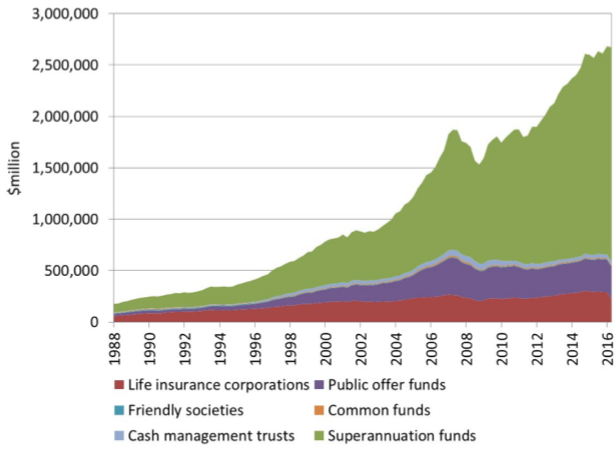
IPM Exam Notes

Week 1 – Introduction

- Funds have different taste for risk
- Value stocks riskier than growth stocks
- Small cap stocks riskier than large cap stocks
- Assets you can invest in
    - House
    - Shares etc
- Market participant are not all trying to do the same thing
    - Funds – make money for other
    - Insurance companies – Take premiums and try to make money in order to pay out claims
    - Superannuation – Long term investments
    - Banks – Bank bill swap rate
    - Govt – issue bonds
    - Sovereign wealth funds – future fund
- **Investment fund industry**
    - Types of managed funds
        - **Unit trust**
            - Investors' funds are pooled into specific type of assets
            - Investors are assigned units in the fund which are traded
            - Invest in tradable units
            - Unlisted trust can issue new units any time
            - Value of each unit depends on value of underlying investment
            - Not liquid to enter
        - **Superannuation**
            - Accept/manages contributions from employers/employees
            - Defined benefit
                - Payout determined by formula and employer takes the risk
            - Defined contribution
                - Based on accumulation of contributions which helps out firms
        - **Hedge funds**
            - Seeks to hedge against risky price movements by any strategy
            - Access is limited as you need to be a high net worth individual
        - **Exchange traded funds**
            - Listed on stock market
            - Traded like a stock
            - Hybrid between listed security and an open-ended fund
            - Provides ease of access and low costs of entry/exit

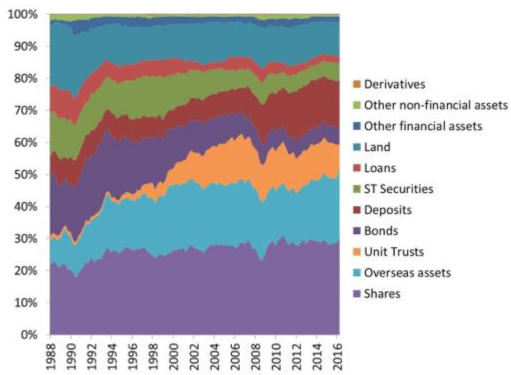
- Often have an explicit objective/benchmark for index tracking
- Easy to access and low costs of entry/exit

Assets under management ($m)



- Strategic asset allocation is benchmark allocation between asset classes:
  - Cash
  - Fixed interest
  - Property
  - Alternative investments
  - Equity
- This sends out a signal of firm's strategy
- Allocation depends on firm's objectives
- Tactical asset allocation allows to move between asset classes. Exploits temporary mispricing by adjusting exposure to different asset classes. You have a min/max on amounts in each class

Asset allocation of Australian managed funds



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Week 2 – Investment decisions under uncertainty

- Investors with high risk aversion wants high risk premiums
- Utility helps us rank choices
- Certain utility functions represent investors with different risk aversion
- If no uncertainty, we can choose how much to consume now or later
- **Risk free asset:** Return is certain across all possible states
- **Risky asset:** Returns are not certain
- **Utility analysis**
  - Allows us to rank alternatives

    - Let E(U) denote the expectation of U

    $$E[U(W)] = \sum_{W} U(W)P(W)$$

    - Weighting function is a utility function U(W)
    - P(W) is a probability
    - The principle is the expected utility theorem
    - Investors choose among alternatives to maximise expected utility
- Investors choose highest expected utility
- **Axioms of expected utility**
  - **Comparability** – investors can say what they prefer and rank them
  - **Transitivity/Consistency** – if a preferred to B, and B preferred to C, then A preferred to C
  - **Independence** – still have same preferences in states of uncertainty. Doesn't matter what other assets you have
  - **Certainty equivalent** – every gamble has a value that investors will be indifferent to
  - Investor prefers W1 to W2 if expected utility of W1 Is higher
  - **Ranking is measurable**
- To calculate: take and find out the utility and then multiply by the probabilities of each utility occurring
- To find certainty equivalent, find the expected utility and take inverse of utility function

  Problems:
- Independence and existence of complements Violates independence
- Individuals do not always rank things in consistent manner
- Ranking of alternatives may depend upon environment they are in
- Non-satiation is not reasonable at extreme levels of consumption

Certainty equivalent rate – Level of risk free return that generates same utility as a prospect

Properties of utility function

- More is preferred to less (non-satiation)
  - First derivative of utility function is positive

$$U'(W) = \frac{dU(W)}{dW} > 0$$

  - Suppose there are two (certain) risk-free investments, one with outcome W_1 dollars, and the other with outcome W_2 dollars. If $W_1 > W_2$ then:

$$U(W_1) > U(W_2)$$

- Adding a constant to a utility function or multiplying utility functions by a constant does not change rankings
  - The same investment is selected

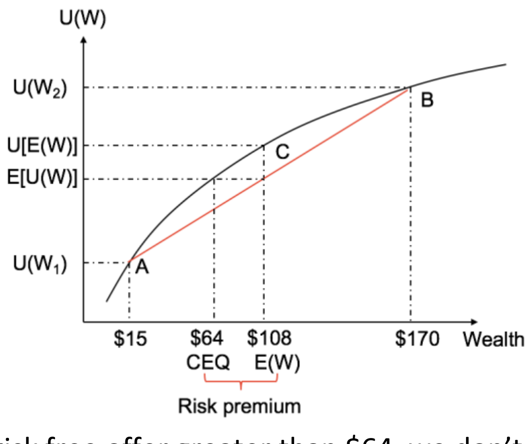
- Fair gamble: Risky investment whose expected return equals risk-free rate of return or risky investment with zero risk premium. E.g. costs $1 to flip coin to win $2 or nothing

If risk free asset and risky asset, the standard deviation of the portfolio is the standard deviation of risky asset * weight
CAL: Is risk-reward line for investors

- 3 types of investors
  - **Risk averse**
  - They will reject a fair gamble and require risk premium

$$E[U(W_R)] < E[U(W_{RF})] = U(W_{RF}) = U[E(W_R)]$$

$$E[U(W)] \neq U[E(W)]$$



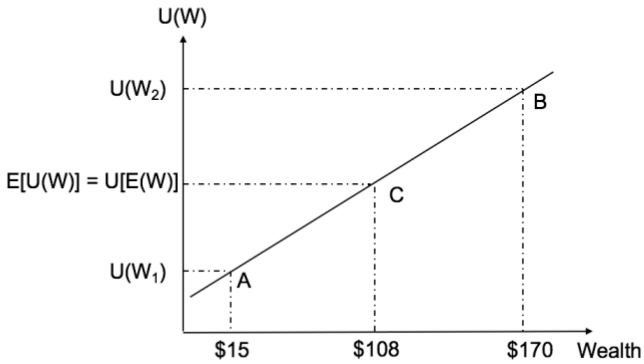
  - If risk free offer greater than $64, we don't gamble
  - If we offer $63, they gamble
  - They prefer certainty
  - The black line curve is certainty

- Risk-aversion implies second derivative is negative
  - Utility is concave in wealth
- Diminishing marginal utility of wealth
  - The utility from an additional dollar of wealth declines as wealth increases

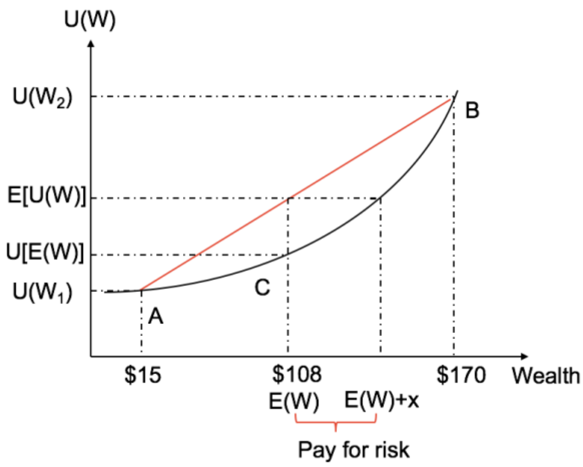
$$U''(W) = \frac{d^2U(W)}{dW^2} < 0$$

- Certainty equivalent wealth (CEQ) is the indifference point
  - How much is a risky outcome worth in risk-free terms?

- **Risk-neutral**
- They are indifferent to fair gamble
- Utility function is linear in wealth
- Second derivate of utility function is 0



- **Risk-seeking**
- They will select a fair gamble
- Prefer fair gamble over risk free investment
- Utility function is convex
- Second derivate is positive
- Pay a premium to take risks (e.g. go to casinos)



- We assume investors are risk averse, maximise expected utility of wealth, prefer more wealth to less, and have diminishing marginal utility of wealth

- **Absolute risk aversion**: Shifts in investor's preferences in response to wealth

- If amount in risky assets increases as wealth increases, investor has decreasing absolute risk aversion. They are still risk averse but decreasing risk aversion as more wealth

$$ARA = A(W) = \frac{-U''(W)}{U'(W)}$$

- (Read book about derivation)
- Generally assumed that investors exhibit decreasing absolute risk aversion

- So as ARA increases, we put more money into risky assets as total wealth increases (if positive ARA)

- **Relative risk aversion**
- % amount of risk aversion changes as wealth changes

$$RRA = R(W) = \frac{-WU''(W)}{U'(W)}$$

$$RRA = R(W) = W \times ARA$$

- No consensus on what RAR is as wealth increases
- Derivative of RRA with W tells us how RRA changes as wealth changes
    - >0, then increasing RRA

How Absolute Risk-Aversion Changes with Wealth

Type of Risk-Aversion	Description	Example of Bernoulli Function
Increasing absolute risk-aversion	As wealth increases, hold fewer dollars in risky assets	w^{-cw^2}
Constant absolute risk-aversion	As wealth increases, hold the same dollar amount in risky assets	$-e^{-cw}$
Decreasing absolute risk-aversion	As wealth increases, hold more dollars in risky assets	$\ln(w)$

How Relative Risk-Aversion Changes with Wealth

Type of Risk-Aversion	Description	Example of Bernoulli Function
Increasing relative risk-aversion	As wealth increases, hold a smaller percentage of wealth in risky assets	$w - cw^2$
Constant relative risk-aversion	As wealth increases, hold the same percentage of wealth in risky assets	$\ln(w)$
Decreasing relative risk-aversion	As wealth increases, hold a larger percentage of wealth in risky assets	$-e^{2w^{-1/2}}$

Table 10.7 Changes in Absolute Risk Aversion with Wealth

Condition	Definition	Property of $A(W)$[a]	Example[b]
Increasing absolute risk aversion	As wealth increases hold fewer dollars in risky assets	$A'(W) > 0$	W^{-CW^2}
Constant absolute risk aversion	As wealth increases hold same dollar amount in risky assets	$A'(W) = 0$	$-e^{-CW}$
Decreasing absolute risk aversion	As wealth increases hold more dollars in risky assets	$A'(W) < 0$	$\ln W$

[a] $A'(W)$ is the first derivative of $A(W)$ with respect to wealth.
[b] The proof is left to the reader.

Types of utility functions

- Log utility function

$$U(W) = \ln(W)$$

- Quadratic utility function

$$U(W) = W - cW^2$$

- Exponential utility

$$U(W) = 1 - e^{-\gamma W}$$

  - γ is the risk aversion coefficient

- Power utility

$$U(W) = \frac{W^{1-A}}{1-A}$$

  - where A is restricted to be greater than zero

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- Check slides for worked examples. LEARN HOW TO DO THESE

- **Mean-variance**
- Distribution of expected returns is normal
  - So we can rank investments according to risk and return
- Utility functions are assumed to be quadratic
  - Expected utility determined by expected mean and variance of expected wealth
    - Approximating investor preferences using expected return and standard deviation from quadratic utility

$$U(W) = W - cW^2$$

    - Take expectations

$$E[U(W)] = E(W) - cE(W^2)$$

    - Note that

$$var(W) = E(W^2) - E(W)^2$$

    - rearrange

$$E(W^2) = var(W) + E(W)^2$$

    - Substitute into expected utility function

$$E[U(W)] = E(W) - c[E(W)^2 + var(W)]$$

$$E[U(W)] = E(W) - cE(W)^2 - c\sigma_W^2$$

- Define expected wealth as one plus the expected return
  - E(W) = E(1+r)

$$E[U(1+r)] = E(1+r) - c[E(1+r)^2 + var(1+r)]$$

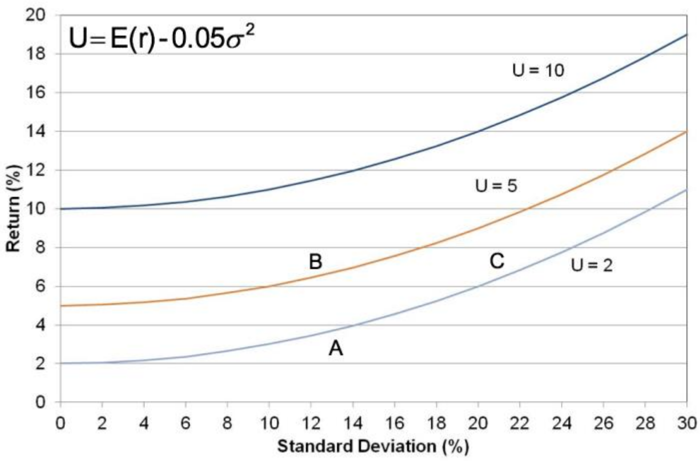
$$E[U(1+r)] = E(1+r) - cE(1+r)^2 - c\sigma_{1+r}^2$$

- Expected utility can be defined by means and variances when utility is quadratic

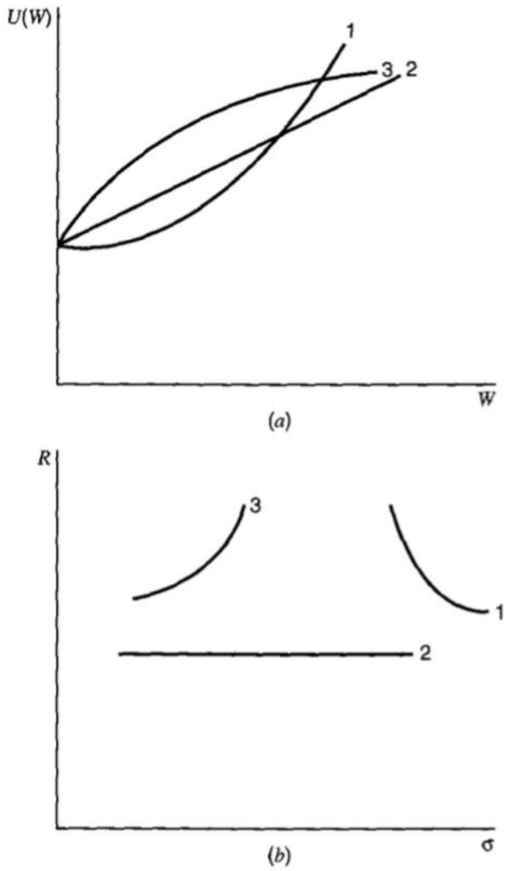
- **Issues with quadratic utility**
  - Implies investors can be satiated. Whereby increase in wealth reduces utility
    - However, people should always prefer more to less
  - Implies increasing absolute risk aversion
    - Risky assets are inferior goods when they should be normal goods. Here, as you get wealthier, you buy less-risky goods since increasing ARA
  - This is due to the fact quadratic utilities have a maximum

$$U = E(r) - \tfrac{1}{2} A\sigma^2$$

- U is the utility derived from an investment with a particular expected return and variance (risk)
- A is the risk aversion parameter
- A=0 is a risk neutral investor
- For a rational investor (risk averse) $A > 0$



- Here, we are indifferent to A and C. Increasing preferences towards North-West region
- Risk aversion coefficient tells us the steepness of the curve
- **Mean-variance criterion:**
  - A is preferred to B if:
    - Expected return on A is equal or higher OR
    - SD of A is equal or lower than B
    - Both equations must hold
  - In other words, we want high return and low risk



10.1 Characteristics of functions with different risk-aversion coefficients. (1) Utility function of a risk-seeking investor. (2) Utility function of a risk-neutral investor. (3) Utility function of a risk-averse investor.

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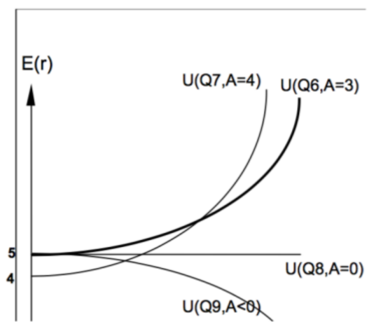
- More risk averse investors will not select portfolios with higher risk (even with higher risk premiums)

Expected returns and standard deviation enough to describe choices between stocks when

1) Quadratic utility functions
2) Returns are normally distributed



Week 3 – Optimal Portfolios

- How to construct risky portfolios
- Mean-variance opportunity set
    - This is combination of risky assets that minimises portfolio variance for a given level of portfolio expected return

$$\text{Minimise} \quad \sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j \sigma_{ij} = \sigma_p^2$$

$$\text{subject to} \quad \sum_{i=1}^{N} x_i E(R_i) = E(R_p)$$

$$\text{and} \quad \sum_{i=1}^{N} x_i = 1$$

- The last term means that the investor is fully invested into the market
- Once investor identifies these portfolios, they pick one to maximise expected utility

- **2 risky asset Portfolio**

    - Expected return is:

$$E(R_p) = E(R_1)x_1 + E(R_2)x_2$$

- Variance is:

$$\sigma_p^2 = x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + 2x_1x_2\sigma_{12}$$

$$\sigma_p^2 = x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + 2x_1x_2\rho_{12}\sigma_1\sigma_2$$

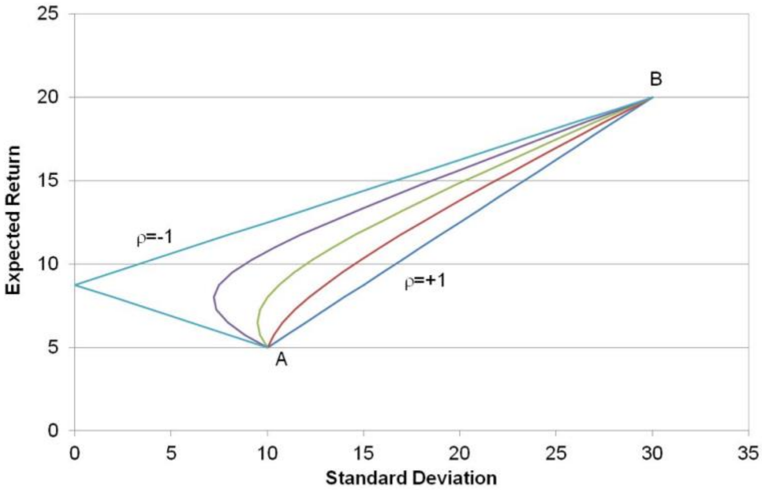
Whereby:

$$r_{ij} = \frac{\sigma_{ij}}{\sigma_i \sigma_j}$$

- Correlation coefficient is the covariance between two assets divided by their standard deviations
- Investors interested in minimising variance of returns
- Risk neutral investors will pick assets with highest return without regards on the risks

- As the correlation falls, then risks of a portfolio falls

- It is possible to achieve a portfolio variance of 0 if one of the assets are perfectly negatively correlated



- It is possible to move past point B whereby we short B and invest in A

- The lower the correlation is, the lower the risk and we see an increase in utility on the mean-variance plane

- **Minimising portfolio variance for 2 assets**

$$\sigma_p^2 = x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + 2x_1x_2\sigma_{12}$$

- Using the formula for two-asset portfolio variance

$$\sigma_p^2 = x_1^2\sigma_1^2 + (1-x_1)^2\sigma_2^2 + 2x_1(1-x_1)\sigma_{12}$$

- We can rearrange and collect terms

$$= x_1^2\left(\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}\right) + \sigma_2^2 + 2x_1\left(\sigma_{12} - \sigma_2^2\right)$$

- Differentiating this equation and setting its first derivative equal to zero

$$\frac{\partial\sigma^2}{\partial x_1} = 2x_1\left(\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}\right) + 2\sigma_{12} - 2\sigma_2^2 = 0$$

- Rearranging we find the weight that minimises portfolio variance

$$x_1 = \frac{\sigma_2^2 - \sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}}$$

- Differentiating it allows us to find the minimum variance of the portfolio. Allows for F.O.C

- **3 risky asset portfolio**

$$\sigma_p^2 = x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + x_3^2\sigma_3^2 + 2x_1x_2\sigma_{12} + 2x_1x_3\sigma_{13} + 2x_2x_3\sigma_{23}$$

$$E(R_p) = E(R_1)x_1 + E(R_2)x_2 + E(R_3)x_3$$

- **Refer to lecture notes for derivations**

Constraint: x’e = 1 - investor must be fully invested into the market

$$\sigma_p^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j \sigma_{ij}$$

$$\sigma_p^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j \rho_{ij}\sigma_i\sigma_j$$

- Portfolios with n-risky assets requires correlation coefficient
- Covariance matrix represents pairwise combinations of all portfolio components
- Requires estimates of n expected return and n variance terms
- **Required number of Covariances of a portfolio is (N^2-N)/2**

Gains from diversification

- Smaller correlation between 2 risky assets, leads to higher gains in diversification
- Investors only gain from this provided securities are not perfectly correlated
- This occurs due to offsetting firm specific risks among assets

$$2x_1(1-x_1)\rho_{12}\sigma_1\sigma_2$$

- More specifically, it is due the p falling

$$\sigma_p^2 = \sum^{n} x_i\sigma_i^2 + \sum\sum_{j} x_i x_j \sigma_{ij}$$

$x_i = \frac{1}{n}$ (this means equally weighted portfolio)

$$\sigma_p^2 = \sum_{i} \frac{1}{n}\frac{1}{n}\sigma_i^2 + \sum\sum\sigma_{ij}\frac{1}{n^2}$$

$$\sigma_p^2 = \frac{1}{N}\bar{\sigma}^2 + \frac{1}{N^2}\sum\sum\sigma_{ij}$$

$$\sigma_p^2 = \frac{1}{N}\,\bar{\sigma}^2 + (N^2 - N)\,\frac{1}{N^2 - N}\,\frac{1}{N^2}\sum\sum\sigma_{i_j}$$

$$\sigma_p^2 = \frac{1}{N}\,\bar{\sigma}^2 + (N^2 - N)\,\frac{1}{N^2}\,\bar{\sigma}_{i_j}$$

$$\boldsymbol{\sigma_p^2 = \frac{1}{N}\,\bar{\sigma}_i{}^2 + \frac{N-1}{N}\bar{\sigma}_{i_j}}$$

- From this, variance has 2 components:
  - (2nd term) Systematic risk which cannot be diversified away such as the economy. N-1/N = 1 as n goes to infinity, so this can't be invested away. However, **if covariance between securities are 0, term on right becomes 0**
  - (1st term) Idiosyncratic risk which is firm specific. Can be removed via diversification. We can invest this away.

- More assets added to a portfolio causes the portfolios' variance to decline on average since as N → infinity

- However, portfolio variance doesn't always decline as the volatility may be greater. However, it declines as you add an infinite amount of stocks

- **Opportunity set**

- Efficient portfolio: No other portfolios will have same expected return and a lower variance of return

$$\text{Minimise} \qquad \sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j \sigma_{ij} = \sigma_p^2$$

$$\text{subject to} \qquad \sum_{i=1}^{N} x_i E(R_i) = E(R_p)$$

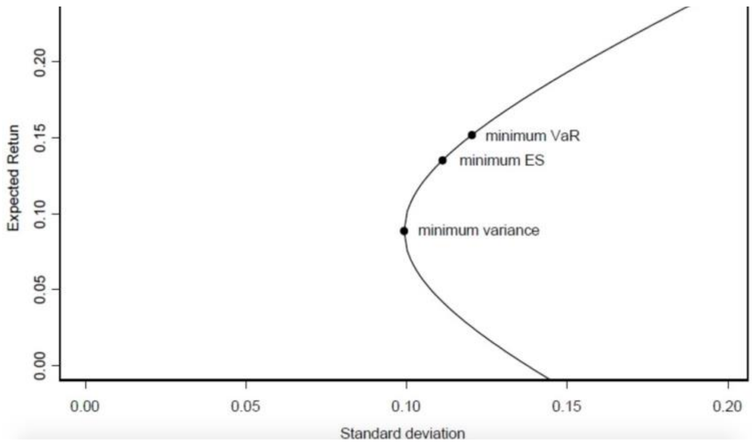
$$\text{and} \qquad \sum_{i=1}^{N} x_i = 1$$

Minimisation problem since

$$c = \sum_{i=1}^{n} x_i^2\sigma_i^2 + 2\sum_{i=1}^{n}\sum_{\substack{j=1\\ i\neq j}}^{n} x_i x_j \rho_{ij}\sigma_i\sigma_j + \lambda_1\left[1 - \sum_{i=1}^{n} x_i\right] + \lambda_2\left[E(R_p) - \sum_{i=1} x_i E(R_i)_i\right]$$

$$\sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j \sigma_{ij} = \sigma_p^2 \rightarrow \sigma_p^2 = x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + 2x_1x_2\sigma_{12}$$

- Take partial derivatives w.r.t to each variable. Generates N+2 equations which can be solved
- Minimising portfolio variance at each expected level of portfolio return is called the min-variance set. We minimise variance, at a given level of return



- Here, anything above minimum variance point is the efficient set/frontier
- The efficient frontier is what investors care about
- Any points in the set itself is considered inefficient and suboptimal
- Where an investor invest on efficient frontier depends upon their risk appetite
- All the individual assets lie to the right of the minimum variance frontier

2 steps

1) Find the optimal risky portfolios which is the same for all investors regardless of risk aversion. We find the CAL tangential to efficient frontier
2) Capital allocation then depends on the personal preference of investor

Question

- A pension fund manager is considering two mutual funds. The first is a stock fund, the second is a long-term government and corporate bond fund. The risk-free rate is 8%. The probability distribution of the risky funds is as follows:

	Expected return	Standard deviation
Stock fund	20%	30%
Bond fund	12%	15%

- The correlation between the fund returns is 0.10

- What are the investment proportions in the minimum-variance portfolio of the two risky funds, and what is the expected value and standard deviation of its rate of return?

(Answer: x = 0.174, 0.826, and return = 13.92%)

- **Capital allocation between risk-free and risky asset**

- Risky portfolio investor chooses depends upon preferences whereby their utility is dependant upon expected return-standard deviation space

- Optimal portfolio selected will lie on efficient frontier and provides maximum level of return for given level of risk. Shifts to the North-East quadrant is increasing utility

- **Risky + risk free asset**

    - Risk free rate generates new stuff

$$
\begin{aligned}
E(R_c) &= (1-x)E(R_f) + xE(R_p) \\
&= (1-x)R_f + xE(R_p) \\
&= R_f + x[E(R_p) - R_f]
\end{aligned}
$$

    -
    - R_P is the return on the portfolio on the efficient frontier. They can allocate capital between that specific point (which was derived from risk-return analysis)
    - R_C *and* $var(R_c)$ is known as a combined portfolio of both the portfolio + risk free asset

$$\begin{aligned} Var(R_c) &= (1-x)^2 Var(R_f) + x^2 Var(R_p) + 2(1-x)xCov(R_p,R_f) \\ &= x^2 Var(R_p) \\ \therefore\ \sigma_c &= x\sigma_p \end{aligned}$$

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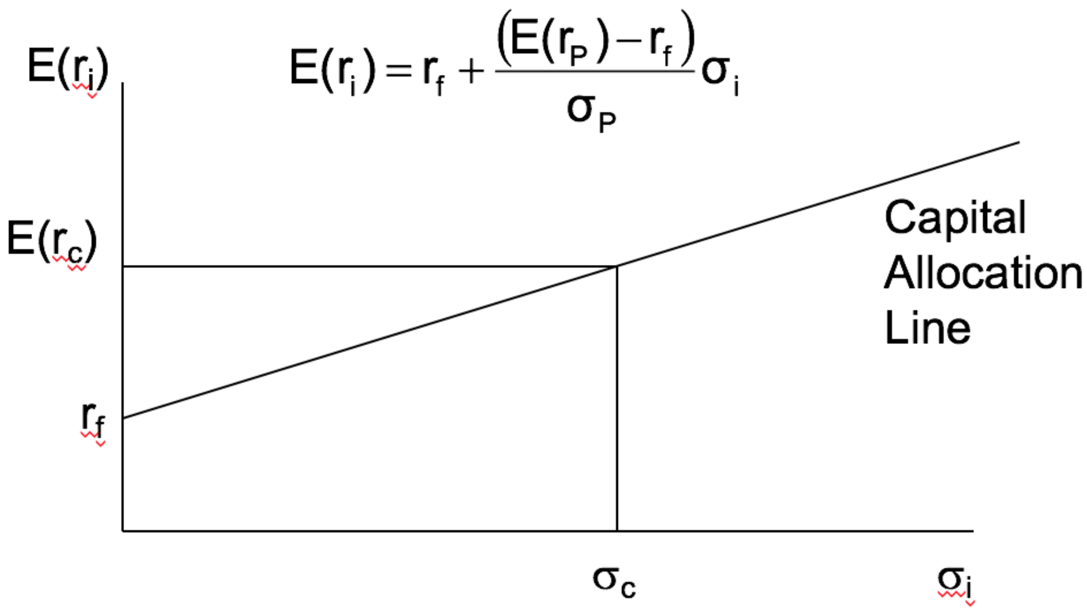
- Therefore, the risk of portfolio only depends upon the riskiness of portfolio
- No correlation between risk-free and risky asset

$$E(R_i) = R_F + x[E(R_p - R_F)$$
$$Where\ x = \frac{\sigma_i}{\sigma_P}$$
$$E(R_i) = R_F + \frac{\sigma_c}{\sigma_p}[E(R_p - R_F)]$$
$$E(R_i) = R_F + \left[\frac{E(R_p) - R_F}{\sigma_p}\right]\sigma_i$$

- **Sharpe ratio = Excess return over risky portfolio is divided by standard deviation of portfolio**

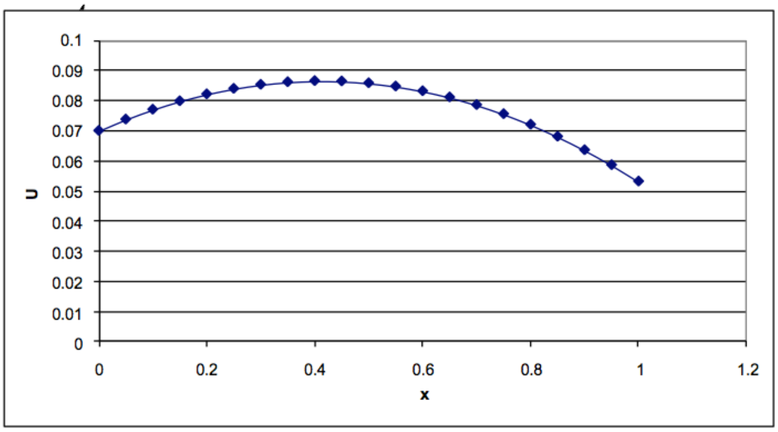


The equation in yellow is known as the **Capital Allocation line**

The slope of this line is known as the **Sharpe Ratio (Reward – variability ratio)**

- A steep CAL is desirable. Every portfolio has an associated CAL with it
- It is desirable to maximise Sharpe ratio for highest return per unit risk
- The Sharpe ratio tells us the additional expected return an investor takes on for each additional unit of risk that the investor takes on
- If different borrowing/investing rates, Sharpe ratio reduces when one invests more than 100% of wealth into risky fund

- We can then combine utility function + asset allocation decision to determine where along CAL is investor likely to select.



This plot shows how utility of the portfolio changes as the proportion invested in risky assets changes ($A = 4$)

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- This tells us how much into risky asset an investor will undertake. Look at optimal value.

- **Risk aversion and asset allocation**

- We can see a point where utility is maximised, which we can rewrite as optimisation problem

$$\max_x U = \mathbf{E}[r_C] - \tfrac{1}{2} A\sigma_C^2$$
$$= r_f + x(\mathbf{E}[r_P - r_f]) - \tfrac{1}{2} Ax^2\sigma_P^2$$

- Differentiate w.r.t to x to find how much into risky asset we should invest into

$$\frac{dU}{dx} = \mathbf{E}[r_P] - r_f - Ax\sigma_P^2 = 0$$
$$x = \frac{\mathbf{E}[r_P] - r_f}{A\sigma_P^2}$$

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- As A increases, we invest less into risky asset since we are more risk-averse.

- The risk aversion parameter depends where along the CAL we move

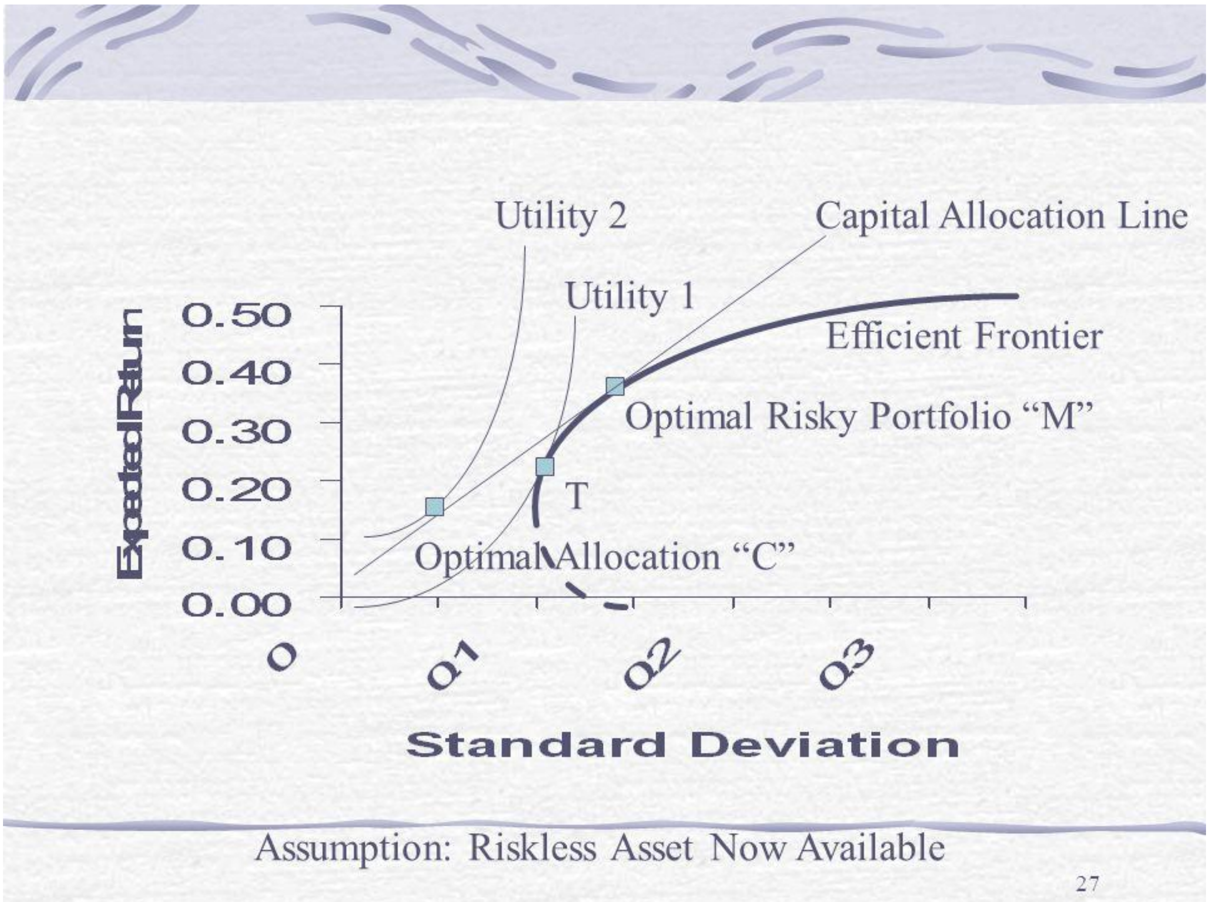
N-risky assets and 1 risk free asset

- We can use mean-variance theory to choose which risky portfolio of N-assets to invest in and combine with risk free asset

$$\mathbf{E}[r_C] = r_f + x(\mathbf{E}[r_P] - r_f)$$

$$\sigma_C = x\sigma_P$$

- **One fund theorem:** When risk free asset available, only 1 risky fund people want to invest in (so we have our point on efficient frontier + risk free rate)
- Efficient frontier is now straight line tangential to efficient frontier
- To get this, we can maximise sharpe ratio



- With a risk free asset, our utility has increased from utility 1 to utility 2
- Everyone holds same risky portfolio with risk free rate

When calculating opportunity set, doesn't matter if we know expected returns etc with certainty. We can estimate these with standard error so estimate accuracies varies across securities. However, if estimates are prone to error, we end up with error maximization scenario which sees portfolios concentrate on subset of assets (securities with highest return/lowest variances that are entered incorrectly). We need accurate measures of expected returns etc, and can hope to derive approximately optimal portfolio.

When all stocks have same expected rate of return, optimal portfolio for risk averse agent is global minimum variance portfolio.
Covariances of global minimum variance portfolio with all other assets are identical and equal to its own variances

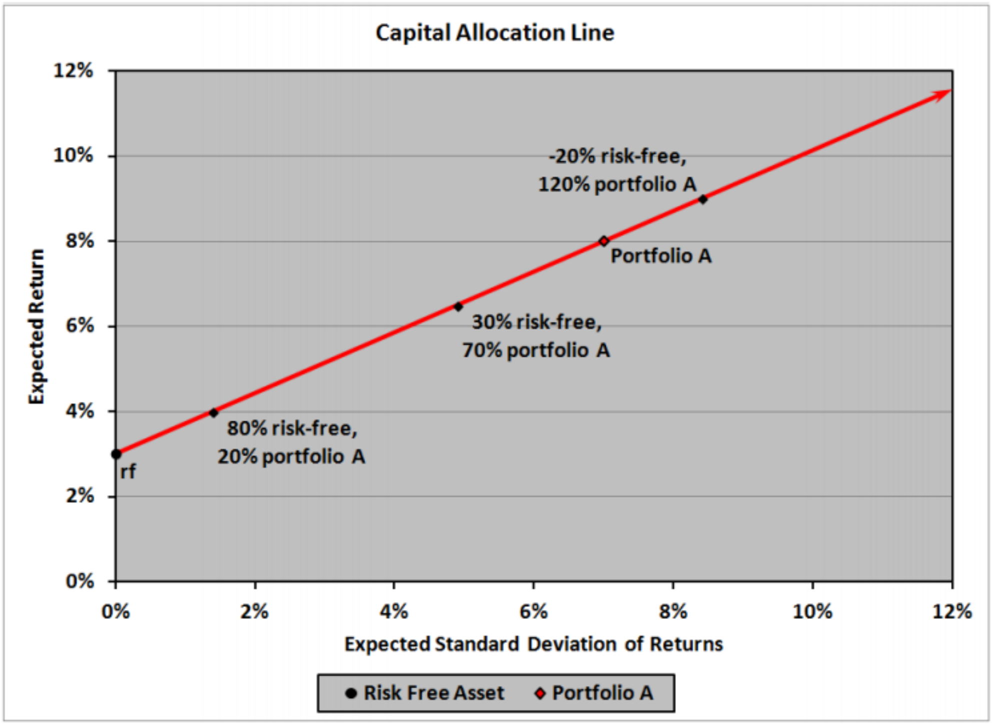
Capital allocation line vs Capital Market Line vs Security Market line

3 assumptions for them

- Risk free asset and risky assets present
- Investor can borrow/lend at risk free rate
- Investor can buy any amount of risky asset

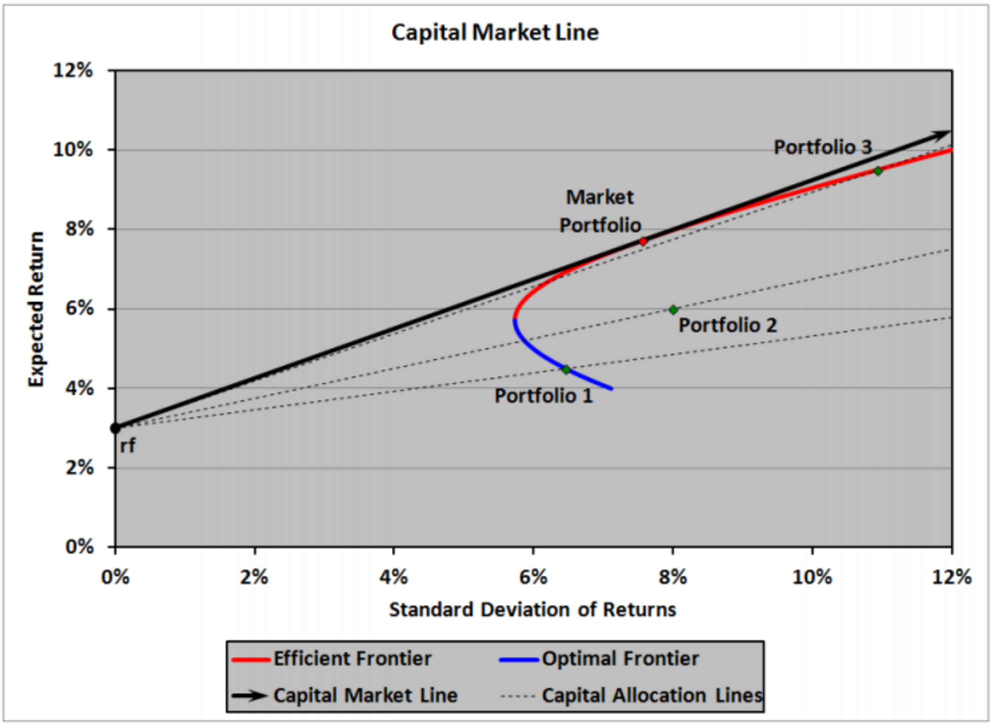
Capital Allocation Line

- Combination of risky asset + risk free asset
- Measures return per unit of total risk
- The gradient is the **Sharpe ratio**



Capital Market Line

- We have a risky portfolio (market portfolio) with the highest sharpe ratio
- **The capital market line is the capital allocation line for the market portfolio**
- Applies to only efficient portfolios



- Slope is the **Market risk Premium/Standard deviation of the market**
- **Formula for Optimal Capital Allocation Line**

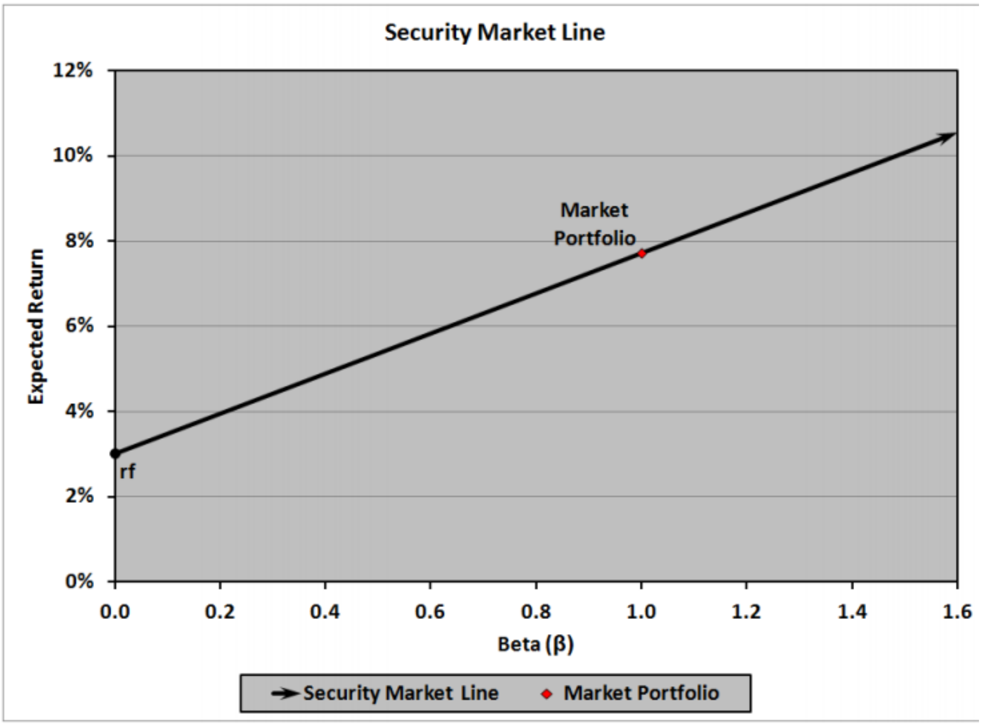
$$r_p = r_f + \sigma_p \left(\frac{r_m - r_f}{\sigma_m} \right)$$

- Can't use CML if not efficient portfolio

Security Market line

- SML is derived from CML except SML only looks at systematic risk (since that's what should be rewarded for)
- The SML is applicable to all portfolios and securities regardless if they are optimal or not

-



If 2 assets has perfectly negative correlation, risk free portfolio can be created

$$\sigma_P = \text{Absolute value } [w_A\sigma_A - w_B\sigma_B]$$

$$\boldsymbol{\sigma_p = \sigma_a x_a - ((1 - x_a) * \sigma_b)}$$

or

$$\boldsymbol{\sigma_p = absolute\ value\ (x_a\sigma_a - x_b\sigma_b)}$$

This comes from taking the perfect square of variance formula

We can set this = 0 to get perfect hedge (if correlation between them is -1

$$\boldsymbol{0 = x_a\sigma_a - x_b\sigma_b}$$

$$\boldsymbol{x_a = \frac{\sigma_b}{\sigma_a + \sigma_b}}$$

Covariance of 2 stocks

$$Cov(r_A, r_B) = \beta_A \beta_B \sigma_M^2$$

Beta of a portfolio is just a weighting of the beta of components in the portfolio

Week 4 – SIM

Flaws with Markowitz

- Requires significant number of inputs N(N-1) + 2n
- Doesn't provide guideline to forecasting security risk premium

- **Simplify by decomposing risk of security into firm specific risk and market wide risk**

- **SIM:** This is a generalised asset pricing model used to price an asset.
- This model suggests that a stock's return is dependant upon market (beta), firms specific expected value (alpha), and firm-specific unexpected component. The stock is further influenced by return on the market portfolio whereby some stocks are more affected by the market than others, therefore leading to a higher beta

$$r_i - r_f = \alpha_i + \beta_i(r_m - r_f) + \epsilon_i$$
$$where : \epsilon_i \sim N(0, \sigma_i)$$

-

Assumptions of SIM

- Residuals are uncorrelated across asset and time.
    - Firm specific risks are uncorrelated
    - No correlation across time
    - Not correlated to the market
- ZCM (on market factor return)
- Not related to factor return, mean 0 and constant variance

This means that returns on the market factor is a constant + residual at each point in time

$$R_{mt} = A + c_t$$

Therefore, expected return on Market factors is

Rm = A

Standard deviation is $\sigma_m = \sigma_c$

Industry effects are ruled out in this model. Since A is constant, no standard deviation effects from it

Expected return on asset i:

$$E(R_i) = \alpha_i + \beta_i E(R_m)$$
$$\sigma_i^2 = \sigma_m^2 \times \beta_i^2 + \sigma_{\varepsilon_i}^2$$

From this, first term of variance is the market risk and 2nd term is firm specific risk

$$R_{it} = \alpha_i + \beta_i R_{Mt} + \varepsilon_{it}$$

$$\sigma_i^2 = 0 + \beta_i^2 \sigma_m^2 + \sigma_{\varepsilon_i}^2$$

The covariance between asset I and j is:

$$\begin{aligned}
cov(R_i, R_j) &= cov(\alpha_i + \beta_i R_{mt} + \varepsilon_{it}, \alpha_j + \beta_j R_{mt} + \varepsilon_{jt}) \\
&= cov(\beta_i R_{mt} + \varepsilon_{it}, \beta_j R_{mt} + \varepsilon_{jt}) \\
&= cov(\beta_i R_{mt}, \beta_i R_{mt}) \\
&= \beta_i \beta_j \sigma_m^2
\end{aligned}$$

Subbing this into the variance of the portfolio and taking the variance of the entire portfolios (therefore we need sigma) gets us:

$$\sigma_p^2 = \sigma_m^2 \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \beta_i \beta_j + \sum_{i=1}^{n} x_i^2 \sigma_{\varepsilon_i}^2$$

First component is systematic risk and 2nd one is idiosyncratic risk
We only need 3n + 2 variables

SIM:
Returns of a security can be expressed as expected component and unanticipated component:

Ri = E(Ri) + Ui

Where Ui is iid ~ N(0,sigma)

Individual returns are generally correlated with each other due to common response to market changes (residuals are however not correlated!) This holds if security returns are well approximated by normal distributions (joint normally distributed)

Assume all stocks are influenced by one influence (market return)

- We are breaking up the earlier equation's uncertainty into firm specific uncertainty + economy uncertainty

$$R_i = E(R_i) + m + z_i$$

Z is firm specific surprise
M is macroeconomic system
E(M) = 0 since it affects all stocks. Also deviation of sigma m
Therefore, 2 surprises that causes firm returns

$$\sigma_i = \sigma_m + \sigma_e$$

- **Some firms are more affected and sensitive to economy, so we can use beta to take this into account (single-factor model)**

$$r_i = E(R_i) + \beta_i m + e_i$$
$$\sigma_i = \beta_i \sigma_m + \sigma_e$$

- **When we run a regression, we can use the market as a proxy for common economic factor and then we get the SIM**
  SIM

- SIM describes an asset return as made up of a constant and a sensitivity to a factor
  - This factor is often the 'market' index (e.g. ASX200, S&P500, FTSE100)

$$R_{it} = \alpha_i + \beta_i R_{mt} + \varepsilon_{it}$$

- Where
  - R_{it} is the return on asset i in period t
  - R_{mt} is the return on the factor in period t
  - α_i is the constant component of asset i's return
  - β_i is the sensitivity of asset i's return with the factor
  - ε_{it} is residual component of asset i's return that is not explained by the factor

The difference between this and CAPM is that this uses any 1 factor, whilst CAPM only uses 1 specific factor, which is the market portfolio

Let's assume the factor is the return on the ASX300 index (R_m)

It is also common to work in 'excess return' form and deduct the risk-free rate from the both the asset's returns and the market's return:

$$R_{it} - R_{ft} = \alpha_i + \beta_i (R_{mt} - R_{ft}) + \varepsilon_{it}$$

- If the excess return on the market is zero, then α_i represents the expected excess return on the stock
- The sensitivity of the securities return is given by β_i (the larger β_i is, the greater the swings in a stocks return when the market moves)

Given that $E(\varepsilon_i) = 0$, we can similarly express expected returns as

$$E(R_{it} - R_{ft}) = \alpha_i + \beta_i E(R_{mt} - R_{ft})$$

- This is the expected return-beta relationship

- **Second term tells us the security's risk premium due to risk premium of the index (which is a proxy for economic factors)**

ASSUME NOW THAT EQUALLY WEIGHTED PORTFOLIO WHERE xi = 1/N, then the alpha of portfolio is just the average alpha of assets

The SIM also provides intuition about the benefits of diversification

Let's assume that portfolios are equally weighted ($x_i = 1/N$)

Alpha, beta and residual risk are therefore the averages:

$$\alpha_p = \frac{1}{N}\sum_{i=1}^{N}\alpha_i$$

$$\beta_p = \frac{1}{N}\sum_{i=1}^{N}\beta_i$$

$$\varepsilon_p = \frac{1}{N}\sum_{i=1}^{N}\varepsilon_i$$

- This allows us to these the alpha etc of the portfolio of equally weighted assets are

- Here, we can see that

$$\begin{aligned} R_p &= \sum x_i R_i \\ &= \sum x_i(\alpha_i + \beta_i R_m + \epsilon_i) \\ &= \sum \alpha_i x_i + R_m \sum \beta_i x_i + \sum \epsilon x_i \end{aligned}$$

$$R_P = \alpha_p + \beta_p R_m + \epsilon_p$$

The return of a portfolio under SIM

- Now look at variance of returns on portfolio

$$\sigma_p^2 = \beta_p^2\, \sigma_m^2 + \sigma_{e_p}^2$$

This looks at market + firm specific risk
Since all firm specific risks are considered to be independent, we can then rewrite this for the portfolio
We can rewrite

$$\sigma_p^2 = \sigma_m^2 \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \beta_i \beta_j + \sum_{i=1}^{n} x_i^2 \sigma_{\varepsilon_i}^2$$

into:

$$\sigma_{\varepsilon_p}^2 = \sum_{i=1}^{N} x_i^2 \sigma_{\varepsilon_i}^2$$

$$= \frac{1}{N} \frac{1}{N} \Sigma \sigma_{\varepsilon_i}^2$$

$$= \frac{1}{N} \bar{\sigma}_{\varepsilon_i}^2$$

The above equation says that the portfolio risk from firms is the average firm specific average X 1/N
Subbing this into above equation, we get

$$\sigma_p^2 = \beta_p^2\, \sigma_m^2 + \frac{1}{N}\bar{\sigma}_e^2$$

as N → infinity, we see that second term goes to 0.
This means that for large portfolio we get

$$\sigma_p^2 = \beta_p^2\, \sigma_m^2$$

This says we only have systematic risks leftover
This is unrealistic since assumes no industry risks.

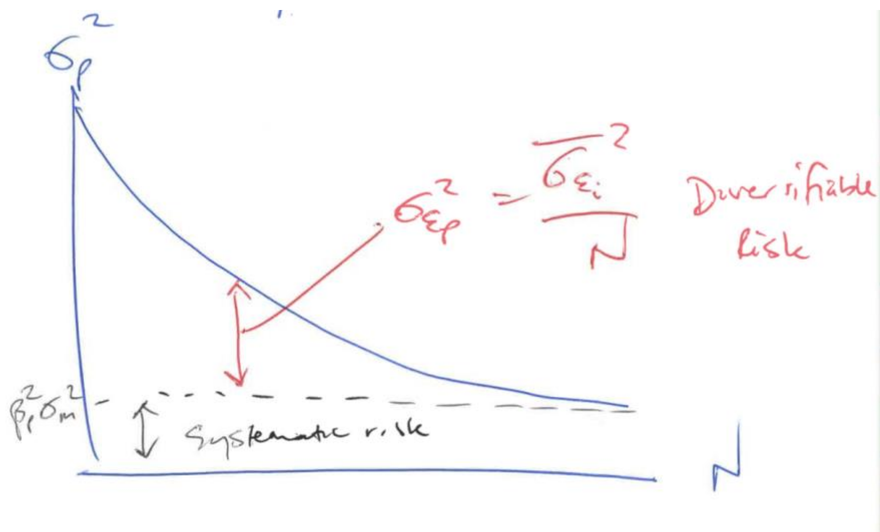
Single index model variance for a non-well diversified portfolio

$$\sigma^2 = \beta^2 \sigma^2{}_M + \sigma^2(e)$$

If it is well-diversified, last term disappears
Covariance between 2 stocks in SIM is:

$$cov(r_i, r_j) = \sigma_{i_j} = \beta_i \beta_j \sigma_m^2$$

$$Correlation\ with\ market = \rho = \sqrt{(R^2)}$$



__Firm specific risk__

1) **Standard deviation of residuals from a regression**
2)

$$R^2 = \frac{Systematic\ risk}{Total\ risk} = \frac{\beta_i^2 \sigma_m^2}{\sigma_i^2} = \frac{\beta_i^2 \sigma_m^2}{\beta_i^2 \sigma_m^2 + \sigma_{\epsilon_i}^2} = 1 - \frac{\sqrt{\frac{RSS}{N-1}}}{\beta_i^2 \sigma_m^2 + \sigma_{\epsilon_i}^2}$$

2nd term is idiosyncratic risk over total risk
The level of R-square tells us how much of the risk is systematic

3)

$$R^2 = \sqrt{(\ SER^2\ X\ \frac{N-K-1}{N-1})}$$

Beta can be estimated using the following regression:

$$R_{i,t} - R_{f,t} = \alpha_i + \beta_i (R_{m,t} - R_{f,t}) + \varepsilon_{i,t}$$

4)

Square root(1 – R^2 *variance(Total raw returns))

Beta is imprecise because
- They vary according to data used for estimation

- Thin trading creates bias. If share traded infrequently, then beta estimates using this approach can lead to bias due to first order serial correlation. So if news occur and market changes, but stock doesn't since infrequently traded, then next trade, share adjust to news but market return and thus there is correlation and bias. We can use a **DIMSOM adjustment** for this

  - Dimson (1979) adjustment :

$$R_{i,t} = a_i + \beta_{i,k} R_{m,t-k} + \ldots + \beta_i R_{m,t} + \ldots + \beta_{i,j} R_{m,t+j} + \varepsilon_{i,t}$$

  - Where
    - $R_{i,t}$ is excess return on stock i at time t
    - $R_{m,t}$ is excess return on the market portfolio at time t
    - $\varepsilon_{i,t}$ is the residual from the regression
    - k is the number of lagged market returns
    - j is the number of leading market returns

  - The stock's beta is

$$\beta_i^{Dim} = \beta_{i,k} + \ldots + \beta_i + \ldots + \beta_{i,j}$$

-
- Can vary over time for same firm.

Alpha and security analysis

- We don't hold same portfolios because people have different opinions
- We can construct portfolios using single index model
-
- **Treynor-Black Model**

- This is for investor who believes most stocks are efficiently price yet there are some stocks they believe aren't. Therefore, they have a choice to invest in the active portfolio where they can find alpha and a market portfolio. The TB model allows for investor to invest between the 2 portfolios

  - Use macroeconomic analysis to estimate risk premium and risk of market index
  - You can invest some portion in market portfolio and some into active portfolio
  - If only care about diversification, hold market portfolio
  - If you can find alpha through security analysis, don't hold market
  - Model trades between alpha and cost of not diversifying efficiently

- Tradeoff for some alpha for some risk. Depends on return vs risk

- So if you think Dominos has some alpha, then hold more dominos in addition to market portfolio but also note that you'll then hold more firm-specific risk

- Risk premium on passive market portfolio: Rm – Rf

- We can estimate standard deviation of passive market portfolio

- There are n+1 assets to invest in

- The N assets refer to assets we believe we have found alpha for and 1 more for market portfolio

$$E(R_p) = \alpha_p + \beta_p E(R_m) = \sum_{i=1}^{n+1} x_i \alpha_i + E(R_m) \sum_{i=1}^{n+1} x_i \beta_i$$

$$\sigma_p^2 = \beta_p^2 \sigma_m^2 + \sigma_{\varepsilon p}^2 = \sigma_m^2 \left(\sum_{i=1}^{n+1} x_i \beta_i \right)^2 + \sum_{i=1}^{n+1} x_i^2 \sigma_{\varepsilon i}^2$$

- Alpha for market = 0 and beta = 1

- **Set all returns to be excess returns**

- Investors want to maximize **Sharpe ratio**

  - $S_p = \frac{E(R_p)}{\sigma_p}$

OPTIMAL WEIGHTS IF YOU HAVE 2 ASSETS. This maximises Sharpe ratio for 2 asset portfolio

$$x_a = \frac{E(R_a)\sigma_m^2 - E(R_m)Cov(R_a, R_m)}{E(R_a)\sigma_m^2 + E(R_m)\sigma_a^2 - [E(R_a) + E(R_m)]Cov(R_a, R_m)}$$

$$x_m = 1 - x_a$$

After derivation, we get this

$$x_a = \frac{\alpha_a}{\alpha_a(1-\beta_a) + \frac{R_m \sigma_{\varepsilon a}^2}{\sigma_m^2}}$$

Assume beta of active portfolio is 1. Term at bottom disappears and we get

$$x_a = \frac{\alpha_a}{\frac{R_m \sigma_{\varepsilon a}^2}{\sigma_m^2}} = \frac{\alpha_a}{\sigma_{\varepsilon a}^2} \frac{\sigma_m^2}{R_m} = \frac{\alpha_a / \sigma_{\varepsilon a}^2}{R_m / \sigma_m^2}$$

Weight of active portfolio is alpha divided by idiosyncratic variance

Trying to maximise sharpe ratio and we can decide to bring in some alpha but we also get some idiosyncratic risk. We need to weigh these against each other. How much alpha per unit of idiosyncratic variance.

If alpha = 0, weight goes to 0, and so you just invest in market portfolio
If not, need to scale it by variance since we adding extra risk
Top line: **Information/appraisal ratio :** More alpha per unit of idiosyncratic risk, you want to hold more of it.
We then take the optimal weight we found above, and readjust for its level of beta. The higher beta it is, less diversification benefits we get, so we need to hold more of it.

But beta isn't always equal to 1

$$x_a^* = \frac{x_a}{1 + (1-\beta_a)x_a}$$

We now need to make adjustments for the fact that active portfolio's beta isn't 1

Beta > 1

- Diversification benefits are lower. Active portfolio not useful for diversification benefits, so hold more of it, so x* increase
- **Note that the more stocks we invest into active portfolio, we get a more diversified portfolio**

- **2 ways to measure covariance**

$$\sigma_A \sigma_M \rho_{AM} = \beta_A \beta_M \sigma_M^2$$

$$\sigma_A \rho_{AM} = \beta_A \beta_M \sigma_M$$

$$\beta_A = \frac{\sigma_A}{\sigma_M} \rho_{AM}$$

- Beta of active portfolio is this. As beta goes up, correlation goes up, so diversification benefits falls. So benefits of diversification with active portfolio falls, and you need to adjust for that by holding a higher weight. No diversification benefit if you hold it, so need to hold more of it.
- So for any idiosyncratic risk, correlation between active portfolio and market portfolio increases as beta increases

- If negative view of stock (so alpha is negative) and therefore active position is now negative

- **Conclusively, we have a passive portfolio and an active portfolio**

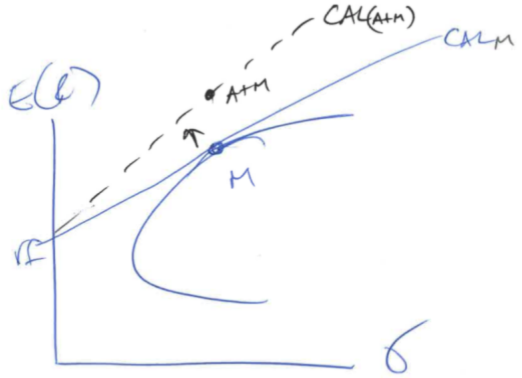
- **Now if you have multiple stocks how do u decide**

$$x_i^* = x_a^* \left(\frac{\frac{\alpha_i}{\sigma_{\varepsilon i}^2}}{\sum_{i=1}^{n} \frac{\alpha_i}{\sigma_{\varepsilon i}^2}} \right)$$

- Weight is information ratio divided by sum of other stocks you want to have a position in
- If 10 stocks have same appraisal ratio, 10% into each
- First calculate individual stock, then allocation between the 2 portfolios

$$Sharpe\ Ratio = S_M^2 + \left(\frac{\alpha_a}{\sigma_{\epsilon_a}} \right)^2$$

The contribution of active portfolio to Sharpe ratio of overall risky portfolio is determined by ratio of alpha to residual standard deviation.
We the nonmarket risk premium (from alpha) but we also get increase in variance through idiosyncratic/ firm-specific risk from residuals



- We can now increase our CAL and Sharpe ratio by adding more value via the appraisal ratio and get a higher utility. Alpha is therefore good
- Beta affects both risk and return
- So from the equation

$$\frac{R_m}{\sigma_m^2} = \frac{\beta_i R_m + \alpha_i}{\beta_i \sigma_m^2 + \sigma_\epsilon^2} = \frac{R_i}{\sigma_i^2}$$

- If we increase Beta, increases both risk and return
- However, if we increase alpha, we increase the return, and only idiosyncratic risk, which is why we only focus on the latter part of the term

$$\frac{R_m}{\sigma_m^2} \Rightarrow \frac{\cancel{\beta_i} R_m + \alpha_i}{\cancel{\beta_i^2 \sigma_m^2} + \sigma_{\epsilon_i}^2} = \frac{R_i}{\sigma_i^2}$$

Beta of a security affects both risk and risk premium, so doesn't make it bad per se.
Passive portfolio is efficient if alphas are all 0

Summary of Optimization Procedure

Once security analysis is complete, the optimal risky portfolio is formed from the index-model estimates of security and market index parameters using these steps:

1. Compute the initial position of each security in the active portfolio as $w_i^0 = \alpha_i/\sigma^2(e_i)$.
2. Scale those initial positions to force portfolio weights to sum to 1 by dividing by their sum, that is, $w_i = \dfrac{w_i^0}{\sum_{i=1}^{n} w_i^0}$.
3. Compute the alpha of the active portfolio: $\alpha_A = \sum_{i=1}^{n} w_i \alpha_i$.
4. Compute the residual variance of the active portfolio: $\sigma^2(e_A) = \sum_{i=1}^{n} w_i^2 \sigma^2(e_i)$.
5. Compute the initial position in the active portfolio: $w_A^0 = \left[\dfrac{\alpha_A / \sigma^2(e_A)}{E(R_M)/\sigma_M^2}\right]$.
6. Compute the beta of the active portfolio: $\beta_A = \sum_{i=1}^{n} w_i \beta_i$.
7. Adjust the initial position in the active portfolio: $w_A^* = \dfrac{w_A^0}{1 + (1 - \beta_A) w_A^0}$.
8. Note: the optimal risky portfolio now has weights: $w_M^* = 1 - w_A^*$; $w_i^* = w_A^* w_i$.
9. Calculate the risk premium of the optimal risky portfolio from the risk premium of the index portfolio and the alpha of the active portfolio: $E(R_P) = (w_M^* + w_A^* \beta_A)E(R_M) + w_A^* \alpha_A$. Notice that the beta of the risky portfolio is $w_M^* + w_A^* \beta_A$ because the beta of the index portfolio is 1.
10. Compute the variance of the optimal risky portfolio from the variance of the index portfolio and the residual variance of the active portfolio: $\sigma_P^2 = (w_M^* + w_A^* \beta_A)^2 \sigma_M^2 + [w_A^* \sigma(e_A)]^2$.

Markowitz vs SIM

Markowtiz	SIM
Requires more data and difficult to calculate	Easier to calculate since only need beta for common factor
No assumption regarding return generating process	Makes assumption about return generating process where one common source of risk
	Omits industry risk
Considers correlation between firm specific risks	Ignores correlation among firm specific risks
Requires N(N-1)/2 + 2N inputs	Requires (3N+2) inputs
Takes correlated residuals into account	Does not take this into account

- Markowitz makes sense in principle but:
    - Too much computation
    - Cumulative errors can result in terrible portfolio vs SIM
    - SIM is more practical and decentralizes macro and security analysis

To solve portfolio choice, need 3 things

- Expected returns
- Variance
- Covariance

Markowtiz requires a covariance matrix

- Requires N(N-1)/2 covariance terms + N returns and N variances

Sharpe reduces number of parameters required

- Requires 3 estimates for every asset + 2 more market
- **Requires 3N+2 estimates**

Sharpe better since less inputs and therefore lower errors overall

Issues with SIM

- Places restriction on structure of asset return uncertainty by breaking it up into macro vs micro risk
    - No industry risk
    - So if residuals of 2 stocks are correlated, SIM ignores this since it assumes it is 0 whilst Markowitz takes this into account

Week 5 – CAPM

- **CAPM gives prediction of relationship between risk of an asset and expected return**
    - Provides a benchmark rate of return for evaluating investments
    - Allows us to have an educated guess regarding returns

- Assets: Bundles of risks (e.g. industry risks etc)
    - Each different factor defines a different set of bad times like recession
    - Investors exposed to losses during bad times are compensated by risk premiums in good times
    - When you buy an asset, you aim to make a return from it
- Risk: Exposure to bad things. The probability something bad happens
- Factors are what matters, not assets
    - Need to understand what the factors are
    - Investors require different risk factors
- **CAPM** defines bad times as the return on the market
    - In bad times, this is when market returns are low. We want them to be high during these bad times

- **Derivation of CAPM**
    1) All investors using Markowitz approach, construct the same efficient frontiers (since they all have the same available stocks to them)
    2) They all have same risk free rate, and then draw CAL's to this efficient frontier (and these CAL are identical too)
    3) All these CAL's will all be tangential to the same point of efficient frontier and therefore all have the same portfolio on the efficient frontier
    4) Investors then (depending on their risk aversion) chooses a point along the CAL to invest upon
    5) **The aggregation of all these identical risky portfolios is the market portfolio and have the same weights. This is the value weighted portfolio of all assets in the universe**
    6) This means that **each investor's optimal risky portfolio will also be on capital market line**

When we sum over all individual lenders/borrowers, the value of this market portfolio is the entire wealth of the economy. (Borrowers cancel out with lenders). Proportion of each stock in this portfolio equals the market value of the stock. If BangkokBank is 1% in each optimal risky portfolio, it will also be 1% of the market portfolio. If BangkokBank was not in the optimal portfolio, everyone avoids this stock until price is cheaper and then eventually people will include it into optimal stock portfolio. This leads to all stocks being in the optimal and therefore the market portfolio. The CAPM is an equilibrium model.

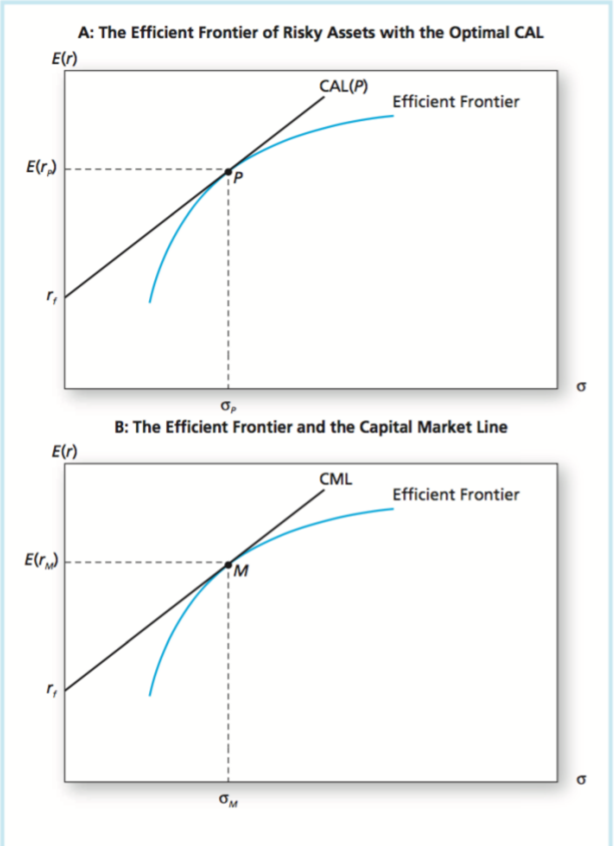


Figure 9.1 Capital allocation line and the capital market line

Assumptions

a. Many investors each with level of wealth that is small compared to total wealth. Perfect competition. Price takers.
b. Investors all plan to hold stocks for single period of same length
c. Investment are restricted to universe of publicly available assets. They can borrow/invest at risk-free rate
d. Investors are mean-variance optimisers. They all use **Markowitz model** to select portfolio. **They have quadratic utilities and returns have multivariate normal distribution**
e. Investors have homogenous expectations. Same estimates for probability distribution of asset returns.
f. No taxes/transactions costs

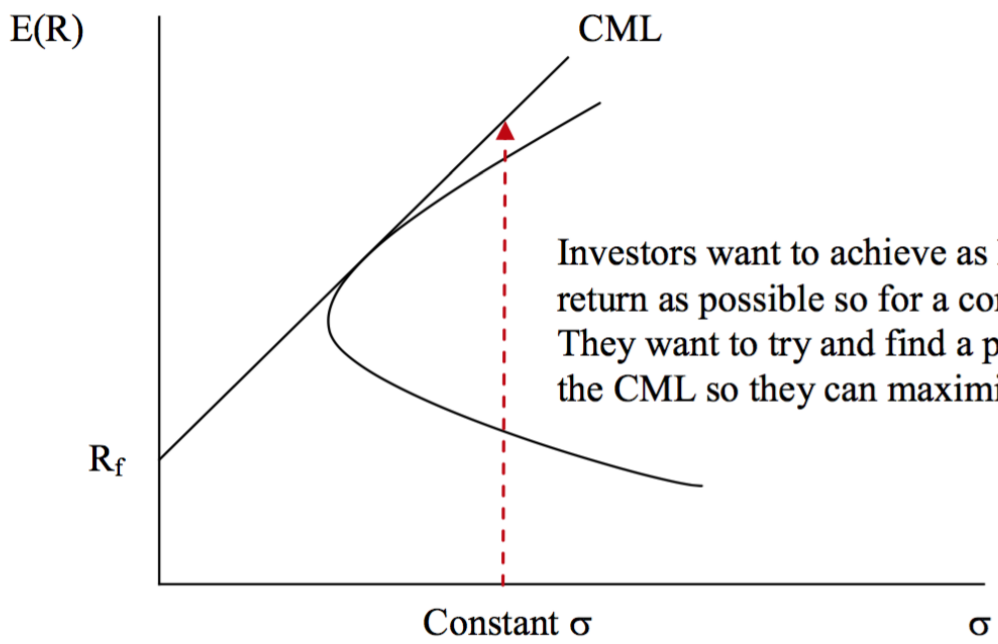
6 assumptions and they are mostly inaccurate

- If investors have same information and form beliefs in same way, all investors have the **same efficient frontier.** They all hold the same optimal risky portfolio. Same weight of asset. E.g. everyone hold 8% of CBA in their portfolio
    - The market portfolio contains all assets (every short has a long) so cancels out

    - Furthermore, if a stock not included in market portfolio, its price falls until attractive enough to be included. Therefore, every stock is in market portfolio, but we just don't know what price they'll be
- All investor will have same U(.) function with different risk aversion
    - This risk aversion determines the allocation between risky portfolio and risk free rate
- With risk-free rate, **efficient frontier is the CML. Not on the bullet curve anymore**
    - The tangent portfolio is the **market portfolio M** which contains all assets (each assets weighting is representative of its relative value to other assets as a result of prices adjusting in equilibrium)
    - **Market portfolio is optimal risky portfolio**

CAPM Derivation

- Efficient portfolios are on CAL which is the tangential line between risk free rate and optimal risky portfolio (Market portfolio)
- Portfolios on CML maximise utility
- For a fixed level of variance, investor determines portfolio weights to determine where on CML to be on



- **3 asset case**
- We maximise the weightings in risky portfolio between it and risk free rate with the constraints of a given variance level

$$\max_{\mathbf{x}} \quad \mathbf{x}'\mathbf{e} + \left(1 - \mathbf{x}'\underline{\mathbf{1}}\right)r_f + \lambda\left(c - \frac{1}{2}\mathbf{x}'\mathbf{V}\mathbf{x}\right)$$

- We take first order conditions to solve

Where:

Vector of weights in three risky assets ($\mathbf{x}$) = $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

Expected returns of three risky assets ($\mathbf{e}$) = $\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}$

Covariance matrix of risky asset returns ($\mathbf{V}$) = $\begin{bmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_2^2 & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_3^2 \end{bmatrix}$

- X'VX = variance of a portfolio
- From F.O.C, we get:

$$\mathbf{e} = \underline{\mathbf{r_f}} + \lambda \mathbf{V}\mathbf{x}$$

- Satisfying this equation, means we are on the CML
- Market are efficient, so weights of 3 assets in portfolio must satisfy f.o.c for maximum and therefore implies that:

$$\mathbf{x_m} = \begin{bmatrix} x_{m1} \\ x_{m2} \\ x_{m3} \end{bmatrix}$$

So the following must be optimal:

$$\mathbf{e} = \underline{\mathbf{r_f}} + \lambda \mathbf{V}\mathbf{x_m}$$

-

- Multiply both sides by X'm

$$E(R_m) = r_f + \lambda Var(R_m)$$

- **Single assets**
  - We can also do it for individual assets

- Multiply both sides by a weight vector which means having 100% invested into asset 1 [1 0 0]

$$\mathbf{x_1'e} = \mathbf{x_1'}\underline{\mathbf{r_f}} + \mathbf{x_1'}\lambda\mathbf{Vx_m}$$

- After rearranging, we get:

- For any asset I (whereby we only invest into it):

$$E(R_i) = r_f + \lambda Cov(R_i, R_m)$$

- **This last term looks at covariance between asset I and the market. Looks at stock's contribution to market variance**

- Taking the formula derived earlier, we can manipulate it

$$E(R_m) = r_f + \lambda Var(R_m)$$

- Which then becomes

$$\lambda = \frac{E(R_m) - r_f}{Var(R_m)}$$

- Lambda is the price of market risk. This tells us the price of risk. The price one gets per unit of risk
- We can sub this into the asset I formula mentioned earlier

$$E(R_i) = r_f + \frac{E(R_m) - r_f}{Var(R_m)} Cov(R_i, R_m)$$

$$E(R_i) = r_f + \frac{Cov(R_i, R_m)}{Var(R_m)} E(R_m) - r_f$$

$$E(R_i) = r_f + \beta\left[E(R_m) - r_f\right]$$

- **THIS GIVES US THE CAPM.** Beta is the covariance of asset with market divided by the variance of the market's return

Lessons

- Don't hold individual asset, hold factor
    - Diversify idiosyncratic risk away
    - Hold market as it is efficiently diversified and has maximum sharpe ratio
    - Same expectations imply same portfolio of risky assets in different weightings depending on individual investor's risk aversion
    - Market is the factor
    - In equilibrium, prices change such that supply=demand
- Each investor has optimal exposure to risk factor and therefore portfolio betas will differ, since each investor has different appetite for risk
- The average investor holds the market portfolio
- **Capm looks at the risk premium of an asset in proportion to its contribution to the portfolio's variance. This looks at how much does it contribute.**
- **Factor risk premium has an economic story**

$$E(R_m) - R_f = \lambda\sigma_m^2$$

    - As market risk increases, the expected value on market goes up. We have higher expected returns from the equation

$$E(R) = \frac{E(P_1) - P_0}{P_0}$$

- If expected return increases, either E(p1) increases or P0 falls
- Therefore, in recession where current prices fall, this causes expected returns to increase
- In volatile times, prices in markets fall.
- Prices drop when lots of risks around. Expected returns increase
- Ignore rf since constant

- **Risk is factor exposure via beta**

$$\beta_i = \rho_{i,m}\frac{\sigma_i}{\sigma_m}$$

- Higher beta means lower diversification benefits since assuming standard deviations are constant, correlation goes up and this leads to lower diversification benefits.
- Low beta assets have diversification benefits and good for diversification.
    - If higher beta, we want higher returns from this since no diversification benefits

- Lower expected return implies that investors are willing to pay a higher price for these assets
- In bad times, low beta assets pays off well and high beta assets pays off poorly
  - You want these low beta assets as they pay off during bad times and it's something you want to hedge yourself against
  - We want to pay for this

- **Risk premium in CAPM is reward for investing in an asset that pays off in bad times**
  - Consider a high beta asset
    - High market return → high asset return
    - Low market return → low asset return
  - Investors are risk-averse and want a risk premium for taking on this risk in bad times
  - Low beta assets are a hedge that pays off well in bad times
  - Utility depends on market and therefore we want assets that do well in markets that do well

$$\frac{Market\ Risk\ Premium}{Market\ variance} = \frac{E(R_m)}{\sigma_m^2}$$

- This is known as the **market price of risk**
  - Quantifies extra return investors demand to bear portfolio risk
- For each stock we add, we look at it's covariance with the market and therefore see its contribution to the portfolio whilst for market portfolio itself, we look at its variance to measure risk
- For a stock, we look at

$$\frac{Stock's\ contribution\ to\ risk\ premium}{Stock's\ contribution\ to\ variance} = \frac{E(R_i)}{Cov(R_i, R_m)}$$

This is a stock's reward to risk ratio

Equate the 2 terms together since in equilibrium, all investments should offer same reward-risk ratio. If the stock's ratio is better, people would rearrange portfolio by buying more of the stock and selling market portfolio, causing prices and therefore returns to change until they are equal.

$$\frac{E(R_i)}{Cov(R_i, R_m)} = \frac{E(R_m)}{\sigma_m^2}$$

$$E(R_i) = \frac{Cov(R_i, R_m)}{\sigma_m^2} E(R_m)$$

Redefine it as Beta and take excess returns

$$E(R_i) = r_f + \beta_i \left(E\left(R_m - r_f\right)\right)$$

Beta measures the contribution of a stock to variance of market portfolio as a fraction of total variance of market portfolio

Security market line (This is plotting out the CAPM)

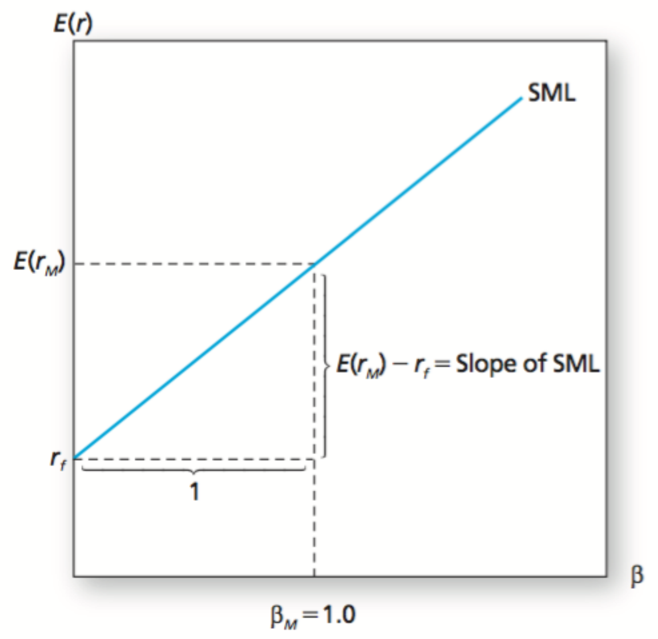


Figure 9.2 The security market line

- Equation for CAPM plots relationship between stock/portfolio's beta and its expected return
- **Beta is the reward-risk equation mentioned earlier**
    - **Beta measures a security's contribution to the risk of an optimal portfolio. Risk premium depends on assets contribution to risk of market portfolio**
- CAPM says all stocks lie on the security market line which is not the case
- Difference between SML and stock's actual return is known as a alpha
- If stock above SML, stock is under-priced since earning a higher return than expected. Therefore, should buy this.
- Only thing that matters is a stock's systematic risk
- CAPM is an equilibrium model and therefore says that alpha should equal 0

Note that CML graphs risk premiums for efficient portfolios whilst SML graphs individual asset risk premiums (can be for both efficient and individual assets)

- The SML line is benchmark required return for a security

Investors only require a risk premium for bearing systematic risk (not total)
Note that no portfolios can be better than the market portfolio

Implications of CAPM

- Market portfolio is efficient
- Risk premium on a risky asset is proportional to its beta

CAPM Extensions

- We can now relax CAPM assumptions
  - Cover taxes, differing borrowing etc
  - We still get same basic structure of
    - Linearity
    - Covariance of returns with risk factors. Only systematic risk is what matters
    - Multi-factor model emerges

Black-CAPM (0 Beta CAPM)

- Any combinations of 2 frontier portfolios are also on the frontier (2 fund theorem)
- Every portfolio on efficient frontier has a companion portfolio on inefficient frontier
  - We can combine these 2 portfolios to get a new portfolio on the frontier which has 0-beta. Useful for when there is no borrowing

- This is what happens if no risk free rate
- Investors choose different points on the efficient frontier
- If no risk free asset, investors now hold different portfolios rather than same optimal risky portfolio
- Here we assume investors want to hold mean-variance efficient portfolios
- Similar to solving CAPM except all the weights will be in risky assets

$$\max_{x} \; x'e + \lambda\left(c - \frac{1}{2}x'Vx\right) + \gamma(1 - x'\underline{1})$$

- **Solving the optimisation problem yields the following first order conditions for a given efficient portfolio x:**

$$e = \gamma\underline{1} + \lambda Vx$$

$$c = \frac{1}{2}x'Vx$$

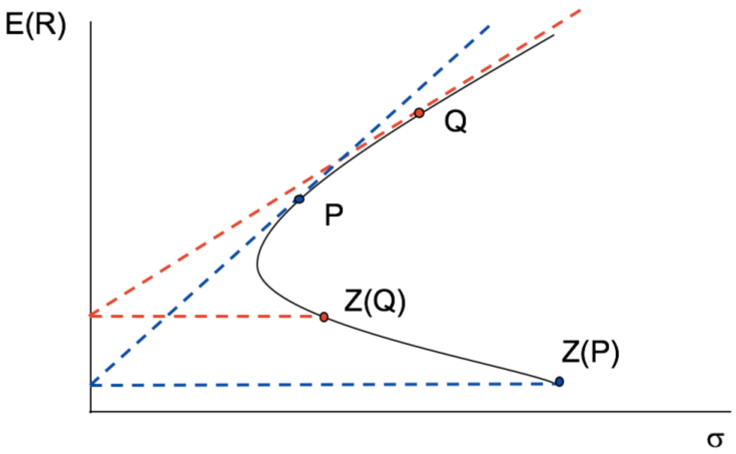
$$1 = x'\underline{1}$$

- Market portfolio has to be efficient. If we hold 2 different portfolios on the efficient frontier, then the combined portfolio is also on the efficient frontier.
- We want to find weight of a portfolio that is uncorrelated to the market portfolio and therefore has 0 beta

BLACK CAPM

$$E(R_i) = E(R_z) + \beta_i E(R_m - R_z)$$

- Unlike the risk free rate, this 0 beta portfolio has a standard deviation and that needs to be considered in calculations



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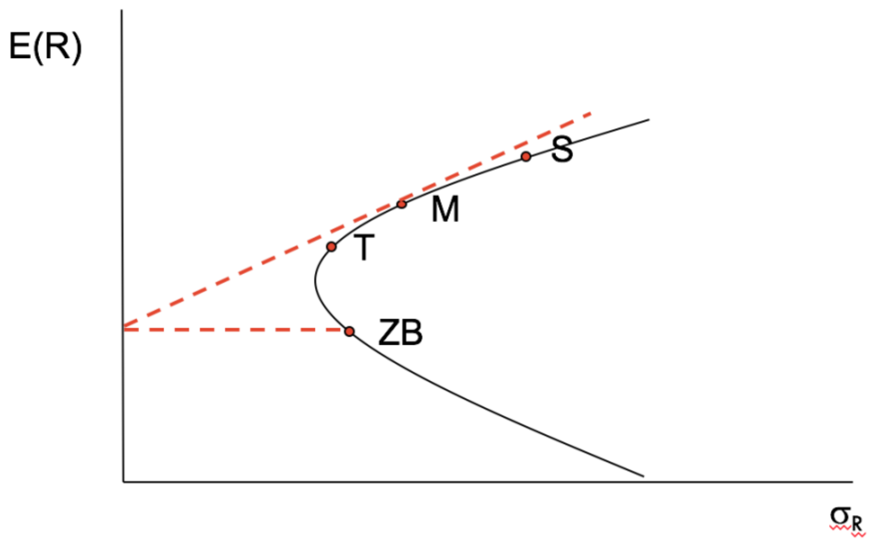
- To find portfolio Z(P), find tangent to P and extract it to 0 risk axis. Then extend horizontal line. This is the appropriate portfolio with 0 beta to original portfolio
- **The zero beta portfolio refers to the companion to the market portfolio**

$$E(R_i) = E(R_{ZB}) + \beta_i [E(R_M) - E(R_{ZB})]$$



- Expected return can be expressed as function of expected returns of any 2 frontier portfolios

<u>2 fund theorem</u>

$$E(R_i) = E(R_{zerobeta}) + \left[E\left(R_{portfolio}\right) - E(R_{zerobeta})\right]\beta_i$$

Simply, we look at the return of companion portfolio's return + portfolio's in questions excess return (with regards to the companion portfolio)

Key Predictions of CAPM

- Completely explains and determines stock's return
- Explains variation of returns to the exclusion of other variables
- Linear form which generates returns
- Beta coefficient is equal to market risk premium (Rm-Rf)
- Over long periods of time, market return is greater than risk free rate to compensate for greater risk associated with market portfolio

Testing CAPM

- ***Things we can test regarding CAPM***
- Return and beta are linearly related
- Slope of SML is positive
    - Higher beta, higher return
- Beta is only variable that explains expected return. No residuals
- Alphas are 0
    - CAPM should explain all of the returns
- Market risk premium should be positive

Problems with testing if CAPM is true

- CAPM is model of equilibrium expected return but we can only observe actual. Returns. Does expected returns equal actual returns?
- Beta estimation
    - Stability: historical stock betas do not predict future betas well
    - Thin trading problems
- Standard CAPM is one period model yet applications and tests use multiperiod data

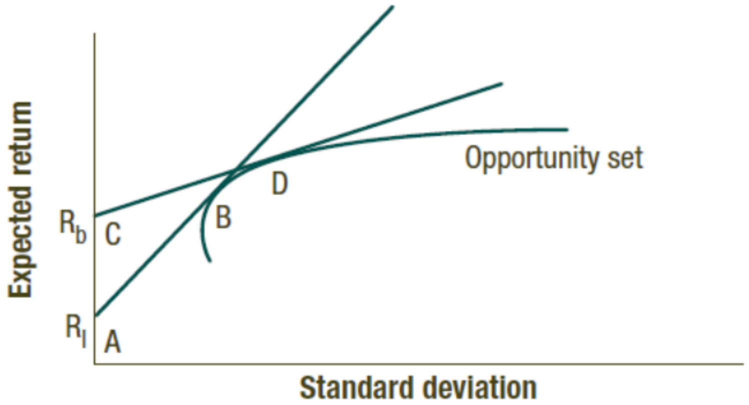
Problems with testing CAPM

- We can only use ex-ante model but use ex-post returns (assumption is made that past information reflects expectations)
- Difficulty estimating returns on a market portfolio (do we have a proxy for it)
- Survivorship bias since funds that close down drop out of databases
- Lots of assumptions are violated such as transaction costs

Roll's critique – CAPM is all about capital assets. However, we don't actually observe all assets and only see shares (not fixed income etc). If proxy for market portfolio is on efficient set, there should be perfect linear relation between returns and beta measured relative to that portfolio. In the end, we are just testing if that market portfolio is efficient or not

We cannot observe market portfolio for all capital assets (some examples include human capital)

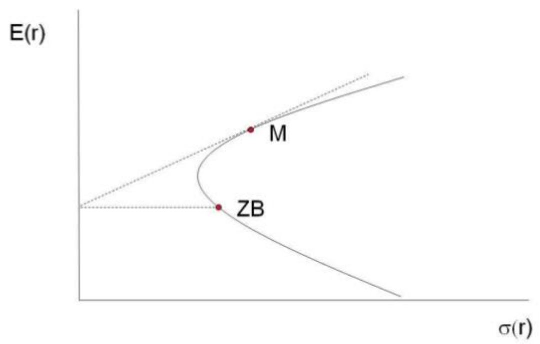
If different borrowing and lending rates exist, CML fails
We don't have one market portfolio anymore and instead we have an infinite amount of portfolios to choose from. Can't price assets anymore



However, zero beta CAPM still holds

However, the zero-beta CAPM will still apply in this case, so the CAPM can survive, albeit in a slightly different form.

$$E(r_i) = E(r_{ZB}) + \beta_i[E(r_M) - E(r_{ZB})]$$



CAPM fails due to unreasonable assumptions

- Investors have only financial wealth
- Irrational
- Not a single period investment
- Heterogeneous expectations
- There are taxes/transaction costs
- Some investors are price setter
- Information asymmetry and incomplete information

- Fama and French found that controlling for size, there's a negative relationship between beta and realised return

With CAPM, short sell asset that is returning expected returns lower than expected whilst buying ones with higher expected returns according to CAPM. Arbitrage profit is the difference between expected returns.

Calculating price of a stock

- We can estimate E(r) using the CAPM

$$E(r) = \frac{D_1 + P_1 - P_0}{P_0}$$

Week 6 – Asset Pricing Model

- **Arbitrage:** Exploiting security mispricing in a risk-free manner

- Using CAPM, we can extend and decompose risk further into market vs firm-specific risks

    - Allows us to deal with different natures of systematic risk
- **Arbitrage Pricing Theory = Factor Models + No arbitrage Condition**

- Covariance between an asset's return and market return is rewarded
- The CAPM has unrealistic assumptions so we add in more risk factors
- We need better models of expected returns as it allows to build better covariance matrices of stock returns
- Investment managers choose which risk factors they believe are important. This is how managers add value
- CAPM is a 1 factor model which we can adjust the portfolios to match systematic risks

- We can add in more market risk factors such as inflation so that assets that pays off well in high inflation is also compensated

- We've also changed it so that the factors measure information coming regarding that factor and how it changes things

- **Note that searching for mispriced securities that aren't risk free is known as risk arbitrage**

Arbitrage Pricing Theory
- This is used for:
    - Overcoming CAPM
        - Less restrictive than CAPM
        - We don't need market portfolio, just a well diversified portfolio on SML
        - APT makes no assumptions either regarding utility functions and allows for many risk factors
    - APT is empirically testable
- Arbitrage is making profits with no risks involved
    - Based on the law of one price. 2 goods with the same cash flow should sell for the same price as they converge. 2 identical assets should sell for same price
- Market portfolio not involved in APT

Assumptions
- **Large asset market. N needs to be large enough to remove any non-systematic risks**
- **Asset returns have a linear factor structure**

- Security returns can be described a factor model
- **Market allows no arbitrage opportunities**
  - This means that securities are not mispriced
  - This occurs since we can form portfolios that have 0 correlation with risk factors and therefore we can arbitrage

(We do not require quadratic utility or normally distributed returns)

Returns are generated via k risk factors

- Risk factors are common systematic sources of risk
- Return generated in similar method to SIM

$$R_i = \alpha_i + \beta_1 R_{p1} + \cdots + \beta_k R_{pk} + \epsilon_i$$

- Beta is the sensitivity of an asset to factor k. It is 0 with other factors.

- Take expectations, as these are the expectations we would expect

$$E(R_i) = \alpha_i + \beta_1 E(R_{p1}) + \cdots + \beta_k E(R_{pk})$$

- Subtract the 2 terms
-

$$R_i = E(R_i) + \beta_1 F_{p1} + \cdots + \beta_k F_{pk} + \epsilon_i$$

- F = R – E(R) is known as a zero-mean risk factor
- Return deviate from expectation if risk factor deviate from expectation
- **In other words, risk factors are known as surprises**

Actual return = expected return + surprise
Surprise = actual return – expected return

- Such examples of risk factor are systematic surprises

One factor APT model with no residual risk

- We have 2 sources of uncertainty, common factor and firm specific events
-

$$R_i = E(R_i) + \beta_i f_1 + \epsilon_i$$

- We can redefine **F as the deviation of the common factor from its expected value, whilst beta measures sensitivity to that factor**

$$R_i = E(R_i) + \beta_i f_1$$

- Consider a portfolio with 2 assets whose beta do not equal

$$R_p = x_i\left(E(R_i) + \beta_i f\right) + x_j\left(E(R_j) + \beta_j f\right)$$

$$R_p = x_i\left(E(R_i) + \beta_i f\right) + (1 - x_i)\left(E(R_j) + \beta_j f\right)$$

$$R_p = x_i\left(E(R_i) - E(R_j)\right) + E(R_j) + \left(x_i\left(\beta_i - \beta_j\right) + \beta_j\right)f$$

- First part has no risk since no betas are present
- The last part measures the systematic risk
- This allows us to pick a weight which removes systematic risks
- If we set the weight of the portfolio to be

$$x_i = \frac{-\beta_j}{\beta_i - \beta_j}$$

- By picking a weight x, we remove the systematic risk and gives us a risk-free return
- This allows for the last term to cancel out and then this then allows for portfolio return should equal risk free rate to prevent arbitrage.

$$R_p = x_i\left(E(R_i) - E(R_j)\right) + E(R_j) = R_f$$

- After subbing in the new term for xi, and doing rearrangements, we get:

$$\frac{E(R_i) - R_f}{\beta_i} = \frac{E(R_j) - R_f}{\beta_j}$$

- This says that excess return per unit of risk should be same across all stocks or else arbitrage will arise. This result due to arbitrage
- We can then set the ratio to be:

$$\lambda = \frac{E(R_i) - R_f}{\beta_i}$$

- Re-arrange to get

$$E(R_i) = R_f + \beta_i \lambda$$

- Lambda is factor risk premium and expected return on a portfolio with a factor loading equal to one (which means that beta =1)
- This means lambda is expected return on a portfolio with a beta of 1 to the factor. Gives us back the return the factor generates
- Sub it back in

$$\lambda = E(R_{\beta=1}) - R_f$$

$$E(R_i) = R_f + \beta_i(E(R_{\beta=1}) - R_f)$$

- Similar to utility. This model is driven from arbitrage. Ensures that asset is priced and reflected on the risk of the asset
- People trade to ensure this happens that assets are priced efficiently

ARBITRAGE PRICING THEORY

Ross (1976) shows that asset's expected return and systematic risk has a linear relationship

$$E(R_i) = \lambda_0 + \beta_{i1}\lambda_1 + \beta_{i2}\lambda_2 + \ldots + \beta_{ik}\lambda_k$$

- **First term is expected return on portfolio with beta of 0 wrt every factor. If risk free factor is present, then the first term becomes this**
- This equation holds for well diversified portfolios
- **Factor portfolio:** Beta of 1 with its factor and beta of 0 with others. This is equivalent to **tracking portfolio**
- Lambda = Expected return – risk free rate
    - **Factor risk premium :** This is the risk premium on a portfolio that has beta = 1 wrt to factor k and beta 0 with all other factors
- No residual risks since diversified away by holding portfolios with multiple assets
- We can measure beta of risk factors given they are uncorrelated with systematic risk factors

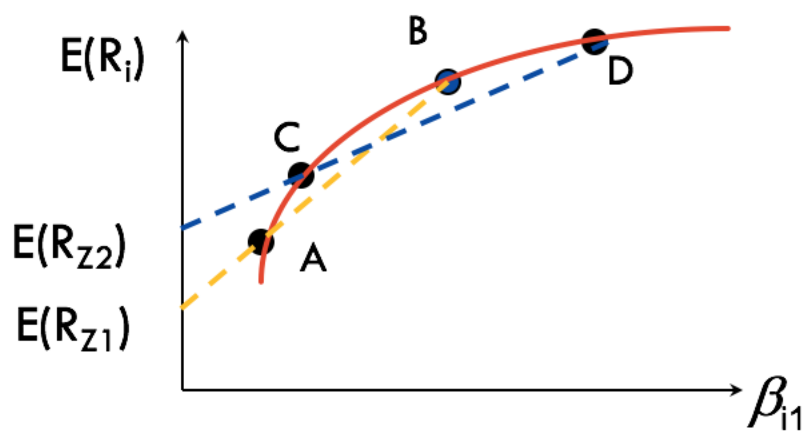
$$\beta_{ik} = \frac{cov(R_i, R_{pk})}{var(R_{pk})}$$

- **When calculating returns from APT, do not forget risk free rate**

APT and Linear Relationship

- Assume that
    - Unlimited number of securities
    - Unlimited short selling
    - One factor price assets
- Suppose relationship between expected return on an asset and its beta to a risk factor is non-linear. This allows for arbitrage opportunity

EXAM QUESTION
Security Market Line



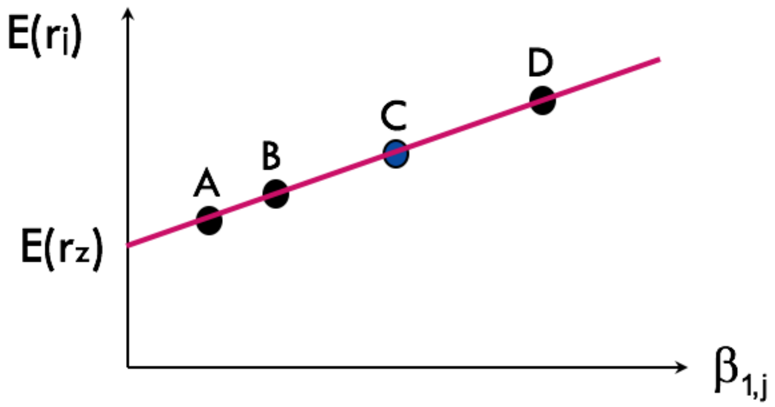
- Combine A and B to make zero beta portfolio by long A and short B. We get portfolio return of E(Rz1)
- Do the same for C and D for (Rz2)
- Now both beta = 0 ro risk free
- Here, return on Z2 > Z1
- Therefore makes sense to buy Z2 and short Z1
- Buying Z2 means that:
  - Short D and buy C
- Shorting Z1 mean that:
  - Short A and buy D

- Buy z2 and short z1 to get an arbitrage profit without any risk
  - Selling A causes the price to fall and Expected return to rise
  - Buying B causes price to rise so expected return fall
  - Selling D causes price to fall so return rise and buying C causes price to rise and return to fall

$$R = \frac{E(R1) - P_0}{P_0}$$

- This strategy causes the portfolio of Z1 to rise whilst price of Z2 falls and they converge as a result
  - Shorting Z1 pushes return on portfolio of A and B up (since price falls)
  - Buying Z2 pushes return on this portfolio of C and D to fall (since price rises)

- Arbitrage causes asset prices to converge to an equilibrium so that risk free portfolio yield same return



- Resultantly, we get this
- Portfolio residual variance will be close to 0 since we have infinite number of assets scattered along curved line which allows us to form many 0 beta portfolios
- Arbitrage ensure linear relationship between risk/return
- **We have just derived a SML from APT!**
- Since residuals between non-systematic residual are uncorrelated, variance of residuals do not contain a correlation coefficient term
- **Arbitrage ensures a linear relationship between risk and return in equilibrium**

$$\sigma^2_{\varepsilon_p} = \sum_{i=1}^{\infty} w_i^2 \sigma^2_{\varepsilon_i} \approx 0$$

APT and the linear relationship

- Consider the following data for a one-factor economy. All portfolios are well diversified.

Portfolio	E(R)	Beta
A	12%	1.2
F	6%	0.0

- Suppose that another portfolio, portfolio E, is well diversified with a beta of 0.6 and expected return of 8%. Would an arbitrage opportunity exist? If so, what would be the arbitrage strategy?

Shortcomings of APT

- Limited applications when number of securities in market is small
- Not practical to hold 1000's of portfolios

- Does not identify what those factors are
- Pricing errors can be small but unable to be exploited due to transaction costs etc
- If APT applies to portfolio of securities, then each portfolio should roughly lie on the SML too. However, not quite exactly true
- Not exact pricing model since based on large number of assets which could be mispriced. Difficulty is variance of pricing errors goes to 0 given arbitrage

APT vs CAPM

$$E(R_P) = R_f + \beta_P \left[E(R_m) - R_f \right]$$

- We were able to derive this earlier from different set of assumptions and still lead us to CAPM if we set the risk factor wrt to the market factor
- Market here just simply needs to be a well-diversified portfolio which lies on SML

CAPM	APT
We know market portfolio and all asset returns are linearly related to covariance of asset with market portfolio	We have factor which drives price movements. Assets are linearly related to covariance of asset with factor
	No assumption regarding utility or distribution.
	Allows for many risk factor
Requires the market portfolio	Holds for subset of risky assets and does not require universe. We can use an index
Better prediction	Better fit with data
	Allows for linear returns wrt to risk factors
Identifies the risk factor	Allows violation of risk-return relation for securities

Consumption CAPM (Rubinstein 1978)

- Intertemporal model whereby investor maximize expected lifetime utility based on consumption
- Investors maximize utility from consumption where utility value of consumption today = utility value of consumption tomorrow
- Financial assets allow for smooth lifetime consumption
    - Sell assets when consumption low since need funds for consumption
    - Buy assets when consumption high

- **Assets with negative covariance with consumption → low risk premium**
    - Allows you to hedge in bad times
    - We want assets that pays off well in bad times, so we are willing to pay high price for them. Therefore, they do not require a risk premium whilst assets that pay off badly in bad times, we need extra compensation to hold them

$$E(R_i) = E(R_Z) + \beta_i\gamma_c$$

- $E(R_Z)$ = the return on a portfolio with zero consumption beta
- $\beta_i = cov_{ic}/\sigma^2_c$ and is a measure of the relationship between portfolio i and the growth rate in aggregate consumption (c)
- γ_c = the premium for consumption risk

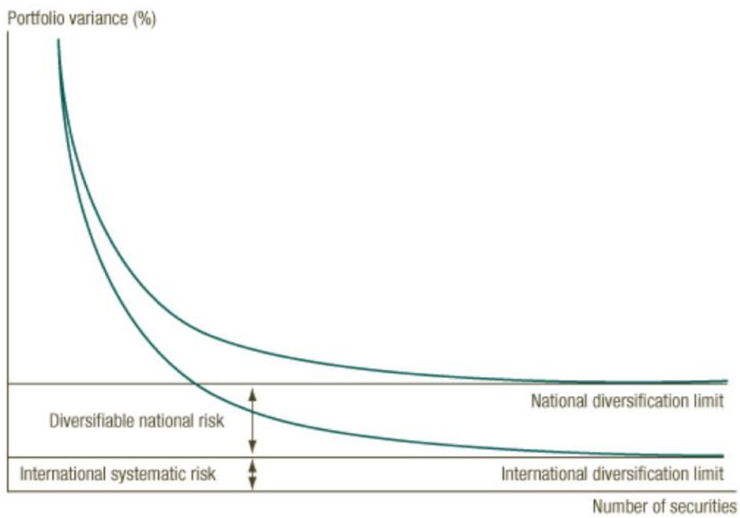
- Issues with CCAPM
  - Consumption data is infrequent
  - There are measurements errors to measure consumption

Empirical results

- Performs as well as CAPM
- Actually explain better than CAPM since tax year ends in December so consumption and investment decisions are more likely to be made simultaneously
- We can use garbage or CO_2 as a proxy variables

International CAPM

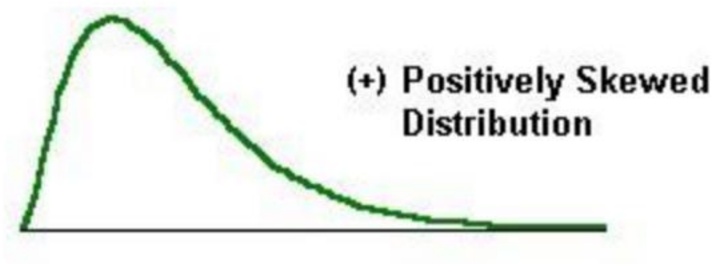
- This prices stocks as if no national/political boundaries
- Assumes complete integrated global financial market
- No investment restriction/barrier to capital flow
- National influences on stock is diversifiable
  - Therefore, only global risk factors affect asset prices

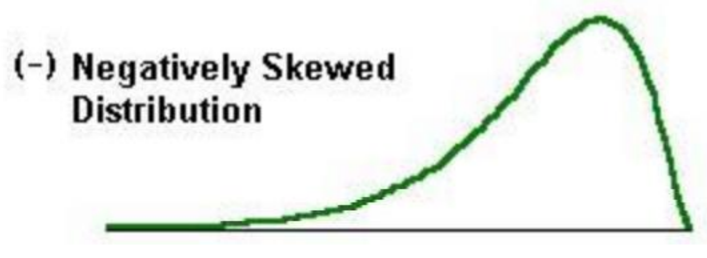


$$E(R_i) = R_f + \beta_{i,World} E(R_{m,World} - R_{f,World})$$

- $E(R_i)$ = expected return of security i (in local currency)
- R_f = the national risk-free rate (i.e. in the same country as security i)
- $\beta_{i,World}$ = the world market beta of security i
- $E(R_{m,World})$ = the expected return on the world market portfolio, defined as the expected return on a value-weighted index of all securities from all countries
- $R_{f,World}$ = the world risk-free rates, defined as the value-weighted average of risk-free rates (using the same weights as in world market portfolio)

- Significant benefits from international diversification
- However, this assumes capital market integration
- As markets are more integrated, correlation between markets increase and ICAPM is better
- Home bias has effect of increasing local influences on asset prices





Even if returns aren't skewed, more likely to be negatively skewed whereby we can adjust the CAPM for this
However, we can still adjust for skewness therefore
- Only systematic part of skewness matters (coskewness)
- Covariance is contribution of security to variance of portfolio
- Coskewness contributes to skewness of well diversified portfolio

Multi-factor models

- Factors can be classified as:
  - Macroeconomic – These are generally not tradeable
    - Oil price
    - Economic growth
    - Unexpected inflation etc
  - Fundamental/characteristic/dynamic – Assets exhibiting certain characteristic
  - Statistical – principal components

Macroeconomic factors

How to compute them:

- Estimate eta from time-series regression of asset return on factors
- Estimate factor premium from cross-sectional regression of asset returns on betas in given month

Fundamental-based factors

- Identify firm characteristics as proxies for risk
- E.g. Fama-french 3 factor model

Fama and French (1993) 3-factor model:

$$E(R_i) = R_f + \beta_i E(R_m - R_f) + s_i E(R_{SMB}) + h_i E(R_{HML})$$

- R_f is the risk-free rate
- R_m-R_f is the market risk-premium
- SMB is the size premium
- HML is the book-to-market premium

- Fama French suggests that SMB and HML can be proxies for more fundamental risk factor variables
  - HML can be a proxy for firms in financial distress if high book to market value
  - SMB can be proxy for small stocks, which are more sensitive to changes in macroeconomy
    - In recessions, smaller stocks are worse off, so investors require higher compensation for this. Pushes price down but causes returns to rise

- Numerous factors to utilise

Summary

- Expected return of an asset can be modelled as linear function of various macro-economic factors
- APT holds because
  - Portfolio formed with no risk and requires no investment will have 0 expected return

  - Arbitraged portfolios can be formed which does not correlate with underlying factors
  - No systematic risk associated with arbitraged portfolios and no systematic returns can be earned
  - Large number of securities used in these arbitraged portfolios to remove diversifiable risk
- However, APT is not an exact pricing model since it's possible for one asset to be mispriced whilst total arbitraged portfolio has expected return of 0
  - Huge assumption we make that variance of pricing errors goes to 0 with more assets added to the model
- **Fama French**
- SMB is returns of small stock portfolio in excess of large stock portfolio
  - Fama French found this to be positive
  - Small firms correlated with SMB
- HMB is returns of growth stocks in excess of value stocks portfolio\
- Flaw in the model is that if small firms merge, despite their operations not changing, they are expected to earn less for no apparent reason
  - However, when small firms become large, the characteristics of stock returns change
  - Stocks of large merged firms behave differently to small ones
  - Large firm has smaller risk premium
    - However, these small companies can have more growth opportunities or due to more volatile environment, can lead to price appreciation
- Different valuation of return on equity can lead to projects being over/understated

Week 7 – Market Efficiency (Efficiency)

Efficient market hypothesis: Prices reflect all available information.
This means that it is impossible to generate alpha

Anomalies: Things that should not be the case

- Number of well known trading strategies that can generate positive returns
- However, are these anomalies performing well on a risk-adjusted basis rather than just simply returns

A share value is the present value of expected future cash flows

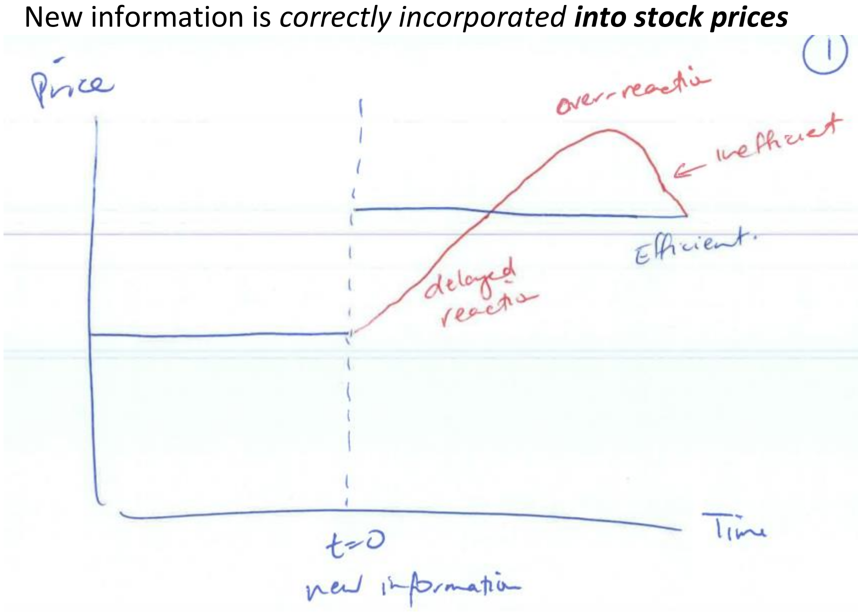
$$Value\ of\ Share = \sum^{T} \frac{E(C_T)}{\prod^{t}[1 + E(r_s)]}$$

- The top term is the expected cash flow in the future whilst the bottom term measures the time period specific rate. This is not a constant rate
- PV model requires information on
    - Cash flow
    - Risk via cost of capital
- Information changes expectations and therefore prices

Efficiency

New information is unpredictable and prices that respond to new information should also move unpredictably

- **Informational efficiency (how quick market adjusts)**
    - Reflect speed at which information is incorporated into prices
- **Market rationality (Is the market biased)**
    - New information is *correctly incorporated* ***into stock prices***



- Efficient markets sees new information incorporated in instantaneous and unbiased manner. Can't earn excess return on information
- New information is unpredictable, if they were, then they would be incorporated into today's prices
- An example is on a time-Price graph and see if there is a step function at information announcement plus whether is it a straight line (if it goes up and down etc, shows that it is biased)
- MB of finding info = MC of finding info and therefore, no more surpluses to be made from this. Inefficiencies that are not economically exploitable is still consistent with efficient market
- **We can't use information set to earn excess returns**
- Consequences of inefficient markets

    - Firms with overvalued companies raise capital too cheaply and vice versa (so undervalued firms lose out)
    - Inefficient allocation of resources

- Inefficiency implies predictability in the market
- Price are not random. It is information that is random which then makes prices random
- Large movements in prices is consistent with efficiency if information arriving is high in frequency
- **Expected returns does not need to equal actual returns (due to random info arriving)**
- Some investors will perform well and some won't

Classes of efficient market

- **Weak form**
    - Do market prices reflect past price and volume
- **Semi-strong**
    - Does it incorporate publicly available information
- **Strong form**
    - Is insider trading happening? Is private information used for excess returns

Equity Market anomalies

- **Anomalies:** Empirical results that seem to be inconsistent with maintained theories of asset pricing behaviour **(Schwert's 2003).** It is something deviation from what is normal. This is something that pricing models can't explain

To test anomalies, sort them and subtract difference between the returns of the most extreme groups of the portfolios to see if there's a premium

- **Joint test problem**
    - Any test of market efficiency is compared to a benchmark model. However, alpha will be different depending on which model you use

    - One benchmark to use is the CAPM.
    - To create excess returns, a model for expected return is required
    - **Any test of market efficiency is therefore a joint test of market efficiency and model of expected return**

Anomalies

- Small cap stocks earn more
    - Outperform even on risk-adjusted basis
    - Relationship between firm size and abnormal returns is not linear
    - Keim (1983) finds half of annual size premium occurs in January. This is since at the end of the year, people offload small stocks to gain tax benefits and then buy them back again at beginning of year.
- Price momentum
    - Stocks with returns in past, continue to do so
    - Jegadeesh and Titman (1993) found that buying winners and selling losers work (from last 3-12 months)
- Longer term price reversals
    - deBondt and thaler (1985) found that stocks with lowest returns over 5 years outperforms winners
- These are consistent since one is long term and one is short term

- **Growth vs value shares**

    - Growth has low B/M whilst value has high ones
    - In long run, value outperforms growth
    - Value firms are riskier than growth firms and therefore morel likely to default

- **Small value stocks appears to be the best in Australian markets**

- **Idiosyncratic volatility anomaly (**ang, hodrick, xing, and zhang)
    - Idiosyncratic volatility is volatility of residual from Fama-French 3 factor model estimated osn daily data for each month
    - Stock with high idiosyncratic volatility have lower returns on average. **This is weird since it should not be priced**
    - IVOL also exists internationally

3-factor model

$$R_i - R_f = \alpha_i + \beta_i(R_m - R_f) + s_i SMB + h_i HML + \varepsilon_i$$

$$\boxed{\sigma^2_{\varepsilon_i}}$$

- There appears to be size, value, and momentum effects in North American, European and Asia Pacific equities and elsewhere

- **Abnormal returns are difference between actual returns and expected returns**
    - **Abnormal returns are risk-adjusted returns**

- We can test for semi-strong EMH via looking at news announcement of:
    - Takeovers
    - Accounting information

CARs of target firms before takeover attempts

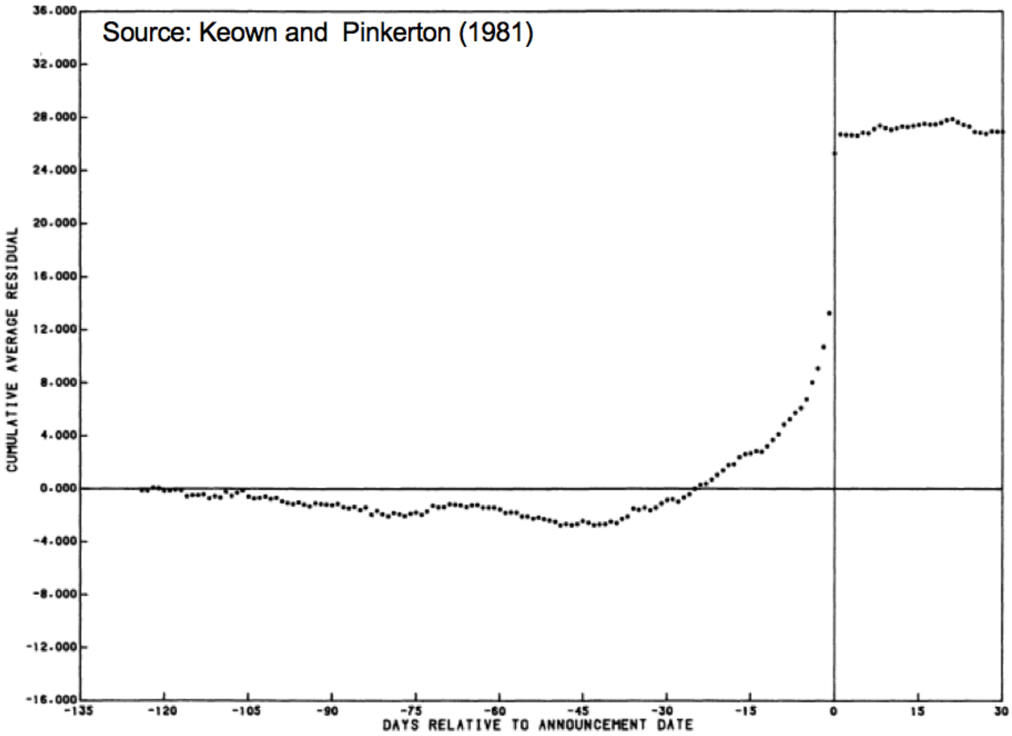


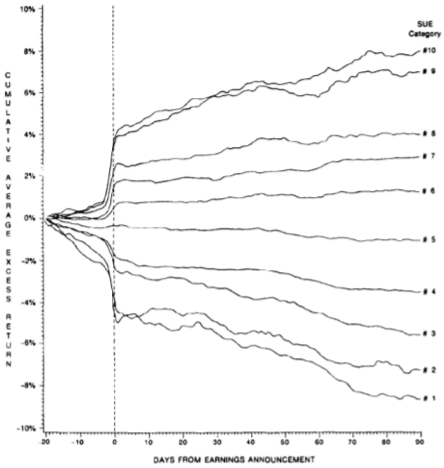
Figure 1. Cumulative Average Residuals–Market Model–Entire Sample

- Here you can see rumours being leaked. Fast adjustment to the news and a slight undervaluation.
    - Instantaneous and unbiased

Post-earnings announcement drift

- SUE is the earnings surprise
- Category 10 is the highest positive surprise
- Category 1 is the most negative surprise

- Source: Rendleman, Jones and Latane (1982)



- Here, we expected all these lines to be straight but it is not the case. Therefore, there is some bias here. Markets aren't responding to the news accurately enough.

- Chordia and Tong has found that returns from anomaly trading strategies reduce over time
- Improvements in liquidity has reduced trading frictions and improve market efficiency. Things are easier to trade and quicker to arbitrage

- Around 97 factors

- Found that academic publishing also reduces the returns on these anomalies after they are reported

  - However, this is more likely due to mispricing rather risk explains anomaly returns
  - Therefore, it is behavioural factors that are driving prices rather than risk.

Beta backtest

  - We see lower beta securities outperforming other ones
  - Volatility increases as you increase beta
  - Drawdown (% between peak and trough)
    - Lower beta takes longer to get back
  - Quintile spread: What happens if you long small beta but short high beta
    - Huge performance

Value backtest

- Value also does well for a while but then reverses whereby growth securities outperform value stocks
- Every factor has negative alpha
- Volatile is quite similar to each other
- **Average turnover:** What % of portfolio to buy and sell each month. Can see how costly it is

Size backtest

- Smallest stocks outperform poorly
- Doesn't hold in Australia landscape
- Lots of stability in large securities with low turnover
- Volatility across this factor to be quite similar

Momentum backtest

- Momentum does quite well
- Momentum beats market
- Winners keep winning and losers keep losing

Looking for recommendations, look at historical performance and hope it performs well into the future
Portfolio with multiple factors are much more robust against portfolio with 1 factor

When constructing portfolio via market weights for a factor portfolio, if you use market cap, then your weightings can be heavily skewed if lots of large market cap stocks are present
Also need to be careful about the industries we use to weight and make sure not heavily exposed to any particular industry bias

You can't have expected returns to be predictable, **but it is ok to have predictable volatility**
Abnormal return is the expected – actual return

Week 8 – Market efficiency 2 (Behavioural)

Behavioural finance
- Behavioural biases of market participants influence asset prices

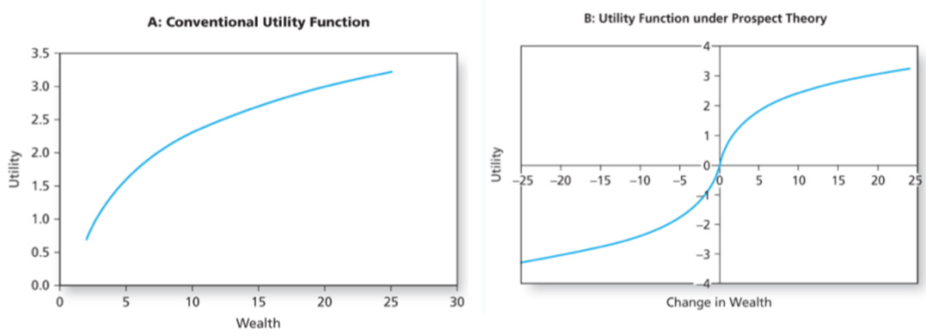
- In conventional finance we have

    - Risk averse utility maximisers
    - Rational investors
    - Incorporate all info into decision making
    - Resources are allocated efficiently
    - Price are at their intrinsic value

THIS IS CLEARLY FALSE
- Investors are humans with their own biases
- Systematic psychological biases will not disappear in aggregate
    - Perhaps anomalies arise in markets due to investors not being rational

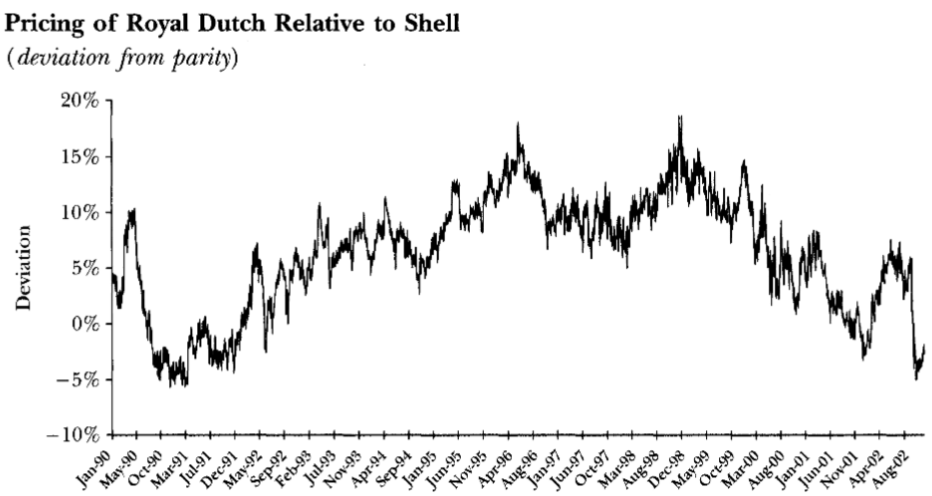
2 categories of irrationalities
- **Investors do not process information correctly**
    - This leads to incorrect probability distributions of future returns
    - Errors can lead to mismatches in estimating probabilities
    - **Forecasting error (memory/availability bias)**
        - Too much weight on recent experiences
    - **Overconfidence**
        - Investors overestimate confidence and precision of forecasts
            - E.g. everyone thinks they are above average at driving
    - **Conservatism**
        - Investors slow to update beliefs and under react to new information
        - Belief perseverance
    - **Sample size neglect and representativeness**
        - Investor quick to infer pattern from small sample
        - Stereotypes
        - Rational investors consider population probabilities

- **Investors make suboptimal decisions**
    - This is due to behavioural biases
    - **Framing things can lead to different decisions** (prospect theory). People prefer gains to losses



- **Conventional view:** Utility depends on **level** of wealth
- **Behavioural view:** Utility depends on **change** in wealth

- **Limits to arbitrage**
  - Behavioural biases don't matter if rational arbitrageurs could exploit these mistakes
  - **Fundamental risks**
    - Markets remain irrational longer than you can remain solvent. Takes too long to converge to intrinsic price
  - **Implementation costs**
    - Transactions costs and restrictions on short selling can limit arbitrage activities. You can't trade on anomalies as a result
  - **Model risk**
    - What if your model is wrong and price is actually correct



- **LOOP** is not holding here

- Not all anomalies are traded equally

- Anomalies may not disappear since too expensive to trade away

Engelberg, Mclead, and Pontiff (2015)
- Anomalies earn more on earnings/news announcement

    - This is likely due to error from analysts
    - Anomaly returns are results of biased expectations which are corrected upon news arrival

Jacobs and Muller (2016)

- Also found across the world
- International markets also face this issue

Explanations for momentum effect:

- Winner stocks have high transaction cost
- Higher liquidity risk
- Earnings momentum
- Can't arbitrage since constraints on shorting
- Under-reaction to positive news about the firm
- Investors extrapolate expectations and expect good times to continue
- Conservative investors on updating expectations
- Overconfidence and biased self-attribution
- Disposition effect (sell winners and hold onto losers)
- Information moves slowly in market
- Momentum profits are negatively skewed

<u>Betting against Beta anomaly</u>

- Frazzini, Andrea, and Pederson came up with this theory

- Model with leverage and margin constraints varying across investor and time

- SML is too flat in empirical data
    - Most likely due to the fact some people can't borrow at risk-free rate

- If you can't access borrowing, how to get a portfolio you want?
    - You can simply purchase assets with a high beta. Need to buy a portfolio with beta of 1.5 so you can get your beta up.
    - This pushes price of high beta asset up and causes returns to fall
    - This causes low beta stock's prices to fall but returns to rise
    - People do this if they believe market is going to go up
- Therefore, rather than moving up CML to maximize utility, they hold high beta stocks to maximise utility (similar to zero beta CAPM)
- Holding high beta assets is a form of leverage

- Investors can also have investor long low beta asset and short high beta asset using leverage

- **Alpha for high beta assets is low**

- The factor is when you:
    - Long low beta assets and short high beta assets. Statistical arbitrage tactic as high beta assets are overpriced and vice versa. These will converge.
    - We want 0 investment and 0 risk, so we then need to leverage/deleverage it so each part has a beta of 1. Combine risky assets with risk-free asset to get a beta for one for long/short leg
- Dollar position in long leg must equal short leg

- **Empirical results show difference in positive returns between high and low beta**

- When funding constraints tighten, return of BAB factor is lower

- More constrained you are, the riskier assets you hold

BAB support

- Anomaly explained that typical institutional investor needs to beat benchmark which discourages activity
    - **Investment managers are not incentivized to find and exploit such mispricing**
- **Institutional investor prefer high beta stock to minimise tracking error**. They are also subjected to short-selling and leverage constraints.
- **Funds need to hold cash so people can redeem their money**
- **In Japan, foreigners over-weight high beta stocks.**
    - BAB anomaly increases when investments increases and strengthens when it decreases.
        - They buy more high-beta stocks when increasing investment and sell more high-beta stocks when divesting
- **Speculative overpricing**. There is a disagreement in the market. Friction costs too since you can't short sell as much as you can long. People with bearish views won't be reflected in the market, causing prices to rise and returns to fall
- **Mispricing's by expectations and biased beliefs**

BAB reject

- **Anomaly isn't real. Investors like to buy stocks with highest returns that month.** Nothing to do with beta. It's to do with volatility. 'Lottery effect' – stocks with highest return will underperform in next month. BAB driven by this. Lots of these lottery stocks are high beta stock
- **IVOL is the actual factor.** Not BAB. IVOL correlated with BAB. Actually picking up signal from BAB, not IVOL
- **Measuring alpha incorrectly** – other ways to measure things with benchmark. Should use conditional CAPM, not normal CAPM. Using a wrong model to measure the performance of BAB strategy

- **Driven by skewness**. Just momentum. Skewness is a form of risk. Compensated for returns' skewness
- **Other factors should also be included to control for factors**
- **High beta stocks are mainly small growth firms**. This explains why bad results for high beta stocks
- **Other firm characteristics such as momentum etc explain BAB instead.** BAB is driven by systematic mispricing that arises because of investor's preferences in firm's characteristics.

NASDAP has high weight of tech stocks.
Huge market sentiment for tech stocks but then speculation that Y2k will happen caused tech stocks to fall.

Explanations for dotcom
- Just overflated
- Bloated valuations with things like relative valuation further making the problem worse
- If stocks overpriced, relative valuation is bad

Statistical arbitrage: Based on mathematical relationship. An example is currency exchange whereby different currencies have different liquidity, so information carried over slowly.
Fundamental arbitrage: Intuitive relationship based on relative under/over valuation

When looking at company, identify what will make market realise it's wrong
Behavioural biases affects market efficiency due to people's perceptions obscuring valuations

Fund managers undertake research since they believe markets to be inefficient.
Results of this can lead to profitable opportunities and then are rewarded on basis of their performance
Even if managers believe in efficiency, they need to be seen as they believe in inefficiency so that they can get paid
Also, undertake research to ensure to themselves that market is efficient and not missing out
However, if everyone stops doing research since they believe markets are efficient, then markets won't be efficient anymore.
- People who believe in market inefficiencies is what make the markets efficient

Even if stock prices follow a random walk, there is compensation for bearing market risk and time value of money over the long run (investors are compensated for this risk since the market generally goes up in long run)
Even if future prospect's of a company are predictable, this has already been incorporated into stock price and we can still see stock price change due to idiosyncratic risks

However, we can still hire a manager to ensure we have
- Well diversified portfolios
- Asset risk tolerance of investors
- Provide assistance with regards to taxation

If stocks outperform expected returns, then there are other factors which has led to rise in stock price.

Week 9 – Investment style and Smart Beta

- Investment styles mainly focuses on equities
- Smart beta investing is an alternative to passive portfolio construction

- Optimal risky portfolio is market portfolio which is a market cap index

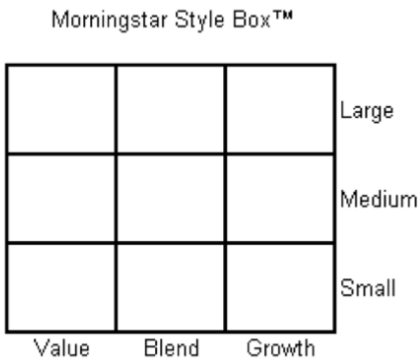
- **Index tracking**
    - **Portfolio should track its designated index**
    - Should exhibit same risk characteristic as designated benchmark
    - Hold each security in same weight as benchmark
    - Factors that affect tracking includes
        - Liquidity
        - Index additions and deletion. Should have the same beta as the benchmark you track

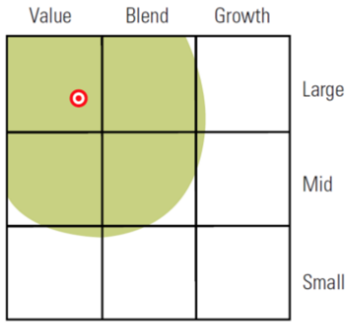
- **Passive investment**
    - Aims to match return performance of a specified benchmark index
    - Should track the return of benchmark before fund fees and expenses
    - Should match risk characteristics of index whilst minimising transaction costs and manage index changes. Fund offers tax-efficient solution for investors seeking to gain broad exposure to share market at low cost

- **Active investment**
    - Aims to beat benchmark
    - Achieve this via
        - Stock selection
            - Identify mispriced securities
        - Market timing
            - Shift between asset classes on expectations on which one is going to perform well

- **Investment style**
    - These are based on common characteristics (e.g. ethical investment, industry, asset class etc)
    - Styles have emerged
        - Growth vs value
        - Large vs small cap
        - Momentum
        - Liquidity

- **Style box from morningstar**
    - Small vs large
    - Growth vs value





Typical Large-Cap Value Fund

- Half of investments are in large cap funds
- Bad idea to hold lots of small cap stocks due to liquidity issues leading to high transaction costs
- Fund managers are not required to disclose what is in their portfolios

Types of benchmarks

- PIC Wholesale Aus Growth Style – large growth against S&P
- Goldman Sachs Emerging leaders fund – Small growth against ASX small ords
- Perennial value smaller companies – Small value benchmark against ASX small ords
- Schroder equity opp fund – unconstrained benchmark against ASX200

- Are style labels reflective of managers?
    - Growth fund goes after high momentum stocks
    - Value funds buy things cheap
    - Growth stocks hold winners
    - Value stocks hold losers

POTENTIAL EXAM Q

- **Impact of investment style in financial markets (Cooper, Gulen and Rau 2005)**
    - Name change towards hot style or away cold style based on returns of styles
        - If value beating growth, include value into name
    - Mutual funds attract an abnormal amount of fund inflow (20% increase or a $60million for given fund)
    - No change in performance of the fund itself
    - This hsows investors not that rational and influenced by cosmetic effects

- Found that characteristics capture style related trends in equity returns
- Additionally, there is evidence for returns reversal for different styles
    - Short good performing style and long bad performing style to profit from this
- Individual investors systematically shift preferences across extreme style portfolios e.g. small vs large

    - Past style return, earning differential, and advice impacts decisions of individual investors to shift between styles
- Some evidence that styles are also based on biological basis, behavioural biases, and what macroeconomic experiences they faced growing up e.g. great depression

Smart beta

- **Benefits of market cap weighting**
    - Passive strategy requiring little trading
    - Concentrated in high liquidity stocks and reduce trading costs

- Instead of using market cap weighted, use other things instead. Take factors into account

- **Smart beta**

    - Investment products that track indices that attempts to increase returns or decrease risk compared to market cap
    - Mix of passive+active
    - Rules based transparent, and low cost manner

- 2 different kinds of smart beta
    - Factor based
        - Tilt to certain factors
    - Diversification based
        - Equal-weight portfolio
        - Min variance portfolio
        - Max diversification portfolio (via maximizing sharpe ratio)

- There are other types too: return-oriented, risk-oriented, weighting-oriented

- **Results oriented strategies**

    - Dividend screened/weighted
        - Aim to overweight dividend paying firms
        - Weight on market cap/dividend yield size
    - Value
        - Overweigh value stocks based on characteristics (low P/E etc)
    - Growth
        - Overweight firms with above average long-term projected earnings growth, historical earnings growth, sales, cash flow growth

- **Risk-oriented strategies**
    - Low min volatility/variance
    - Low/high beta
    - Risk-weighted
        - Weight them according to risk contribution

- We have seen large demand for ETF's too
- Large demand for equities since high returns

Selected smart beta ETFs listed on the ASX

Name	Strategic Beta Attributes	AUM ($m)	Inception Date
Vanguard Australian Shares High Yld	Dividend Screened/Weighted	620.9	23/05/2011
Russell High Div Australian Shares	Dividend Screened/Weighted	280.5	14/05/2010
iShares S&P/ASX Dividend Opportunities	Dividend Screened/Weighted	238.1	6/12/2010
SPDR® MSCI Australia Sel Hi Div Yld	Dividend Screened/Weighted	142.0	24/09/2010
Market Vectors Australian Equal Wt	Equal Weighted	103.7	4/03/2014
Market Vectors MSCI Wld ex Aus Qlty	Quality	95.4	29/10/2014
BetaShares FTSE RAFI Australia 200	Fundamentals Weighted	87.0	10/07/2013
SPDR® S&P Global Dividend	Dividend Screened/Weighted	75.1	1/11/2013
Market Vectors Small Cap Div Payers	Dividend Screened/Weighted	37.8	26/05/2015
UBS IQ Morningstar Australia Div Yld	Dividend Screened / Weighted Multi-Factor; Quality	23.3	14/01/2014
Russell Australian Value	Value	22.6	18/03/2011
Russell Australian Rspnb Investment	Dividend Screened/Weighted	22.2	1/04/2015

Fundamental indexation

- Are indices measured by other measures other than size?
- Use other criteria to create index based on fundamentals
- Reason for this is that it overweights companies that are highly valued
    - However, market cap can represent the fair value for a company
- Weighting by variables other than size requires constant rebalancing
    - Therefore, we need annual reweighting
- Data mining could be an issue here
- Is there a value bias in fundamental indices?
    - Seems to load on value factor
- Passive portfolio based on well known anomalies can outperform a fundamental index

Therefore, if we believe markets are efficient, we would not resort to active investing

Note that these anomalies outperform, **on average,** not every point in time

Dividend paying stocks have done well in past years but risk that they are overvalued and may stop doing well

When investing, you want to
1) Diversify the factors you're expose to
2) Diversify skills of the managers
3) Manager risk is also mitigated since it lower manager-specific risk

Anomalies
- Value vs growth: Value stocks perform better
- Smaller firms outperform bigger firms
- Momentum whereby winners keep on being winners and vice versa
- Liquidity whereby low liquidity have higher returns
- Quality whereby high quality firms offer high returns since people tend to undervalue these firms with solid fundamentals (evidence against efficient markets)
- Yield/dividend whereby higher dividends offer better returns
- Lower beta/volatility stocks earn better returns

NOTE THAT THE ANOMALIES EARN HIGHER ABNORMAL RETURNS

These factors persist due to
1) Behavioural biases
2) Risk premium
3) Structural impediments such as long only

Week 10 – Performance evaluation

- Top funds differ every year
  - Depends on both the manager and also exogenous luck
- Numerous metrics to evaluate performance of managers
- 2 sources of manager's skill
  - Stock selection
  - Market timing
    - Adjust beta of portfolio depending on expectations of the market
- We need a fair comparison when comparing markets

- What factors drive performance?

- Don't ignore past performance when making decision

  - A lot of past performance stats based on only 1 year of data
  - Past performance highly positively correlated with future fund flows

Performance evaluation menu

Tracking error

$$TE = Standard\ Deviaiton\ (R_p - R_b)$$

- This ideally should be 0
- How much does the return deviate from benchmark.
- Should be 0.
- If active investing, you can put limits on your tracking error
- Allows you to see risk of the firm

Adjusting returns for risk

  - You can compare returns to comparison universe. Look at other things with similar risk characteristics
  - Compare comparable assets

Sharpe Ratio

$$\frac{\bar{R}_p - \bar{R}_f}{\sigma_p}$$

- This is the slope of the capital allocation line
- Benchmark value is Sharpe ratio for market
- **Does not rely on asset pricing model**
- **Captures both risk and return**
- What is the excess return per unit of risk
- **Risk measure in Sharpe ratio is total risk and it's appropriated if portfolio represents investor's complete portfolio of assets**

Treynor's Ratio

$$\frac{\bar{R}_p - \bar{R}_f}{\boldsymbol{\beta_p}}$$

- Excess return per unit of beta
- Slope of capital market line
- Every asset should lie on the same line
- Measures portfolio's excess return per unit of systematic risk
- Superior performance if Treynor index exceeds market risk premium (then top part is positive)
- Problems include:
  - Finding correct value of MRP. It relies on CAPM. Don't use this if we aren't using CAPM. However, if we are simply comparing, it's fine
  - Need to estimate beta. What type of beta? Monthly ?
  - Appropriateness of CAPM
- Use this when portfolio is a subportfolio

Jensen's Alpha

$$R_{p,t} - R_{f,t} = \alpha_p + \beta_p(R_{m,t} - R_{f,t})$$

- R_p = Return on the portfolio
- β_p = Beta of portfolio
- R_f = Risk free rate
- R_m = Return on market index portfolio

- Alpha needs to be positive
- Drawback is that it relies on SML and CAPM being correct model
- If performing to expectations, alpha = 0
- Good to use when portfolio is a sub-portfolio, but treynor ratio would be better for this

Information Ratio

$$IR = \frac{\alpha_p}{\sigma_{\varepsilon p}}$$

i.e. how much return was generate from diversifiable risk?

The information ratio is sometimes referred to as:

$$IR = \frac{\sum_{t=1}^{T}(R_{p,t} - R_{b,t})}{TE}$$

- First one is the appraisal ratio
- Risk from return

- Second one is whereby values close to 1 indicate good performance

- DON'T USE IR FOR PASSIVE FUNDS SINCE = 0 (TE should equal 0)

- IR should be positive so that we have excess returns and large number such that small tracking errors

The appropriate measure depends upon investment assumptions

If portfolio represents entire risky investment, use the Sharpe measure

  - Sharpe ratio uses total risk which is fine for an entire portfolio

If portfolio is one of many combined into a largest one, use Jensen or Treynor

- Treynor is better since it weights excess returns against systematic risk (since this is what we compensated for)

Measure	P	Q	Mkt
Sharpe Ratio	0.45	0.51	0.19
Alpha	1.63	5.28	0.00
Beta	0.69	1.40	1.00
Treynor Index	4.00	5.40	1.63
Information Ratio	0.84	0.59	0.00

If P or Q represents the entire investment, Q is better because of its higher Sharpe measure

If P and Q are competing for a role as one of a number of subportfolios, Q also dominates because its Treynor measure is higher

If we seek an active portfolio to mix with an index portfolio, P is better due to its higher information ratio

- People hold cash so that they have liquidity to invest
- Hold cash if you think market is going to go down
- Cash has a beta of 0 which helps lower portfolio's beta
    - However, it takes time and money to put cash into market
        - Liquidity and transaction cost

- A future has a beta of negative 1

Market timing

- Treynor and Mazuy:

$$r_P - r_f = \alpha + \beta(r_M - r_f) + \Psi(r_M - r_f)^2 + e_p$$

- A positive value of α_p is indicative of superior stock selection performance
- A positive value for Ψ_p indicates superior market timing ability

- *Market timing involves shifting funds between market index portfolio and safe asset*
- *If positive candle stick, return goes up greater than proportional to the market*

Merton Model

Option is valuable when equity return falls below risk-free rate: $$r_P - r_f = \alpha + \beta(r_M - r_f) + \phi Max[0, r_M - r_f] + e_p$$

Positive values of ϕ_p indicate market timing ability

- Merton defines market timing as performance relative to risk free rate
- Managers can switch between equity and bonds
  - Portfolio return is comprised of return on equity and put on equity market via investing in bound
- Similar to put option
- Option is valuable when equity return falls below risk free rate

Role of style in performance evaluation

$$r_{it} = \alpha_i + b_i RMRF_t + s_i SMB_t + h_i HML_t + p_i PR1YR_t + e_{it}$$

- RMRF: excess return on market index
- SMB: size factor
- HML: value factor
- PR1YR: 1-year momentum factor

- Market anomalies are also used in performance evaluation
- See how well they do and is returns just simply due to anomalies
- We can see if stock is actually good by if positive alpha whilst holding and accounting for factors

Buffet's alpha
- Compare funds when considering style effects
  - Large cap stocks may perform better in general, but doesn't mean the fund is better
- BAB reflects tendency to buy safe beta stocks whilst staying away from high beta
- QMJ means buy high quality companies

Alpha	**12.1%**	**9.2%**	6.3%
	(3.19)	(2.42)	(1.58)
MKT	**0.84**	**0.83**	**0.95**
	(11.65)	(11.70)	(10.98)
SMB	**-0.32**	**-0.32**	-0.15
	-(3.05)	-(3.13)	-(1.15)
HML	**0.63**	**0.38**	**0.46**
	(5.35)	(2.79)	(3.28)
UMD	0.06	-0.03	-0.05
	(0.90)	-(0.40)	-(0.71)
BAB		**0.37**	**0.29**
		(3.61)	(2.67)
QMJ			**0.43**
			(2.34)
R2 bar	0.25	0.27	0.28

- Things you can use to hold constant to see if alpha still significant and shows investors skill

Performance attribution

- Where did outperformance come from

- Decompose relative performance into security selection and asset allocation
  1. Market: Equity, fixed income, alternative investments, etc
  2. Industry: Mining, Financials, Retail, etc
  3. Security: RIO, BHP, FMG, WPL etc

- Return on benchmark:

$$r_B = \sum_{i=1}^{N} w_{Bi} r_{Bi}$$

- Return on portfolio:

$$r_P = \sum_{i=1}^{N} w_{Pi} r_{Pi}$$

- Net return:

$$r_P - r_B = \sum_{i=1}^{N} w_{Pi} r_{Pi} - \sum_{i=1}^{N} w_{Bi} r_{Bi} = \sum_{i=1}^{N} (w_{Pi} r_{Pi} - w_{Bi} r_{Bi})$$

- Total contribution from asset class i

$$w_{Pi}r_{Pi} - w_{Bi}r_{Bi}$$

- Contribution from asset allocation

$$(w_{Pi} - w_{Bi})r_{Bi}$$

- Contribution from security selection

$$w_{Pi}(r_{Pi} - r_{Bi})$$

- Contribution from asset allocation
  - Given return for that industry, are we overweighting in that. Are we investing in industries performing well
  - How much we invest in asset vs the index weight
- Contribution from security selection
  - Given allocation, did we pick right stocks in that industry

Asset allocation
- Active weight:
  - What is the differential between our portfolios industry weights vs industry's industry weights?
    - Market uses market cap for their weight
  - **To calculate, get our stock's weight – market weight**
  - Allows you to see which industries you under/overweight compared to market portfolio
  - **Asset allocation differences/sector weight differences**
- Net return:
  - Our portfolio's return – market return
    - What is the weight in that industry * return of that industry
    - What is the difference between them
  - You can add this up to see the sum of the **net return**
  - You can see which industry led to more returns and which industries led to under/over performance

- **Asset allocation (industry decision)**

        - Take difference of our portfolio's industry weights vs market's weights whilst holding returns the same. Multiply this by market returns to keep that constant
        - Take summation to see how well this industry decision is
        - See the weighting of this to see whether did we weigh it correctly
            - If industry has negative returns, but we had less weighting in it compared to market, this means we had less exposure to that bad return which is good!
            - If industry has positive return, we want more overweighting in those industry
        - Keep returns same but weightings different

- **Security selection (stock decision)**
    - Looks inside those industry.
    - Keep industry weights the same but see the returns of the stocks we picked in those industry (e.g. we may both invest in finance industry, but which firms within finance industry? Market uses largest market cap finance firms but we used other stuff)
    - Take difference in net returns whilst multiplying industry weights of our portfolio (keeps weightings same but returns different)
    - So we may have invested 16% in finance industry vs 5% in market
        - But we picked stocks that are worse and therefore earns us lower net returns
        - Negatives here shows that our security selection approach is inferior to market cap approach to selecting stocks in market portfolio
        - We can see how much weighting we had in industries that we picked good stocks for as well

- So we can see whether are we doing well because we pick right stocks or right industry weightings

Performance evaluation summary

- 2 key issues
    - Need long observation time to measure performance whilst requiring return distribution to be stable with constant mean and variance
    - What if mean and variance not constant? We have issues

Week 11 – Hedge Funds

Mutual Funds	Hedge Funds
Regulations require public disclose of strategy and portfolio composition	Only requires minimal disclose of strategy and portfolio composition
Unlimited number of investors	No more than 100 *sophisticated* (high net worth) investors
Fees are fixed % from 0.5%-1.5%	2% of asset and incentive fee of 20%
Predictable stable strategy	Very flexible and act opportunistically
Limited use of shorting, leverage, and options	Can use anything
Quite liquid to withdraw	Have lockup periods and require redemption notices

Hedge Fund strategies

- **Directional:** Bets that one sector or asset class will outperform other sectors
- **Non-directional:** Exploit temporary misalignment in relative valuation across sector. Long one and short one. Strives to be market neutral. Expect convergence of price.

Different hedge fund strategies

Equity Hedge
- Take long and short positions in under and over valued things

- **Equity Market Neutral (EMN)**
    - Exploits equity market inefficiencies by shorting both and long assets of same size, beta, and industry
    - Maximum of 10% short and long

- **Short-based (SSE)**
    - Identify overvalued companies
    - Degree of short exposure vary depending on market cycle

- **Long/short equity**
    - Long or shorting equity but not being market neutral

Event-driven
- Driven by anticipated corporate events
- Driven by fundamental analysis

- **Merge arbitrage (MEA)**

    - Simultaneously long target and short acquirer (still a tiny gap since unsure if deal will actually go through)

- **Distressed Securities (DSE)**
    - Invest in debt/equity of financially distressed companies
    - Allows HF to be involved in bankruptcy negotiations and deal-making
    - Vulture funds tend to do this

Relative value

- Take positions expecting that valuation differentials will converge

- **Fixed Income Convertible Arbitrage (COA)**

    - Profit from convergence of valuation difference between securities where one is a convertible fixed income instrument
    - Systematic drivers of return are issuer credit quality, volatility, and interest rate
    - Here, we purchase convertible securities and short sell common stock
    - There is reason to believe that the convertible security is priced incorrectly

- **Volatility**
    - View volatility as own asset class
    - Generate return through arbitrage, direction, market neutral or any combination of strategies

Global Macro (MAC)

- Carry long/short positions in capital/derivative markets. Reflects views by economic trends. Big picture

Managed futures

- Invest in futures

Emerging market (EMT)

- Focus on emerging markets
- Very likely to be priced incorrectly due to lower coverage

Value of $1 invested in Feb 1991

- Quite a few strategies manage to beat the market

Hedge funds should not be correlated with S&P500 as they should be aiming to beat it

- We can analyse the factor model regression of a hedge fund's returns

$$r_P = \alpha + \beta_1 r_{S\&P500} + \beta_2 Size + \beta_3 Credit + \beta_4 Bond + \beta_5 r_{EM} + e_p$$

- $r_{S\&P500}$: Excess return on the S&P500
- Size: Return on Russell 2000 index – return on S&P500 index
- Credit: Yield on Baa corporate bonds – 10-yr U.S. Treasury yield
- Bond: 10-yr U.S. Treasury yield
- r_{EM}: Return on MSCI Emerging Market index

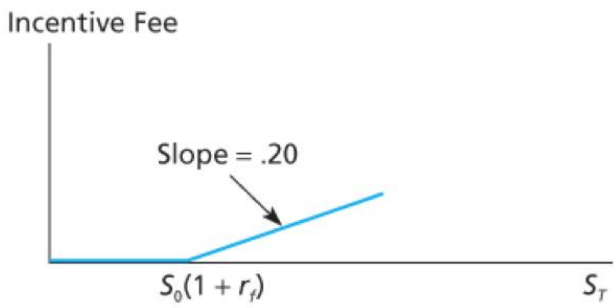
- When considering these factors, we find a lot of the alpha from hedge funds disappear
- What isn't in the R-squared is idiosyncratic
- In aggregate, HF's don't do that well. Most likely due to the poor performers bringing average down.

Fee structure

- 2% of assets plus 20% of investment profits
- Incentive fees are call options

  X = (portfolio value) * (1 + benchmark return)

- Managers get nothing if lose money



- **High water mark**
  - If fund experience losses, can't charge incentive fee unless recovers its losses
  - Deep losses makes this very unlikely and therefore HF can shut down
  - However, reputations of managers are damaged if they do this repeatedly

Funds of funds

- Funds that invest into hedge funds for you
  - Allows for diversification across HF's
  - Does its due diligence by screening out HF's
- Spread risk across several funds
  - **However, must make sure that the HF's have different styles or else not diversified**
  - **Fund of funds are also highly leveraged**
- Optionality can have big impact on fees whereby fund of funds pay incentive fees to each underlying fund that outperforms benchmark even if aggregate performance is bad

- The aggregate portfolio of the fund of funds is -5%
- Still pays incentive fees of $0.12 for every $3 invested

	Fund 1	Fund 2	Fund 3	Fund of Funds
Start of year (millions)	$1.00	$1.00	$1.00	$3.00
End of year (millions)	$1.20	$1.40	$0.25	$2.85
Gross rate of return	20%	40%	-75%	-5%
Incentive fee (millions)	$0.04	$0.08	$0.00	$0.12
End of year, net of fee	$1.16	$1.32	$.25	$2.73
Net rate of return	16%	32%	-75%	-9%

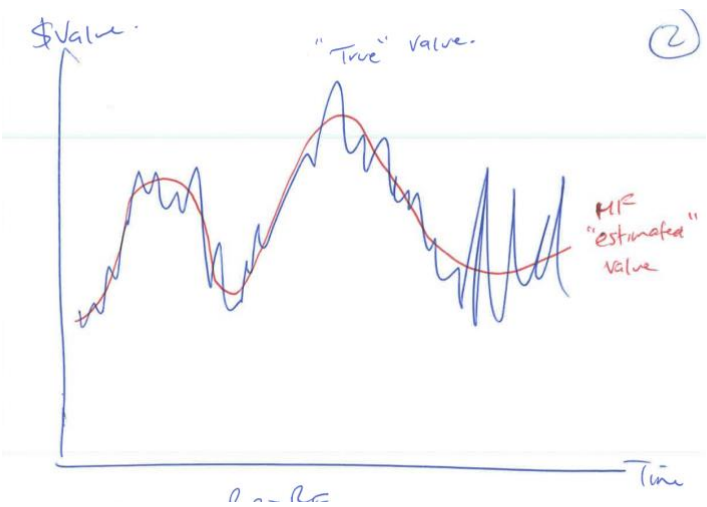
- HF charges incentive fee of 20%
- Liquidity Hedge
    - 10 year bonds issued 5 years ago should be the same as a 5 year bond issued today
    - However, 5 year bond today more liquid and cheaper
    - Short 5 year bond and buy 10 year bond and these should converge and make a liquidity premium
- **Note that fund of funds have lower returns because**
    - **Higher liquidity offered (**pays off well in bad times, so require lower return)
    - **Extra fees**

Things to learn from HF failure

- **To measure risk is not to control it**
- **Need for transparency and information disclosure**
- **Dangers of generous credit limits for trading**
- **Unexpected black Swan events**
- **Moral hazard whereby taking big bets and being too big to fail means government will *have* to bail you out**

Issues with evaluating Hedge Funds

- Hedge funds hold more illiquid assets which makes measuring performance tricky
    - They can make up values
    - They can compare to other favourable instruments
- Alpha may be an equilibrium liquidity premium as a result rather than stock picking ability
- They can smooth out the illiquid asset



- **Hedge fund returns have serial correlation** which biases the Sharpe ratio upwards
    - They can lie about the volatility of their performance
- **Backfill bias**
    - HF's can choose to report when to report. They'll only report when returns are good. Used as a marketing tool

- **Survivorship bias**
    - Failed funds drop out of the database
    - Hedge fund failure rates are twice as high as mutual funds

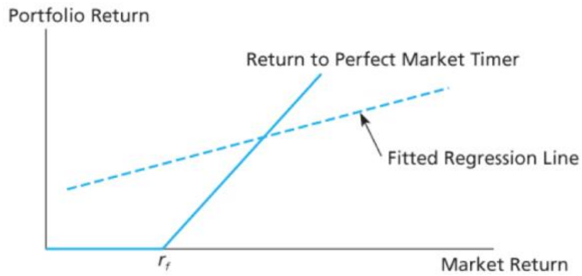
- Hedge funds are opportunistic and frequently change risk profiles
- If risk not constant, alpha will be biased if we used linear index model
- If risk profile changes in systematic matter too, then evaluation is even more difficult

- Should be able to time the market similar to an option

- Adjust beta in the right manner in response to the market

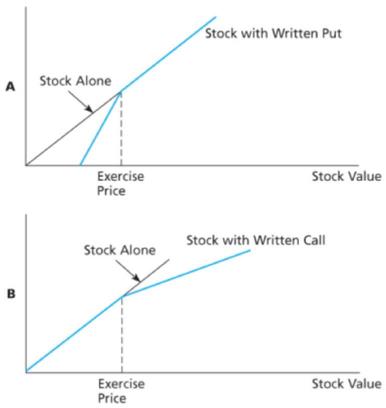
- Moves funds from risk free asset into market portfolio only when market will outperform risk free rate

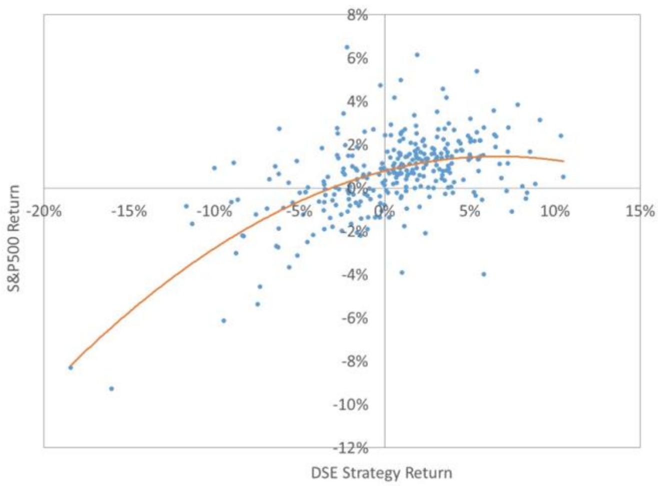
- Characteristic line has slope of 0 when market excess return is negative but slope of 1 when it is positive



- Hedge funds tend to write a lot of options and generate premiums from this
- Blue line is if you own stock and you writing options, how much you make
- A shows that if you write puts but stock prices fall, you lose out
- Same for B in call options
- **Therefore, we can see there is a non-linear relationship in the profits of HF's**

Portfolios with written options





- Not what we want since opposite effect of market timing.
- Here, beta falls when market return is high and vice versa
- Funds have higher down market betas (higher slopes) than upmarket betas. We want linear relationship. We don't want high market sensitivity when market is weak
- Higher slope (means higher beta) when market it doing badly.

- Ability to perfectly time markets give funds non-linear characteristic line similar to call option
- Fund has greater sensitivity to rising markets

$$r_P - r_f = \alpha + \beta(r_M - r_f) + \phi \text{Max}[0, r_M - r_f] + e_p$$

- Using this model, we can see alpha is present.
  - Therefore, this is a better model to use

	All HFs	EQH	EMN	SSE	FoHFs
Alpha	0.009**	0.008**	0.005**	0.005	0.007**
	(7.26)	(5.38)	(6.07)	(1.42)	(5.69)
r_m-r_f	0.409**	0.507**	0.068**	-0.782**	0.310**
	(13.03)	(12.36)	(3.29)	(-8.98)	(9.83)
Max[0,r_m-r_f]	-0.153**	-0.117	-0.012	-0.130	-0.184**
	(-2.7)	(-1.59)	(-0.33)	(-0.83)	(-3.24)
Adj. R^2	0.546	0.554	0.079	0.485	0.355

- Beta here is 0.409
- Down is -0.153
- **Upmarket beta = .256**
- **Downmarket beta = .409** (since coefficient of max is negative, this goes to 0)
- Not what we want, we want it other way around

Performance persistence: Degree to which able to maintain consistent performance across time. Examine correlation of individual fund's return across time. Therefore, poorly performing funds will continue to do badly. This doesn't necessarily contradict with efficient markets (even though there is predictability involved since you can guess which funds will continue to do well). Investors who chase these well performing funds are not diversifying themselves across the market and therefore since expected return is higher, diversifiable risk is higher. This is still consistent with efficient markets whereby higher expected returns comes with higher diversifiable costs.

Alpha: Regression intercept
Information ratio: Alpha/Residual standard deviation. This is known as **active return divided by tracking error.** We look at difference between security and relevant selected benchmark
Sharpe Ratio: Excess return over standard deviation

$$*\text{Sharpe measure} = \frac{r_P - r_f}{\sigma_P}$$

Index model regression estimates	$1\% + 1.2(r_m\text{-}r_f)$	$2\% + 0.8(r_m\text{-}r_f)$

- **If the regression model is given to us as this, it already computes for us rp-rf. Therefore no need to minus rf again**

Treynor ratio: excess return over beta

- Sharpe measure is the best to evaluate a **sole risky asset in a portfolio**

- Appraisal/information ratio is the best measure for evaluating a stock being added to a portfolio composing solely of holdings in market portfolio since overall contribution to Sharpe ratio depends on appraisal ratio

- Treynor ratio is best if the fund is one of many funds. How much you can earn on an investment with no diversifable risk (only market risk). Only used as a ranking criteria. If the portfolios aren't part of a larger fully diversified portfolio, then portfolios with identical systematic risks but different total risks will be rated the same

Statistical arbitrage is not true arbitrage since it does not establish risk-free positions based on security mispricing. It is essentially a portfolio of risky bets hoping some of them will pay off. Doesn't exploit risk-free position. Uses quantitative trading systems to seek out many temporary and modest misalignments in prices among securities. Law of averages says they will profit from this in the end (making it a statistical certainty). Pair trading is to pair up companies that are similar but one is overpriced. Then short expensive one and buy cheap one.
Fund of funds charges incentive fees if each individual fund does well or not
However, standalone fund only charges a fee if aggregate portfolio does well

Higher watermarks are less valuable for hedge funds since it means they need to achieve a higher return in order to be compensated for that performance.
High water mark for hedge fund is equivalent to a call option on an asset with current market value equal to net asset value of the fund

Hedge fund fees:
Level – Fixed fee charged on assets under management
Shape – Performance incentive

Incentive fees are a weak mechanism to ensure HF managers do well

If hedge funds had to instead beat a predetermined market beta, it is better. Since it's possible for HF to have positive returns, but the market had a much higher return.
This makes a higher hurdle rate for hedge funds.
Therefore, for hedge fund to get performance compensation, they need to outperform the index too

Smaller hedge funds are younger. Charge lower fees and puts pressure on large funds
Outperformance becomes harder.
Equilibrium occurring with more firms entering the market and leading to a decrease in price (fees)

Do note that the **average hedge fund** does not beat the market
The issue is trying to identify hedge funds with actual skills and persistence in beating the market. The issue is
1) Finding these HF's
2) Being allowed to invest into these hedge funds

Should use a multifactor model as a benchmark for hedge fund evaluation

Week 12 – Illiquid Assets

Liquidity

- Being able to trade the quantity of securities when you want to at the price you want to
- This does not impact the price (enough volume to trade whilst bid-ask spread is small)

Proxies used for liquidity

- **Bid ask spread** (people specify prices to buy/sell. If small gap, then large demand). The spread tells us, how much must we pay if we want to trade right away. Therefore, smaller spread means easier to trade it right away
- **Price impact** – If large change in price, then not many things being traded
- **Volume/value** of shares traded
- **Depth** in limit order book – How many people want to trade
- **Time between transaction**
- **Market capitalisation** can also be a proxy
- **Reputation** and how well the stock is known

Illiquid assets makes up a large share

- 90% of individuals' wealth is in illiquid assets

Asset Class	Typical Time between Transactions	Turnover (p.a.)
Public Equities	Within seconds	Well over 100%
OTC Equities	Within a day, but many stocks over a week	Approx 35%
Corporate Bonds	Within a day	25–35%
Municipal Bonds	Approx 6 months, with 5% of muni bonds trading more infrequently than once per decade	Less than 10%
Private Equity	Funds last for 10 years; the median investment duration is 4 years; secondary trade before exit is relatively unusual	Less than 10%
Residential Housing	4–5 years, but ranges from months to decades	Approx 5%
Institutional Real Estate	8–11 years	Approx 7%
Institutional Infrastructure	50–60 years for initial commitment, some as long as 99 years	Negligible
Art	40–70 years	Less than 15%

- Turnover is less important compared to typical time between transactions

Sources of illiquidity

- Participation cost
  - Takes a lot of time/money to be skilled enough to trade such as art
- Transaction cost
  - Commissions, taxes, due diligence, etc
- Search friction
  - Finding a counterparty to trade with can take time
- Asymmetric information
  - Seller knows something buyer doesn't know. Therefore, buyer is worried about trading
- Price impact
  - Large trades move markets because they consume liquidity. If massive price jump when trade occurs, people less willing to trade
- Funding constraints
  - Firms are quite leveraged. Many vehicles used to invest in illiquid assets are highly leveraged and therefore, if access to funding is impaired, difficult to transact in illiquid asset

Always be sceptical of illiquid assets

3 key biases cause returns of illiquid assets to be overstated and risk understated

Survivorship bias
- Poorly performing funds tend to stop reporting to databases
- Large proportion of failed funds is not in the database
- Volunteering reporting is an important aspect
- In illiquid markets, we never observe the full universe

Infrequent trading
- Returns are essentially smoothed out and therefore not see the true volatility
- Don't see numerous trades and therefore don't see true volatility of the asset
- Volatility, correlation and beta is understated since can't see true returns of the asset
- Seems like no risk since such infrequent trades

Selection bias
- Investors don't sell assets when their value is low
- Only sell when value is high and therefore over reports the asset's return and value
- Beta and volatility is underestimated whilst expected returns are overestimated

Reported returns of illiquid assets are not true

Illiquidity risk premiums

- These premiums reward people for:
  - Unable to access capital immediately
  - Withdrawal of liquidity during illiquidity crises (pays off badly in bad times, need compensation for that)
    - Investors want to be compensated in good times for assets that pay off badly in bad times
    - Therefore, expected returns on assets with positive relationship with liquidity should be higher
      - Pricing of these assets will fall but this leads to expected returns being higher
    - Assets that pay off well in bad times are good and priced higher since they provide a hedge against bad times. They have a higher price which implies lower expected return.

- You can calculate this via certainty equivalent comparing liquid and illiquid asset

4 ways to capture illiquidity premiums

1) Set passive allocation to illiquid asset class like real estate
2) Choose securities in an asset class that are more illiquid via liquidity security selection
3) Act as a market maker on individual security level
4) Engage in dynamic strategies at aggregate portfolio level

Illiquidity risk premium across asset classes

- Difficult to determine if illiquid asset classes deliver excess returns
- How do we adjust for risk?
  - What is market portfolio to compare to for an illiquid asset?
- Manager's skill can't be disentangled from risk factors like in liquid markets
- Therefore, difficult to see relationship between returns and illiquidity

- Depending on time horizon, ability to find skilled managers, and risk aversion, this tells us how much of portfolio should be in illiquid asset and premium requirement

- Illiquidity risk causes investor to behave in more risk-averse fashion
  - Consumption is lower when illiquid asset are held in portfolio
  - Risk can't be diversified since illiquid asset cant be traded
- Solvency ratio of illiquid to liquid wealth drives investor risk aversion

Portfolio choice with illiquid assets

Average Time Between Liquidity Events	Optimal Portfolio Allocation	Illiquidity Risk Premium
10 Years	5%	6.0%
5 Years	11%	4.3%
2 Years	24%	2.0%
1 Year	37%	0.9%
½ Year	44%	0.7%
Continuous Trading	59%	0.0%

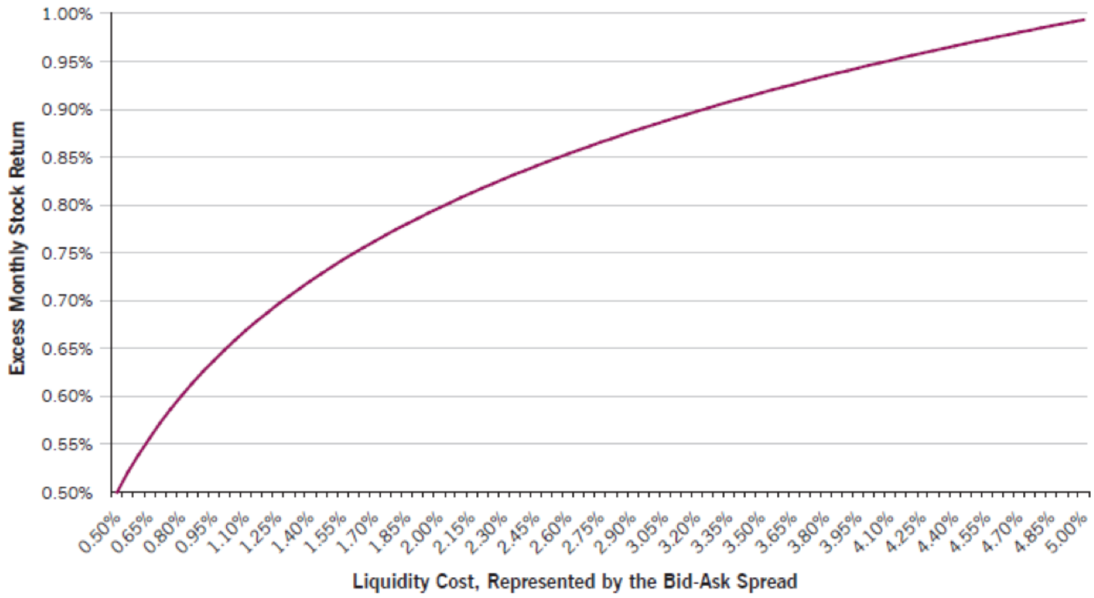
- If you want illiquid asset, have a long holding period
- However, they don't offer high risk-adjusted returns
- Illiquid investing can lead to agency problems as difficult to monitor agent
- High idiosyncratic risk since no market portfolio
- If you can find a skilled manager, it's good because
  - Illiquid assets most likely have information asymmetry and transaction cost, meaning alpha to be earned
  - Skilled investor can find these opportunities and calls for due diligence

Liquidity impact on asset prices

- If assets incorporates transaction cost, there is a demand for compensation
- Pay a lower price and receive a discount

- Higher trading cost, the higher illiquidity discount required (in the form of bid-ask spread)

Liquidity costs and excess returns



- On the left, is short term investor and long term as you go right
- Costs are amortised over longer time so less compensation required over time
- Non-linear since people have different time horizon
- HFT only trades in low spreads whilst long term investor trade in high spreads

Liquidity risk

- Should receive compensation for non-diversifiable liquidity risk
- If asset positively correlated with liquidity, should get higher expected return. This means that if illiquid markets, we have an asset that pays off badly. This pushes down price and cause return to rise.
- If negatively correlated, gives us a good hedge mechanism
- With this, we can estimate liquidity risk at security level and average across asset to get a market-level measure of liquidity risk
  - o Stocks with higher liquidity (**factor is liquidity risk)** betas should have higher expected return to compensate for risk

Liquidity adjusted CAPM

- Expected return of security increases with expected liquidity and net beta
- Relax assumption about ease to trade. **There are now transaction costs.**
- We have stock returns but now there’s a cost to trade stock (c)
- We have return on market but also cost of trading in the market
- CAPM is covariance between ri and rm, which is the beta of the CAPM

- Net beta proportional to covariance of stock return and liquidity cost with market portfolio net returns

- **Net Beta now comprised of liquidity risk can be decomposed into**

    - Commonality in liquidity with market liquidity (costly to trade stock and trade in market) (2^{nd} term)
        - How costly is it to trade
    - Return sensitivity to market liquidity
        - Do we want assets that pays off high when markets are high? YES. Therefore, we don't mind having lower returns
    - Liquidity sensitivity to market return

$Cov(R_i, R_m) \Rightarrow$ Beta in CAPM (divided by σ_m^2)

$$Cov(R_i - C_i, R_m - C_m)$$

$$+ \, Cov(R_i, R_m)$$

$$+ \, Cov(C_i, C_m)$$

$$- \, Cov(C_i, R_m)$$

$$- \, Cov(R_i, C_m)$$

- **It's the net returns on the stock and the stock market is what matter**
- From this, we get 4 covariance term
    1) Normal beta
    2) **Commonality in liquidity with market liquidity:** If stock costly to trade and market is costly, then positive return. If stock costly to trade when market is costly to trade, higher return is expected.
    3) **Liquidity sensitivity to market returns:** If stock is cheap to trade, and market return is low, this is a good thing. This is good for portfolio and you'll take lower return on that. Since if market is bad, easy to offload our stock.
    4) **Return sensitivity to market liquidity:** If cost of trading market is high, and return on asset is high, this is good and we don't mind taking lower return for this. Helps us hedge. Since if we can't get rid of our assets since high cost to trade in market, doesn't matter since our stock earns us a high return.

- if first term is highly correlated, higher return. First term is normal beta whilst last 3 are **liquidity beta**

- This leads to an asset pricing model of the form:

$$E(R_{it} - R_{ft}) = E(c_{it}) + \lambda\beta_{i1} + \lambda\beta_{i2} - \lambda\beta_{i3} - \lambda\beta_{i4}$$

  - β represents market risk and the three liquidity risks
  - λ represents the risk premium for each of these risks
- Illiquid securities have high liquidity risk
  - Flight to liquidity in bad times

- First term is the cost. This is non-linear relationship
- Beta 3 is the most important
- This result is seen internationally

- **Is this why CAPM doesn't work? Since missing 3 terms that should be there**

Hedge funds with higher liquidity beta has higher returns compared to those with low liquidity risk

- However, it appears this result is driven by small cap stocks
  - When controlling for size of stock, alpha becomes lower

Liquidity timing
- Investment managers can try add liquidity timing by timing risk factor
- Do they decrease liquidity exposure when payoff is low
- Funds with this ability earn 2% higher returns
- Long only mutual fund managers and hedge funds appear to be able to do this