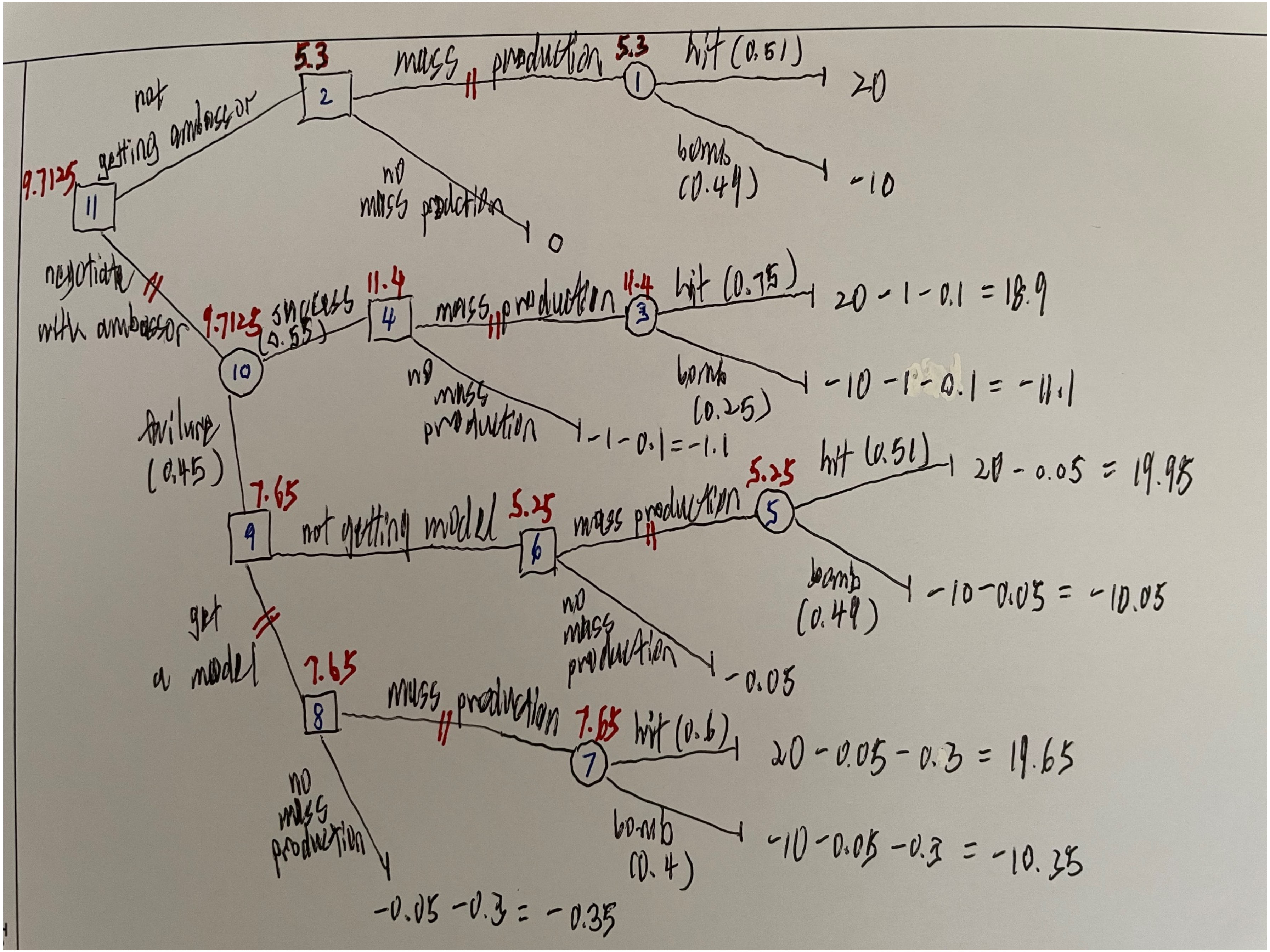
Decision tree question



Node	Expected value at node (in mil)
1	$(20)(0.51) + (-10)(0.49) = 5.3$
2	5.3
3	$(18.9)(0.75) + (-11.1)(0.25) = 11.4$
4	11.4
5	$(19.95)(0.51) + (-10.05)(0.49) = 5.25$
6	5.25
7	$(19.65)(0.6) + (-10.35)(0.4) = 7.65$
8	7.65
9	7.65
10	$(7.65)(0.45) + (11.4)(0.55) = 9.7125$
11	9.7125

Recommend the following strategy :

1, Negotiate to get ambassor JJ through an agent,

2, If it is successful to get JJ as ambassor, mass produce the collection for sale.

3, If it is not successful to get JJ as ambassor, get a model instead, and mass produce the collection for sale.

maximum expected profit of the above strategy = $9.7125 mil #

Possible actual profits of the above strategy are $18.9 mil, -$11.1 mil, $19.65 mil, -$10.35 mil #

suppose the negotiation and appointment of ambassador is costless. Then, the expected value with sample information (EVWSI) $= 9.7125 + 1 + 0.1$

$= \$10.8125$ mil

The expected value with original information (EVW

$= \$5.3$ mil

Hence, the expected value of sample information (E

$= EVWSI - EVWOI$

$= 10.8125 - 5.3 = \$5.5125$ mil

The fashion house can pay at most \$5.5125 mil to negotiate and appoint ambassador JJ. #

AHP question

(a) overall goal : select the best girlfriend

Criteria : Soul mate (S) | Earning power (E) | Liked by par

Decision alternatives : Anne (A) Jane (J) | Anne (A) Jane (J) | Anne (A) Jane (J)

(b) step 1 sum the values in each column

	S	E	P
S	1	5	6
E	$\frac{1}{5}$	1	2
P	$\frac{1}{6}$	$\frac{1}{2}$	1
sum	1.367	6.5	9

Total

Solution

step 2 Divide by column total, then calculate row average

	S	E	P	Priority
S	0.732	0.769	0.667	0.723
E	0.146	0.154	0.222	0.174
P	0.122	0.077	0.111	0.103

From the above table, we see that S is the most important criterion, E is the next important criterion and P is the least important criterion. #

(c) step 1

$$\begin{bmatrix} 1 & 5 & 6 \\ \frac{1}{5} & 1 & 2 \\ \frac{1}{6} & \frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} 0.723 \\ 0.174 \\ 0.103 \end{bmatrix} = \begin{bmatrix} 2.211 \\ 0.5246 \\ 0.3105 \end{bmatrix}$$

step 2

S : $2.211 / 0.723 = 3.058$

E : $0.5246 / 0.174 = 3.015$

P : $0.3105 / 0.103 = 3.015$

step 3 $\quad \lambda_{max} = \frac{1}{3}(3.058 + 3.015 + 3.015)$

$= 3.029$

step 4 $\quad$ consistency index (CI) $= \frac{\lambda_{max} - 3}{3-1} = 0.0145$

step 5 $\quad$ consistency ratio (CR) $= \frac{CI}{RI} = \frac{0.0145}{0.58}$

$= 0.025 < 0.1$

Since $CR < 0.1$, the consistency of the pairwise comparisons of S, E, P is acceptable. ∗

(d) Soul mate (S) :

step 1

	Jane	Anne
Jane	1	4
Anne	$\frac{1}{4}$	1
Sum	1.25	5

step 2

	Jane	Anne	Priority
Jane	0.8	0.8	0.8
Anne	0.2	0.2	0.2

Earning power (E):

step 1

	Jane	Anne
Jane	1	$\frac{1}{6}$
Anne	6	1
sum	7	1.167

step 2

	Jane	Anne	Prior
Jane	0.143	0.143	0.143
Anne	0.857	0.857	0.857

Liked by parents (P):

step 1

	Jane	Anne
Jane	1	$\frac{1}{7}$
Anne	7	1
sum	8	1.143

step 2

	Jane	Anne	Priority
Jane	0.125	0.125	0.125
Anne	0.875	0.875	0.875

Combining the 3 sets of priorities gives

	S	E	P
Jane	0.8	0.143	0.125
Anne	0.2	0.857	0.875

We see that

Jane is the preferred choice based on S

Anne " " " " " " E

Anne " " " " " " P

The composite priority is calculated as follows

$$\begin{bmatrix} 0.8 & 0.143 & 0.125 \\ 0.2 & 0.857 & 0.875 \end{bmatrix} \begin{bmatrix} 0.723 \\ 0.174 \\ 0.103 \end{bmatrix} = \cancel{0.012875} \begin{bmatrix} 0.616 \\ 0.384 \end{bmatrix}$$

	Priority
Jane	0.616
Anne	0.384

Jane has a higher priority, so Jane is the preferred choice. #

DTMC question

state 0 : low sales in the current week

state 1 : high sales in the current week

(A): Advertise when sales are low, not to advertise when sales are high : $P_{00} = \frac{1}{5}$, $P_{10} = \frac{1}{3}$

$$P = \begin{array}{c|cc} \text{state} & 0 & 1 \\ \hline 0 & \frac{1}{5} & 1-\frac{1}{5} \\ 1 & \frac{1}{3} & 1-\frac{1}{3} \end{array} = \begin{bmatrix} \frac{1}{5} & \frac{4}{5} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$$

#

(B): Advertise when sales are high, not to advertise when sales are low : $P_{00} = \frac{1}{2}$, $P_{10} = \frac{1}{10}$

$$P = \begin{array}{c|cc} \text{state} & 0 & 1 \\ \hline 0 & \frac{1}{2} & 1-\frac{1}{2} \\ 1 & \frac{1}{10} & 1-\frac{1}{10} \end{array} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{10} & \frac{9}{10} \end{bmatrix}$$

#

Total

Let T_i = sojourn time of state i, $i = 0, 1$.

$$E(T_i) = \frac{1}{1 - P_{ii}}$$

(A): $E(T_0) = \frac{1}{1-P_{00}} = \frac{1}{1-\frac{1}{5}} = \frac{5}{4}$

$E(T_1) = \frac{1}{1-P_{11}} = \frac{1}{1-\frac{2}{3}} = 3$ #

(B): $E(T_0) = \frac{1}{1-P_{00}} = \frac{1}{1-\frac{1}{2}} = 2$

$E(T_1) = \frac{1}{1-P_{11}} = \frac{1}{1-\frac{9}{10}} = 10$ #

strategy (A) has the lowest $E(T_0) = \frac{5}{4}$ while strategy (B) has the highest $E(T_1) = 10$. Hence, if the co. wants to have short low sales period on the average, choose strategy (A). On the other hand, if the co. wants to have long high sales period on the average, choose strategy (B). #

Total

(c) Let $Y = [y_0 \quad y_1]$ be the steady-state probabili

$$\begin{cases} Y = YP \\ y_0 + y_1 = 1 \end{cases}$$

(A): $[y_0 \quad y_1] = [y_0 \quad y_1]\begin{bmatrix} \frac{1}{5} & \frac{4}{5} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$

$\Rightarrow \frac{4}{5} y_0 = \frac{1}{3} y_1$

$\therefore$ Solving gives $y_0 = \frac{5}{17}$, $y_1 = \frac{12}{17}$ #

(B): $[y_0 \quad y_1] = [y_0 \quad y_1]\begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{10} & \frac{9}{10} \end{bmatrix}$

$\Rightarrow \frac{1}{2} y_0 = \frac{1}{10} y_1$

$\therefore$ Solving gives $y_0 = \frac{1}{6}$, $y_1 = \frac{5}{6}$ #

(d) (A): Long run expected profit in a week

$= (2,000,000)\, y_1 + (500,000 - 200,000)\, y_0$

$= (2,000,000)\frac{12}{17} + (300,000)\frac{5}{17}$

$= \$1,500,000$

(B) : Long run expected profit in a week

$$= (2{,}000{,}000 - 100{,}000)\,\pi_1 + (500{,}000)\,\pi_0$$

$$= (1{,}900{,}000)\,\frac{5}{6} + (500{,}000)\,\frac{1}{6}$$

$$= \$1{,}666{,}666.67$$

Hence, strategy (B) gives higher long run expected profit in a week. #