NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2019-2020

EE6401– ADVANCED DIGITAL SIGNAL PROCESSING

November / December 2019 Time Allowed: 3 hours

INSTRUCTIONS

1. This paper contains 5 questions and comprises 5 pages.
2. Answer all 5 questions.
3. All questions carry equal marks.
4. This is a closed book examination.

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1. (a) Plot the continuous-time Fourier transform (CTFT) of the signal

$$x(t) = 2\cos(10\pi t) + \cos(20\pi t)$$

and the discrete-time Fourier transforms (DTFTs) of $x[n]$ that is obtained by sampling $x(t)$ at frequencies of 15, 20 and 25 Hz, respectively. Determine which DTFT can be used to recover $x(t)$.

(12 Marks)

(b) The multi-rate system in Figure 1 contains the up-samplers by the factors of 5 and 3, respectively, and a down sampler by a factor of 15. Find the expression of the output, $y[n]$, in terms of the input, $x[n]$.

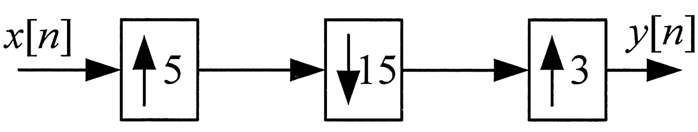

Figure 1

(8 Marks)

2. (a) Derive the transfer function of the system in Figure 2 and prove that this system meets the condition of perfect reconstruction.

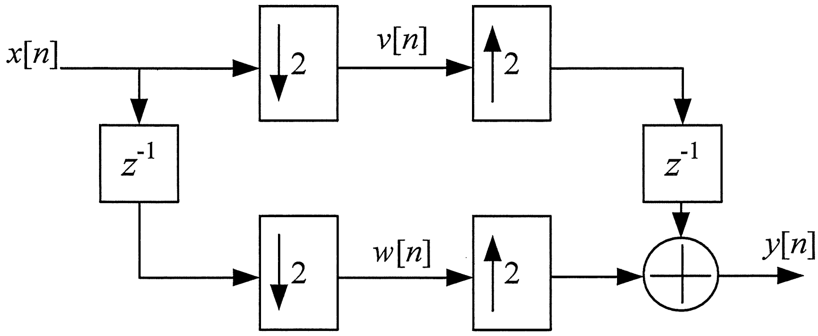

Figure 2

(8 Marks)

(b) A linear phase FIR filter is designed to satisfy the following specifications based on a single-stage and a two-stage multi-rate structures, respectively.

Passband ripple: $< 10^{-1}$ Passband: 0 to 60 Hz
Stopband ripple: $< 10^{-3}$ Stopband: $>$ 65 Hz
Sampling frequency: 10 kHz

(i) By determining a suitable stop band frequency for an integer decimation factor, calculate the orders of filters needed in the single-stage and two-stage multi-rate structures, respectively.

(ii) Calculate the number of multiplications per second used in each of the structures obtained from part (b)(i) with any assumption clearly stated.

(12 Marks)

3. (a) Figure 3 on page 3 shows the block diagram of a recursive system.

(i) Derive the system transfer function $H(z)$.

(ii) Is the system canonic?

(iii) If $a = 2$ and $b = 1$ in Figure 3, verify whether the system is canonic.

Note: Question No. 3 continues on page 3.

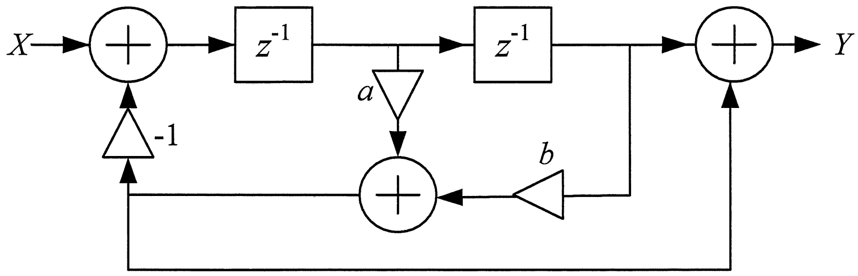

Figure 3

(10 Marks)

(b) Assume the analysis filters of a three-band perfect reconstruction filter bank are given by

$$H_0(z) = z^{-2} + 6z^{-1} + 4$$
$$H_1(z) = z^{-1} + 2$$
$$H_2(z) = 1$$

Determine its synthesis filters.

(10 Marks)

4. (a) In an adaptive noise cancelling system, the primary input to the system is $x[n] + w_1[n] + w_2[n]$, where $x[n]$ is a desired signal corrupted by an additive noise $w_1[n]$ and another additive noise (interference) $w_2[n]$. Assume that $w_2[n]$ is passed to an unknown linear system $G(z)$ and then serves as the secondary input to the system. Both $w_1[n]$ and $w_2[n]$ are zero mean and uncorrelated. The objective of the adaptive noise cancelling system is to design an adaptive algorithm to cancel $w_2[n]$. Draw a block diagram for this adaptive noise cancelling system and briefly explain how this system works. Can the system be used to cancel $w_1[n]$? Justify your answer.

(5 Marks)

Note: Question No. 4 continues on page 4.

(b) The weight update equation of the least-mean-square (LMS) algorithm is given by

$$\mathbf{h}_M[n+1] = \mathbf{h}_M[n] + \mu\, e[n]\,\mathbf{X}_M[n]$$

where $\mathbf{h}_M[n]$, $e[n]$ and $\mathbf{X}_M[n]$ are the filter weight vector, error signal and the input signal vector at the n^{th} time index, respectively, and μ is the step size parameter for the weight update. Derive the convergence condition for the LMS algorithm.

(5 Marks)

(c) If Γ_M is an input correlation matrix having distinct real eigenvalues, show that the eigenvectors of Γ_M are orthogonal to each other. Explain how Γ_M can be diagonalized by the eigen-decomposition.

(5 Marks)

(d) The matrix form of the Wiener-Hopf equation is given by $\Gamma_M \mathbf{h}_M = \gamma_d$, where Γ_M is the input correlation matrix and γ_d is the input/desired output cross-correlation vector. Let $E(d^2)$ be the mean-square value of the desired output d. Given that

$$\Gamma_M = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix},\ \gamma_d = \begin{bmatrix} 7 \\ 8 \end{bmatrix},\ \text{and } E(d^2) = 42,$$

obtain the Wiener filter coefficient vector $\mathbf{h}_M$ and also the resulting mean-square-error.

(5 Marks)

5. Let $x[n] = s[n] + e[n]$, where $s[n]$ is a wide sense stationary process with the auto-correlation function $r_s[m]$, and $s[n]$ is orthogonal with $e[n]$. Suppose that $e[n]$ is an auto-regressive AR(1) random process given by

$$e[n] - \beta e[n-1] = v[n]$$

where β is a real-valued constant, with $|\beta| < 1$, and the white noise $v[n]$ has a variance $\sigma_v^2 = 0.01$.

Note: Question No. 5 continues on page 5.

(a) Derive the expression for the auto-correlation function $r_x[m]$.

(10 Marks)

(b) Assume that $r_x[0]=0.5229$, $r_x[1]=0.3707$, and $r_x[2]=0.0129$. If $x[n]$ is an auto-regressive AR(2) random process, estimate the peak location(s) of the power spectrum density function $P_x(f)$ obtained using the Yule-Walker method.

(10 Marks)

END OF PAPER

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Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.