NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 2 EXAMINATION 2022-2023

EE7403 – IMAGE ANALYSIS AND PATTERN RECOGNITION

April / May 2023 Time Allowed: 3 hours

INSTRUCTIONS

1. This paper contains 4 questions and comprises 3 pages.
2. Answer all 4 questions.
3. All questions carry equal marks.
4. This is a closed book examination.
5. Unless specifically stated, all symbols have their usual meanings.

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1. An image is modeled by $f(x,y) = h(\omega x - \sqrt{3}\omega y)$, where $h(t)$ is a differentiable function, ω is a constant, x and y are continuous variables.

   (a) Determine the orientation $\phi, -\frac{\pi}{2} < \phi \le \frac{\pi}{2}$, along which $f(x,y)$ is a constant.

   (5 Marks)

   (b) Compute the orientation ϕ based on the direction of image gradient given by:

   $$\nabla f(x,y) = [\frac{\partial f(x,y)}{\partial x}, \frac{\partial f(x,y)}{\partial y}].$$

   (5 Marks)

   (c) A discrete image $F(m, n)$ is obtained by sampling the image $f(x,y)$ with a sampling interval Δ. The gradient of the discrete image is approximated by the finite difference as

   $$\nabla F(m,n) = [F(m+1,n) - F(m-1,n), F(m,n+1) - F(m,n-1)].$$

   Given $h(t) = \sin(t)$, find the image orientation using the approximated gradient $\nabla F(m,n)$ for $\Delta = \pi/(2\omega)$ and for $\Delta = \pi/(8\omega)$.

   (10 Marks)

Note: Question No. 1 continues on page 2.

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(d) Suggest how the estimation error in part (c) can be reduced.

(5 Marks)

[**Hint**: $\sin\alpha - \sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$]

2. Suppose that the class prior probability and the class-conditional probability function are given as $p(\omega_i)$ and $p(\mathbf{x} \mid \omega_i)$ for $\omega_i, i = 1, 2, \ldots, c$. The class-conditional PDF can be modeled by Gaussian function,

$$p(\mathbf{x} \mid \omega_i) = \frac{1}{(2\pi)^{d/2}|\boldsymbol{\Sigma}_i|^{1/2}} \exp\left[-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu}_i)^T \boldsymbol{\Sigma}_i^{-1}(\mathbf{x}-\boldsymbol{\mu}_i)\right]$$

(a) Derive a classifier that can achieve the highest classification accuracy.

(13 Marks)

(b) Suppose you have 6 training samples $\mathbf{x}_1 = (1, 1)^T$, $\mathbf{x}_2 = (1, 2)^T$, $\mathbf{x}_3 = (2, 1)^T$, $\mathbf{x}_4 = (2, 2)^T$, $\mathbf{x}_5 = (2, 3)^T$, and $\mathbf{x}_6 = (3, 2)^T$. The first three samples are from class 1 and the last three samples are from class 2. Design your derived classifier in part (a) by using the training samples.

(12 Marks)

3. A dataset contains 4 samples $\mathbf{x}_1 = (1, 6)^T$, $\mathbf{x}_2 = (4, 7)^T$, $\mathbf{x}_3 = (2, 9)^T$, and $\mathbf{x}_4 = (5, 10)^T$. Its covariance matrix has eigenvectors $\phi_1 = (\frac{1}{\sqrt{2}} \quad \frac{1}{\sqrt{2}})^T$ and $\phi_2 = (-\frac{1}{\sqrt{2}} \quad \frac{1}{\sqrt{2}})^T$.

(a) Compute the one-dimensional representations a_i and b_i, i=1, 2, 3, 4, of these 4 samples along each of the two eigenvectors.

(8 Marks)

(b) Compute the two eigenvalues.

(7 Marks)

(c) Compute the two-dimensional samples $\hat{\mathbf{x}}_i$ reconstructed from single one-dimensional representation (a_i or b_i) which minimize the reconstruction error.

(10 Marks)

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4. Inputs of a layer of typical neural network for image processing are denoted by x_{ijk} for $0 < i < I+1$, $0 < j < J+1$, $0 < k < K_1+1$, and its outputs before the activation function are denoted by y_{ijk} for $1 < i < I$, $1 < j < J$, $0 < k < K_2+1$, where i and j are spatial indexes of the image or feature map and k is the index of the channels.

(a) If the inputs and the outputs are fully connected by network parameters $w(.)$, express the outputs y_{ijk} in terms of its inputs x_{ijk} and network parameters $w(.)$. In the expression, you need to specify the parameters by giving proper variables (indexes) in the bracket of $w(.)$. What is the number of parameters of this network that needs to be learnt? What is the number of multiplications needed to compute the outputs from the inputs?

(7 Marks)

(b) If the outputs are computed by K_2 spatial filters of size $3\times3\times K_1$ (3×3 in the spatial domain) denoted by $h(.)$, express the outputs y_{ijk} in terms of its inputs x_{ijk} and the filter $h(.)$. In the expression, you need to specify the parameters by giving proper variables (indexes) in the bracket of $h(.)$. What is the number of parameters of this network that needs to be learnt? What is the number of multiplications needed to compute the outputs from the inputs?

(10 Marks)

(c) Briefly discuss two most significant differences between the networks in 4(a) and 4(b) in network training and inference.

(8 Marks)

END OF PAPER

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Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.

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