Advanced Digital Signal Processing Exams

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Personal answers.

May not always be right.

2018-2019

1.

(a)

(b)

2.

(a)

(i)

(ii)

(b)

3.

(a)

(b)

(c)

4.

(a)

(b)

(c)

5.

(a)

(b)

(c)

2019-2020

1.

(a)

(b)

$\uparrow 5 \ \downarrow 5 \ \downarrow 3 \ \uparrow 3$

$\downarrow 3 \ \uparrow 3$

$$y(n) = \begin{cases} x(n) & n = 0, \pm 3, \pm 6... \\ 0 & \text{otherwise} \end{cases}$$

2.

(a)

$V(z) = \frac{1}{D}\sum_{k=0}^{D-1} H_D(e^{-j2\pi\frac{k}{D}} z^{\frac{1}{D}}) X(e^{-j2\pi\frac{k}{D}} z^{\frac{1}{D}}) = \frac{1}{2}[X(z^{\frac{1}{2}}) + X(-z^{\frac{1}{2}})]$

$W(z) = \frac{1}{2}z^{-\frac{1}{2}}[X(z^{\frac{1}{2}}) - X(-z^{\frac{1}{2}})]$

$V_n(z) = \frac{1}{2}[X(z) + X(-z)]$

$W_n(z) = \frac{1}{2}z^{-1}[X(z) - X(-z)]$

$Y(z) = W_n(z) + z^{-1}V_n(z)$

(b)

(i)

$F_{SC} = 100$

$D = \frac{F}{2F_{SC}} = 50$

$\Delta f = \frac{f_s - f_p}{F} = \frac{65-60}{10\times 10^3}$

$M = \frac{-10\log_{10}\delta_s\delta_p - 13}{14.6\Delta f} + 1 = 5000$

??? (ii)

3.

(a)

(i)

(ii)

(iii)

(b)

$$\begin{bmatrix} H_0(z) \\ H_1(z) \\ H_2(z) \end{bmatrix} = \begin{bmatrix} 4 & 6 & 1 \\ 2 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ z^{-1} \\ z^{-2} \end{bmatrix}$$

$$P(z) = \begin{bmatrix} 4 & 6 & 1 \\ 2 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$Q(z) = P^{-1}(z) = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -2 \\ 1 & -6 & 8 \end{bmatrix}$$

$$G(z) = \begin{bmatrix} G_0(z) \\ G_1(z) \\ G_2(z) \end{bmatrix} = z^{-2} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -2 \\ 1 & -6 & 8 \end{bmatrix} \begin{bmatrix} 1 \\ z^{-1} \\ z^{-2} \end{bmatrix} = \begin{bmatrix} 1 \\ z^{-1} - 2 \\ z^{-2} - 6z^{-1} + 8 \end{bmatrix}$$

4.

(a)

(b)

(c)

(d)

5.

(a)

(b)

2020-2021

1.

(a)

(i)

(ii)

(b)

2.

(a)

(b)

3.

(a)

(b)

4.

(a)

(b)

(i)

(ii)

(iii)

2021-2022

1.

(a)

(i)

$x(n) = x_a(nT_s) = x_a(\frac{n}{20}) = 2cos(\frac{3\pi n}{10}) + sin(\frac{3\pi n}{5})$

(ii)

$V(z) = \frac{1}{D}\sum_{k=0}^{D-1} X(z^{\frac{1}{D}}e^{-j\frac{2\pi k}{D}}) = \frac{1}{2}[X(z^{\frac{1}{2}}) + X(-z^{\frac{1}{2}})]$

$z = e^{j\omega}$

$V(\omega) = \frac{1}{2}[X(e^{j\frac{\omega}{2}}) + X(e^{j(\frac{\omega}{2}-\pi)})] = \frac{1}{2}[X(\frac{\omega}{2}) + X(\frac{\omega}{2} - \pi)]$

(iii)

(iv)

???

(b)

(i)

$\Delta f = \frac{f_s - f_p}{F}$

$N = \frac{-10\log_{10}\delta_s\delta_p - 13}{14.6\Delta f} + 1 = 762$

$762 \div 6 = 127$

(ii)

Week7 41:16

2.

(a)

(i)

$V = X + \beta V z^{-2}$

$Y = \alpha V + V z^{-1} + V z^{-2}$

$\frac{V}{X} = \frac{1}{1-\beta z^{-2}}$

$\frac{Y}{V} = \alpha + z^{-1} + z^{-2}$

$P(z) = \frac{Y}{X} = \frac{\alpha+z^{-1}+z^{-2}}{1-\beta z^{-2}}$

(ii)

$P(z) = \frac{Y}{X} = \frac{(1-\frac{1}{2}z^{-1})(\alpha-2z^{-1})}{(1-\frac{1}{2}z^{-1})(1+\frac{1}{2}z^{-1})}$

$P(z) = \frac{Y}{X} = \frac{(1+\frac{1}{2}z^{-1})(\alpha+2z^{-1})}{(1-\frac{1}{2}z^{-1})(1+\frac{1}{2}z^{-1})}$

$\alpha = -2 \quad \text{or} \quad \alpha = -6$

(iii)

$P(z) = 2 + z^{-1} + z^{-2}$

$Q(z) = z^{-2}P(z^{-1}) = 1 + z^{-1} + 2z^{-2}$

$H(z) = P(z)Q(z)$

$H(\omega) = e^{-j2\omega}(6 + 6cos\omega + 4cos2\omega)$

$e^{-j2\omega}$ is the phase of $H(\omega)$

(b)

(i)

$V_1(z) = V(z^4)$

$V_2(z) = V_1(z)R(z)$

$V_3(z) = z^{-1}V_2(z)$

$W(z) = \frac{1}{4}\sum_{k=0}^{3} V_3(e^{-j2\pi\frac{k}{4}}z^{\frac{1}{4}})$

$H(z) = \frac{W(z)}{V(z)}$

Due to downsampling operation, this system is not time-invariant.

(ii)

???

3.

(a)

(i)

(ii)

(iii)

(b)

(i)

(ii)

4.

(a)

(i)

(ii)

(b)

(i)

$H_1(z) = 1 - z^{-2}$

$\Gamma_{h_1h_1}(f) = H_1(z)H_1(z^{-1})|_{z=e^{j2\pi f}} = 2 - z^2 - z^{-2}$

$\Gamma_{ww}(f) = \sigma_w^2$

$\Gamma_{xx}(f) = \Gamma_{h_1h_1}(f)\Gamma_{ww}(f) = \sigma_w^2(2 - z^2 - z^{-2})$

$H_2(z) = \frac{1}{1+0.81z^{-2}}$

$\Gamma_{h_2h_2}(f) = H_2(z)H_2(z^{-1})|_{z=e^{j2\pi f}} = \frac{1}{1.6561+0.81z^2+0.81z^{-2}}$

$\Gamma_{yy}(f) = \Gamma_{h_2h_2}(f)\Gamma_{ww}(f) = \frac{\sigma_w^2}{1.6561+0.81z^2+0.81z^{-2}}$

(ii)

(iii)

$\Gamma_{yy}(f) = \frac{\sigma_w^2}{1.6561+0.81z^2+0.81z^{-2}} = \sigma_w^2 \frac{z^2}{0.81z^4+1.6561z^2+0.81}$

5.

(a)

$A_2(z) = 1 + 2z^{-1} + \frac{1}{3}z^{-2}$

$K_2 = \frac{1}{3}$

$B_2(z) = \frac{1}{3} + 2z^{-1} + z^{-2}$

$A_1(z) = \frac{A_2(z)-K_2B_2(z)}{1-K_2^2} = 1 + \frac{3}{2}z^{-1}$

$K_1 = \frac{3}{2}$

(b)

$H_{sv}(z) = \frac{1}{1-0.8z^{-1}}$

$\Gamma_{ss}(\omega) = \sigma_v^2 H_{sv}(\omega) H_{sv}(-\omega)$

12.14