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ANALYSIS AND SIMULATION OF A QPSK SYSTEM
EQ2310 Digital Communications

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the text is easy to follow and the language correct	yes ☐	mostly ☐	sometimes ☐	no ☐
results and conclusions are clearly stated	yes ☐	most of them ☐	only a few ☐	no ☐
results are correct	yes ☐	most of them ☐	only a few ☐	no ☐
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the results can be reproduced	yes ☐	most of them ☐	no ☐	
the mathematics/theory is correct	yes ☐	mostly ☐	no ☐	hard to tell ☐
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Analysis and Simulation of a QPSK System

1. Abstract

This project delves into the simulation and analysis of a Quadrature Phase Shift Keying (QPSK) system. The simulation framework involves completing essential functions for modulation, synchronization, phase estimation, and demodulation. The study encompasses tasks such as verifying simulation results against theoretical error performances, investigating constellation diagrams at different signal-to-noise ratios (SNR, or E_b/N_0), assessing the impact of pulse shaping, analyzing QPSK performance in multipath channels, determining efficient training bit lengths, and evaluating system behavior in a time-varying phase shift channel. The results demonstrate the accuracy of theoretical predictions, showcase the impact of varying SNR on constellation diagrams, and reveal the effectiveness of pulse shaping techniques. Additionally, the study sheds light on the system's performance in challenging multipath channels and explores the optimal configuration of training bits for efficient communication. The investigation into time-varying phase shift channels underscores the importance of adaptive modulation schemes.

2. Background and Problem Formulation

In the realm of digital communications, the Quadrature Phase Shift Keying (QPSK) system presents itself as a fundamental and widely implemented modulation scheme. As a simple but efficient method, QPSK makes it an ideal candidate for exploration and study. A baseband equivalent QPSK system consists of three core components: Transmitter, Channel, and Receiver. Transmitter firstly concatenate the data bits with guard bits and training bits, and then modulate them to QPSK symbols based on Grey Mapping, with each QPSK symbol differs one bit and 90 degrees with its neighbor. After pulse shaping and up sampling, the transmitted signal is input to the channel. Channel will introduce distortions and delays to the signal. In most of our simulations, the channel is AWGN channel. In real-world scenarios, it is likely to be multipath (fading) channel. The distorted signal captured by Receiver is fed to the matched filter and synchronization algorithm is applied to find the optimal sampling time of the symbols. The phase shift of the symbols, resulted from channel effect, is rectified via phase estimation algorithm. Finally, the transmitted data bits can be recovered after QPSK demodulation.

This project focuses on simulating such a QPSK system. The authors are provided with a simulation framework with missing functions, *qpsk.m*, *sync.m*, *phase_estimation.m*, and *detect.m*. Our primary task is to complete the functions.

a) *qpsk.m* returns the modulated QPSK symbols of input bits.
b) *sync.m* determines the sample time by maximizing the cross-correlation between matched filter output and training sequence.

c) *phase_estimation.m* estimates the phase shift via minimizing the norm of the difference between received and original version of training sequence.
d) *detect.m* demodulate the received symbols based on real and imaginary parts.

With the complete simulation workflow in hand, one can easily explore the properties of QPSK system. The authors' further work focuses on studying the following tasks and questions:

1. Verify the result of the QPSK simulation are consistent with the theory in error performances.
2. Study the properties of constellation diagram of QPSK, given different SNR (E_b/N_0).
3. How does different pulse shapes affect the transmission.
4. The performance of QPSK system in multipath channel, compared to AWGN channel.
5. Finding the efficient length of training bits.
6. How does time-varying phase shift channel alter the performance of the system.

3. Methodology

For the missing functions, it is sufficient to complete them with descriptions in project manual and comments within each function file. In the Appendix, the content and the functions are exhibited. Though it is not related to the results of the simulations, but the author has tried to implement the functions in the vectorized manner (preferred in MATLAB), leading to simplicity and high execution efficiency. For the raised tasks and questions in previous section, they are studied in the below manner:

1. The figure of both theoretical BER and experimental BER in QPSK simulations are plotted, by which verification is conducted.
2. The constellation diagrams of received symbols are plotted for different values of E_b/N_0, from which the differences can be observed.
3. Estimate the spectrums of transmitted signal after it is shaped by 1. Rectangular pulse, and 2. Root Raised Cosine pulse. Compare the spectrums yielding how pulse shaping impacts the transmission.
4. The eye diagrams of received symbols are plotted to observe how the performance of QPSK in 2-path channel is differed from AWGN channel. How multipath effect influences phase estimation and synchronization is studied.
5. Trying different lengths of training bits, evaluate how phase estimation, synchronization and BER are related to this variable.
6. We introduce a linearly increasing phase shift in the channel, ranging from 0 to $\pi/4$, and study the changes in BER performance.

4. Results

Unless specified, the results are obtained in a QPSK system with 1000 data bits, 100 training bits, and 8 samples per symbol with rectangular pulse shaping, in the transmission.

4.1 BER Performance

From course slide, the exact BER of Grey Mapping QPSK system is $\sqrt{2E_b/N_0}$ [1]. The experimental BER is simulated with E_b/N_0 ranging from 0-10dB. Figure 1 compares the theoretical and experimental results, and they match perfectly. Results indicate that as SNR increases, it is likely to have less bit errors. This verifies the correctness of the four missing functions, and the consistency between theory and experiment.

4.2 Constellation Diagram

The constellation diagrams of E_b/N_0 at 2, 6, 10dB are shown in Figure 2. Ideally, in the case where no distortion occurs, QPSK symbols should only appear in four points, with 90 degrees phase shift of each, on the constellation diagram. The existence of Gaussian noise reshapes the distribution of constellations in both real axis and imaginary axis, which results in the diagrams in Figure 2. Generally, as SNR increases, the four "clusters" becomes more distinct on the diagram. On the other hand, as SNR decreases, the constellations will become more spread-out visually, in which some of them are close to or exceed the decision boundaries, accounting for higher BER. In the authors' simulations, the phase estimation accuracy is satisfactory. In scenario with a false phase estimation, the constellation will be a rotated version of QPSK (see Appendix Figure 1), which will lead to significant bit errors.

4.3 Spectrum of Transmitting Signal using Different Shaping Pulse

Pulse shaping is to modify the shape of transmitted signals to control their bandwidth and reduce intersymbol interference (ISI). Most simulations in this project is conducted with rectangular pulse for pulse shaping. However, in real-world systems, most of them utilize (Root) Raised Cosine pulse (RRC). Here, the authors compared the performances of two types of pulses by inpecting the power spectral density (estimated by Welch method with 50% overlap) of transmitted symbols after pulse shaping, as shown in Figure 3. Compared to rectangular pulse, the PSD plot of RRC shows a smoother main lobe that decays fast at the cut-off frequency. Moreover, the side lobes of RRC are in extremely low level, which reduces spectral leakage and interference to adjacent channels.

4.4 Performance in Two-Path Channel

In real world, scenarios, it is unlikely that the channel only contains Gaussian white noise. Hence, the authors also study the performance of QPSK system in a 2-path channel (1 path of LOS, and 1 path delay 6, with equal gain). By interpreting the eye diagram (Figure 4) for both AWGN and the 2-path channel. Clearly, the "eye" can be seen in diagram of AWGN. But for that of 2-path channel, the plot is severely distorted by ISI. The BER plot (Appendix Figure 2(a)) in both scenarios agrees with the result. QPSK system performs much better in AWGN channel than in 2-path channel.

Appendix Figure 2(b) shows the phase estimation for both cases under different SNR. For AWGN case, the expected phase shift caused by channel should equal to 0. We can see that the blue line vibrates around 0, which symbolics a good accuracy for phase estimation. For 2-path channel, the delay certainly causes the phase shift of the received signal, which can be seen that the orange line vibrates more intensively than the blue line. The phase shift due to delayed path adds difficulties in phase estimation.

Appendix Figure 2(c) reflects the synchronization in the Receiver for both channel conditions. "k" in the vertical axis of the plot corresponds to the delay that maximizes the cross-correlation between matched filter output and training bits. "k" is also the estimated ideal sample point. There are fractional values of k because we use the averaged k obtained from loops. For AWGN case, k converges to 8 as SNR goes up. For 2-path case, the distribution of k shows a large variance. This indicates synchronization in 2-path channel is less accurate than that in AWGN channel. This can be further verified by the Appendix Figure 3, which are the cross-correlation plots of both scenarios. A "steep" peak in AWGN case shows better correlation properties, than the "flat" peek in 2-path case, which may bring problems in determining $argmax_k(correlation)$.

4.5 Exploration of Efficient Length of Training Bits

It is simple to understand that the longer the training bits, the lower the BER. By adjusting the number of training bits, the BER plot in Figure 5 is obtained. The plot agrees the theory. However, the continuous growth in number of training bits will add extra overhead that reduces the communication efficieny. Therefore, we make a reasonable choice from the plot that 20 bits of training sequence is an effective option, as the BER does not increase much compared to 100 bits case.

One can have an inpection on how the length of the training bits affects the synchronization and phase estimation in Appendix Figure 4. In this simulation, E_b/N_0 is fixed 6 dB. Appendix Figure 4(a) shows the estimated k that maximize correlation converges to $k = 8$ when the number of training bits is 20, which is good. In Appendix Figure 4(b), a wider range of training bits is applied. Results shows that, indeed, the phase offset estimation is more accurate when the number of trainig bits is larger. However, the estimated phase when the number of training bits is 20 does not deviates much to the true value, which is 0. The tiny error in phase estimation will not drastically decrease the BER performance. Hence, the number of training bits = 20 is an efficient choice for the given QPSK system. The training overhead is $20/1020 = 1.96\%$.

4.6 Performance in Time-Varying Phase Shift Channel

The channel with time-varying phase response is very common in real life. If the author is watching online stream on his phone on the moving bus, it is likely that that communication channel experienced is a time-varying phase shift channel, caused by the Doppler effect. In this simulation, we build a simple such channel, whose phase response switches from 0 to $\pi/4$ during one transmission.

Figure 6 demonstrates the comparison of BER performance between pure AWGN channel and the channel with both Gaussian noise and the phase-varying property. As depicted in the plot, the varying phase response will significantly affect the accuracy of correct detection. Theoretically, at the beginning of the transmission, the phase of the channel is close to zero, which will not cause much deviation in phase to the received symbols. Over time, the phase of the channel becomes larger, the received symbols' constellation will rotate nearly 90 degrees, resulting in the symbols close to the original decision boundaries, for which plenty of errors occur. Hence the error bits will mostly appear at the tail of the data bits. The experimental results shown in Appendix Figure 5 agrees the theory.

If the QPSK system is replaced with Differential-QPSK system, the existence of Time-Varying Phase Shift Channel will no longer be a problem. Since the D-QPSK system modulates bits according to its phase difference with its predecessor, and the same for the demodulation. D-QPSK will largely alleviate the large amount of bit errors due to the slow-varied phase of channel.

5. Conclusions

The study on QPSK system in digital communications provides key insights into its performance under various conditions. It confirms the consistency between theoretical and experimental BER, with improved accuracy at higher SNR. The constellation diagrams illustrate the impact of noise, where higher SNR leads to more distinct symbol clusters. Pulse shaping, particularly using Root Raised Cosine pulses, shows better performance in controlling bandwidth and reducing intersymbol interference compared to rectangular pulses. The system's performance in multipath channels is notably poorer than in AWGN channels, highlighting the challenges of real-world conditions. The study suggests that an efficient length for training bits is around 20, balancing error reduction and communication efficiency. In time-varying phase shift channels, common in mobile scenarios, the performance drops significantly. Overall, this project provides a detailed understanding of QPSK systems, analyzing their performance in various scenarios and suggesting improvements.

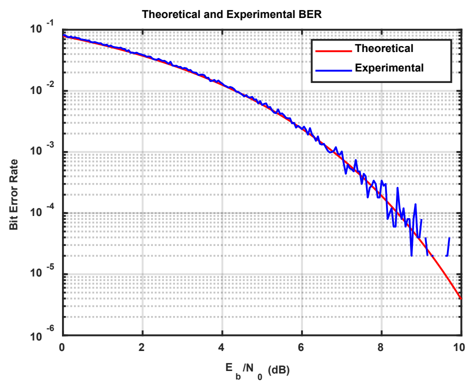


Figure 1. Theoretical BER vs. Experimental BER, of QPSK

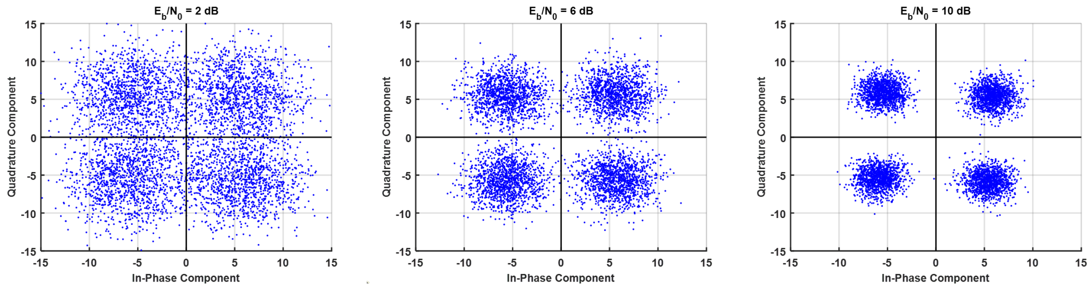


Figure 2. Constellation diagrams for different E_b/N_0

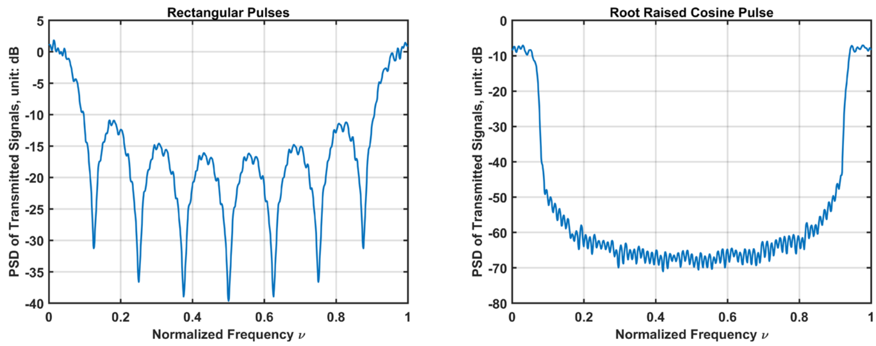


Figure 3. The spectrums of transmitted signal after pulse shaping

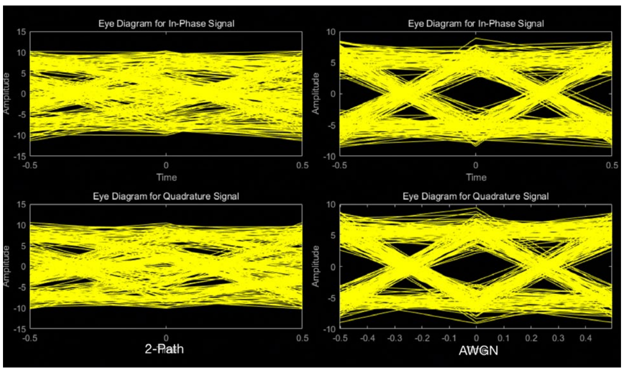


Figure 1. Eye diagrams for AWGN channel (right) and for 2-path channel (left)

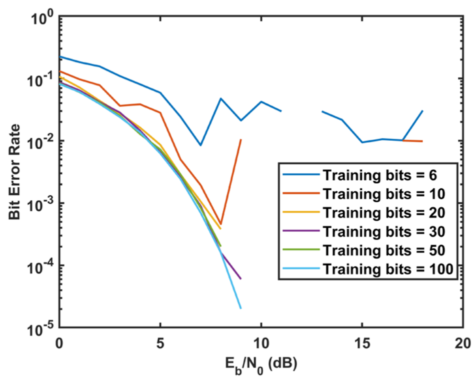


Figure 5. BER performances for different number of training bits

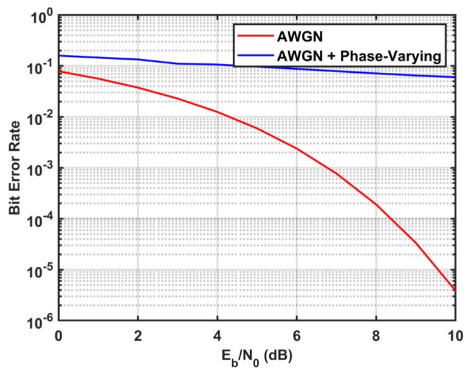
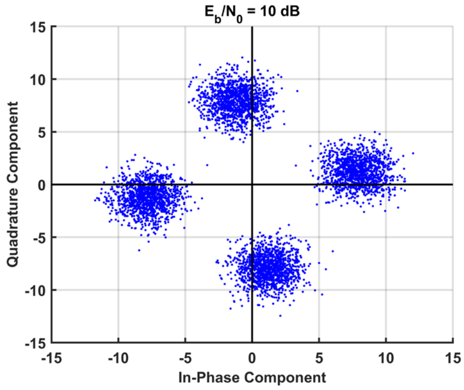
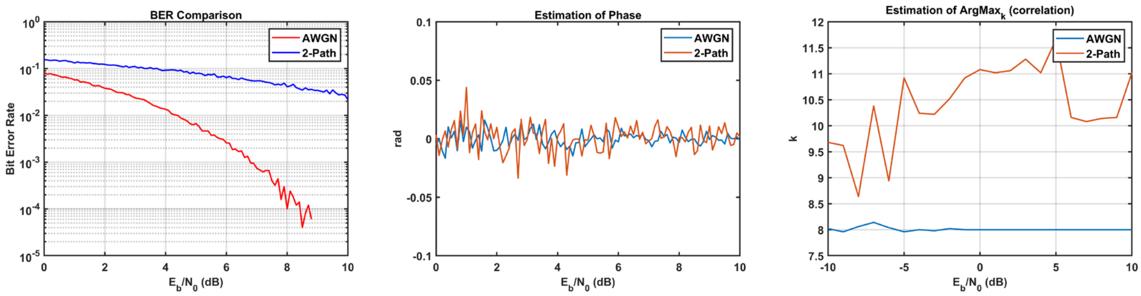


Figure 6. BER for time-varying phase shift channel

6. Appendix

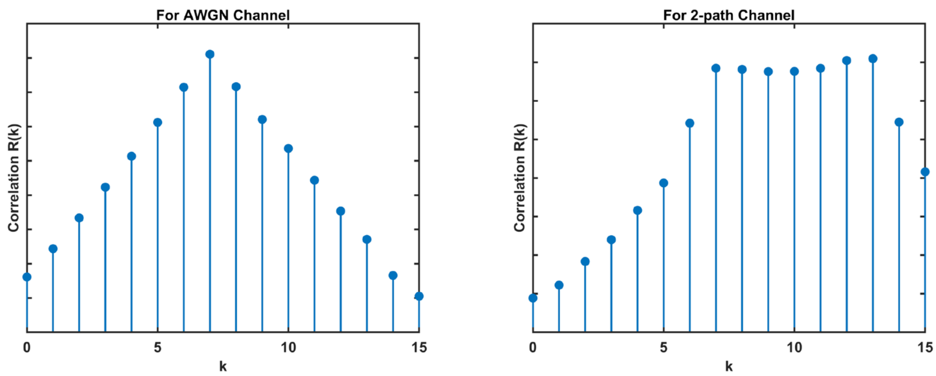


Appendix Figure 1. Rotated Constellation



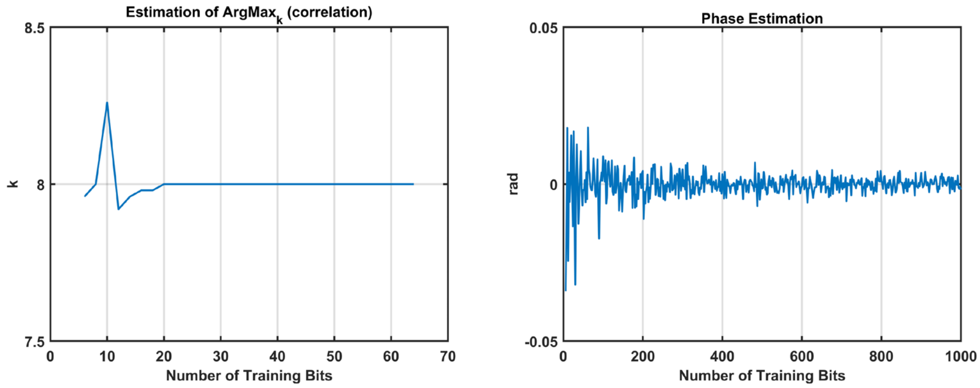
(a) (b) (c)

Appendix Figure 2. System performance in 2-path channel and in AWGN channel



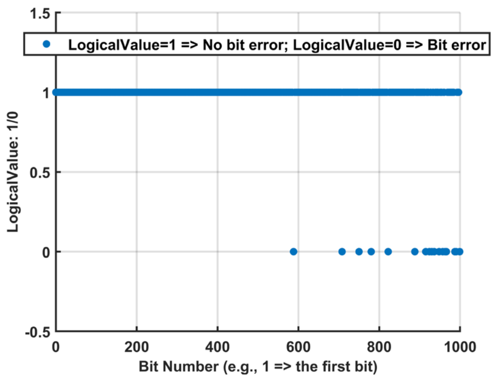
(a) (b)

Appendix Figure 3. Correlation function measured in Synchronization



(a) (b)

Appendix Figure 4. The influence of the number of training bits on synchronization and phase estimation



Appendix Figure 5. Position of bit error

a) *qpsk.m*

```
function d = qpsk(b)
    % Compute the number of symbols
    N = length(b)/2;
    % Reshape b into two rows, for convenience
    b = reshape(b, 2, N);
    % Map bits to QPSK symbols by Gray Mapping, boosted using the
    % sign of real and imaginary parts
    d = ((1 - 2*b(1,:)) + 1j*(1 - 2*b(2,:)))/sqrt(2);
end
```

b) *sync.m*

```
function t_samp = sync(mf, b_train, Q, t_start, t_end)
    % Modulate the training sequence
    c = qpsk(b_train);
    % Initialize correlation set
```

```
    corr = zeros(1, t_end-t_start+1);
    % Iterate over the range to find the best t_samp
    for t = t_start:t_end
        % According to (28), we have the critical two lines of code
below
        kQ_t = [0:length(c)-1] * Q + t;
        mf_downsamp = mf(kQ_t);
        % t-t_start+1 since matlab index starts at 1, c' is the
conjugate
        % transprose
        corr(t-t_start+1) = mf_downsamp * c';
    end
    [max_val, arg_max] = max(abs(corr));
    t_samp = t_start + (arg_max - 1); % Also because matlab index
starts at 1
end
```

c) *phase_estimation.m*

```
function phihat = phase_estimation(r, b_train)
    % Modulate the training sequence
    d_train = qpsk(b_train);
    % Compute # of symbols in the training sequence
    N = length(d_train);
    % Extract the corresponding part within the received signal
    r_train = r(1:N);
    % Compute the phase difference between received and expected
symbols,
    % based on formula (30)
    phase_diff = angle(d_train .* conj(r_train));
    % Take the Average value
    phihat = mean(phase_diff);
end
```

d) *detect.m*

```
function bhat = detect(r)
    % Compute # of symbols
    N = length(r);
    % Initialize the bit set
    bhat = zeros(1, 2*N);
    % Direct mapping using sign of real and imaginary parts
    bhat(1:2:end) = real(r) < 0;
    bhat(2:2:end) = imag(r) < 0;
end
```

For rest of the code, please visit the link:

https://drive.google.com/drive/folders/12SzVrntsDcURqmRUSrIfIoJWA0zmX7jf?usp=drive_link

There are .m files titled “pX”, X corresponds to the problem number.

7. References

[1] J. Proakis and M. Salehi, *Digital Communications*. New York, NY: McGraw-Hill Education, 2007.

[2] J. G. Proakis, *Digital Signal Processing: Principles, Algorithms, and Applications*. Upper Saddle River, NJ: Pearson Prentice Hall; London: Pearson Education, Ltd., 2007.