# EE6401-2022 : Assignment

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## Question 1:

### a.

We have $F_0 = 21$kHz and $F_{final} = 10$kHz

So we have two configuration:

1. $I = I_1 I_2 = 5 \times 2$ , $D = D_1 D_2 = 7 \times 3$
2. $I = 10$ , $D = D_1 D_2 = 7 \times 3$

The most efficient filtering system can only be the above two configurations like $I_1 I_2 D_1 D_2$ *or* $ID_1 D_2$ as $IDID$ system will have more filters and multiplications

We will first calculate configuration 1:

calculating the transposed version of it with the same result

$\delta_1 = 0.01 \quad \delta_2 = 0.0001 \quad F_{pc} = 4$kHz $\quad F_{pc} = 5$kHz

First stage:

$F_0 = 210$kHz $\quad F_1 = 105$kHz

Passband $\quad 0 \leq F < 4$kHz

Transation band $\quad$ 4kHz $\leq$ F $<$ 100kHz

$\Delta f = \frac{100-4}{210} = 0.4571$

$\delta_{11} = \frac{\delta_1}{3} = 0.0033 \quad \delta_{12} = \delta_2 = 0.0001$

$N_1 = \frac{-10\log_{10}\delta_1\delta_2 - 13}{14.6\Delta f} + 1 = 9$

Second stage:

$F_2 = 21\text{kHz}$

Passband $\quad 0 \leq F < 4\text{kHz}$

Transation band $\quad$ 4kHz $\leq$ F $<$ 16kHz

$\Delta f = \frac{16-4}{105} = 0.1143$

$\delta_{21} = \frac{\delta_1}{3} = 0.0033 \quad \delta_{22} = \delta_2 = 0.0001$

$N_2 = \frac{-10\log_{10}\delta_1\delta_2 - 13}{14.6\Delta f} + 1 = 33$

We then calculate configuration 2:

$\delta_1 = 0.01 \quad \delta_2 = 0.0001 \quad F_{pc} = 4\text{kHz} \quad F_{pc} = 5\text{kHz}$

Single stage:

$F_0 = 210\text{kHz} \quad F_1 = 21\text{kHz}$

Passband $\quad 0 \leq F < 4\text{kHz}$

Transation band $\quad$ 4kHz $\leq$ F $<$ 16kHz

$\Delta f = \frac{16-4}{210} = 0.0571$

$\delta_{11} = \frac{\delta_1}{2} = 0.005 \quad \delta_{12} = \delta_2 = 0.0001$

$N = \frac{-10\log_{10}\delta_1\delta_2 - 13}{14.6\Delta f} + 1 = 61$

Complexity in the down-sampling part for configuration 1 and 2 is identical, so we only need to calculate the number of multiplication and addition in interpolation part.

For configuration 1:

$$\begin{aligned}
\text{Number of multiplication} &= \frac{1}{2}\left(N_1+1\right)F_1 + \frac{1}{2}\left(N_2+1\right)F_2 \\
&= \frac{1}{2}\left(33+1\right)\times 105\text{k} + \frac{1}{2}\left(9+1\right)\times 21\text{k} \\
&= 1.89\times 10^6 \\
\text{Number of addition} &= N_1F_1 + N_2F_2 \\
&= 33\times 105\text{k} + 9\times 21\text{k} \\
&= 3.654\times 10^6
\end{aligned}$$

For configuration 2:

$$\begin{aligned}
\text{Number of multiplication} &= \frac{1}{2}\left(N+1\right)F \\
&= \frac{1}{2}\left(61+1\right)\times 210\text{k} \\
&= 6.51\times 10^6 \\
\text{Number of addition} &= NF \\
&= 61\times 210\text{k} \\
&= 1.281\times 10^7
\end{aligned}$$

Therefore, configuration 1 is more efficient than configuation 2.

The block diagram is below, where $I_1 = 5, I_2 = 2, D_1 = 7, D_2 = 3$

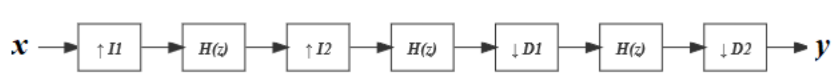


Figure 1: block diagram

**b.**

Filter $I_1$:

Passband frequency = 4kHz

$\Omega = \frac{8\pi}{21}$

Transition band $\quad 4\text{kHz} \leq \text{F} < 16\text{kHz}$

Pass-band ripple = 0.005

Stopband Ripple = 0.0001

Order of the filter = 31

Filter $I_2$:

Passband frequency = 4kHz

Transition band $\quad 4\text{kHz} \leq \text{F} < 100\text{kHz}$

Pass-band ripple = 0.005

Stopband Ripple = 0.0001

Order of the filter = 9

Filter $D_1$:

Passband frequency = 4kHz

Transition band $\quad 4\text{kHz} \leq \text{F} < 25\text{kHz}$

Pass-band ripple = 0.0033

Stopband Ripple = 0.0001

Order of the filter = 37

Filter $D_2$:

Passband frequency = 4kHz

Transition band $\quad 4\text{kHz} \leq \text{F} < 5\text{kHz}$

Pass-band ripple = 0.0033

Stopband Ripple = 0.0001

Order of the filter = 108

**c.**

Filter $I_1$:
$\frac{1}{2}(33+1) \times 105\text{k} = 1.785 \times 10^6$ multiplications per second

Filter $I_2$:
$\frac{1}{2}(9+1) \times 21\text{k} = 1.05 \times 10^5$ multiplications per second

Filter $D_1$:
$\frac{1}{2}(37+1) \times 30\text{k} = 5.7 \times 10^5$ multiplications per second

Filter $D_2$:
$\frac{1}{2}(108+1) \times 10\text{k} = 5.45 \times 10^5$ multiplications per second

Overall multiplication rate $= 3.005 \times 10^6$ multiplications per second

## Question 2:

**a.**

The commutator has similar function with a series of $z^{-1}$ time delay, assuming that $p_i(x)$ is the impulse response of the system $P_i(z)$, so we have:

$y(m) = \sum_{i=0}^{L-1} p\,(i)\,x\,(n-i)$

**b.**

$Y(z) = \sum_{i=0}^{L-1} z^{-i} P_i(z^L) X(z)$

## Question 3:

**a.**

Linear phase low-pass FIR filter with $I = 3$

$y(n) = x(n) + \frac{1}{3}[x(n-1) + x(n+2)] + \frac{2}{3}[x(n-2) + x(n+1)]$

The filter operates zero padding in frequency so that signal in time domain will have interpolation.

The unit impulse response of the low-pass filter $h\,(n) = \begin{cases} 1 - \dfrac{|n|}{3}\ , & |n| \le 3 \\ 0\ , & \text{otherwise} \end{cases}$

And we have:$H(e^{j\omega}) = \dfrac{1}{3}\left(\dfrac{sin(\frac{3\omega}{2})}{sin(\frac{\omega}{2})}\right)^2$

The gain of the filter is 3, and the cutoff frequency is $\frac{\pi}{3}$

**b.**

Low-pass brick filter with sinc function can be considered as ideal low-pass filter, so the frequency response of low-pass brick filter is flatter than that of linear interpolation filter in the interval $[-\frac{\pi}{3}, \frac{\pi}{3}]$. And low-pass brick filter has less out-of-band attenuation and higher up-sampling accuracy.