NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2021-2022

EE7403 – IMAGE ANALYSIS AND PATTERN RECOGNITION

April / May 2022 Time Allowed: 3 hours

INSTRUCTIONS

1. This paper contains 5 questions and comprises 3 pages.
2. Answer all 5 questions.
3. All questions carry equal marks.
4. This is a closed book examination.
5. Unless specifically stated, all symbols have their usual meanings.

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1. A digital image filter impulse response is given by

$$h(x,y)=\delta(x-1,y-1)+\delta(x-1,y)+\delta(x,y-1)$$
$$-\delta(x+1,y)-\delta(x,y+1)-\delta(x+1,y+1).$$

(a) Show the filter mask and indicate your x- and y-axes besides the mask.
(5 Marks)

(b) Express the filter output image $g(x,y)$ in terms of the input image $f(x,y)$.
(5 Marks)

(c) Compute the frequency response of the filter.
(5 Marks)

(d) Analyze the filter properties.
(5 Marks)

2. (a) An algorithm has detected n pixels from an image and now we want to detect lines from the image based on the detected points by applying Hough transform. Suppose there is no error caused by the discrete accumulator cells in the parameter space. What are the possible numbers of peaks in the accumulator cells of the parameter space for n=5? For each possible number of peaks, give the value of each peak and state what kinds of lines (the number of pixels on the line) are detected. (Note that a peak means a point whose value is larger than its eight-neighbors.)

(10 Marks)

(b) Four gradients are presented by complex numbers as $s_1 = a + ja$, $s_2 = b$, $s_3 = -a - ja$, $s_4 = -jb$. Let $c = s_1 + s_2 + s_3 + s_4$ and $d = s_1^2 + s_2^2 + s_3^2 + s_4^2$.

(i) Compute the angle of c and compare it with angles of s_1, s_2, s_3 and s_4.

(ii) Compute the angle of d and compare it with angles of s_1, s_2, s_3 and s_4.

(10 Marks)

3. You are in an office on the school campus. A student of height 1.67m knocks on the door. You need to judge whether the student is male or female with the minimal probability of wrong decision.

(a) What are your decision, and the probability that your decision is wrong, if you only know that there are 300 male students and 700 female students in the school?

(5 Marks)

(b) What is your decision if you know that there are an equal number of male and female students in the school, and the average height of male students is 1.7m and that of female students is 1.62m?

(6 Marks)

(c) Suppose there are 300 male and 700 female students in the school. What is your decision if you know that the average height and standard deviation of male students are 1.7m and 0.2m respectively, and those of female students are 1.62m and 0.3m respectively?

(9 Marks)

[Hints: Derive your solutions under the Gaussian assumption if it is needed.]

4. Given a database of c classes, each class has n samples and each sample has m features. Each element of the database is represented by x_{kij}, $1 \le i \le m$, $1 \le j \le n$, $1 \le k \le c$.

(a) Compute the class-conditional mean μ_{ki} and the element σ_{kpq} of the covariance matrix.

(5 Marks)

Note: Question No. 4 continues on page 3.

(b) Construct the column feature vector $\mathbf{x}_{kj}$ of each sample from the given x_{kij}, and compute the class-conditional mean vector $\boldsymbol{\mu}_k$ and the covariance matrix $\boldsymbol{\Sigma}_k$ using $\mathbf{x}_{kj}$.

(5 Marks)

(c) Construct the class-conditional data matrix $\mathbf{X}_k$ using $\mathbf{x}_{kj}$ and $\boldsymbol{\mu}_k$ such that the mean feature vector of $\mathbf{X}_k$ is zero, and compute $\boldsymbol{\Sigma}_k$ using $\mathbf{X}_k$.

(5 Marks)

(d) We want to extract a one-dimensional feature. The unit length column vector that spans this dimension in the feature space is $\mathbf{a}$. Derive the class-conditional variance v_k of this one-dimensional feature from the definition of the variance. Express the final result in terms of $\mathbf{a}$ and $\boldsymbol{\Sigma}_k$.

(5 Marks)

5. A gray-level digital image $\boldsymbol{X}$ of size 100×100 is cropped into small images of size 3×3 by sliding the center of a 3×3 window to every pixel of $\boldsymbol{X}$. If the center of the window is on the boundary of the image $\boldsymbol{X}$, zero padding is used to obtain images of size 3×3 so that we get 10000 small images of size 3×3. Each 3×3 image is flattened into a column vector and is expressed as $\mathbf{z}^k = (z_1^k, \ldots, z_9^k, 1)^T$, $k = 1, \ldots, 10000$, and is inputted to a typical fully-connected layer of multilayer perceptron (MLP) with linear activation function to generate the outputs $\mathbf{y}^k = (y_1^k, \ldots, y_6^k)^T$. The network parameters of this layer are denoted by w_{ij} and b_j, $0 < i < 10$, $0 < j < 7$.

(a) Express the outputs y_j^k in terms of the inputs z_i^k.

(5 Marks)

(b) Construct the matrix $\boldsymbol{W}$ that contains all network parameters, and express the output $\mathbf{y}^k$ in terms of the input $\mathbf{z}^k$.

(5 Marks)

(c) Six images $\boldsymbol{Y}_j$ are constructed by the 10000 outputs $\mathbf{y}^k$, $k = 1, \ldots, 10000$. Express the output images $\boldsymbol{Y}_j$ in terms of the input image $\boldsymbol{X}$.

(5 Marks)

(d) Suppose this network is trained by 100 images of size 100×100, What is the number of training samples used to train the network parameters $\mathbf{W}$ or w_{ij} and b_j?

(5 Marks)

END OF PAPER

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Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.