

EE7401 Probability and Random Processes

Homework 1

To be submitted by 24 September 2022, 23:59

1. (35 marks) (Estimation vs. Detection) Let

$$X = \begin{cases} 1 & \text{with probability } \frac{1}{2}, \\ -1 & \text{with probability } \frac{1}{2}, \end{cases}$$

and the noise $Z \sim \text{Unif}(-2, 2)$ be independent random variables. Their sum $Y = X + Z$ is observed.

- Find the conditional pmf $p_{X|Y}(x | y)$. Find the MMSE of X given Y and its MSE.
- Suppose we use a decoder to decide whether $X = 1$ or -1 . Using the pmf $p_{X|Y}(x | y)$ found in part (a), find the MAP decoder and its probability of error. Compare the MAP decoder's MSE to the minimum MSE.

2. (45 marks) Given a Gaussian random vector $\mathbf{X} = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, where

$$\boldsymbol{\mu} = \begin{bmatrix} 1 \\ 5 \\ 2 \end{bmatrix}, \quad \boldsymbol{\Sigma} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 9 \end{bmatrix}.$$

- What are the pdfs of
 - X_1 ,
 - $X_2 + X_3$,
 - $2X_1 + X_2 + X_3$, and
 - X_3 given (X_1, X_2) ?
- What is $\mathbb{P}(2X_1 + X_2 + X_3 < 0)$?
- What is the joint pdf of $\mathbf{Y} = \mathbf{A}\mathbf{X}$, where

$$\mathbf{A} = \begin{bmatrix} 2 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}?$$