Data Supplement to an Analysis of an Open Source Binomial Random Variate Generation Algorithm

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Abstract

This technical report is a data supplement to "An Analysis of an Open Source Binomial Random Variate Generation Algorithm" published in ASEC 2023. The dataset concerns the acceptance-rejection iterations of an algorithm for generating binomial random variates. This report includes additional experimental data that was excluded from the original ASEC 2023 paper in the interests of brevity. The source code of the experiments that produced the data is open source and available via GitHub.

Keywords: binomial, BTPE, data, open source, random variate

ACM Classes: G.3; G.4

MSC Classes: 60-04; 60-11; 60E05

1 Introduction

The paper "An Analysis of an Open Source Binomial Random Variate Generation Algorithm" concerns the average case behavior of an algorithm for generating binomial random variates [2]. The algorithm that is the subject of that paper is the BTPE Algorithm (Binomial, Triangle, Parallelogram, Exponential) of Kachitvichyanukul and Schmeiser [3], and specifically the implementation of BTPE as provided by the open source Java library $\rho\mu$ [1]. This technical report is a supplement to the ASEC 2023 paper [2], providing additional experimental data that was excluded from the prior paper in the interests of brevity. Section 2 explains the experimental methodology, and Section 3 provides the data.

2 Experimental Data

The BTPE algorithm uses acceptance–rejection sampling when generating binomial random variates. The experiments concern the average rejection sampling iterations for various binomial distributions $B(n, p)$. The implementation of BTPE in the $\rho\mu$ library [1] was instrumented to track this behavior, specifically

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counting the number of uniform random numbers are generated, which provides insight into the number of iterations since exactly two uniform random variates are generated per iteration [3].

The experiments used OpenJDK 17 on a Windows 10 PC with a 3.4 GHz AMD A10-5700 CPU and 8 GB RAM. The version of $\rho\mu$ is 3.1.1. The code of the experiments is on GitHub at `https://github.com/cicirello/btpe-iterations`. The source of $\rho\mu$ is also on GitHub at `https://github.com/cicirello/rho-mu`.

In the experiments, we consider $n \in \{2^5, 2^6, \ldots, 2^{20}\}$. For each n, we consider $p \in \{\frac{10}{n}, \frac{16}{n}, \frac{32}{n}, \ldots, \frac{1}{2}, \ldots, \frac{n-32}{n}, \frac{n-16}{n}, \frac{n-10}{n}\}$. The minimum p considered is $\frac{10}{n}$ as it is the minimum supported by BTPE. For lower values of p an alternative algorithm is required. The average number of uniform variates per binomial random variate is calculated averaged over the generation of 10,000 binomial random variates for each combination of p and n. We compute 95% confidence intervals. We additionally use the analytical formula determined by Kachitvichyanukul and Schmeiser for the expected number of iterations [3] to predict the rejection sampling behavior, and test the significance of the difference between predicted and observed averages with a t-test.

For complete experimental methodology, see the ASEC 2023 paper [2]. That paper, for the sake of brevity, included only the results for $n \in \{2^5, 2^{10}, 2^{15}, 2^{20}\}$. Here in this report, we include all of the results in Tables 1–16. The tables show the observed averages, the predictions, and p-values from t-tests. No statistically significant differences are found between observed and predicted behavior in the vast majority of cases. The few instances where differences appear significant at the 0.05 level are explained by random chance (e.g., the number of such cases, relative to the huge number of cases tested, is consistent with what should be expected at the 0.05 level).

3 The Data

Table 1: Average number of calls to $U(0,1)$ by $\rho\mu$'s BTPE implementation for $n = 2^5$

	$\rho\mu$		T-test
p	mean	predicted	p-value
0.3125	3.83 ± 0.052	3.84	0.76
0.5	3.58 ± 0.046	3.60	0.46
0.6875	3.78 ± 0.050	3.84	0.03

Table 2: Average number of calls to $U(0,1)$ by $\rho\mu$'s BTPE implementation for $n = 2^6$

	$\rho\mu$		T-test
p	mean	predicted	p-value
0.15625	3.79 ± 0.050	3.79	0.80
0.25	3.68 ± 0.049	3.66	0.45
0.5	3.33 ± 0.042	3.34	0.89
0.75	3.67 ± 0.048	3.66	0.84
0.84375	3.80 ± 0.051	3.79	0.64

Table 3: Average number of calls to $U(0,1)$ by $\rho\mu$'s BTPE implementation for $n = 2^7$

	$\rho\mu$		T-test
p	mean	predicted	p-value
0.078125	3.81 ± 0.051	3.79	0.45
0.125	3.56 ± 0.046	3.54	0.33
0.25	3.24 ± 0.039	3.21	0.06
0.5	3.00 ± 0.034	2.98	0.27
0.75	3.21 ± 0.040	3.21	0.88
0.875	3.55 ± 0.046	3.54	0.56
0.921875	3.80 ± 0.051	3.79	0.66

Table 4: Average number of calls to $U(0,1)$ by $\rho\mu$'s BTPE implementation for $n = 2^8$

	$\rho\mu$		T-test
p	mean	predicted	p-value
0.0390625	3.76 ± 0.051	3.79	0.27
0.0625	3.48 ± 0.045	3.49	0.61
0.125	3.07 ± 0.036	3.06	0.50
0.25	2.77 ± 0.029	2.74	0.04
0.5	2.64 ± 0.025	2.63	0.66
0.75	2.72 ± 0.027	2.74	0.09
0.875	3.04 ± 0.035	3.06	0.36
0.9375	3.47 ± 0.045	3.49	0.54
0.9609375	3.82 ± 0.052	3.79	0.34

Table 5: Average number of calls to $U(0,1)$ by $\rho\mu$'s BTPE implementation for $n = 2^9$

	$\rho\mu$		T-test
p	mean	predicted	p-value
0.01953125	3.80 ± 0.051	3.80	0.99
0.03125	3.47 ± 0.044	3.47	0.87
0.0625	2.97 ± 0.033	3.00	0.15
0.125	2.73 ± 0.028	2.76	0.06
0.25	2.59 ± 0.024	2.58	0.27
0.5	2.41 ± 0.019	2.42	0.41
0.75	2.55 ± 0.023	2.58	0.03
0.875	2.76 ± 0.029	2.76	0.71
0.9375	3.03 ± 0.035	3.00	0.11
0.96875	3.44 ± 0.043	3.47	0.27
0.98046875	3.83 ± 0.051	3.80	0.21

Table 6: Average number of calls to $U(0,1)$ by $\rho\mu$'s BTPE implementation for $n = 2^{10}$

	$\rho\mu$		T-test
p	mean	predicted	p-value
0.009765625	3.82 ± 0.051	3.80	0.42
0.015625	3.49 ± 0.045	3.46	0.16
0.03125	2.94 ± 0.033	2.97	0.06
0.0625	2.70 ± 0.027	2.69	0.53
0.125	2.52 ± 0.022	2.52	0.38
0.25	2.36 ± 0.018	2.34	0.09
0.5	2.30 ± 0.016	2.32	0.11
0.75	2.35 ± 0.018	2.34	0.36
0.875	2.54 ± 0.022	2.52	0.30
0.9375	2.67 ± 0.026	2.69	0.14
0.96875	3.00 ± 0.034	2.97	0.05
0.984375	3.41 ± 0.043	3.46	0.05
0.990234375	3.79 ± 0.051	3.80	0.74

Table 7: Average number of calls to $U(0,1)$ by $\rho\mu$'s BTPE implementation for $n = 2^{11}$

	$\rho\mu$		T-test
p	mean	predicted	p-value
0.0048828125	3.88 ± 0.052	3.80	0.00
0.0078125	3.46 ± 0.044	3.45	0.65
0.015625	2.97 ± 0.033	2.96	0.54
0.03125	2.66 ± 0.026	2.66	0.68
0.0625	2.46 ± 0.021	2.46	0.59
0.125	2.34 ± 0.018	2.33	0.27
0.25	2.31 ± 0.016	2.30	0.28
0.5	2.31 ± 0.017	2.31	0.85
0.75	2.29 ± 0.016	2.30	0.15
0.875	2.32 ± 0.017	2.33	0.15
0.9375	2.44 ± 0.021	2.46	0.15
0.96875	2.65 ± 0.026	2.66	0.85
0.984375	2.95 ± 0.032	2.96	0.55
0.9921875	3.45 ± 0.044	3.45	0.96
0.9951171875	3.80 ± 0.052	3.80	0.89

Table 8: Average number of calls to $U(0,1)$ by $\rho\mu$'s BTPE implementation for $n = 2^{12}$

	$\rho\mu$		T-test
p	mean	predicted	p-value
0.0024414062	3.81 ± 0.051	3.80	0.72
0.00390625	3.45 ± 0.044	3.45	0.92
0.0078125	2.93 ± 0.033	2.95	0.29
0.015625	2.63 ± 0.026	2.64	0.58
0.03125	2.43 ± 0.020	2.42	0.64
0.0625	2.34 ± 0.018	2.33	0.17
0.125	2.28 ± 0.016	2.29	0.17
0.25	2.28 ± 0.016	2.28	0.89
0.5	2.28 ± 0.015	2.28	0.98
0.75	2.27 ± 0.015	2.28	0.18
0.875	2.29 ± 0.016	2.29	0.51
0.9375	2.33 ± 0.017	2.33	0.90
0.96875	2.45 ± 0.020	2.42	0.03
0.984375	2.63 ± 0.025	2.64	0.37
0.9921875	2.94 ± 0.033	2.95	0.67
0.99609375	3.46 ± 0.044	3.45	0.56
0.9975585938	3.76 ± 0.050	3.80	0.18

Table 9: Average number of calls to $U(0,1)$ by $\rho\mu$'s BTPE implementation for $n = 2^{13}$

	$\rho\mu$		T-test
p	mean	predicted	p-value
0.0012207031	3.81 ± 0.051	3.80	0.70
0.001953125	3.47 ± 0.045	3.45	0.38
0.00390625	2.95 ± 0.033	2.95	0.69
0.0078125	2.63 ± 0.026	2.63	0.55
0.015625	2.53 ± 0.023	2.51	0.03
0.03125	2.36 ± 0.018	2.37	0.29
0.0625	2.27 ± 0.015	2.27	0.90
0.125	2.25 ± 0.015	2.26	0.08
0.25	2.27 ± 0.015	2.27	0.56
0.5	2.29 ± 0.016	2.30	0.13
0.75	2.27 ± 0.015	2.27	0.72
0.875	2.27 ± 0.015	2.26	0.59
0.9375	2.27 ± 0.015	2.27	0.60
0.96875	2.36 ± 0.018	2.37	0.71
0.984375	2.51 ± 0.022	2.51	0.97
0.9921875	2.64 ± 0.026	2.63	0.73
0.99609375	2.93 ± 0.032	2.95	0.26
0.998046875	3.44 ± 0.045	3.45	0.68
0.9987792969	3.78 ± 0.051	3.80	0.51

Table 10: Average number of calls to $U(0,1)$ by $\rho\mu$'s BTPE implementation for $n = 2^{14}$

	$\rho\mu$		T-test
p	mean	predicted	p-value
0.0006103516	3.80 ± 0.051	3.80	0.97
0.0009765625	3.47 ± 0.045	3.45	0.36
0.001953125	2.95 ± 0.033	2.95	0.86
0.00390625	2.61 ± 0.025	2.63	0.13
0.0078125	2.51 ± 0.022	2.50	0.37
0.015625	2.35 ± 0.018	2.35	0.75
0.03125	2.28 ± 0.016	2.28	0.51
0.0625	2.26 ± 0.015	2.26	0.59
0.125	2.25 ± 0.015	2.26	0.54
0.25	2.30 ± 0.017	2.28	0.14
0.5	2.30 ± 0.017	2.30	0.55
0.75	2.29 ± 0.016	2.28	0.24
0.875	2.25 ± 0.015	2.26	0.23
0.9375	2.25 ± 0.015	2.26	0.24
0.96875	2.29 ± 0.016	2.28	0.28
0.984375	2.35 ± 0.018	2.35	0.91
0.9921875	2.50 ± 0.022	2.50	0.79
0.99609375	2.64 ± 0.026	2.63	0.36
0.998046875	2.94 ± 0.032	2.95	0.79
0.9990234375	3.46 ± 0.044	3.45	0.48
0.9993896484	3.82 ± 0.051	3.80	0.45

Table 11: Average number of calls to $U(0,1)$ by $\rho\mu$'s BTPE implementation for $n = 2^{15}$

	$\rho\mu$		T-test
p	mean	predicted	p-value
0.0003051758	3.81 ± 0.052	3.80	0.73
0.0004882812	3.48 ± 0.044	3.45	0.13
0.0009765625	2.96 ± 0.033	2.94	0.50
0.001953125	2.62 ± 0.025	2.63	0.51
0.00390625	2.48 ± 0.021	2.49	0.15
0.0078125	2.34 ± 0.017	2.34	0.81
0.015625	2.27 ± 0.015	2.27	0.55
0.03125	2.26 ± 0.015	2.26	0.92
0.0625	2.25 ± 0.015	2.26	0.57
0.125	2.28 ± 0.015	2.28	0.43
0.25	2.29 ± 0.016	2.28	0.59
0.5	2.29 ± 0.016	2.30	0.08
0.75	2.30 ± 0.016	2.28	0.16
0.875	2.27 ± 0.016	2.28	0.62
0.9375	2.26 ± 0.015	2.26	0.76
0.96875	2.26 ± 0.015	2.26	0.53
0.984375	2.28 ± 0.015	2.27	0.15
0.9921875	2.32 ± 0.017	2.34	0.04
0.99609375	2.48 ± 0.021	2.49	0.22
0.998046875	2.62 ± 0.025	2.63	0.69
0.9990234375	2.98 ± 0.034	2.94	0.08
0.9995117188	3.45 ± 0.044	3.45	0.81
0.9996948242	3.78 ± 0.051	3.80	0.53

Table 12: Average number of calls to $U(0,1)$ by $\rho\mu$'s BTPE implementation for $n = 2^{16}$

	$\rho\mu$		T-test
p	mean	predicted	p-value
0.0001525879	3.83 ± 0.052	3.80	0.27
0.0002441406	3.42 ± 0.043	3.45	0.27
0.0004882812	2.95 ± 0.033	2.94	0.93
0.0009765625	2.65 ± 0.025	2.63	0.17
0.001953125	2.50 ± 0.022	2.49	0.31
0.00390625	2.33 ± 0.017	2.34	0.23
0.0078125	2.25 ± 0.015	2.26	0.23
0.015625	2.27 ± 0.015	2.27	0.55
0.03125	2.27 ± 0.015	2.27	0.47
0.0625	2.26 ± 0.015	2.27	0.16
0.125	2.28 ± 0.016	2.28	0.80
0.25	2.30 ± 0.016	2.29	0.32
0.5	2.31 ± 0.017	2.30	0.65
0.75	2.30 ± 0.017	2.29	0.16
0.875	2.27 ± 0.015	2.28	0.22
0.9375	2.27 ± 0.015	2.27	0.85
0.96875	2.26 ± 0.015	2.27	0.83
0.984375	2.27 ± 0.015	2.27	0.64
0.9921875	2.26 ± 0.015	2.26	0.66
0.99609375	2.34 ± 0.017	2.34	0.98
0.998046875	2.48 ± 0.022	2.49	0.39
0.9990234375	2.63 ± 0.025	2.63	0.88
0.9995117188	2.94 ± 0.033	2.94	0.60
0.9997558594	3.46 ± 0.044	3.45	0.52
0.9998474121	3.84 ± 0.051	3.80	0.10

Table 13: Average number of calls to $U(0,1)$ by $\rho\mu$'s BTPE implementation for $n = 2^{17}$

	$\rho\mu$		T-test
p	mean	predicted	p-value
0.0000762939	3.83 ± 0.052	3.80	0.32
0.0001220703	3.42 ± 0.044	3.45	0.19
0.0002441406	2.91 ± 0.032	2.94	0.03
0.0004882812	2.62 ± 0.025	2.63	0.34
0.0009765625	2.49 ± 0.021	2.49	0.76
0.001953125	2.34 ± 0.017	2.34	0.98
0.00390625	2.26 ± 0.015	2.26	0.66
0.0078125	2.26 ± 0.015	2.26	0.43
0.015625	2.28 ± 0.016	2.27	0.32
0.03125	2.27 ± 0.016	2.27	0.35
0.0625	2.27 ± 0.015	2.29	0.11
0.125	2.30 ± 0.016	2.29	0.27
0.25	2.29 ± 0.016	2.30	0.14
0.5	2.30 ± 0.016	2.31	0.29
0.75	2.29 ± 0.016	2.30	0.28
0.875	2.28 ± 0.016	2.29	0.28
0.9375	2.27 ± 0.015	2.29	0.06
0.96875	2.27 ± 0.015	2.27	0.56
0.984375	2.27 ± 0.015	2.27	0.50
0.9921875	2.26 ± 0.015	2.26	0.35
0.99609375	2.27 ± 0.015	2.26	0.10
0.998046875	2.33 ± 0.017	2.34	0.44
0.9990234375	2.49 ± 0.021	2.49	0.63
0.9995117188	2.63 ± 0.026	2.63	0.70
0.9997558594	2.95 ± 0.032	2.94	0.65
0.9998779297	3.42 ± 0.043	3.45	0.29
0.9999237061	3.84 ± 0.052	3.80	0.13

Table 14: Average number of calls to $U(0,1)$ by $\rho\mu$'s BTPE implementation for $n = 2^{18}$

	$\rho\mu$		T-test
p	mean	predicted	p-value
0.000038147	3.82 ± 0.051	3.80	0.46
0.0000610352	3.42 ± 0.043	3.45	0.24
0.0001220703	2.93 ± 0.032	2.94	0.50
0.0002441406	2.64 ± 0.025	2.63	0.43
0.0004882812	2.49 ± 0.022	2.49	0.73
0.0009765625	2.33 ± 0.017	2.33	0.35
0.001953125	2.31 ± 0.017	2.30	0.30
0.00390625	2.26 ± 0.015	2.26	0.58
0.0078125	2.26 ± 0.015	2.26	0.54
0.015625	2.25 ± 0.015	2.26	0.15
0.03125	2.29 ± 0.016	2.28	0.29
0.0625	2.30 ± 0.016	2.29	0.22
0.125	2.30 ± 0.016	2.30	0.73
0.25	2.31 ± 0.016	2.31	0.81
0.5	2.31 ± 0.017	2.31	0.73
0.75	2.31 ± 0.016	2.31	0.77
0.875	2.30 ± 0.016	2.30	0.51
0.9375	2.29 ± 0.016	2.29	0.90
0.96875	2.30 ± 0.017	2.28	0.03
0.984375	2.26 ± 0.015	2.26	0.64
0.9921875	2.26 ± 0.015	2.26	0.45
0.99609375	2.26 ± 0.015	2.26	0.98
0.998046875	2.30 ± 0.017	2.30	0.52
0.9990234375	2.33 ± 0.017	2.33	0.71
0.9995117188	2.49 ± 0.022	2.49	0.68
0.9997558594	2.61 ± 0.025	2.63	0.15
0.9998779297	2.96 ± 0.033	2.94	0.25
0.9999389648	3.44 ± 0.044	3.45	0.91
0.999961853	3.82 ± 0.052	3.80	0.52

Table 15: Average number of calls to $U(0,1)$ by $\rho\mu$'s BTPE implementation for $n = 2^{19}$

	$\rho\mu$		T-test
p	mean	predicted	p-value
0.0000190735	3.79 ± 0.050	3.80	0.80
0.0000305176	3.46 ± 0.045	3.45	0.48
0.0000610352	2.93 ± 0.032	2.94	0.36
0.0001220703	2.63 ± 0.025	2.63	0.97
0.0002441406	2.51 ± 0.022	2.49	0.18
0.0004882812	2.32 ± 0.017	2.33	0.22
0.0009765625	2.30 ± 0.016	2.30	0.58
0.001953125	2.26 ± 0.015	2.26	0.70
0.00390625	2.26 ± 0.015	2.26	0.76
0.0078125	2.28 ± 0.015	2.27	0.41
0.015625	2.27 ± 0.015	2.28	0.29
0.03125	2.29 ± 0.016	2.29	0.83
0.0625	2.29 ± 0.016	2.30	0.69
0.125	2.30 ± 0.017	2.30	0.51
0.25	2.31 ± 0.017	2.31	0.43
0.5	2.31 ± 0.017	2.31	0.91
0.75	2.30 ± 0.017	2.31	0.71
0.875	2.32 ± 0.017	2.30	0.12
0.9375	2.29 ± 0.016	2.30	0.69
0.96875	2.29 ± 0.016	2.29	0.81
0.984375	2.28 ± 0.015	2.28	0.79
0.9921875	2.27 ± 0.015	2.27	0.77
0.99609375	2.26 ± 0.015	2.26	0.82
0.998046875	2.25 ± 0.015	2.26	0.79
0.9990234375	2.31 ± 0.017	2.30	0.18
0.9995117188	2.34 ± 0.018	2.33	0.50
0.9997558594	2.50 ± 0.022	2.49	0.65
0.9998779297	2.64 ± 0.025	2.63	0.19
0.9999389648	2.95 ± 0.033	2.94	0.89
0.9999694824	3.44 ± 0.044	3.45	0.90
0.9999809265	3.83 ± 0.052	3.80	0.27

Table 16: Average number of calls to $U(0,1)$ by $\rho\mu$'s BTPE implementation for $n = 2^{20}$

	$\rho\mu$		T-test
p	mean	predicted	p-value
0.0000095367	3.81 ± 0.051	3.80	0.74
0.0000152588	3.42 ± 0.043	3.45	0.21
0.0000305176	2.99 ± 0.034	2.94	0.01
0.0000610352	2.60 ± 0.025	2.63	0.06
0.0001220703	2.48 ± 0.021	2.49	0.54
0.0002441406	2.35 ± 0.018	2.33	0.17
0.0004882812	2.29 ± 0.016	2.30	0.41
0.0009765625	2.26 ± 0.015	2.26	0.95
0.001953125	2.25 ± 0.015	2.26	0.48
0.00390625	2.27 ± 0.015	2.27	0.90
0.0078125	2.27 ± 0.015	2.28	0.27
0.015625	2.29 ± 0.016	2.29	0.61
0.03125	2.30 ± 0.016	2.30	0.65
0.0625	2.29 ± 0.016	2.30	0.17
0.125	2.29 ± 0.016	2.31	0.06
0.25	2.30 ± 0.016	2.31	0.24
0.5	2.31 ± 0.017	2.31	0.58
0.75	2.31 ± 0.016	2.31	0.60
0.875	2.31 ± 0.016	2.31	0.87
0.9375	2.30 ± 0.016	2.30	0.75
0.96875	2.29 ± 0.016	2.30	0.57
0.984375	2.28 ± 0.016	2.29	0.10
0.9921875	2.27 ± 0.015	2.28	0.29
0.99609375	2.27 ± 0.015	2.27	0.94
0.998046875	2.26 ± 0.015	2.26	0.58
0.9990234375	2.25 ± 0.015	2.26	0.47
0.9995117188	2.29 ± 0.016	2.30	0.63
0.9997558594	2.33 ± 0.017	2.33	0.62
0.9998779297	2.48 ± 0.021	2.49	0.28
0.9999389648	2.63 ± 0.025	2.63	0.73
0.9999694824	2.95 ± 0.033	2.94	0.78
0.9999847412	3.44 ± 0.043	3.45	0.75
0.9999904633	3.84 ± 0.053	3.80	0.17

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