}$. Use the notation $P(a,b)$ in part (a) to find the transition probability matrix (TPM) of the DTMC.

(ii) Suppose that 2 lucky draw prizes were given out in May 2022. What is the probability of giving out 3 lucky draw prizes in August 2022? How would you find this probability?

(18 Marks)

4. (a) A machine produces a part, one at a time. Each time the machine finishes the production of a part, it has to be set up for the production of the next part. If the machine fails during production, the repair process starts immediately. After being repaired, the machine resumes the production of the unfinished part. The machine does not fail during setup. Assume that the production time, setup time, time between failures and repair time are all mutually independent and exponentially distributed with rates p, s, f and r, respectively. The system can be modeled as a continuous time Markov chain (CTMC) with the following states:

State 0 denotes a machine is being set up

State 1 denotes a machine is producing a part

State 2 denotes a machine has failed, under repair

(i) Draw the state transition diagram.
(ii) Calculate the steady state probabilities and the average production rate.

(6 Marks)

(b) In the system of part (a), after some configuration the machine can now produce N parts at a time, where $N \geq 1$. Each time the machine finishes the production of a batch of N parts, it has to be set up again for the production of the next batch. Suppose all the assumptions of part (a) hold, with p interpreted as the production rate of a single part.

(i) Model the system as a CTMC. Specify the states and draw the state transition diagram.
(ii) Calculate the steady state probabilities and the average production rate.

(11 Marks)

(c) Let $N = 3$, $p = 2$, $s = 1$, $f = 0.1$ and $r = 1$. Compute the average production rates in questions (a) and (b). Comment on your answers.

(3 Marks)

5. A property developer has assets valued at \$50 billion. An old condominium situated at a prime site has put up a tender for en-bloc sale. The reserve price of the old condominium is \$1 billion. Reserve price is the minimum sale price that the sale committee of the old condominium hopes to achieve. If the developer bids in the open tender with a price of \$1 billion, there is a 50% chance that he will succeed. If the developer puts in a higher bid of \$1.05 billion, there is a 65% chance that he will succeed. In negotiating with the sale committee of the condominium, it is confirmed that a private treaty could be done at a price of \$1.3 billion, i.e., it will be a done deal if the developer agrees to buy at a price of \$1.3 billion. The developer needs to consult the architect to do a site study before he bids, the consultancy cost is \$200,000 (=\$0.0002 billion). There is also an interest cost amounting to 3% of the bid price should the developer decides to put in a bid (open tender or private treaty).

Assuming that the developer successfully acquires the site, he has to decide whether to launch the new project now (meaning within the next 3 months) or to launch it 9 months later. The property market may get more bullish. If the new project is launched now, there is a 60% chance that it will be a hit, and a 40% chance that it will be a bomb. If the new project is launched 9 months later, there is a 70% chance that it will be a hit, and a 30% chance that it will be a bomb. The profit/loss of a hit/bomb, for different bid prices and launch timings, are in the table below. All the figures are in billions.

Bid Price	Launch now	Launch later
1	Hit: 0.3, Bomb: –0.4	Hit: 0.25, Bomb: –0.45
1.05	Hit: 0.23, Bomb: –0.5	Hit: 0.2, Bomb: –0.52
1.3	Hit: 0.18, Bomb: –0.7	Hit: 0.13, Bomb: –0.73

(a) Use a decision tree to represent the above problem.

(10 Marks)

(b) Recommend a strategy for the developer to adopt so that his expected final asset position is maximized.

(4 Marks)

(c) Suppose the developer bids at \$ p billion, and he wants to achieve the expected final asset position of \$ $(50+0.1p)$ billion. Assume that the bid price of \$ p billion guarantees the acquisition of the condominium and the conservative profit/loss follows that at bid price of \$1 billion, i.e.,

Bid Price	Launch now	Launch later
p	Hit: 0.3, Bomb: –0.4	Hit: 0.25, Bomb: –0.45

Find the bid price p. Comment on your answer.

(6 Marks)

Appendix A

M/M/1 Queue with Arrival Rate λ and Service Rate μ:

$$\begin{aligned}
\rho &= \frac{\lambda}{\mu} \\
\pi_0 &= 1-\rho \\
\pi_k &= \rho^k(1-\rho), \qquad k \geq 1 \\
L &= \frac{\rho}{1-\rho} = \frac{\lambda}{\mu-\lambda} \\
Q &= \frac{\rho^2}{1-\rho} = \frac{\lambda^2}{\mu(\mu-\lambda)} \\
W &= \frac{1}{\mu(1-\rho)} = \frac{1}{\mu-\lambda} \\
D &= D = W - \frac{1}{\mu} = \frac{\lambda}{\mu(\mu-\lambda)}
\end{aligned}$$

M/M/1/N Queue with Arrival Rate λ and Service Rate μ:

$$\begin{aligned}
\rho &= \frac{\lambda}{\mu} \\
\pi_0 &= \left(\sum_{k=0}^{N} \rho^k\right)^{-1} = \frac{1-\rho}{1-\rho^{N+1}} \\
\pi_k &= \rho^k \pi_0 = \frac{\rho^k(1-\rho)}{1-\rho^{N+1}}, \qquad 0 \leq k \leq N \\
L &= \frac{\rho[1-\rho^N - N\rho^N(1-\rho)]}{(1-\rho)(1-\rho^{N+1})} \\
Q &= \frac{\rho^2[1-\rho^N - N\rho^{N-1}(1-\rho)]}{(1-\rho)(1-\rho^{N+1})} \\
W &= \frac{1-\rho^N - N\rho^N(1-\rho)}{\mu(1-\rho)(1-\rho^{N+1})} \\
D &= \frac{\rho[1-\rho^N - N\rho^{N-1}(1-\rho)]}{\mu(1-\rho)(1-\rho^{N+1})}
\end{aligned}$$

Note: Appendix A continues on page 7.

Appendix A (Continued)

M/M/m Queue with Arrival Rate λ and Service Rate μ:

$$
\begin{aligned}
\rho &= \frac{\lambda}{m\mu} \\
\pi_0 &= \left[\frac{(m\rho)^m}{m!(1-\rho)} + \sum_{k=0}^{m-1}\frac{(m\rho)^k}{k!}\right]^{-1} \\
\pi_k &= \pi_0 \begin{cases} \dfrac{(m\rho)^k}{k!}, & 0 \le k \le m-1 \\[2ex] \dfrac{m^m \rho^k}{m!}, & k \ge m \end{cases} \\
L &= \frac{\rho(m\rho)^m \pi_0}{m!(1-\rho)^2} + \frac{\lambda}{\mu} \\
Q &= \sum_{k=m}^{\infty}(k-m)\pi_k = \frac{\rho(m\rho)^m \pi_0}{m!(1-\rho)^2} \\
W &= \frac{L}{\lambda} = \frac{\rho(m\rho)^m \pi_0}{m!\lambda(1-\rho)^2} + \frac{1}{\mu} \\
D &= W - \frac{1}{\mu} = \frac{\rho(m\rho)^m \pi_0}{m!\lambda(1-\rho)^2}
\end{aligned}
$$

M^b/M/1 Queue with Arrival Rate λ and Service Rate μ:

$$
\begin{aligned}
\rho &= \frac{b\lambda}{\mu} \\
\pi_0 &= 1-\rho \\
\pi_k &= \begin{cases} \left(\dfrac{\lambda+\mu}{\mu}\right)^{k-1} \dfrac{\lambda}{\mu}\pi_0 & 1 \le k \le b \\[2ex] \dfrac{\lambda+\mu}{\mu}\pi_{k-1} - \dfrac{\lambda}{\mu}\pi_{k-b-1} & k \ge b+1 \end{cases} \\
L &= \frac{\rho(1+b)}{2(1-\rho)} \\
Q &= L - \rho = \frac{\rho(b-1+2\rho)}{2(1-\rho)} \\
W &= \frac{L}{\lambda b} = \frac{1+b}{2\mu(1-\rho)} \\
D &= W - \frac{1}{\mu} = \frac{b+2\rho-1}{2\mu(1-\rho)}
\end{aligned}
$$

END OF PAPER

EE6204 SYSTEMS ANALYSIS

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.