204A Problem Set 2

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Problem 1

For the purposes of this exercise, we are assuming that a function is a pdf if the integral over its domain is one and the function is non negative. The gamma distribution is defined as:

$$
\begin{aligned}
f_x =& \frac{1}{\Gamma(a)\beta^a} x^{\alpha-1} e^{-x/\beta} \\
\text{where } \Gamma(\alpha) =& \int_0^\infty y^{\alpha-1} e^{-y} dy, \quad \alpha, \beta, x > 0
\end{aligned}
$$

Clearly, Γ is both real and positive for positive alpha. Therefore, f_x is positive over the domain. We will now argue that its integral is one.

$$
\begin{aligned}
I(k) =& \int_0^k \frac{1}{\Gamma(a)\beta^a} x^{\alpha-1} e^{-x/\beta} dx \\
I(k) =& \frac{1}{\Gamma(a)\beta^a} \int_0^k x^{\alpha-1} e^{-x/\beta} dx \\
& \text{if } \alpha = 1 : \\
\alpha = 1 : \;\; I(k) =& \frac{1}{(\Gamma(a) = 1)\,\beta} \int_0^k e^{-x/\beta} dx = -e^{-x/\beta}|_0^k \\
I(k) =& e^{-k/\beta} + 1 \\
\lim_{k\to\infty} I(k) =& 1 \checkmark \\
\alpha > 0, \alpha \neq 1 : \;\; I(k) =& \frac{1}{\Gamma(a)\beta^a} \int_0^k x^{\alpha-1} e^{-x/\beta} dx \\
u =& \frac{x}{\beta} \implies du = \frac{dx}{\beta} \\
I(k) =& \frac{\beta^\alpha}{\Gamma(a)\beta^a} \int_0^k u^{\alpha-1} e^{-u} dx \\
\lim_{k\to\infty} I(k) =& \frac{1}{\Gamma(a)} \int_0^k u^{\alpha-1} e^{-u} dx \\
I(k) =& \frac{1}{\Gamma(a)} \int_0^\infty u^{\alpha-1} e^{-u} dx \\
\lim_{k\to\infty} I(k) =& \frac{\Gamma(a)}{\Gamma(a)} = 1 \checkmark
\end{aligned}
$$