EE6222: Machine Vision

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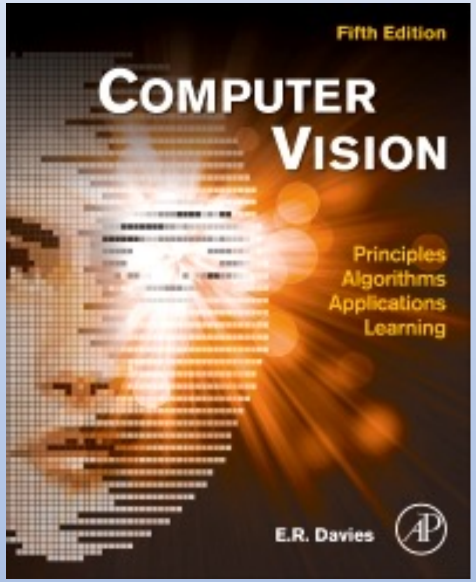
Course Content

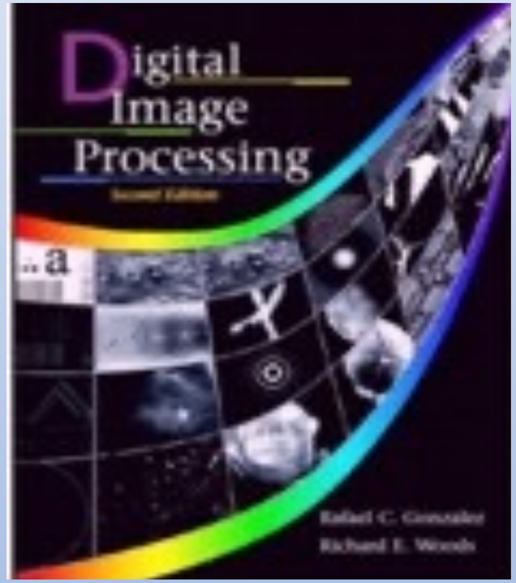
1. Image Fundamentals and Human Perception.
2. LSI Systems and Transforms
3. Image Denoising and Enhancement
4. Morphological Image Processing
5. MAP Decision and Classifiers
6. Statistical Estimation and Machine Learning
7. Eigenvalue and Eigenvector Decomposition of Data Matrix
8. Visual Data Dimensionality Reduction
9. Neural Networks and Deep Machine Learning: from MLP to CNN
10. Deep Learning: from CNN to Transformer
11. Video Analysis
12. Video Recognition
13. Three-dimensional Machine Perception
14. Three-dimensional Machine Vision

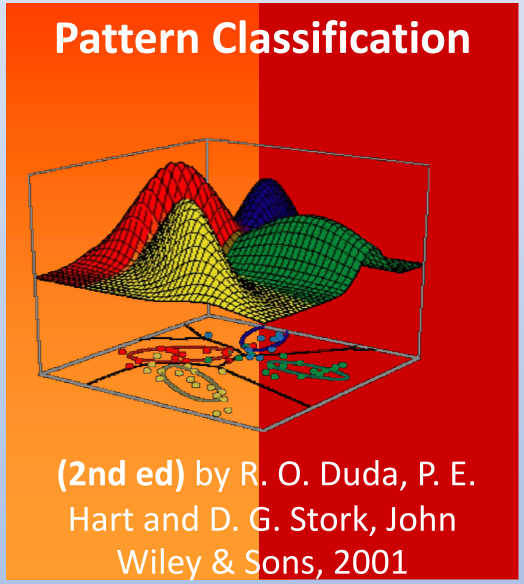
Topics **11, 12, 13, 14** will be lectured by Dr. Xu Yuecong

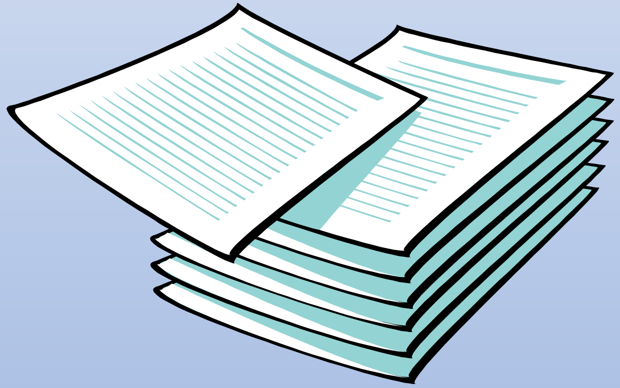
Textbooks, Assessments & Prerequisites

- Davies E. R., Computer Vision: Principles, Algorithms, Applications, Learning, Elsevier Science, Academic Press, 2017.
- Gonzalez, R. C., Woods, R. E., *Digital Image Processing*, Prentice Hall
- R. O. Duda, P. E. Hart, and D. G. Stork, *Pattern Classification*, Wile Inter-science









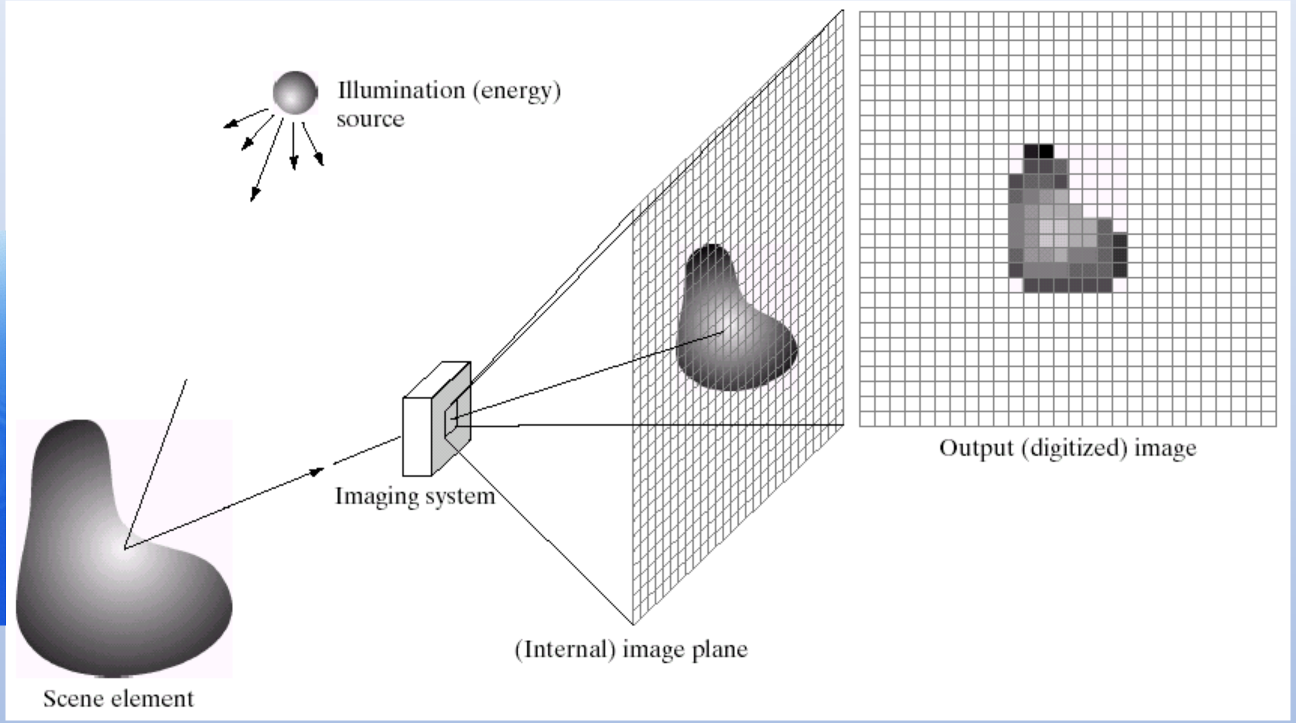
- **Assessment scheme:** Final Examination 60% and CA 40% by one quiz (10%, 1 hour in class at week 6) and two take-home assignments (15 % each)
- **Prerequisites:** Linear Algebra, Probability Theory, Signals & Systems

1. Image Fundamentals & human Perception—Outline

- Image Formation
- Human Perception of Image and Color image
- Digital Image Representation
- Image Histogram
- Image Processing and Its Application

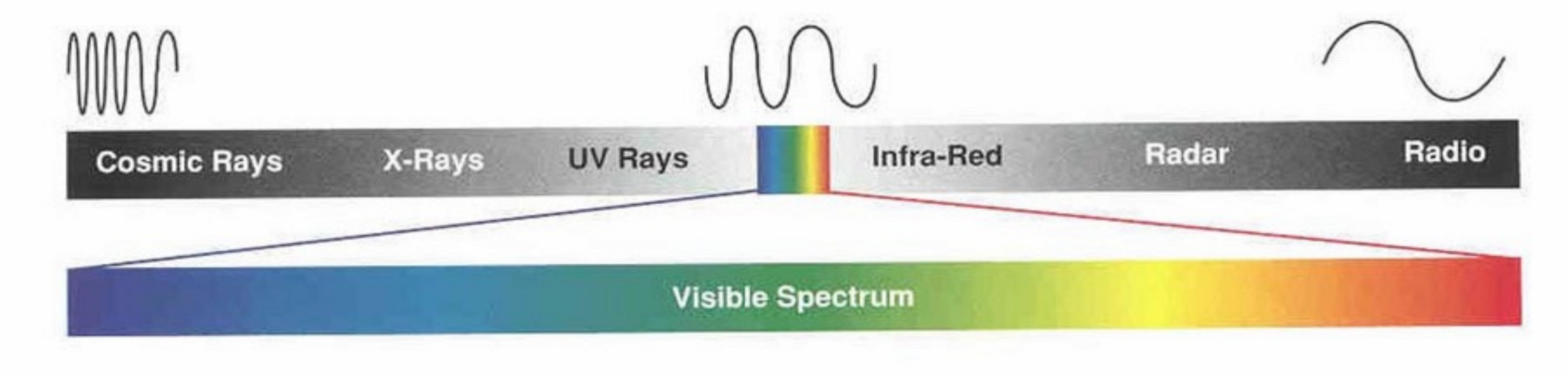
1. Image Fundamentals—image formation

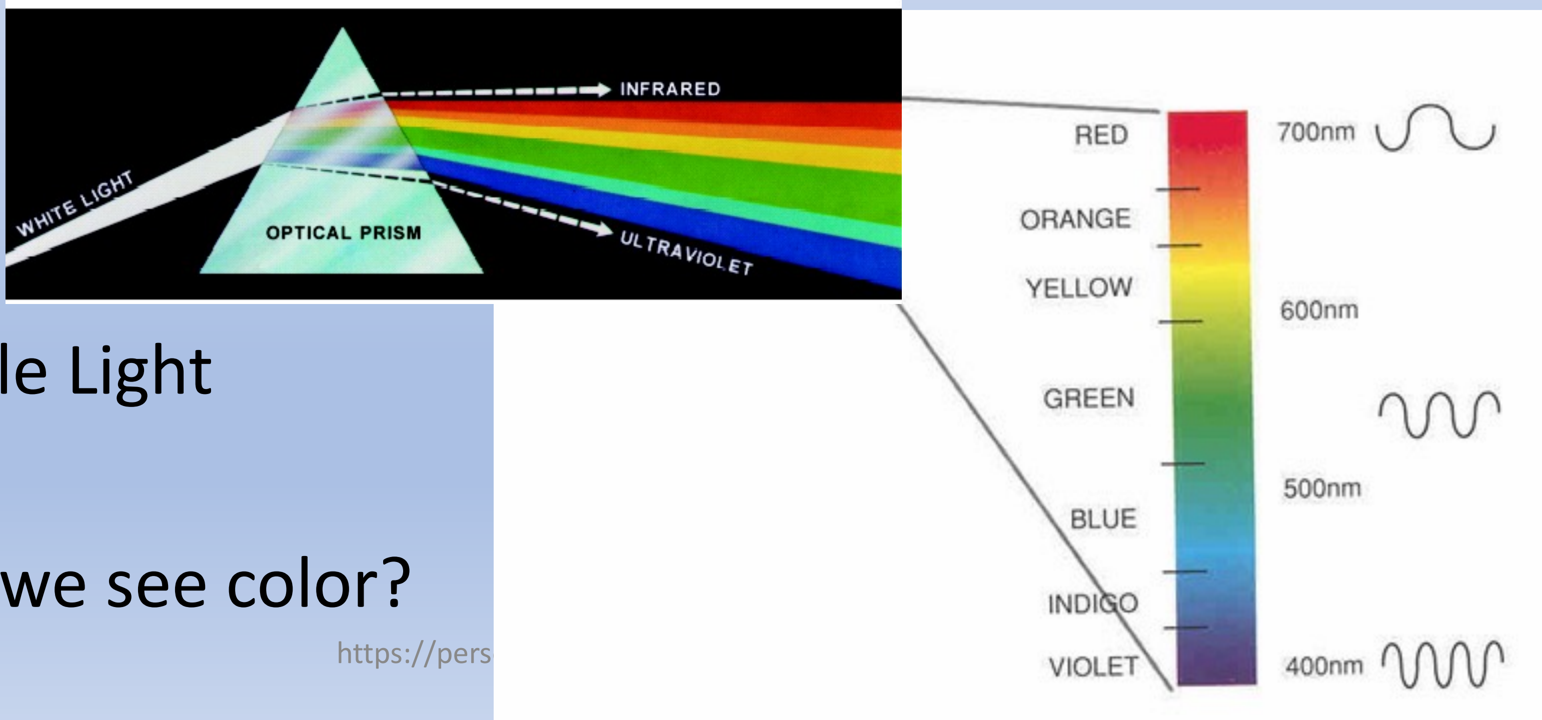
What is an image?



- Physically, an image is a two-dimensional (2-D) projection of a three-dimensional (3-D) scene, a visual representation, a vivid or graphic description of an object or scene.

1. Image Fundamentals—color image





The Visible Light

Why can we see color?

1. Image Fundamentals—Luminance and brightness

Light received/reflected from an object is

$$I(\lambda) = \rho(\lambda)L(\lambda)$$

where $\rho(\lambda)$ is the reflectivity of object, $L(\lambda)$ is the spectral energy distribution of the light source, λ is the wavelength in the visible spectrum, 350nm to 780 nm.

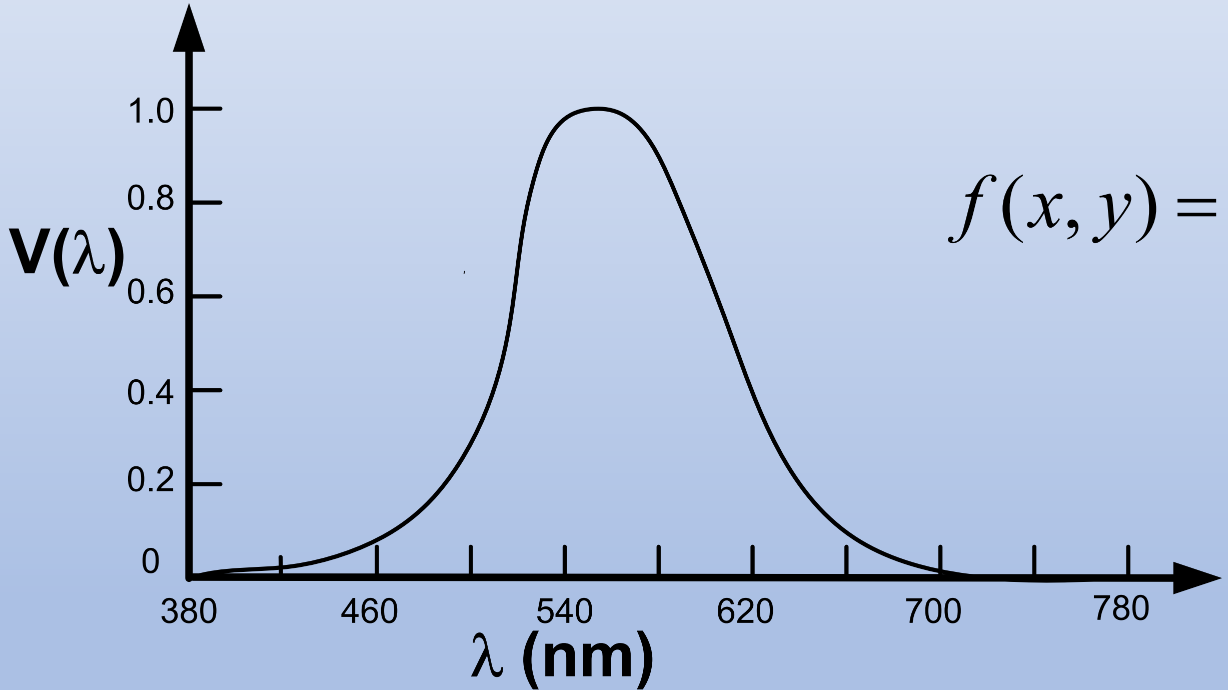
Luminance or intensity of a spatially distributed object with light distribution $I(x,y,\lambda)$ is defined as

$$f(x,y) = \int_0^\infty I(x,y,\lambda)V(\lambda)d\lambda$$

where $V(\lambda)$ is the relative spectral sensitivity function of the visual system

1. Image Fundamentals—Luminance and brightness

$V(\lambda)$ is a bell-shaped curve



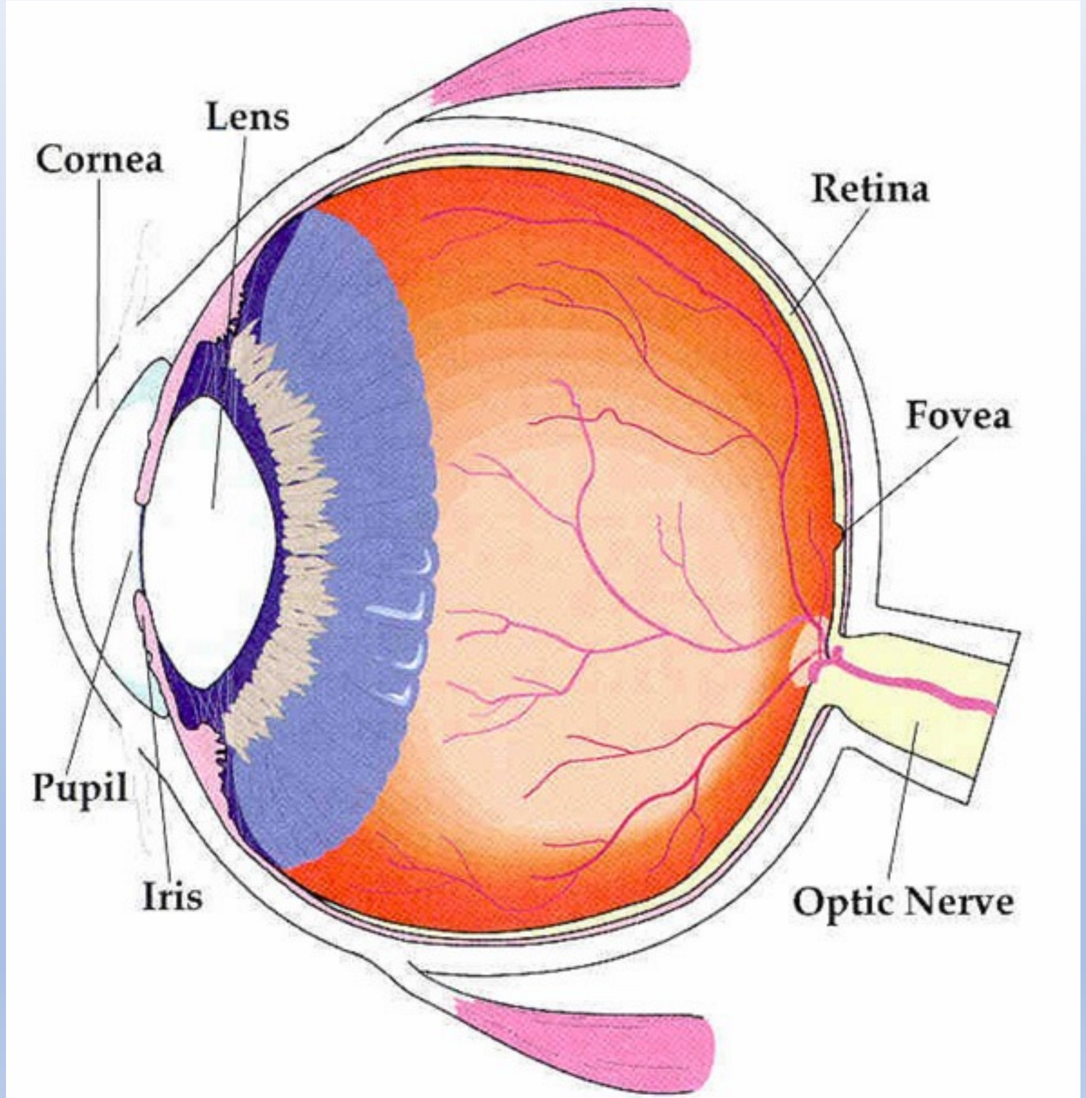
$$f(x,y)=\int_0^\infty I(x,y,\lambda)V(\lambda)d\lambda$$

$V(\lambda)$ is some kind of frequency response

1. Image Fundamentals—Human Perception

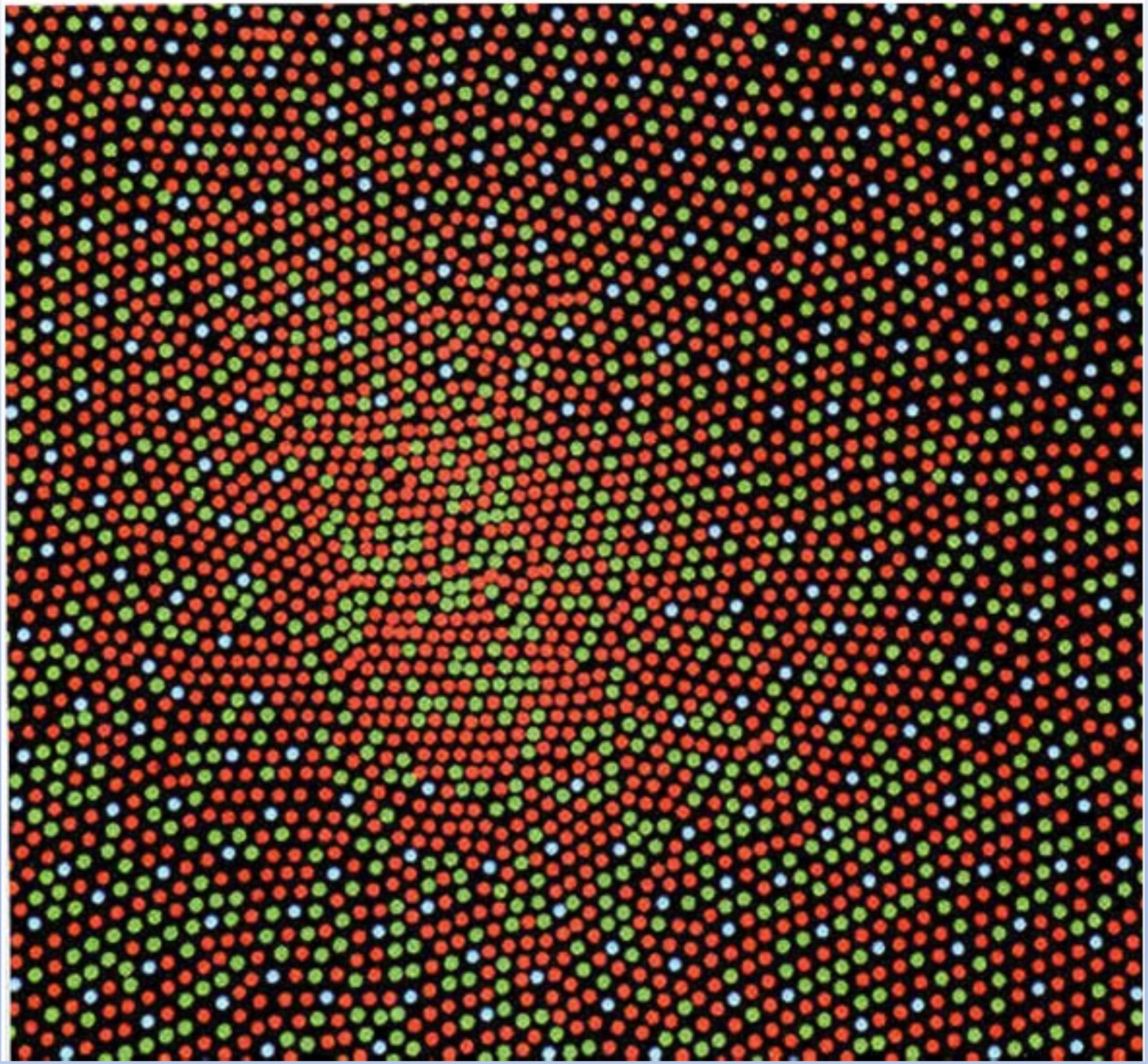
The Human eye

- Human eye has many cells shaped like cones to perceive light.
- There are three different type of cones with different spectral sensitivity function $V(\lambda)$, which help perceive colour.



1. Image Fundamentals—Human Perception

Relative proportions of L (red), M (green), and S (blue) cones in the human retina.



1. Image Fundamentals—Human Perception

- Color is a function of wavelength (frequency)

- Color Primaries: Red(R), Green(G), Blue(B)

$$f_1(x,y) = \int_0^{\infty} I(x,y,\lambda)V_1(\lambda)d\lambda$$

$$f_2(x,y) = \int_0^{\infty} I(x,y,\lambda)V_2(\lambda)d\lambda$$

$$f_3(x,y) = \int_0^{\infty} I(x,y,\lambda)V_3(\lambda)d\lambda$$

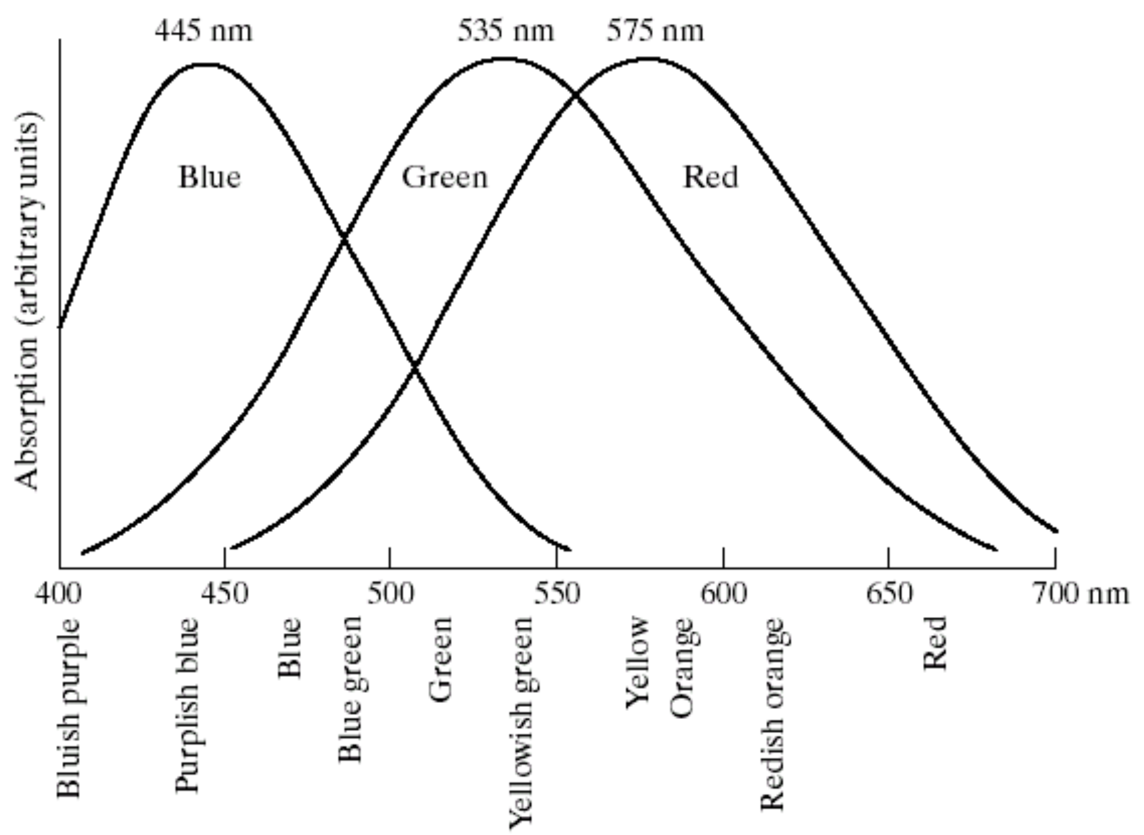
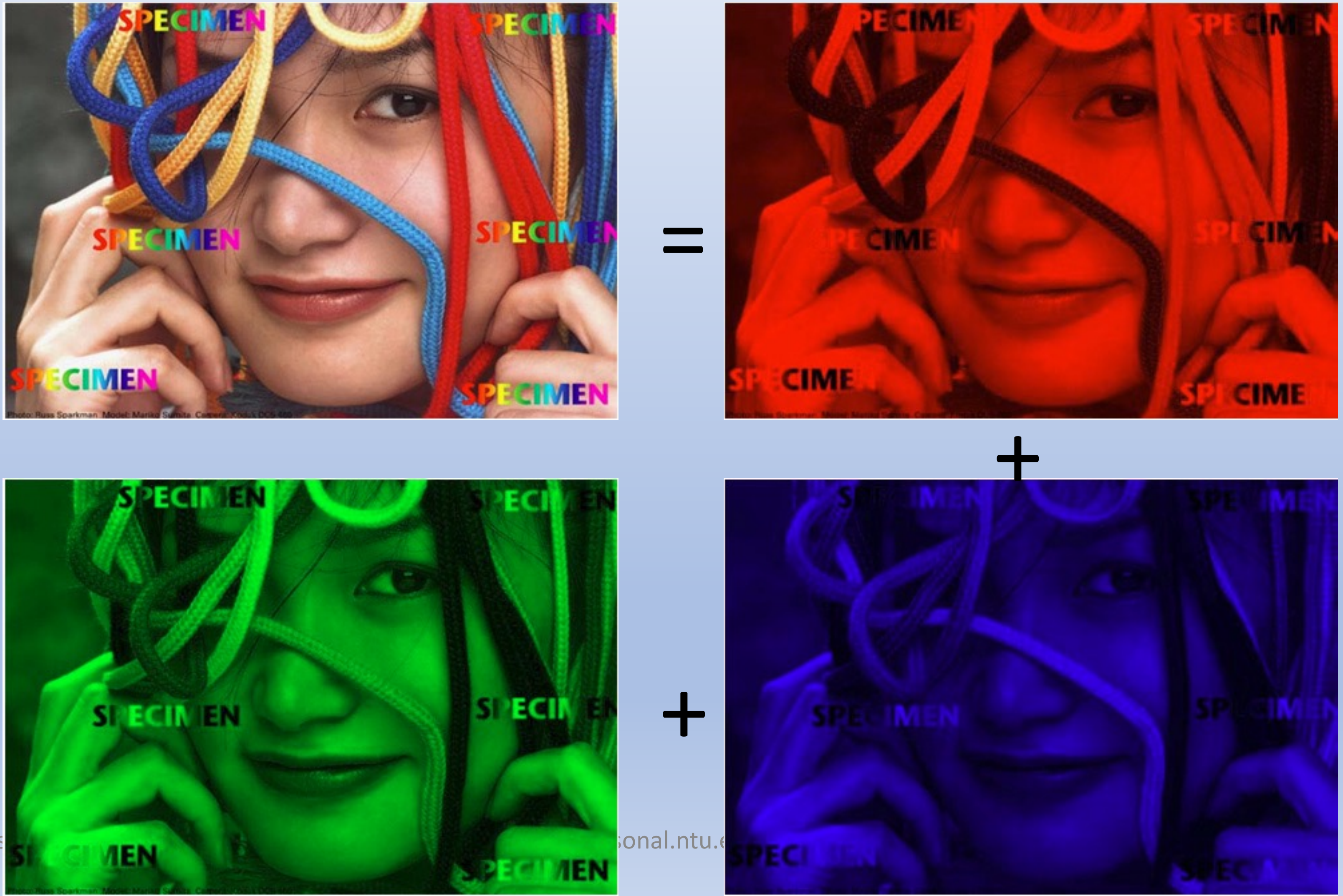


FIGURE 6.3 Absorption of light by the red, green, and blue cones in the human eye as a function of wavelength.

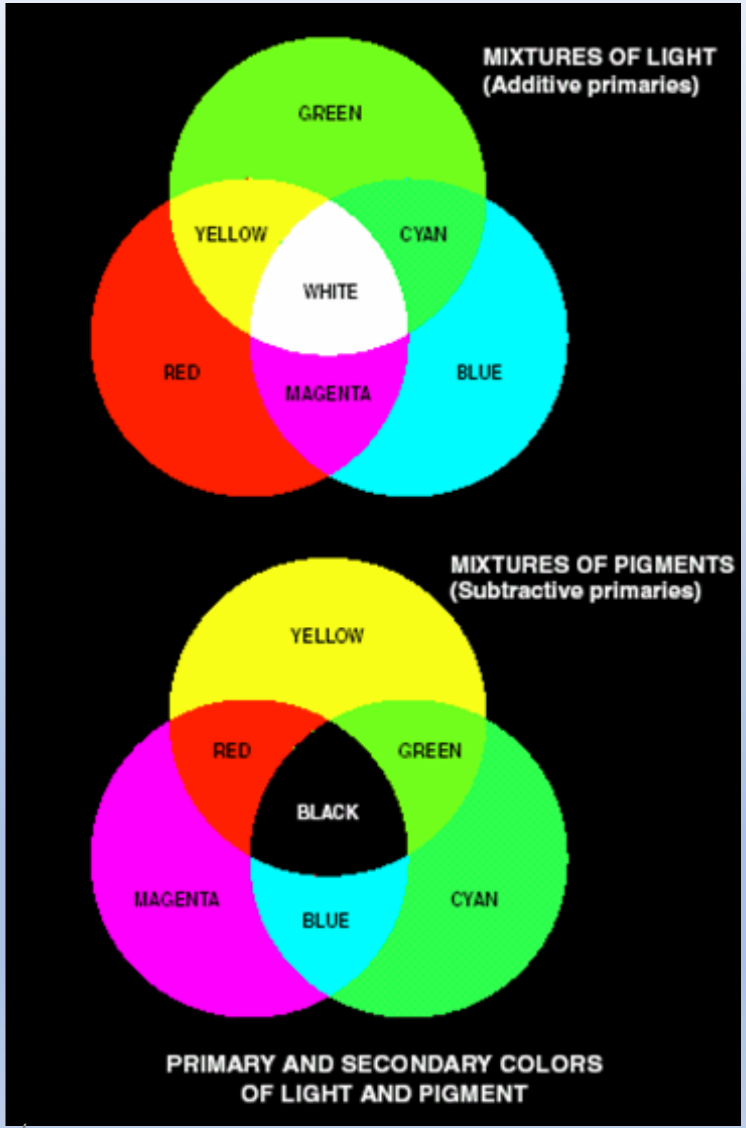
1. Image Fundamentals—color image

Three values per sample (pixel) are required for a color image.



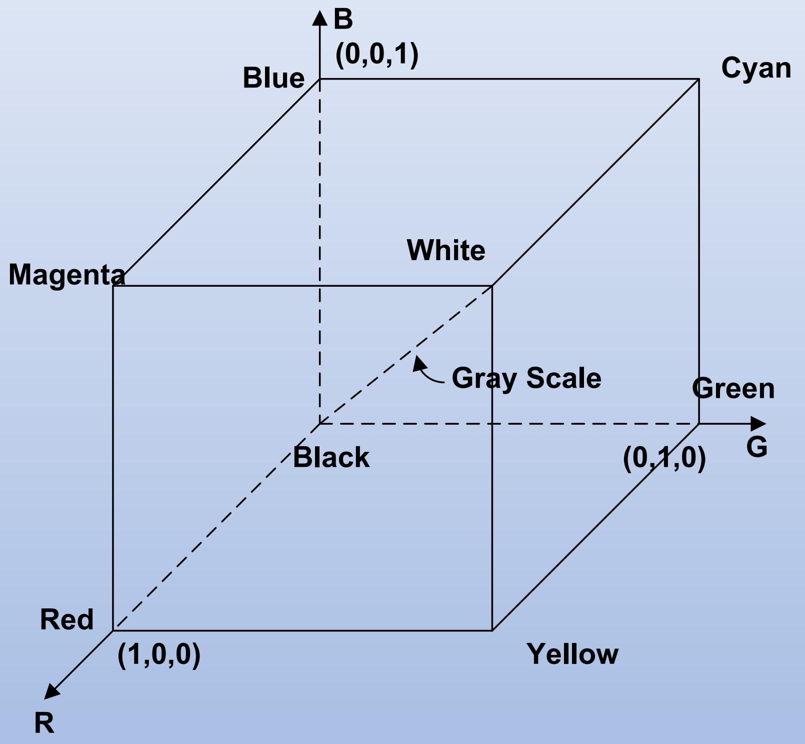
1. Image Fundamentals—color space

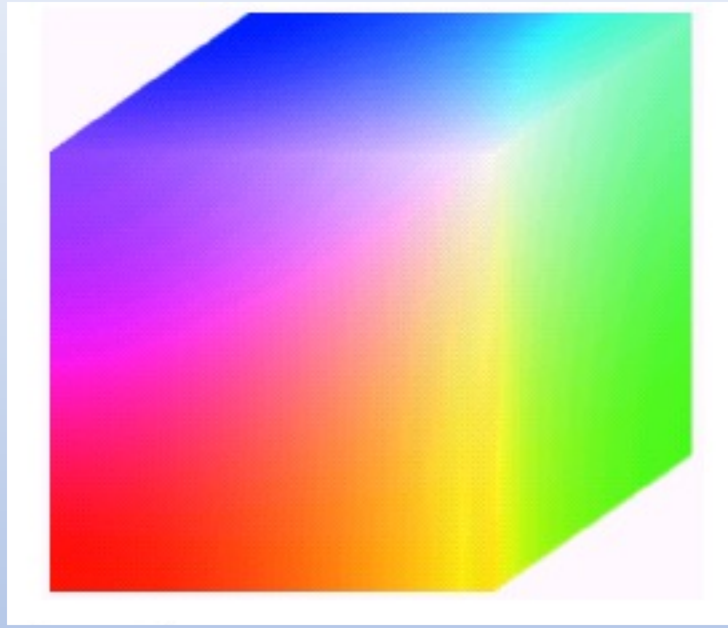
- Additive primaries: Red(R), Green(G), Blue(B)
- Subtractive primaries: Cyan, Magenta, Yellow
- A color can be specified in terms of the amounts of three primaries required: $c = a\text{x}p1 + b\text{x}p2 + c\text{x}p3$, where ($p$1,$p$2,$p$3) is a particular set of primaries.
- A color space is a 3D space, defined to describe color in some standard way.



1. Image Fundamentals—color space

- RGB color spaces:





Z. Lu, X.D. Jiang and A. Kot, "[A Color Channel Fusion Approach for Face Recognition](),"
IEEE Signal Processing letters, vol. 22, no. 11, pp. 1839 - 1843, Nov. 2015.

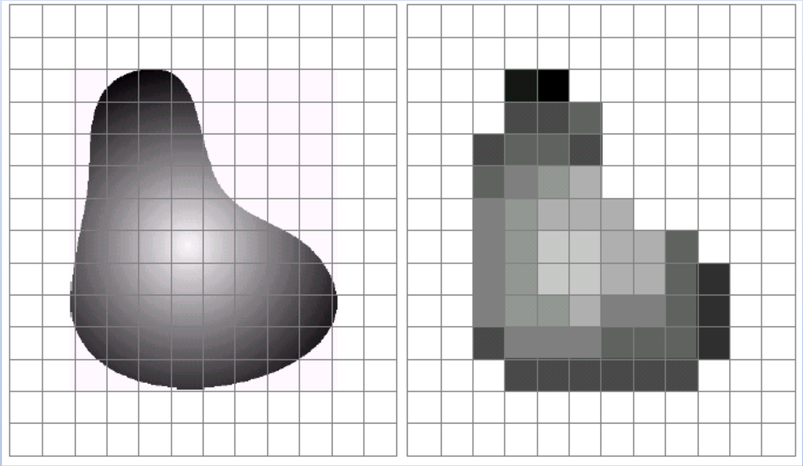
1. Image Fundamentals—color space

- Common color spaces:
  - RGB - hardware oriented, used for monitors, video cameras
  - rgb - Normalized RGB
  - CMY (Cyan-Magenta-Yellow) - used for color printer
  - YIQ (luminance, in-phase, quadrature.) - color TV broadcast
  - HSI (HSV) (Hue, saturation, intensive/value) - used for color manipulation
  - CIE-Luv, CIE-Lab (lightness, red-green, yellow-blue) - used for color differentiation.
  - sRGB – used for device independent digital image display.

Z. Lu, X. Jiang, A. Kot, "Color Space Construction by Optimizing Luminance and Chrominance Components for Face Recognition," ***Pattern Recognition***, vol. 83, pp. 456-468, 2018.

1. Image Fundamentals—representation of image

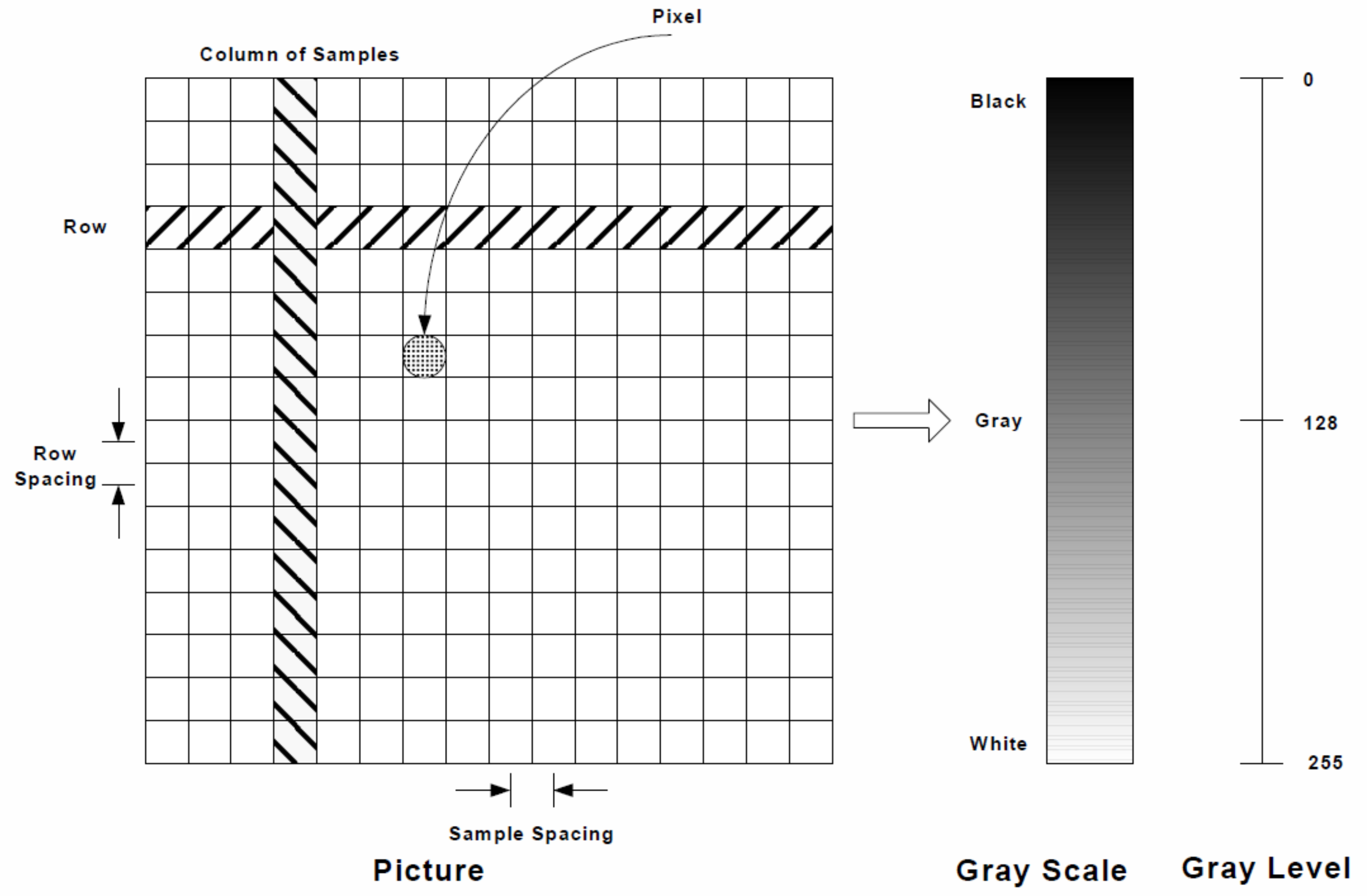
- Mathematically, an Image is a two-dimensional (2-D) function $f(x,y)$, a function of the two spatial coordinates.



- A digital image is a sampled, quantized version of a 2D light-intensity function generated by optical means.
- The function is usually sampled in an equally spaced rectangular grid pattern, with its amplitude quantized in equal intervals.
- A digital image is $f(x,y)$ where x, y and f are all finite and discrete quantities.

1. Image Fundamentals—image digitization

- Digitization of an image: Spatial sampling or discretization and Intensity or gray-level quantization



1. Image Fundamentals—representation of image

- Denote an image, a 2-D light-intensity function, as $f(x,y)$.
- The value or amplitude of f at spatial coordinates (x, y) indicates the intensity (brightness, grey level) of the image at that point.
- $f(x,y)$ must be digitized both spatially and in amplitude for computer processing.
- Digitization of spatial coordinates $f(x,y)$ is referred to as spatial sampling or discretization.
- Digitization of intensity amplitude $f(x,y)$ is referred to as intensity or gray-level quantization.

1. Image Fundamentals—representation of image

- The (spatial) resolution of a digital image refers to the size of the mxn array of which the image is sampled.

$$f(x,y) = e.g.: \sin[2\pi(u\sin(\phi)x + v\sin(\phi)y]$$

$$\begin{bmatrix} f(1,1) & f(1,2) & \cdots & f(1,n) \\ f(2,1) & f(2,2) & \cdots & f(2,n) \\ \vdots & \vdots & \ddots & \vdots \\ f(m,1) & f(m,2) & \cdots & f(m,n) \end{bmatrix} \text{or } \mathbf{F} = \begin{bmatrix} f_{11} & f_{12} & \cdots & f_{1n} \\ f_{21} & f_{22} & \cdots & f_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ f_{m1} & f_{m2} & \cdots & f_{mn} \end{bmatrix}$$

- The gray-level resolution of a digital image refers to the number of gray levels (intensities) $g=2^b$, where b is the number of bits per sample.

1. Image Fundamentals—spatial resolution

600 x 408 pixels

300 x 204 pixels

150 x 102 pixels

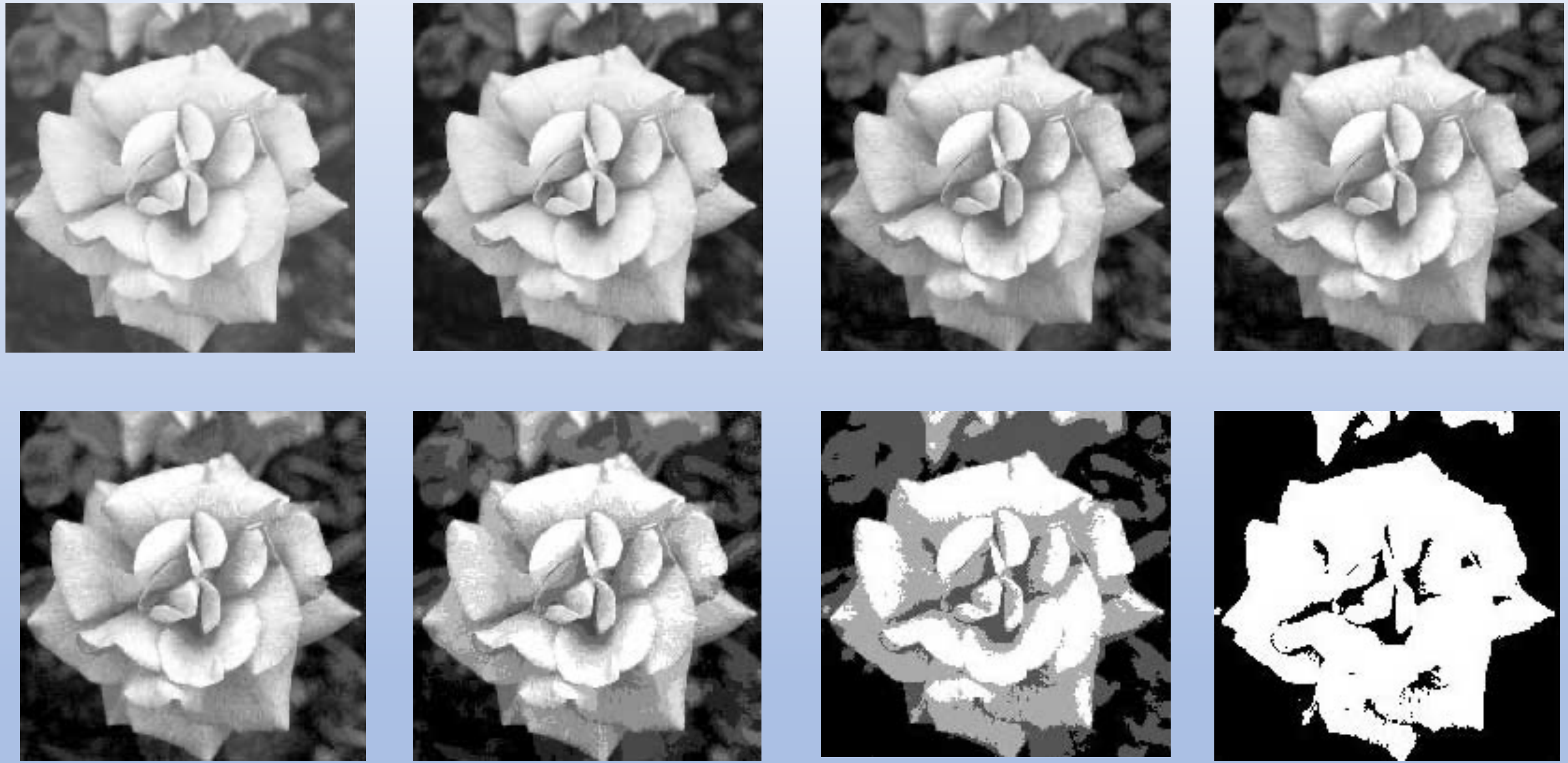
1. Image Fundamentals—spatial resolution

a b c
d e f

FIGURE 2.20 (a) 1024 × 1024, 8-bit image. (b) 512 × 512 image resampled into 1024 × 1024 pixels by row and column duplication. (c) through (f) 256 × 256, 128 × 128, 64 × 64, and 32 × 32 images resampled into 1024 × 1024 pixels.

1. Image Fundamentals—gray-level resolution

256 levels 128 levels 64 levels 32 levels



16 levels 8 levels 4 levels 2 levels

1. Image Fundamentals—image histogram

- The histogram of a digital image $f(x,y)$ with gray level range $[0, L]$ is a discrete function

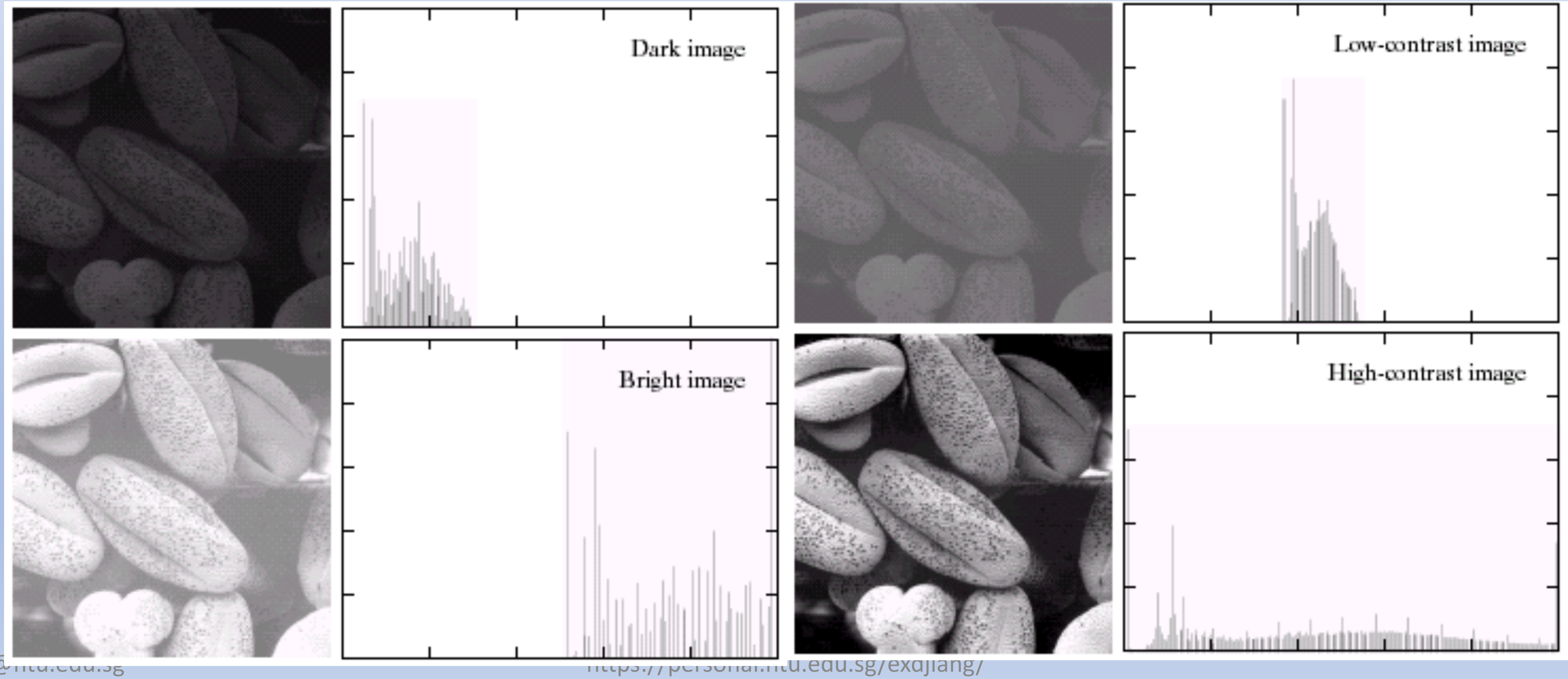
$$p_f(f) = \frac{n_f}{n}$$

- where f is the gray level, f = 0, 1, 2, …, L. n_f is the number of pixels with that gray level. n is the total number of pixels in the region of the image being processed.
- It is clear that the histogram $p_f(f)$ of a digital image is the frequency of occurrence of gray-level f in the image.
- Histogram shows the frequency distribution of gray-level f.
- Obviously, $p_f(f) \geq 0, \quad \text{and} \quad \sum_{f=0}^{L} p_f(f) = 1$
- If we treat the pixel gray-level of an image as a random variable, the histogram $p_f(f)$ is an estimate of the probability of occurrence of gray-level f. over an image.

1. Image Fundamentals—image histogram

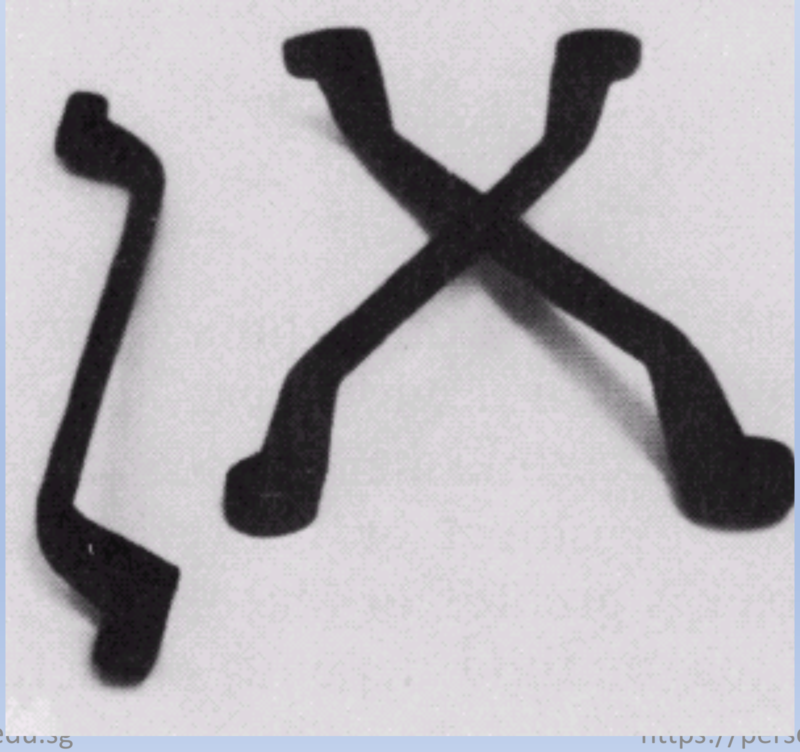
- **Understand the histogram**: The shape of image histogram provides many clues as to the characteristics of image. For example,
  - a narrowly distributed histogram indicates a low-contrast image

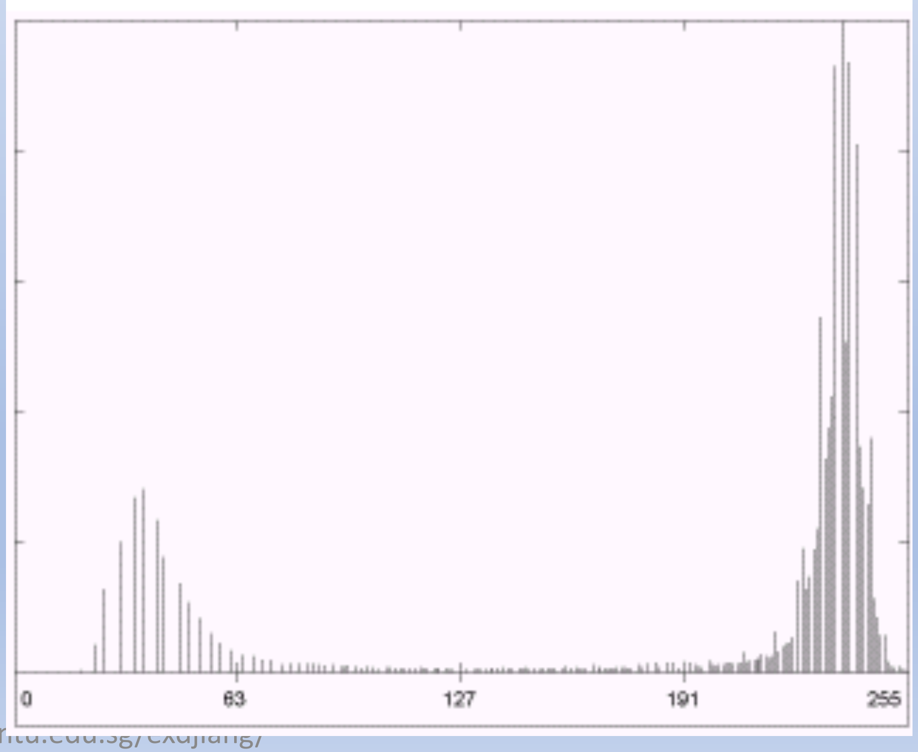
$f(x,y)$ $\quad$ $p_f(f)$



1. Image Fundamentals—image histogram

- Understand the histogram: The shape of image histogram provides many clues as to the characteristics of image. For example,
  - A bimodal histogram suggests that the image contains an object with a narrow amplitude range against a background of differing amplitude.





1. Image Fundamentals—image histogram

➢ Some Journal papers exploring histogram features:

J. Ren, X. Jiang and J. Yuan, "LBP Encoding Schemes Jointly Utilizing the Information of Current Bit and Other LBP Bits," ***IEEE Signal Processing letters***, vol. 22, no. 12, pp. 2373 - 2377, Dec. 2015.

J. Ren, X. Jiang and J. Yuan, "A Chi-Squared-Transformed Subspace of LBP Histogram for Visual Recognition," ***IEEE Trans. Image Processing***, vol. 24, no. 6, pp. 1893-1904, June, 2015.

J. Ren, X. Jiang and J. Yuan, "Learning LBP Structure by Maximizing the Conditional Mutual Information," ***Pattern Recognition*, **vol. 48, no. 10, pp. 3180 - 3190, Oct. 2015.

A. Satpathy, X. Jiang and H. Eng, "LBP Based Edge-Texture Features for Object Recognition," ***IEEE Trans. Image Processing***, vol. 23, no. 5, pp. 1953-1964, May, 2014.

J. Ren, X. Jiang, J. Yuan and W. Gang, "Optimizing LBP Structure for Visual Recognition Using Binary Quadratic Programming," ***IEEE Signal Processing letters***, vol. 21, pp. 1346-1350, Nov. 2014.

A. Satpathy, X. Jiang and H. Eng, "Human Detection by Quadratic Classification on Subspace of Extended Histogram of Gradients," ***IEEE Trans. Image Processing***, vol. 23, pp. 287-297, Jan, 2014.

J. Ren, X. Jiang and J. Yuan, "Noise-Resistant Local Binary Pattern with an Embedded Error-Correction Mechanism," ***IEEE Trans. Image Processing***, vol. 22, no. 10, pp. 4049-4060, Oct, 2013.

1. Image Fundamentals—what is image processing?

- An digital image is a two dimensional numerical representation (a 2-D function $f(x,y)$ or a mxn matrix) of a 3D scene or an object.

What is digital image processing?

- Digital image processing is a series of machine or computer operations leading to some desired results.
- The operations could be, should be, and are desired to be described by mathematics.
- Digital image processing, starting with an image or a set of images, produces a modified version of the image(s) or extract more "meaningful" information (features) from the image(s) or understand (recognize) the meaning of the image content.

1. Image Fundamentals—Why need image processing?

- **Why need image processing?**
  - Visualization :
    - Contrast enhancement, noise removal, visual quality improvement, pseudo colouring
  - Image understanding
    - Extraction of image properties such as colour, shape, texture, edges, lines, curves, corners.
  - Automated Guided vehicle
    - Identifying road, vehicle, pedestrian, traffic signs
  - Visual servicing
    - Automated robot control
  - Security
    - Intrusion detection, biometrics
  - Information retrieval
    - Image content based search

1. Image Fundamentals—image processing examples

- Contrast Enhancement

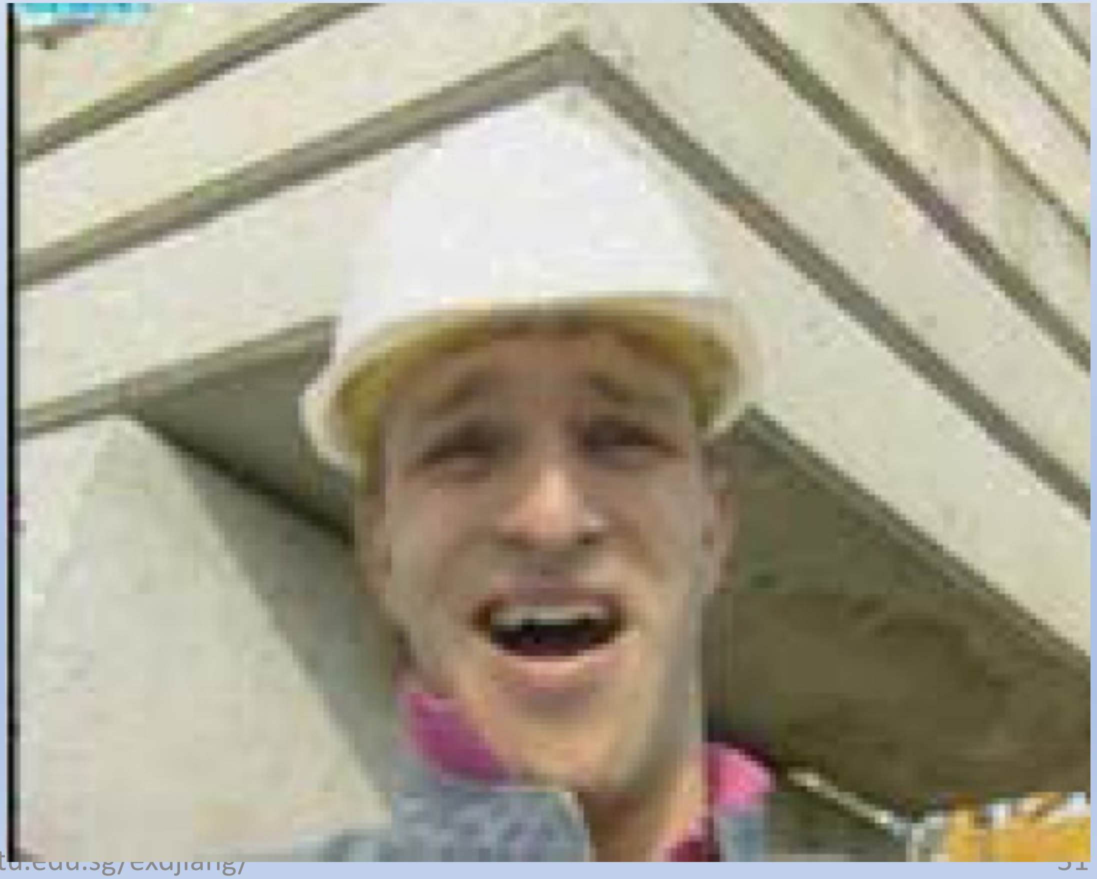
1. Image Fundamentals—image processing examples

➢ Deblurring

1. Image Fundamentals—image processing examples

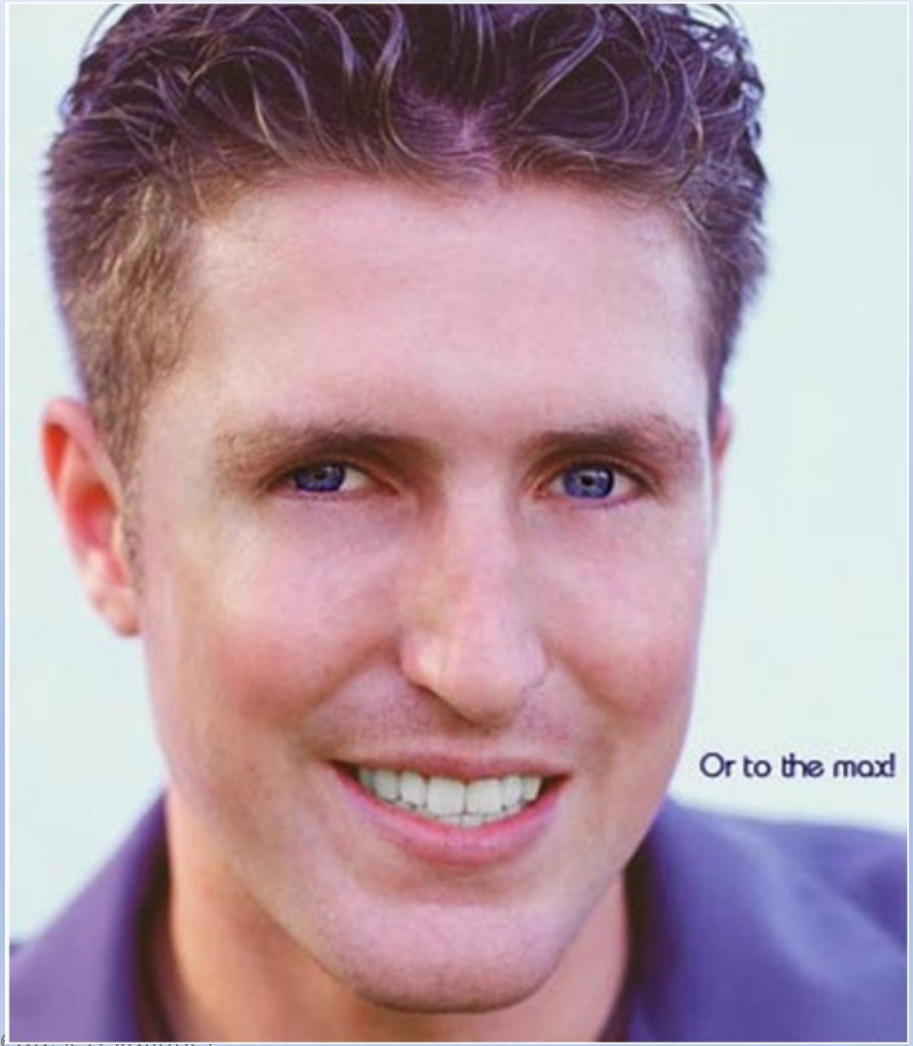
- Denoising: noise attenuation

Noise corrupted image and the image processed by DIP



1. Image Fundamentals—image processing examples

- Beautifying

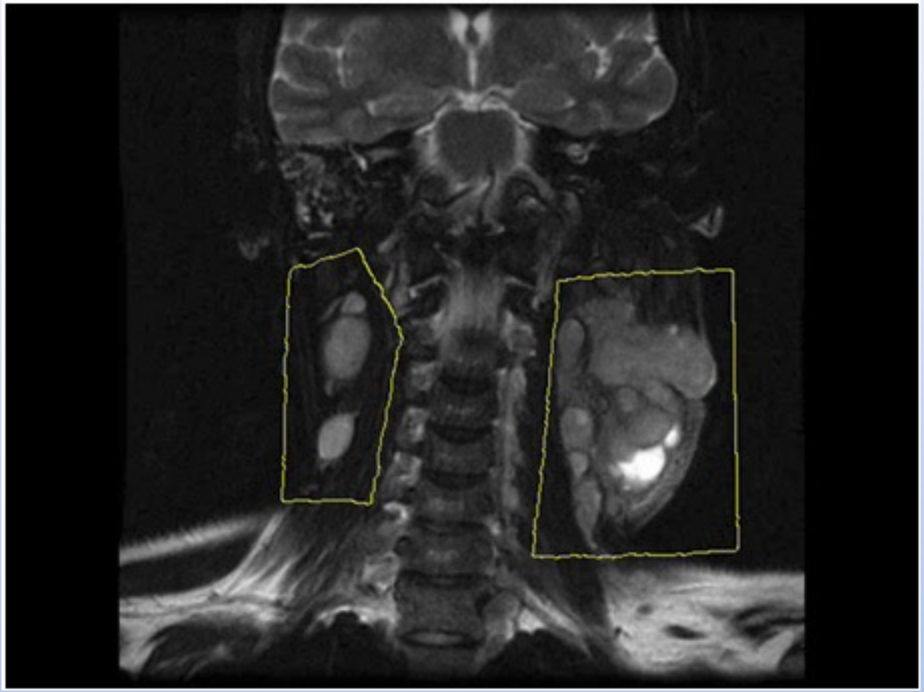


1. Image Fundamentals—image processing examples

➢ Restoration and Retouching

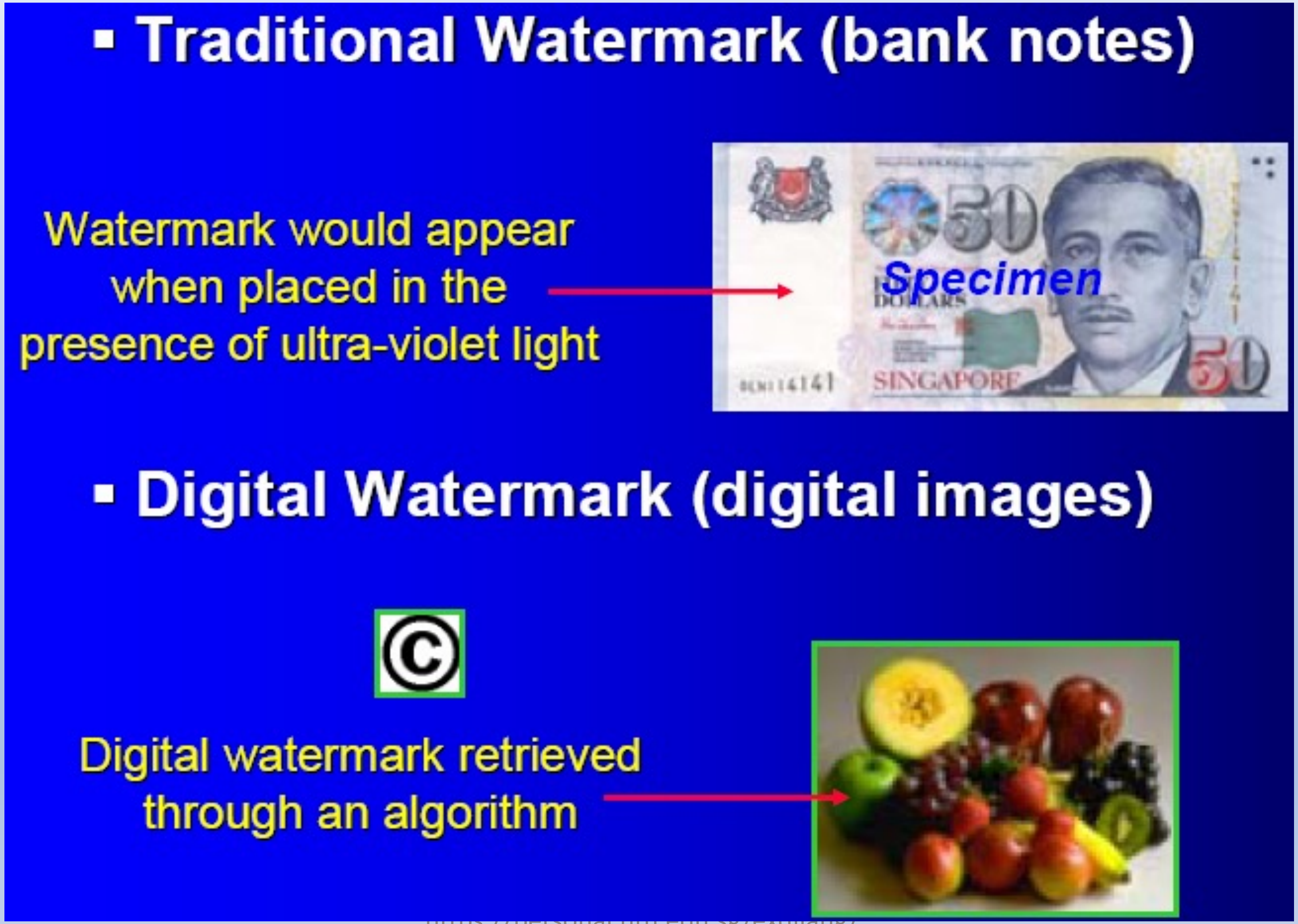
1. Image Fundamentals—image processing examples

- Segmentation



1. Image Fundamentals—image processing examples

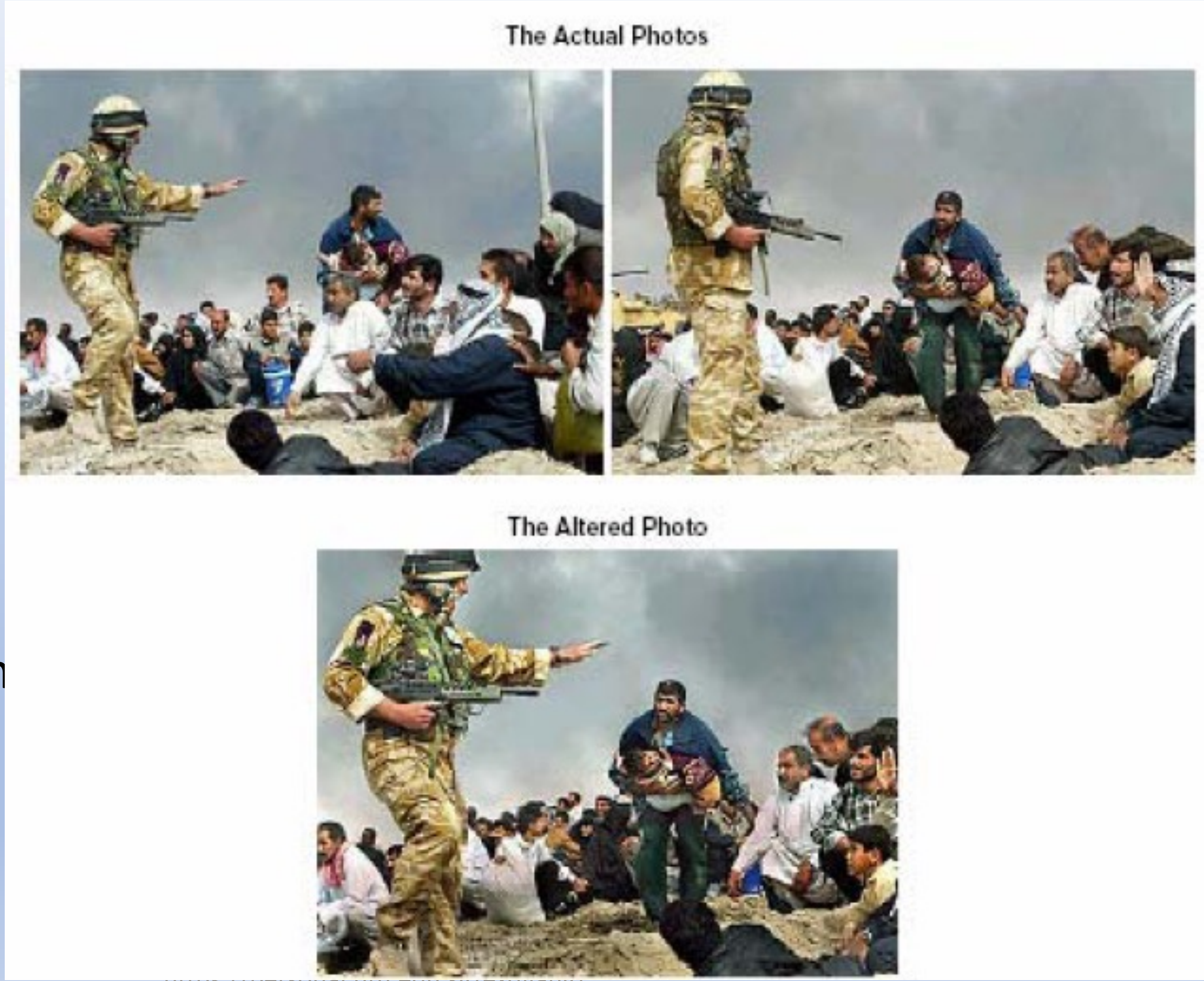
- Digital Watermarking



1. Image Fundamentals—image processing examples

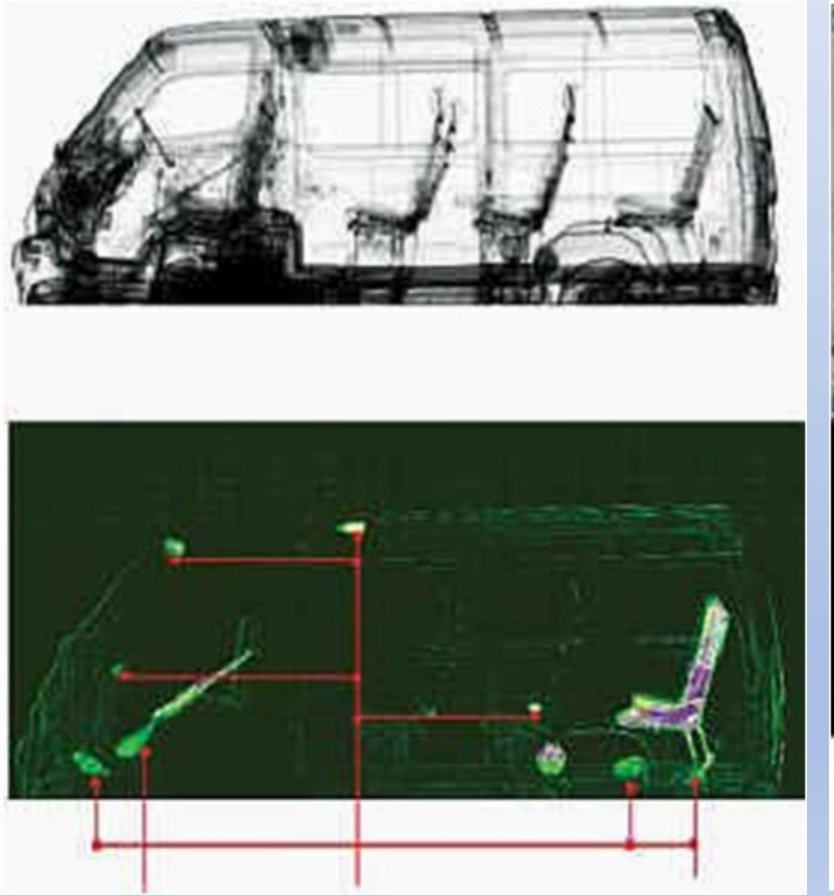
Digital content tampering detection

The photograph is a composite created by ex-composite LA Times photographer Brian Walski.
He was dismissed when the photograph was found be altered.



1. Image Fundamentals—image processing examples

- Objection Detection

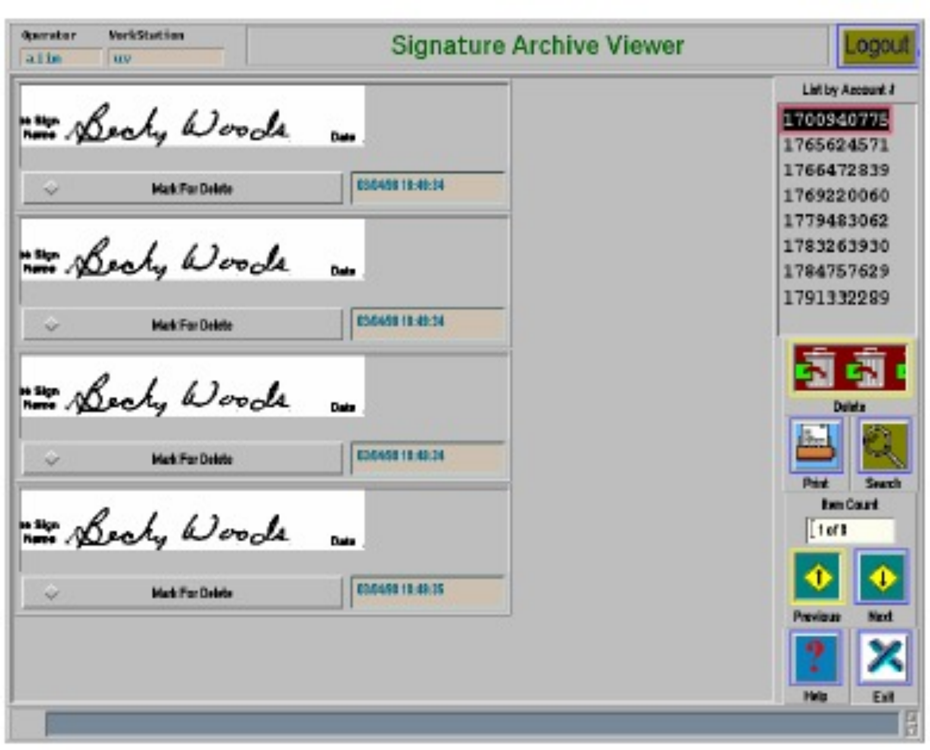


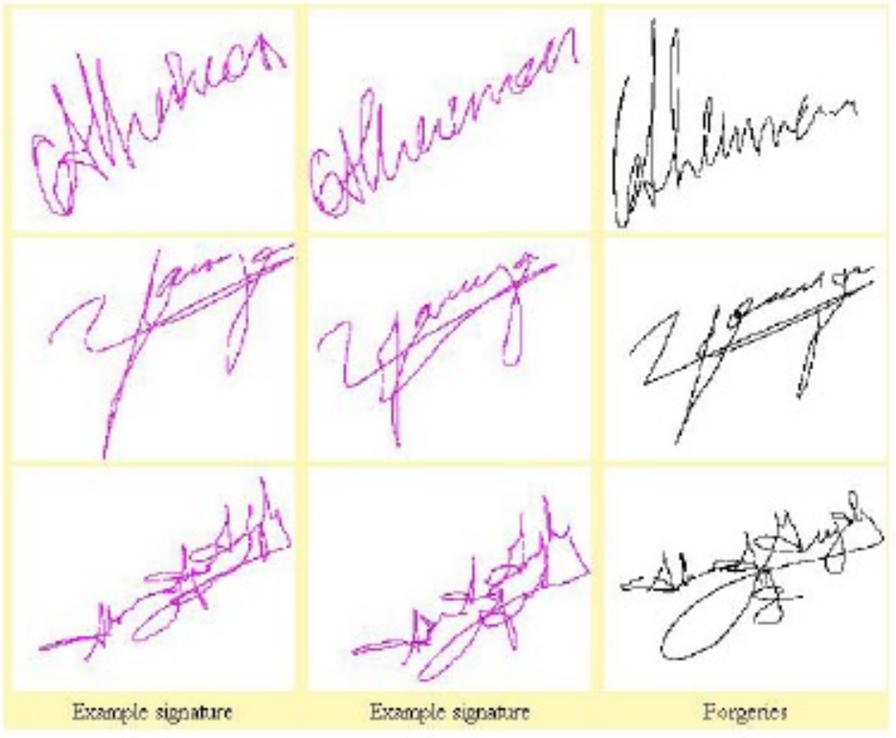
1. Image Fundamentals—image processing examples

- Face Detection

1. Image Fundamentals—image processing examples

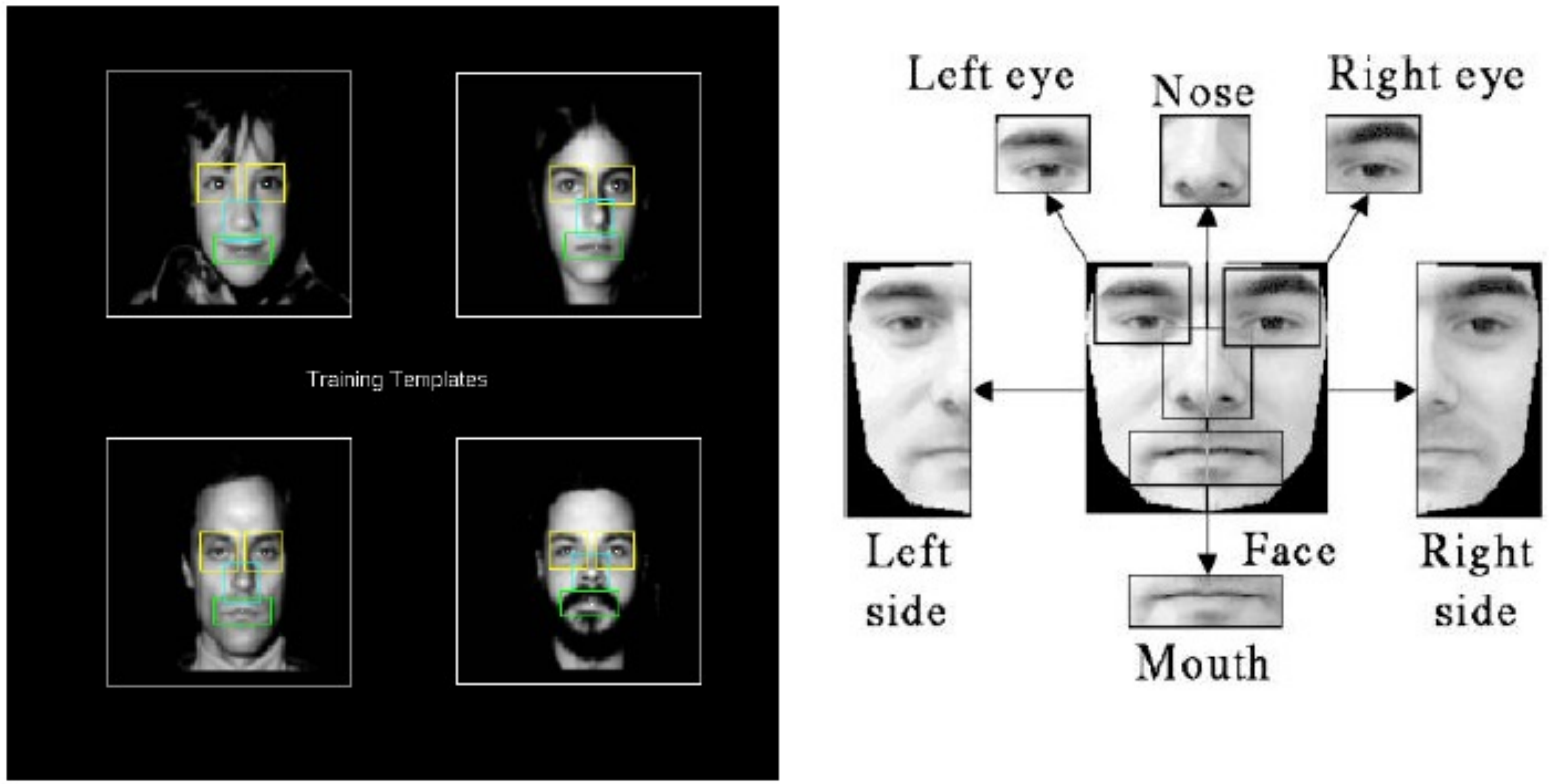
- Automatic signature verification and identification



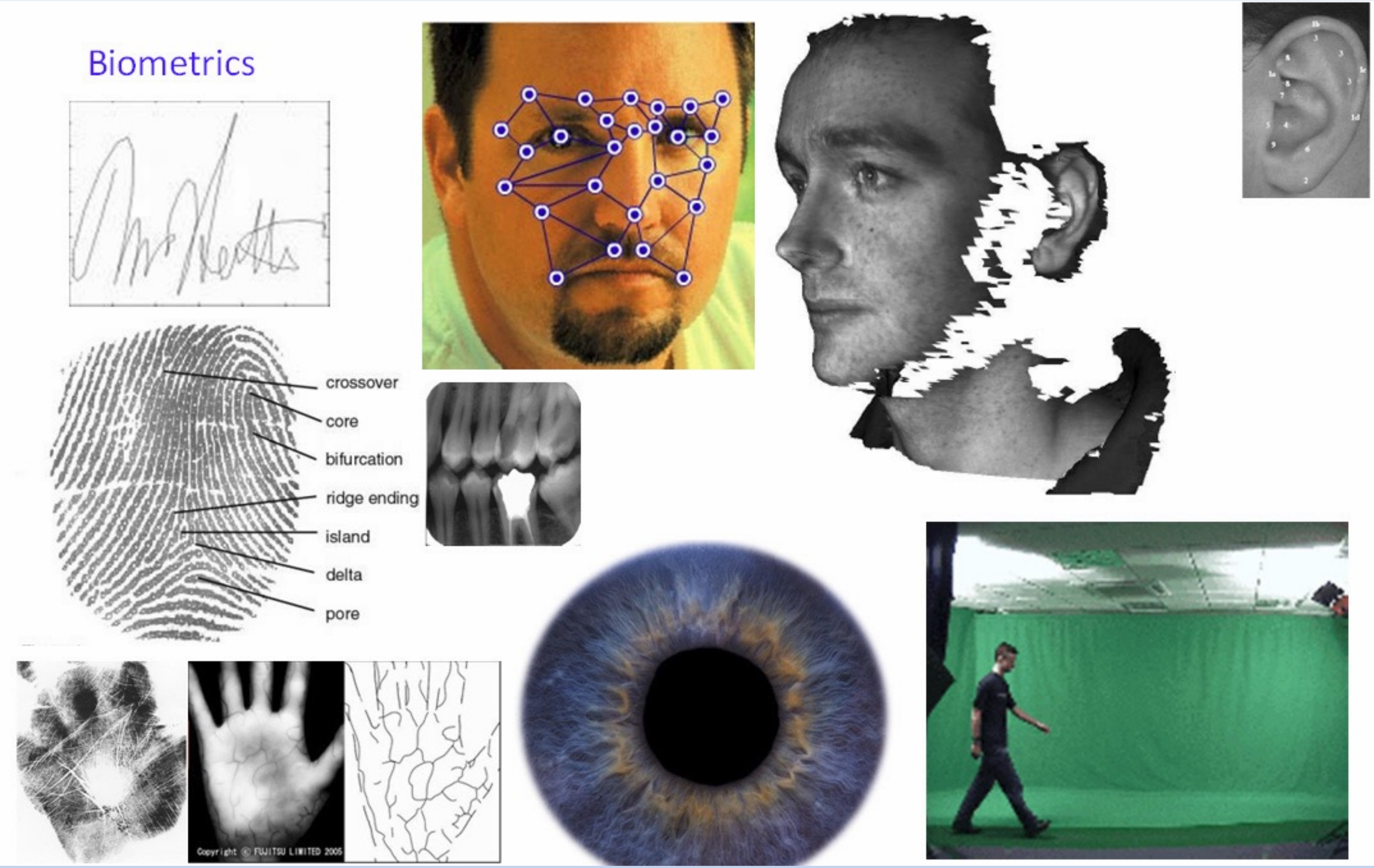


1. Image Fundamentals—image processing examples

Are these feature effective for Automatic Face Recognition?



1. Image Fundamentals—image processing examples



1. Image Fundamentals—image processing examples

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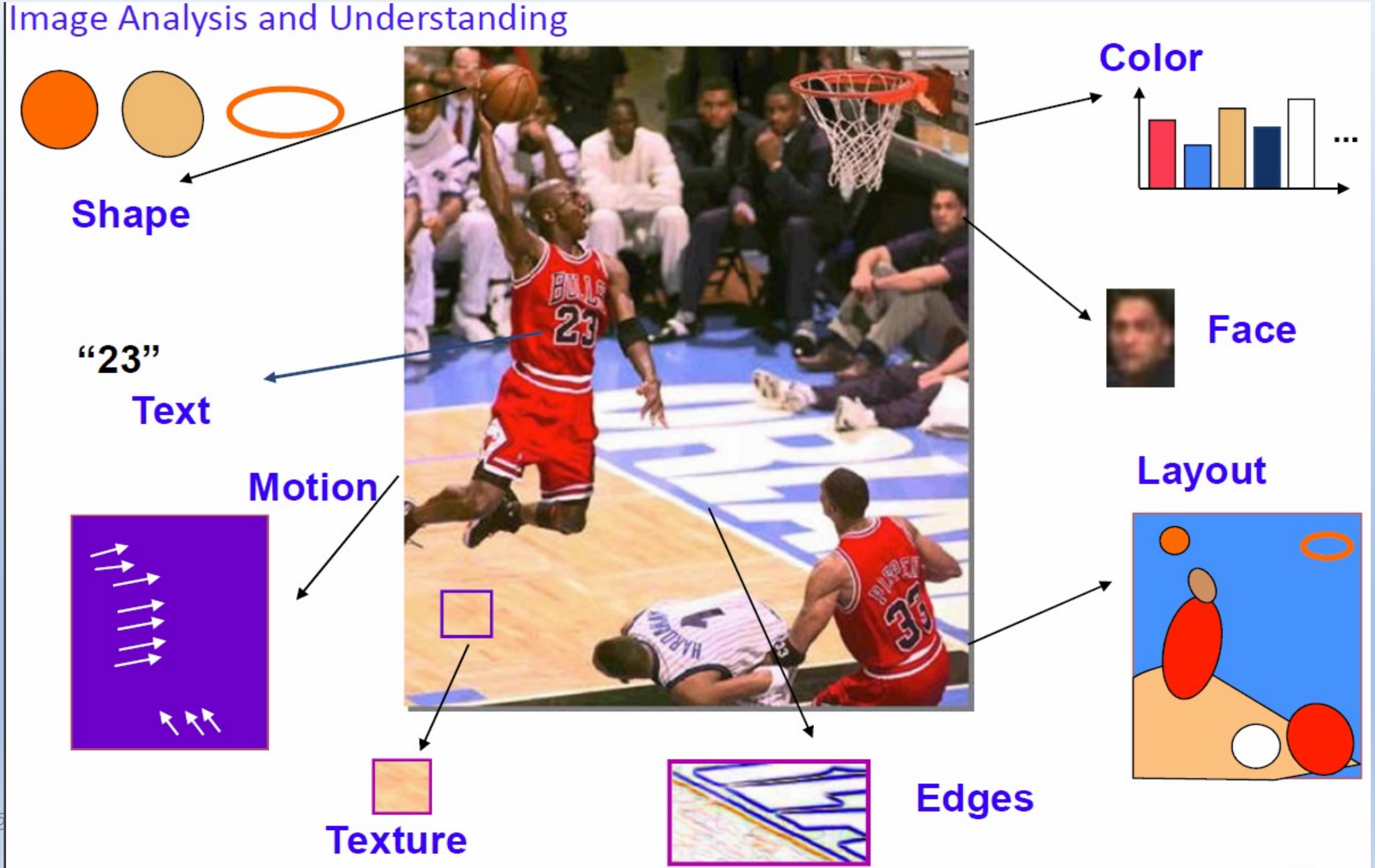
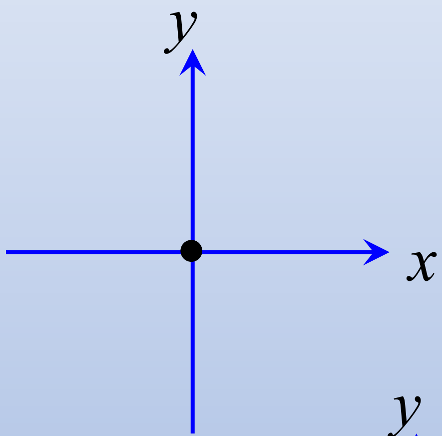


2. LSI Systems & Transforms—Outline

- Image Decomposition and Linear Shift Invariant Image Processing System
- Two-dimensional Convolution and its Properties
- Two-dimensional Fourier Transform and its Properties
- Image Sampling

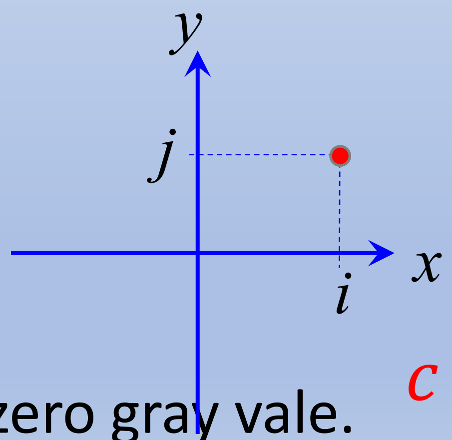
2. LSI Systems & Transforms—basic image element

- A digital image can be represented by a 2-D function with two integer arguments, such as $f(x,y)$ where x and y are integers
- The basic element image is the impulse



$$\delta(x,y)=\begin{cases}1, & x=y=0\\ 0, & \text{otherwise}\end{cases}$$

- Shifted and scaled the impulse



$$f(x,y)=c\delta(x-i,y-j)=\begin{cases}c, & x=i, y=j\\ 0, & \text{otherwise}\end{cases}$$

- It describes any arbitrary pixel with all other pixels zero gray vale.

$$c \Longrightarrow f(i,j)$$

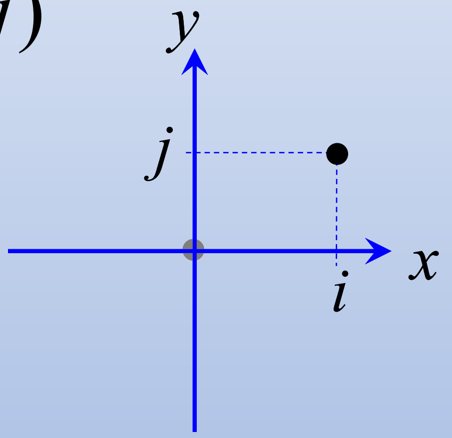
2. LSI Systems & Transforms—image decomposition

- Any image $f(x,y)$ then can be represented by the sum of a number of shifted and scaled impulses

$$f(x,y) = \sum_{j=-\infty}^{\infty} \sum_{i=-\infty}^{\infty} f(i,j)\delta(x-i, y-j)$$

Or

$$f(x,y) = \sum_{j=-n}^{n} \sum_{i=-m}^{m} f(i,j)\delta(x-i, y-j)$$



- For example, a constant gray level 215 square of size 11X11 centered at (0,0) is:

$$f(x,y) = \sum_{j=-5}^{5} \sum_{i=-5}^{5} 215\delta(x-i, y-j)$$

2. LSI Systems & Transforms—2-D convolution

- A processing system relates any input image $f(x,y)$ to a unique output image $g(x,y)$, given by

$$g(x,y)=T\{f(x,y)\}=T\left\{\sum_{j=-\infty}^{\infty}\sum_{i=-\infty}^{\infty}f(i,j)\delta(x-i,y-j)\right\}$$

- If the processing system is linear, then

$$g(x,y)=T\{f(x,y)\}=\sum_{j=-\infty}^{\infty}\sum_{i=-\infty}^{\infty}f(i,j)T\{\delta(x-i,y-j)\}$$

- If define the output image of the input impulse image as impulse response of the system,

$h(x,y)\triangleq T\{\delta(x,y)\}$. Then for shift invariant system:

$$T\{\delta(x-i,y-j)\}=h(x-i,y-j)$$

2. LSI Systems & Transforms—2-D convolution

- Therefore, given an input image $f(x,y)$, a linear and shift-invariant (LSI) image processing system T produces the output image $g(x,y)$ by

$$f \qquad = \qquad \delta$$

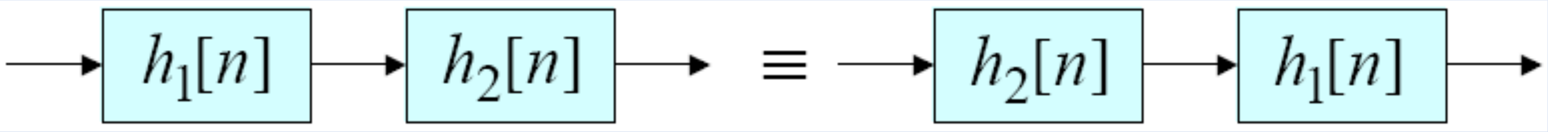
$$g(x,y) = T\{f(x,y)\} = \sum_{j=-\infty}^{\infty}\sum_{i=-\infty}^{\infty} f(i,j)h(x-i,y-j)$$

$$\triangleq f(x,y) * h(x,y) = \sum_{j=-\infty}^{\infty}\sum_{i=-\infty}^{\infty} h(i,j)f(x-i,y-j)$$

- A LSI system is completely characterized by its impulse response $h(x,y)$. $*$ is the convolution operator.
- For any LSI image processing system, the output image equals to the input image convolving with the impulse response of the system.

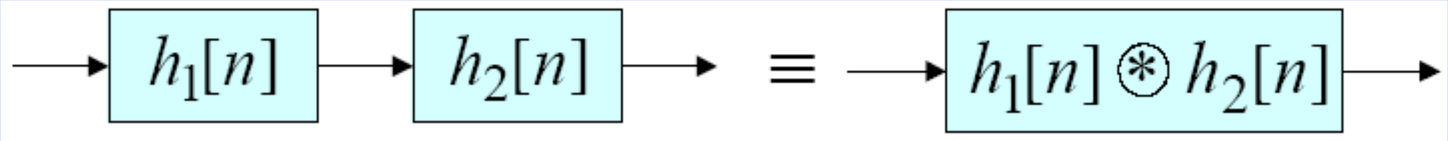
2. LSI Systems & Transforms—convolution property

- **Commutative:** $f(x,y)*h(x,y)=h(x,y)*f(x,y)$



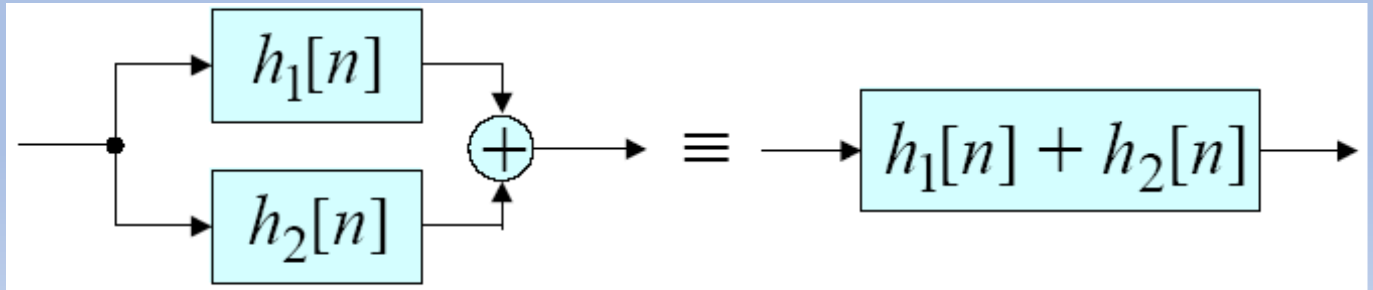
- **Associative:**

$$f(x,y)*\left(h_1(x,y)*h_2(x,y)\right)=\left(f(x,y)*h_1(x,y)\right)*h_2(x,y)$$



- **Distributive:**

$$f(x,y)*\left(h_1(x,y)+h_2(x,y)\right)=f(x,y)*h_1(x,y)+f(x,y)*h_2(x,y)$$



2. LSI Systems & Transforms—what is $h(x,y)$?

Understanding the impulse response:

- Impulse response $h(x,y)$ of an image processing system is the output image when a impulse image is inputted to the system.
- Therefore, the impulse response $h(x,y)$ is also an image, often called spatial representation of a filter or filter mask or filter coefficients or filter parameters.
- Although the impulse response $h(x,y)$ is basically an image, to speed up the image process, it is often a small size of image with size such as 3X3, 5X5, …11X11, comparing with a normal image of size 256X256.

$$g(x,y) = f(x,y) * h(x,y) = \sum_{j=-\infty}^{\infty} \sum_{i=-\infty}^{\infty} h(i,j) f(x-i, y-j)$$

$$= \sum_{j=-3}^{3} \sum_{i=-3}^{3} h(i,j) f(x-i, y-j), \quad \text{if } h(x,y) \neq 0 \;\; \text{only when} -3 < x, y < 3$$

2. LSI Systems & Transforms—understand by example

- Given an input image $f(x,y)$, you want to suppress the pixel random noise or smooth the image so that the image looks "soft". So you do some local average that a pixel in the output image is produced by sum up the corresponding pixel and its 4 neighbor pixels in the input image.
- What is the mathematical expression of this very simple process?

$$g(x,y) = f(x,y) + f(x-1,y) + f(x+1,y) + f(x,y-1) + f(x,y+1)$$

- What is the convolution like? $g(x,y) = \sum_{j=-1}^{1}\sum_{i=-1}^{1} h(i,j) f(x-i,y-j)$
- What is the size of the impulse response? Is the impulse response a constant one within the filter window $h(x,y)=1$?

$$g(x,y) = \sum_{j=-1}^{1}\sum_{i=-1}^{1} f(x-i,y-j)$$

- What is the impulse response analytically?

$$h(x,y) = \delta(x,y) + \delta(x-1,y) + \delta(x+1,y) + \delta(x,y-1) + \delta(x,y+1)$$

- How to represent the impulse response by image, or filter mask or filter coefficient?

0	1	0
1	1	1
0	1	0

2. LSI Systems & Transforms—understand by example

- A numerical example of convolution:

$$g(x,y) = f(x,y) * h(x,y) = \sum_{j=-1}^{1}\sum_{i=-1}^{1} h(i,j) f(x-i, y-j)$$

$$f(x,y) = \begin{array}{cccccc} 2 & 4 & 66 & 7 & 8 & 9 \\ 34 & 5 & 6 & 7 & 2 & 4 \\ 4 & 44 & 5 & 6 & 9 & 10 \\ 3 & 6 & 7 & 8 & 9 & 7 \\ 5 & 7 & 3 & 5 & 7 & 5 \end{array} \quad (x \downarrow,\ y \rightarrow)$$

$$h(x,y) = \begin{array}{ccc} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{array} \quad (x \downarrow,\ y \rightarrow)$$

$$g(4,4) = \sum_{j=-1}^{1}\sum_{i=-1}^{1} h(i,j) f(4-i, 4-j)$$

- $g(4,4)$ = (7*1+5*2+3*3+9*4+8*5+7*6+9*7+6*8+5*9)
- At each point (x,y) the response of the filter at that point is calculated as a sum of products of the filter coefficients and the corresponding image pixels in the area spanned by the filter mask.

2. LSI Systems & Transforms—Fourier transform

- Let $f(x)$ be a continuous function of a single variable x and $F(u)$ be its Fourier transform, then

$$F(u) = \Im\{f(x)\} = \int_{-\infty}^{\infty} f(x)\exp(-j2\pi ux)dx$$

$$\underline{f(x)} = \Im^{-1}\{F(u)\} = \int_{-\infty}^{\infty} F(u)\underline{\exp(j2\pi ux)}du$$

where u is frequency variable and $j = \sqrt{-1}$

- The two dimensional (2-D) Fourier transform and inverse Fourier transform are given by

$$F(u,v) = \Im\{f(x,y)\} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\exp[-j2\pi(ux+vy)]dxdy$$

$$f(x,y) = \Im^{-1}\{F(u,v)\} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(u,v)\exp[j2\pi(ux+vy)]dudv$$

where u and v are 2 frequency variables.

2. LSI Systems & Transforms—Fourier transform

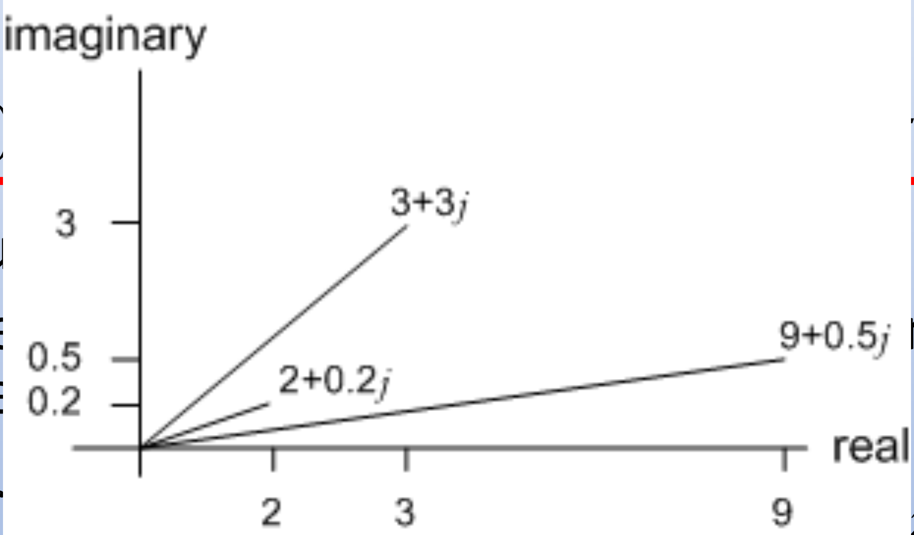
$$\begin{aligned} f(x) &= A\cos(2\pi ux+\varphi) \\ &= (A\cos\varphi)\cos 2\pi ux - (A\sin\varphi)\sin 2\pi ux \\ &= \tfrac{1}{2}A\exp(j2\pi ux + j\varphi) + \tfrac{1}{2}A\exp(-j2\pi ux - j\varphi) \\ &= (\tfrac{1}{2}A\exp j\varphi)\exp j2\pi ux + (\tfrac{1}{2}A\exp(-j\varphi))\exp(-j2\pi ux) \end{aligned}$$

- Let $f(x)$ … x and $F(u)$ be its F…

$f(x)$ … $ux)du$

where u is frequ…

- The two dime… nd inverse Fourier transform are



$F(u,v) = \Im\{f$ … $\pi(ux+vy)]dxdy$

$$c = a + jb = re^{j\varphi}$$

$$a = r\cos(\varphi), b = r\sin(\varphi), r = \sqrt{a^2+b^2}, \varphi = \tan^{-1}(\frac{b}{a})$$

2. LSI Systems & Transforms—Fourier transform

- 2-D Fourier transform is separable

$$F(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\exp[-j2\pi(ux+vy)]dxdy$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\exp(-j2\pi ux)\exp(-j2\pi vy)dxdy$$

$$= \int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} f(x,y)\exp(-j2\pi ux)dx\right]\exp(-j2\pi vy)dy$$

$$= \int_{-\infty}^{\infty} F_x(u,y)\exp(-j2\pi vy)dy$$

$$? = F_x(u)F_y(v) \quad \text{only if } f(x,y) = f_1(x)f_2(y)$$

- Note

$$\exp[-j2\pi(ux+vy)] = \cos[-2\pi(ux+vy)] + j\sin(-2\pi(ux+vy))$$

$$= \cos[2\pi(ux+vy)] - j\sin(2\pi(ux+vy))$$

2. LSI Systems & Transforms—Fourier transform

- Obviously, $F(u,v)$ is in general a complex function that can be represented by

$$F(u,v) = R(u,v) + jI(u,v) = |F(u,v)| \exp[j\varphi(u,v)]$$

where $$|F(u,v)| = \sqrt{R^2(u,v) + I^2(u,v)}$$

$$\varphi(u,v) = \tan^{-1}\left[\frac{I(u,v)}{R(u,v)}\right]$$

$$R(u,v) = |F(u,v)| \cos\varphi(u,v)$$

$$I(u,v) = |F(u,v)| \sin\varphi(u,v)$$

$$c = a + jb = re^{j\varphi}$$
$$a = rcos(\varphi), b = rsin(\varphi)$$
$$r = \sqrt{a^2 + b^2}, \varphi = tan^{-1}(\frac{b}{a})$$